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Research Article

Spherical Fuzzy Soft Topology and Its Application in Group Decision-Making Problems

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The spherical fuzzy soft set is a generalized soft set model, which is more realistic, practical, and accurate. It is an extended version of existing fuzzy soft set models that can be used to describe imprecise data in real-world scenarios. The paper seeks to introduce the new concept of spherical fuzzy soft topology defined on spherical fuzzy soft sets. In this work, we define some basic concepts including spherical fuzzy soft basis, spherical fuzzy soft subspace, spherical fuzzy soft interior, spherical fuzzy soft closure, and spherical fuzzy soft boundary. The properties of these defined set are also discussed and explained with an appropriate examples. Also, we establish certain important results and introduce spherical fuzzy soft separation axioms, spherical fuzzy soft regular space, and spherical fuzzy soft normal space. Furthermore, as an application, a group decision-making algorithm is presented based on the TOPSIS (Technique of Order Preference by Similarity to an Ideal Solution) method for solving the decision-making problems. The applicability of the proposed method is demonstrated through a numerical example. The comprehensive advantages of the proposed work have been stated over the existing methods.

1. Introduction

The human life with all of its complexities, is currently in flux due to the exponential growth of innovation and changing technologies that constantly redefine, reshape, and redesign the way the world is perceived and experienced, and the tools once used to solve problems become obsolete and inappropriate. This is no exception to any discipline of knowledge. Thus, The strategies commonly adopted in classical mathematics are not effective all the time due to the uncertainty and ambiguity it entails. Techniques such as fuzzy set theory [1], vague set theory [2], and interval mathematics [3] are viewed as mathematical models for coping with uncertainty and variability. However, these theories suffer from their own shortcomings and inadequacies to deal with the task at hand more objectively. Zadeh's fuzzy set theory was extensively used in the beginning for many applications. Fuzzy sets are thought to be an extended version of classical sets, where each element has a membership grade. The definition of intuitionistic fuzzy sets was developed by Atanassov [4] to circumvent some limitations of fuzzy sets. Many other fuzzy set extensions have been proposed, including interval-valued intuitionistic fuzzy sets [5], Pythagorean fuzzy sets [6], picture fuzzy sets [7], and so on. These sets were effectively applied in several areas of science and engineering, economics, medical science, and environmental science. Recently, as a generalization of fuzzy set, intuitionistic fuzzy set, and picture fuzzy set, certain authors have developed the concept of spherical fuzzy sets [8] and T-spherical fuzzy sets [9] to enlarge the picture fuzzy sets as it has their restrictions. To address decision-making problems, Ashraf et al. [10] proposed the spherical fuzzy aggregation operators. Akram et al. [11] introduced the complex spherical fuzzy model that excels at expressing ambiguous information in two dimensions. The applications of these sets to solve decision-making problems are prevalent in a variety of fields [12–17].

In 1999, Molodtsov [18] proposed a new type of set, called soft set, to deal with uncertainty and vagueness. The challenge of determining the membership function in fuzzy set theory does not occur in soft set theory, making the theory applicable to multiple fields of game theory, operations research, Riemann integration, etc. Later, Maji et al. [19] studied more on soft sets and used Pawlak's rough mathematics [20] to propose a decision-making problem as an application of soft sets. Also, Maji et al. [21] developed a hybrid structure of soft sets and fuzzy sets, known as fuzzy soft sets, which is a more powerful mathematical model for handling different kinds of real-life situations. Many researchers were interested in this concept and various fuzzy set generalizations such as generalized fuzzy soft sets [22], group generalized fuzzy soft sets [23], intuitionistic fuzzy soft sets [24], Pythagorean fuzzy soft sets [25], interval-valued picture fuzzy soft sets [26] were put forward. In the recent times, Perveen et al. [27] created a spherical fuzzy soft set (SFSS), which is a more advanced form of fuzzy soft set. This newly evolved set is arguably the more realistic, practical and accurate. SFSSs are a new variation of the picture fuzzy soft set that was developed by merging soft sets and spherical fuzzy sets, where the membership degrees satisfy the condition $0 \leq \mu^2_{\aleph(\varpi)}(\varsigma) + \eta^2_{\aleph(\varpi)}(\varsigma) + \vartheta^2_{\aleph(\varpi)}(\varsigma) \leq 1$ rather than $0 \leq \mu_{\aleph(\varpi)}(\varsigma) + \eta_{\aleph(\varpi)}(\varsigma) + \vartheta_{\aleph(\varpi)}(\varsigma) \leq 1$ as in picture fuzzy soft sets. SFSS has more capability in modeling vagueness and uncertainty while dealing with decision-making problems that occur in real-life circumstances. The authors [28] also developed similarity measures of SFSS and applied the proposed spherical fuzzy soft similarity measure in the field of medical science.

These theories have applications in topology and many other fields of mathematics. Chang [29] suggested the concept of a fuzzy topological space in 1968. He extended many basic concepts like continuity, compactness, open set, and closed set in general topology to the fuzzy topological spaces. Again, Lowen [30] conducted an elaborated study of the structure of fuzzy topological spaces. Çoker [31] invented the idea of an intuitionistic fuzzy topological space in 1995. Many other results including continuity, compactness, and connectedness of intuitionistic fuzzy topological spaces were proposed by Coker et al. [32, 33]. The notion of Pythagorean fuzzy topological space was presented by Olgun et al. [34]. Kiruthika and Thangavelu [35] discussed the link between topology and soft topology. Recently, by using elementary operations over a universal set with a set of parameters, Taskopru and Altintas [36] established the elementary soft topology. Tanay and Kandemir [37] defined the idea of fuzzy soft topology. They also introduced fuzzy soft neighbourhood, fuzzy soft basis, fuzzy soft interior, and fuzzy soft subspace topology. Several related works on fuzzy soft topology can be seen in [38–40]. Osmanoglu and Tokat [41] proposed the subspace, compactness, connectedness, and separation axioms of intuitionistic fuzzy soft topological spaces. Also, intuitionistic fuzzy soft topological spaces were examined by Bayramov and Gunduz [42]. They studied intuitionistic fuzzy soft continuous mapping and related properties. Riaz et al. [43] proposed the concept of Pythagorean fuzzy soft topology defined on Pythagorean fuzzy soft sets, and provided an application of Pythagorean fuzzy soft topology in medical diagnosis by making use of TOPSIS method.

Hwang and Yoon [44] developed Technique for order of Preference by Similarity to ideal solution (TOPSIS) as a multi-criteria decision analysis and further studied by Chen et al. [45, 46]. Boran et al. [47] invented the TOPSIS approach based on intuitionistic fuzzy sets for multi-criteria decision-making problems. Chen et al. [48] developed a proportional interval T2 hesitant fuzzy TOPSIS approach based on the Hamacher aggregation operators and the andness optimization models. Further, the fuzzy soft TOPSIS method presented briefly as a multi-criteria decision-making technique by Selim and Karaaslan [49]. They proposed a group decision-making process in a fuzzy soft environment based on the TOPSIS method. Also, many researchers in [50–54] have looked at the TOPSIS approach for solving decision-making problems under the different fuzzy environment.

Topological structures on fuzzy soft sets have application in several areas including medical diagnosis, decision-making, pattern recognition, and image processing. Since SFSS is one of the most generalized versions of the fuzzy soft set, introducing topology on SFSS is highly essential in both theoretical and practical scenarios. There are some basic operations of SFSSs in the literature, more functional operations of SFSSs are derived day by day. The development of topology on SFSSs can be considered as an important contribution to fill the gap in the literature on the theory of SFSS. The aim of this paper is to introduce the notion of spherical fuzzy soft topology (SFS-topology) on SFSS, and to discuss some basic concepts such as SFS-subspace, SFS-point, SFS-nbd, SFS-basis, SFS-interior, SFS-closure, SFS-boundary, SFS-exterior and SFS-separation axioms. Also, through this paper, we use the SFS-topology in group decision-making method based on TOPSIS under spherical fuzzy soft environment.

The rest of the paper is ordered as follows. In Section 2, some fundamental concepts of fuzzy sets, spherical fuzzy sets, soft sets, fuzzy soft sets, and spherical fuzzy soft sets are recalled, and definitions of spherical fuzzy subset, spherical fuzzy union and spherical fuzzy intersection are modified. In Section 3, the concept of SFS-topology is defined on SFSS including some basic definitions. In Section 4, by using the ideas of SFS-points, SFS-open set, and SFS-closed set, SFS-separation axioms are proposed. In Section 5, an algorithm is presented besed on group decision-making method and extension of TOPSIS approach accompanied by a numerical example. This theory will have implications in the discipline of Human resource management, organizational behavior and assessing the rationale of consumer choice. In Section 6, a comparative study is conducted with an already existing algorithm to show the effectiveness of the proposed algorithm. Finally, Section 7 ends with a conclusion and recommendations for future work.

2. Preliminaries

In this section, we recall certain fundamental ideas associated with various kinds of sets including fuzzy sets, spherical fuzzy sets, soft sets, fuzzy soft sets, and spherical fuzzy soft sets. We redefine the definitions of spherical fuzzy subset, spherical fuzzy union, and spherical fuzzy intersection, also propose the notions of null SFSS and absolute SFSS. Let Σ be the initial universal set of discourse and $\mathscr{K}$ be the attribute (or parameter) set in connection with the objects in Σ, and $\mathscr{L} \subseteq \mathscr{K}$.

Definition 1 (see [1]). A fuzzy set $\aleph$ on a universe Σ is an object of the form

$$\aleph = \{(\varsigma, \mu_{\aleph}(\varsigma)) | \varsigma \in \Sigma\}, \quad (1)$$

where $\mu_{\aleph}: \Sigma \longrightarrow [0,1]$ is the membership function of $\aleph$, the value $\mu_{\aleph}(\varsigma)$ is the grade of membership of ς in $\aleph$.

Definition 2 (see [9]). A spherical fuzzy set (SFS) $\mathscr{S}$ over the universal set Σ can be written as

$$\mathscr{S} = \{(\varsigma, \mu_{\mathscr{S}}(\varsigma), \eta_{\mathscr{S}}(\varsigma), \vartheta_{\mathscr{S}}(\varsigma)) | \varsigma \in \Sigma\}, \quad (2)$$

where $\mu_{\mathscr{S}}(\varsigma)$, $\eta_{\mathscr{S}}(\varsigma)$ and $\vartheta_{\mathscr{S}}(\varsigma)$ are the membership functions defined from Σ to $[0, 1]$, indicate the positive, neutral, and negative membership degrees of $\varsigma \in \Sigma$ respectively, with the condition, $0 \le \mu^2_{\mathscr{S}}(\varsigma) + \eta^2_{\mathscr{S}}(\varsigma) + \vartheta^2_{\mathscr{S}}(\varsigma) \le 1$, $\forall \varsigma \in \Sigma$.

Definition 3 (see [9]). Let $\aleph = \{(\varsigma, \mu_{\aleph}(\varsigma), \eta_{\aleph}(\varsigma), \vartheta_{\aleph}(\varsigma)) | \varsigma \in \Sigma\}$ and $\Omega = \{(\varsigma, \mu_{\Omega}(\varsigma), \eta_{\Omega}(\varsigma), \vartheta_{\Omega}(\varsigma)) | \varsigma \in \Sigma\}$ be two SFSs over Σ. Then

(1) $\aleph \subseteq \Omega$ if $\mu_{\aleph}(\varsigma) \le \mu_{\Omega}(\varsigma)$, $\eta_{\aleph}(\varsigma) \le \eta_{\Omega}(\varsigma)$, and $\vartheta_{\aleph}(\varsigma) \ge \vartheta_{\Omega}(\varsigma)$

(2) $\aleph = \Omega$ if and only if $\aleph \subseteq \Omega$ and $\aleph \supseteq \Omega$

(3) $\aleph \cup \Omega = \{(\varsigma, \mu_{\aleph}(\varsigma) \vee \mu_{\Omega}(\varsigma), \eta_{\aleph}(\varsigma) \wedge \eta_{\Omega}(\varsigma), \vartheta_{\aleph}(\varsigma) \wedge \vartheta_{\Omega}(\varsigma)) | \varsigma \in \Sigma\}$

(4) $\aleph \cap \Omega = \{(\varsigma, \mu_{\aleph}(\varsigma) \wedge \mu_{\Omega}(\varsigma), \eta_{\aleph}(\varsigma) \wedge \eta_{\Omega}(\varsigma), \vartheta_{\aleph}(\varsigma) \vee \vartheta_{\Omega}(\varsigma)) | \varsigma \in \Sigma\}$

Where the symbols "$\vee$" and "$\wedge$" represent the maximum and minimum operations respectively.

Definition 4 (see [10]). Let Σ be the initial universal set.

(1) An SFS is said to be an absolute SFS over the universe Σ, denoted by 1^{Σ}, if $\forall \varsigma \in \Sigma$,

$$\mu_{1^{\Sigma}}(\varsigma) = 1, \eta_{1^{\Sigma}}(\varsigma) = 0, \text{and } \vartheta_{1^{\Sigma}}(\varsigma) = 0. \quad (3)$$

(2) An SFS is said to be a null SFS over the universe Σ, denoted by 1_{Σ}, if $\forall \varsigma \in \Sigma$,

$$\mu_{1_{\Sigma}}(\varsigma) = 0, \eta_{1_{\Sigma}}(\varsigma) = 0, \text{and } \vartheta_{1_{\Sigma}}(\varsigma) = 1. \quad (4)$$

Example 1. Let $\Sigma = \{\varsigma_1, \varsigma_2\}$ be the universal set. Let $\aleph$ and Ω be two SFSs over Σ given by,

$$\aleph = \{(\varsigma_1, 0.3, 0.4, 0.5), (\varsigma_2, 0.5, 0.2, 0.4)\}, \quad (5)$$

$$\Omega = \{(\varsigma_1, 0.4, 0.5, 0.2), (\varsigma_2, 0.6, 0.3, 0.3)\}. \quad (6)$$

Then it is clear that $\aleph \subseteq \Omega$, and $\aleph \cup \Omega = \{(\varsigma_1, 0.4, 0.4, 0.2), (\varsigma_2, 0.6, 0.2, 0.3)\}$.

Further, $1^{\Sigma} = \{(\varsigma_1, 1.0, 0.0, 0.0), (\varsigma_2, 1.0, 0.0, 0.1)\}$ and $1_{\Sigma} = \{(\varsigma_1, 0.0, 0.0, 0.0), (\varsigma_2, 0.0, 0.0, 0.1)\}$. Then $\aleph \cup 1_{\Sigma} = \{(\varsigma_1, 0.3, 0.0, 0.5), (\varsigma_2, 0.5, 0.0, 0.4)\}$ and $\aleph \cap 1^{\Sigma} = \{(\varsigma_1, 0.3, 0.0, 0.5), (\varsigma_2, 0.5, 0.0, 0.4)\}$.

From the above example, It can be showed that the following results are not true generally in spherical fuzzy set theory.

(1) $\aleph \subseteq 1^{\Sigma}$

(2) $\aleph \cup 1_{\Sigma} = \aleph$

(3) $\aleph \cap 1^{\Sigma} = \aleph$

(4) If $\aleph \subseteq \Omega$, then $\aleph \cup \Omega = \Omega$

To overcome this difficulty, we modified the definitions of spherical fuzzy subset, spherical fuzzy union, and spherical fuzzy intersection as follows.

Definition 5. Let $\aleph$ and Ω be two spherical fuzzy sets over the universe Σ, where $\aleph = \{(\varsigma, \mu_{\aleph}(\varsigma), \eta_{\aleph}(\varsigma), \vartheta_{\aleph}(\varsigma)) | \varsigma \in \Sigma\}$ and $\Omega = \{(\varsigma, \mu_{\Omega}(\varsigma), \eta_{\Omega}(\varsigma), \vartheta_{\Omega}(\varsigma)) | \varsigma \in \Sigma\}$. Then $\aleph$ is said to be a spherical fuzzy subset (modified) of Ω, denoted by $\aleph \mathrel{\widehat{\subseteq}} \Omega$, if $\forall \varsigma \in \Sigma$

$$\begin{cases} \mu_{\aleph}(\varsigma) \le \mu_{\Omega}(\varsigma), \eta_{\aleph}(\varsigma) \le \eta_{\Omega}(\varsigma), \vartheta_{\aleph}(\varsigma) \ge \vartheta_{\Omega}(\varsigma); \text{ if } \mu_{\Omega}(\varsigma) \ne 1 \\ \mu_{\aleph}(\varsigma) \le \mu_{\Omega}(\varsigma), \eta_{\aleph}(\varsigma) \ge \eta_{\Omega}(\varsigma), \vartheta_{\aleph}(\varsigma) \ge \vartheta_{\Omega}(\varsigma); \text{ otherwise} \end{cases} \quad (7)$$

Definition 6. Let $\aleph = \{(\varsigma, \mu_{\aleph}(\varsigma), \eta_{\aleph}(\varsigma), \vartheta_{\aleph}(\varsigma)) | \varsigma \in \Sigma\}$ and $\Omega = \{(\varsigma, \mu_{\Omega}(\varsigma), \eta_{\Omega}(\varsigma), \vartheta_{\Omega}(\varsigma)) | \varsigma \in \Sigma\}$ be two spherical fuzzy sets over Σ. Then the spherical fuzzy union (modified), denoted by $\aleph \mathbin{\widehat{\cup}} \Omega$, and the spherical fuzzy intersection (modified), denoted by $\aleph \mathbin{\widehat{\cap}} \Omega$, are defined as follows:

(1) $\Lambda = \aleph \mathbin{\widehat{\cup}} \Omega = \{(\varsigma, \mu_{\Lambda}(\varsigma), \eta_{\Lambda}(\varsigma), \vartheta_{\Lambda}(\varsigma)) | \varsigma \in \Sigma\}$, where

$\mu_{\Lambda}(\varsigma) = \mu_{\aleph}(\varsigma) \vee \mu_{\Omega}(\varsigma)$

$\eta_{\Lambda}(\varsigma) = \{ \eta_{\aleph}(\varsigma) \vee \eta_{\Omega}(\varsigma)$; if $(\mu_{\aleph}(\varsigma) \vee \mu_{\Omega}(\varsigma))^2 + (\eta_{\aleph}(\varsigma) \vee \eta_{\Omega}(\varsigma))^2 + (\vartheta_{\aleph}(\varsigma) \wedge \vartheta_{\Omega}(\varsigma))^2 \le 1 \eta_{\aleph}(\varsigma) \wedge \eta_{\Omega}(\varsigma)$; otherwise

$\vartheta_{\Lambda}(\varsigma) = \vartheta_{\aleph}(\varsigma) \wedge \vartheta_{\Omega}(\varsigma)$

(2) $\Pi = \aleph \mathbin{\widehat{\cap}} \Omega = \{(\varsigma, \mu_{\Pi}(\varsigma), \eta_{\Pi}(\varsigma), \vartheta_{\Pi}(\varsigma)) | \varsigma \in \Sigma\}$, where

$\mu_{\Pi}(\varsigma) = \mu_{\aleph}(\varsigma) \wedge \mu_{\Omega}(\varsigma)$

$$\eta_{\Pi}(\varsigma) = \begin{cases} \eta_{\aleph}(\varsigma) \vee \eta_{\Omega}(\varsigma); \text{ if } (\mu_{\aleph}(\varsigma) \text{or} \mu_{\Omega}(\varsigma)) = 1 \\ \eta_{\aleph}(\varsigma) \wedge \eta_{\Omega}(\varsigma); \text{ otherwise} \end{cases}$$

$\vartheta_{\Pi}(\varsigma) = \vartheta_{\aleph}(\varsigma) \vee \vartheta_{\Omega}(\varsigma)$

Definition 7 (see [18]). Let $P(\Sigma)$ denote the power set of the universal set Σ and $\mathscr{K}$ be the set of attributes. A soft set over

Σ is a pair $\langle \aleph, \mathscr{L} \rangle$, where $\aleph$ is a function from $\mathscr{L}$ to $P(\Sigma)$, and $\mathscr{L} \subseteq \mathscr{K}$.

Definition 8 (see [21]). Let $FS(\Sigma)$ denote the collection of all fuzzy subsets over the universal set Σ. A fuzzy soft set (FSS) is a pair $\langle \aleph, \mathscr{L} \rangle$, where $\aleph$ is a mapping given by $\aleph: \mathscr{L} \longrightarrow FS(\Sigma)$ and $\mathscr{L} \subseteq \mathscr{K}$.

Definition 9 (see [27]). Let $SFS(\Sigma)$ be the set of all spherical fuzzy sets over Σ. A spherical fuzzy soft set (SFSS) is a pair $\langle \aleph, \mathscr{L} \rangle$, where $\aleph$ is a mapping from $\mathscr{L}$ to $SFS(\Sigma)$ and $\mathscr{L} \subseteq \mathscr{K}$.

For each $\varpi \in \mathscr{L}$, $\aleph(\varpi)$ is a spherical fuzzy set such that $\aleph(\varpi) = \{(\varsigma, \mu_{\aleph(\varpi)}(\varsigma), \eta_{\aleph(\varpi)}(\varsigma), \vartheta_{\aleph(\varpi)}(\varsigma))\, \varsigma \in \Sigma\}$, where $\mu_{\aleph(\varpi)}(\varsigma), \eta_{\aleph(\varpi)}(\varsigma)$, $\vartheta_{\aleph(\varpi)}(\varsigma) \in [0, 1]$ are the membership degrees which are explained in Definition 2, with the same condition.

Definition 10 (see [27]). Let $\langle \aleph, \mathscr{L} \rangle$ and $\langle \Omega, \mathscr{M} \rangle$ be two SFSSs over Σ, and $\mathscr{L}, \mathscr{M} \subseteq \mathscr{K}$. Then $\langle \aleph, \mathscr{L} \rangle$ is said to be a SFS-subset of $\langle \Omega, \mathscr{M} \rangle$, if

(1) $\mathscr{L} \subseteq \mathscr{M}$
(2) $\forall \varpi \in \mathscr{L}$, $\aleph(\varpi) \widehat{\subseteq} \Omega(\varpi)$

Definition 11 (see [27]). Let $\langle \aleph, \mathscr{L} \rangle$ be a SFSS over the universal set Σ. Then the SFS-complement of $\langle \aleph, \mathscr{L} \rangle$, denoted by $\langle \aleph, \mathscr{L} \rangle^c$, is defined by $\langle \aleph, \mathscr{L} \rangle^c = \langle \aleph^c, \mathscr{L} \rangle$, where $\aleph^c: \mathscr{L} \longrightarrow SFS(\Sigma, \mathscr{K})$ is a mapping given by $\aleph^c(\varpi) = \{(\varsigma, \vartheta_{\aleph(\varpi)}(\varsigma), \eta_{\aleph(\varpi)}(\varsigma), \mu_{\aleph(\varpi)}(\varsigma))\, \varsigma \in \Sigma\}$ for every $\varpi \in \mathscr{L}$.

Definition 12 (see [27]). Let $\langle \aleph, \mathscr{L} \rangle$ and $\langle \Omega, \mathscr{M} \rangle$ be two SFSSs over Σ, and $\mathscr{L}, \mathscr{M} \subseteq \mathscr{K}$. then the SFS-union of $\langle \aleph, \mathscr{L} \rangle$ and $\langle \Omega, \mathscr{M} \rangle$, denoted by $\langle \aleph, \mathscr{L} \rangle \widehat{\cup} \langle \Omega, \mathscr{M} \rangle$, is a SFSS $\langle \Gamma, \mathscr{N} \rangle$, where $\mathscr{N} = \mathscr{L} \cup \mathscr{M}$ and $\forall \varpi \in \mathscr{N}$

$$\Gamma(e) = \begin{cases} \aleph(\varpi), & \text{if} \varpi \in \mathscr{L} - \mathscr{M} \\ \Omega(\varpi), & \text{if} \varpi \in \mathscr{M} - \mathscr{L} \\ \aleph(\varpi) \widehat{\cup} \Omega(\varpi), \text{if} \varpi \in \mathscr{L} \cap \mathscr{M}, \end{cases} \tag{8}$$

Now, we propose the definitions of spherical fuzzy soft restricted intersection, null spherical fuzzy soft, and absolute spherical fuzzy soft, which are essential for further discussions.

Definition 13. Let $\langle \aleph, \mathscr{L} \rangle$ and $\langle \Omega, \mathscr{M} \rangle$ be two SFSSs over Σ, $\mathscr{L}, \mathscr{M} \subseteq \mathscr{K}$. then the SFS-restricted intersection of $\langle \aleph, \mathscr{L} \rangle$ and $\langle \Omega, \mathscr{M} \rangle$, denoted by $\langle \aleph, \mathscr{L} \rangle \widehat{\cap} \langle \Omega, \mathscr{M} \rangle$, is a SFSS $\langle \Gamma, \mathscr{N} \rangle$, where $\mathscr{N} = \mathscr{L} \cap \mathscr{M}$ and $\forall \varpi \in \mathscr{N}$, $\Gamma(\varpi) = \aleph(\varpi) \widehat{\cap} \Omega(\varpi)$

Definition 14. Let $\langle \aleph, \mathscr{K} \rangle$ be a SFSS defined over Σ. $\langle \aleph, \mathscr{K} \rangle$ is said to be a null spherical fuzzy soft set, if for every $\varpi \in \mathscr{K}$, $\aleph(\varpi) = \{(\varsigma, 0, 0, 1) | \varsigma \in \Sigma\}$. That is, $\forall \varsigma \in \Sigma$ and $\varpi \in \mathscr{K}$, $\mu_{\aleph(\varpi)}(\varsigma) = 0$, $\eta_{\aleph(\varpi)}(\varsigma) = 0$ and $\vartheta_{\aleph(\varpi)}(\varsigma) = 1$. It is denoted by $\varnothing_{\mathscr{K}}$.

Definition 15. A SFSS $\langle \aleph, \mathscr{K} \rangle$ over Σ is said to be an absolute spherical fuzzy soft set, if for every $\varpi \in \mathscr{K}$, $\aleph(\varpi) = \{(\varsigma, 1, 0, 0) | \varsigma \in \Sigma\}$. That is, $\forall \varsigma \in \Sigma$ and $\varpi \in \mathscr{K}$, $\mu_{\aleph(\varpi)}(\varsigma) = 1$, $\eta_{\aleph(\varpi)}(\varsigma) = 0$ and $\vartheta_{\aleph(\varpi)}(\varsigma) = 0$. It is denoted by $\Sigma_{\mathscr{K}}$.

3. Spherical Fuzzy Soft Topology

In this section, we define the notion of spherical fuzzy soft topological space (SFS-topological space) so as to differentiate the concept from the existing fuzzy models and to mark the boundaries and deliberate the basic properties thereof. Further, we define SFS-subspace, SFS-point, SFS-nbd, SFS-basis, SFS-interior, SFS-closure, SFS-boundary and SFS-exterior with the support of befitting numerical illustrations.

Definition 16. Let $SFSS(\Sigma, \mathscr{K})$ be the collection of all spherical fuzzy soft sets over the universal set Σ and the parameter set $\mathscr{K}$. Let $\mathscr{L}, \mathscr{M} \widehat{\subseteq} \mathscr{K}$. Then a sub-collection $\mathscr{T}$ of $SFSS(\Sigma, \mathscr{K})$ is said to be a spherical fuzzy soft topology (SFS-topology) on Σ, if

(1) $\varnothing_{\mathscr{K}}, \Sigma_{\mathscr{K}} \in \mathscr{T}$
(2) If $\langle \aleph_1, \mathscr{L} \rangle$, $\langle \aleph_2, \mathscr{M} \rangle \in \mathscr{T}$, then $\langle \aleph_1, \mathscr{L} \rangle \widehat{\cap} \langle \aleph_2, \mathscr{M} \rangle \in \mathscr{T}$
(3) If $\langle \aleph_i, \mathscr{L}_i \rangle \in \mathscr{T}\, \forall i \in I$, an index set, then $\widehat{\cup}_{i \in I} \langle \aleph_i, \mathscr{L}_i \rangle \in \mathscr{T}$

The binary $(\Sigma_{\mathscr{K}}, \mathscr{T})$ is known as a spherical fuzzy soft topological space over Σ. Each member of $\mathscr{T}$ is considered as spherical fuzzy soft open sets and their complements are considered as spherical fuzzy soft closed sets.

Example 2. Let $\Sigma = \{\varsigma_1, \varsigma_2, \varsigma_3\}$ be the universal set with the attribute set $\mathscr{K} = \{\varpi_1, \varpi_2, \varpi_3, \varpi_4\}$. Let $\mathscr{L}, \mathscr{M} \subseteq \mathscr{K}$, where $\mathscr{L} = \{\varpi_1, \varpi_2\}$ and $\mathscr{M} = \{\varpi_1, \varpi_2, \varpi_3\}$. Consider the following SFSSs

$$\langle \aleph_1, \mathcal{L}\rangle = \begin{array}{c} \\ \varsigma_1 \\ \varsigma_2 \\ \varsigma_3 \end{array} \begin{array}{c} \begin{array}{cc} \varpi_1 & \varpi_2 \end{array} \\ \begin{pmatrix} (0.5, 0.2, 0.4) & (0.7, 0.2, 0.3) \\ (0.6, 0.3, 0.5) & (0.4, 0.2, 0.6) \\ (0.9, 0.2, 0.5) & (0.9, 0.1, 0.1) \end{pmatrix} \end{array}$$
$$\langle \aleph_2, \mathcal{M}\rangle = \begin{array}{c} \\ \varsigma_1 \\ \varsigma_2 \\ \varsigma_3 \end{array} \begin{array}{c} \begin{array}{ccc} \varpi_1 & \varpi_2 & \varpi_3 \end{array} \\ \begin{pmatrix} (0.6, 0.3, 0.2) & (0.8, 0.3, 0.1) & (0.1, 0.2, 0.9) \\ (0.8, 0.3, 0.4) & (0.6, 0.2, 0.5) & (0.3, 0.1, 0.7) \\ (1.0, 0.0, 0.0) & (0.9, 0.2, 0.1) & (0.5, 0.2, 0.3) \end{pmatrix} \end{array} \quad (9)$$

Then $\mathcal{T} = \{\sigma_{\mathcal{K}}, \varnothing_{\mathcal{K}}, \langle \aleph_1, \mathcal{L}\rangle, \langle \aleph_2, \mathcal{M}\rangle\}$ is a SFS-topology on Σ.

Definition 17. Let $(\Sigma_{\mathcal{K}}, \mathcal{T})$ be a SFS-topology on Σ and let $Z \subseteq \Sigma$ and $\mathcal{L} \subseteq \mathcal{K}$. Then $\mathcal{T}_Z = \{\langle \Omega, \mathcal{L}\rangle : \langle \Omega, \mathcal{L}\rangle = \langle \aleph, \mathcal{L}\rangle \widehat{\cap} Z_{\mathcal{K}}, \langle \aleph, \mathcal{L}\rangle \in \mathcal{T}\}$ is called the SFS-subspace topology of $\mathcal{T}$, where $Z_{\mathcal{K}}$ is the absolute SFSS on Z. The doublet $(Z_{\mathcal{K}}, \mathcal{T}_Z)$ is known as the SFS-subspace of the SFS-topological space $(\Sigma_{\mathcal{K}}, \mathcal{T})$.

Example 3. Consider Example 2. Suppose $Z = \{\varsigma_1, \varsigma_3\} \widehat{\subseteq} \Sigma$. Now,

$$Z_{\mathcal{K}} = \begin{array}{c} \\ \varsigma_1 \\ \varsigma_3 \end{array} \begin{array}{c} \begin{array}{cccc} \varpi_1 & \varpi_2 & \varpi_3 & \varpi_4 \end{array} \\ \begin{pmatrix} (1.0, 0.0, 0.0) & (1.0, 0.0, 0.0) & (1.0, 0.0, 0.0) & (1.0, 0.0, 0.0) \\ (1.0, 0.0, 0.0) & (1.0, 0.0, 0.0) & (1.0, 0.0, 0.0) & (1.0, 0.0, 0.0) \end{pmatrix} \end{array}$$
$$\langle \Omega_1, \mathcal{L}\rangle = \langle \aleph_1, \mathcal{L}\rangle \widehat{\cap} Z_{\mathcal{K}} = \begin{array}{c} \\ \varsigma_1 \\ \varsigma_3 \end{array} \begin{array}{c} \begin{array}{cc} \varpi_1 & \varpi_2 \end{array} \\ \begin{pmatrix} (0.5, 0.2, 0.4) & (0.7, 0.2, 0.3) \\ (0.9, 0.2, 0.5) & (0.9, 0.1, 0.1) \end{pmatrix} \end{array} \quad (10)$$
$$\langle \Omega_2, \mathcal{M}\rangle = \langle \aleph_2, \mathcal{M}\rangle \widehat{\cap} Z_{\mathcal{K}} = \begin{array}{c} \\ \varsigma_1 \\ \varsigma_3 \end{array} \begin{array}{c} \begin{array}{ccc} \varpi_1 & \varpi_2 & \varpi_3 \end{array} \\ \begin{pmatrix} (0.6, 0.3, 0.2) & (0.8, 0.3, 0.1) & (0.1, 0.2, 0.9) \\ (1.0, 0.0, 0.0) & (0.9, 0.2, 0.1) & (0.5, 0.2, 0.3) \end{pmatrix} \end{array}$$

Then $\mathcal{T}_Z = \{Z_{\mathcal{K}}, \varnothing_{\mathcal{K}}, \langle \Omega_1, \mathcal{L}\rangle, \langle \Omega_2, \mathcal{M}\rangle\}$ is a SFS-subspace topology of $\mathcal{T}$.

Definition 18. Let $(K_{\mathcal{K}}, \mathcal{T})$ be a SFS-topological space with $\mathcal{T} = \{\varnothing_{\mathcal{K}}, \Sigma_{\mathcal{K}}\}$, then $\mathcal{T}$ is said to be the indiscrete SFS-topology on Σ and $(\Sigma_{\mathcal{K}}, \mathcal{T})$ is called the indiscrete SFS-topological space. The indiscrete SFS-topology is the smallest SFS-topology on Σ.

Definition 19. Let $(\Sigma_{\mathcal{K}}, \mathcal{T})$ be a SFS-topological space with $\mathcal{T} = SFS(\Sigma, \mathcal{K})$, then $\mathcal{T}$ is called the discrete SFS-topology on Σ and $(\Sigma_{\mathcal{K}}, \mathcal{T})$ is said to be the discrete SFS-topological space. The discrete SFS-topology is the largest SFS-topology on Σ.

Example 4. Let Σ be the universal set and $\mathcal{K}$ be the parameter set, where $\Sigma = \{\varsigma_1, \varsigma_2\}$ and $\mathcal{K} = \{\varpi_1, \varpi_2, \varpi_3, \varpi_4\}$. Let $\mathcal{L}_1, \mathcal{L}_2, \mathcal{M}_1, \mathcal{M}_2 \subseteq \mathcal{K}$ with $\mathcal{L}_1 = \mathcal{M}_2 = \{\varpi_1, \varpi_2\}$, $\mathcal{L}_2 = \{\varpi_1\}$, $\mathcal{M}_1 = \{\varpi_1, \varpi_2, \varpi_3\}$. Consider the following SFSSs;

$$\langle \aleph_1, \mathcal{L}_1\rangle = \begin{array}{c} \\ \varsigma_1 \\ \varsigma_2 \end{array} \begin{array}{c} \begin{array}{cc} \varpi_1 & \varpi_2 \end{array} \\ \begin{pmatrix} (0.9, 0.2, 0.1) & (0.6, 0.2, 0.2) \\ (0.7, 0.1, 0.3) & (0.4, 0.3, 0.1) \end{pmatrix} \end{array}$$
$$\langle \aleph_2, \mathcal{L}_2\rangle = \begin{array}{c} \\ \varsigma_1 \\ \varsigma_2 \end{array} \begin{array}{c} \varpi_1 \\ \begin{pmatrix} (0.7, 0.2, 0.2) \\ (0.5, 0.1, 0.6) \end{pmatrix} \end{array}$$
$$\langle \Omega_1, \mathcal{M}_1\rangle = \begin{array}{c} \\ \varsigma_1 \\ \varsigma_2 \end{array} \begin{array}{c} \begin{array}{ccc} \varpi_1 & \varpi_2 & \varpi_3 \end{array} \\ \begin{pmatrix} (0.7, 0.2, 0.3) & (0.8, 0.2, 0.2) & (0.5, 0.3, 0.6) \\ (0.2, 0.1, 0.9) & (0.5, 0.2, 0.3) & (0.8, 0.2, 0.1) \end{pmatrix} \end{array} \quad (11)$$
$$\langle \Omega_2, \mathcal{M}_2\rangle = \begin{array}{c} \\ \varsigma_1 \\ \varsigma_2 \end{array} \begin{array}{c} \begin{array}{cc} \varpi_1 & \varpi_2 \end{array} \\ \begin{pmatrix} (0.6, 0.1, 0.4) & (0.8, 0.0, 0.3) \\ (0.1, 0.0, 0.9) & (0.4, 0.1, 0.7) \end{pmatrix} \end{array}$$

Then $\mathcal{T}_1 = \{\varnothing_{\mathcal{K}}, \Sigma_{\mathcal{K}}, \langle \aleph_1, \mathcal{L}_1\rangle, \langle \aleph_2, \mathcal{L}_2\rangle\}$ and $\mathcal{T}_2 = \{\varnothing_{\mathcal{K}}, \Sigma_{\mathcal{K}}, \langle \Omega_1, \mathcal{M}_1\rangle, \langle \Omega_2, \mathcal{M}_2\rangle\}$ are two SFS-topologies. Consider $\mathcal{T}_1 \cup \mathcal{T}_2 = \{\varnothing_{\mathcal{K}}, \Sigma_{\mathcal{K}}, \langle \aleph_1, \mathcal{L}_1\rangle, \langle \aleph_2, \mathcal{L}_2\rangle, \langle \Omega_1, \mathcal{M}_1\rangle, \langle \Omega_2, \mathcal{M}_2\rangle\}$. Now.

$$\langle \aleph_1, \mathcal{L}_1\rangle \,\widehat{\cap}\, \langle \Omega_1, \mathcal{M}_1\rangle = \begin{matrix} \\ \varsigma_1 \\ \varsigma_3 \end{matrix} \begin{pmatrix} \varpi_1 & \varpi_2 \\ (0.7, 0.2, 0.3) & (0.6, 0.2, 0.2) \\ (0.2, 0.1, 0.9) & (0.4, 0.2, 0.3) \end{pmatrix} \tag{12}$$

Thus, $\langle \aleph_1, \mathcal{L}_1\rangle, \langle \Omega_1, \mathcal{M}_1\rangle \in \mathcal{T}_1 \cup \mathcal{T}_2$, but $\langle \aleph_1, \mathcal{L}_1\rangle \,\widehat{\cap}\, \langle \Omega_1, \mathcal{M}_1\rangle \notin \mathcal{T}_1 \cup \mathcal{T}_2$. Therefore, $\mathcal{T}_1 \cup \mathcal{T}_2$ is not a SFS-topology on Σ.

Theorem 1. *Suppose $\mathcal{T}_1$ and $\mathcal{T}_2$ are two SFS-topologies on Σ, then $\mathcal{T}_1 \cap \mathcal{T}_2$ is also a SFS-topology on Σ. But, $\mathcal{T}_1 \cup \mathcal{T}_2$ need not be a SFS-topology on Σ.*

Proof. Suppose that, $\mathcal{T}_1$ and $\mathcal{T}_2$ are two SFS-topologies on Σ.

Since $\varnothing_{\mathcal{K}}, \Sigma_{\mathcal{K}} \in \mathcal{T}_1$ and $\varnothing_{\mathcal{K}}, \Sigma_{\mathcal{K}} \in \mathcal{T}_2$, then $\varnothing_{\mathcal{K}}, \Sigma_{\mathcal{K}} \in \mathcal{T}_1 \cap \mathcal{T}$.

Let $\langle \aleph, \mathcal{L}\rangle, \langle \Omega, \mathcal{M}\rangle \in \mathcal{T}_1 \cap \mathcal{T}_2 \Rightarrow \langle \aleph, \mathcal{L}\rangle, \langle \Omega, \mathcal{M}\rangle \in \mathcal{T}_1$ and $\langle \aleph, \mathcal{L}\rangle, \langle \Omega, \mathcal{M}\rangle \in \mathcal{T}_2 \Rightarrow \langle \aleph, \mathcal{L}\rangle \,\widehat{\cap}\, \langle \Omega, \mathcal{M}\rangle \in \mathcal{T}_1$ and $\langle \aleph, \mathcal{L}\rangle \,\widehat{\cap}\, \langle \Omega, \mathcal{M}\rangle \in \mathcal{T}_2 \Rightarrow \langle \aleph, \mathcal{L}\rangle \,\widehat{\cap}\, \langle \Omega, \mathcal{M}\rangle \in \mathcal{T}_1 \cap \mathcal{T}_2$.

Let $\langle \aleph_i, \mathcal{L}_i\rangle \in \mathcal{T}_1 \cap \mathcal{T}_2$, $i \in I$, an index set.

$\Rightarrow \langle \aleph_i, \mathcal{L}_i\rangle \in \mathcal{T}_1$ and $\langle \aleph_i, \mathcal{L}_i\rangle \in \mathcal{T}_2$, $\forall i \in I \Rightarrow \widehat{\cup}_{i \in I} \langle \aleph_i, \mathcal{L}_i\rangle \in \mathcal{T}_1$ and $\widehat{\cup}_{i \in I} \langle \aleph_i, \mathcal{L}_i\rangle \in \mathcal{T}_2 \widehat{\cup}_{i \in I} \langle \aleph_i, \mathcal{L}_i\rangle \in \mathcal{T}_1 \cap \mathcal{T}_2$

Thus $\mathcal{T}_1 \cap \mathcal{T}_2$ satisfies all requirements of SFS-topology on Σ. □

Definition 20. Consider the two SFS-topologies $\mathcal{T}_1$ and $\mathcal{T}_2$ on Σ. $\mathcal{T}_1$ is called weaker or coarser than $\mathcal{T}_2$ or $\mathcal{T}_2$ is called finer or stronger than $\mathcal{T}_1$ if and only if $\mathcal{T}_1 \subseteq \mathcal{T}_2$.

Remark 3.1. If either $\mathcal{T}_1 \subseteq \mathcal{T}_2$ or $\mathcal{T}_2 \subseteq \mathcal{T}_1$, then $\mathcal{T}_1$ and $\mathcal{T}_2$ are comparable. Otherwise $\mathcal{T}_1$ and $\mathcal{T}_2$ are not comparable.

Example 5. Consider $\Sigma = \{\varsigma_1, \varsigma_2\}$ as the universal set with the attribute set $\mathcal{K} = \{\varpi_1, \varpi_2, \varpi_3, \varpi_4\}$. Let $\mathcal{L}_1, \mathcal{L}_2, \mathcal{L}_1 \subseteq \mathcal{K}$, where $\mathcal{L}_1 = \{\varpi_1, \varpi_2, \varpi_3\}$, $\mathcal{L}_2 = \{\varpi_1, \varpi_2\}$ and $\mathcal{L}_3 = \{\varpi_1\}$. The SFSSs $\langle \aleph_1, \mathcal{L}_1\rangle, \langle \aleph_2, \mathcal{L}_2\rangle, \langle \aleph_3, \mathcal{L}_3\rangle$ are given as follows:

$$\begin{aligned} \langle \aleph_1, \mathcal{L}_1\rangle &= \begin{matrix} \\ \varsigma_1 \\ \varsigma_2 \end{matrix} \begin{pmatrix} \varpi_1 & \varpi_2 & \varpi_3 \\ (0.3, 0.1, 0.7) & (0.4, 0.2, 0.3) & (0.7, 0.1, 0.3) \\ (0.8, 0.2, 0.3) & (0.9, 0.1, 0.1) & (0.6, 0.2, 0.4) \end{pmatrix} \\ \langle \aleph_2, \mathcal{L}_2\rangle &= \begin{matrix} \\ \varsigma_1 \\ \varsigma_2 \end{matrix} \begin{pmatrix} \varpi_1 & \varpi_2 \\ (0.2, 0.1, 0.8) & (0.2, 0.1, 0.4) \\ (0.7, 0.1, 0.4) & (0.7, 0.1, 0.3) \end{pmatrix} \\ \langle \aleph_3, \mathcal{L}_3\rangle &= \begin{matrix} \\ \varsigma_1 \\ \varsigma_2 \end{matrix} \begin{pmatrix} \varpi_1 \\ (0.1, 0.1, 0.9) \\ (0.6, 0.0, 0.5) \end{pmatrix} \end{aligned} \tag{13}$$

Here, $\mathcal{T}_1 = \{\varnothing_{\mathcal{K}}, \Sigma_{\mathcal{K}}, \langle \aleph_1, \mathcal{L}_1\rangle, \langle \aleph_2, \mathcal{L}_2\rangle, \langle \aleph_3, \mathcal{L}_3\rangle\}$ and $\mathcal{T}_2 = \{\varnothing_{\mathcal{K}}, \Sigma_{\mathcal{K}}, \langle \aleph_1, \mathcal{L}_1\rangle\}$ are two SFS-topologies on Σ. It is clear that $\mathcal{T}_2 \subseteq \mathcal{T}_1$. Thus $\mathcal{T}_1$ is finer than $\mathcal{T}_2$ or $\mathcal{T}_2$ is weaker than $\mathcal{T}_1$.

Definition 21. A SFSS $\langle \aleph, \mathcal{L}\rangle$ is said to be a spherical fuzzy soft point (SFS-point), denoted by $\varpi(\aleph)$, if for every $\varpi \in \mathcal{L}$, $\aleph(\varpi) \neq \{(\varsigma, 0, 0, 1) | \varsigma \in \Sigma\}$ and $\aleph(\widehat{\varpi}) = \{(\varsigma, 0, 0, 1) | \varsigma \in \Sigma\}$, $\forall \widehat{\varpi} \in \mathcal{L} - \{\varpi\}$. Note that, any SFS-point $\varpi(\aleph)$ (say) is also considered as a singleton SFS-subset of the SFSS $\langle \aleph, \mathcal{L}\rangle$.

Definition 22. A SFS-point $\varpi(\aleph)$ is said to be in the SFSS $\langle \Omega, \mathcal{L}\rangle$, that is, $\varpi(\aleph) \in \langle \Omega, \mathcal{L}\rangle$, if $\aleph(\varpi) \,\widehat{\subseteq}\, \Omega(\varpi)$, for every $\varpi \in \mathcal{L}$.

Example 6. Suppose that $\Sigma = \{\varsigma_1, \varsigma_2, \varsigma_3\}$ and $\mathcal{L} = \{\varpi_1, \varpi_2, \varpi_3\} \subseteq \mathcal{K} = \{\varpi_1, \varpi_2, \varpi_3, \varpi_4\}$. Consider the SFSS

$$\langle \aleph, \mathcal{L}\rangle = \begin{matrix} \\ \varsigma_1 \\ \varsigma_2 \\ \varsigma_3 \end{matrix} \begin{pmatrix} \varpi_1 & \varpi_2 & \varpi_3 \\ (0.0, 0.0, 1.0) & (0.0, 0.0, 1.0) & (0.5, 0.2, 0.8) \\ (0.0, 0.0, 1.0) & (0.0, 0.0, 1.0) & (0.9, 0.1, 0.1) \\ (0.0, 0.0, 1.0) & (0.0, 0.0, 1.0) & (0.4, 0.2, 0.6) \end{pmatrix} \tag{14}$$

Here, $\varpi_3 \in \mathscr{L}$ and $\aleph(\varpi_3) \neq \{(\varsigma, 0, 0, 1) | \varsigma \in \Sigma\}$. But, for $\mathscr{L} - \{\varpi_3\} = \{\varpi_1, \varpi_2\}$, $\aleph(\varpi_1) = \aleph(\varpi_2) = \{(\varsigma, 0, 0, 1) | \varsigma \in \Sigma\}$. Thus, $\langle \aleph, \mathscr{L} \rangle$ is a SFS-point in Σ and denoted by $\varpi_3(\aleph)$.

Let

$$\langle \Omega, \mathcal{L} \rangle = \begin{array}{c} \\ \varsigma_1 \\ \varsigma_2 \\ \varsigma_3 \end{array} \begin{array}{c} \begin{array}{ccc} \varpi_1 & \varpi_2 & \varpi_3 \end{array} \\ \begin{pmatrix} (0.1, 0.2, 0.8) & (0.0, 0.0, 1.0) & (0.6, 0.3, 0.4) \\ (0.3, 0.1, 0.4) & (0.0, 0.0, 1.0) & (0.9, 0.1, 0.1) \\ (0.9, 0.1, 0.1) & (0.0, 0.0, 1.0) & (0.7, 0.2, 0.3) \end{pmatrix} \end{array} \tag{15}$$

Here, $\aleph(\varpi_3) \widehat{\subseteq} \Omega(\varpi_3)$. Thus, we can say that $\varpi_3(\aleph) \in \langle \Omega, \mathscr{L} \rangle$.

Definition 23. Let $\langle \Gamma, \mathscr{L} \rangle$ be a SFSS over Σ. $\langle \Gamma, \mathscr{L} \rangle$ is said to be a spherical fuzzy soft neighbourhood (SFS-nbd) of the SFS-point $\varpi(\aleph)$ over Σ, if there exist a SFS-open set $\langle \Omega, \mathscr{M} \rangle$ such that $\varpi(\aleph) \in \langle \Omega, \mathscr{M} \rangle \widehat{\subseteq} \langle \Gamma, \mathscr{L} \rangle$.

Definition 24. Let $\langle \Gamma, \mathscr{L} \rangle$ be a SFSS over Σ. $\langle \Gamma, \mathscr{L} \rangle$ is said to be a spherical fuzzy soft neighbourhood (SFS-nbd) of the SFSS $\langle \aleph, \mathscr{M} \rangle$, if there exist a SFS-open set $\langle \Omega, \mathscr{N} \rangle$ such that $\langle \aleph, \mathscr{M} \rangle \widehat{\subseteq} \langle \Omega, \mathscr{N} \rangle \widehat{\subseteq} \langle \Gamma, \mathscr{L} \rangle$.

Theorem 2. *Let $(\Sigma_{\mathscr{K}}, \mathscr{T})$ be a SFS-topological space. A SFSS $\langle \aleph, \mathscr{L} \rangle$ is open if and only if for each SFSS $\langle \Omega, \mathscr{M} \rangle$ such that $\langle \Omega, \mathscr{M} \rangle \widehat{\subseteq} \langle \aleph, \mathscr{L} \rangle$, $\langle \aleph, \mathscr{L} \rangle$ is a SFS-nbd of $\langle \Omega, \mathscr{M} \rangle$.*

Proof. Suppose that the SFSS $\langle \aleph, \mathscr{L} \rangle$ is SFS-open. That is, $\langle \aleph, \mathscr{L} \rangle \in \mathscr{T}$.

Thus for each $\langle \Omega, \mathscr{M} \rangle \widehat{\subseteq} \langle \aleph, \mathscr{L} \rangle$, $\langle \aleph, \mathscr{L} \rangle$ is a SFS-nbd of $\langle \Omega, \mathscr{M} \rangle$.

Conversely, suppose that, for each $\langle \Omega, \mathscr{M} \rangle \widehat{\subseteq} \langle \aleph, \mathscr{L} \rangle$, $\langle \aleph, \mathscr{L} \rangle$ is A SFS-nbd of $\langle \Omega, \mathscr{M} \rangle$.

Since $\langle \aleph, \mathscr{L} \rangle \widehat{\subseteq} \langle \aleph, \mathscr{L} \rangle$, $\langle \aleph, \mathscr{L} \rangle$ is a SFS-nbd of $\langle \aleph, \mathscr{L} \rangle$ itself.

Therefore, there exist an open set $\langle \Gamma, \mathscr{N} \rangle$ such that $\langle \aleph, \mathscr{L} \rangle \widehat{\subseteq} \langle \Gamma, \mathscr{N} \rangle \widehat{\subseteq} \langle \aleph, \mathscr{L} \rangle \Rightarrow \langle \aleph, \mathscr{L} \rangle = \langle \Gamma, \mathscr{N} \rangle \Rightarrow \langle \aleph, \mathscr{L} \rangle$ is open. □

Definition 25. Let $(\Sigma_{\mathscr{K}}, \mathscr{T})$ be a SFS-topological space. A sub-collection $\mathscr{B}$ of the SFS-topology $\mathscr{T}$ is referred as a spherical fuzzy soft basis (SFS-basis) for $\mathscr{T}$, if for each $\langle \aleph, \mathscr{L} \rangle \in \mathscr{T}$, $\exists B \in \mathscr{B}$ such that (Tex translation failed).

Example 7. Let $\Sigma = \{\varsigma_1, \varsigma_2\}$ and $\mathscr{K} = \{\varpi_1, \varpi_2, \varpi_3\}$. Let $\mathscr{L}_i \subseteq \mathscr{K}$ $i = 1$ to 11 with $\mathscr{L}_1 = \mathscr{L}_2 = \mathscr{L}_3 = \mathscr{L}_4 = \mathscr{L}_5 = \mathscr{L}_6 = \mathscr{L}_7 = \mathscr{K}$, $\mathscr{L}_8 = \mathscr{L}_9 = \{\varpi_1, \varpi_2\}$, and $\mathscr{L}_{10} = \mathscr{L}_{11} = \{\varpi_1\}$. Consider the following SFSSs;

$$
\begin{aligned}
\langle \aleph_1, \mathcal{L}_1 \rangle &= \begin{array}{cccc} & \varpi_1 & \varpi_2 & \varpi_3 \\ \varsigma_1 & (1.0, 0.0, 0.0) & (0.6, 0.2, 0.4) & (1.0, 0.0, 0.0) \\ \varsigma_2 & (0.6, 0.3, 0.2) & (1.0, 0.0, 0.0) & (0.7, 0.3, 0.2) \end{array} \\
\langle \aleph_2, \mathcal{L}_2 \rangle &= \begin{array}{cccc} & \varpi_1 & \varpi_2 & \varpi_3 \\ \varsigma_1 & (0.9, 0.1, 0.2) & (0.6, 0.2, 0.4) & (0.3, 0.4, 0.5) \\ \varsigma_2 & (0.6, 0.3, 0.2) & (0.8, 0.2, 0.1) & (0.7, 0.3, 0.2) \end{array} \\
\langle \aleph_3, \mathcal{L}_3 \rangle &= \begin{array}{cccc} & \varpi_1 & \varpi_2 & \varpi_3 \\ \varsigma_1 & (0.9, 0.1, 0.2) & (1.0, 0.0, 0.0) & (0.3, 0.4, 0.5) \\ \varsigma_2 & (1.0, 0.0, 0.0) & (0.8, 0.2, 0.1) & (1.0, 0.0, 0.0) \end{array} \\
\langle \aleph_4, \mathcal{L}_4 \rangle &= \begin{array}{cccc} & \varpi_1 & \varpi_2 & \varpi_3 \\ \varsigma_1 & (1.0, 0.0, 0.0) & (0.6, 0.2, 0.4) & (1.0, 0.0, 0.0) \\ \varsigma_2 & (0.6, 0.3, 0.2) & (1.0, 0.0, 0.0) & (1.0, 0.0, 0.0) \end{array} \\
\langle \aleph_5, \mathcal{L}_5 \rangle &= \begin{array}{cccc} & \varpi_1 & \varpi_2 & \varpi_3 \\ \varsigma_1 & (0.9, 0.1, 0.2) & (1.0, 0.0, 0.0) & (0.3, 0.4, 0.5) \\ \varsigma_2 & (1.0, 0.0, 0.0) & (0.8, 0.2, 0.1) & (0.7, 0.3, 0.2) \end{array} \\
\langle \aleph_6, \mathcal{L}_6 \rangle &= \begin{array}{cccc} & \varpi_1 & \varpi_2 & \varpi_3 \\ \varsigma_1 & (1.0, 0.0, 0.0) & (1.0, 0.0, 0.0) & (1.0, 0.0, 0.0) \\ \varsigma_2 & (1.0, 0.0, 0.0) & (1.0, 0.0, 0.0) & (0.7, 0.3, 0.2) \end{array} \\
\langle \aleph_7, \mathcal{L}_7 \rangle &= \begin{array}{cccc} & \varpi_1 & \varpi_2 & \varpi_3 \\ \varsigma_1 & (0.9, 0.1, 0.2) & (0.6, 0.2, 0.4) & (0.3, 0.4, 0.5) \\ \varsigma_2 & (0.6, 0.3, 0.2) & (0.8, 0.2, 0.1) & (1.0, 0.0, 0.0) \end{array} \\
\langle \aleph_8, \mathcal{L}_8 \rangle &= \begin{array}{ccc} & \varpi_1 & \varpi_2 \\ \varsigma_1 & (0.9, 0.1, 0.2) & (1.0, 0.0, 0.0) \\ \varsigma_2 & (1.0, 0.0, 0.0) & (0.8, 0.2, 0.1) \end{array} \\
\langle \aleph_9, \mathcal{L}_9 \rangle &= \begin{array}{ccc} & \varpi_1 & \varpi_2 \\ \varsigma_1 & (0.9, 0.1, 0.2) & (0.6, 0.2, 0.4) \\ \varsigma_2 & (0.6, 0.3, 0.2) & (0.8, 0.2, 0.1) \end{array} \\
\langle \aleph_{10}, \mathcal{L}_{10} \rangle &= \begin{array}{cc} & \varpi_1 \\ \varsigma_1 & (0.3, 0.4, 0.5) \\ \varsigma_2 & (1.0, 0.0, 0.0) \end{array} \\
\langle \aleph_{11}, \mathcal{L}_{11} \rangle &= \begin{array}{cc} & \varpi_1 \\ \varsigma_1 & (0.3, 0.4, 0.5) \\ \varsigma_2 & (0.7, 0.3, 0.2) \end{array}
\end{aligned} \tag{16}
$$

Then the sub-collection

$$\mathcal{B} = \{B_1 = \langle \aleph_1, \mathcal{L}_1, \rangle, B_2 = \langle \aleph_2, \mathcal{L}_2 \rangle, B_3 = \langle \aleph_8, \mathcal{L}_8 \rangle, B_4 = \langle \aleph_9, \mathcal{L}_9 \rangle, B_5 = \langle \aleph_{10}, \mathcal{L}_{10} \rangle, B_6 = \langle \aleph_{11}, \mathcal{L}_{11} \rangle\}$$

is a SFS-basis for the SFS-topology $\mathscr{T} = \{\varnothing_{\mathscr{K}}, \Sigma_{\mathscr{K}}, \langle \aleph_i, \mathscr{L}_i \rangle, i = 1 \text{ to } 11\}$.

Theorem 3. *Let $\mathscr{B}$ be a SFS-basis for a SFS-topology $\mathscr{T}$, then for each $\varpi \in \mathscr{L}, \mathscr{L} \subseteq \mathscr{K}$, $\mathscr{B}_\varpi = \{\aleph(\varpi): \langle \aleph, \mathscr{L} \rangle \in \mathscr{B}\}$ acts as a spherical fuzzy basis for the spherical fuzzy topology $\mathscr{T}(\varpi) = \{\aleph(\varpi): \langle \aleph, \mathscr{L} \rangle \in \mathscr{T}\}$.*

Proof. Suppose that $\aleph(\varpi) \in \mathscr{T}(\varpi)$ for some $\varpi \in \mathscr{L}$
$\Rightarrow \langle \aleph, \mathscr{L} \rangle \in \mathscr{T}$

Since $\mathscr{B}$ is a SFS-basis for the SFS-topology $\mathscr{T}$, $\exists B' \widehat{\subseteq} B$ such that $\langle \aleph, \mathscr{L} \rangle = \widehat{\cup} B \Rightarrow \aleph(\varpi) = \widehat{\cup} B_\varpi$, where $B_\varpi = \{\aleph(\varpi): \langle \aleph, \mathscr{L} \rangle \in B\} \widehat{\subseteq} \mathscr{B}_\varpi$ $\aleph(\varpi) = \widehat{\cup} \mathscr{B}_\varpi \Rightarrow \mathscr{B}_\varpi$ is a spherical fuzzy basis for the spherical fuzzy topology $\mathscr{T}(\varpi)$. □

Theorem 4. *Let $(\Sigma_E, \mathscr{T})$ be a SFS-topological space. Let $\mathscr{B} = \{\langle \aleph_i, \mathscr{L}_i \rangle : i \in I\}$ be a sub-collection of SFS-topology $\mathscr{T}$. $\mathscr{B}$ is a SFS-basis for $\mathscr{T}$ if and only if for any SFS-open set $\langle \Omega, \mathscr{M} \rangle$ and a SFS-point $\varpi(\Gamma) \in \langle \Omega, \mathscr{M} \rangle$, there exist a $\langle \aleph_i, \mathscr{L}_i \rangle \in \mathscr{B}$ for some $i \in I$, such that $\varpi(\Gamma) \in \langle \aleph_i, \mathscr{L}_i \rangle \widehat{\subseteq} \langle \Omega, \mathscr{M} \rangle$.*

Proof. Suppose that, $\mathscr{B} = \{\langle \aleph_i, \mathscr{L}_i\rangle : i \in I\} \subseteq \mathscr{T}$ is a SFS-basis for the SFS-topology $\mathscr{T}$.

For any SFS-open set $\langle \Omega, \mathscr{M}\rangle$, there exists SFSSs $\langle \aleph_j, \mathscr{L}_j\rangle$, $j \in J \subseteq I$, where (Tex translation failed)

Thus, for any SFS-point $\varpi(\Gamma) \in \langle \Omega, \mathscr{M}\rangle$, there exist a $\langle \aleph_j, \mathscr{L}_j\rangle \in \mathscr{B}$ such that $\varpi(\Gamma) \in \langle \aleph_j, \mathscr{L}_j\rangle \widehat{\subseteq} \langle \Omega, \mathscr{M}\rangle$.

Conversely, suppose for any SFS-open set $\langle \Omega, \mathscr{M}\rangle$ and a SFS-point $\varpi(\Gamma) \in \langle \Omega, \mathscr{M}\rangle$, there exist a $\langle \aleph_i, \mathscr{L}_i\rangle \in \mathscr{B}$ such that $\varpi(\Gamma) \in \langle \aleph_i, \mathscr{L}_i\rangle \widehat{\subseteq} \langle \Omega, \mathscr{M}\rangle$

Thus, $\langle \Omega, \mathscr{M}\rangle \widehat{\subseteq} \widehat{\bigcup}_{\varpi(\Gamma) \in \langle \Omega, \mathscr{M}\rangle} \langle \aleph_i, \mathscr{L}_i\rangle \widehat{\subseteq} \langle \Omega, \mathscr{M}\rangle \langle \Omega, \mathscr{M}\rangle \widehat{\bigcup}_{\varpi(\Gamma) \in \langle \Omega, \mathscr{M}\rangle} \langle \aleph_i, \mathscr{L}_i\rangle$

Since $\langle \aleph_i, \mathscr{L}_i\rangle \in \mathscr{B}$, $\mathscr{B}$ is a SFS-basis for the SFS-topology $\mathscr{T}$. □

Definition 26. Suppose $(\Sigma_{\mathscr{K}}, \mathscr{T})$ is a SFS-topological space and $\langle \aleph, \mathscr{L}\rangle$ is a SFSS over Σ, where $\mathscr{L} \subseteq \mathscr{K}$. Then

(1) The SFS-union of all SFS-open subsets of $\langle \aleph, \mathscr{L}\rangle$ is known as spherical fuzzy soft interior (SFS-interior) of $\langle \aleph, \mathscr{L}\rangle$, symbolized by $\langle \aleph, \mathscr{L}\rangle$. It is the largest SFS-open set contained in $\langle \aleph, \mathscr{L}\rangle$. That is, $\langle \aleph, \mathscr{L}\rangle^{\circ} \widehat{\subseteq} \langle \aleph, \mathscr{L}\rangle$.

(2) The SFS-intersection of all SFS-closed supersets of $\langle \aleph, \mathscr{L}\rangle$ is known as spherical fuzzy soft closure (SFS-closure) of $\langle \aleph, \mathscr{L}\rangle$, symbolized by $\overline{\langle \aleph, \mathscr{L}\rangle}$. It is the smallest SFS-closed set containing $\langle \aleph, \mathscr{L}\rangle$. That is, $\langle \aleph, \mathscr{L}\rangle \widehat{\subseteq} \overline{\langle \aleph, \mathscr{L}\rangle}$.

(3) The spherical fuzzy soft boundary (SFS-boundary) of $\langle \aleph, \mathscr{L}\rangle$, denoted by $\partial\langle \aleph, \mathscr{L}\rangle$, is defined as follows: $\partial\langle \aleph, \mathscr{L}\rangle = \overline{\langle \aleph, \mathscr{L}\rangle} \widehat{\cap} \overline{\langle \aleph, \mathscr{L}\rangle^c}$

(4) The spherical fuzzy soft exterior (SFS-exterior) of $\langle \aleph, \mathscr{L}\rangle$, denoted by $Ext\langle \aleph, \mathscr{L}\rangle$, is defined as follows: $Ext\langle \aleph, \mathscr{L}\rangle = (\langle \aleph, \mathscr{L}\rangle^c)^{\circ}$

Example 8. Suppose that $\Sigma = \{\varsigma_1, \varsigma_2\}$ is the universal set with the attribute set $\mathscr{K} = \{\varpi_1, \varpi_2, \varpi_3\}$. Consider the SFS-topology $\mathscr{T} = \{\varnothing_{\mathscr{K}}, \Sigma_{\mathscr{K}}, \langle \aleph_1, \mathscr{K}\rangle, \langle \aleph_2, \mathscr{K}\rangle, \langle \aleph_3, \mathscr{K}\rangle\}$, where

$$\begin{aligned}
\langle \aleph_1, \mathcal{K}\rangle &= \begin{matrix} \\ \varsigma_1 \\ \varsigma_2 \end{matrix} \overset{\begin{matrix}\varpi_1 & \varpi_2 & \varpi_3\end{matrix}}{\begin{pmatrix} (0.8,0.2,0.3) & (0.6,0.3,0.4) & (0.7,0.2,0.4) \\ (0.4,0.1,0.5) & (0.5,0.1,0.5) & (0.3,0.2,0.4) \end{pmatrix}} \\
\langle \aleph_2, \mathcal{K}\rangle &= \begin{matrix} \\ \varsigma_1 \\ \varsigma_2 \end{matrix} \overset{\begin{matrix}\varpi_1 & \varpi_2 & \varpi_3\end{matrix}}{\begin{pmatrix} (0.7,0.1,0.4) & (0.5,0.2,0.5) & (0.7,0.1,0.5) \\ (0.3,0.1,0.6) & (0.4,0.0,0.6) & (0.2,0.1,0.8) \end{pmatrix}} \\
\langle \aleph_3, \mathcal{K}\rangle &= \begin{matrix} \\ \varsigma_1 \\ \varsigma_2 \end{matrix} \overset{\begin{matrix}\varpi_1 & \varpi_2 & \varpi_3\end{matrix}}{\begin{pmatrix} (0.5,0.0,0.6) & (0.4,0.2,0.6) & (0.6,0.1,0.7) \\ (0.1,0.1,0.9) & (0.2,0.0,0.8) & (0.1,0.0,0.9) \end{pmatrix}}
\end{aligned} \quad (17)$$

Clearly, the members of $\mathscr{T}$ are the SFS-open sets. Now, the corresponding closed sets are given as follows: $(\varnothing_{\mathscr{K}})^c = \Sigma_{\mathscr{K}}$ $(\Sigma_{\mathscr{K}})^c = \varnothing_{\mathscr{K}}$

$$\begin{aligned}
(\emptyset_{\mathcal{K}})^c &= \Sigma_{\mathcal{K}} \\
(\Sigma_{\mathcal{K}})^c &= \emptyset_{\mathcal{K}} \\
\langle \aleph_1, \mathcal{K}\rangle^c &= \begin{matrix} \\ \varsigma_1 \\ \varsigma_2 \end{matrix} \overset{\begin{matrix}\varpi_1 & \varpi_2 & \varpi_3\end{matrix}}{\begin{pmatrix} (0.3,0.2,0.8) & (0.4,0.3,0.6) & (0.4,0.2,0.7) \\ (0.5,0.1,0.4) & (0.5,0.1,0.5) & (0.4,0.2,0.3) \end{pmatrix}} \\
\langle \aleph_2, \mathcal{K}\rangle^c &= \begin{matrix} \\ \varsigma_1 \\ \varsigma_2 \end{matrix} \overset{\begin{matrix}\varpi_1 & \varpi_2 & \varpi_3\end{matrix}}{\begin{pmatrix} (0.4,0.1,0.7) & (0.5,0.2,0.5) & (0.5,0.1,0.7) \\ (0.6,0.1,0.3) & (0.6,0.0,0.4) & (0.8,0.1,0.2) \end{pmatrix}} \\
\langle \aleph_3, \mathcal{K}\rangle^c &= \begin{matrix} \\ \varsigma_1 \\ \varsigma_2 \end{matrix} \overset{\begin{matrix}\varpi_1 & \varpi_2 & \varpi_3\end{matrix}}{\begin{pmatrix} (0.6,0.0,0.5) & (0.6,0.2,0.4) & (0.7,0.1,0.6) \\ (0.9,0.1,0.1) & (0.8,0.0,0.2) & (0.9,0.0,0.1) \end{pmatrix}}
\end{aligned} \quad (18)$$

Consider the following SFSS.

$$\langle \aleph, \mathcal{K}\rangle = \begin{matrix} \\ \varsigma_1 \\ \varsigma_2 \end{matrix} \overset{\begin{matrix}\varpi_1 & \varpi_2 & \varpi_3\end{matrix}}{\begin{pmatrix} (0.7,0.1,0.3) & (0.6,0.2,0.4) & (0.7,0.1,0.5) \\ (0.3,0.1,0.7) & (0.4,0.1,0.5) & (0.2,0.2,0.6) \end{pmatrix}} \quad (19)$$

Thus.

$$\langle \aleph, \mathcal{K}\rangle^c = \begin{matrix} & \varpi_1 & \varpi_2 & \varpi_3 \\ \varsigma_1 & (0.3, 0.1, 0.7) & (0.4, 0.2, 0.6) & (0.5, 0.1, 0.7) \\ \varsigma_2 & (0.7, 0.1, 0.3) & (0.5, 0.1, 0.4) & (0.6, 0.2, 0.2) \end{matrix} \quad (20)$$

Then, the SFS-interior of $\langle \aleph, \mathscr{K}\rangle$,

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$$\langle \aleph, \mathcal{K}\rangle^\circ = \emptyset_{\mathcal{K}} \,\widehat{\cup}\, \langle \aleph_2, \mathcal{K}\rangle \,\widehat{\cup}\, \langle \aleph_3, \mathcal{K}\rangle = \langle \aleph_2, \mathcal{K}\rangle$$

The SFS-closure of $\langle \aleph, \mathscr{K}\rangle$, $\overline{\langle \aleph, \mathscr{K}\rangle} = \Sigma_{\mathscr{K}}$

$$\begin{aligned} \overline{\langle \aleph, \mathcal{K}\rangle^c} &= \Sigma_{\mathcal{K}} \,\widehat{\cap}\, \langle \aleph_2, \mathcal{K}\rangle^c \,\widehat{\cap}\, \langle \aleph_3, \mathcal{K}\rangle^c \\ &= \begin{matrix} & \varpi_1 & \varpi_2 & \varpi_3 \\ \varsigma_1 & (0.4, 0.0, 0.7) & (0.5, 0.2, 0.5) & (0.5, 0.1, 0.7) \\ \varsigma_2 & (0.6, 0.1, 0.3) & (0.6, 0.0, 0.4) & (0.8, 0.0, 0.2) \end{matrix} \end{aligned} \quad (21)$$

So that the SFS-boundary of $\langle \aleph, \mathscr{K}\rangle$,

$$\begin{aligned} \partial\langle \aleph, \mathcal{K}\rangle &= \overline{\langle \aleph, \mathcal{K}\rangle} \,\widehat{\cap}\, \overline{\langle \aleph, \mathcal{K}\rangle^c} \\ &= \begin{matrix} & \varpi_1 & \varpi_2 & \varpi_3 \\ \varsigma_1 & (0.4, 0.0, 0.7) & (0.5, 0.2, 0.5) & (0.5, 0.1, 0.7) \\ \varsigma_2 & (0.6, 0.1, 0.3) & (0.6, 0.0, 0.4) & (0.8, 0.0, 0.2) \end{matrix} \end{aligned} \quad (22)$$

The SFS-exterior of $\langle \aleph, \mathscr{K}\rangle$, $Ext\langle \aleph, \mathscr{K}\rangle = (\langle \aleph, \mathscr{K}\rangle^c)^\circ = \varnothing_{\mathscr{K}}$.

Theorem 5. *Suppose that $(\Sigma_{\mathscr{K}}, \mathscr{T})$ is a SFS-topological space and $\langle \aleph, \mathscr{L}\rangle$ is a spherical fuzzy soft set over Σ, where $\mathscr{L} \subseteq \mathscr{K}$. Then we have*

(1) $(\langle \aleph, \mathscr{L}\rangle^\circ)^c = \overline{\langle \aleph, \mathscr{L}\rangle^c}$

(2) $\overline{\langle \aleph, \mathscr{L}\rangle}^c = (\langle \aleph, \mathscr{L}\rangle^c)^\circ$

Proof. Proof is direct □

Theorem 6. *Suppose that $(\Sigma_{\mathscr{K}}, \mathscr{T})$ is a SFS-topological space and $\langle \aleph, \mathscr{L}\rangle$ is a spherical fuzzy soft set over Σ, where $\mathscr{L} \subseteq \mathscr{K}$. Then $\partial\langle \aleph, \mathscr{K}\rangle = \partial\langle \aleph, \mathscr{K}\rangle^c$*

Proof. Proof is direct. □

Definition 27. Let $\varpi(\Xi)$ and $\varpi(\Psi)$ be two SFS-points. $\varpi(\Xi)$ and $\varpi(\Psi)$ are said to be distinct, denoted by $\varpi(\Xi) \neq \varpi(\Psi)$, if their corresponding SFSSs $\langle \Xi, \mathscr{L}\rangle$ and $\langle \Psi, \mathscr{M}\rangle$ are disjoint. That is, $\langle \Xi, \mathscr{L}\rangle \,\widehat{\cap}\, \langle \Psi, \mathscr{M}\rangle = \varnothing_{\mathscr{L}\cap\mathscr{M}}$.

4. Spherical Fuzzy Soft Separation Axioms

In this section, we define SFS-separation axioms by using the concepts SFS-point, SFS-open sets and SFS-closed sets.

Definition 28. Let $(\Sigma_{\mathscr{K}}, \mathscr{T})$ be a SFS-topological space and let $\varpi(\Xi)$ and $\varpi(\Psi)$ be any two distinct SFS-points over Σ. If there exist SFS-open sets $\langle \aleph, \mathscr{L}\rangle$ and $\langle \Omega, \mathscr{M}\rangle$ such that $\varpi(\Xi) \in \langle \aleph, \mathscr{L}\rangle$ and $\varpi(\Psi) \notin \langle \aleph, \mathscr{L}\rangle$ or $\varpi(\Psi) \in \langle \Omega, \mathscr{M}\rangle$ and $\varpi(\Xi) \notin \langle \Omega, \mathscr{M}\rangle$, then $(\Sigma_{\mathscr{K}}, \mathscr{T})$ is known as SFS T_0-space.

Example 9. All discrete SFS-topological spaces are SFS T_0-spaces. Because, for any two distinct SFS-points $\varpi(\Xi)$ and $\varpi(\Psi)$ over Σ, there exist a SFS-open set $\{\varpi(\Xi)\}$, such that $\varpi(\Xi) \in \{\varpi(\Xi)\}$ and $\varpi(\Psi) \notin \{\varpi(\Xi)\}$.

Definition 29. Let $(\Sigma_{\mathscr{K}}, \mathscr{T})$ be a SFS-topological space and let $\varpi(\Xi), \varpi(\Psi)$ be two SFS-points over Σ with $\varpi(\Xi) \neq \varpi(\Psi)$. If there exist two SFS-open sets $\langle \aleph, \mathscr{L}\rangle$ and $\langle \Omega, \mathscr{M}\rangle$ such that $\varpi(\Xi) \in \langle \aleph, \mathscr{L}\rangle$, $\varpi(\Psi) \notin \langle \aleph, \mathscr{L}\rangle$ and $\varpi(\Psi) \in \langle \Omega, \mathscr{M}\rangle$, $\varpi(\Xi) \notin \langle \Omega, \mathscr{M}\rangle$, then $(\Sigma_{\mathscr{K}}, \mathscr{T})$ is known as SFS T_1-space.

Example 10. Every discrete SFS-topological space is a SFS T_1-space. Because, for any two distinct SFS -points $\varpi(\Xi)$ and

$\varpi(\Psi)$ over Σ, there exist SFS-open sets $\{\varpi(\Xi)\}$ and $\{\varpi(\Psi)\}$, such that $\varpi(\Xi) \in \{\varpi(\Xi)\}$, $\varpi(\Psi) \notin \{\varpi(\Xi)\}$ and $\varpi(\Xi) \notin \{\varpi(\Psi)\}$, $\varpi(\Psi) \in \{\varpi(\Psi)\}$.

Definition 30. Let $(\Sigma_{\mathscr{K}}, \mathscr{T})$ be a SFS-topological space and let $\varpi(\Xi)$ and $\varpi(\Psi)$ be any two distinct SFS-points over Σ. If there exist two SFS-open sets $\langle \aleph, \mathscr{L} \rangle$ and $\langle \aleph, \mathscr{L} \rangle$ such that $\varpi(\Xi) \in \langle \aleph, \mathscr{L} \rangle$ and $\varpi(\Psi) \in \langle \Omega, \mathscr{M} \rangle$, and $\langle \aleph, \mathscr{L} \rangle \widehat{\cap} \langle \Omega, \mathscr{M} \rangle = \varnothing_{\mathscr{L} \cap \mathscr{M}}$, then $(\Sigma_{\mathscr{K}}, \mathscr{T})$ is said to be SFS T_2-space or SFS-Hausdorff space.

Example 11. Suppose that $(\Sigma_{\mathscr{K}}, \mathscr{T})$ is a discrete SFS-topological space. If $\varpi(\Xi)$ and $\varpi(\Psi)$ are any two distinct SFS-points over Σ. Then there exists distinct SFS-open sets $\{\varpi(\Xi)\}$ and $\{\varpi(\Psi)\}$ such that $\varpi(\Xi) \in \{\varpi(\Xi)\}$ and $\varpi(\Psi) \in \{\varpi(\Psi)\}$. Therefore, $(\Sigma_{\mathscr{K}}, \mathscr{T})$ is a SFS-Hausdorff space.

Theorem 7. *Let $(\Sigma_{\mathscr{K}}, \mathscr{T})$ be a SFS-topological space with attribute set $\mathscr{K}$. $(\Sigma_{\mathscr{K}}, \mathscr{T})$ is a SFS-Hausdorff space if and only if for any two distinct SFS-points $\varpi(\Xi)$ and $\varpi(\Psi)$, there exist SFS-closed sets $\langle \Omega_1, \mathscr{K} \rangle$ and $\langle \Omega_2, \mathscr{K} \rangle$ such that $\varpi(\Xi) \in \langle \Omega_1, \mathscr{K} \rangle$, $\varpi(\Psi) \notin \langle \Omega_1, \mathscr{K} \rangle$ and $\varpi(\Xi) \notin \langle \Omega_2, \mathscr{K} \rangle$, $\varpi(\Psi) \in \langle \Omega_2, \mathscr{K} \rangle$, and also $\langle \Omega_1, \mathscr{K} \rangle \widehat{\cup} \langle \Omega_2, \mathscr{K} \rangle = \Sigma_{\mathscr{K}}$.*

Proof. Suppose that $(\Sigma_{\mathscr{K}}, \mathscr{T})$ is a SFS-Hausdorff space, $\varpi(\Xi)$ and $\varpi(\Psi)$ are any two distinct SFS-points over Σ. That is, $\varpi(\Xi) \widehat{\cap} \varpi(\Psi) = \varnothing_{\mathscr{K}}$.

Since $(\Sigma_{\mathscr{K}}, \mathscr{T})$ is SFS-Hausdorff space, there exist two SFS-open sets $\langle \aleph_1, \mathscr{K} \rangle$ and $\langle \aleph_2, \mathscr{K} \rangle$ such that $\varpi(\Xi) \in \langle \aleph_1, \mathscr{K} \rangle$, $\varpi(\Psi) \notin \langle \aleph_1, \mathscr{K} \rangle$ and $\varpi(\Xi) \notin \langle \aleph_2, \mathscr{K} \rangle$, $\varpi(\Psi) \in \langle \aleph_2, \mathscr{K} \rangle$. And also $\langle \aleph_1, \mathscr{K} \rangle \widehat{\cap} \langle \aleph_1, \mathscr{K} \rangle = \varnothing_{\mathscr{K}} \Rightarrow \langle \aleph_1, \mathscr{K} \rangle^c \widehat{\cup} \langle \aleph_1, \mathscr{K} \rangle^c = \Sigma_{\mathscr{K}}$ and also both $\langle \aleph_1, \mathscr{K} \rangle^c$ and $\langle \aleph_2, \mathscr{K} \rangle^c$ are SFS-closed sets.

Let $\langle \aleph_1, \mathscr{K} \rangle^c = \langle \Omega_1, \mathscr{K} \rangle$ and $\langle \aleph_2, \mathscr{K} \rangle^c = \langle \Omega_2, \mathscr{K} \rangle$

Then, $\varpi(\Xi) \notin \langle \Omega_1, \mathscr{K} \rangle, \varpi(\Psi) \in \langle \Omega_1, \mathscr{K} \rangle$ and $\varpi(\Xi) \notin \langle \Omega_2, \mathscr{K} \rangle, \varpi(\Psi) \in \langle \Omega_2, \mathscr{K} \rangle$.

Conversely, suppose that for any two distinct SFS-points $\varpi(\Xi)$ and $\varpi(\Psi)$, there exist SFS-closed sets $\langle \Omega_1, \mathscr{K} \rangle$ and $\langle \Omega_2, \mathscr{K} \rangle$ such that $\varpi(\Xi) \in \langle \Omega_1, \mathscr{K} \rangle$, $\varpi(\Psi) \notin \langle \Omega_1, \mathscr{K} \rangle$ and $\varpi(\Xi) \notin \langle \Omega_2, \mathscr{K} \rangle$, $\varpi(\Psi) \in \langle \Omega_2, \mathscr{K} \rangle$, and also $\langle \Omega_1, \mathscr{K} \rangle \widehat{\cup} \langle \Omega_2, \mathscr{K} \rangle = \Sigma_{\mathscr{K}}$.

$\Rightarrow \langle \Omega_1, \mathscr{K} \rangle^c$ and $\langle \Omega_2, \mathscr{K} \rangle^c$ are SFS-open sets and $\langle \Omega_1, \mathscr{K} \rangle^c \widehat{\cap} \langle \Omega_2, \mathscr{K} \rangle^c = \varnothing_{\mathscr{K}}$

Also, $\varpi(\Xi) \notin \langle \Omega_1, \mathscr{K} \rangle^c$, $\varpi(\Psi) \in \langle \Omega_1, \mathscr{K} \rangle^c$ and $\varpi(\Xi) \in \langle \Omega_2, \mathscr{K} \rangle^c$, $\varpi(\Psi) \notin \langle \Omega_2, \mathscr{K} \rangle^c$.

Thus, $(\Sigma_{\mathscr{K}}, \mathscr{T})$ is a SFS-Hausdorff space. □ □

Definition 31. Let $(\Sigma_{\mathscr{K}}, \mathscr{T})$ be a SFS-topological space, $\langle \Omega, \mathscr{M} \rangle$ be a SFS-closed set $\varpi(\Xi)$ and $\varpi(\Psi)$, be a SFS-point over Σ such that $\varpi(\Xi) \notin \langle \Omega, \mathscr{M} \rangle$. If there is SFS-open sets $\langle \aleph_1, \mathscr{L}_1 \rangle$ and $\langle \aleph_2, \mathscr{L}_2 \rangle$ such that $\varpi(\Xi) \in \langle \aleph_1, \mathscr{L}_1 \rangle$, $\langle \Omega, \mathscr{M} \rangle \widehat{\subseteq} \langle \aleph_2, \mathscr{L}_2 \rangle$ and $\langle \aleph_1, \mathscr{L}_1 \rangle \widehat{\cap} \langle \aleph_2, \mathscr{L}_2 \rangle = \varnothing_{\mathscr{L}_1 \cap \mathscr{L}_2}$, then $(\Sigma_{\mathscr{K}}, \mathscr{T})$ is called a SFS-regular space.

Example 12. Let $(\Sigma_{\mathscr{K}}, \mathscr{T})$ be a SFS-topological space over $\Sigma = \{\varsigma_1, \varsigma_2\}$ with SFS-topology $\mathscr{T} = \{\Sigma_{\mathscr{K}}, \varnothing_{\mathscr{K}}, \langle \aleph_1, \mathscr{K} \rangle, \langle \aleph_2, \mathscr{K} \rangle\}$, where,

$$\langle \aleph_1, \mathcal{K} \rangle = \begin{matrix} & \varpi_1 & \varpi_2 \\ \varsigma_1 & (0.0, 0.0, 1.0) & (1.0, 0.0, 0.0) \\ \varsigma_2 & (0.0, 0.0, 1.0) & (1.0, 0.0, 0.0) \end{matrix}$$
$$\langle \aleph_2, \mathcal{K} \rangle = \begin{matrix} & \varpi_1 & \varpi_2 \\ \varsigma_1 & (1.0, 0.0, 0.0) & (0.0, 0.0, 1.0) \\ \varsigma_2 & (1.0, 0.0, 0.0) & (0.0, 0.0, 1.0) \end{matrix} \quad (23)$$

Then $(\Sigma_{\mathscr{K}}, \mathscr{T})$ is a SFS-regular space.

Definition 32. Let $(\Sigma_{\mathscr{K}}, \mathscr{T})$ be a SFS-topological space. If $(\Sigma_{\mathscr{K}}, \mathscr{T})$ is a SFS-regular T_1-space, then it is called a SFS T_3-space.

Definition 33. Let $(\Sigma_{\mathscr{K}}, \mathscr{T})$ be a SFS-topological space and let $\langle \Omega_1, \mathscr{M}_1 \rangle$ and $\langle \Omega_2, \mathscr{M}_2 \rangle$ be two disjoint SFS-closed sets in $(\Sigma_{\mathscr{K}}, \mathscr{T})$. If there exist SFS-open sets $\langle \aleph_1, \mathscr{L}_1 \rangle$ and $\langle \aleph_2, \mathscr{L}_2 \rangle$ such that $\langle \Omega_1, \mathscr{M}_1 \rangle \widehat{\subseteq} \langle \aleph_1, \mathscr{L}_1 \rangle$, $\langle \Omega_2, \mathscr{M}_2 \rangle \widehat{\subseteq} \langle \aleph_2, \mathscr{L}_2 \rangle$ and $\langle \aleph_1, \mathscr{L}_1 \rangle \widehat{\cap} \langle \aleph_2, \mathscr{L}_2 \rangle = \varnothing_{\mathscr{L}_1 \cap \mathscr{L}_2}$, then $(\Sigma_{\mathscr{K}}, \mathscr{T})$ is called a SFS-normal space.

Example 13. Let $(\Sigma_{\mathscr{K}}, \mathscr{T})$ be a SFS-topological space over $\Sigma = \{\varsigma_1, \varsigma_2\}$ with SFS-topology

$$\mathcal{T} = \{\Sigma_{\mathcal{K}}, \emptyset_{\mathcal{K}}, \langle \aleph_1, \mathcal{K} \rangle, \langle \aleph_2, \mathcal{K} \rangle, \langle \aleph_3, \mathcal{K} \rangle, \langle \aleph_4, \mathcal{K} \rangle, \langle \aleph_5, \mathcal{K} \rangle, \langle \aleph_6, \mathcal{K} \rangle\}, \text{ where,}$$

$$\langle \aleph_1, \mathcal{K} \rangle = \begin{matrix} \\ \varsigma_1 \\ \varsigma_2 \end{matrix} \begin{matrix} \varpi_1 & \varpi_2 \\ \begin{pmatrix} (0.7, 0.3, 0.2) & (1.0, 0.0, 0.0) \\ (0.9, 0.2, 0.1) & (1.0, 0.0, 0.0) \end{pmatrix} \end{matrix}$$

$$\langle \aleph_2, \mathcal{K} \rangle = \begin{matrix} \\ \varsigma_1 \\ \varsigma_2 \end{matrix} \begin{matrix} \varpi_1 & \varpi_2 \\ \begin{pmatrix} (0.5, 0.3, 0.4) & (0.0, 0.0, 1.0) \\ (0.3, 0.2, 0.5) & (0.0, 0.0, 1.0) \end{pmatrix} \end{matrix}$$

$$\langle \aleph_3, \mathcal{K} \rangle = \begin{matrix} \\ \varsigma_1 \\ \varsigma_2 \end{matrix} \begin{matrix} \varpi_1 & \varpi_2 \\ \begin{pmatrix} (1.0, 0.0, 0.0) & (0.8, 0.4, 0.3) \\ (1.0, 0.0, 0.0) & (0.6, 0.3, 0.4) \end{pmatrix} \end{matrix}$$

$$\langle \aleph_4, \mathcal{K} \rangle = \begin{matrix} \\ \varsigma_1 \\ \varsigma_2 \end{matrix} \begin{matrix} \varpi_1 & \varpi_2 \\ \begin{pmatrix} (0.0, 0.0, 1.0) & (0.6, 0.4, 0.4) \\ (0.0, 0.0, 1.0) & (0.5, 0.3, 0.5) \end{pmatrix} \end{matrix} \quad (24)$$

$$\langle \aleph_5, \mathcal{K} \rangle = \begin{matrix} \\ \varsigma_1 \\ \varsigma_2 \end{matrix} \begin{matrix} \varpi_1 & \varpi_2 \\ \begin{pmatrix} (0.7, 0.3, 0.2) & (0.8, 0.4, 0.3) \\ (0.9, 0.2, 0.1) & (0.6, 0.3, 0.4) \end{pmatrix} \end{matrix}$$

$$\langle \aleph_6, \mathcal{K} \rangle = \begin{matrix} \\ \varsigma_1 \\ \varsigma_2 \end{matrix} \begin{matrix} \varpi_1 & \varpi_2 \\ \begin{pmatrix} (0.5, 0.3, 0.4) & (0.6, 0.4, 0.4) \\ (0.3, 0.2, 0.5) & (0.5, 0.3, 0.5) \end{pmatrix} \end{matrix}$$

Then $(\Sigma_{\mathscr{K}}, \mathscr{T})$ is a SFS-normal space.

Definition 34. Let $(\Sigma_{\mathscr{K}}, \mathscr{T})$ be a SFS-topological space. If $(\Sigma_{\mathscr{K}}, \mathscr{T})$ is a SFS-normal T_1-space, then it is known as SFS T_4-space.

Theorem 8. *Suppose that $(\Sigma_{\mathscr{K}}, \mathscr{T})$ is a SFS-topological space and Z is a non-empty subset of Σ.*

(1) If $(\Sigma_{\mathscr{K}}, \mathscr{T})$ is a SFS T_0-space, then $(Z_{\mathscr{K}}, \mathscr{T}_Z)$ is also a SFS T_0-space.

(2) If $(\Sigma_{\mathscr{K}}, \mathscr{T})$ is a SFS T_1-space, then $(Z_{\mathscr{K}}, \mathscr{T}_Z)$ is also a SFS T_1-space.

(3) If $(\Sigma_{\mathscr{K}}, \mathscr{T})$ is a SFS T_2-space, then $(Z_{\mathscr{K}}, \mathscr{T}_Z)$ is also a SFS T_2-space.

Proof. Here we provide the proof if (1). (2) and (3) can be proved in the similar way.

Suppose that $\varpi(\Xi)$ and $\varpi(\Psi)$ are two distinct SFS-points over Z.

Since $(\Sigma_{\mathscr{K}}, \mathscr{T})$ is a SFS T_0-space, there is SFS-open sets $\langle \aleph, \mathscr{L} \rangle$ and $\langle \Omega, \mathscr{M} \rangle$ such that $\varpi(\Xi) \in \langle \aleph, \mathscr{L} \rangle$, $\varpi(\Psi) \notin \langle \aleph, \mathscr{L} \rangle$ or $\varpi(\Psi) \in \langle \Omega, \mathscr{M} \rangle$, $\varpi(\Xi) \notin \langle \Omega, \mathscr{M} \rangle$

Thus, $\varpi(\Xi) \in \langle \aleph, \mathscr{L} \rangle \widehat{\cap} Z_{\mathscr{K}}$, $\varpi(\Psi) \notin \langle \aleph, \mathscr{L} \rangle \widehat{\cap} Z_{\mathscr{K}}$ or $\varpi(\Psi) \in \langle \Omega, \mathscr{M} \rangle \widehat{\cap} Z_{\mathscr{K}}$, $\varpi(\Xi) \notin \langle \Omega, \mathscr{M} \rangle \widehat{\cap} Z_{\mathscr{K}}$

Therefore, $(Z_{\mathscr{K}}, \mathscr{T}_Z)$ is also a SFS T_0-space. □

5. Group Decision Algorithm and Illustrative Example

In this section, we utilize the proposed SFS-topology to the group decision-making (GDM) process under the spherical fuzzy soft environment. For it, we presented the concept of TOPSIS method and embedding it into the proposed SFS-topology.

5.1. Proposed Algorithm with TOPSIS Method. Consider a GDM process which consist a certain set of alternatives $K = \{\varsigma_1, \varsigma_2, \ldots, \varsigma_m\}$. Each alternative is evaluated under the different set of attributes denoted by $\mathscr{K} = \{\varpi_1, \varpi_2, \ldots, \varpi_n\}$ by the different "p" decision-makers (or experts), say $\mathscr{DM}_1, \mathscr{DM}_2, \ldots, \mathscr{DM}_p$. Each expert has evaluated the given alternatives and provide their ratings in terms of linguistic variables such as "Excellent," "Good" etc. All the linguistic variables and their corresponding weights are considered in this work from the list which is summarized in Table 1.

Then to access the finest alternative(s) from the given alternative, we summarize the following steps of the proposed approach as below.

Step 1: Create a weighted SFS parameter matrix $A_w = [\alpha_{ij}]_{p\times m}$ by considering the linguistic terms from Table 1. That is,

$$A_w = [\alpha_{ij}]_{p\times n} = \begin{pmatrix} \alpha_{11} & \alpha_{12} & \cdots & \alpha_{1n} \\ \alpha_{21} & \alpha_{22} & \cdots & \alpha_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{p1} & \alpha_{p2} & \cdots & \alpha_{pn} \end{pmatrix} \quad (25)$$

Table 1: Linguistic terms to determine the alternatives.

Linguistic terms	Weights
Excellent	0.90
Very good	0.70
Good	0.50
Bad	0.30
Very bad	0.10

where each element α_{ij} is the linguistic rating given by the decision-maker $\mathscr{D}\mathscr{M}_i$ to the attribute ϖ_j.

Step 2: Create the weighted normalized SFS parameter matrix N_w as follows:

$$N_w = [\rho_{ij}]_{p\times n} = \begin{pmatrix} \rho_{11} & \rho_{12} & \cdots & \rho_{1n} \\ \rho_{21} & \rho_{22} & \cdots & \rho_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{p1} & \rho_{p2} & \cdots & \rho_{pn} \end{pmatrix} \tag{26}$$

where, $\rho_{ij} = \alpha_{ij}/\sqrt{\sum_{i=1}^{p} (\alpha_{ij})^2}$

Step 3: Compute the weight vector $\Theta = \{\theta_1, \theta_2, \ldots, \theta_n\}$, where θ_i's are obtained as

$$\theta_i = \frac{w_i}{\sum\limits_{q=1}^{n} w_q} \quad ; \quad w_j = \frac{\sum\limits_{i=1}^{p} \rho_{ij}}{p} \tag{27}$$

Step 4: Construct a SFS-topology by aggregating the SFSSs $\langle \mathscr{D}\mathscr{M}_i, \mathscr{K} \rangle$, $i = 1, 2, \ldots, p$, accorded by each decision-makers in the matrix form as their evaluation value. The matrix corresponding to the SFSS $\langle \mathscr{D}\mathscr{M}_i, \mathscr{K} \rangle$ is denoted by $\mathbb{DM}_i$ for all $i = 1, 2, \ldots, p$ and it is called the SFS-decision matrix, where the rows and columns of each $\mathbb{DM}_i$ represents the alternatives and the attributes respectively.

Step 5: Compute the aggregated SFS matrix $\mathbb{DM}_{Agg}$ given as follows:

$$\mathbb{DM}_{Agg} = \frac{\mathbb{DM}_1 + \mathbb{DM}_2 + \ldots + \mathbb{DM}_p}{p} = [d_{pq}]_{m\times n} \tag{28}$$

Step 6: Construct the weighted SFS-decision matrix

$$B = [\beta_{pq}]_{m\times n} = \begin{pmatrix} \beta_{11} & \beta_{12} & \cdots & \beta_{1n} \\ \beta_{21} & \beta_{22} & \cdots & \beta_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \beta_{m1} & \beta_{m2} & \cdots & \beta_{mn} \end{pmatrix} \tag{29}$$

where $\beta_{pq} = \theta_q \times d_{pq}$ and each $\beta_{pq} = (\mu_{\varpi_q}(\varsigma_p), \eta_{\varpi_q}(\varsigma_p), \vartheta_{\varpi_q}(\varsigma_p))$, $p = 1, 2, \ldots, m$ and $q = 1, 2, \ldots, n$.

Step 7: Obtain SFS-valued positive ideal solution ($SFSV^+$) and SFS-valued negative ideal solution ($SFSV^-$), where

$$\begin{aligned} SFSV^{+} &= \{\beta_1^{+}, \beta_2^{+}, ..., \beta_n^{+}\} \\ &= \{(\max_p \mu_{\varpi_q}(\varsigma_p), \min_p \eta_{\varpi_q}(\varsigma_p), \min_p \vartheta_{\varpi_q}(\varsigma_p)); q = 1, 2, ..., n\} \\ &= \{(\mu_q^{+}, \eta_q^{+}, \vartheta_q^{+}); q = 1, 2, ..., n\} \end{aligned} \tag{30}$$

$$\begin{aligned} SFSV^{-} &= \{\beta_1^{-}, \beta_2^{-}, ..., \beta_n^{-}\} \\ &= \{(\min_p \mu_{\varpi_q}(\varsigma_p), \min_p \eta_{\varpi_q}(\varsigma_p), \max_p \vartheta_{\varpi_q}(\varsigma_p)); q = 1, 2, ..., n\} \\ &= \{(\mu_q^{-}, \eta_q^{-}, \vartheta_q^{-}); q = 1, 2, ..., n\} \end{aligned} \tag{31}$$

Step 8: Compute the SFS-separation measurements Ed_p^{+} and Ed_p^{-}, $\forall\, p = 1, 2, \ldots, m$, defined as follows:

$$Ed_p^{+} = \sqrt{\sum_{q=1}^{n} \left\{ \left(\mu_{\varpi_q}(\varsigma_p) - \mu_q^{+}\right)^2 + \left(\eta_{\varpi_q}(\varsigma_p) - \eta_q^{+}\right)^2 + \left(\vartheta_{\varpi_q}(\varsigma_p) - \vartheta_q^{+}\right)^2 \right.}, \tag{32}$$

$$Ed_p^{-} - = \sqrt{\sum_{q=1}^{n} \left\{ \left(\mu_{\varpi_q}(\varsigma_p) - \mu_q^{-}\right)^2 + \left(\eta_{\varpi_q}(\varsigma_p) - \eta_q^{+}\right)^2 + \left(\vartheta_{\varpi_q}(\varsigma_p) - \vartheta_q^{-}\right)^2 \right.}. \tag{33}$$

Step 9: Obtain the SFS-closeness coefficient $\widehat{\mathbf{C}_p}$ of each alternatives. Where

$$\widehat{\mathbf{C}_p} = \frac{Ed_p^{-}}{Ed_p^{+} + Ed_p^{-}} \in [0, 1]. \tag{34}$$

provided $Ed_p^{+} \neq 0$.

Step 10: Based on the SFS-closeness coefficient, rank the alternatives in decreasing (or increasing) order and choose the optimal object from the alternatives.

5.2. Illustrative Example. An international company conducted a campus recruitment in a college and shortlisted four students $\Sigma = \{\varsigma_1, \varsigma_2, \varsigma_3, \varsigma_4\}$ through the first round of recruitment. There is only one vacancy and they have to select one student as their candidate out of these five students. Suppose there are six decision-makers $\mathscr{DM} = \{\mathscr{DM}_1, \mathscr{DM}_2, \mathscr{DM}_3, \mathscr{DM}_4, \mathscr{DM}_5, \mathscr{DM}_6\}$ for the final round and they must have select the candidate based on the parameter set $\mathscr{K} = \{\varpi_1, \varpi_2, \varpi_3, \varpi_4, \varpi_5\}$. For $i = 1, 2, 3, 4, 5$, the parameters ϖ_j stand for "educational discipline," "English speaking," "writing skill," "technical discipline," and "general knowledge" respectively. Then the steps of the proposed approach have been executed to find the best alternative(s) as follows.

Step 1: The weighted SFS parameter matrix A_w is formulated on the basis of equation (25) as follows:

$$A_w = \begin{pmatrix} 0.70 & 0.90 & 0.50 & 0.70 & 0.30 \\ 0.30 & 0.10 & 0.70 & 0.90 & 0.90 \\ 0.90 & 0.70 & 0.30 & 0.50 & 0.30 \\ 0.50 & 0.30 & 0.70 & 0.90 & 0.50 \\ 0.90 & 0.70 & 0.10 & 0.30 & 0.90 \\ 0.10 & 0.90 & 0.30 & 0.50 & 0.70 \end{pmatrix} \tag{35}$$

Step 2: The weighted normalized SFS parameter matrix N_w is computed by using equation (26).

$$N_w = \begin{pmatrix} 0.45 & 0.55 & 0.41 & 0.43 & 0.18 \\ 0.19 & 0.06 & 0.59 & 0.55 & 0.56 \\ 0.57 & 0.43 & 0.25 & 0.30 & 0.18 \\ 0.32 & 0.18 & 0.59 & 0.55 & 0.31 \\ 0.57 & 0.43 & 0.08 & 0.18 & 0.56 \\ 0.06 & 0.55 & 0.25 & 0.30 & 0.44 \end{pmatrix} \tag{36}$$

Step 3: By using equation (27), the weight vector of the given attributes are computed as

$$\Theta = \{0.195, 0.200, 0.195, 0.210, 0.200\} \tag{37}$$

Step 4: For each decision-maker $\mathbb{DM}_i, i = 1$ to 6 and their corresponding SFS-decision matrices, we get a SFS-topology on Σ as

$$\begin{aligned}
\langle \mathcal{DM}_1, \mathcal{K} \rangle = \mathbb{DM}_1 &= \begin{matrix} & \varpi_1 & \varpi_2 & \varpi_3 & \varpi_4 & \varpi_5 \\ \varsigma_1 & (0.4, 0.1, 0.5) & (0.5, 0.2, 0.1) & (0.9, 0.1, 0.2) & (0.8, 0.1, 0.3) & (0.9, 0.2, 0.2) \\ \varsigma_2 & (0.8, 0.1, 0.2) & (0.7, 0.2, 0.3) & (0.6, 0.2, 0.4) & (0.7, 0.2, 0.2) & (0.3, 0.1, 0.8) \\ \varsigma_3 & (0.7, 0.1, 0.3) & (0.2, 0.1, 0.5) & (0.3, 0.2, 0.7) & (0.6, 0.3, 0.4) & (0.7, 0.2, 0.5) \\ \varsigma_4 & (0.6, 0.2, 0.4) & (0.4, 0.2, 0.8) & (0.5, 0.3, 0.6) & (0.5, 0.2, 0.6) & (0.4, 0.3, 0.5) \end{matrix} \\
\langle \mathcal{DM}_2, \mathcal{K} \rangle = \mathbb{DM}_2 &= \begin{matrix} & \varpi_1 & \varpi_2 & \varpi_3 & \varpi_4 & \varpi_5 \\ \varsigma_1 & (1.0, 0.0, 0.0) & (1.0, 0.0, 0.0) & (1.0, 0.0, 0.0) & (1.0, 0.0, 0.0) & (1.0, 0.0, 0.0) \\ \varsigma_2 & (1.0, 0.0, 0.0) & (1.0, 0.0, 0.0) & (1.0, 0.0, 0.0) & (1.0, 0.0, 0.0) & (1.0, 0.0, 0.0) \\ \varsigma_3 & (1.0, 0.0, 0.0) & (1.0, 0.0, 0.0) & (1.0, 0.0, 0.0) & (1.0, 0.0, 0.0) & (1.0, 0.0, 0.0) \\ \varsigma_4 & (1.0, 0.0, 0.0) & (1.0, 0.0, 0.0) & (1.0, 0.0, 0.0) & (1.0, 0.0, 0.0) & (1.0, 0.0, 0.0) \end{matrix} \\
\langle \mathcal{DM}_3, \mathcal{K} \rangle = \mathbb{DM}_3 &= \begin{matrix} & \varpi_1 & \varpi_2 & \varpi_3 & \varpi_4 & \varpi_5 \\ \varsigma_1 & (0.5, 0.2, 0.6) & (0.7, 0.1, 0.2) & (0.3, 0.2, 0.2) & (0.2, 0.1, 0.9) & (0.9, 0.0, 0.0) \\ \varsigma_2 & (0.9, 0.0, 0.3) & (0.8, 0.3, 0.4) & (0.5, 0.2, 0.7) & (0.6, 0.2, 0.4) & (1.0, 0.0, 0.0) \\ \varsigma_3 & (0.4, 0.2, 0.5) & (0.7, 0.1, 0.5) & (0.9, 0.1, 0.1) & (0.8, 0.3, 0.2) & (0.2, 0.1, 0.8) \\ \varsigma_4 & (0.6, 0.3, 0.5) & (0.4, 0.1, 0.7) & (0.7, 0.3, 0.1) & (0.1, 0.1, 0.4) & (0.6, 0.3, 0.4) \end{matrix} \\
\langle \mathcal{DM}_4, \mathcal{K} \rangle = \mathbb{DM}_4 &= \begin{matrix} & \varpi_1 & \varpi_2 & \varpi_3 & \varpi_4 & \varpi_5 \\ \varsigma_1 & (0.5, 0.2, 0.5) & (0.7, 0.2, 0.1) & (0.9, 0.2, 0.2) & (0.8, 0.1, 0.3) & (0.9, 0.2, 0.0) \\ \varsigma_2 & (0.9, 0.1, 0.2) & (0.8, 0.3, 0.3) & (0.6, 0.2, 0.4) & (0.7, 0.2, 0.2) & (1.0, 0.0, 0.0) \\ \varsigma_3 & (0.7, 0.2, 0.3) & (0.7, 0.1, 0.5) & (0.9, 0.2, 0.1) & (0.8, 0.3, 0.2) & (0.7, 0.2, 0.5) \\ \varsigma_4 & (0.6, 0.3, 0.4) & (0.4, 0.2, 0.7) & (0.7, 0.3, 0.1) & (0.5, 0.2, 0.4) & (0.6, 0.3, 0.4) \end{matrix} \\
\langle \mathcal{DM}_5, \mathcal{K} \rangle = \mathbb{DM}_5 &= \begin{matrix} & \varpi_1 & \varpi_2 & \varpi_3 & \varpi_4 & \varpi_5 \\ \varsigma_1 & (0.0, 0.0, 0.1) & (0.0, 0.0, 0.1) & (0.0, 0.0, 0.1) & (0.0, 0.0, 0.1) & (0.0, 0.0, 0.1) \\ \varsigma_2 & (0.0, 0.0, 0.1) & (0.0, 0.0, 0.1) & (0.0, 0.0, 0.1) & (0.0, 0.0, 0.1) & (0.0, 0.0, 0.1) \\ \varsigma_3 & (0.0, 0.0, 0.1) & (0.0, 0.0, 0.1) & (0.0, 0.0, 0.1) & (0.0, 0.0, 0.1) & (0.0, 0.0, 0.1) \\ \varsigma_4 & (0.0, 0.0, 0.1) & (0.0, 0.0, 0.1) & (0.0, 0.0, 0.1) & (0.0, 0.0, 0.1) & (0.0, 0.0, 0.1) \end{matrix} \\
\langle \mathcal{DM}_6, \mathcal{K} \rangle = \mathbb{DM}_6 &= \begin{matrix} & \varpi_1 & \varpi_2 & \varpi_3 & \varpi_4 & \varpi_5 \\ \varsigma_1 & (0.4, 0.1, 0.6) & (0.5, 0.1, 0.2) & (0.3, 0.1, 0.2) & (0.2, 0.1, 0.9) & (0.9, 0.0, 0.2) \\ \varsigma_2 & (0.8, 0.0, 0.3) & (0.7, 0.2, 0.4) & (0.5, 0.2, 0.7) & (0.6, 0.2, 0.4) & (0.3, 0.1, 0.8) \\ \varsigma_3 & (0.4, 0.1, 0.5) & (0.2, 0.1, 0.5) & (0.3, 0.1, 0.7) & (0.6, 0.3, 0.4) & (0.2, 0.1, 0.8) \\ \varsigma_4 & (0.6, 0.2, 0.5) & (0.4, 0.1, 0.8) & (0.5, 0.3, 0.6) & (0.1, 0.1, 0.6) & (0.4, 0.3, 0.5) \end{matrix}
\end{aligned} \tag{38}$$

Thus, the collection $\{\mathbb{DM}_1, \mathbb{DM}_2, \mathbb{DM}_3, \mathbb{DM}_4, \mathbb{DM}_5, \mathbb{DM}_6\}$ gives a SFS-topology on Σ.

Step 5: The aggregated SFS matrix $\mathbb{DM}_{Agg}$ is obtained by using equation (28) and summarized as

$$\mathbb{DM}_{Agg} = \begin{matrix} & \varpi_1 & \varpi_2 & \varpi_3 & \varpi_4 & \varpi_5 \\ \varsigma_1 & (0.47, 0.10, 0.53) & (0.57, 0.10, 0.27) & (0.57, 0.10, 0.30) & (0.50, 0.07, 0.57) & (0.77, 0.07, 0.23) \\ \varsigma_2 & (0.73, 0.03, 0.33) & (0.67, 0.17, 0.40) & (0.53, 0.13, 0.53) & (0.60, 0.13, 0.37) & (0.60, 0.03, 0.43) \\ \varsigma_3 & (0.53, 0.10, 0.43) & (0.47, 0.07, 0.50) & (0.57, 0.10, 0.43) & (0.63, 0.20, 0.37) & (0.47, 0.10, 0.60) \\ \varsigma_4 & (0.57, 0.17, 0.47) & (0.43, 0.10, 0.67) & (0.57, 0.20, 0.40) & (0.37, 0.10, 0.50) & (0.50, 0.20, 0.47) \end{matrix} \tag{39}$$

Step 6: The weighted SFS-decision matrix B is obtained by using equation (29) and written as

$$B = \begin{matrix} & \varpi_1 & \varpi_2 & \varpi_3 & \varpi_4 & \varpi_5 \\ \varsigma_1 & (0.091, 0.019, 0.103) & (0.114, 0.020, 0.054) & (0.111, 0.019, 0.059) & (0.105, 0.015, 0.120) & (0.154, 0.014, 0.046) \\ \varsigma_2 & (0.142, 0.005, 0.064) & (0.134, 0.034, 0.080) & (0.103, 0.025, 0.103) & (0.126, 0.027, 0.018) & (0.120, 0.006, 0.086) \\ \varsigma_3 & (0.103, 0.019, 0.084) & (0.094, 0.014, 0.100) & (0.111, 0.019, 0.084) & (0.132, 0.042, 0.078) & (0.094, 0.020, 0.120) \\ \varsigma_4 & (0.111, 0.033, 0.091) & (0.086, 0.020, 0.134) & (0.111, 0.039, 0.078) & (0.078, 0.021, 0.105) & (0.100, 0.040, 0.094) \end{matrix} \tag{40}$$

Step 7: From the weighted matrix B and utilizing equations (30), (31), we obtain ideal solutions $SFSV^+$ and $SFSV^-$ are

$$\begin{aligned} SFSV^+ &= \left\{ \begin{array}{c} (0.142, 0.005, 0.064), (0.134, 0.014, 0.054), (0.111, 0.019, 0.059), \\ (0.132, 0.015, 0.078), (0.154, 0.006, 0.046) \end{array} \right\} \\ SFSV^- &= \left\{ \begin{array}{c} (0.091, 0.005, 0.103), (0.086, 0.014, 0.134), (0.103, 0.019, 0.103), \\ (0.078, 0.015, 0.120), (0.094, 0.006, 0.120) \end{array} \right\} \end{aligned} \tag{41}$$

Step 8: For each $p = 1, 2, 3, 4$, the SFS-separation measurements Ed_p^+ and Ed_p^- are calculated by using equations (32), (33) as

$$\begin{matrix} Ed_1^+ = 0.0855 & ; & Ed_2^+ = 0.0982 & ; & Ed_3^+ = 0.1283 & ; & Ed_4^+ = 0.1484 \\ Ed_1^- = 0.1389 & ; & Ed_2^- = 0.1564 & ; & Ed_3^- = 0.0892 & ; & Ed_4^- = 0.0677 \end{matrix} \tag{42}$$

Step 9: Using equation (34), compute the SFS-closeness coefficients $\widehat{\mathbf{C}}_p$, for each $p = 1, 2, 3, 4$ and get

$$\hat{\mathbf{C}}_1 = 0.6190 \quad ; \quad \hat{\mathbf{C}}_2 = 0.6142 \quad ; \quad \hat{\mathbf{C}}_3 = 0.4101 \quad ; \quad \hat{\mathbf{C}}_4 = 0.3132 \tag{43}$$

Step 10: Based on the ratings of $\widehat{\mathbf{C}}_p$'s, we can obtain the ordering of the given alternatives as

$$\hat{\mathbf{C}}_1 > \hat{\mathbf{C}}_2 > \hat{\mathbf{C}}_3 > \hat{\mathbf{C}}_4 \tag{44}$$

Which corresponds to the alternatives ratings as $\varsigma_1 > \varsigma_2 > \varsigma_3 > \varsigma_4$. This, we conclude that the international company should select the student ς_1 as their candidate.

6. Comparison Analysis

In this section, the proposed algorithm is compared to the existing algorithm (Algorithm 1: Decision making based on adjustable soft discernibility matrix) [27]. Since the optimal solution of the study discussed in Section 5.2 using Algorithm 1 is also "ς_1," it can be seen that the proposed algorithm based on the group decision-making method and the extension of TOPSIS approach is comparable to previously known method, which validates the reliability and dependability of the proposed algorithm.

The advantages of the work drawn in earlier sections can be summarized as follows:

(i) Topological structures on fuzzy soft sets are used in a variety of applications, including medical diagnosis, decision-making, pattern recognition, image processing, and so on.

(ii) SFSS is one of the most generalized version of fuzzy soft set and it is arguably the more realistic, practical and accurate.

(iii) Introducing topology on SFSS is seem to be highly important in both theoretical and practical scenarios.

(iv) While dealing with group decision-making problems of SFSS, the proposed algorithm is more reliable and expressive.

7. Conclusions

The spherical fuzzy soft set is the most generalized version of all other existing fuzzy soft set models. This newest concept is more precise, accurate, and sensible and the models are thus capable of solving myriad problems more deftly and practically. In this paper, we probed into certain basic aspects of spherical fuzzy soft topological space. SFS-topology is developed by using the notions of SFS-union and SFS-intersection. The paper has also provided certain fundamental definitions pertaining to the SFS-topology including SFS-subspace, SFS-point, SFS-nbd, SFS-basis, SFS-interior, SFS-closure, SFS-boundary, and SFS-exterior and on the basis of the said definitions mooted, we have proven a few theorems. Further, SFS-separation axioms are presented by using the concepts of SFS-point, SFS-closed sets, and SFS-open sets on the basis of which an algorithm is also proposed as an application with vivid implications in group decision-making method. The model is presented as an extension of TOPSIS approach as well. A numerical example is used to illustrate the efficiency of the proposed algorithm.

In the future, we will explore algebraicproperties of SFSSs and investigate their applications in decision making, medical diagnosis, clustering analysis, pattern recognition, and information science. Also relationship between SFSSs and T-SFSSs, and the algebraic and topological structures of T-SFSSs can be studied as future work.

Data Availability

No data were used to support this study.

Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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