Research Article

An Improved Prediction Method of Soil-Water Characteristic Curve by Geometrical Derivation and Empirical Equation

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Much attention has been paid on the soil-water characteristic curve (SWCC) during decades because it plays great roles in unsaturated soil mechanics. However, it is time-consuming and costly to obtain a series of entire saturation-suction data by experiments. The curves acquired by directly fitting empirical equations to limited experimental data are greatly different from the actual SWCC, and the relevant soil parameters obtained by inaccurate curve are also incorrect. Thus, an improved prediction method for more accurate entire SWCC was established. This novel method was based on the analysis of shape characteristics of SWCC with three critical points S , C_1 , and C_2 under the hypothesis of geometrical symmetric relation. The theoretical computation was specifically deduced under conventional Gardner, VG, and FX models, respectively, and then inferred on different soil types of 45 collected SWCC datasets. This geometrical symmetric relation exhibited well in all these three conventional empirical equations, especially in Gardner equation. Finally, a series of filter paper tests on sand, silt, and clay were also carried out to acquire entire SWCC curve for the verification and evaluation of the proposed geometrical method. Results show that this improved prediction method effectively decreases deviation resulting from directly fitting empirical equations to limited data of wide types of soils. The averaged improvement was larger under VG equation than under Gardner and FX equation. It proved that the accuracy of predicting greatly depends on the shape characteristic point of maximum curve curvature (point C_2), other than the number of points. This research provides a novel computation method to improve prediction accuracy even under relative less experimental data.

1. Introduction

Most of the sedimentary soils widely distributed on the surface belong to unsaturated soils. With the acceleration of human development, more and more engineering practices are related to unsaturated soils, such as the rainfall-induced slope failures, construction about the road and railway subgrades [1], and even the realization of “sponge cities” [2]. Since the soil-water characteristic curve (SWCC) is used as the basis for the prediction of other unsaturated soil parameters, such as the permeability [3, 4], shear-strength functions [5], and diffusion coefficient [6], consequently, it is important to have a reasonably accurate characterization of soil-water characteristic curve. And a great number of

research studies have focused on the problem about the acquisition of SWCC.

There are two common methods to obtain an entire SWCC experimental dataset. The first method is to directly measure suction through complex and time-consuming experiments. Actually, the pressure plate method is commonly used in the suction measurement of unsaturated soils [7], but the maximum measurement range of this method only can reach 2000 kPa [8], which is not enough for the soil that has a great quantity of fine particles, especially for clay. Other conventional experimental methods such as Tempe cell, solution method [9], filter paper method, and axis translation method are tedious and have narrow measurement range. Some monitors could also be used to

simultaneously measure soil suction in the model test. Recently, some of newer equipment for determination of SWCC comprise HYPROP, small-scale centrifuge, and dew-point chilled mirror [10, 11]. However, each technique has its own limitation in terms of suction. Sometimes, multiple measurement methods are used in combination to acquire an entire SWCC. Consequently, the excessive costs associated with direct measurement of SWCC have encouraged the pursuit of a great quantities of indirect methods to acquiring SWCC based on some empirical relationship and statistical regularity of known soil parameters or limited experimental data, including pedotransfer functions [12], physical-empirical equation [13, 14], fractal equation [15, 16], and empirical formula method [17]. In order to get more accurate properties of SWCC, these methods are usually combined with each other [18, 19]. Additionally, the machine learning or intelligent algorithms are also applied with these methods [20, 21].

Besides, considering the relation between the grain-size distribution and SWCC [22–24], some equations can be used to estimate unimodal SWCC with unimodal grain-size distribution [11, 25]. Pore size distribution function (PSDF) and volumetric shrinkage curve (VSC) also play an important role in estimating SWCC. Fredlund and Xing [26] proposed the SWCC equation by integrating the assumed pore size distribution function. The PSDF also had a significant effect on the saturated coefficient of permeability for sandy soils. The volumetric shrinkage curve (VSC) is commonly used to convert the SWCC in the form of gravimetric water content (w -SWCC) into a curve that is in the form of degree of saturation (S -SWCC) [27]. Lin and Cerato [28] showed that the VSC bears a relationship to the soil-water characteristic curve during drying from a near-saturated condition. Zhai et al. [29] put forward a framework used to measured w -SWCCs, and the VSC of soil in a loose condition is used as the input information for the estimation of the w -SWCCs of soil under different void ratios.

Because of its wide application and the simplicity of the calculation process, the empirical equation method has become the most common method for estimating SWCC. The common empirical equations that have been proposed include Gardner equation [30], van Genuchten equation (VG equation) [17], Fredlund and Xing equation (FX equation) [31], and Brooks–Corey equation [32]. Recently, some new equations were put forward. Considering the existence of bimodal SWCC and the limitation of conventional equations, Wijaya and Leong [33] proposed a new equation for both unimodal and bimodal SWCCs. However, there is an evident problem that final fitting curves would greatly deviate from the actual SWCC when directly fitting empirical equation to limited data acquired by limitation of suction measurement range of the frequently used suction measure method (Figure 1). Siller and Fredlund [35] stated that the lowest measured water contents and soil suctions appear to influence the magnitude of the residual water content. The fitting routine tends to overestimate the residual water content if the experimental data do not extend well into the zone of desaturation. Consequently, the important values of unsaturated soil predicted by the incorrect SWCC also greatly differ from

FIGURE 1: A typical SWCC and an incorrect SWCC obtained by directly fitting to Gardner equation (data from [34]).

actual values. Moreover, the related design and evaluation results based on the inaccurate values will be unsafe.

However, there are few scholars currently studying the above problems. Wang et al. [36] combined statistical algorithms to predict entire SWCC from the Bayesian perspective, nevertheless, that approach is relatively complicated. Ren et al. [37] proposed a method for estimating SWCC with limited data, but they did not quantitatively testify the feasibility of that method.

This paper tried to acquiring more accurate SWCC from the perspective of geometry. Observing Figure 1, there seems to be a symmetric relation between the point S which is the largest absolute value of slope and two points C_1 and C_2 which are two extreme points of curvature on SWCC obtained by Gardner equation in semilogarithmic coordinate. Therefore, in the subsequent part of this paper, the geometrical relation between three points would be discussed, and a geometrical method based on this critical relation for preprocessing limited data before directly fitting with empirical equation was put forward.

2. Methodology

2.1. The Geometrical Relationship of S , C_1 , and C_2 of SWCC under Gardner Equation and Validation. The special geometrical relation is found on the SWCC fitting by Gardner equation; thus, the characteristics of Gardner equation need to be studied. Gardner used a function of three parameters (equation (1)) to describe the soil-water characteristic curve, and it is one of the simplest empirical equations.

$$\theta = \theta_r + \frac{\theta_s - \theta_r}{1 + (\psi/a)^b}, \quad (1)$$

where θ is the volumetric water content; θ_s is the saturated water content; θ_r is the residual water content as the ratio of the volumetric water gradient to suction approaches zero; a is the fitting parameter related to the air-entry value of value (bubbling pressure); b is the parameter that has relation with desorption rate when the matric suction is bigger than the air-entry value of value; and ψ is the suction of soil, kPa.

Considering the possible geometrical symmetry of three points S , C_1 , and C_2 which have specific mathematical significance on SWCC in semilogarithmic coordinate, theoretical derivation and checking calculation of substantial datasets were done to prove whether this relation exists.

Firstly, a great number of SWCC datasets of different type of soils (sand, silt, and clay) have been collected to do numerical calculation in order to qualitatively prove that assumption is supported by a quantity of practical experimental data. Every parameter in equation (1) and the suction value of three points are displayed in Table 1, and the value of α and β (equations (2) and (3)) is used to judge the geometrical relation between three points (Table 2). The calculation results of 45 datasets show that ψ_s is just equal with a , and the values of α and β in all datasets are 1, which means

the symmetric relation between the point S and two points C_1 and C_2 may exist.

$$\alpha = \frac{\theta(\psi_s) - \theta(\psi_{C_2})}{\theta(\psi_{C_1}) - \theta(\psi_s)}, \quad (2)$$

$$\beta = \frac{\log(\psi_{C_2}) - \log(\psi_s)}{\log(\psi_s) - \log(\psi_{C_1})}. \quad (3)$$

Although the possible relation between three points have been testified by actual data, a series of mathematical derivations also are needed in order to completely and truly prove that geometrical symmetric relation. Thus, the curves of absolute value of slope in semilogarithmic coordinate can be acquired by the following equation:

$$\begin{aligned} |\dot{\theta}(\psi)_{\log}| &= |\dot{\theta}(\psi) \cdot \ln(10) \cdot \psi| \\ &= \frac{b(\theta_s - \theta_r)(\psi/a)^{b-1}}{a(1 + (\psi/a)^b)^2} \cdot \ln(10) \cdot \psi, \end{aligned} \quad (4)$$

and the point $S(\psi_s, \theta(\psi_s))$ is the extreme point of the curves of absolute value of slope in semilogarithmic that can be derived when the second derivation of $\theta(\psi)$ (equation (5)) is zero.

$$\ddot{\theta}(\psi)_{\log} = \psi \cdot (\ln(10))^2 \cdot (\ddot{\theta}(\psi) \cdot \psi + \dot{\theta}(\psi)) = -\frac{b^2(\theta_s - \theta_r)(\psi/a)^b(-1 + (\psi/a)^b)(\ln(10))^2}{(1 + (\psi/a)^b)^3}. \quad (5)$$

It is obvious that $\ddot{\theta}(\psi)_{\log}$ achieves zero when $\psi = a$, so the location of point $S(\psi_s, \theta(\psi_s))$ can be determined as $(a, \theta_s + \theta_r/2)$. Then, the expression of curvature of SWCC in

semilogarithmic coordinate can also be defined as the following equation:

$$k(\psi) = \frac{\ddot{\theta}(\psi)_{\log}}{(1 + \dot{\theta}(\psi)_{\log}^2)^{1.5}} = \frac{b^2(\theta_s - \theta_r)(\psi/a)^b(-1 + (\psi/a)^b)(\ln(10))^2(1 + (\psi/a)^b)^3}{\left((1 + (\psi/a)^b)^4 + b^2(\theta_s - \theta_r)^2(\psi/a)^{2b}(\ln(10))^2\right)^{1.5}}. \quad (6)$$

The points C_1 and C_2 are two extreme points of curvature on SWCC in semilogarithmic coordinate; consequently, the points meet the conditions that $\dot{k}(\psi_{C_1})_{\log} = 0$ or $\dot{k}(\psi_{C_2})_{\log} = 0$. Although it is difficult to find all the analytic solution of $\dot{k}(\psi)_{\log} = 0$, two solutions $\psi_{C_1} = (2 - \sqrt{3})^{1/b} a \cdot t$ and $\psi_{C_2} = (2 + \sqrt{3})^{1/b} a \cdot 1/t$ can be found out, where t is a complicate function about b , θ_s , and θ_r . Thus, the points C_1 and C_2 can be determined as $C_1((2 - \sqrt{3})^{1/b} a \cdot t, \theta_s + (\theta_s - \theta_r)/(1 + (2 - \sqrt{3}) \cdot t^b))$ and $C_2((2 + \sqrt{3})^{1/b} a \cdot 1/t, \theta_r + (\theta_s - \theta_r)/(1 + (2 + \sqrt{3}) \cdot t^{-b}))$. Then, the symmetric relation between the point S and points C_1 and C_2 in semilogarithmic coordinate can be verified by equations (7) and (8). Plugged the specific

coordinates of point S and points C_1 and C_2 acquired above into equations (7) and (8), and it was found that those equalities held.

θ_r of FX equation can be acquired by the equation $\theta = \theta_r + (\theta_s - \theta_r)/(\{\ln[e + (\psi/a)^b]\}^c)$ [56].

$$\log \psi_{C_1} + \log \psi_{C_2} = 2 \log \psi_s \Rightarrow \psi_{C_1} \cdot \psi_{C_2} = \psi_s^2, \quad (7)$$

$$\theta(\psi_{C_1}) + \theta(\psi_{C_2}) = 2\theta(\psi_s). \quad (8)$$

Through the above deduction, the symmetric relation between three points can be theoretically proved. In conclusion, the symmetric relation between point C_2 and points C_1

TABLE 1: The parameters of 45 SWCC datasets fitting to Gardner equation.

Type	Data resource	θ_s	Gardner equation			VG equation			FX equation					$\bar{\theta}_r$	θ_r^s
		θ_r	a	b	θ_r	a	b	c	θ_r	ψ_r	a	b	c		
Sand	[38]	0.2106	0.0339	7.50	3.14	0.0360	4.51	100.56	0.01	0.0347	45.52	5.09	12.82	0.40	0.0349
	[38]	0.4246	0.0153	1.11	6.79	0.0142	1.00	8.18	0.59	0.0110	1.21	1.10	32.79	0.80	0.0135
	[26]	0.3884	0.0735	22.14	4.12	0.1044	58.69	3.37	21.24	0.1043	30.72	18.64	5.12	0.95	0.0941
	[39]	0.3464	0.0655	2.86	3.02	0.0559	1.64	196.12	0.01	0.0607	6.62×10^7	1.92	5.48	0.65	0.0607
	[40]	0.3263	0.0802	23.71	4.47	0.0678	20.26	5.20	0.56	0.0340	77.34	18.96	5.91	0.74	0.0607
	[40]	0.2656	0.0862	1.71	4.35	0.0568	1.23	6.50	0.26	0.0656	6.67×10^{10}	1.29	5.46	0.62	0.0695
	[41]	0.3418	0.0860	2.17	7.86	0.0741	1.68	282.43	0.01	0.0901	7.31×10^{10}	1.80	20.32	0.44	0.0834
	[42]	0.2959	0.0854	0.51	0.43	0.0740	0.03	17.09	0.01	0.0756	3.02	0.07	32.04	0.12	0.0783
	[20]	0.2499	0.1109	5.88	4.52	0.0829	3.76	35.21	0.04	0.0987	23.37	4.35	10.78	0.29	0.0975
	[20]	0.3448	0.1004	4.30	1.77	0.0842	1.71	26.44	0.03	0.0623	7.68×10^{10}	2.16	4.25	0.46	0.0823
	[43]	0.4527	0.0263	2.51	1.14	0.0193	1.40	25.11	0.03	0.0123	7.27×10^7	1.54	1.69	1.28	0.0193
	[44]	0.3523	0.0437	180.15	2.17	0.0406	76.67	115.40	0.01	0.0367	104.89	109.29	10.40	0.33	0.0403
	[44]	0.3598	0.0447	183.69	2.33	0.0377	104.53	3.99	0.30	0.0110	88.64	120.65	6.06	0.53	0.0311
	[45]	0.1838	0.0192	2.96	1.03	0.0146	0.86	69.99	0.01	0.0172	6.42	1.30	4.31	0.55	0.0170
	[45]	0.2444	0.0356	3.64	1.55	0.0283	1.48	37.79	0.02	0.0219	17.68	3.07	15.09	0.41	0.0286
	[34]	0.4028	0.0377	1769.98	0.91	0.0404	2.75×10^4	0.84	9.86	0.0362	333.00	5152.19	0.90	4.27	0.0381
	[34]	0.4078	0.0391	1280.15	1.22	0.0694	1.32×10^7	0.98	6488.41	0.0694	165.45	867.80	2.35	0.65	0.0593
	[7]	0.4226	0.0769	742.25	1.94	0.0754	541.70	2.21	0.64	0.0710	259.60	611.82	99.27	0.14	0.0744
	[7]	0.3841	0.0340	1993.11	0.64	0.0928	2.32×10^8	0.60	1034.88	0.0461	5.27×10^5	250.67	0.86	0.97	0.0576
	[38]	0.3300	0.0156	3927.64	0.63	0.0382	9117.25	0.59	1.52	0.0196	1415.78	1329.20	0.57	1.10	0.0245
	[38]	0.3041	0.0017	6104.89	0.50	0.0127	101.35	0.96	0.07	0.0076	47.28	1.13×10^4	7.62	0.13	0.0073
	[46]	0.4804	0.1186	6.08	2.82	0.1218	7.78	2.50	1.61	0.1198	0.74	4.82	7.90	0.39	0.1201
Silt	[47]	0.4556	0.0341	514.29	0.65	0.0788	53.34	1.51	0.38	0.0511	8532.04	36.42	2.02	0.50	0.0547
	[47]	0.4417	0.0141	877.77	0.57	0.0207	33.60	1.58	0.20	0.0179	1823.62	33.72	1.79	0.45	0.0176
	[48]	0.3794	0.0361	70.11	0.91	0.0397	61.19	0.95	0.94	0.0387	213.49	29.23	1.01	1.18	0.0382
	[44]	0.3220	0.0325	513.71	0.75	0.0397	847.53	0.72	1.35	0.0252	14.16	317.82	1.99	0.39	0.0325
	[45]s	0.3456	0.0456	63.62	1.27	0.0165	21.19	2.05	0.28	0.0681	191.38	28.54	1.64	0.92	0.0434
	[45]s	0.4371	0.0923	23.36	1.22	0.0964	44.96	1.10	1.70	0.0949	1.64	28.09	62.51	0.16	0.0945
	[49]	0.3991	0.1160	21.97	3.11	0.1992	12.96	161.64	0.01	0.1136	2146.41	14.03	8.07	0.23	0.1429
	[50]	0.4368	0.1273	25.97	1.16	0.1339	5.62	1.83	0.15	0.1336	1.81×10^8	7.45	1.53	0.35	0.1316
	[42]	0.4956	0.0498	302.63	0.96	0.0532	281.19	0.99	0.97	0.0665	37.96	183.24	1.25	0.85	0.0565
	[42]	0.4364	0.0662	945.31	0.52	0.0445	2741.66	0.52	2.22	0.0796	4.08	641.51	1.73	0.97	0.0634
	[51]	0.6945	0.0102	2477.66	0.62	0.0039	4397.95	0.59	1.32	0.0080	33.37	3141.60	1.50	0.61	0.0074

TABLE 1: Continued.

Type	Data resource	θ_s	Gardner equation			VG equation			FX equation			$\bar{\theta}_r$	c	θ_r^s
		θ_r	a	b	θ_r	a	b	c	θ_r	ψ_r	a	b	c	
Clay	[7]	0.6018	0.0343	2363.76	0.34	0.0373	4.80×10^5	0.39	0.0338	1.72×10^5	122.36	1.05	0.48	0.079245
	[7]	0.4482	0.0737	7430.10	0.67	0.0647	2858.87	0.77	0.0835	235.38	3.83×10^5	2.02	-2.62	0.0740
	[14]	0.5204	0.0108	2961.29	0.88	0.0132	3007.03	0.89	0.0143	297.14	2599.03	1.12	1.07	0.0128
	[43]	0.4913	0.2434	66.27	0.62	0.2762	2.14×10^6	0.53	0.2926	2.68	1017.68	135.80	-0.03	0.2707
	[44]	0.4405	0.1567	256.59	1.01	0.0854	34.76	69.13	0.1480	4702.93	53.19	2.83	0.30	0.1300
	[44]	0.5530	0.1594	432.81	0.72	0.1179	153.42	0.83	0.1725	16.30	226.37	17.84	0.05	0.1499
	[52]	0.4600	0.1647	11.42	1.56	0.0989	4.62	2.30	0.1209	96.73	5.19	1.95	0.56	0.1282
	[53]	0.3781	0.0915	2916.26	1.31	0.0432	981.45	1.18	0.0616	5925.74	1024.26	2.71	0.40	0.0654
	[53]	0.4888	0.1196	2569.65	1.32	0.0841	758.27	3.39	0.0728	3787.67	866.21	3.43	0.33	0.0922
	[54]	0.9031	0.0620	3535.16	0.73	0.0537	1463.29	0.87	0.0613	7600.92	492.90	1.10	0.65	0.0590
	[54]	0.6467	0.0294	8007.59	0.74	0.0288	4786.72	0.81	0.0249	1.91×10^4	1268.51	0.94	0.76	0.0277
	[55]	0.4702	0.0159	1757.23	0.77	0.0171	1909.35	0.76	0.0161	60.42	1264.73	1.66	0.57	0.0164

TABLE 2: The ψ of point S, C_1 , and C_2 of 45 SWCC datasets fitting to Gardner equation, VG equation, and FX equation.

Type	Data resource	Gardner equation					VG equation					FX equation				
		ψ_S	ψ_{C_1}	ψ_{C_2}	α	β	ψ_S	ψ_{C_1}	ψ_{C_2}	α	β	ψ_S	ψ_{C_1}	ψ_{C_2}	α	β
Sand	[38]	7.50	4.81	11.69	1.00	1.00	4.70	4.50	4.95	1.34	1.17	5.62	4.86	6.44	1.01	0.94
	[38]	1.11	0.80	1.54	1.00	1.00	1.07	0.79	1.50	1.15	1.16	1.13	0.96	1.32	1.30	0.92
	[26]	22.14	14.82	33.09	1.00	1.00	23.69	15.96	37.28	0.92	1.15	21.66	15.26	30.50	1.02	0.98
	[39]	2.86	1.75	4.68	1.00	1.00	2.86	2.04	5.08	0.93	1.70	2.32	1.68	3.16	1.01	0.95
	[40]	23.71	16.67	33.74	1.00	1.00	22.66	16.09	32.42	1.07	1.05	22.26	16.50	29.69	1.01	0.96
	[40]	1.71	1.22	2.39	1.00	1.00	1.51	1.10	2.10	1.11	1.05	1.57	1.15	2.09	0.99	0.94
	[41]	2.17	1.69	2.78	1.00	1.00	1.91	1.67	2.31	0.98	1.47	1.91	1.68	2.20	1.23	1.15
	[42]	0.51	0.02	10.92	1.00	1.00	0.49	0.07	6.82	0.83	1.39	0.08	0.07	0.08	1.04	0.94
	[20]	5.88	4.30	8.03	1.00	1.00	4.73	3.74	6.61	1.10	1.42	4.95	4.18	5.77	0.98	0.92
	[20]	4.30	1.99	9.28	1.00	1.00	3.95	1.69	8.89	0.52	0.96	3.87	1.93	9.14	0.71	1.23
	[43]	2.51	0.75	8.35	1.00	1.00	2.06	0.92	6.26	1.77	1.38	2.18	0.92	4.97	0.98	0.95
	[44]	180.15	94.04	345.12	1.00	1.00	104.56	80.12	140.33	0.89	1.11	124.03	103.02	147.96	1.02	0.95
	[44]	183.69	98.81	341.48	1.00	1.00	140.36	85.91	234.85	1.11	1.05	144.38	107.77	191.00	1.01	0.96
	[45]	2.96	0.82	10.71	1.00	1.00	2.91	1.23	10.96	0.82	1.54	1.68	1.06	3.06	1.29	1.30
	[45]	3.64	1.53	8.65	1.00	1.00	3.61	1.88	8.76	0.75	1.36	3.33	2.92	3.80	1.05	0.98
	[34]	1769.98	400.89	7814.73	1.00	1.00	1786.54	511.89	6112.25	0.93	0.98	1937.77	419.46	7910.79	0.90	0.92
	[34]	1280.15	416.53	3934.36	1.00	1.00	1707.71	606.61	4685.02	0.91	0.98	1204.32	624.03	2315.77	1.01	0.99
	[7]	742.25	356.39	1545.85	1.00	1.00	679.01	333.14	1396.41	1.00	1.01	669.57	469.32	1209.23	0.74	1.66
	[7]	1993.11	249.91	1.59×10^4	1.00	1.00	2318.54	458.22	1.16×10^4	0.93	0.99	632.86	115.83	2923.60	0.94	0.90
	[38]	3927.64	470.87	3.28×10^4	1.00	1.00	4914.27	619.07	3.99×10^4	0.95	1.00	3532.32	500.16	2.55×10^4	1.03	1.01
	[38]	6104.89	431.30	8.64×10^4	1.00	1.00	6651.27	482.94	8.31×10^4	0.88	0.96	6488.40	472.30	8.47×10^4	1.31	0.98
	[46]	6.08	3.52	10.52	1.00	1.00	6.44	3.64	11.19	0.96	0.97	5.63	4.40	7.16	1.04	0.98
Silt	[47]	514.29	65.78	4020.82	1.00	1.00	101.68	32.10	325.64	1.05	1.01	65.13	29.16	135.12	0.96	0.91
	[47]	877.77	84.28	9141.81	1.00	1.00	93.37	24.84	355.60	1.08	1.01	149.07	26.90	3686.97	1.02	1.87
	[48]	70.11	16.15	304.37	1.00	1.00	65.53	15.57	275.97	1.00	1.00	57.10	13.74	219.80	0.98	0.95
	[44]	513.71	87.46	3017.35	1.00	1.00	549.34	96.48	3123.20	1.00	1.00	501.14	103.32	3107.87	1.13	1.16
	[45]	63.62	21.83	185.38	1.00	1.00	39.56	15.63	101.32	1.07	1.01	47.09	18.84	109.72	0.96	0.92
	[45]	23.36	7.65	71.37	1.00	1.00	27.71	9.01	84.62	0.97	0.99	28.78	27.44	30.67	1.44	1.33
	[49]	21.97	19.55	24.70	1.00	1.00	13.30	12.94	13.76	1.41	1.20	16.82	13.38	20.72	0.98	0.91
	[50]	25.97	8.25	81.73	1.00	1.00	15.83	4.58	55.08	1.09	1.01	17.74	6.02	47.15	0.96	0.90
	[42]	302.63	73.52	1245.69	1.00	1.00	291.44	72.39	1174.08	1.00	1.00	279.61	77.88	911.76	0.95	0.92
	[42]	945.32	73.25	1.22×10^4	1.00	1.00	596.69	65.22	5426.52	0.97	1.00	808.67	370.40	1918.41	1.08	1.11
	[51]	2477.66	270.50	2.27×10^4	1.00	1.00	3137.80	189.46	2.80×10^4	0.79	0.78	2012.34	135.66	2.16×10^4	1.20	0.88
Clay	[7]	2363.75	50.31	1.11×10^5	1.00	1.00	1.61×10^4	943.47	2.69×10^5	0.96	0.99	2385.66	73.03	1.10×10^5	0.78	1.10
	[7]	7430.10	1014.25	5.44×10^4	1.00	1.00	5347.42	765.62	3.75×10^4	1.02	1.00	1.40×10^4	232.70	2.53×10^5	0.67	0.71
	[14]	2961.28	629.33	1.39×10^4	1.00	1.00	2945.55	634.51	13669.10	1.00	1.00	3020.23	662.40	1.15×10^4	0.90	0.88
	[43]	66.27	7.92	554.33	1.00	1.00	91.27	14.69	563.92	0.94	1.00	64.34	6.96	527.20	1.07	0.95
	[44]	256.59	67.82	970.84	1.00	1.00	237.60	64.70	940.80	0.72	1.06	148.00	47.51	908.48	0.87	1.60
	[44]	432.81	67.72	2766.34	1.00	1.00	233.85	25.60	2179.71	1.26	1.01	248.25	223.76	272.49	0.94	0.90
	[52]	11.42	4.74	27.53	1.00	1.00	10.64	4.64	25.77	0.94	1.07	9.26	4.03	19.43	0.94	0.89
	[53]	2916.26	1035.33	8214.35	1.00	1.00	2694.63	967.33	7548.71	0.88	1.01	2651.25	978.06	7808.75	0.75	1.08
	[53]	2569.65	903.52	7308.18	1.00	1.00	2281.71	871.06	7791.75	0.84	1.28	2284.17	865.65	7995.74	0.75	1.29
	[54]	3535.16	513.19	2.44×10^4	1.00	1.00	2560.38	419.47	1.60×10^4	1.03	1.01	1348.69	297.34	4066.30	0.83	0.73
	[54]	8007.59	1254.26	5.11×10^4	1.00	1.00	6715.74	1105.24	4.11×10^4	1.01	1.00	4612.57	733.92	4.04×10^4	1.23	1.18
	[55]	1757.23	304.32	1.01×10^4	1.00	1.00	1877.85	325.12	1.08×10^4	0.98	1.00	1836.05	349.76	1.18×10^4	1.06	1.12

and S in semilogarithmic coordinate can be quantitatively and qualitatively proved based on numerical checking of 45 SWCC experimental datasets and mathematical derivations. Consequently, the specific position of point C_2 can be acquired when the points C_1 and S are known by making use of this relation.

2.2. The Geometrical Relationship of Points S, C_1 , and C_2 of SWCC under VG Equation and FX Equation. Besides Gardner equation, VG equation put forward by van Genuchten (equation (9)) and FX equation (equation (10)) raised by Fredlund are also the popular empirical equations used to acquire SWCC. Considering the VG equation is one of the most frequently used empirical equations and the FX

equation can give the best fit among the equations [56], they are selected to testify whether this graphic symmetric relationship exists in other equations.

$$\theta = \theta_r + \frac{\theta_s - \theta_r}{[1 + (\psi/a)^b]^{c_s}} \quad (9)$$

$$\theta = \left[1 - \frac{\ln(1 + \psi/\psi_r)}{\ln(1 + 1000000/\psi_r)} \right] \frac{\theta_s}{\{\ln[e + (\psi/a)^b]\}^{c_s}} \quad (10)$$

$$C(\psi) = 1 - \frac{\ln(1 + \psi/\psi_r)}{\ln(1 + 1000000/\psi_r)} \quad (11)$$

The same calculations were done to 45 selected experimental datasets in order to find three special points S , C_1 , and C_2 whether have the same relation on the curve fitting to the VG equation and FX equation. ψ of points S , C_1 , and C_2 is shown in Table 2, and the value of α and β (equations (2) and (3)) is also used to judge the geometrical symmetric relation of three points. It is found that α and β of Gardner equation and FX equation are not exactly equal to 1, but they are still very close to 1. In consideration of the fact that these empirical equations are all used to fit with the same dataset of discrete points, despite their slightly different forms, the curves acquired are roughly same. Thus, the positions of points S , C_1 , and C_2 on the curves obtained by VG and FX equations are similar to those on the curve obtained by Gardner equation. In conclusion, the graphic symmetric relation between three special points S , C_1 , and C_2 can also be considered generally valid on the curves fitting by VG equation and FX equation.

2.3. The Physical Meaning of Points S , C_1 , and C_2 . Although the mathematical significance and geometrical symmetric relation of three points S , C_1 , and C_2 have been proved by theoretical derivation and checking calculation of substantial datasets, the specific physical meaning of these points is also needed to be discussed. Some researchers have already studied SWCC from the geometrical perspective. Siller and Fredlund [35] used two points of extreme values of curvature to divide SWCC into three zones (capillary saturation zone, funicular zone, and zone of residual saturation). Fredlund and Xing [26] put forward a geometric solution to determine parameters a , b , and c in equation (12) by drawing a tangent line through point S and, respectively, intersecting the approximate straight lines of low-suction part (capillary saturation zone) and high suction part (zone of residual saturation) of SWCC. They considered that two points S_1 and S_2 related with the air-entry value ψ_a (the matric suction where air starts to enter the largest pores in the soil) and the residual water content θ_r are the intersection points. However, the points C_1 and C_2 put forward in this paper are on the SWCC which are the extreme values of curvature. The points S_1 and S_2 are the intersection points of tangent line of point S and lines of low-suction part which are out of SWCC. In view of graph, they are all around the curvy part of SWCC and close to each other (Figure 2). Additionally, the points C_1 and C_2 are on the SWCC, so they may have stronger physical connection with SWCC than points S_1 and S_2 , especially the change of curvature. Therefore, points C_1 and C_2 also have relationship with the air-entry value ψ_a and the residual water content θ_r . The air-entry value ψ_a and the residual water content θ_r are closely associated with the mineral composition, pore structure, and dense state; among them, the mineral composition and pore structure are the basic factors. Moreover, these two points are crucial for determining the position and shape of SWCC because of their important mathematical meaning. In other words, SWCC is a typical sigmoid curve; once the locations of mutation point of curvature are confirmed, the rough shape and position of SWCC can be obtained.

The point S is the inflection point of SWCC (the point is largest absolute value of slope), from the perspective of pore size distribution; it corresponds to the point with the maximum value of frequency in the pore size distribution curve [57]. Because of the log-normal distribution of pore size of most soils, the maximum value of frequency in the pore size distribution curve can be considered as the mean pore size of the soil sample [37]. Besides this, Ren and Santamaina [58] and Ren et al. [59] mentioned that the mean pore size can be expressed by void ratio and specific surface area. Therefore, the position of point S , void ratio, and specific surface area may have some inner link.

2.4. The Significance of Point C_2 . When the empirical equations are used to fitting available datasets, it is significant to have an effective and precise standard to judge whether datasets can generate accurate outcomes. Leong and Rahardjo [56] mentioned that the data used to obtain SWCC should include points after θ_r , which is not directly available value before fitting. Fredlund and Xing [26] considered that the magnitude of ψ_r will generally be in the range of 1500 to 3000 kPa but not an exact value of ψ_r .

According to the above discussion about the physical and geometrical relation between the point C_2 and value of θ_r , this paper desired to quantitatively check whether point C_2 is also important to acquiring accurate SWCC. The SWCC datasets of three types of soils were chosen. Then, the half and quarter points of datasets were used to fit to equation (12), which is the empirical equation of low sensitivity according to the analyses carried out by Leong [56]. Moreover, the points before the point C_2 and the dataset including the point just larger than point C_2 were also been calculated. Because all the empirical equations are the overparameterized functions (the parameters of the functions are not independent), some different fitting parameter combinations result in the same curves for the same dataset. Thus, equation (13) was used to evaluate the difference between the curves obtained by different datasets, where $f(x)$ and $g(x)$ are two different SWCC suction-saturation equation needed to be compared and η is the proximity of the two curves in some extent; the smaller η is, the closer the two curves are. The calculating results of different types of soils are shown in Figure 3 and Table 3.

$$\theta = \frac{\theta_s}{\left\{ \ln \left[e + (\psi/a)^b \right] \right\}^c}, \quad (12)$$

$$\eta = \int_{0.01}^{10^6} |f(x) - g(x)| d(\log x). \quad (13)$$

From Figure 3 and Table 3, it was easy to drive a conclusion that curves acquired by half points and a quarter points were almost coincided with the SWCC obtained by original dataset. However, the difference between the curves acquired by the points before point C_2 and the original SWCC was relatively large. And the gap reduced when the fitting dataset just including the point C_2 . In some occasions, the SWCC obtained by the dataset including point C_2 is almost similar to the original curve. Therefore, a conclusion

FIGURE 2: The difference between the points S_1 and S_2 and points C_1 and C_2 (two examples of SWCC acquired by FX equation): (a) sand (data from [38]); (b) silt (data from [38]).

FIGURE 3: Continued.

FIGURE 3: The results fitting to equation (13) with datasets of different situations (a half data, a quarter data, and the data points before point C_2): (a) sand (data from [38]); (b) silt (data from [38]); (c) clay (data from [53]).

TABLE 3: The results of fitting to equation (13) with datasets of different situations.

Soil type	Points situation	a	b	c	η
Sand [38]	24 points	5.037	7.421	0.551	
	13 points	4.885	6.146	0.586	0.00508
	6 points	4.982	6.612	0.590	0.01032
	Before C_2	5.025	11.552	0.417	0.04922
	Including C_2	5.072	10.799	0.452	0.02913
Silt [38]	20 points	10394.203	0.480	3.354	
	11 points	8089.230	0.508	3.348	0.03059
	6 points	9028.341	0.485	3.393	0.02031
	Before C_2	2095.411	0.547	1.880	0.02828
	Including C_2	7254.044	0.525	2.223	0.01836
Clay [53]	20 points	1097.902	1.756	0.740	
	11 points	1120.315	1.767	0.750	0.00575
	6 points	1149.266	1.747	0.775	0.02988
	Before C_2	878.877	2.653	0.493	0.20576
	Including C_2	903.982	2.048	0.629	0.16672

could be drawn that the accuracy of empirical equations fitting depends on the dataset whether it includes point C_2 other than the number of points. Then, in this paper, the dataset which is of sigmoid shape and at least reaches the maximum curvature (point C_2) can be regarded as a “entire” SWCC dataset.

2.5. A Geometrical Method for Entire SWCC. As what are discussed above, the experimental datasets sometimes miss the points of part of funicular zone and zone of residual saturation because of the limitation of the range of the

measuring instruments. Thus, this paper puts forward a method to predict the position of point C_2 and point S based on the graphic symmetric relationship of points S , C_1 , and C_2 to acquire more accurate SWCC.

Firstly, the first critical point $C_1(\psi_{C_1}, \theta(\psi_{C_1}))$ is the first extreme value of curvature on the SWCC that is obtained by directly fitting empirical equations to limited data. because the upper part of SWCC acquired by fitting to limited dataset is almost coincident with that of actual SWCC. In most cases, the first point of extreme value of curvature ($C_1(\psi_{C_1}, \theta(\psi_{C_1}))$) on these two curves is almost the same. The inflection point S on the SWCC of Gardner equation is

$(a, \theta_s + \theta_r/2)$, and the inflection point S acquired by other empirical equations is very close. Therefore, point $(a, \theta_s + \theta_r/2)$ can be seen as the inflection point S on the SWCC, and θ_s can be easily obtained from the saturated samples. The value of θ_r is closely associated with the soil characteristics, like mineral composition, pore structure, and dense state. In consideration of that, θ_r of different datasets under different empirical equations was calculated (Table 1), and the average values of same dataset under three different empirical equations were gained. Afterwards, these 15 values of different types of soils were also averaged; thus, the statistical values of θ_r of three types of soils were obtained. It was found that these three statistical values of θ_r were in line with the characteristics of different soils that the finer particle size is, the higher values of θ_r is. Consequently, the empirical values

of θ_r of sand, silt, and clay were, respectively, chosen as 0.054082, 0.062445, and 0.079245. The points S , C_1 , and C_2 are almost in a line, considering the coincidence of the upper part of SWCC acquired from limited dataset and that of actual SWCC; therefore, the line $C_1'C_2'$ of the SWCC acquired by limited data almost coincides with the line C_1SC_2 of the actual SWCC. In other words, predicting point S is on the line $C_1'C_2'$. Consequently, $\theta(\psi_s)$ could be derived by the empirical θ_r and known θ_s (equation (14)), and the value of ψ_s could be gained by the line $C_1'C_2'$ and the obtained value of $\theta(\psi_s)$ (equation (15)). Eventually, the point C_2 could be acquired by predicting point S and known point C_1 (equations (16) and (17)). Moreover, point C_2 was added to limited dataset, and the new dataset was fitted to empirical equations to get a more accurate SWCC.

$$\theta(\psi_s) = \frac{(\theta_s + \theta_r)}{2}, \quad (14)$$

$$\log(\psi_s) = \frac{(\theta(\psi_{C_2'}) - \theta(\psi_s))(\log(\psi_{C_2'}) - \log(\psi_{C_1'}))}{(\theta(\psi_{C_1'}) - \theta(\psi_{C_2'}))} + \log(\psi_{C_2'}), \quad (15)$$

$$\log(\psi_{C_2}) = 2 \log(\psi_s) - \log(\psi_{C_1}), \quad (16)$$

$$\theta(\psi_{C_2}) = 2\theta(\psi_s) - \theta(\psi_{C_1}). \quad (17)$$

However, in some cases, the lowest value of θ of limited dataset may be lower than the empirical value of θ_r this paper adopted. Then, $\theta(\psi_s)$ could be chosen as $\theta_s/2$. An example shows procedures and processing result (Figure 4), and diagrammatic sketch about specific procedures of this geometrical predicting method is shown in Figure 5.

3. Materials and Experiment

3.1. Materials. The soil sample used in this test was remolded soil. Sand, silt, and clay are all configured by forging kaolin and standard sand according to the predetermined mass ratio. Two kinds of particle sizes were used (0.045 mm and 0.001 mm). Standard sand is made according to the standard named GB/T17671-1999, and the grain diameter ranges from 0.08 mm to 2 mm. The specific ratio is shown in Table 4, and the grain-size distribution curves of three kinds of sample are shown in Figure 6.

3.2. Filter Paper Method. In the measurement of suction of unsaturated soils, compared with the pressure plate method or other methods, the filter paper method is more time-consuming and cost-saving, and it also has larger measuring range [60]. Therefore, this paper uses the filter paper method

to obtain the entire suction-saturation datasets of sand, silt, and clay.

This method was first proposed by Gardner [61], and since then, many researchers have been conducting further research. Feuerharmel et al. [62] and Agus and Schanz [63] compared and analyzed the filter paper method and other methods and found out that the value obtained by the filter paper method has higher accuracy. Marinho and Oliveira [64] also made a deeper discussion on the operation procedures and control conditions of the filter paper method.

In filter paper experiment, the most important procedure is to determine the calibration curve. Fawcett and Collis [65], Leong and Rahardjo [56], and other scholars, respectively, used the solution method or pressure plate instrument and other methods to calibrate the Whatman No. 42 filter paper calibration curve and the applicable range. The specific calibration equation is given. Scholars in China such as Tang et al. [66], Bai et al. [67], and Wang et al. [68] have measured the calibration curves of domestic double-loop filter papers. The ASTM (American Society for Testing and Materials) has formulated the relevant test equipment and operating procedures of the filter paper method into a relevant standard-ASTM D5298-10, which also specifies the calibration curve of Whatman No. 42 filter paper. This paper used the calibration curve equation (equations (18) and (19))

FIGURE 4: Example of the procedures and result of this method (data of sand sample from [46]).

FIGURE 5: Procedures of the new method to predict SWCC based on the limited datasets.

TABLE 4: Specific ratio of kaolin and sand in different types of soil.

Type	Specific gravity of soil solids	Dry density	Ingredients		
			Sand (%)	Silt (%)	Clay (%)
Sand sample	2.63	1.65	77	15	8
Silt sample	2.66	1.30	40	50	10
Clay sample	2.69	1.20	23	20	57

FIGURE 6: Grain-size distribution curves of sand, silt, and clay.

put forward by Leong to calculate suction, where m_a is the mass of wet filter paper; m_b is the mass of dry filter paper; and w_f is the water content of processed filter paper:

$$w_f = \frac{m_a - m_b}{m_b}, \quad (18)$$

$$\begin{cases} \log \psi = 2.909 - 0.0229w_f (w_f \geq 47), \\ \log \psi = 4.945 - 0.0673w_f (w_f < 47). \end{cases} \quad (19)$$

In addition, considering the hysteresis characteristics of SWCC, Houston et al. [69] and Leong et al. [70] have demonstrated through a large number of experiments that there is no significant difference in matrix suction during the hygroscopic process and dehumidifying process through a large number of experiments. Muñoz et al. [71] proposed that using wet filter paper can eliminate the hysteresis characteristic of SWCC, but the test results show that the results obtained with dry filter paper and wet filter paper are not much different. In summary, this paper uses the dry Whatman No. 42 filter to measure the suction of soil samples under different water content. The sketch of the ring-knife layout and the photos of experimental process are shown in Figure 7. And the results are shown in Table 5.

4. Results and Discussion

4.1. Verification Using the Experimental Results of Filter Paper Experiments. The datasets acquired by filter paper experiment covered entire suction range; thus, the curves they formed could be considered as the actual SWCC. Firstly, a part of the data of filter paper experiment were selected in order to make the curve they formed did not have two points of extreme value of curvature. Therefore, this processed dataset could be regarded as limited experimental data. The proposed method was applied to these limited data and compared quantitatively performance of this method applied to the experimental data by equations (13) and (20). In equation (20), φ is the index parameter that quantifies improvement degree of the proposed method; η_1 is the deviation between the curve obtained by directly fitting limited experimental datasets to empirical equation and actual SWCC; and η_2 is the deviation between the curve obtained by the proposed method and the actual SWCC. The larger value φ is, the greater the improvement the method makes. The specific comparison charts and calculation results charts are shown in Figure 8 and Table 6.

$$\varphi = \frac{\eta_1 - \eta_2}{\eta_1}. \quad (20)$$

FIGURE 7: The sketch of the ring-knife layout and the photos of experimental process. (a) The sketch of ring-knife layout. (b) The Whatman No. 42 filter. (c) The wrapped ring-knife samples under different water content. (d) The precision balance measuring the filter paper.

TABLE 5: Results of the filter paper method.

Water content of filter paper (%)	Sand		Silt			Clay		
	Water content of soil (%)	Suction (kPa)	Water content of filter paper (%)	Water content of soil (%)	Suction (kPa)	Water content of filter paper (%)	Water content of soil (%)	Suction (kPa)
11.851	0.005	14042.388	23.917	0.014	2164.499	8.869	0.005	22288.953
23.674	0.009	2247.753	24.988	0.014	1833.610	12.623	0.009	12458.153
28.523	0.018	1060.181	28.558	0.030	1054.535	24.301	0.021	2039.631
38.268	0.035	234.182	35.968	0.058	334.448	30.290	0.043	806.218
46.567	0.052	64.722	41.279	0.089	146.865	35.422	0.085	363.979
58.214	0.070	37.661	46.860	0.118	61.851	40.870	0.128	156.466
74.665	0.088	15.818	52.807	0.147	50.087	48.520	0.172	62.789
87.594	0.106	8.000	58.404	0.177	37.287	55.947	0.215	42.443
101.226	0.115	3.899	66.698	0.206	24.078	63.047	0.258	29.190
116.186	0.128	1.771	74.722	0.239	15.771	72.785	0.300	17.467
126.357	0.142	1.036	75.993	0.254	14.749	76.154	0.322	14.624
142.341	0.150	0.446	86.114	0.269	8.650	99.900	0.343	4.181
155.374	0.160	0.224	135.265	0.280	0.648	187.996	0.427	0.040
			145.300	0.295	0.382			

From Table 6 and Figure 7, it is found out that the improvement this method made on three empirical equations ranges from 0.948 to 0.459; in other words, the curves

gained from the method are much closer to the actual SWCC than the curves obtained by directly fitting limited data to empirical equations. Thus, this method performs well when

- Complete experimental data
- Limited experimental data
- SWCC fitting with complete data
- - - SWCC fitting with limited data
- · - · - SWCC fitting with limited data and predicted C_2
- ◇ S on the SWCC fitting with complete data
- ◆ S on the SWCC fitting limited data
- △ C_1 on the SWCC fitting with complete data
- ▲ C_1 on the SWCC fitting with limited data
- C_2 on the SWCC fitting with complete data
- Predicted C_2

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- Limited experimental data
- SWCC fitting with complete data
- - - SWCC fitting with limited data
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- Limited experimental data
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- - - SWCC fitting with limited data
- · - · - SWCC fitting with limited data and predicted C_2
- ◇ S on the SWCC fitting with complete data
- ◆ S on the SWCC fitting limited data
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- ▲ C_1 on the SWCC fitting with limited data
- C_2 on the SWCC fitting with complete data
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- Limited experimental data
- SWCC fitting with complete data
- - - SWCC fitting with limited data
- · - · - SWCC fitting with limited data and predicted C_2
- ◇ S on the SWCC fitting with complete data
- ◆ S on the SWCC fitting limited data
- △ C_1 on the SWCC fitting with complete data
- ▲ C_1 on the SWCC fitting with limited data
- C_2 on the SWCC fitting with complete data
- Predicted C_2

FIGURE 8: Continued.

- Complete experimental data
- Limited experimental data
- SWCC fitting with complete data
- - - SWCC fitting with limited data
- · - · - SWCC fitting with limited data and predicted C_2
- ◇ S on the SWCC fitting with complete data
- ◆ S on the SWCC fitting limited data
- △ C_1 on the SWCC fitting with complete data
- ▲ C_1 on the SWCC fitting with limited data
- C_2 on the SWCC fitting with complete data
- Predicted C_2

(e)

- Complete experimental data
- Limited experimental data
- SWCC fitting with complete data
- - - SWCC fitting with limited data
- · - · - SWCC fitting with limited data and predicted C_2
- ◇ S on the SWCC fitting with complete data
- ◆ S on the SWCC fitting limited data
- △ C_1 on the SWCC fitting with complete data
- ▲ C_1 on the SWCC fitting with limited data
- C_2 on the SWCC fitting with complete data
- Predicted C_2

(f)

- Complete experimental data
- Limited experimental data
- SWCC fitting with complete data
- - - SWCC fitting with limited data
- · - · - SWCC fitting with limited data and predicted C_2
- ◇ S on the SWCC fitting with complete data
- ◆ S on the SWCC fitting limited data
- △ C_1 on the SWCC fitting with complete data
- ▲ C_1 on the SWCC fitting with limited data
- C_2 on the SWCC fitting with complete data
- Predicted C_2

(g)

- Complete experimental data
- Limited experimental data
- SWCC fitting with complete data
- - - SWCC fitting with limited data
- · - · - SWCC fitting with limited data and predicted C_2
- ◇ S on the SWCC fitting with complete data
- ◆ S on the SWCC fitting limited data
- △ C_1 on the SWCC fitting with complete data
- ▲ C_1 on the SWCC fitting with limited data
- C_2 on the SWCC fitting with complete data
- Predicted C_2

(h)

FIGURE 8: Continued.

FIGURE 8: The results of applying the method to data of filter paper experiment (sand, silt, and clay): (a–c) sand (data from filter paper method); (d–f) silt (data from filter paper method); (g–i) clay (data from filter paper method).

TABLE 6: Calculation of effectiveness of the prediction method.

Data resource	VG equation			Gardner equation			FX equation		
	η_1	η_2	φ	η_1	η_2	φ	η_1	η_2	φ
Sand (filter paper)	0.247	0.066	0.735	0.158	0.008	0.948	0.124	0.007	0.943
Silt (filter paper)	0.561	0.210	0.626	0.355	0.183	0.484	0.187	0.037	0.800
Clay (filter paper)	1.012	0.468	0.538	0.942	0.407	0.568	0.354	0.192	0.459
Sand [46]	0.758	0.308	0.593	0.790	0.146	0.815	0.129	0.117	0.097
Silt [47]	0.639	0.124	0.806	0.444	0.297	0.331	0.015	0.189	−11.694
Clay [51]	0.720	0.172	0.761	0.612	0.168	0.725	0.334	0.157	0.529

it is applied to the data from filter paper experiments on three different types of soils. However, the performance of this method on these three datasets is not stable, namely, it does not show any preference of applicability to types of soil. In the perspective of empirical equations, the value of η_1 and η_2 of FX equation is lower than that of VG equation and Gardner equation, which means the curves obtained by FX equation were closer to the actual SWCC. That is because the FX equation has a restriction point (1,000,000, 0) and the deviation resulting from directly fitting FX equations to limited data relatively decreases. When this method is used on FX equation, there two points are added to the limited data, besides point C_2 , so that the final obtained curve is closer to actual SWCC.

4.2. Verification Using the Previous Experimental Data. Considering the limitedness and occasionality of only three experimental datasets, this paper selected and collected the entire previous experimental data according to the standard mentioned above to verify the effectiveness of the proposed method. Similarly, the entire data were selected as above processes, and in the dataset of silt and clay, the points whose suction were lower than 2,000 kPa and were selected in order to simulate limitation of the measurement range. The calculation results and comparison charts are shown in Table 6 and Figure 9.

Comparing with the performance of this method on filter paper experimental data, its application of silt on FX equation was not well. Generally, some improvement of this

- Complete experimental data
- Limited experimental data
- SWCC fitting with complete data
- - - SWCC fitting with limited data
- · - · - SWCC fitting with limited data and predicted C_2
- ◇ S on the SWCC fitting with complete data
- ◆ S on the SWCC fitting limited data
- △ C_1 on the SWCC fitting with complete data
- ▲ C_1 on the SWCC fitting with limited data
- C_2 on the SWCC fitting with complete data
- Predicted C_2

(a)

- Complete experimental data
- Limited experimental data
- SWCC fitting with complete data
- - - SWCC fitting with limited data
- · - · - SWCC fitting with limited data and predicted C_2
- ◇ S on the SWCC fitting with complete data
- ◆ S on the SWCC fitting limited data
- △ C_1 on the SWCC fitting with complete data
- ▲ C_1 on the SWCC fitting with limited data
- C_2 on the SWCC fitting with complete data
- Predicted C_2

(b)

- Complete experimental data
- Limited experimental data
- SWCC fitting with complete data
- - - SWCC fitting with limited data
- · - · - SWCC fitting with limited data and predicted C_2
- ◇ S on the SWCC fitting with complete data
- ◆ S on the SWCC fitting limited data
- △ C_1 on the SWCC fitting with complete data
- ▲ C_1 on the SWCC fitting with limited data
- C_2 on the SWCC fitting with complete data
- Predicted C_2

(c)

- Complete experimental data
- Limited experimental data
- SWCC fitting with complete data
- - - SWCC fitting with limited data
- · - · - SWCC fitting with limited data and predicted C_2
- ◇ S on the SWCC fitting with complete data
- ◆ S on the SWCC fitting limited data
- △ C_1 on the SWCC fitting with complete data
- ▲ C_1 on the SWCC fitting with limited data
- C_2 on the SWCC fitting with complete data
- Predicted C_2

(d)

FIGURE 9: Continued.

- Complete experimental data
- Limited experimental data
- SWCC fitting with complete data
- - - SWCC fitting with limited data
- · - · - SWCC fitting with limited data and predicted C_2
- ◇ S on the SWCC fitting with complete data
- ◆ S on the SWCC fitting limited data
- △ C_1 on the SWCC fitting with complete data
- ▲ C_1 on the SWCC fitting with limited data
- C_2 on the SWCC fitting with complete data
- Predicted C_2

(e)

- Complete experimental data
- Limited experimental data
- SWCC fitting with complete data
- - - SWCC fitting with limited data
- · - · - SWCC fitting with limited data and predicted C_2
- ◇ S on the SWCC fitting with complete data
- ◆ S on the SWCC fitting limited data
- △ C_1 on the SWCC fitting with complete data
- ▲ C_1 on the SWCC fitting with limited data
- C_2 on the SWCC fitting with complete data
- Predicted C_2

(f)

- Complete experimental data
- Limited experimental data
- SWCC fitting with complete data
- - - SWCC fitting with limited data
- · - · - SWCC fitting with limited data and predicted C_2
- ◇ S on the SWCC fitting with complete data
- ◆ S on the SWCC fitting limited data
- △ C_1 on the SWCC fitting with complete data
- ▲ C_1 on the SWCC fitting with limited data
- C_2 on the SWCC fitting with complete data
- Predicted C_2

(g)

- Complete experimental data
- Limited experimental data
- SWCC fitting with complete data
- - - SWCC fitting with limited data
- · - · - SWCC fitting with limited data and predicted C_2
- ◇ S on the SWCC fitting with complete data
- ◆ S on the SWCC fitting limited data
- △ C_1 on the SWCC fitting with complete data
- ▲ C_1 on the SWCC fitting with limited data
- C_2 on the SWCC fitting with complete data
- Predicted C_2

(h)

FIGURE 9: Continued.

FIGURE 9: The results of applying the method to previous experimental data of sand, silt, and clay samples: (a–c) sand (data from [46]); (d–f) silt (data from [47]); (g–i) clay (data from [51]).

method has been made at different degree ranging from 0.815 to 0.097, expect the result of silt on FX equation. However, the value of η_1 and η_2 under FX equation was also lower than that under VG equation and Gardner equation. From Figure 10, the performance of this method did not greatly vary with soil types, and the averaged improvement was the largest on VG equation among three empirical equations.

4.3. Discussion. Overall, the data processed by this method could acquire more accurate SWCC to some extent, comparing with the limited data. From a series of calculation, it is found that the values of η_1 and η_2 under FX equation are lower than those under VG equation and Gardner equation. That is because the restriction point (1,000,000, 0) makes the fitting results of FX equation more approximate than those of other empirical equations. For sake of this, the improvement of adding point C_2 on FX equation is not stable. Among three empirical equations, the averaged improvement of this method is the largest on VG equation. Therefore, it is suggested this method would achieve the better effectiveness when it is combined with VG equation.

In addition, this study was focussed on the prediction about typical unimodal SWCC. The empirical equations this method based on and discussed in this paper were not applicable to both unimodal SWCC and bimodal SWCC. Therefore, the proposed method is not suitable for the prediction of bimodal

FIGURE 10: The improvements of this method on different soil types and different empirical equations (VG equation, Gardner equation, and FX equation).

SWCC. SWCCs of residual and colluvial soils commonly show bimodal behavior; thus, the applicability of the proposed method on those soils is limited.

What this method needs to be improved in the future is how to make the value of $\theta(\psi_s)$ or residual water content θ_r closer to reality because the value of θ_r is the averaged statistics of a quantity of entire datasets in this paper, other than some regularity of the soil characteristics. According to the analysis about the physical meaning of three points S, C_1 , and C_2 and the regularity presented by calculation above, the value of θ_r may have some qualitative relations with soil properties (mineral composition, pore structure, and dense state). Thus, the future work can be focused on finding some specific relation between some soil index and the value of residual water content θ_r , aiming at obtaining more accurate θ_r whose physical meaning are also more specific.

5. Conclusions

This paper proposed a geometrical method based on a symmetric relation of three critical points (point S which is the largest absolute value of slope; two points C_1 and C_2 which are two extreme points of curvature) on SWCC in semi-logarithmic coordinate to preprocess limited data before directly fitting with conventional empirical equations. Meantime, a series of theoretical derivation and confirmation computation proved this symmetric relation of three critical points exhibited well in all three conventional empirical equations, especially in Gardner equation. The accuracy of SWCC obtained by empirical equations depends on the dataset whether the formed curve includes the critical shape characteristic point of maximum curvature (point C_2). If this point has not been reached in experimental data, even many numbers of points obtained, the fitting results directly by the empirical model could be large error. Three datasets of sand, silt, and clay acquired by the filter paper method were used to testify applicability of this method. The performance of this method did not greatly vary with soil types, but it was floating a lot on different empirical equations. Through quantitative analysis, the method generally performed well on sand, silt, and clay. In perspective of empirical equations, the evident improvement can be made when the proposed method is combined with VG equation, Gardner equation, and FX equation, but averaged improvement was the largest on the VG equation among three empirical equations. Generally, this method provides good evidence of wide application potential. And future work could be concentrated on finding out the relation between residual water content θ_r and soil characteristics instead of using averaged statistical value.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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