Research Article

Transient Flow of Jeffrey Fluid over a Permeable Wall

Sana Bajwa,¹ Saif Ullah ,¹ Ilyas Khan ,² and Md. Fayz-Al-Asad ³

¹Department of Mathematics, Government College University, Lahore 54000, Pakistan

²Department of Mathematics, College of Science Al-Zulfi, Majmaah University, Al-Majmaah 11952, Saudi Arabia

³Department of Mathematics, Bangladesh University of Engineering and Technology (BUET), Dhaka 1000, Bangladesh

Correspondence should be addressed to Ilyas Khan; i.said@mu.edu.sa and Md. Fayz-Al-Asad; fayzmath.buet@gmail.com

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Transient incompressible flows of Jeffrey fluids over a permeable, flat, and infinite plate have been investigated. The plate motion is an oscillatory translation along the x -axis. Using the Laplace transform and perturbation method, the analytical solution for the velocity field in the transform domain has been obtained. The velocity field in the real domain has been determined by using the numerical Stehfest's algorithm for the Laplace transform inversion. To have a validation of the obtained solution, we have determined the analytical solution for the flow without transpiration. It was found that when the transpiration parameter approaches zero, the solution for the flow with transpiration tends to the solution corresponding to the case without perspiration. The influence of the system parameters on fluid motion has been investigated by numerical simulations and graphical illustrations prepared with the software package Mathcad.

1. Introduction

The investigation of fluids such as non-Newtonian has currently increased extraordinary attention of the engineers, analysts, and researchers. Such inspiration of scientists is because of different usages of non-Newtonian fluids in innovation and mechanical territories. Not at all like the viscous materials, cannot the non-Newtonian fluids be clarified by utilizing Navier–Stokes hypothesis. A solitary constitutive relationship cannot depict the attributes of non-Newtonian fluids. The specifics of non-Newtonian fluids are particular than the facts of viscous materials. As compared with the differential equations of viscous material, the order of differential equations of non-Newtonian fluids is greater. There are countless types of non-Newtonian fluids by means of differing properties. These fluids have been grouped into three classifications recognized as the differential, integral, and rate type. The most widely recognized and least difficult type of non-Newtonian fluids is Jeffrey fluid. Such fluid has time-derivative rather than convected derivative. Trepathi et al. studied MHD flow in a cylindrical tube of Jeffrey fluid [1]. Das examined impacts of heat transfer and

slip on MHD (magnetohydro dynamic) peristaltic flow of Jeffrey fluid [2]. In the occurrence of thermal jump, the variable thermal conductivity of Jeffrey fluid has been considered by Hamad et al. [3]. Nonlinear thermal radiation influence of Jeffrey nanofluid in three-dimensional flows is presented by Shehzad et al. [4]. Hussain and Ellahi [5] studied slip features of the flow of a Jeffrey fluid. [6] Hayyat et al. considered stagnation points flow in existence of Newtonian heating of Jeffrey nanofluid. Reddy et al. [7] studied Jeffrey fluid's flow as a result of oscillation of disks. Farooq et al. explored MHD flows of a Jeffrey fluid in occurrence of a Newtonian heating [8].

In contrast, an inclusive argument of a Jeffrey fluid on steady flows using stretching sheet can be originated in [9–14]. The analytical solution over a shrinking sheet of the Jeffrey fluid for boundary layer steady flow has been underlined by Nadeem et al. [15] whereas Hamad et al. [16] suggested a finite difference study to analyze the thermal jump effects near a stagnation points on the shrinking sheet using variable thermal conductivity on boundary layer flow of a Jeffrey fluid. References [17–29] are delivered for several thought-provoking outcomes of a Jeffrey fluid peristaltic flow.

For the Jeffrey fluid, a more wide-ranging solution can be obtained when a fluid transpiration is considered on the wall. The leading equations of motion for Jeffrey fluid with wall transpiration are far complex as related with Jeffrey fluid without wall transpiration. This complexity of governing equations of Jeffrey fluid with wall transpiration creates the inconvenience to attain the exact solution, and it is challenging to handle. In the study of wall transpiration, most of the concern is by reason of its substantial uses in aeronautical system, controlled boundary layer, mechanical and chemical engineering processes, and for instance manufacturing techniques. Cruz and Lins [30] are associated to a recent published article on this line, which examines the unsteady flow of Newtonian fluid for Stokes' first and second problems, bounded by plate with wall transpiration. In their research, an analytical solution is presented for the fluid's behavior over the flat plates with oscillating and impulsive motions, beginning from rest and using wall transpiration.

This paper is extended by Hameed et al. [31]; they derived exact solution for unsteady flow of a second-grade fluid above flat plates with oscillating and impulsive motions, initially at rest, and by means of wall transpiration by Laplace transform, perturbation method, and an extended variable separation technique.

Stokes problems have countless importance in abundant fields of life, which contain the industry manufacturing, geophysical flows, chemical engineering, and a heat conduction phenomenon. Initially, in 1851, Stokes [32] presented a famous research study on pendulum in which the problems related to the oscillatory and impulsive motions of a plane were considered. Initially, Stokes' problems are boundary value one-dimensional problems to a viscous flows inspired by a moving plane. After simplified Navier–Stokes equations, the resulted partial differential equation is momentum equation that contains differentiation with respect to (w.r.t) two variables only, the coordinates which is normal to the time and plane. This work assists that the viscous flows of fluid are mathematically solvable. The related solution for both steady and transient state has been well deliberated [33–38].

The aim of this paper is to study the transient incompressible flows of Jeffrey fluids over a permeable, flat, and infinite plate. The plate motion is an oscillatory translation along the x -axis. Using the Laplace transform and perturbation method, the analytical solution for the nondimensional velocity field in the transform domain has been obtained. The velocity field in the real domain has been determined by using the numerical Stehfest's algorithm for the Laplace transform inversion. The particular case of the flow without transpiration is also investigated, and the analytical solution of the velocity field is determined. To have a validation of the obtained solution by the perturbation method, we have compared the analytical solution for the flow with transpiration with the solution corresponding to

the flow without transpiration. It was found that for the transpiration parameter tending to zero, the solution for the flow with transpiration tends to the solution corresponding to the case without perspiration. The influence of the system parameters on fluid motion has been investigated by numerical simulations and graphical illustrations prepared with the software package Mathcad.

2. Mathematical Modeling of the Problem

Let us consider incompressible unsteady flows of a Jeffrey fluid over a flat porous plate situated in the (x, z) -plane of a Cartesian coordinate system $Oxyz$; therefore, the y -axis is perpendicular to the plate (Figure 1). Initially, at $t=0$, the plate and fluid are at the rest. In this paper, we seek a velocity field of the form $\vec{V}(y, t) = u(y, t)\vec{e}_x + v(y, t)\vec{e}_y$, where $\vec{e}_x$ and $\vec{e}_y$ are the unit vectors in the x - and y -direction, respectively. The incompressibility condition implies that the velocity component $v(y, t)$ has to satisfy condition $\partial v(y, t)/\partial y = 0$, so $v(y, t) = V(t)$. In this paper, for velocity component $v(y, t)$ to be constant, therefore, hereafter we consider $v(y, t) = -V_0$. After the instant $t = 0$, the plate translates along x -axis with velocity $u(0, t) = U_0 \exp(i\omega t)$, where ω is the oscillations frequency.

Under the above consideration, the governing equation for the flow of unsteady incompressible Jeffrey fluid over a porous plate has the following form [3, 11]:

$$\rho \left(\frac{\partial u}{\partial t} - V_0 \frac{\partial u}{\partial y} \right) = \frac{\mu}{1 + \lambda_1} \left(\frac{\partial^2 u}{\partial y^2} + \lambda_2 \frac{\partial^3 u}{\partial y^2 \partial t} + \lambda_2 V_0 \frac{\partial^3 u}{\partial y^3} \right), \quad y > 0, t > 0, \quad (1)$$

where μ denotes the dynamic viscosity of the fluid, ρ represents the constant fluid's density, V_0 symbolizes the uniform transpiration (suction for $V_0 > 0$, respectively, blowing for $V_0 < 0$), λ_1 represents the ratio of relaxation time to the retardation time, whereas λ_2 is the retardation time. It is noteworthy that by particularizing values of the parameters λ_1 and λ_2 , the governing equations of Newtonian and second-grade fluids can be obtained.

Along with equation (1), the following initial and boundary conditions are considered:

$$u(y, 0) = 0, \quad y \geq 0, \quad (2)$$

$$u(0, t) = u_0(t) = U_0 e^{i\omega t}, \quad t > 0, \quad (3)$$

$$u(y, t) = 0 \text{ as } y \longrightarrow \infty, \quad t \geq 0. \quad (4)$$

Now, to obtain the dimensionless form of the above-mentioned equations, we use the set of dimensionless variables as follows:

FIGURE 1: Geometry of the problem.

$$\begin{aligned}
 u^* &= \frac{u}{U_0}, \\
 y^* &= \frac{U_0 y}{\nu}, \\
 t^* &= \frac{U_0^2 t}{\nu}, \\
 \omega^* &= \frac{\omega \nu}{U_0^2}, \\
 \beta &= \frac{U_0^2}{\nu} \lambda_2, \\
 2\gamma &= \frac{V_0}{U_0},
 \end{aligned} \tag{5}$$

into equations (1)–(4). The dimensionless initial-boundary value problem (dropping the * notation) is

$$\frac{1}{1 + \lambda_1} \left(\frac{\partial^2 u}{\partial y^2} + \beta \frac{\partial^3 u}{\partial y^2 \partial t} + 2\beta\gamma \frac{\partial^3 u}{\partial y^3} \right) + 2\gamma \frac{\partial u}{\partial y} - \frac{\partial u}{\partial t} = 0, \tag{6}$$

$$u(y, 0) = 0, \quad y \geq 0, \tag{7}$$

$$u(0, t) = u_0(t) = e^{i\omega t}, \quad t > 0, \tag{8}$$

$$u(y, t) = 0 \text{ as } y \longrightarrow \infty, \quad t \geq 0, \tag{9}$$

where β is the Jeffrey fluid parameter and γ represents wall transpiration parameter ($\gamma > 0$ for suction and $\gamma < 0$ for injection).

3. Solution of the Problem

To determine analytical solution of the problem formulated in Section 2, we use the Laplace transform with respect to time t and the regular perturbation method. Applying the Laplace transform to equations (6), (8), and (9) and using the initial condition (7), the following problem in the transformed domain is obtained:

$$\frac{1}{1 + \lambda_1} (1 + \beta s) \frac{\partial^2 \bar{u}}{\partial y^2} + \frac{2\beta\gamma}{1 + \lambda_1} \frac{\partial^3 \bar{u}}{\partial y^3} + 2\gamma \frac{\partial \bar{u}}{\partial y} - s\bar{u} = 0, \tag{10}$$

$$\bar{u}(0, s) = \frac{1}{s - i\omega}, \quad \bar{u}(y, s) = 0, \quad \text{as } y \longrightarrow \infty, t \geq 0, \tag{11}$$

where $\bar{u}(y, s) = \int_0^\infty u(y, t) \exp(-st) dt$ is the Laplace transform of function $u(y, t)$ and s is known as the transform parameter.

3.1. The Perturbed Analytical Solution. To solve equation (10) along with boundary conditions (11), we suppose that the Jeffrey parameter β has small values; therefore, we suppose that $\beta \ll 1$ and the transformed velocity field $\bar{u}(y, s)$ is expressed as

$$\bar{u}(y, s) = \bar{u}_0(y, s) + \beta \bar{u}_1(y, s) + O(\beta^2). \tag{12}$$

Now, substituting equation (12) into equations (10) and (11) and then comparing the powers of β , we obtain the following problems:

(i) The zero order problem:

$$\begin{aligned}\bar{u}_0'' + 2\gamma(1 + \lambda_1)\bar{u}_0' - (1 + \lambda_1)s\bar{u}_0 &= 0, \\ u_0(0, s) &= \frac{1}{s - i\omega}, \\ u(\infty, s) &= 0.\end{aligned}\quad (13)$$

(ii) The first order problem:

$$\begin{aligned}\bar{u}_1'' + 2\gamma(1 + \lambda_1)\bar{u}_1' - (1 + \lambda_1)s\bar{u}_1 &= -\left(2\gamma\bar{u}_0''' + s\bar{u}_0'''\right), \\ u_1(0, s) &= 0, u_1(\infty, s) = 0.\end{aligned}\quad (14)$$

The solutions to the above equations are given by

$$\begin{aligned}\bar{u}_0(y, s) &= \frac{1}{s - i\omega}e^{-y\varphi(s)}, \\ \bar{u}_1(y, s) &= A(s)ye^{-y\varphi(s)},\end{aligned}\quad (15)$$

where

$$\begin{aligned}u(y, t) &\cong \frac{\ln(2)}{t} \sum_{k=1}^P Q_k \bar{u}\left(y, \frac{k \ln(2)}{t}\right), \\ Q_k &= (-1)^{k+(P/2)} \sum_{j=[k+1/2]}^{\min(k, (P/2))} \frac{j^{(P/2)} (2j)!}{((P/2) - j)! j! (j - 1)! (k - j)! (2j - k)!}.\end{aligned}\quad (18)$$

3.2. Analytical Solution of the Problem without Transpiration ($\gamma = 0$). In this case, equation (6) becomes

$$\left(1 + \beta \frac{\partial}{\partial t}\right) \frac{\partial^2 u(y, t)}{\partial y^2} = (1 + \lambda_1) \frac{\partial u(y, t)}{\partial t}. \quad (19)$$

Applying the Laplace transform to equation (19) and using the initial condition (7), we obtain the transformed equation as follows:

$$\frac{\partial^2 \bar{u}(y, s)}{\partial y^2} = \frac{(1 + \lambda_1)s}{(1 + \beta s)} \bar{u}(y, s). \quad (20)$$

Function $\bar{u}(y, s)$ in (20) has to satisfy the boundary condition (11). The analytical solution of the problem described by equations (20) and (11) is given by

$$\begin{aligned}\bar{u}(y, s) &= \frac{1}{s - i\omega} \exp\left(-y \sqrt{\frac{a_0 s}{s + b_0}}\right), \\ a_0 &= \frac{1 + \lambda_1}{\beta}, \\ b_0 &= \frac{1}{\beta}.\end{aligned}\quad (21)$$

Transform (21) can be written in the equivalent form as follows:

$$\begin{aligned}A(s) &= \frac{-\varphi^2(s)(2\gamma\varphi(s) - s)}{2\delta_1 \sqrt{1 + s/\delta_1 \gamma}(s - i\omega)}, \\ \varphi(s) &= \delta_1 \left(1 + \sqrt{1 + \frac{s}{\delta_1 \gamma}}\right),\end{aligned}\quad (16)$$

$$\delta_1 = \gamma(1 + \lambda_1).$$

The transformed velocity field is given by

$$\bar{u}(y, s) = \frac{1}{s - i\omega} \left[1 + \frac{\beta\gamma(s - 2\gamma\varphi(s)\varphi^2(s))}{2\delta_1 \sqrt{1 + s/\delta_1 \gamma}}\right] e^{-y\varphi(s)}. \quad (17)$$

The inverse Laplace of expression (17) is difficult to determine. For this reason, the numerical values of the velocity field in the real domain will be obtained using Stehfest's numerical algorithm. According to this algorithm, the numerical values of the function $u(y, t)$ are approximated by the following expression [37]:

$$\bar{u}(y, s) = \left(1 + \frac{i\omega}{s - i\omega}\right) F(y, s), \quad (22)$$

$$F(y, s) = \frac{1}{s} \exp\left(-y \sqrt{\frac{a_0 s}{s + b_0}}\right).$$

Using the following inversion formula:

$$\begin{aligned}f(y, t) &= L^{-1}\{F(y, s)\} = L^{-1}\left\{\frac{1}{s} \exp\left(-y \sqrt{\frac{a_0 s}{s + b_0}}\right)\right\} \\ &= 1 - \frac{2a_0}{\pi} \int_0^\infty \frac{\sin(yz)}{z(a_0 + z^2)} \exp\left(\frac{-b_0 z^2 t}{a_0 + z^2}\right) dz,\end{aligned}\quad (23)$$

we obtain the velocity field given by

$$\begin{aligned}u(y, t) &= (\delta(t) + i\omega e^{i\omega t}) * f(y, t) \\ &= f(y, t) + i\omega \int_0^t e^{i\omega(t-\tau)} f(y, \tau) d\tau,\end{aligned}\quad (24)$$

where $\delta(t)$ is the Dirac distribution.

It is important to emphasize that solution (24) can be used to validate the solution given by (17) and (18) for the general case of the flow with transpiration on the wall. For this, we show in Figure 2 that for $\gamma \rightarrow 0$, the graphs of solution (17) tend to overlap the graph of function (24). To draw the curves in Figure 2, we used the following values of

FIGURE 2: Comparison between solutions (17) and (24) for the transpiration parameter tending to zero.

the parameters: $\lambda_1 = 0.45$, $\omega = \pi/2$, $\beta = 0.01$, and $\gamma \in \{0.50, 0.25, 0.01, 0.00\}$. It is observed in Figure 2 that when the values of the transpiration parameter tend to zero, graphs of curves given by equation (1) tend to the graph of the solution given by (24). This shows that the perturbed solution of the problem is correct.

4. Results and Discussion

In the current work, approximate analytical solutions for the unsteady flows of a Jeffrey fluid over a porous plate have been obtained by using Laplace transform and perturbation techniques. The variant behavior of the velocity of Jeffrey fluid with different values of resulting parameters is displayed graphically in Figures 3 –7. These nondimensional emerging parameters are ratio of relaxation and retardation time λ_1 , Jeffrey fluid parameter β , wall transpiration parameter γ , oscillating frequency ω , and time t . The following figures show the velocity profiles for different values of dimensionless parameters of an unsteady flow of Jeffrey fluid above the porous plate.

The effect of ratio of relaxation and retardation time λ_1 on velocity distribution can be observed in Figure 3. We see that velocity decreased with the increasing values of λ_1 . The fluid flow is therefore noticeably decelerated with an increase in relaxation time (or decrease in retardation time) because increment in λ_1 leads to increment in relaxation time; it means particles need much more time to come back from the perturbed system to equilibrium system in which subsequently fluid velocity decreases.

In Figure 4, we observe the effect of Jeffrey fluid parameter β on velocity. It is observed that with the increment in Jeffrey fluid parameter, the velocity profile decreases. It is found that the momentum boundary layer thickness increases as the Jeffrey fluid parameter effect is strengthened. Hence, the velocity profile decreases due to this fact.

FIGURE 3: The influence of parameter λ_1 on velocity when $\beta = 0.01$, $\omega = \pi/2$, $\gamma = 0.1$, and $t = 0.5$.

The behavior of velocity under the effect of wall transpiration parameter γ is observed with the help of Figure 5. It is perceived that the velocity of Jeffrey fluid flow is decreased along with increasing wall transpiration (injection/suction parameter) parameter. In fact, fluid experiences greater viscosity with the porous boundaries and hence offers resistance to flow resulting reduction in the velocity. For large values of parameter, the decay is more.

In order to study the effect of frequency of oscillation ω , we have plotted Figure 6, where it shows that the velocity of the Jeffrey fluid is also a strong function of ω . The effect of

FIGURE 4: Influence of β on velocity for $\lambda_1 = 0.2$, $\omega = \pi/2$, $\gamma = 0.1$, and $t = 0.5$.

FIGURE 5: Influence of γ on velocity when $\lambda_1 = 0.2$, $\omega = \pi/2$, $\beta = 0.01$, and $t = 0.5$.

FIGURE 6: Influence of ω on velocity when $\lambda_1 = 0.2$, $\gamma = 0.1$, $\beta = 0.01$, and $t = 0.5$.

FIGURE 7: Influence of (t) on velocity when $\lambda_1 = 0.2$, $\omega = \pi/2$, $\beta = 0.01$, and $\gamma = 0.1$.

frequency of oscillation on the velocity profile is opposite as that of the ratio of relaxation to retardation time λ_1 .

Figure 7 shows the relationship between velocity of the fluid and time. We see that the velocity enhances by increasing the value of dimensionless time t .

5. Conclusion

This work presents approximate analytical solutions for the unsteady flows of Jeffrey fluids over a porous, oscillating plate. Approximate analytical solutions for the nondimensional velocity field in the transformed domain have been obtained using the Laplace transform and perturbation method. The velocity field in the real domain has been determined by using the numerical Stehfest's algorithm for the Laplace transform inversion. The particular case of flow without transpiration has been studied. The comparison between both solutions,

namely, the solution with transpiration when the transpiration parameter approaches zero and the solution without transpiration has shown that for the transpiration parameter tending to zero, the solution for the flow with transpiration tends to the solution corresponding to the case without perspiration. The influence of the system parameters on fluid motion has been investigated by numerical simulations and graphical illustrations prepared with the software package Mathcad.

Data Availability

No data were used to support this research.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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