Research Article

Cooperative Advertising in Dual Channel Supply Chain System with Different Contracting Schemes

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Received 19 January 2022; Revised 28 June 2022; Accepted 4 July 2022; Published 29 September 2022

1. Introduction

The beverage industry features a highly competitive nature including price and advertising competitions, especially the soft drink market is almost a duopoly market. The average advertising expenditures by Coca-Cola or Pepsi are about $4 billion each year from 2015 to 2020 [1]. In the fast food industry, it is natural to see products of Coca or Pepsi alongside a burger on restaurant billboards. This case is a typical cooperative program, in which Coke or Pepsi often offer some incentive schemes to their partners who advertise to boost their sales. Cost-sharing (CS) has often been implemented in the form of cooperative advertising. In reality, most manufacturers set the cost-sharing rate arbitrarily and without detailed analysis to 50% or 100% [2]. However, according to Dutta et al. [3]; they pointed out that participation rates are higher only when there are fewer competitors. Bergen and John [4] demonstrate that the competition between channels has a significant impact on the participation rates. A question arises, is a higher cost-sharing rate always beneficial to the retailers? Meantime, a taste experiment sponsored by Pepsi Cola shows that consumers prefer Pepsi Cola over Coca-Cola if they are only guided by taste. Interestingly, the market share of Coca-Cola in the soft drink market accounts for 42.5%, which is larger than Pepsi’s [5]. The other question arises, how do market shares between two competing firms influence the efficiency of advertising? Furthermore, are there some other forms of cooperative advertising schemes that can be used to coordinate the supply chain?

The cost-sharing contract is very prevalent in cooperative advertising, such as Aust and Buscher [6]; Aust and Buscher [7]; Huang et al. [8]; Szmerekovsky and Zhang [9]; SeyyedEsafahani et al. [10]; Xie and Ai [11]; Xie and Neyret [12]; Zhao et al. [13]. The literature listed above is limited by the assumption that the cost-sharing rate is an endogenous variable in the single channel and show that the cost-sharing contract can coordinate the entire supply chain. This is because they adopt a centralized decision-making approach to characterize the cooperative advertising efficiency, in which...
they assume a unique decision maker announces both the manufacturer and the retailer advertising levels. It is obvious that the coordination can be achieved under such cooperative advertising assumption. However, the hypothesis of a centralized control is not realistic in marketing practices when supply chain members are independent economic entities [14]. Notably, we find that the coordination mechanism proposed by the aforementioned literature does not work well by comparing the scenarios with and with cost sharing. Moreover, Zhang et al. [15] use a very general function to demonstrate that cost-sharing is not an effective coordination mechanism when the cost-sharing rate is assumed to be endogenous. Additionally, all of the above-cited literature are limited to a cooperative advertising scheme in a single channel. Different from the above literature, Li et al. [16] examine the efficacy of different advertising structures in a dual-exclusive channel system and find that the endogenous cost-sharing cannot coordinate the entire supply chain. It remains unknown whether the cost-sharing contract can coordinate the competing supply chains when the cost-sharing rate is determined exogenously.

Some literature investigates the cost-sharing efficiency for cooperative advertising in competing supply chains. For instance, Karray and Zaccour [17] discuss a duopoly common retailer channel and suggest that the higher competition leads to a Prisoner’s Dilemma for manufacturers. Yan and Pei [18] investigate channel conflict and coordination in a dual channel and find that the improved retail services effectively alleviate the channel conflict. Pei and Yan [19] investigate the effect of manufacturer advertising on alleviating channel conflictions in a dual channel. Liu et al. [20] discuss manufacturer advertising, retailer advertising and cooperative advertising in a dual-exclusive channel. They find the cost-sharing contract can coordinate the dual channel system under manufacturer advertising, while does not work well under retailer advertising. Most of the literature listed above, however, considers the cost-sharing rate as an exogenous variable and suggest that cost-sharing contract is an effective coordination mechanism. Among the exceptions, Karray and Amin [21] investigate the cost-sharing efficiency in a monopoly common manufacturer channel by assuming the cost-sharing rate to be an endogenous variable. However, the above literature assumes the cost-sharing rate to be endogenous, likewise, it remains unknown whether the cost-sharing contract can coordinate the competing supply chains when the cost-sharing rate is determined exogenously.

According to Cachon and Lariviere [22]; they point out that the revenue-sharing contract (RS) particularly suits for the video rental industry. Although the popularity of the revenue-sharing contract, limited literature applies this contract into the cooperative advertising scheme in a dual-exclusive channel. In practice, Apple has revenue-sharing arrangements with its retailers who advertise its products [23]. There are some literatures investigating the efficacy of revenue sharing in a single supply chain. He et al. [24] consider a dynamic advertising strategy in a single supply chain and find that it is not always profitable for the manufacturer to offer a cost-sharing contract. Kunter [23] investigates the efficiency of revenue-sharing and cost-sharing contracts in a single channel. Note that the revenue-sharing contract is not popular to deal with the conflict of cooperative advertising in the offline business. Our work follows this trend but from a different perspective under the revenue-sharing rate assumed to be endogenous or exogenous in a dual-exclusive channel system, and tries to explain why the revenue-sharing contract is far less common in the marketing practices.

Furthermore, our study is closely related to Karray and Amin [21] who investigate how competition affects the cost-sharing efficiency in a monopoly common manufacturer channel. Our paper is also closely related to He et al. [24] and Kunter [23] who consider cooperative advertising with cost sharing and revenue sharing in a single channel. However, none of the literature to date has considered the cost-sharing and revenue-sharing efficacies in a dual-exclusive channel. To differ from the previous literature, this study develops two models—one in which each manufacturer determines the sharing rate endogenously or exogenously under the Stackelberg game approach and the other in which each manufacturer and its exclusive retailer cooperatively determine the sharing rate under the Nash bargaining approach, in a dual-exclusive channel. We first investigate a non-cooperative advertising program. Our findings indicate that advertising is never a competitive tool for the small firm if its market share is sufficiently small. We then study the efficiency of cost sharing and revenue sharing with the sharing rate either as an endogenous variable or as an exogenous variable. If the sharing rate is endogenous, neither the cost-sharing contract nor the revenue-sharing contract is an effective coordination mechanism; if the sharing rate is exogenous, both cost-sharing and revenue-sharing are effective coordination mechanisms when the competition is sufficiently small. Further, an optimal sharing rate can be used to deal with the conflict for the cooperative advertising framework under the Nash bargaining approach. Most notably, we find that the cost-sharing contract is more beneficial to the cooperative advertising mechanism than the revenue-sharing contract.

The most related studies are presented in Table 1. For simplicity, we use symbols, WP, RP, MA, RA, and SR to represent wholesale price, retail price, manufacturer advertising, retailer advertising level, and sharing rate, respectively.

### 2. Model Formulation and Notation

Consider a dual-exclusive channel where each manufacturer sells its goods through an exclusive retailer. The manufacturer and the retailer in each channel engage in price and advertising competitions with rivals. The dual-exclusive channel system is not rare in real life. In China, McDonald and Coca-Cola constitute a supply chain to compete with other supply chains such as Kentucky Fried Chicken and Pepsi-Cola. Advertising is a crucial tool to improve a firm’s competitiveness and market share, and it may come from both manufacturers and retailers. To model the influence of
advertising on sales, we employ the framework of Xie and Wei [14] and Liu et al. [20] to characterize the demand for channel \( i \),

\[
D_i = \frac{Y_i - \theta Y_{3-i} - p_i + \theta p_{3-i}}{1 - \theta^2},
\]

where \( Y_i = A_i + H_{mi}K_{mi}e_{mi} + H_{ri}K_{ri}e_{ri} \). The impact of advertising is embedded in \( Y_i \). \( A_i \) is channel \( i \)'s initial base market share; To obtain simplicity, a new parameter called "the demand ratio," \( \Omega \equiv A_1/A_2 \), is added to measure the degree of asymmetry between the two channels. The demand ratio is symmetric if \( \Omega = 1 \); otherwise, it is asymmetric. \( \Omega > 1 \) means channel 1’s market size is larger than channel 2’s. \( H_{mi} \) and \( H_{ri} \) are equal to 0 or 1, which is the indicator of whether manufacturer (retailer) \( i \) advertises; \( K_{mi} \) and \( K_{ri} \) represent the advertising efficacy of manufacturer \( i \) and retailer \( i \). For simplicity, \( K_{mi} \) and \( K_{ri} \) are normalized to 1. \( e_{mi} \) and \( e_{ri} \) represent manufacturer \( i \)'s advertising level and retailer \( i \)'s advertising level; \( p_i \) is the retail price and \( w_i \) is the wholesale price. \( \theta (0 < \theta < 1) \) captures the competition between channels 1 and 2. It is worth noting that, the above demand function is different from the general demand function adopted by Barman et al. [25]; Sana noting that, the above demand function is different from the general demand function adopted by Barman et al. [25]; Sana

To obtain simplicity, we use the subscripts NN, AA, RAA, CC, and RR to denote the corresponding incentive scheme, respectively. Scenario NN: neither manufacturers nor retailers advertise (\( H_{mi} = 0, H_{ri} = 0, \Phi_i = 0, \psi_i = 0 \)); Scenario AA: both manufacturers and retailers advertise (\( \Phi_i = 0, \psi_i = 0 \)); Scenario RAA: Retailers advertise solely (\( H_{mi} = 0, \Phi_i = 0, \psi_i = 0 \)); Scenario CC: cooperative advertising with cost-sharing contract (\( \psi_i = 0 \)); RR: cooperative advertising with revenue-sharing contract (\( \Phi_i = 0 \)).

3. Advertising without Sharing

In order to examine how the advertising effort affects the profitability of advertisers, we first study the advertising program without any incentive schemes.

3.1. Comparison: AA versus NN. In Scenario AA, each manufacturer first announces its wholesale price and advertising level using the response functions of retailers 1 and 2, and then each retailer determines its retail price and advertising level given the wholesale prices and manufacturer advertising levels. Through the comparison of the equilibrium outcomes with and without advertising, we can check whether a channel member has an incentive to advertise.

**Theorem 1.** Although advertising is a dominant strategy and equilibrium, advertising is never a viable competitive tool for small firms; The advertising Pareto zone for retailers is larger than that for manufacturers. Such that,

(i) AA benefits a channel member with a larger base demand (e.g., region C or D);

(ii) AA benefits manufacturers and retailers in region A, and benefits retailers in region B.

Theorem 1 tells us an interesting result that both manufacturers and retailers have incentives to advertise, and these incentives critically depend on the relative demand ratio. Advertising benefits the channel member with a larger base demand but hurts the one with a smaller base demand. Note that only the market share gap between two channels is sufficiently large can the small advertiser be worse off. In practice, the market share gap between Pepsi Cola and Coca-
Cola is small in the soft drink market. Hence, Pepsi Cola consistently advertises to maintain its market share; otherwise, it will lose the market share to the other large competitors, such as Coca-Cola. Interestingly, Theorem 1 also reveals that retailers have a larger advertising Pareto zone than manufacturers, which implies that retailers at the end of the market and thereby undertake more effective advertising. This observation can be used to explain the prevalence of retailer advertising without manufacturer subsidies in current business practices. Note that if we extend our discussion to the symmetry case (ω = 1), the conclusion in Theorem 1 still holds for manufacturers. For the retailer’s perspective, they always get the best performance in Scenario AA. It is evident that the asymmetric market share does change the nature of equilibrium in our model.

As shown in Figure 1, it is mutually beneficial for manufacturers and retailers to advertise in region A. This is because \( \omega^{NN} < \omega^M \), \( p_i^{NN} < p_i^M \), \( D_i^{NN} < D_i^M \) in region A, and thereby increases the revenue of channel members. In region B, advertising only benefits retailers. Figure 1 also shows that advertising benefits channel member 1 (2) but hurts channel member 2 (1) in region C (region D).

To explain why a channel member with a smaller base market does not advertise in equilibrium, we will explore properties for price elasticity.

**Corollary 1.** Under Scenario AA, a channel member’s price increases with its own base demand/advertising effort but decreases with its rival’s (e.g., \( \partial j_i^{AA}/\partial \omega > 0 \) and \( \partial j_i^{AA}/\partial \omega < 0 \) ).

Corollary 1 indicates that the large advertising expenditure/base demand unambiguously tends to increase its own price, but to decrease the rival’s (e.g., \( \partial p_i^{AA}/\partial \omega > 0 \) and \( \partial p_i^{AA}/\partial \omega < 0 \) ). This observation means that the advertiser with a smaller base demand triggers a heightened price competition, and thereby the small firm cut price but the large firm raise price, which can be best explained using Figure 2 (e.g., \( p_1^{NN} > p_1^{AA} \) for \( \Omega < \Omega^{AA-NN}_{p1} \)). Consequently, advertising is never a competitive tool for a small advertiser if its market share are sufficiently different from rivals.

3.2. Comparison: AA versus RAA. In the aforementioned analysis, we present the analysis of advertising performed by all channel members. As a matter of fact, it is natural to see that large retailers, such as Walmart and Target, often advertise certain products so that many of their suppliers do not need to (Note that the result in Theorem 2 continues to hold if advertising is performed solely by manufacturers. Therefore, we omit it for brevity). In this regard, compare profits between scenarios AA and RAA to explore if it is necessary for manufacturers to advertise.

![Figure 1](image1.png)

**Figure 1:** Manufacturers’ (M) and retailers’ (R) preferences with and without advertising given \( \Omega \neq 1 \).

![Figure 2](image2.png)

**Figure 2:** Channel members’ pricing decisions with and without advertising given \( \Omega \neq 1 \).

**Theorem 2.** There is no need for manufacturer \( i \) to advertise if and only if \( \Omega_i < \Omega \leq \Omega^{AA-RAA}_{m1} \).

Theorem 2 answers the research question under what market conditions should manufacturer \( i \) advertise. Intuitively, one might think that manufacturer \( i \) has an incentive to advertise, as advertising is a useful tool for demand creation. However, Theorem 1 indicates that the profitability for manufacturers depends on their base market share. For instance, when a channel obtain the larger basic market share (\( \Omega^{AA-RAA}_{m1} < \Omega < \Omega^{AA-RAA}_{m1-1} \)), it is always profitable for the manufacturer with the larger base market to advertise. The
reason can be referred to Corollary 1. Further, Theorem
Theorem 2 continues to hold when the relative demand ratio
is symmetric. Combining Theorems 1 and 2, we obtain
Insight 1:

Insight 1: Advertising is never a competitive tool for the
small firm in competing supply chains.

4. Advertising with Endogenous Sharing Rate

Cost-sharing and revenue-sharing contracts have been
commonly used to coordinate the conflict in cooperative
advertising schemes. In this section, we will explore the
efficacy of cost sharing and revenue sharing under the
sharing scheme offered by its partner. f"heorem 3, however,
and decreases with the product substitutability.

Proposition 1 tells us that manufacturer \( i \)' sharing policy
depends on its wholesale price, as well as advertising effort.
In particular, if the wholesale price is relatively high such
that \( 1 - e_{CC}^{CS} < w_{i}^{CS} < 1 \), manufacturers will offer advertis-
ing allowance to the retailers, otherwise they will offer
nothing, which is sharply contrasted with Huang et al. [8]
and SeyedEsfahani et al. [10]; as they assume prices as ex-
genous variables. This finding implies that the higher
product substitutability will induce channel members to
lower their share rates. Additionally, the cost-sharing rate
decreases with the degree of competition and is in the range
of 0 to 33%. Proposition 1 also indicates that Scenario CC is
equivalent to Scenario AA if the wholesale price is relatively
small (e.g., \( 0 < w_{i}^{CC} < 1 - C_{CC}/3 \)).

We now investigate how the cost-sharing contract im-
pacts the channel member’s profitability.

Theorem 3. Under cooperative advertising with cost sharing,
manufacturer \( i \) always prefers cost-sharing contract, whereas
retailer \( i \) always prefers no cost-sharing contract.

Intuitively, an advertiser should be benefited from the
sharing scheme offered by its partner. Theorem 3, however,

shows a counter-intuitive result that retailer \( i \) does not have
an incentive to participate in the cooperative advertising
program, though manufacturer \( i \) bears some portion of the
advertising expenditures. This finding is in line with Karray
and Amin [21] in which cost sharing is not beneficial to
competing retailers under certain conditions, but from a
different perspective with manufacturer advertising in a
dual-exclusive channel. We also depict the result in Figures 3
and 4; cost-sharing benefits manufacturer \( i \) but hurts retailer
\( i \). In summary, the cost-sharing contract is not an effective
coordination mechanism when the cost-sharing rate is de-
cided by the manufacturer.

4.2. Advertising with Endogenous Revenue-Sharing Rate.
The cost-sharing contract cannot coordinate the entire
supply chain system, which motivates us to investigate
whether the revenue-sharing contract can work well. Simi-
larly, the optimal revenue-sharing rate is given as follows:

Proposition 2. Under cooperative advertising with revenue
sharing, there is an optimal revenue-sharing rate, which
depends critically on the parameters, such that the wholesale price and
the manufacturer advertising effort. The optimal sharing rate is
\[
\begin{align*}
\Phi_{CS}^{RR} &= 1 - \frac{9 - 12d^2 + 4d^4}{36d - 64d^3 + 16d^5} \quad \text{if } 0 < w_{i}^{RR} < 1 - \frac{e_{m}^{RR} + \Lambda}{4}, \\
\Phi_{CS}^{RR} &= 0, \quad \text{otherwise}
\end{align*}
\]
and increases and then decreases with product substitutability. Where \( \Lambda = \sqrt{1 + 2e_{mi} - (11 + 8\theta - 8\theta^2)e_{mi}^2} \).

Similar to Propositions 1, Proposition 2 demonstrates that manufacturer \( i \)' revenue-sharing policy depends on their wholesale price, as well as the advertising efforts. However, contrary to Proposition 1, the optimal revenue-sharing rate increases and then decreases with product substitutability. This is because revenue-sharing drives up the wholesale price (e.g., \( w_{mi}^{RR} > w_{mi}^{AA} \)) and the demand (e.g., \( D_{m1}^{RR} > D_{m1}^{AA} \)), which in turn drives up manufacturer advertising level. Further, the revenue-sharing encourages retailer \( i \) to raise its advertising effort and thereby intensifies the competition between two channels. The competition erodes manufacturers' profits. Consequently, manufacturers cut the revenue-sharing rate when the competition is relatively high.

Similar to the scenario with cost sharing, comparing profits between Scenarios RR and AA gives the following result:

**Theorem 4.** Under cooperative advertising with revenue sharing, manufacturer \( i \) prefers revenue sharing when product substitutability is relatively small, whereas retailer \( i \) always prefers no revenue sharing.

Similar to the scenario with cost sharing, Theorem 4 shows that retailer \( i \) has no incentive to participate in the cooperative advertising program. But for manufacturer \( i \), the incentive depends on the degree of substitution between the two channels. Specifically, the higher product substitutability will lead manufacturer \( i \) to violate the cooperative advertising program, as shown in Figures 5 and 6 (e.g., \( \pi_{mi}^{RR} < \pi_{mi}^{AA} \) for \( \theta \in [0.6, 0.75] \)). Combined with Theorem 3, the insight can be derived as follows:
5. Advertising with Exogenous Sharing Rate

From Theorems 3 and 4, we find that if the manufacturer is allowed to decide both its sharing rate and wholesale price in the first stage, the game will have unreasonable outcomes. In the following, the major concern is whether the sharing parameter determined exogenously is capable of leading channel members to achieve Pareto improvement in their profits.

5.1. Advertising with Exogenous Cost-Sharing Rate. The equilibrium threshold value is difficult to be solved analytically for the retailer $i$ if the cost-sharing rate is decided exogenously, we will use Plot3D (Mathematica procedure) to analyze the efficiency of cost sharing on channel members’ profitability. Combining Figures 7 and 8, there must be a threshold $\Phi_{ri} \in [CC-AA \leq \Omega] \subset (0, \Phi_{mi}]$ to coordinate the overall supply chain.

**Theorem 5.** Under cooperative advertising with cost sharing, a cost-sharing threshold value can be used to coordinate the entire supply chain system when the cost-sharing rate is relatively low.

Although we might have expected retailer $i$ could be better off if its partners bear some portion of advertising cost, Theorem 5 is somewhat counterintuitive that retailer $i$ could be worse off when the cost-sharing rate is high. In particular, it is mutually beneficial for both manufacturer $i$ and retailer $i$ to adopt the cost-sharing contract when the cost-sharing rate is sufficient low, since cost-sharing leads all channel members to invest more on advertising expenditures and in turn expand the market demand. However, if the cost-sharing rate becomes high, retailer $i$ will violate the coordination. This is because the advertising subsidy provided by manufacturer $i$ result in a higher retailer advertising level, which intensifies the supply chain competition and in turn erodes retailer $i$’s profit. Moreover, manufacturer $i$ can recover its advertising subsidy by increasing the wholesale price. Hence, only manufacturer $i$ prefers the cost-sharing mechanism. As the cost-sharing rate becomes insufficiently larger such that $\Phi_{ri}^{RR-AA} < \Phi < \Phi_{mi}$, neither manufacturer $i$ nor retailer $i$ have incentives to participate in the coordinate mechanism, as shown in Figures 7 and 8.

Given the asymmetric setting, there exists a cost-sharing threshold value to coordinate the confliction in cooperative advertising program. As shown in Figure 9, when the competition is low, CC is a dominated strategy for manufacturers and retailers in region A, which implies there is a common feasible area to guarantee all channel members to be better off. As the competition intensifies, CC benefits the manufacturers, while hurting the retailers. It is worth noting that the advertising with cost-sharing Pareto zone is larger for manufacturers.

5.2. Advertising with Exogenous Revenue-Sharing Rate. The game rule is similar to the scenario with cost-sharing when the revenue-sharing rate is exogenous. By comparing channel members’ profits with and without revenue sharing, we obtain the following theorem:

**Theorem 6.** Under cooperative advertising with revenue sharing, a revenue-sharing threshold value $\psi_{ri}^{RR-AA}$ can be used to coordinate the entire supply chain as long as $0 < \psi_{ri} < \psi_{ri}^{RR-AA}$. Such that both manufacturer $i$ and retailer $i$ prefer revenue sharing to no revenue sharing as long as $0 < \psi_{ri} < \psi_{ri}^{RR-AA}$, where $\psi_{ri}^{RR-AA} = 45 - 72 \theta - 264 \theta^2 + 144 \theta^3 + 452 \theta^4 - 64 \theta^5 - 304 \theta^6 + 64 \theta^7 / (3 - 2\theta)(99 + 36 \theta - 294 \theta^2 - 72 \theta^3 + 264 \theta^4 + 32 \theta^5 - 64 \theta^6)$. 

*Insight 1: when the sharing rate is endogenous, cooperative advertising with cost or revenue-sharing contract cannot coordinate the dual-exclusive channel system.*

*Insight 2: when the sharing rate is endogenous, cooperative advertising with cost or revenue-sharing contract cannot coordinate the dual-exclusive channel system.*
Theorem 6, is similar to Theorem 5, it also reveals that the revenue-sharing contract can coordinate the entire supply chain when the demand ratio is symmetric. The reason is identical to the scenario with cost sharing. We also depict Theorem 5 in Figure 10, both manufacturer $i$ and retailer $i$ benefit from the revenue-sharing contract in Region A, while only manufacturer $i$ prefer the contract in Region B. Note that in region C, if the revenue-sharing rate is high and the competition is moderate, both manufacturer $i$ and retailer $i$ will violate the cooperation. This result implies that the higher revenue-sharing rate intensifies the horizontal competition.

Given the asymmetric setting, there is also a Pareto improvement zone for both manufacturers and retailers. As shown in Figure 11, RR is a dominated strategy for the manufacturers and the retailers in region A. RR is a dominated strategy for the manufacturers in region A. It is worth noting that the revenue-sharing contract fails to solve the confliction for the cooperative advertising framework when the competition becomes fierce. Additionally, the advertising with revenue sharing or cost-sharing Pareto zone is larger for manufacturers, which is sharply contracted with Theorem 1. Combing Theorems 5 and 6 gives us insight 3:

Insight 3: Given the sharing rate exogenous, cooperative advertising with cost or revenue-sharing contract is capable of coordinating the dual-exclusive channel system when the competition is low. Nerveless, neither the cost-sharing nor the revenue-sharing works well when the completion is fierce.
5.3. Comparison: Revenue Sharing versus Cost-Sharing. So far, we have discussed the cost-sharing and the revenue-sharing efficiencies for channel members in the aforementioned sections. In the foregoing section, we will check which contract is better for the whole channel. Given the exogenous sharing rate assumption, we fail to analytically solve the equilibrium results. For simplicity, given \( \theta = 0.2, \theta = 0.6, \) and \( \Omega = 1 \), which can deliver us more management insights.

For a channel perspective, as vividly shown in Figures 12 and 13, the cost-sharing contract is more beneficial to channel \( i \), which means that cost-sharing contract more suits for coordinating the conflict in the cooperative advertising program. This observation can be used to explain the reason why the cost-sharing is widely used in market practices. Note that Li et al. [29] demonstrate that the revenue-sharing contract is suite for the green supply chain.

Insight 4: Given the sharing rate exogenous, the cost-sharing efficiency outperforms the revenue-sharing efficiency under the cooperative advertising framework.

6. Advertising with Bargaining

In marketing practices, it is common to find a manufacturer and a retailer bargain on the sharing rate, in which the sharing rate is cooperatively determined by the manufacturer and the retailer instead of a single-channel member. To investigate the impact of bargaining mechanism on cooperative advertising efficiency, we employ the egalitarian bargaining structure first proposed by Nash [30]:

\[
\text{argmax}_i \{ B(L) \} = \text{argmax}_i \{ \pi^\text{CC} - \pi^\text{RR} \} \equiv \left\{ \pi^\text{CC} > \pi^\text{RR} \right\},
\]

where \( L = \{ \Phi, \psi \}, J = \{ \text{CC, RR} \}. \) The symbols BRR and BCC represent manufacturer \( i \) and retailer \( i \) bargaining on the revenue-sharing rate and the cost-sharing rate, respectively. To obtain tractability, we will investigate the bargaining case under the symmetric demand ratio. Since we fail to analytically solve the equilibrium results for retailers when the cost sharing is assumed to be exogenous, it is difficult for us to obtain the explicit constrains for the bargaining case. For simplicity, we only investigate the revenue sharing under bargaining case in the following analysis. It is worth noting that the advantage of the bargaining efficiency lies on that it is a Pareto improvement for all participants. Specifically, if \( J = \text{RR} \), then the constraints: \( 0 < \psi \leq \psi^\text{RR-CC} \).

Substituting the equilibrium solutions derived in Scenarios RR and AA into equation (5) gives us the following results:

**Proposition 3.** Under the bargaining framework, an optimal revenue-sharing rate can be used to coordinate the entire supply chain. Such that:

\[
\psi^\text{RR} = \frac{B_1 + \sqrt{B_1^2/4(3 + 2\theta - 2\theta^2)}}{B_3},
\]

where

\[
B_1 = 14175 - 1998\theta - 86850\theta^2 + 5652\theta^3 + 202788\theta^4 - 5208\theta^5 - 229720\theta^6 + 2032\theta^7 + 131776\theta^8 - 704\theta^9 - 3584\theta^{10} + 256\theta^{11} + 3584\theta^{12},
\]

\[
B_2 = 119574225 + 96446700\theta - 1364518872\theta^2 - 1061272368\theta^3 + 6766919820\theta^4 + 4957338672\theta^5 - 19307303856\theta^6 - 12967269696\theta^7 + 3536079088\theta^8 + 2111160684\theta^9 - 43818663168\theta^{10} - 22467335424\theta^{11} + 37695627584\theta^{12} + 15954508032\theta^{13} - 22679449344\theta^{14} + 7568171876\theta^{15} + 9473607680\theta^{16} + 2354139136\theta^{17} - 2680696832\theta^{18} - 458113024\theta^{19} + 4879482880\theta^{20} + 502661120\theta^{21} - 51314688\theta^{22} - 2359296\theta^{23} + 2359296\theta^{24},
\]

\[
B_3 = 2511 - 1377\theta - 11754\theta^2 + 5160\theta^3 + 19932\theta^4 - 6692\theta^5 - 15192\theta^6 + 3408\theta^7 + 5216\theta^8 - 576\theta^9 - 640\theta^{10}.
\]
result is not surprising since revenue-sharing results in lower retail prices when product substitutability is low, which leads consumers to pay less and buy more.

Insight 5: when channel members cooperatively determine the revenue-sharing rate, the revenue-sharing contract can coordinate the advertising conflict.

7. Conclusions

This work investigates the efficiency of the cost-sharing and revenue-sharing contracts for the cooperative advertising program in a dual-exclusive channel, in which the sharing rate is assumed to be an endogenous variable or an exogenous variable. We first explore the advertising efficiency without cost-sharing and revenue-sharing contracts. Our findings indicate that advertising is never a competitive tool for small firms and the advertising efficiency is more beneficial to retailers. We next explore the advertising efficiency with cost-sharing and revenue-sharing contracts. Our findings indicate that when the sharing rate is assumed to be endogenous, the sharing rate cannot be determined by a single-channel member, this is because neither the cost-sharing contract nor the revenue-sharing contract can coordinate the advertising conflict. When the sharing rate is assumed to be exogenous, it will give us some interesting results. For instance, supply chain coordination critically depends on parameters, such as the basic demand ratio and product substitution. We finally explore the advertising efficiency with bargaining. Our findings indicate that when the channel members cooperatively determine the cost-sharing rate or the revenue-sharing rate, both the cost-sharing contract and the revenue-sharing contract can coordinate the advertising confliction. Most notably, compared to the revenue-sharing contract, we find that the cost-sharing contract is particular suite for the cooperative advertising mechanism. It is should be note that it is difficult for manufacturers to monitor the retailers’ advertising spending. Our analysis also offers some management insights for enterprises, cooperative advertising does work well, but there are some drawbacks of which to be aware. For instance, the higher sharing rate may not be beneficial to retailers.

Our analysis might have some limitations. On the one hand, to obtain more management insights, we assume the two channels have symmetric demand share in the bargaining case. On the other hand, our work demonstrates that the cost-sharing contract outperforms the revenue-sharing contract for the cooperative advertising program. Nevertheless, the latter contract is popular in Internet marketing, which accounts for about 80% of the online advertising scheme. It is natural to see that large online retailers, such as Amazon and eBay, adopt the revenue-sharing contract to distribute advertising revenues with their suppliers, and further studies could conduct the revenue-sharing efficiency for the online advertising program. Moreover, Yu et al. [31] point out that the horizontal channel members may cooperate to advertise, and further studies could investigate the horizontal cooperative advertising issue in a dual-exclusive channel system.

Appendix

A. The derivation of equilibrium solutions

Due to space limitations, we do not detail the optimal solutions in each sub-game, the derivation for scenarios CC, RR, and BB are presented as follows:

**Scenario NN:** We solve the scenario without advertising backward. Thus, firm i’s optimal solutions.

\[
\begin{align*}
\pi^{\text{NN}}_i &= \left(8 - 9\theta^2 + 2\theta^4\right)A_i - \theta\left(2 - \theta^2\right)A_{z_{-i}} / 16 - 17\theta^2 + 4\theta^4, \\
L^{\text{NN}}_i &= \left(2 - \theta^2\right)\left(8 - 9\theta^2 + 2\theta^4\right)A_i - \theta\left(2 - \theta^2\right)A_{z_{-i}} \left(4 - \theta^2\right)\left(16 - 17\theta^2 + 4\theta^4\right), \\
\pi^{\text{NN}}_{m_i} &= \left(2 - \theta^2\right)\left(8 - 9\theta^2 + 2\theta^4\right)A_i - \theta\left(2 - \theta^2\right)A_{z_{-i}} \left(4 - 5\theta^2 + \theta^4\right)\left(16 - 17\theta^2 + 4\theta^4\right)^2, \\
\pi^{\text{NN}}_{r_i} &= \left(2 - \theta^2\right)^2 \left(8 - 9\theta^2 + 2\theta^4\right)A_i - \theta\left(2 - \theta^2\right)A_{z_{-i}} \left(4 - \theta^2\right)^2 \left(16 - 17\theta^2 + 4\theta^4\right)^2.
\end{align*}
\]

**Scenario AA:** Similar to Scenario NN, we obtain firm i’s optimal solutions.
\[
\begin{align*}
\psi_{i}^{AA} &= \left(9 - 16\theta^{2} + 4\theta^{4}\right)\left(15 + 26\theta^{2} + 8\theta^{4}\right)A_{i} - 2\theta\left(3 + 2\theta^{2}\right)A_{3-i}/225 - 816\theta^{2} + 964\theta^{4} - 432\theta^{6} + 64\theta^{8}, \\
e_{mi}^{AA} &= \left(3 - 2\theta^{2}\right)\left(15 + 26\theta^{2} + 8\theta^{4}\right)A_{i} - 2\theta\left(3 + 2\theta^{2}\right)A_{3-i}/225 - 816\theta^{2} + 964\theta^{4} - 432\theta^{6} + 64\theta^{8}, \\
e_{ri}^{AA} &= \left(3 - 2\theta^{2}\right)\left(15 + 26\theta^{2} + 8\theta^{4}\right)A_{2} - 2\theta\left(3 + 2\theta^{2}\right)A_{3-i}/225 - 816\theta^{2} + 964\theta^{4} - 432\theta^{6} + 64\theta^{8}, \\
D_{i}^{AA} &= \left(3 + 2\theta^{2}\right)\left(15 + 26\theta^{2} + 8\theta^{4}\right)A_{i} - 2\theta\left(3 - 2\theta^{2}\right)A_{3-i}/225 - 816\theta^{2} + 964\theta^{4} - 432\theta^{6} + 64\theta^{8}, \\
p_{mi}^{AA} &= \left(3 - 2\theta^{2}\right)\left(5 - 30\theta^{2} + 8\theta^{4}\right)\left(15 + 26\theta^{2} + 8\theta^{4}\right)A_{i} - 2\theta\left(3 - 2\theta^{2}\right)A_{3-i}/225 - 816\theta^{2} + 964\theta^{4} - 432\theta^{6} + 64\theta^{8}, \\
p_{ri}^{AA} &= \left(3 - 2\theta^{2}\right)\left(3 - 4\theta^{2}\right)\left(15 + 26\theta^{2} + 8\theta^{4}\right)A_{i} - 2\theta\left(3 - 2\theta^{2}\right)A_{3-i}/225 - 816\theta^{2} + 964\theta^{4} - 432\theta^{6} + 64\theta^{8}. \\
&\text{(A.2)}
\end{align*}
\]

**Scenario CC** (exogenous the Cost-sharing rate): First, we regard the cost-sharing rate as an exogenous variable. This case is similar to scenario AA, a similar process obtains.

\[
\begin{align*}
\psi_{i}^{CC} &= \left(9 + 4\theta^{4}\left(1 - \Phi\right)^{3} - 30\Phi + 360\Phi^{2} - 16\Phi^{3} - \theta^{2}\left(16 - 50\Phi + 54\Phi^{2} - 20\Phi^{3}\right)\right)\left(S_{1}A_{i} + S_{2}A_{3-i}\right)/S, \\
e_{mi}^{CC} &= \left(3 - 2\theta^{2}\left(1 - \Phi\right) - 4\Phi\right)\left(1 - \Phi\right)^{2}\left(S_{1}A_{i} + S_{2}A_{3-i}\right)/S, \\
e_{ri}^{CC} &= \left(3 - 2\theta^{2}\left(1 - \Phi\right) - 4\Phi\right)\left(1 - \Phi\right)^{2}\left(S_{1}A_{i} + S_{2}A_{3-i}\right)/S, \\
D_{i}^{CC} &= \left(3 - 2\theta^{2}\left(1 - \Phi\right) - 4\Phi\right)\left(1 - \Phi\right)^{2}\left(S_{1}A_{i} + S_{2}A_{3-i}\right)/S, \\
p_{mi}^{CC} &= \left(1 - 4\left(1 + \theta^{2}\right)\left(1 - \Phi\right)\right)\left(3 - 2\theta^{2}\left(1 - \Phi\right) - 4\Phi\right)^{2}\left(-1 + \Phi\right)^{3}\left(S_{1}A_{i} + S_{2}A_{3-i}\right)^{2}/S. \\
&\text{(A.3)}
\end{align*}
\]

where,

\[
S = 4\theta^{2}\left(1 - \Phi\right)^{2}\left(241 + 1078\Phi^{2} + 1839\Phi^{4} - 1418\Phi^{6} + 417\Phi^{8}\right) - 4\theta^{2}\left(\frac{204 - 1379\Phi + 3927\Phi^{2} - 6031\Phi^{3}}{+5268\Phi^{4} - 2481\Phi^{5} + 492\Phi^{6}}\right)^{6}, \quad \text{(A.4)}
\]

\[
S_{1} = 15 + 8\theta^{4}\left(1 - \Phi\right)^{3} - 53\Phi + 65\Phi^{2} - 28\Phi^{3} - \theta^{2}\left(26 - 84\Phi + 92\Phi^{2} - 34\Phi^{3}\right), \\
S_{2} = 2\theta^{2}\left(3 - 2\theta^{2}\left(1 - \Phi\right) - 4\Phi\right)\left(1 - \Phi\right)^{2}.
\]

**Scenario CC** (endogenous the Cost-sharing rate): Then, we regard the Cost-sharing rate as an endogenous variable. Since \(\pi_{ri}\) should be strictly concave in \(\{p_{i}, e_{ri}\}\), the constrains \(\frac{\partial^{2}\pi_{ri}}{\partial p_{i}^{2}}\pi_{ri} - \frac{\partial^{2}\pi_{ri}}{\partial p_{i} \partial e_{ri}}\pi_{ri} = 3 - 4\theta^{2}\left(1 - \Phi\right) - 4\Phi\left(1 - \theta^{2}\right)^{2} > 0\) must be satisfied. We obtain,

\[
0 < \Phi_{i} < \frac{3 - 4\theta^{2}}{4} < \frac{3}{4}\quad \text{and} \quad 0 < \theta < 0.86. \quad \text{(A.5)}
\]
where,

\[
\begin{align*}
E_1 &= -12\theta^2\Phi^2 + 4\theta^2\Phi^2 + 4\theta^4\Phi^2 + 2(-1 + \theta^3)(3 + 2\theta^2(-1 + \Phi) - 4\Phi)(-1 + \Phi)e_{mi}, \\
E_2 &= 4\theta(-1 + \theta^3)(-1 + \Phi)^2 e_{m3-i} + (3 - 6\theta^2 - 10\Phi + 14\theta^2\Phi + 8\Phi^2 - 8\theta^2\Phi^2)\omega_i \\
&\quad + (4\theta - 4\theta^3 - 8\theta\Phi + 8\theta^3\Phi + 4\theta^2\Phi - 4\theta^2\Phi^2)\omega_{j-i}, \\
E_3 &= 3 - 2\theta - 2\theta^2 - 4\Phi + 2\theta\Phi + 2\theta^2\Phi + (3 + 2\theta^2(-1 + \Phi) - 4\Phi)e_{mi}, \\
E_4 &= 2\theta(-1 + \Phi)e_{m3-i} - (3 - 2\theta^2 - 4\Phi + 2\theta^2\Phi)\omega_i + (2\theta - 2\theta\Phi)\omega_{j-i},
\end{align*}
\]

(A.7)

It can be easily find that \(w_1 = w_2\) and \(e_{m1} = e_{m2}\) due to the symmetry assumption \((A_1 = A_2 = 1)\). Combining with equation (A.6), the cost-sharing rate is determined by taking the first-order conditions of \(M_i\)'s profit maximization problem subject to the constraints imposed by equation (A.5), we obtain:

\[
\Phi_i = \frac{\left(3 + 2\theta - 2\theta^2\right)(3w_i - e_{mi} - 1)}{\left(2 + \theta - \theta^2\right)(1 + e_{mi} + w_i)}.
\]

(A.8)

In line with Xie and Neyret [12]; we also consider two different cases: either \(1 - e_{mi}/3 < w_i < 1\) and \(\Phi_i \neq 0\) or \(0 < w_i < 1 - e_{mi}/3\) and \(\Phi_i = 0\), since \(\Phi_i\) should be non-negative in equation (A.8). Note that, the second case \((\Phi_i = 0)\) is equivalent to Scenario AA, and hence, we only investigate the first case. Combining equations (A.6) and (A.8), after tedious algebraic calculations we can obtain:

\[
\begin{align*}
\Phi_1 &= \frac{13 - 8\theta - 8\theta^2 + \sqrt{25 + 32\theta - 40\theta^2 + 16\theta^4}}{2\left(12 - 4\theta - 6\theta^2\right)} \\
\Phi_2 &= \frac{13 - 8\theta - 8\theta^2 - \sqrt{25 + 32\theta - 40\theta^2 + 16\theta^4}}{2\left(12 - 4\theta - 6\theta^2\right)} \\
\Phi_3 &= \frac{13 - 8\theta - 8\theta^2 + \sqrt{25 + 32\theta - 40\theta^2 + 16\theta^4}}{2\left(12 - 4\theta - 6\theta^2\right)}
\end{align*}
\]

(A.9)

It is easily verified that \(\Phi_1 > \Phi_2 > 3 - 4\theta^2/4 - 4\theta^2 > \Phi_3\), and hence, \(\Phi_1\) and \(\Phi_2\) should be discarded. Moreover, the other solutions can be derived as follows (the solutions are uniformly very complicated so we omit some of them).

\[
\begin{align*}
\omega_i^{CC} &= \frac{4(F_5 + F_4\Delta + F_2\Delta^2 + F_3\Delta^3)}{T_1 + T_2\Delta + T_3\Delta^2 + T_4\Delta^3},
\end{align*}
\]

(A.10)
where

\[
\Delta \\
F_1 = 2660 + 240 - 8455\theta^5 + 692\theta^5 - 8437\theta^6 - 672\theta^7 - 3860\theta^8 + 192\theta^9 + 848\theta^{10}, \\
F_2 = 732 - 48\theta - 1767\theta^2 + 24\theta^3 + 1375\theta^4 - 432\theta^5 + 48\theta^6, \\
F_3 = 20 + 8\theta - 47\theta^2 - 4\theta^3 + 25\theta^4 - 4\theta^6, \\
F_4 = 4 - 5\theta^2 + \theta^4, \\
F_5 = 1935 + 254\theta - 6059\theta^2 + 6339\theta^4 - 128\theta^5 - 3104\theta^6 + 64\theta^7 + 736\theta^8 - 64\theta^{10}, \\
F_6 = 597 + 20\theta - 1446\theta^2 - 48\theta^3 + 1175\theta^4 + 16\theta^5 - 396\theta^6 + 48\theta^8, \\
F_7 = 57 + 14\theta - 123\theta^2 - 8\theta^3 + 69\theta^4 - 12\theta^6, \\
F_8 = 3 - 4\theta^2 + \theta^4, \\
T_1 = 6310 + 3744\theta - 19657\theta^2 - 6098\theta^3 + 21812\theta^4 + 3704\theta^5 - 11312\theta^6 - 992\theta^7 + 2816\theta^8 + 128\theta^9 - 256\theta^{10}, \\
T_2 = 2082 + 1252\theta - 5087\theta^2 - 1758\theta^3 + 4372\theta^4 + 752\theta^5 - 1536\theta^6 - 96\theta^7 + 192\theta^8, \\
T_3 = 234 + 184\theta - 487\theta^2 - 142\theta^3 + 276\theta^4 + 24\theta^5 - 48\theta^6, \\
T_4 = (14 + 4\theta - 17\theta^2 - 2\theta^3 + 4\theta^5).
\]

**Scenario RR (exogenous the revenue-sharing rate):**

When the cost-sharing rate is assumed to be an exogenous variable, it is easy to obtain \( R_i \)'s reaction functions,

\[
\rho_i \\
e_{ri} = \frac{3 - 2\theta(1 + \theta) + (3 - 2\theta^2)e_{mi} - \theta e_{m3-i} - (1 - \psi)(3 - 2\theta^2)w_i - 2\theta w_{i-1}}{9 - 16\theta^2 + 4\theta^4} \\
\nu_{i}^{RR} = \frac{3 - 2\theta^2}{2\theta(\theta(13 + 2\theta(1 - 2\theta)(1 - \psi) - 14\psi - 3(1 - \psi)) - 3(5 - 6\psi))}.
\]
Furthermore, the optimal profits for manufacture \( i \) and retailer \( i \) are as follows:

\[
\pi_{mi}^{\text{RR}} = \frac{\left(3 - 2\theta^2\right)^2 \left(3 - 14n\theta^2 q(1 - \psi)^3 h - \gamma x\psi 7 + C3; \psi^2\right)}{\theta^2 (26 - 28\psi) - 6\theta (1 - \psi) + 4\theta^3 (1 - \psi) - 8\theta^4 (1 - \psi) - 3(5 - 6\psi)^2}.
\]  
(A.14)

**Scenario RR** (endogenous the revenue-sharing rate): When the revenue-sharing rate is an endogenous variable. It is easy to find that \( \pi_{ri} \) is strictly concave in \( \{p_i, e_{ri}\} \) and retailer \( i \)'s reaction function is given by:

\[
p_i = \frac{3 - 2\theta - 2\theta^2 + (3 - 2\theta^2)e_{mi} - 2\theta e_{mi} - (1 - \psi)((3 - 2\theta^2)w_i - 2\theta w_i)}{9 - 16\theta^2 + 4\theta^3}.
\]  
(A.15)

Likewise, the optimal revenue-sharing rate can be derived as follows:

\[
\psi_i = \frac{(3 + 2\theta - 2\theta^2)e_{mi}^2 - 2\theta e_{mi} - 2\theta e_{mi} - (1 - 2\theta)w_i}{4\theta^2}.
\]  
(A.16)

Further, the second-order condition with respect to \( \psi_i \) gives, \( \partial^2 \pi_{mi}/\partial \psi_i^2 = -4\theta^2/3 + 2\theta - 2\theta^2 < 0 \). Hence, we obtain:

\[
\pi_{mi}^{\text{RR}} = \frac{243 + 324\theta - 2268\theta^2 - 1008\theta^3 + 6120\theta^4 + 1056\theta^5 - 6400\theta^6 - 4480\theta^7 + 2480\theta^8 + 64\theta^9 - 320\theta^{10}}{4(3 + 2\theta - 2\theta^2)^2(9 - 8\theta^2 + 6\theta^3 + 4\theta^4)^2}.
\]  
(A.17)

**Scenario BB**: Substituting the equilibrium solutions derived in Scenarios RR and AA into the bargaining solution function, it can be easily find that equation (5) is strictly concave in \( \psi \). Solving the first-order conditions with respect to \( \psi \) gives us:

\[
\psi_1 = \frac{B_1 + \sqrt{B_2}}{4(3 + 2\theta - 2\theta^2) B_3}.
\]  
(A.18)

where
Since the optimal revenue-sharing rate should be satisfied $0 < \psi \leq \psi_{r1}^{R-R-A_A}$, it is obvious that $\psi_3$ is the optimal revenue-sharing rate.

**B: Proof of Propositions and Theorems**

To ensure a fair comparison, the common feasible area is obtained from the non-negative demand constraints, such that $\Omega_L = 2\theta(3 - 2\theta)/(15 - 26\theta + 8\theta^2)$, $\Omega_U = 2\theta(3 - 2\theta)/(15 - 26\theta + 8\theta^2)$ and $\psi_U = 15 - 30\theta + 8\theta^2/18 - 32\theta + 8\theta^2$. For brevity, we will omit some of the $\Omega$ terms shown in the analytical results, as they are a little complicated.

**Proof of Theorem 1.** $\Delta \pi_{AA-NN}$ is defined as manufacturer 1’s profit under AA minus the one under NN. Reorganizing $A_1$ and $A_2$ into $\Omega$ and solving $\Delta \pi_{AA-NN} = 0$ yields two roots: $\Omega_{m1-2}^{AA-NN} = B_1 + B_2 + B_3$ and $\Omega_{m1-2}^{AA-NN} = B_1 + B_2 - B_3$.

**Proof of Theorem 2.** To compare Scenarios between AA and RAA, an approach similar to Theorem 1 is adopted here, we can easily demonstrate that $\Omega_{m1-2}^{AA-RAA} < \Omega < \Omega_{m1-2}^{NN-RAA} < \Omega_U$, and $\partial^2 \Delta \pi_{m1-2}^{AA-RAA} / \partial \Omega^2 > 0$ in dominant feasible area, and thereby $\pi_{m1}^{AA} \geq \pi_{m1}^{RAA}$ if $\Omega_{m1-2}^{AA-RAA} < \Omega < \Omega_U$, otherwise $\pi_{m1}^{AA} < \pi_{m1}^{RAA}$.

**Proof of Theorem 3:** The common feasible area is $\Omega_U = 3 - 2\theta^3/4 - 2\theta^4$. Comparing the profits with and without cost sharing for manufacturer 1. Solving $\Delta \pi_{m1-2}^{CC-CC} = 0$ yields two roots: $\Phi_{m1-2}^{CC} = 15 - 15\theta + 6\theta^3 + \sqrt{9 - 4\theta^2 + 45\theta^4 - 20\theta^6 + 4\theta^8/8(3 - 3\theta^2 + \theta^4)}$ and $\Phi_{m1-2}^{CC-CC} = 15 - 15\theta + 6\theta^3 - \sqrt{9 - 4\theta^2 + 45\theta^4 - 20\theta^6 + 4\theta^8/8(3 - 3\theta^2 + \theta^4)}$. Since $\Phi_{m1-2}^{CC} < \Phi_{m1-2}^{CC-CC} < \Omega_U$, and $\partial^2 \Delta \pi_{m1}^{CC-CC} / \partial \Phi^2 > 0$, and thereby we have $\pi_{m1}^{CC} > \pi_{m1}^{CC-CC}$ if $0 < \Omega < \Phi_{m1-2}^{CC-CC}$. However, the equilibrium result is difficult to be solved analytically for retailer 1. By contour plotting, we find that retailer 1 benefits from the cost-sharing contract when the cost-sharing rate and product substitutability are sufficiently small. Combining the equilibrium results for the manufacturer, there must have a threshold
Proof of Theorem 6. Given the symmetric setting, the common feasible area is \(\Psi = 15 - 30\theta^2 + 8\theta^3/18 - 32\theta^2 + 8\theta^3\). Solving \(\Delta \pi_{\text{RR}} = 0\) yields two roots: 
\[
\Psi_{\text{RR}}^{-1} = 225 - 936\theta - 489\theta + 1172\theta^2 + 32\theta^3 - 496\theta^4 + 64\theta^5/270 - 1056\theta^2 - 489\theta + 1256\theta^2 + 32\theta^3 - 512\theta^4 + 64\theta^5 \quad \text{and} \quad \Psi_{\text{m1}}^{-1} = 0.
\]
Since \(\Psi_{\text{RR}}^{-1} < \Omega_{\text{UV}}, \text{and } \frac{\partial^2 \Delta \pi_{\text{RR}}}{\partial \psi} > 0\), consequently we have \(\pi_{m1} > \pi_{AA}\) if \(0 < \psi < \Psi_{m1}^{-1}\). A similar process obtains \(\pi_{m1} > \pi_{AA}\) if \(0 < \Omega < \Psi_{m1}^{-1}\) and \(\pi_{AA} > \pi_{m1}^{-1}\). Note that \(\Psi_{m1}^{-1} < \Psi_{\text{RR}}^{-1} < \Psi_{m1}^{-1}\).

Proof of Propositions 1–3: The proof can be easily obtained from Appendix A. Thus, the details are skipped.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

This research was supported by Shandong Provincial Natural Science Foundation of China (ZR2020QG001); Shandong Provincial Social Science Foundation of China (22DGLJ028); the Central Government to Support the Reform and Development of Local Universities of China (2021); Philosophy and Social Science Research Planning Project of Heilongjiang Province (21GLB065); and National Nature Science Foundation of China (71971134, 71701056, and 72174045).

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