Research Article

Multiple Attribute Decision-Making Problem Using Picture Fuzzy Graph

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In a picture fuzzy environment, almost all multiple attribute decision-making (MADM) methods have been discussed a type of problem in which there is no relationship among the attributes. Although the relationship among the attributes should be considered in the actual applications, so we need to pay attention to that important issue. This article applied graph theory to the picture fuzzy set (PFS) and obtained a new method, MADM, to solve complicated problems under a picture fuzzy environment. The developed method can capture the relationship among the attributes that cannot be handled well by any existing methods. This study introduces union, intersection, sum, Cartesian product, the composition of picture fuzzy graphs (PFGs), and their important properties. Finally, by considering the importance of relationships among attributes in the determination process, two algorithms, based on PFG, have developed to solve complicated problems using picture fuzzy information. Also, two numerical examples have introduced to explain how to deal with the MADM problem under picture fuzzy environment.

1. Introduction

At present, graphs do not disclose all the systems properly because of the uncertainty of the parameters within a system. For instance, a social network can be uttered as the graph, where nodes denote an account (such as institution or person) and edges express the connection between the accounts. If the connections among accounts are measurable as bad or good according to the recurrence rate of contacts among the accounts, fuzziness should be added to representation. In 1975, Rosenfeld first defined the fuzzy graph considering fuzzy relations on fuzzy sets [1]. A PFS is a generalization of intuitionistic fuzzy set (IFS) [2]. The picture fuzzy model gives more precision, flexibility, and compatibility than the intuitionistic fuzzy model.

The concept of PFS was first introduced by Coung [3] in 2013. In addition to IFS, Coung appended new components which determine the neutral membership degree. IFS gives an element's membership and nonmembership degree, while PFS gives positive membership degree, neutral membership degree, and negative membership degree of an element. These memberships are almost independent and the sum of these three membership degrees is ≤ 1 . Basically, PFS-based models may be adequate in situations, where we counter several opinions that involve more answers of types: yes, no, abstain, and refusal. If we take voting as an example, human voters may be separated into four possible groups with distinct opinions: vote for, vote against, abstain, and refusal of the voting. Picture fuzzy sets have several interesting applications in system analysis, operation research, economics, medicine, computer science, engineering,

mathematics, etc. Some properties of PFS and its operators have been studied in [4, 5].

1.1. Review of Literature. After invention of fuzzy graph, it develops with its different branches, such as fuzzy threshold graph [6], balanced interval-valued fuzzy graphs [7], cubic graph [8], m -step fuzzy competition graphs [9], fuzzy planar graphs [10,11], and fuzzy k -competition graph [12]. Pramanik et al. defined interval-valued fuzzy threshold graph and studied several properties [13]. They also have considered planarity in bipolar fuzzy graph, and they extended it to bipolar fuzzy planar graphs [14]. Also, Pramanik et al. have extended fuzzy planar graph to interval-valued fuzzy planar graph [15] and interval-valued fuzzy graph [16]. Voskoglou et al. [17] have discussed and characterized several fuzzy graph theoretic structure and fuzzy hypergraphs. Sahoo et al. [18] have studied the intuitionistic fuzzy competition graph. Balanced intuitionistic fuzzy graphs are discussed by Karunambigai et al. [19]. Also, Sahoo et al. have studied some problems regarding IFG [18, 20, 21]. Recently, some researchers have carried out study regarding picture fuzzy graphs and its applications [22], regular picture fuzzy graph and its application [23], and edge domination in picture fuzzy graphs [24]. Many related problems such as a study on picture Dombi fuzzy graph [25], q -rung picture fuzzy graphs [26], interval-valued picture uncertain linguistic generalized Hamacher aggregation operators and their application in multiple attribute decision-making process [27], multiple attribute decision-making algorithm via picture fuzzy nanotopological spaces [28], decision-making model under complex picture fuzzy Hamacher aggregation operators [29], and fuzzy aggregation operators and their applications to multicriteria decision-making [30] are investigated. In 2018, Ullah et al. [31] have studied similarity measures for T-spherical fuzzy sets with applications in pattern recognition. They also have studied policy decision-making based on some averaging aggregation operators of T-spherical fuzzy sets [32]. The concept of spherical fuzzy set and T-spherical fuzzy set is introduced as a generalization of fuzzy set, intuitionistic fuzzy set, and picture fuzzy set by Mahmood et al. [33].

In 2020, Devaraj et al. [34] have studied picture fuzzy labelling graphs, and they also have presented an application of picture fuzzy labelling graphs; also, Mahmood et al. [35] have studied a lot of results regarding the fuzzy cross entropy for picture hesitant fuzzy sets and their application in multicriteria decision-making. Also, T. Mahmood [36] has studied a novel approach towards bipolar soft sets and their applications. Applications of the generalized picture fuzzy soft set in concept selection have been studied by Khan et al. [37]. Exponential operational laws and new aggregation operators for the intuitionistic multiplicative set in the multiple attribute group decision-making process have been studied by Garg [27]. In 2021, Amanathulla et al. have studied a lot of results regarding balanced picture fuzzy graphs [38]. Recently, many researchers have applied various related concepts of the current study on graphs in different aspects (see, for e.g., [39–45]).

1.2. Motivation. Most of MADM methods with picture fuzzy environment are to discuss a type of problem that there is no relationship among attributes. Although this relationship should be considered in the actual applications, so we need to pay attention to that issue. From this point of view, we consider MADM problem using picture fuzzy graph. This article applies graph theory to PFS and obtained a new method, MADM, to solve complicated problems under a picture fuzzy environment. The proposed method can capture the relationship among the attribute that cannot be handled well by any existing technique. Also, we have been given two examples to show that our decision-making algorithm is original. The remaining parts of this article are organized as follows. Some preliminaries are presented in Section 2. In Section 3, PFG and some of its properties are presented. In Section 4, two algorithms based on multiple attribute decision-making for complicated problems are presented. In Section 5, two numerical examples for PFGMADM problem with picture fuzzy information are used to present the applications of the proposed decision-making algorithm. Section 6 is for the brief conclusion.

2. Preliminaries

PFS is an extension of IFS. Some definitions related to PFS are presented below, which we have used later to develop the paper.

Definition 1 (see [4]). Let $S = (p_S, q_S, r_S)$ and $T = (p_T, q_T, r_T)$ be two PFSs. Then, the union and the intersection of the PFSs S and T are defined by

- (i) $(S \cup T)(t) = \{(t, p_S(t) \vee p_T(t), q_S(t) \vee q_T(t), r_S(t) \vee r_T(t)): t \in X\}$
- (ii) $(S \cap T)(t) = \{(t, p_S(t) \wedge p_T(t), q_S(t) \wedge q_T(t), r_S(t) \wedge r_T(t)): t \in X\}$

A picture fuzzy number is defined by $f_n = (\alpha, \beta, \gamma)$.

Definition 2 (see [4]). Let $f_n = (\alpha, \beta, \gamma)$ be a picture fuzzy number; then, the score function of f_n is denoted by $\text{scor}(f_n)$ and is defined by $\text{scor}(f_n) = ((1 + \alpha - 2\beta - \gamma)/2)$.

Observation 1. Let f_1 and f_2 be two picture fuzzy numbers; then, $\text{scor}(f_1) > \text{scor}(f_2) \Rightarrow f_1 \Rightarrow f_2$.

Definition 3 (see [4]). A PF relation ρ in a universe $A \times B$ is a PFS defined by $\rho(a, b) = \{((a, b), p_\rho(a, b), q_\rho(a, b), r_\rho(a, b)): (a, b) \in A \times B\}$, where $p_\rho: A \times B \rightarrow [0, 1]$, $q_\rho: A \times B \rightarrow [0, 1]$, $r_\rho: A \times B \rightarrow [0, 1]$, and $0 \leq p_\rho(a, b) + q_\rho(a, b) + r_\rho(a, b) \leq 1$, for all $(a, b) \in A \times B$.

Definition 4 (see [4]). Let $S = (p_S, q_S, r_S)$ and $T = (p_T, q_T, r_T)$ be two PFSs on a set X . If S be a PFR on X , then S is also a PFR on T if $p_S(a, b) \leq p_T(a) \wedge p_T(b)$, $q_S(a, b) \geq q_T(a) \vee q_T(b)$, and $r_S(a, b) \geq r_T(a) \vee r_T(b)$, for all $a, b \in X$.

3. Picture Fuzzy Graph

In this section, the PFG and some properties and theorems of PFG have been described.

Definition 5. A PFG of a graph $G^* = (V, E)$ is a pair $G = (S, T)$, where $S = (p_S, q_S, r_S)$ is a PFS on V and $T = (p_T, q_T, r_T)$ is a PFR on E such that $p_T(a, b) \leq p_S(a) \wedge p_{(b)}$, $q_T(a, b) \geq q_S(a) \vee q_{(b)}$, $r_T(a, b) \geq r_S(a) \vee r_{(b)}$, and $0 \leq p_T(a, b) + q_T(a, b) + r_T(a, b) \leq 1$.

Here, S is the picture fuzzy node set of G and T is a picture fuzzy edge set on G . Also, $p_S(a)$, $q_S(a)$, and $r_S(a)$, respectively, denote the positive, neutral, and negative membership degree of the node a and $p_T(a, b)$, $q_T(a, b)$, and $r_T(a, b)$ denote that of edge (a, b) .

Now, we give some properties of PFG such as composition, Cartesian product, union, and intersection.

Let $G_1 = (S_1, T_1)$ and $G_2 = (S_2, T_2)$ be two PFGs, where $S_1 = (p_{S_1}, q_{S_1}, r_{S_1})$, $S_2 = (p_{S_2}, q_{S_2}, r_{S_2})$, $T_1 = (p_{T_1}, q_{T_1}, r_{T_1})$, and $T_2 = (p_{T_2}, q_{T_2}, r_{T_2})$.

Definition 6. Let G_1 and G_2 be two PFGs; then, the cartesian product $G_1 \times G_2$ of G_1 and G_2 is defined by $G_1 \times G_2 = (S_1 \times S_2, T_1 \times T_2)$, where

- (i) $(p_{S_1} \times p_{S_2})(a_1, a_2) = p_{S_1}(a_1) \wedge p_{S_2}(a_2)$
 $(q_{S_1} \times q_{S_2})(a_1, a_2) = q_{S_1}(a_1) \vee q_{S_2}(a_2)$
 $(r_{S_1} \times r_{S_2})(a_1, a_2) = r_{S_1}(a_1) \vee r_{S_2}(a_2), (a_1, a_2) \in V_1 \times V_2 = V.$
- (ii) $(p_{T_1} \times p_{T_2})((a, a_2), (a, b_2)) = p_{S_1}(a) \wedge p_{T_2}(a_2, b_2)$
 $(q_{T_1} \times q_{T_2})((a, a_2), (a, b_2)) = q_{S_1}(a) \vee q_{T_2}(a_2, b_2)$
 $(r_{T_1} \times r_{T_2})((a, a_2), (a, b_2)) = r_{S_1}(a) \vee r_{T_2}(a_2, b_2), a \in V_1, (a_2, b_2) \in E_2.$
- (iii) $(p_{T_1} \times p_{T_2})((a_1, c), (b_1, c)) = p_{T_1}(a_1, b_1) \wedge p_{S_2}(c)$
 $(q_{T_1} \times q_{T_2})((a_1, c), (b_1, c)) = q_{T_1}(a_1, b_1) \vee q_{S_2}(c)$
 $(r_{T_1} \times r_{T_2})((a_1, c), (b_1, c)) = r_{T_1}(a_1, b_1) \vee p_{S_2}(c), c \in V_2, (a_1, b_1) \in E_1.$

Theorem 1. Let G_1 and G_2 be two PFGs; then, $G_1 \times G_2$ is a PFG.

Proof. Let $a \in V_1, (a_2, b_2) \in E_2$. Then, we obtain

$$\begin{aligned}
 (p_{T_1} \times p_{T_2})((a, a_2), (a, b_2)) &= p_{S_1}(a) \wedge p_{T_2}(a_2, b_2) \\
 &\leq p_{S_1}(a) \wedge (p_{S_2}(a_2) \wedge p_{S_2}(b_2)) \\
 &= (p_{S_1}(a) \wedge p_{S_2}(a_2)) \wedge (p_{S_1}(a) \wedge p_{S_2}(b_2)) \\
 &= (P_{S_1} \times P_{S_2})(a, a_2) \wedge (P_{S_1} \times P_{S_2})(a, b_2), \\
 (q_{T_1} \times q_{T_2})((a, a_2), (a, b_2)) &= q_{S_1}(a) \vee q_{T_2}(a_2, b_2) \\
 &\geq q_{S_1}(a) \vee (q_{S_2}(a_2) \vee q_{S_2}(b_2)) \\
 &= (q_{S_1}(a) \vee q_{S_2}(a_2)) \vee (q_{S_1}(a) \vee q_{S_2}(b_2)) \\
 &= (q_{S_1} \times q_{S_2})(a, a_2) \vee (q_{S_1} \times q_{S_2})(a, b_2), \\
 (r_{T_1} \times r_{T_2})((a, a_2), (a, b_2)) &= r_{S_1}(a) \vee r_{T_2}(a_2, b_2) \\
 &\geq r_{S_1}(a) \vee (r_{S_2}(a_2) \vee r_{S_2}(b_2)) \\
 &= (r_{S_1}(a) \vee r_{S_2}(a_2)) \vee (r_{S_1}(a) \vee r_{S_2}(b_2)) \\
 &= (r_{S_1} \times r_{S_2})(a, a_2) \vee (r_{S_1} \times r_{S_2})(a, b_2).
 \end{aligned} \tag{1}$$

Again, let $c \in V_2$ and $(a_1, b_1) \in E_1$. Then, we obtain

$$\begin{aligned}
 (p_{T_1} \times p_{T_2})((a_1, c), (b_1, c)) &= p_{T_1}(a_1, b_1) \wedge p_{S_2}(c) \\
 &\leq p_{S_2}(c) \wedge (p_{S_1}(a_1) \wedge p_{S_1}(b_1)) \\
 &= (p_{S_1}(a_1) \wedge p_{S_2}(c)) \wedge (p_{S_1}(b_1) \wedge p_{S_2}(c)) \\
 &= (P_{S_1} \times P_{S_2})(a_1, c) \wedge (P_{S_1} \times P_{S_2})(b_1, c),
 \end{aligned}$$

$$\begin{aligned}
(q_{T_1} \times q_{T_2})((a_1, c), (b_1, c)) &= q_{T_1}(a_1, b_1) \vee q_{S_2}(c) \\
&\geq q_{S_2}(c) \vee (q_{S_1}(a_1) \vee q_{S_1}(b_1)) \\
&= (q_{S_1}(a_1) \vee q_{S_2}(c)) \vee (q_{S_1}(b_1) \vee q_{S_2}(c)) \\
&= (q_{S_1} \times q_{S_2})(a_1, c) \vee (q_{S_1} \times q_{S_2})(b_1, c), \\
(r_{T_1} \times r_{T_2})((a_1, c), (b_1, c)) &= r_{T_1}(a_1, b_1) \vee r_{S_2}(c) \\
&\geq r_{S_2}(c) \vee (r_{S_1}(a_1) \vee r_{S_1}(b_1)) \\
&= (r_{S_1}(a_1) \vee r_{S_2}(c)) \vee (r_{S_1}(b_1) \vee r_{S_2}(c)) \\
&= (r_{S_1} \times r_{S_2})(a_1, c) \vee (r_{S_1} \times r_{S_2})(b_1, c).
\end{aligned} \tag{2}$$

The above results proves that $G_1 \times G_2$ is a PFG. $\square$

Definition 7. Let G_1 and G_2 be two PFGs; the composition of G_1 and G_2 is defined by $G_1[G_2] = (S_1 \cdot S_2, T_1 \cdot T_2)$, where

- (i) $(p_{S_1} \cdot p_{S_2})(a_1, a_2) = p_{S_1}(a_1) \wedge p_{S_2}(a_2)$
 $(q_{S_1} \cdot q_{S_2})(a_1, a_2) = q_{S_1}(a_1) \vee q_{S_2}(a_2)$
 $(r_{S_1} \cdot r_{S_2})(a_1, a_2) = r_{S_1}(a_1) \vee r_{S_2}(a_2), \quad (a_1, a_2) \in V_1 \times V_2 = V.$
- (ii) $(p_{T_1} \cdot p_{T_2})((a, a_2), (a, b_2)) = p_{S_1}(a) \wedge p_{T_2}(a_2, b_2)$
 $(q_{T_1} \cdot q_{T_2})((a, a_2), (a, b_2)) = q_{S_1}(a) \vee q_{T_2}(a_2, b_2)$
 $(r_{T_1} \cdot r_{T_2})((a, a_2), (a, b_2)) = r_{S_1}(a) \vee r_{T_2}(a_2, b_2), \quad a \in V_1, (a_2, b_2) \in E_2.$
- (iii) $(p_{T_1} \cdot p_{T_2})((a_1, c), (b_1, c)) = p_{T_1}(a_1, b_1) \wedge p_{S_2}(c)$
 $(q_{T_1} \cdot q_{T_2})((a_1, c), (b_1, c)) = q_{T_1}(a_1, b_1) \vee q_{S_2}(c)$

$$\begin{aligned}
(r_{T_1} \cdot r_{T_2})((a_1, c), (b_1, c)) &= r_{T_1}(a_1, b_1) \vee p_{S_2}(c), \quad c \in V_2, (a_1, b_1) \in E_1. \\
\text{(iv)} \quad (p_{T_1} \cdot p_{T_2})((a_1, a_2), (b_1, b_2)) &= p_{S_2}(a_2) \wedge p_{S_2}(b_2) \wedge p_{T_1}(a_1, b_1) \\
(q_{T_1} \cdot q_{T_2})((a_1, a_2), (b_1, b_2)) &= q_{S_2}(a_2) \vee q_{S_2}(b_2) \vee q_{T_1}(a_1, b_1) \\
(r_{T_1} \cdot r_{T_2})((a_1, a_2), (b_1, b_2)) &= r_{S_2}(a_2) \vee r_{S_2}(b_2) \vee r_{T_1}(a_1, b_1), \quad (a_1, a_2), (b_1, b_2) \in E - \bar{E}, \text{ where } E = \bar{E} \\
&\cup \{(a_1, a_2), (b_1, b_2) : (a_1, b_1) \in E_1, a_2 \neq b_2\}.
\end{aligned}$$

Theorem 2. Let G_1 and G_2 be two PFGs; then, $G_1[G_2]$ is a PFG.

Proof. Let $a \in V_1, (a_2, b_2) \in E_2$. Then, we obtain

$$\begin{aligned}
(p_{T_1} \cdot p_{T_2})((a, a_2), (a, b_2)) &= p_{S_1}(a) \wedge p_{T_2}(a_2, b_2) \\
&\leq p_{S_1}(a) \wedge ((p_{S_2}(a_2) \wedge p_{S_2}(b_2))) \\
&= (p_{S_1}(a) \wedge p_{S_2}(a_2)) \wedge (p_{S_1}(a) \wedge p_{S_2}(b_2)) \\
&= (p_{S_1} \cdot p_{S_2})(a, a_2) \wedge (p_{S_1} \cdot p_{S_2})(a, b_2), \\
(q_{T_1} \cdot q_{T_2})((a, a_2), (a, b_2)) &= q_{S_1}(a) \vee q_{T_2}(a_2, b_2) \\
&\geq q_{S_1}(a) \vee (q_{S_2}(a_2) \vee q_{S_2}(b_2)) \\
&= (q_{S_1}(a) \vee q_{S_2}(a_2)) \vee (q_{S_1}(a) \vee q_{S_2}(b_2)) \\
&= (q_{S_1} \cdot q_{S_2})(a, a_2) \vee (q_{S_1} \cdot q_{S_2})(a, b_2), \\
(r_{T_1} \cdot r_{T_2})((a, a_2), (a, b_2)) &= r_{S_1}(a) \vee r_{T_2}(a_2, b_2) \\
&\geq r_{S_1}(a) \vee (r_{S_2}(a_2) \vee r_{S_2}(b_2)) \\
&= (r_{S_1}(a) \vee r_{S_2}(a_2)) \vee (r_{S_1}(a) \vee r_{S_2}(b_2)) \\
&= (r_{S_1} \cdot r_{S_2})(a, a_2) \vee (r_{S_1} \cdot r_{S_2})(a, b_2).
\end{aligned} \tag{3}$$

Again, let $c \in V_2$ and $(a_1, b_1) \in E_1$. Then, we obtain

$$\begin{aligned}
 (p_{T_1} \cdot p_{T_2})((a_1, c), (b_1, c)) &= p_{T_1}(a_1, b_1) \wedge p_{S_2}(c) \\
 &\leq p_{S_2}(c) \wedge (p_{S_1}(a_1) \wedge p_{S_1}(b_1)) \\
 &= (p_{S_1}(a_1) \wedge p_{S_2}(c)) \wedge (p_{S_1}(b_1) \wedge p_{S_2}(c)) \\
 &= (p_{S_1} \cdot p_{S_2})(a_1, c) \wedge (p_{S_1} \cdot p_{S_2})(b_1, c), \\
 (q_{T_1} \cdot q_{T_2})((a_1, c), (b_1, c)) &= q_{T_1}(a_1, b_1) \vee q_{S_2}(c) \\
 &\geq q_{S_2}(c) \vee (q_{S_1}(a_1) \vee q_{S_1}(b_1)) \\
 &= (q_{S_1}(a_1) \vee q_{S_2}(c)) \vee (q_{S_1}(b_1) \vee q_{S_2}(c)) \\
 &= (q_{S_1} \cdot q_{S_2})(a_1, c) \vee (q_{S_1} \cdot q_{S_2})(b_1, c), \\
 (r_{T_1} \cdot r_{T_2})((a_1, c), (b_1, c)) &= r_{T_1}(a_1, b_1) \vee r_{S_2}(c) \\
 &\geq r_{S_2}(c) \vee (r_{S_1}(a_1) \vee r_{S_1}(b_1)) \\
 &= (r_{S_1}(a_1) \vee r_{S_2}(c)) \vee (r_{S_1}(b_1) \vee r_{S_2}(c)) \\
 &= (r_{S_1} \cdot r_{S_2})(a_1, c) \vee (r_{S_1} \cdot r_{S_2})(b_1, c).
 \end{aligned} \tag{4}$$

Again, let $(a_1, a_2), (b_1, b_2) \in E - E$, so $(a_1, b_1) \in E_1$, $a_2 \neq b_2$. Then, we obtain

$$\begin{aligned}
 (p_{T_1} \cdot p_{T_2})((a_1, a_2), (b_1, b_2)) &= p_{S_2}(a_2) \wedge p_{S_2}(b_2) \wedge p_{T_1}(a_1, b_1) \\
 &\leq p_{S_2}(a_2) \wedge p_{S_2}(b_2) \wedge (p_{S_1}(a_1) \wedge p_{S_1}(b_1)) \\
 &= p_{S_1}(a_1) \wedge p_{S_2}(a_2) \wedge (p_{S_1}(b_1) \wedge p_{S_2}(b_2)) \\
 &= (p_{S_1} \cdot p_{S_2})(a_1, a_2) \wedge (p_{S_1} \cdot p_{S_2})(b_1, b_2), \\
 (q_{T_1} \cdot q_{T_2})((a_1, a_2), (b_1, b_2)) &= q_{S_2}(a_2) \vee q_{S_2}(b_2) \vee q_{T_1}(a_1, b_1) \\
 &\leq p_{S_2}(a_2) \wedge p_{S_2}(b_2) \wedge (p_{S_1}(a_1) \wedge p_{S_1}(b_1)) \\
 &\geq q_{S_2}(a_2) \vee q_{S_2}(b_2) \vee (q_{S_1}(a_1) \vee q_{S_1}(b_1)) \\
 &= q_{S_1}(a_1) \vee q_{S_2}(a_2) \vee (q_{S_1}(b_1) \vee q_{S_2}(b_2)) \\
 &= (q_{S_1} \cdot q_{S_2})(a_1, a_2) \vee (q_{S_1} \cdot q_{S_2})(b_1, b_2), \\
 (r_{T_1} \cdot r_{T_2})((a_1, a_2), (b_1, b_2)) &= r_{S_2}(a_2) \vee r_{S_2}(b_2) \vee r_{T_1}(a_1, b_1) \\
 &\leq p_{S_2}(a_2) \wedge p_{S_2}(b_2) \wedge (p_{S_1}(a_1) \wedge p_{S_1}(b_1)) \\
 &\geq r_{S_2}(a_2) \vee r_{S_2}(b_2) \vee (r_{S_1}(a_1) \vee r_{S_1}(b_1)) \\
 &= r_{S_1}(a_1) \vee r_{S_2}(a_2) \vee (r_{S_1}(b_1) \vee r_{S_2}(b_2)) \\
 &= (r_{S_1} \cdot r_{S_2})(a_1, a_2) \vee (r_{S_1} \cdot r_{S_2})(b_1, b_2).
 \end{aligned} \tag{5}$$

The proves that $G_1[G_2]$ is a PFG. $\square$

Definition 8. Let G_1 and G_2 be two PFGs; then, the union of G_1 and G_2 is defined by $G = G_1 \cup G_2 = (S_1 \cup S_2, T_1 \cup T_2)$, where

(i)

$$(p_{S_1} \cup p_{S_2})(a) = \begin{cases} p_{S_1}(a), & \text{if } a \in V_1 \cap V_2^c, \\ p_{S_2}(a), & \text{if } a \in V_2 \cap V_1^c, \\ p_{S_1}(a) \vee p_{S_2}(a), & \text{if } a \in V_1 \cap V_2. \end{cases} \quad (6)$$

(ii)

$$(q_{S_1} \cup q_{S_2})(a) = \begin{cases} q_{S_1}(a), & \text{if } a \in V_1 \cap V_2^c, \\ q_{S_2}(a), & \text{if } a \in V_2 \cap V_1^c, \\ q_{S_1}(a) \wedge q_{S_2}(a), & \text{if } a \in V_1 \cap V_2. \end{cases} \quad (7)$$

(iii)

$$(r_{S_1} \cup r_{S_2})(a) = \begin{cases} r_{S_1}(a), & \text{if } a \in V_1 \cap V_2^c, \\ r_{S_2}(a), & \text{if } a \in V_2 \cap V_1^c, \\ r_{S_1}(a) \wedge r_{S_2}(a), & \text{if } a \in V_1 \cap V_2. \end{cases} \quad (8)$$

(iv)

$$(p_{T_1} \cup p_{T_2})(a, b) = \begin{cases} p_{T_1}(a, b), & \text{if } (a, b) \in E_1 \cap E_2^c, \\ p_{T_2}(a, b), & \text{if } (a, b) \in E_2 \cap E_1^c, \\ p_{T_1}(a, b) \vee p_{T_2}(a, b), & \text{if } (a, b) \in E_1 \cap E_2. \end{cases} \quad (9)$$

(v)

$$(q_{T_1} \cup q_{T_2})(a, b) = \begin{cases} q_{T_1}(a, b), & \text{if } (a, b) \in E_1 \cap E_2^c, \\ q_{T_2}(a, b), & \text{if } (a, b) \in E_2 \cap E_1^c, \\ q_{T_1}(a, b) \wedge q_{T_2}(a, b), & \text{if } (a, b) \in E_1 \cap E_2. \end{cases} \quad (10)$$

(vi)

$$(r_{T_1} \cup r_{T_2})(a, b) = \begin{cases} r_{T_1}(a, b), & \text{if } (a, b) \in E_1 \cap E_2^c, \\ r_{T_2}(a, b), & \text{if } (a, b) \in E_2 \cap E_1^c, \\ r_{T_1}(a, b) \wedge r_{T_2}(a, b), & \text{if } (a, b) \in E_1 \cap E_2. \end{cases} \quad (11)$$

Theorem 3. Let G_1 and G_2 be two PFGs; then, $G_1 \cup G_2$ is also a PFG.

Proof. If $(a, b) \in E_1 \cap E_2$, then we obtain

$$\begin{aligned} (p_{T_1} \cup p_{T_2})(a, b) &= p_{T_1}(a, b) \vee p_{T_2}(a, b) \\ &\leq (p_{S_1}(a) \wedge p_{S_1}(b)) \vee (p_{S_2}(a) \wedge p_{S_2}(b)) \\ &= (p_{S_1}(a) \vee p_{S_2}(a)) \wedge (p_{S_1}(b) \vee p_{S_2}(b)) \\ &= (p_{S_1} \cup p_{S_2})(a) \wedge (p_{S_1} \cup p_{S_2})(b), \\ (q_{T_1} \cup q_{T_2})(a, b) &= q_{T_1}(a, b) \wedge q_{T_2}(a, b) \\ &\geq (q_{S_1}(a) \vee q_{S_1}(b)) \wedge (q_{S_2}(a) \vee q_{S_2}(b)) \\ &= (q_{S_1}(a) \wedge q_{S_2}(a)) \vee (q_{S_1}(b) \wedge q_{S_2}(b)) \\ &= (q_{S_1} \cup q_{S_2})(a) \wedge (q_{S_1} \cup q_{S_2})(b), \\ (r_{T_1} \cup r_{T_2})(a, b) &= r_{T_1}(a, b) \wedge r_{T_2}(a, b) \\ &\geq (r_{S_1}(a) \vee r_{S_1}(b)) \wedge (r_{S_2}(a) \vee r_{S_2}(b)) \\ &= (r_{S_1}(a) \wedge r_{S_2}(a)) \vee (r_{S_1}(b) \wedge r_{S_2}(b)) \\ &= (r_{S_1} \cup r_{S_2})(a) \wedge (r_{S_1} \cup r_{S_2})(b). \end{aligned} \quad (12)$$

If $(a, b) \in E_1 \cap E_2^c$, then

$$\begin{aligned} (p_{T_1} \cup p_{T_2})(a, b) &= p_{T_1}(a, b) \\ &\leq (p_{S_1}(a) \wedge p_{S_1}(b)) \\ &= (p_{S_1} \cup p_{S_2})(a) \wedge (p_{S_1} \cup p_{S_2})(b), \\ (q_{T_1} \cup q_{T_2})(a, b) &= q_{T_1}(a, b) \\ &\geq (q_{S_1}(a) \vee q_{S_1}(b)) \\ &= (q_{S_1} \cup q_{S_2})(a) \vee (q_{S_1} \cup q_{S_2})(b), \\ (r_{T_1} \cup r_{T_2})(a, b) &= r_{T_1}(a, b) \\ &\geq (r_{S_1}(a) \vee r_{S_1}(b)) \\ &= (r_{S_1} \cup r_{S_2})(a) \vee (r_{S_1} \cup r_{S_2})(b). \end{aligned} \quad (13)$$

If $(a, b) \in E_2 \cap E_1^c$, then we have

$$\begin{aligned} (p_{T_1} \cup p_{T_2})(a, b) &= p_{T_2}(a, b) \\ &\leq (p_{S_2}(a) \wedge p_{S_2}(b)) \\ &= (p_{S_1} \cup p_{S_2})(a) \wedge (p_{S_1} \cup p_{S_2})(b), \\ (q_{T_1} \cup q_{T_2})(a, b) &= q_{T_2}(a, b) \\ &\geq (q_{S_2}(a) \vee q_{S_2}(b)) \\ &= (q_{S_1} \cup q_{S_2})(a) \vee (q_{S_1} \cup q_{S_2})(b), \\ (r_{T_1} \cup r_{T_2})(a, b) &= r_{T_2}(a, b) \\ &\geq (r_{S_2}(a) \vee r_{S_2}(b)) \\ &= (r_{S_1} \cup r_{S_2})(a) \vee (r_{S_1} \cup r_{S_2})(b). \end{aligned} \quad (14)$$

This shows that $G_1 \cup G_2$ is a PFG. $\square$

Corollary 1. Let $\{G_\lambda: \lambda \in \Lambda\}$ be a family of PFGs; then, $\bigcup_{\lambda \in \Lambda} G_\lambda$ is a PFG.

Definition 9. Let G_1 and G_2 be two PFGs; then, the intersection of G_1 and G_2 is defined by $G = G_1 \cap G_2 = (S_1 \cap S_2, T_1 \cap T_2)$, where

$$\begin{aligned} \text{(i)} \quad & (p_{S_1 \cap S_2})(a) = p_{S_1}(a) \wedge p_{S_2}(a), \quad a \in V_1 \cap V_2, \\ & (q_{S_1 \cap S_2})(a) = q_{S_1}(a) \vee q_{S_2}(a), \quad a \in V_1 \cap V_2, \\ & (r_{S_1 \cap S_2})(a) = r_{S_1}(a) \vee r_{S_2}(a), \quad a \in V_1 \cap V_2. \end{aligned} \quad (15)$$

$$\begin{aligned} \text{(ii)} \quad & (p_{T_1 \cap T_2})(a, b) = p_{T_1}(a, b) \wedge p_{T_2}(a, b), \quad (a, b) \in E_1 \cap E_2, \\ & (q_{T_1 \cap T_2})(a, b) = q_{T_1}(a, b) \vee q_{T_2}(a, b), \quad (a, b) \in E_1 \cap E_2, \\ & (r_{T_1 \cap T_2})(a, b) = r_{T_1}(a, b) \vee r_{T_2}(a, b), \quad (a, b) \in E_1 \cap E_2. \end{aligned} \quad (16)$$

Theorem 4. Let G_1 and G_2 be two PFGs; then, $G_1 \cap G_2$ is also a PFG.

Proof. For $u, v \in V$, we obtain

$$\begin{aligned} (p_{T_1 \cap T_2})(a, b) &= p_{T_1}(a, b) \wedge p_{T_2}(a, b) \\ &\leq (p_{S_1}(a) \wedge p_{S_1}(b)) \wedge (p_{S_2}(a) \wedge p_{S_2}(b)) \\ &= (p_{S_1}(a) \wedge p_{S_2}(a)) \wedge (p_{S_1}(b) \wedge p_{S_2}(b)) \\ &= (p_{S_1 \cap S_2})(a) \wedge (p_{S_1 \cap S_2})(b), \\ (q_{T_1 \cap T_2})(a, b) &= q_{T_1}(a, b) \vee q_{T_2}(a, b) \\ &\geq (q_{S_1}(a) \vee q_{S_1}(b)) \vee (q_{S_2}(a) \vee q_{S_2}(b)) \end{aligned}$$

$$\begin{aligned} &= (q_{S_1}(a) \vee q_{S_2}(a)) \vee (q_{S_1}(b) \vee q_{S_2}(b)) \\ &= (q_{S_1 \cap S_2})(a) \vee (q_{S_1 \cap S_2})(b), \\ (r_{T_1 \cap T_2})(a, b) &= r_{T_1}(a, b) \vee r_{T_2}(a, b) \\ &\geq (r_{S_1}(a) \vee r_{S_1}(b)) \vee (r_{S_2}(a) \vee r_{S_2}(b)) \\ &= (r_{S_1}(a) \vee r_{S_2}(a)) \vee (r_{S_1}(b) \vee r_{S_2}(b)) \\ &= (r_{S_1 \cap S_2})(a) \vee (r_{S_1 \cap S_2})(b). \end{aligned} \quad (17)$$

This shows that $G_1 \cap G_2$ is a PFG. $\square$

Corollary 2. Let $\{G_\lambda : \lambda \in \Lambda\}$ be a family of PFGs; then $\bigcap_{\lambda \in \Lambda} G_\lambda$ is a PFG.

Definition 10. Let G_1 and G_2 be two PFGs; then, the sum of G_1 and G_2 is defined by $G = G_1 + G_2 = (S_1 + S_2, T_1 + T_2)$, where

$$\begin{aligned} \text{(i)} \quad & (p_{S_1 + S_2})(a) = (p_{S_1} \cup p_{S_2})(a), \\ & (q_{S_1 + S_2})(a) = (q_{S_1} \cap q_{S_2})(a), \\ & (r_{S_1 + S_2})(a) = (r_{S_1} \cap r_{S_2})(a), \quad \text{if } a \in V_1 \cup V_2. \end{aligned} \quad (18)$$

(ii)

$$\begin{aligned} (p_{T_1 + T_2})(a, b) &= (p_{T_1} \cup p_{T_2})(a, b) = p_{T_1}(a, b), \\ (q_{T_1 + T_2})(a, b) &= (q_{T_1} \cap q_{T_2})(a, b) = q_{T_1}(a, b), \\ (r_{T_1 + T_2})(a, b) &= (r_{T_1} \cap r_{T_2})(a, b) = r_{T_1}(a, b), \quad \text{if } (a, b) \in E_1 \cap E_2. \end{aligned} \quad (19)$$

(iii)

$$\begin{aligned} (p_{T_1 + T_2})(a, b) &= p_{S_1}(a) \vee p_{S_2}(a), \\ (q_{T_1 + T_2})(a, b) &= q_{S_1}(a) \wedge q_{S_2}(a), \\ (r_{T_1 + T_2})(a, b) &= (r_{S_1}(a) \wedge r_{S_2}(a)) \quad \text{if } (a, b) \in E', \text{ where } E' \text{ is the set of edges connecting the nodes of } V_1. \end{aligned} \quad (20)$$

Theorem 5. Let G_1 and G_2 be two PFGs'; then, $G_1 + G_2$ is also a PFG.

4. Picture Fuzzy Graph-Based Multiple Attribute Decision-Making

PFS is an important tool to solve real-world problems. PFS deals with inconsistent, incomplete, and indeterminate information or fact. Nowadays, PFS has become an exciting topic for its wide applications. So, PFG can efficiently solve such type of real-world problem.

Here, the concept of the graph is applied to MADMP with a picture fuzzy environment, and we proposed two algorithms. Also, to illustrate our proposed decision-making algorithm, we have been given two examples. Let $A = \{A_1, A_2, A_3, \dots, A_m\}$ be an arrangement of alternatives and $C = \{C_1, C_2, C_3, \dots, C_n\}$ be the arrangement of attribute. $w = \{w_1, w_2, w_3, \dots, w_n\}$ be the weight vector of the attribute C_i , $i = 1, 2, \dots, n$, where $w_i \geq 0$, for $i = 1, 2, \dots, n$, and $\sum_{i=1}^n w_i = 1$.

Let $M = (b_{kj})_{m \times n} = (p_{p_{kj}, q_{kj}, r_{kj}})_{m \times n}$ be a picture fuzzy decision matrix, where p_{kj} is the positive membership degree for which alternative A_j satisfies the attribute C_j , which was given by the decision makers, q_{kj} is the neutral membership degree so that alternative A_k does not satisfies the attribute C_j , and r_{kj} is the degree that the alternatives A_k does not fulfill the attribute C_j which was given by the decision maker, where $p_{kj} \in [0, 1]$, $q_{kj} \in [0, 1]$, $r_{kj} \in [0, 1]$, and $0 \leq p_{kj} + q_{kj} + r_{kj} \leq 1$, $k = 1, 2, \dots, m$. The picture fuzzy relation between two attributes $C_i = (p_i, q_i, r_i)$ and $C_j = (p_j, q_j, r_j)$ is defined by $f_{ij} = (p_{ij}, q_{ij}, r_{ij})$, where $p_{ij} \leq p_i \wedge p_j$, $q_{ij} \geq q_i \vee q_j$, and $r_{ij} \geq r_i \vee r_j$, $i, j = 1, 2, \dots, m$, otherwise, $f_{ij} = (0, 0, 1)$.

We proposed two algorithms to develop the graph structure and solve multiattribute decision-making (MADM) problems using PFG (Algorithms 1 and 2).

Let $A = (p_j, q_j, r_j)$ be a decision solution, for $j = 1, 2, \dots, n$. Now, we develop an algorithm that is based on PFG and the similarity measure between picture fuzzy numbers. Here, the main advantage is that it can compute the relationship among multiple-input arguments through the graph theory approach.

5. Numerical Example

In this part, numerical examples for the PFGMADM problem with picture fuzzy information are used to present the application of the proposed algorithms. Here, we consider a MADMP taken from S. Ashraf et al. [46].

Example 1. An investment company wants to invest money in the best choice. There are four measurable alternatives:

- A_1 : a car company
- A_2 : a food company
- A_3 : a computer company
- A_4 : an arms company

The investment company makes a decision based on the three attributes:

- C_1 : the risk analysis
- C_2 : the growth analysis
- C_3 : the environmental impact analysis

The growth vector of the attribute is given by $w = (0.35, 0.25, 0.40)$.

The four possible alternatives are to be measured under the three attributes and are given in the form of picture fuzzy information by decision-making according to three attributes C_1, C_2 , and C_3 and the evaluation information on the alternative A_1, A_2, A_3 , and A_4 under the factors C_1, C_2 , and C_3 can be shown in the following picture fuzzy decision matrix M :

$$M = \begin{pmatrix} (0.6, 0.2, 0.2) & (0.8, 0.1, 0.1) & (0.6, 0.1, 0.3) \\ (0.5, 0.3, 0.2) & (0.5, 0.2, 0.3) & (0.8, 0.1, 0.1) \\ (0.4, 0.2, 0.4) & (0.6, 0.3, 0.1) & (0.4, 0.2, 0.4) \\ (0.3, 0.1, 0.6) & (0.7, 0.1, 0.2) & (0.7, 0.1, 0.2) \end{pmatrix}. \quad (21)$$

Also, we assume that the relationship among the attribute C_1, C_2 , and C_3 can be described by a complete graph $G = (V, E)$, where $V = \{C_1, C_2, C_3\}$ and $E = \{(C_1, C_2), (C_1, C_3), (C_2, C_3)\}$, see Figure 1.

From equation (1), we get all the impact coefficient to find out the relationships among the attribute. Now, the picture fuzzy edges denoting the connections among the attributes are described as

$$\begin{aligned} f_{12} &= (p_{12}, q_{12}, r_{12}) = (0.3, 0.4, 0.5), \\ f_{13} &= (p_{13}, q_{13}, r_{13}) = (0.3, 0.3, 0.4), \\ f_{23} &= (p_{23}, q_{23}, r_{23}) = (0.2, 0.4, 0.4). \end{aligned} \quad (22)$$

Notice that here $G = (V, E)$ is a PFG according to the relationship among the attribute for every alternatives. To find the best alternatives, we perform the following steps:

Step 1: the impact coefficient between the attribute C_j , $j = 1, 2, 3$, are as follows:

$$\begin{aligned} \eta_{12} &= \frac{p_{12} + (1 - q_{12})(1 - r_{12})}{3} \\ &= \frac{0.3 + (1 - 0.4)(1 - 0.4)}{3} \\ &= 0.22, \\ \eta_{13} &= \frac{p_{13} + (1 - q_{13})(1 - r_{13})}{3} \\ &= \frac{0.3 + (1 - 0.3)(1 - 0.4)}{3} \\ &= 0.24, \\ \eta_{23} &= \frac{p_{23} + (1 - q_{23})(1 - r_{23})}{3} \\ &= \frac{0.2 + (1 - 0.4)(1 - 0.4)}{3} \\ &= 0.187. \end{aligned} \quad (23)$$

Step 1: calculate the impact coefficient between the attributes C_i and C_j by $\eta_{ij} = ((p_{ij} + (1 - q_{ij})(1 - r_{ij}))/3)$ for $i, j = 1, 2, \dots, n$, where $\eta_{ij} = (p_{ij}, q_{ij}, r_{ij})$ is the picture fuzzy edge between the nodes C_i and C_j , for $i, j = 1, 2, \dots, n$. We have $\eta_{ij} = 1$ and $\eta_{ij} = \eta_{ji}$ if $i = j$.

Step 2: find the attribute of the alternative A_k by $\widetilde{A}_k = (\widetilde{p}_k, \widetilde{q}_k, \widetilde{r}_k) = (1/3) \sum_{j=1}^n w_j (\sum_{s=1}^n \eta_{sj} b_{ks})$, where $f_{sj} = (p_{sj}, q_{sj}, r_{sj})$.

Step 3: calculate the score function of the alternative $\widetilde{A}_k$ by $\text{scor}(\widetilde{A}_k) = (1/2)[1 + \widetilde{p}_k - 2\widetilde{q}_k - \widetilde{r}_k]$.

Step 4: rank all the alternative A_k depending on $\text{scor}(\widetilde{A}_k)$ and then select the best alternative.

Step 5: stop.

ALGORITHM 1: Computation of best alternative.

Step 1: calculate the impact coefficient between the attributes C_i and C_j by $\eta_{ij} = ((p_{ij} + (1 - q_{ij})(1 - r_{ij}))/3)$ for $i, j = 1, 2, 3, \dots, n$, where $\eta_{ij} = (p_{ij}, q_{ij}, r_{ij})$ is the picture fuzzy edge between the nodes C_i and C_j for $i, j = 1, 2, \dots, n$. We have $\eta_{ij} = 1$ and $\eta_{ij} = \eta_{ji}$ if $i = j$.

Step 2: compute the associated weighted value of attribute C_j , for $j = 1, 2, 3, \dots, n$, over the other criteria by $\widetilde{b}_{kj} = (\widetilde{p}_{kj}, \widetilde{q}_{kj}, \widetilde{r}_{kj}) = (1/3)w_j \sum_{s=1}^n \eta_{sj} b_{ks}$.

Step 3: find the similarity measure between the decision solution $A = (p_j, q_j, r_j)$, $j = 1, 2, 3, \dots, n$, and every alternative A_k , $k = 1, 2, 3, \dots, m$, by $\text{scor}(A, A_k) = 1 - (1/3n) \sum_{j=1}^n |p_j - \widetilde{p}_{kj}| + |q_j - \widetilde{q}_{kj}| + |r_j - \widetilde{r}_{kj}|$.

Step 4: rank all the alternative A_k according to $\text{scor}(A, A_k)$, $k = 1, 2, 3, \dots, m$.

Step 5: stop.

ALGORITHM 2: Computation of best alternative using similarity measure.

FIGURE 1: The graph relationship among the three attributes.

Step 2: the attribute of the alternative A_1 is calculated below:

$$\begin{aligned}
 \widetilde{A}_1 &= \frac{1}{3} [w_1 (\eta_{11} b_{11} + \eta_{21} b_{12} + \eta_{31} b_{13}) + w_2 (\eta_{12} b_{11} + \eta_{22} b_{12} + \eta_{32} b_{13}) + w_3 (\eta_{13} b_{11} + \eta_{23} b_{12} + \eta_{33} b_{13})] \\
 &= \frac{1}{3} [0.35\{1 \times (0.6, 0.2, 0.2) + 0.22 \times (0.8, 0.1, 0.1) + 0.24 \times (0.6, 0.1, 0.3)\} \\
 &\quad + 0.25\{0.22 \times (0.6, 0.2, 0.2) + 1 \times (0.8, 0.1, 0.1) + 0.187 \times (0.6, 0.1, 0.3)\} \\
 &\quad + 0.4\{0.24 \times (0.6, 0.2, 0.2) + 0.187 \times (0.8, 0.1, 0.1) + 1 \times (0.6, 0.1, 0.3)\}] \\
 &= (0.314, 0.065, 0.1), \\
 \widetilde{A}_2 &= \frac{1}{3} [w_1 (\eta_{11} b_{21} + \eta_{21} b_{22} + \eta_{31} b_{23}) + w_2 (\eta_{12} b_{21} + \eta_{22} b_{22} + \eta_{32} b_{23}) + w_3 (\eta_{13} b_{21} + \eta_{23} b_{22} + \eta_{33} b_{23})] \\
 &= \frac{1}{3} [0.35\{1 \times (0.5, 0.3, 0.2) + 0.22 \times (0.5, 0.2, 0.3) + 0.24 \times (0.8, 0.1, 0.1)\} \\
 &\quad + 0.25\{0.22 \times (0.5, 0.3, 0.2) + 1 \times (0.5, 0.2, 0.3) + 0.187 \times (0.8, 0.1, 0.1)\}
 \end{aligned}$$

$$\begin{aligned}
& + 0.4\{0.24 \times (0.5, 0.3, 0.2) + 0.187 \times (0.5, 0.2, 0.3) + 1 \times (0.8, 0.1, 0.1)\} \\
& = (0.292, 0.088, 0.078), \\
\widetilde{A}_3 &= \frac{1}{3} [w_1 (\eta_{11}b_{31} + \eta_{21}b_{32} + \eta_{31}b_{33}) + w_2 (\eta_{12}b_{31} + \eta_{22}b_{32} + \eta_{32}b_{33}) + w_3 (\eta_{13}b_{31} + \eta_{23}b_{32} + \eta_{33}b_{33})] \\
&= \frac{1}{3} [0.35\{1 \times (0.4, 0.2, 0.4) + 0.22 \times (0.6, 0.3, 0.1) + 0.24 \times (0.4, 0.2, 0.4)\} \\
&\quad + 0.25\{0.22 \times (0.4, 0.2, 0.4) + 1 \times (0.6, 0.3, 0.1) + 0.187 \times (0.4, 0.2, 0.4)\} \\
&\quad + 0.4\{0.24 \times (0.4, 0.2, 0.4) + 0.187 \times (0.6, 0.3, 0.1) + 1 \times (0.4, 0.2, 0.4)\}] \\
&= (0.218, 0.109, 0.151), \\
\widetilde{A}_4 &= \frac{1}{3} [w_1 (\eta_{11}b_{41} + \eta_{21}b_{42} + \eta_{31}b_{43}) + w_2 (\eta_{12}b_{41} + \eta_{22}b_{42} + \eta_{32}b_{43}) + w_3 (\eta_{13}b_{41} + \eta_{23}b_{42} + \eta_{33}b_{43})] \\
&= \frac{1}{3} [0.35\{1 \times (0.3, 0.1, 0.6) + 0.22 \times (0.7, 0.1, 0.2) + 0.24 \times (0.7, 0.1, 0.2)\} \\
&\quad + 0.25\{0.22 \times (0.3, 0.1, 0.6) + 1 \times (0.7, 0.1, 0.2) + 0.187 \times (0.7, 0.1, 0.2)\} \\
&\quad + 0.4\{0.24 \times (0.3, 0.1, 0.6) + 0.187 \times (0.7, 0.1, 0.2) + 1 \times (0.7, 0.1, 0.2)\}] \\
&= (0.268, 0.048, 0.162).
\end{aligned} \tag{24}$$

Step 3: now, we compute the score functions as follows:

$$\begin{aligned}
\text{scor}(\widetilde{A}_1) &= \frac{1}{2} [1 + \widetilde{p}_1 - 2\widetilde{q}_1 - \widetilde{r}_1] \\
&= \frac{1}{2} [1 + 0.314 - 2 \times 0.065 - 0.1] = 0.542, \\
\text{scor}(\widetilde{A}_2) &= \frac{1}{2} [1 + \widetilde{p}_2 - 2\widetilde{q}_2 - \widetilde{r}_2] \\
&= \frac{1}{2} [1 + 0.292 - 2 \times 0.088 - 0.078] = 0.519, \\
\text{scor}(\widetilde{A}_3) &= \frac{1}{2} [1 + \widetilde{p}_3 - 2\widetilde{q}_3 - \widetilde{r}_3] \\
&= \frac{1}{2} [1 + 0.218 - 2 \times 0.109 - 0.151] = 0.425, \\
\text{scor}(\widetilde{A}_4) &= \frac{1}{2} [1 + \widetilde{p}_4 - 2\widetilde{q}_4 - \widetilde{r}_4] \\
&= \frac{1}{2} [1 + 0.264 - 2 \times 0.048 - 0.162] = 0.505.
\end{aligned} \tag{25}$$

Step 4: therefore, we rank these alternatives as $A_1 > A_2 > A_4 > A_3$. From the above numerical observation we have, A_1 is the best choice in the decision-making problem.

Example 2. In this example, we consider medical diagnosis problem adapted from Ye [47]. Let us consider a set of diagnosis as $A = \{A_1 \text{ (viral fever)}, A_2 \text{ (malaria)}, A_3 \text{ (typhoid)}, A_4 \text{ (gastritis)}, A_5 \text{ (stenocardia)}\}$ and set of symptoms as $C = \{C_1 \text{ (temperature)}, C_2 \text{ (headache)}, C_3 \text{ (stomach pain)}, C_4 \text{ (cough)}, C_5 \text{ (sttenocardia)}\}$

Let the weight vector of the symptoms be $w = (0.25, 0.15, 0.10, 0.20, 0.30)$. Also, the performance values of the considered diseases are characterized by PFS, and results are shown in Table 1.

Suppose a patient A having all symptoms is represented by the following picture fuzzy information:

$$\begin{aligned}
A(\text{patient}) &= \{\langle C_1, 0.8, 0.1, 0.1 \rangle, \langle C_2, 0.6, 0.2, 0.1 \rangle, \\
&\quad \cdot \langle C_3, 0.2, 0.0, 0.8 \rangle, \langle C_4, 0.6, 0.1, 0.1 \rangle, \\
&\quad \cdot \langle C_5, 0.1, 0.1, 0.6 \rangle\}
\end{aligned} \tag{26}$$

Let the picture fuzzy edges denote the connection among the symptoms (see Figure 2), which is described as $f_{12} = (0.2, 0.3, 0.1)$, $f_{13} = (0.1, 0.1, 0.8)$, $f_{14} = (0.3, 0.4, 0.2)$, $f_{15} = (0.1, 0.2, 0.6)$, $f_{23} = (0.0, 0.2, 0.8)$, $f_{24} = (0.4, 0.3, 0.1)$, $f_{25} = (0.1, 0.2, 0.6)$, $f_{34} = (0.1, 0.1, 0.8)$, $f_{35} = (0.1, 0.1, 0.8)$, and $f_{45} = (0.1, 0.2, 0.6)$. The impact coefficient between symptoms is calculated below:

$$\begin{aligned}
\eta_{12} &= \frac{1}{3} [p_{12} + (1 - q_{12})(1 - r_{12})] \\
&= \frac{1}{3} [0.2 + (1 - 0.3)(1 - 0.1)] = 0.277, \\
\eta_{13} &= \frac{1}{3} [p_{13} + (1 - q_{13})(1 - r_{13})] \\
&= \frac{1}{3} [0.1 + (1 - 0.1)(1 - 0.8)] = 0.273, \\
\eta_{14} &= \frac{1}{3} [p_{14} + (1 - q_{14})(1 - r_{14})] \\
&= \frac{1}{3} [0.3 + (1 - 0.4)(1 - 0.2)] = 0.260.
\end{aligned} \tag{27}$$

TABLE 1: Performance values of the disease.

	C_1 (temperature)	C_2 (headache)	C_3 (stomach pain)	C_4 (cough)	C_5 (chestpain)
A_1 (viral fever)	(0.052, 0.037, 0.042)	(0.028, 0.014, 0.037)	(0.010, 0.011, 0.310)	(0.042, 0.023, 0.042)	(0.028, 0.024, 0.101)
A_2 (malaria)	(0.079, 0.016, 0.038)	(0.032, 0.010, 0.038)	(0.010, 0.005, 0.039)	(0.062, 0.027, 0.027)	(0.032, 0.008, 0.114)
A_3 (typhoid)	(0.049, 0.026, 0.067)	(0.039, 0.015, 0.028)	(0.012, 0.006, 0.037)	(0.034, 0.015, 0.060)	(0.031, 0.010, 0.124)
A_4 (gastritis)	(0.038, 0.021, 0.091)	(0.018, 0.019, 0.047)	(0.030, 0.006, 0.016)	(0.026, 0.017, 0.074)	(0.049, 0.020, 0.095)
A_5 (stenocardia)	(0.027, 0.012, 0.122)	(0.011, 0.007, 0.068)	(0.016, 0.002, 0.039)	(0.024, 0.005, 0.091)	(0.090, 0.013, 0.066)

FIGURE 2: The graph relationship among the five attributes.

Similarly, $\eta_{15} = 0.140$, $\eta_{23} = 0.053$, $\eta_{24} = 0.343$, $\eta_{25} = 0.140$, $\eta_{34} = 0.093$, $\eta_{35} = 0.280$, and $\eta_{45} = 0.140$.

Now, the associated weighted values of disease are obtained by $\widetilde{b}_{kj} = (w_j/3) \sum_{s=1}^5 \eta_{sj} b_{ks}$, where $\widetilde{b}_{kj} = (\widetilde{p}_{kj}, \widetilde{q}_{kj}, \widetilde{r}_{kj})$ is a PFN.

Therefore,

$$\begin{aligned}
 \widetilde{b}_{11} &= \frac{w_1}{3} \sum_{s=1}^5 \eta_{s1} b_{1s} \\
 &= \frac{w_1}{3} (\eta_{11} b_{11} + \eta_{21} b_{12} + \eta_{31} b_{13} + \eta_{41} b_{14} + \eta_{51} b_{15}) \\
 &= \frac{0.25}{3} [1 \times (0.4, 0.3, 0.0) + 0.277 \times (0.3, 0.1, 0.5) + 0.273 \times (0.1, 0.2, 0.7) \\
 &\quad + 0.260 \times (0.4, 0.2, 0.3) + 0.140 \times (0.1, 0.1, 0.7)] \\
 &= (0.052, 0.037, 0.042), \\
 \widetilde{b}_{12} &= \frac{w_2}{3} \sum_{s=1}^5 \eta_{s2} b_{1s} \\
 &= \frac{w_2}{3} (\eta_{12} b_{11} + \eta_{22} b_{12} + \eta_{32} b_{13} + \eta_{42} b_{14} + \eta_{52} b_{15}) \\
 &= \frac{0.15}{3} [0.277 \times (0.4, 0.3, 0.0) + 1 \times (0.3, 0.1, 0.5) + 0.053 \times (0.1, 0.2, 0.7) \\
 &\quad + 0.343 \times (0.4, 0.2, 0.3) + 0.140 \times (0.1, 0.1, 0.7)] \\
 &= (0.028, 0.014, 0.037),
 \end{aligned}$$

$$\begin{aligned}
\widetilde{b}_{13} &= \frac{w_1}{3} \sum_{s=1}^5 \eta_{s3} b_{1s} \\
&= \frac{w_3}{3} (\eta_{13} b_{11} + \eta_{23} b_{12} + \eta_{33} b_{13} + \eta_{43} b_{14} + \eta_{53} b_{15}) \\
&= \frac{0.10}{3} [0.273 \times (0.4, 0.3, 0.0) + 0.053 \times (0.3, 0.1, 0.5) + 1 \times (0.1, 0.2, 0.7) \\
&\quad + 0.093 \times (0.4, 0.2, 0.3) + 0.280 \times (0.1, 0.1, 0.7)] \\
&= (0.010, 0.011, 0.031), \\
\widetilde{b}_{14} &= \frac{w_4}{3} \sum_{s=1}^5 \eta_{s4} b_{1s} \\
&= \frac{w_4}{3} (\eta_{14} b_{11} + \eta_{24} b_{12} + \eta_{34} b_{13} + \eta_{44} b_{14} + \eta_{54} b_{15}) \\
&= \frac{0.20}{3} [0.26 \times (0.4, 0.3, 0.0) + 0.034 \times (0.3, 0.1, 0.5) + 0.093 \times (0.1, 0.2, 0.7) \\
&\quad + 1 \times (0.4, 0.2, 0.3) + 0.140 \times (0.1, 0.1, 0.7)] \\
&= (0.042, 0.023, 0.042), \\
\widetilde{b}_{15} &= \frac{w_5}{3} \sum_{s=1}^5 \eta_{s5} b_{1s} \\
&= \frac{w_5}{3} (\eta_{15} b_{11} + \eta_{25} b_{12} + \eta_{35} b_{13} + \eta_{45} b_{14} + \eta_{55} b_{15}) \\
&= \frac{0.30}{3} [0.140 \times (0.4, 0.3, 0.0) + 0.140 \times (0.3, 0.1, 0.5) + 0.280 \times (0.1, 0.2, 0.7) \\
&\quad + 0.140 \times (0.4, 0.2, 0.3) + 1 \times (0.1, 0.1, 0.7)] \\
&= (0.028, 0.024, 0.101), \\
\widetilde{b}_{21} &= \frac{w_1}{3} \sum_{s=1}^5 \eta_{s1} b_{2s} \\
&= \frac{w_1}{3} (\eta_{11} b_{21} + \eta_{21} b_{22} + \eta_{31} b_{23} + \eta_{41} b_{24} + \eta_{51} b_{25}) \\
&= \frac{0.25}{3} [1 \times (0.7, 0.1, 0.0) + 0.277 \times (0.2, 0.1, 0.6) + 0.273 \times (0.0, 0.1, 0.9) \\
&\quad + 0.260 \times (0.7, 0.2, 0.0) + 0.140 \times (0.1, 0.0, 0.8)] \\
&= (0.079, 0.016, 0.038), \\
\widetilde{b}_{22} &= \frac{w_2}{3} \sum_{s=1}^5 \eta_{s2} b_{2s} \\
&= \frac{w_2}{3} (\eta_{12} b_{21} + \eta_{22} b_{22} + \eta_{32} b_{23} + \eta_{42} b_{24} + \eta_{52} b_{25}) \\
&= \frac{0.15}{3} [0.277 \times (0.7, 0.1, 0.0) + 1 \times (0.2, 0.1, 0.6) + 0.053 \times (0.0, 0.1, 0.9) \\
&\quad + 0.343 \times (0.7, 0.2, 0.0) + 0.140 \times (0.1, 0.0, 0.8)] \\
&= (0.032, 0.010, 0.038).
\end{aligned}$$

(28)

Similarly, $\widetilde{b}_{23} = (0.010, 0.005, 0.039)$, $\widetilde{b}_{24} = (0.062, 0.027, 0.027)$, and $\widetilde{b}_{25} = (0.032, 0.008, 0.114)$. Again,

$$\begin{aligned}
 \widetilde{b}_{31} &= \frac{w_1}{3} \sum_{s=1}^5 \eta_{s1} b_{3s} \\
 &= \frac{w_1}{3} (\eta_{11} b_{31} + \eta_{21} b_{32} + \eta_{31} b_{33} + \eta_{41} b_{34} + \eta_{51} b_{35}) \\
 &= \frac{0.25}{3} [1 \times (0.3, 0.2, 0.3) + 0.277 \times (0.6, 0.2, 0.1) + 0.273 \times (0.2, 0.1, 0.7) \\
 &\quad + 0.260 \times (0.2, 0.1, 0.6) + 0.140 \times (0.1, 0.0, 0.9)] \\
 &= (0.049, 0.026, 0.067).
 \end{aligned} \tag{29}$$

Similarly, $\widetilde{b}_{32} = (0.039, 0.015, 0.028)$, $\widetilde{b}_{33} = (0.012, 0.006, 0.037)$, $\widetilde{b}_{34} = (0.034, 0.015, 0.060)$, and $\widetilde{b}_{35} = (0.031, 0.010, 0.124)$. Also,

$$\begin{aligned}
 \widetilde{b}_{41} &= \frac{w_1}{3} \sum_{s=1}^5 \eta_{s1} b_{4s} \\
 &= \frac{w_1}{3} (\eta_{11} b_{41} + \eta_{21} b_{42} + \eta_{31} b_{43} + \eta_{41} b_{44} + \eta_{51} b_{45}) \\
 &= \frac{0.25}{3} [1 \times (0.1, 0.1, 0.7) + 0.277 \times (0.2, 0.3, 0.4) + 0.273 \times (0.8, 0.1, 0.0) \\
 &\quad + 0.260 \times (0.2, 0.1, 0.7) + 0.140 \times (0.2, 0.1, 0.7)] \\
 &= (0.038, 0.021, 0.091).
 \end{aligned} \tag{30}$$

Similarly, $\widetilde{b}_{42} = (0.018, 0.019, 0.047)$, $\widetilde{b}_{43} = (0.030, 0.006, 0.016)$, $\widetilde{b}_{44} = (0.026, 0.017, 0.074)$, and $\widetilde{b}_{45} = (0.049, 0.020, 0.095)$. Finally,

$$\begin{aligned}
 \widetilde{b}_{51} &= \frac{w_1}{3} \sum_{s=1}^5 \eta_{s1} b_{5s} \\
 &= \frac{w_1}{3} (\eta_{11} b_{51} + \eta_{21} b_{52} + \eta_{31} b_{53} + \eta_{41} b_{54} + \eta_{51} b_{55}) \\
 &= \frac{0.25}{3} [1 \times (0.1, 0.1, 0.8) + 0.277 \times (0.0, 0.1, 0.8) + 0.273 \times (0.2, 0.0, 0.8) \\
 &\quad + 0.260 \times (0.2, 0.0, 0.8) + 0.140 \times (0.8, 0.1, 0.1)] \\
 &= (0.027, 0.012, 0.0122).
 \end{aligned} \tag{31}$$

Similarly, $\widetilde{b}_{52} = (0.011, 0.007, 0.068)$, $\widetilde{b}_{53} = (0.016, 0.002, 0.039)$, $\widetilde{b}_{54} = (0.024, 0.005, 0.091)$, and $\widetilde{b}_{55} = (0.090, 0.013, 0.066)$.

Therefore, the results obtained are shown in Table 2:
The similarity measure between the ideal solution A and each diseases A_k , $k = 1, 2, 3, 4, 5$, are calculated below:

TABLE 2: The associated weighted values of the disease.

	C_1	C_2	C_3	C_4	C_5
A_1	(0.052, 0.037, 0.042)	(0.028, 0.014, 0.037)	(0.010, 0.011, 0.310)	(0.042, 0.023, 0.042)	(0.028, 0.024, 0.101)
A_2	(0.079, 0.016, 0.038)	(0.032, 0.010, 0.038)	(0.010, 0.005, 0.039)	(0.062, 0.027, 0.027)	(0.032, 0.008, 0.114)
A_3	(0.049, 0.026, 0.067)	(0.039, 0.015, 0.028)	(0.012, 0.006, 0.037)	(0.034, 0.015, 0.060)	(0.031, 0.010, 0.124)
A_4	(0.038, 0.021, 0.091)	(0.018, 0.019, 0.047)	(0.030, 0.006, 0.016)	(0.026, 0.017, 0.074)	(0.049, 0.020, 0.095)
A_5	(0.027, 0.012, 0.122)	(0.011, 0.007, 0.068)	(0.016, 0.002, 0.039)	(0.024, 0.005, 0.091)	(0.090, 0.013, 0.066)

$$\begin{aligned}
\text{scor}(A, A_1) &= 1 - \frac{1}{15} [|p_1 - p_{11}| + |q_1 - q_{11}| + |r_1 - r_{11}| + |p_2 - p_{12}| + |q_2 - q_{12}| + |r_2 - r_{12}| \\
&\quad + |p_3 - p_{13}| + |q_3 - q_{13}| + |r_3 - r_{13}| + |p_4 - p_{14}| + |q_4 - q_{14}| + |r_4 - r_{14}| \\
&\quad + |p_5 - p_{15}| + |q_5 - q_{15}| + |r_5 - r_{15}|] \\
&= 1 - \frac{1}{15} [0.869 + 0.821 + 0.691 + 0.693 + 0.647] = 0.7519, \\
\text{scor}(A, A_2) &= 1 - \frac{1}{15} [|p_1 - p_{21}| + |q_1 - q_{21}| + |r_1 - r_{21}| + |p_2 - p_{22}| + |q_2 - q_{22}| + |r_2 - r_{22}| \\
&\quad + |p_3 - p_{23}| + |q_3 - q_{23}| + |r_3 - r_{23}| + |p_4 - p_{24}| + |q_4 - q_{24}| + |r_4 - r_{24}| \\
&\quad + |p_5 - p_{25}| + |q_5 - q_{25}| + |r_5 - r_{25}|] \\
&= 1 - \frac{1}{15} [0.867 + 0.820 + 0.956 + 0.684 + 0.646] = 0.7351, \\
\text{scor}(A, A_3) &= d1 - \frac{1}{15} [|p_1 - p_{31}| + |q_1 - q_{31}| + |r_1 - r_{31}| + |p_2 - p_{32}| + |q_2 - q_{32}| + |r_2 - r_{32}| \\
&\quad + |p_3 - p_{33}| + |q_3 - q_{33}| + |r_3 - r_{33}| + |p_4 - p_{34}| + |q_4 - q_{34}| + |r_4 - r_{34}| \\
&\quad + |p_5 - p_{35}| + |q_5 - q_{35}| + |r_5 - r_{35}|] \\
&= 1 - \frac{1}{15} [0.858 + 0.818 + 0.957 + 0.691 + 0.635] = 0.7361, \\
\text{scor}(A, A_4) &= 1 - \frac{1}{15} [|p_1 - p_{41}| + |q_1 - q_{41}| + |r_1 - r_{41}| + |p_2 - p_{42}| + |q_2 - q_{42}| + |r_2 - r_{42}| \\
&\quad + |p_3 - p_{43}| + |q_3 - q_{43}| + |r_3 - r_{43}| + |p_4 - p_{44}| + |q_4 - q_{44}| + |r_4 - r_{44}| \\
&\quad + |p_5 - p_{45}| + |q_5 - q_{45}| + |r_5 - r_{45}|] \\
&= 1 - \frac{1}{15} [0.850 + 0.816 + 0.960 + 0.717 + 0.636] = 0.7347, \\
\text{scor}(A, A_5) &= 1 - \frac{1}{15} [|p_1 - p_{51}| + |q_1 - q_{51}| + |r_1 - r_{51}| + |p_2 - p_{52}| + |q_2 - q_{52}| + |r_2 - r_{52}| \\
&\quad + |p_3 - p_{53}| + |q_3 - q_{53}| + |r_3 - r_{53}| + |p_4 - p_{54}| + |q_4 - q_{54}| + |r_4 - r_{54}| \\
&\quad + |p_5 - p_{55}| + |q_5 - q_{55}| + |r_5 - r_{55}|] \\
&= 1 - \frac{1}{15} [0.883 + 0.815 + 0.947 + 0.680 + 0.631] = 0.7363.
\end{aligned} \tag{32}$$

a Therefore, $\text{scor}(A, A_1) > \text{scor}(A, A_5) > \text{scor}(A, A_3) > \text{scor}(A, A_2) > \text{scor}(A, A_4)$. The rank of the attributes are $A_1 > A_5 > A_3 > A_2 > A_4$.

Thus, the patient A can be diagnosed with the diseases A_1 (viral fever) according to the recognition principle. The ranking is the same as J.Ye [2011]. The above example indicates that this type of decision-making algorithm is well suitable for picture fuzzy environment and is a useful technique that provides a different respective than others for picture fuzzy environment.

6. Conclusion and Future Directions

Graph theory is a needful tool for solving MADMP in different areas. PFG is a new dimension of graph theory which is a useful tool for solving real-world problems. Most of MADM algorithms with picture fuzzy environment discuss a type of problem with no relationship among attributes. Although this relationship should be considered in the actual applications, so we need to pay attention to that issue. This article applies graph theory to PFS and obtained a new method for solving complicated problems under picture fuzzy information. The proposed method can capture the relationship among the attributes that cannot be handled well by any available methods. In this study, we introduce union, intersection, sum, Cartesian product, and the composition of PFG. Finally, by considering the importance of relationships among attributes in the decision process, two new techniques based on single-valued PFG were developed to solve complicated problems using picture fuzzy information. Also, two numerical examples were presented to explain how to deal with the MADMP under a picture fuzzy environment. In the future, we can solve this type of MADM problem using soft sets, picture fuzzy hesitant fuzzy sets, and spherical and T-spherical fuzzy sets.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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