Research Article

Investigation of Heat Transfer from Convective and Radiative Stretching/Shrinking Rectangular Fins

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We study the efficiency of shrinking/stretching radiative fins to improve heat transfer rate. To evaluate the competence of suggested fins, the influence of shrinking/stretching, thermogeometric parameters, surface temperature, convection conduction, radiation conduction, and Peclet number is investigated. The problem is solved numerically using a shooting method. To validate the numerical solution, the results are compared with the solution of a differential transform method. Temperature distribution increases with a rise in convection and radiation conduction parameters when Peclet number, stretching/shrinking, ambience, and surface temperatures are raised. The temperature of the fin’s tip increases as ambient temperature, Peclet number, and surface temperature increase, and decreases for enhanced radiation and convection conduction parameters. Radiation and convection cause the efficiency of the fin to increase for shrinking and decrease for stretching, which shows an important role in heat transfer analysis in mechanical engineering. The formulated model is also studied analytically, and the result is compared to numerical solution, which shows qualitatively good agreement.

1. Introduction

Enhancing heat transfer rate from hot surfaces of various appliances provides significant applications in technological, industrial, and engineering machinery such as air-cooled craft engines, air conditions, computer equipment, automobile radiators, CPU, gas turbines, and refrigerators [1]. An optimized extended surface or fin is applied to enhance the heat exchange between the warm surface and the neighbouring fluid. It is the fin’s job to transfer heat from the base to its surface by conduction and then to disperse it to the surrounding air by convection and radiation. Various efforts have been performed to analyze numerous properties for a variety of fins, where transfer of heat has been studied by considering convection and radiation with different geometries and nonlinearities [2].

The temperature distribution is investigated using a variety of fin shapes. The heat flux, heat transfer coefficient, and weight of annular fins and fins of star geometry are compared, and it is revealed that fins of star geometry dominate annular fins in terms of performance [3]. In the T-shape porous fin, heat transfer increases as the porosity parameter increases and decreases as the Biot number increases [4]. The use of a Y-shaped fin in a hollow is analyzed, and it is observed that a Y-shaped fin is more useful in a hollow [5]. Experimental and mathematical studies have found that the W-type fin has a significantly greater cooling effect than a straight parallel plate fin [6]. The numerical simulation of two-dimensional heat transfer by convection in a partially heated, tilted rectangular channel with many porous fins connected to a hot surface was carried out [7]. The heat exchange from a trapezoidal straight fin is investigated [8]. In the study of longitudinal fins, parabolic, trapezoidal, and rectangular geometries are mathematically analyzed [9]. By curved fins and heat transfer through natural convection, heat sinks are investigated, and it is revealed that the mass of the fins can be reduced by up to 60% and that the total thermal effectiveness of the suggested fins is around 98%. It is also revealed that curved fin heat sinks increase heat transfer performance by 12.52% [10].
The mass and heat transfer mechanism with optimization and efficiency of straight fins with different geometries has been extensively investigated under the influence of atmospheric pressure [11]. The vertical tubes with branching pin fins are explored experimentally, and it is observed that the forked pin fins have lower heat resistance than plate fins [12]. The thermal effectiveness of an air-cooled heat tubes used for cooling the CPU is examined [13]. The outcomes show that the thermosiphon position has the lowest thermal resistance, and it is less influenced by the heat input. Besides the analysis of straight fins, the thermal characteristics of moving fins have also been studied by several researchers. The study of fin set into motion is applicable during manufacturing processes [14], where the heat transfer in moving fins shows more impact on convection and radiation than the stationary fin [15]. The impact of fins and conducting walls on heat exchange rate inside hot cavities has been investigated numerically [16]. It is concluded that the inclusion of a fin improves the overall heat exchange rate by 21% for conductive horizontal walls and 18% for adiabatic walls. The effects of thermal conductivity through radiation and convection in a moving fin that dissipate heat by radiation and convection to its environments are investigated [17]. The importance of heat exchangers at high temperatures is greatly influenced by the issue of radiation. Thermal radiation and convection show more impacts on nanofluids in annulus fins [18]. Furthermore, the effect of natural convection heat exchange has also been extensively studied in 3D finned tubes [19]. A numerical and theoretical investigation of the behavior of electrically conducting water Fe3O4/CNT hybrid nanofluids within a trapezoidal enclosure of wavy-wall under a transverse magnetic field was conducted [20]. And it is observed that at intermediate Rayleigh numbers, a growth in the undulation number produces a small increase in the Bejan number. However, the Nusselt number increases considerably, and the influence is enhanced at very low high Rayleigh numbers. A finite-element model is used to investigate the thermal efficiency of varnishes in high-temperature gas turbine blade protection [21]. Based on the thermal results, the coating strains seemed to be controlled by the strain state of the building element near the coating interface.

The temperature distribution, radiation, and convection in longitudinal fins with heat transfer and heat generation have been extensively studied [22–24]. It is concluded that the temperature at the tip of the fin increases with the decrease in the radiation-conduction parameter and Peclet number with a rise in the heat production gradient. Furthermore, the ideal temperature profile for a straight rectangular fin is obtained at a static fin having internal heat generation in the absence of radiation. The flow of radiative nanofluid and the temperature distribution between two parallel disks with stretching and penetrating walls considering heat flux has also been studied, considering the Cattaneo–Christov model [25]. An examination of heat exchange in rectangular fins with convection and thermal radiation is obtained through variable conductivity and optimal linearization [26]. The exact solution for the fin with varying thermal conductivity and heat transfer is derived [27]. The performance of porous fins is examined in relation to convection and heat generation/absorption [28].

In Reference [29], the problem of mixed convection flows of Williamson nanofluid across a radial stretching surface under convective boundary conditions is explored. The magnetic field parameter appears to have opposing effects on velocity and temperature profiles, according to research. The velocity profile and boundary layer thickness decrease as the first-order velocity slip and Williamson parameter are increased. Using convective boundary conditions and second-order slip, a boundary layer flow of a dusty nanofluid which passed a centrifugally expanding surface was examined [30]. It has been observed that in both fluids and dusty phases, the magnetic field parameter influences the velocity profile similarly. Moreover, Cu-water and dusty nanoparticle volume fraction have parallel effects on dusty particle and temperature of nanofluid. A threedimensional mixed convection flow of Oldroyd-B nanofluid across a stretching surface is studied using the Cattaneo–Christov mass and heat flux model with third-order slip and convection boundary conditions [31, 32]. As the thermal relaxation parameter and the Prandtl number Pr increase, thickness and temperature of the thermal layer decrease.

To study the characteristics and geometry of the fin, many analytical and numerical methods are used. The homotopy perturbation method (HPM), spectral collocation method, Adomian decomposition method (ADM), and spectral element method are used to study the efficiency, heat transfer, convective radiation, thermal conductivity, and temperature distribution in a variety of fins [33, 34]. The differential transform method (DTM) considered herein is also very useful to study such models. It was first presented to study both linear/nonlinear DEs [35]. The significant advantage of the DTM is that it does not require any discretization/linearization or perturbation. It has also been extensively used to study nonlinear oscillations, DEs, and its convergence for both linear and nonlinear DEs, eigenvalue and engineering problems, and heat transfer analysis [36–38].

It is worth mentioning that, the stretching/shrinking fin of the exponential shape problem has been investigated before numerically by Gireesha et al. [39]. Similarly, the influence of the stretching/shrinking technique on the energy profile of a moving fin of various shapes has been analysed by Mosavat et al. [40]. Furthermore, a rectangular fin exposed to a convective environment is considered in an analytical study of the stretching/shrinking moving fin problem [41]. Here, we study the temperature distribution in stretching/shrinking straight fins having a rectangular geometry. The proposed model is studied analytically using DTM. Furthermore, the influence of radiation, temperature profile, and fin’s tip temperature under the influence of different parameters involved in the considered model is studied numerically.

1.1. Description of the Flow Parameters in the Considered Model of Heat Transfer Analysis

(i) Peclet numbers: the ratio of the rate of advection of a given substance by the flow to diffusion rate of the
same quantity cause by a suitable gradient is known as the Peclet number. For heat transfer, Peclet number is defined as \( Pe = UL/\lambda \).

(ii) Stefan–Boltzmann law: Stefan–Boltzmann law states that radiant energy radiated from any hot surface increases with its absolute temperature by the fourth power. Stefan–Boltzmann constant is represented by the Greek letter \( \sigma \). It value is \( 5.670374419 \times 10^{-8} \).

(iii) Heat transfer coefficient: a fluid’s ability to transfer heat by convection is described by its heat transfer coefficient.

(iv) Thermal conductivity: in thermodynamics, thermal conductivity is an indicator of how well a material conducts heat.

(v) Specific heat: heat required for increasing the temperature of a unit mass of a material by one degree is known as specific heat.

(vi) Emissivity: the emissivity of a material is the amount of energy that is released by its surface relative to the amount of energy that is emitted by a blackbody at the same wavelength and temperature.

2. Mathematical Formulation

We consider a heat exchange due to radiation and convection in a rectangular fin of length \( L \) and cross-section area \( A \), as presented in Figure 1. Let \( T_a \), \( T_b \), represent the ambient temperature, \( T_a \) and \( T_s \) are the base and surface temperatures, \( h \) denotes the convective coefficient, and \( T \) is the temperature distribution. Convective heat transfer coefficient \( h \) and thermal conductivity \( k \) of the fin are assumed to be constants, with \( \rho \) being the density, \( \epsilon \) being the emissivity of the material, and \( \sigma \) is the Stefan–Boltzmann constant. The fin is considered both moving and stretching/shrinking horizontally with velocity \( U(1 + S^* x) \), where \( U \) is the local speed, \( x \) is the distance from the base, and \( S^* \) represents the rate of stretching/shrinking. The governing equation for heat transfer through the fin is written as [41] follows:

\[
\frac{d^2T}{dx^2} + \frac{hP}{kA} (T - T_a) + \frac{\rho U C_p}{kA} (1 + S^* x) \frac{dT}{dx} - \frac{\epsilon \sigma P L^2}{kA} (T - T_b^4) = 0.
\]

Using dimensionless variables,

\[
X = \frac{x}{L},
\]

\[
\theta = \frac{T}{T_b},
\]

\[
\theta_a = \frac{T_a}{T_b},
\]

\[
\theta_s = \frac{T_s}{T_b},
\]

\[
N^2 = \frac{hPL^2}{kA},
\]

\[
Pe = \frac{UL}{\lambda},
\]

\[
N_r = \frac{\epsilon \sigma P L^2 r_b^4}{kA},
\]

\[
S = S^* L,
\]

where \( \lambda = k/\rho C_p \) represents diffusivity, \( c_p \) is the specific heat, \( \rho \) is the density of the material, \( \theta_a \) and \( \theta_s \) are the temperature ratios, \( N \) is the convection-conduction parameter, \( Pe \) is the Peclet number, \( N_r \) represents radiation conduction, \( h \) denotes the heat transfer coefficient, and \( S \) is the stretching/shrinking parameter. Hence, equation (1) becomes

\[
\frac{d^2\theta}{dx^2} - N^2 (\theta - \theta_a) + Pe(1 + Sx) \frac{d\theta}{dx} - N_r (\theta^4 - \theta_s^4) = 0,
\]

which gives

\[
\frac{d^2\theta}{dx^2} + Pe(1 + Sx) \frac{d\theta}{dx} - N^2 \theta - N_r \theta^3 + N^2 \theta_a + N_r \theta_s^4 = 0.
\]

with adiabatic boundary conditions

\[
\theta(1) = 1, \quad \frac{d\theta}{dx}|_{X=0} = 0.
\]

The fin efficiency justifies minimization in temperature difference between the fin and surrounding fluid. The fin

\[\text{Figure 1: Schematic diagram of stretching/shrinking longitudinal fin having rectangular profile.}\]
efficiency represented by $\eta$ is defined as the ratio of actual total heat dissipation of the fin to the maximum heat dissipation. Mathematically, efficiency can be defined as [42] follows:

$$\eta = \frac{\theta (1)}{N^2 (1 - \theta_a)}$$  \hspace{1cm} (6)

Equation (3) shows that the distribution of temperature $\theta(x)$, and fin's efficiency $\eta$ depends on $N_r$, $N$, $S$, $Pe$, and temperature ratios $\theta_a$ and $\theta_s$.

3. Differential Transform Method (DTM)

For understanding fundamentals of DTM, let $\phi(x)$ is an analytic function in the domain $D$, and $x = x_i$ represents an arbitrary point. A function $\phi(x)$ can be expressed by a power series with its centre at $x_i$. The expansion of $\phi(x)$ can be written as [43] follows:

$$\phi(x) = \sum_{m=0}^{\infty} (x-x_i)^m \Phi(m) \quad \forall x \in D.$$  \hspace{1cm} (8)

The differential transformation of $\phi(x)$ gives [36]

$$\Phi(m) = \sum_{m=0}^{\infty} \frac{(x-x_i)^m}{m!} \frac{d^m \phi(x)}{dx^m} \bigg|_{x=x_i},$$  \hspace{1cm} (9)

where $\phi(x)$ represents a primary function, and $\Phi(m)$ is the transform function. The differential spectrum of $\Phi(m)$ is bounded by an interval $x \in [0, M]$, where $M \in \mathcal{R}$.


To evaluate model equation (3), we apply DTM as discussed in previous section, and we obtain

$$\Theta(m + 2) = \frac{1}{(m + 1)(m + 2)} \left[ N^2 \Theta(m) - N^2 \theta_a \delta(m) + N_s \sum_{\ell=0}^{m} \sum_{k=0}^{\ell} \phi(j)\phi(k-j)\phi(\ell-k)\phi(m-\ell) \right.$$  \hspace{1cm} (10)

$$+ PeS \sum_{\ell=0}^{m} \delta(\ell-1)(m-\ell+1)\Theta(m-\ell+1) + Pe(m+1)\Theta(m+1) + N_s \theta_s \delta(m) \bigg].$$

By rearranging, we get

$$\Theta(0) = a,$$

$$\Theta(1) = 1.$$  \hspace{1cm} (12)
Similarly,

\[ \Theta(2) = \frac{1}{2} \left[ Nr a^4 - Nr^2 \theta_s^4 + N^2 a - N^2 \theta_a + Pe \right], \]

\[ \Theta(3) = \frac{1}{6} \left[ N^2 + N r (a^4 + 4a^3) - 2 Pe \Theta(2) - Pe + S \right], \]

\[ \Theta(4) = \frac{1}{12} \left[ N^2 \Theta(2) + N r (4a^3 \Theta(2) + 4a^3 + 6a^2 + a^4) - 2 Pe \Theta(2) - 3 Pe \Theta(3) \right], \]

\[ \Theta(5) = \frac{1}{20} \left[ N^2 - 3 Pe S + 4 N r a^3 \right] \Theta(3) + N r \left[ 12a^3 + 4a^3 \right] \Theta(2) + a^4 + 4a^3 + 6a^2 + 4a] - 4 Pe \Theta(4)]. \]
The final solution can be written as
\[ \theta(x) = \sum_{m=0}^{n} \Theta(m)x^m. \] (14)

The DTM and numerical solutions are compared in Figure 2, where good agreement is obtained.

4. Results and Discussion

Here, we investigate numerically heat transfer profile, temperature of fin’s tip, \( \theta(0) \), and efficiency \( \eta \) of fin under the influence of radiation and convection in stretching/shrinking rectangular fins.

The temperature distribution \( \theta(x) \) for the considered parameters is presented in Figures 3 and 4. The influence of dimensionless speed \( Pe \) on temperature profile is demonstrated in Figure 3(a). It is observed that temperature distribution gradually rises by increasing Peclet number. On the one hand, by adding the stretching mechanism to the fin motion, the exposure of the fin is further decreased. On the other hand, the shrinking mechanism increases the fin exposure time, which increases heat loss through convection. The influences of stretching/shrinking parameter on the distribution of temperature are presented in Figure 3(b), and it is observed that as the fin shrinks, the temperature decreases, while as the fin stretches, the temperature increases.

The effect of convection-conduction number is shown in Figure 3(c). The rate of heat transmission through the fin increases as the convection-conduction numbers increase, as seen in Figure 3(c). The temperature of the fin drops faster due to the strong heat flow from the fin’s base.
The influence of radiation parameters on temperature profiles when \( N_r \) is to change while other parameters remain constant is presented in Figure 4(a). As radiative transportation grows, radiative cooling has become very efficient, resulting to lower temperatures in the fin. Figure 4(b) demonstrates how variations in the surrounding temperature affect temperature distribution in the fin while other parameters are kept constant. The influence of \( \theta_a \) and \( \theta_s \) on temperature distribution is illustrated in Figures 4(b) and 4(c), which show that the temperature profile increases as \( \theta_a \) and \( \theta_s \) rise. Convection heat loss from the moving fin’s surface decreases as \( \theta_a \) and \( \theta_s \) grow, resulting in higher temperatures on the fin’s exterior.

Fin’s tip temperatures \( \theta(0) \) of stretching/shrinking fins are presented in Figures 5 and 6. It is illustrated that during a stretching/shrinking process, the temperature at the fin’s tip increases, indicating the general improvement in temperature distribution across fin surfaces. We conclude that, by increasing the stretching parameter, the surface temperature increases. The effect of raising \( N \) and \( N_r \) is shown to lower the fin’s temperature as well as \( \theta_a(0) \), whereas the influence of \( Pe \) shows opposite behaviour, as shown in Figure 5. As can be seen, increasing \( Pe \) results in an increase of fin heat loss. With the stretching/shrinking parameter \( S \) sliding from \( S = -1 \) towards \( S = 1 \), the effect becomes more evident.

From Figure 6, it is observed that the fin’s tip temperature along the stretching/shrinking parameter increases with the influence of increasing ambient temperature \( \theta_a \) and surface temperature \( \theta_s \). Figures 7 and 8 present the influence of parameters \( N_r, N, Pe, \theta_a, \) and \( \theta_s \) involved in stretching/shrinking fin on fin’s efficiency. It is clear that the efficiency of the fin \( \eta \) grows for shrinking and decreases for enhancing...
stretching considering radiation and convection. The effect of radiation on fin efficiency is shown in Figure 7(a), where a gradual increase in fin's efficiency $\eta$ is observed, while the value of $\eta$ diminishes by increasing $N$, $Pe$, $\theta_s$, and $\theta_0$, as presented in Figures 7(b) and 8.

Temperature profile $\theta(x)$, temperature of fin's tip $\theta(0)$, and efficiency $\eta$ indicate that shrinking fins with radiation provide better efficiency as compared to stretching fins with radiation. According to the results, a stretching/shrinking mechanism applied to longitudinal rectangular fins with radiation impact can yield a more efficient and more economical heat transfer solution.

5. Conclusion

We have studied the combined effect of radiation and convection on stretching/shrinking longitudinal fins having a rectangular profile. The temperature distribution, fin tip temperature and efficiency are studied. Analytical and numerical solutions are considered for the model. With increasing Peclet number, stretching/shrinking, ambient and surface temperatures, and increasing convection and radiation-conduction parameters, it appears that temperature distribution increases. With an increase in ambient temperature, Peclet number, and surface temperature, the tip temperature of a fin increases against stretching/shrinking. This increases against stretching/shrinking while diminishing with an increase in radiation conduction and convection. The efficiency of the fin under the influence of radiation and convection initiates an increase for shrinking and a decrease for stretching the fin. Furthermore, the efficiency versus stretching/shrinking grows with increasing radiation-conduction parameter and declines with higher values of convection number, Peclet number, and surface and ambient temperatures. Using the considered parameters of longitudinal rectangular fins, the effect of stretching/shrinking with radiation increases the fin's heat transfer rate and efficiency. We conclude that considering the shrinking mechanism in fin with radiation provides more efficiency and better economic benefits as compared to stretching with radiation. [44]

Abbreviations

- $A$: Area of fin’s surface (m$^2$)
- $k$: Thermal conductivity (W/mK)
- $N$: Convection parameter
- $S$: Stretching/shrinking parameter
- $S^*$: Rate of stretching/shrinking (m$^1$)
- $U$: Speed of moving fin (m/s)
- $X$: Dimensionless distance
- $N_r$: Radiation parameter
- $T_a$: Dimensional surrounding temperature (K)
- $a$: Fin shape parameter
- $\varepsilon$: Fin’s emissivity
- $\alpha$: Surface emissivity
- $\theta$: Dimensionless temperature of the fin
- $\theta_0$: Dimensionless base temperature
- $L$: Fin’s length (m)

$h$: Convection coefficient (W/m$^2$K)
$P$: Fin’s perimeter (m)
$T$: Fin temperature (K)
$W$: Fin width (m)
$x$: Dimensional distance (m)
$C_p$: Specific heat (J/kgK)
$T_b$: Fin base temperature (K)
$T_s$: Dimensional surface temperature (K)
$\lambda$: Thermal diffusivity (m$^2$/s)
$\rho$: Density of material (kg/m$^3$)
$\sigma$: Stefan–Boltzmann constant W/m$^2$K$^4$
$\theta_0$: Dimensionless surrounding temperature
$\theta_b$: Dimensionless base temperature

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest regarding this manuscript.

Authors’ Contributions

It is also declared that all the authors have equal contribution in the manuscript. Furthermore, the authors have checked and approved the final version of the manuscript.

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