Hindawi
Mathematical Problems in Engineering
Volume 2022, Article ID 1265204, 21 pages
https://doi.org/10.1155/2022/1265204

Research Article

Efficient Auxiliary Information–Based Control Charting Schemes for the Process Dispersion with Application of Glass Manufacturing Industry

Muhammad Aslam,[1] **Syed Masroor Anwar**,[2] **Majid Khan**,[3] **Nafiu Lukman Abiodun**,[4] **and Zahid Rasheed**[5]

[1]*Department of Mathematics and Statistics, Riphah International University, Islamabad, Pakistan*
[2]*Department of Statistics, University of Azad Jammu and Kashmir, Muzaffarabad, Pakistan*
[3]*Government Post Graduate College for Boys, Huripur, Pakistan*
[4]*Department of Economics and Statistics, Faculty of Economics and Management Sciences, Kabale University, Kabale, Uganda*
[5]*Department of Mathematics, Women University of Azad Jammu and Kashmir, Bagh, Pakistan*

Correspondence should be addressed to Nafiu Lukman Abiodun; lanafiu@kab.ac.ug

Received 9 February 2022; Accepted 30 April 2022; Published 30 May 2022

Academic Editor: A. M. Bastos Pereira

The hybrid exponentially weighted moving average (HEWMA) control chart is an enhanced version of the EWMA control chart that monitors the process parameters effectively. Similarly, the auxiliary information-based (AIB) EWMA control charts are very efficient for monitoring process parameters. The purpose of this paper is to propose two new control charts for the improved monitoring of process dispersion referred to as $\text{HEWMA}_{\text{AIB}}^{(1)}$ and $\text{HEWMA}_{\text{AIB}}^{(2)}$ control charts. A simulation study is carried out to assess the performance of the proposed $\text{HEWMA}_{\text{AIB}}^{(1)}$ and $\text{HEWMA}_{\text{AIB}}^{(2)}$ control charts. Average run length, extra quadratic loss, relative average run length, and performance comparison index are used to compare the performance of the proposed control charts against the existing counterparts. The comparisons reveal the superiority of the proposed control charts against other competing control charts, particularly for small to moderate shifts in the process dispersion. Finally, a real-life data set from the glass industry is used to demonstrate the practical implementation of the proposed control charts.

1. Introduction

There are two types of variations in manufacturing and service processes; common (random) cause variations and special (assignable) cause variations. The common cause variations are an inherent part of every process and cannot be removed entirely. However, the special cause variations are harmful and may distract the processes from their target which results a shifts in the process parameter(s) (location and/or scale). As a result, the practitioner needs to identify and eliminate the assignable cause variations in the process. The statistical control chart is a primary tool in the statistical process control (SPC) toolkit that identifies and rectifies the special cause variations in the process. Memory-less control charts introduced by Shewhart [1] are used to monitor large shifts in the process. On the other hand, the classical memory control charts like cumulative sum (CUSUM) control chart designed by Page [2] and exponentially weighted moving average (EWMA) control chart offered by Roberts [3] are used to monitor small to moderate shifts in the process.

Generally, the classical EWMA control chart has been used to detect small shifts in the process mean. However, in many practical situations, the shifts may also occur in the process variance (or standard deviation); when the process variance increases, the productivity and capability of the process may be damaged. If the process variance decreases, more units will be closer to their target value, resulting in improved process functionality. If these changes in the process dispersion are not rectified quickly, unnecessary losses may occur. Numerous authors have constructed different EWMA control charts for the process variance. For example, Crowder and Hamilton [4] used a suitable log transformation to S^2 $(\ln(S^2/\sigma^2_{Y0}))$, and designed the EWMA control chart for monitoring the process standard deviation, where σ^2_{Y0} is the in-control (IC) process variance. Following the same lines, Shu and Jiang [5] suggested the new EWMA control chart in which $\ln(S^2/\sigma^2_{Y0})$ truncated to its IC mean on every occasion whenever it is less than zero. Similarly, Chang and Gan [6] constructed a one-sided optimal EWMA for monitoring the process variance. Likewise, Khoo [7] discussed the double sampling variance control chart features for monitoring process variability. Similarly, Castagliola [8] used three parameters logarithmic transformation to S^2 to improve normality and hence the proposed bilateral EWMA control chart for monitoring the process dispersion. Later, Castagliola, et al. [9] constructed a new S^2-EWMA control chart for flexible sampling intervals. Likewise, Eyvazian [10] proposed the EWMSV control chart for process dispersion when the sample size equals 1. In the same direction, Huwang, et al. [11] suggested the EWMA control charts named HHW1 and HHW2 for the process dispersion. They demonstrated that their control charts perform uniformly better against the control charts by Crowder and Hamilton [4] and Shu and Jiang [5]. Razmy and Peiris [12] designed an standardized EWMA control chart for monitoring of process dispersion. Later, Yang and Arnold [13] constructed an unbiased EWMA-p control chart for monitoring the process dispersion. Currently, Castagliola, et al. [14] proposed the double sampling S^2-chart for the process variance and elaborated various features of the proposed control chart. Further related work on dispersion control charts, see Abbasi and Miller [15], Ali and Haq [16] and Saghir, et al. [17], etc.

Combining the features of the different memory control charts enhances the performance of the ultimate control charts. For example, Haq [18] has constructed a new hybrid exponentially weighted moving average (HEWMA) control chart using one EWMA statistic as an input for another EWMA statistic. Subsequently, several other authors discussed some of the innovations in the HEWMA control chart. For instance, Azam, et al. [19] presented a HEWMA control chart for the process mean under repetitive sampling. Similarly, Aslam, et al. [20] proposed a HEWMA control chart for the COM-Poisson distribution. Likewise, Ali and Haq [16] suggested the generally weighted moving average and the HEWMA control charts for process dispersion. Recently, Chan, et al. [21] suggested improved double EWMA and homogeneously weighted moving average lepage schemes with real life applications. For a more detailed study, see Aslam, et al. [22], Noor-ul-Amin, et al. [23], Mukherjee, et al. [24], Song, et al. [25], and references therein.

Several auxiliary information-based (AIB) control charts have been suggested for efficient monitoring of process parameters. For example, Abbas, et al. [26] introduced AIB EWMA for the process mean, named as EWMA_{AIB} control chart. Likewise, Haq [27] recommended two new AIB EWMA control charts named as $\text{EWMA-I}_{\text{AIB}}$ and $\text{EWMA-II}_{\text{AIB}}$ control charts that efficiently monitor the process dispersion. Similarly, Haq [28] provided AIB maximum EWMA control chart for process location and dispersion. Hussain, et al. [29] suggested EWMA control chart based on dual auxiliary information-based estimator for the monitoring of process location. Similarly, Abbasi and Riaz [30] provides the control chart using dual auxiliary information under different ranked set sampling schemes. On the same lines, Riaz, et al. [31] suggested variability control chart using dual auxiliary information-based estimators under different ranked set sampling techniques and different runs rules. Besides, Noor-ul-Amin, et al. [32] suggested the $\text{Max EWMA}_{\text{AIB}}$ control chart for the simultaneous monitoring of the process mean and coefficient of variation. Recently, Anwar, et al. [33] introduced an AIB combined mixed EWMA-CUSUM control chart for joint monitoring of process paramters.

As mentioned before, Ali and Haq [16] proposed the HEWMA control chart for the process dispersion which is more sensitive than the classical EWMA control chart. Sometimes, researchers, engineers, and practitioners are interested in utilizing the features of HEWMA control charts when the original variable carries other information, such as an auxiliary variable, to improve the process's effectiveness. In this case, the HEWMA control chart will remain inefficient. So, to address this deficiency, this study introduces two auxiliary information-based HEWMA, symbolized as $\text{HEWMA}^{(1)}_{\text{AIB}}$ and $\text{HEWMA}^{(2)}_{\text{AIB}}$ control charts to monitor the small shifts in the process dispersion. To evaluate the performance of the proposed $\text{HEWMA}^{(1)}_{\text{AIB}}$ and $\text{HEWMA}^{(2)}_{\text{AIB}}$ control charts against other control charts, specific performance evaluation measures such as average run length (ARL), extra quadratic loss (EQL), performance comparison index (PCI), and relative ARL (RARL) measures are considered. Besides, an algorithm is designed in R using the Monte Carlo simulations method to calculate the performance evaluation measures. Existing control charts such as HEWMA, adaptive EWMA (AEWMA), HHW1, HHW2, $\text{EWMA-I}_{\text{AIB}}$, and $\text{EWMA-II}_{\text{AIB}}$ control charts are considered for comparison. Moreover, the proposed control charts are also implemented with real-life applications to show the significance for practical importance.

The article's remainder is organized as follows: variable of interest, auxiliary information, transformation based on auxiliary information, and the existing HEWMA control chart are highlighted in Section 2. Section 3 presents the design structure of the proposed $\text{HEWMA}^{(1)}_{\text{AIB}}$ and $\text{HEWMA}^{(2)}_{\text{AIB}}$ control charts. Besides, Section 4 highlights the performance evaluation measures. Furthermore, Section 5 consists of the performance comparison of the proposed $\text{HEWMA}^{(1)}_{\text{AIB}}$ and $\text{HEWMA}^{(2)}_{\text{AIB}}$ control charts against the existing control charts. Similarly, the real-life application of the proposed control charts is provided in Section 6. The last section presents an overall summary and conclusions.

2. Existing Method

This section provides insight into the variable of interest and transformation in Subsection 2.1. Likewise, the methodology of the HEWMA control charts are presented in Subsection 2.2.

2.1. Variable of Interest and Transformation. Suppose Y be normally distributed process variable, that is, $Y \sim N(\mu_Y, \sigma_Y^2)$. It is assumed that over a certain period, the underlying process remains IC with variance σ_Y^2, but afterwards, it becomes out-of-control (OOC) with variance $\sigma_{Y,1}^2$. Let $\tau = \sigma_{Y,1}/\sigma_Y$ be the amount of shift in process standard deviation σ_Y. In the case of the IC process, $\tau = 1$ and OOC process, $\tau \neq 1$. Also, $\overline{Y}$ represents the the sample mean and S_Y^2 denotes sample variance of the process variable Y.

2.2. Transformation. Suppose, X be an auxiliary information variable of Y variable, then X and Y follow a bivariate normal distribution. Suppose (Y_i, X_i), for $i = 1, 2, \ldots, n$ be a random sample of size n. Let $\overline{X}$ and S_X^2 be the sample mean and the sample variance of X, respectively. According to Garcia and Cebrian [34], the unbiased regression estimator of σ_Y^2 say S_Y^{2*}, is given by

$$S_Y^{2*} = S_Y^2 + \rho^2\left(\frac{\sigma_Y^2}{\sigma_X^2}\right)\left(\sigma_X^2 - S_X^2\right). \quad (1)$$

Where ρ is the correlation coefficient. The mean and variance of S_Y^{2*} are given as $E(S_Y^{2*}) = \sigma_Y^2, V(S_Y^{2*}) = (2\sigma_Y^4/n-1)(1-\rho^4)$.

Similarly, Haq [27] suggested the difference estimator for process dispersion given as

$$D_t = M_{Y,t} - \rho^* M_{X,t}, \quad (2)$$

where $M_Y = \Phi^{-1}(G((n-1)S_Y^2/\sigma_Y^2))$, $M_X = \Phi^{-1}(G((n-1)S_X^2/\sigma_X^2))$ and t represents the sample number. Also, $G(\cdot; n-1)$ is the cumulative distribution function (CDF) of chi-square distribution at $n-1$ degrees of freedom, and $\Phi^{-1}(\cdot)$ denotes the inverse CDF of the standard normal distribution. The ρ^* is the correlation between M_Y and M_X. The mean and variance of D_t given by $E(D_t) = 0$ and $V(D_t) = 1 - \rho^*$.

2.3. HEWMA Control Chart for Process Dispersion. Ali and Haq [16] proposed the HEWMA control chart for the monitoring of process dispersion. Let $\{HE_t\}$ for $t \geq 1$ be the sequence of independentely and identically distributed (IID) observations based on the other sequence $\{M_{Y,t}\}$, then the plotting statistic HE_t, for the HEWMA control chart is defined as:

$$\begin{aligned} E_t &= (1-\lambda_1)E_{t-1} + \lambda_1 M_{Y,t},\ 0 < \lambda_1 \leq 1, \\ HE_t &= (1-\lambda_2)HE_{t-1} + \lambda_2 E_t,\ 0 < \lambda_2 \leq 1, \end{aligned} \quad (3)$$

where $HE_0 = E_0 = 0$, and λ_1 and λ_2 are smoothing constants. The mean and variance of HE_t are, respectively, given as $E(HE_t) = 0$, and $V(HE_t) = (\lambda_1\lambda_2/\lambda_1 - \lambda_2)^2 [\sum_{k=1}^{2}((1-\lambda_k)^2\{1-(1-\lambda_k)^{2t}\}/1-(1-\lambda_k)^2) - (2(1-\lambda_1)(1-\lambda_2)\{1-(1-\lambda_1)^t(1-\lambda_2)^t\}/1-(1-\lambda_1)(1-\lambda_2))]$. The lower and upper control limits of the HEWMA control chart at the time t, are presented as

$$\left(\mathrm{UCL}_{(\mathrm{HEWMA})t}, \mathrm{LCL}_{(\mathrm{HEWMA})t}\right) = \pm \mathrm{L}_{\mathrm{HEWMA}}\sqrt{V(\mathrm{HE}_t)}, \quad (4)$$

where $\mathrm{L}_{\mathrm{HEWMA}}$ control chart coefficient is used to adjust the IC ARL of the HEWMA control chart at a pre-specified desired level. The HEWMA control chart triggers an OOC signal whenever HE_t fall outside of the control limits $(\mathrm{UCL}_{(\mathrm{HEWMA})t}, \mathrm{LCL}_{(\mathrm{HEWMA})t})$.

3. Proposed Methods

This section contains the methodology of the proposed $\mathrm{HEWMA}_{\mathrm{AIB}}^{(1)}$ and $\mathrm{HEWMA}_{\mathrm{AIB}}^{(2)}$ control charts for monitoring the process dispersion. Subsection 3.1 covers the design structure of the proposed $\mathrm{HEWMA}_{\mathrm{AIB}}^{(1)}$ control chart, whereas, the $\mathrm{HEWMA}_{\mathrm{AIB}}^{(2)}$ control chart are given in Subsection 3.2.

3.1. $HEWMA_{AIB}^{(1)}$ Control Chart. Let $\{S_{Y,t}^{2*}\}$ for $t \geq 1$ be the sequence of IID random variables, then the plotting statistic $H_{1,t}$, for $\mathrm{HEWMA}_{\mathrm{AIB}}^{(1)}$ control chart using the recurrence formula, given by

$$\begin{aligned} E_{1,t} &= \lambda_1 S_{Y,t}^{2*} + (1-\lambda_1)E_{1,t-1},\ E_{1,0} = \sigma_Y^2, \\ H_{1,t} &= (1-\lambda_2)H_{1,t-1} + \lambda_2 E_{1,t},\ H_{1,0} = \sigma_Y^2. \end{aligned} \quad (5)$$

Here λ_1 and λ_2 $\in\in (0,1]$ are smoothing constants. The mean and variance of the $H_{1,t}$ can be given by the expression as $E(H_{1,t}) = \sigma_Y^2$ and $V(H_{1,t}) = (2(1-\rho^4)/n-1)(\lambda_1\lambda_2/\lambda_1-\lambda_2)^2[\sum_{k=1}^{2}((1-\lambda_k)^2\{1-(1-\lambda_k)^{2t}\}/1-(1-\lambda_k)^2) - (2(1-\lambda_1)(1-\lambda_2)\{1-(1-\lambda_1)^t(1-\lambda_2)^t\}/1-(1-\lambda_1)(1-\lambda_2))]\sigma_Y^4$, respectively. The control limits of the proposed $\mathrm{HEWMA}_{\mathrm{AIB}}^{(1)}$ control chart are given by

$$\begin{aligned} &\left(UCL_{(\mathrm{HEWMA}_{\mathrm{AIB}}^{(1)})t}, LCL_{(\mathrm{HEWMA}_{\mathrm{AIB}}^{(1)})t}\right) \\ &\quad = \sigma_Y^2 \pm \mathrm{L}_{\mathrm{HEWMA}_{\mathrm{AIB}}^{(1)}}\sqrt{V(H_{1,t})}, \end{aligned} \quad (6)$$

where $\mathrm{L}_{\mathrm{HEWMA}_{\mathrm{AIB}}^{(1)}}$ is the coefficient for the $\mathrm{HEWMA}_{\mathrm{AIB}}^{(1)}$ control chart at a pre-specified false alarm rate. The $H_{1,t}$ statistic is plotted against the $\mathrm{UCL}_{(\mathrm{HEWMA}_{\mathrm{AIB}}^{(1)})t}$ and $\mathrm{LCL}_{(\mathrm{HEWMA}_{\mathrm{AIB}}^{(1)})t}$. The process is considered to be OOC when $H_{1,t} > \mathrm{UCL}_{(\mathrm{HEWMA}_{\mathrm{AIB}}^{(1)})t}$ or $H_{1,t} < \mathrm{LCL}_{(\mathrm{HEWMA}_{\mathrm{AIB}}^{(1)})t}$; otherwise, it is IC.

3.2. $\mathrm{HEWMA}_{\mathrm{AIB}}^{(2)}$ *Control Chart.* Let $\{D_t\}$ for $t \geq 1$ be the sequence of IID random variables, based on $\{D_t\}$, we defined a new sequence $H_{2,t}$, using the recurrence formula, given by

$$E_{2,t} = \lambda_1 D_t + (1-\lambda_1)E_{2,t-1},\ E_{2,0} = 0,$$
$$H_{2,t} = (1-\lambda_2)H_{2,t-1} + \lambda_2 E_{2,t},\ H_{2,0} = 0. \tag{7}$$

Here λ_1 and λ_2 $\in\in (0,1]$ are smoothing constants. The mean and variance of $H_{2,t}$ can be given by the expression as $E(H_{2,t}) = 0$ and $V(H_{2,t}) = (1-\rho^*)(\lambda_1 \quad \lambda_2/\lambda_1 - \lambda_2)^2[\sum_{k=1}^{2}((1-\lambda_k)^2\{1-(1-\lambda_k)^{2t}\}/1-(1-\lambda_k)^2) - (2(1-\lambda_1)(1-\lambda_2)\{1-(1-\lambda_1)^t(1-\lambda_2)^t\}/1-(1-\lambda_1)(1-\lambda_2))]$. The control limits for the $\text{HEWMA}_{\text{AIB}}^{(2)}$ control chart are given by

$$\left(\text{UCL}_{(\text{HEWMA}_{\text{AIB}}^{(2)})t}, \text{LCL}_{(\text{HEWMA}_{\text{AIB}}^{(2)})t}\right) = \pm\, \text{L}_{\text{HEWMA}_{\text{AIB}}^{(2)}} \sqrt{V(H_{2,t})}, \tag{8}$$

where $\text{L}_{\text{HEWMA}_{\text{AIB}}^{(2)}}$ is the control chart coefficient for $\text{HEWMA}_{\text{AIB}}^{(2)}$ control chart at a pre-specified false alarm rate. The $\text{HEWMA}_{\text{AIB}}^{(2)}$ statistic $H_{2,t}$ is plotted against the $\text{LCL}_{(\text{HEWMA}_{\text{AIB}}^{(2)})t}$ and $\text{LCL}_{(\text{HEWMA}_{\text{AIB}}^{(2)})t}$, the process is considered to be OOC when $H_{2,t} < \text{LCL}_{(\text{HEWMA}_{\text{AIB}}^{(2)})t}$ or $H_{2,t} > \text{UCL}_{(\text{HEWMA}_{\text{AIB}}^{(2)})t}$; otherwise, IC.

4. Performance Evaluation Measures

Quality experts use different performance evaluation measures to evaluate the control charts' performance. ARL is mostly used for a single shift, while EQL, RARL, and PCI are used to assess the overall performance of control charts. An algorithm is developed in R software, and the Monte Carlo simulation technique is used to compute the numerical results. Monte Carlo simulation with 20000 iterations is performed at each shift τ, where $\tau = 0.5, 0.6, 0.7, 0.8\ 0.9, 1.0, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9$, and 2.0. The details of these performance measures are given in the following subsections.

4.1. ARL Measure. The ARL is defined as the average number of sample points plotted until the OOC signal is detected. The ARL is categorized as IC ARL (ARL_0) and OOC ARL (ARL_1). If the process is IC state, the ARL_0 needed to be large enough to avoid frequent false alarms. However, the ARL_1 should be small enough that it quickly detects the shift(s) in the process parameters. It is necessary for the better performance of the control chart that it should have a smaller ARL_1 with fixed ARL_0 at the desired level.

4.2. Overall Performance Measures. The EQL, RARL, and PCI performance evaluation measures evaluate a control chart's overall effectiveness by comparison method. The EQL evaluates the overall performance of control charts over a specific range of shifts (Raza, et al. [35]). It is based on the loss function and is defined as:

$$\text{EQL} = (\tau_{\max} - \tau_{\min})^{-1} \int_{\tau_{\min}}^{\tau_{\max}} \tau^2 ARL(\tau)\,\text{d}\tau, \tag{9}$$

where $\text{ARL}(\tau)$ is the ARL of a particular control chart at shift τ. The EQL is a weighted average ARL over the entire shift domain ($\tau_{\min} < \tau < \tau_{\max}$) using the square of shift (τ^2) as weight. A control chart with a minimum EQL value is preferred over other control charts (Anwar, et al. [36]).

The RARL is the average of the ratios among the ARL of a particular control chart with the ARL of a benchmark control chart for all desired shifts.

$$\text{RARL} = (\tau_{\max} - \tau_{\min})^{-1} \int_{\tau_{\min}}^{\tau_{\max}} \frac{\text{ARL}(\tau)}{\text{ARL}_{\text{BM}}(\tau)}\,\text{d}\tau, \tag{10}$$

where $\text{ARL}(\tau)$ and $\text{ARL}_{\text{BM}}(\tau)$ symbolize the ARL of a particular control chart and a benchmark control chart for the desired shift, respectively. The benchmark control chart is the control chart with the least EQL. The RARL value for the benchmark control chart is one, and for the other control charts, it is greater than 1.

The PCI evaluates the performance of the best control chart. It is defined as the ratio between the EQL of a control chart and the EQL of the benchmark control chart.

$$\text{PCI} = \frac{\text{EQL}}{\text{EQL}_{\text{BM}}}, \tag{11}$$

where EQL_{BM} is the EQL of the best-performing control chart. The PCI for the benchmark control chart is 1, while the other control chart's PCI is greater than 1.

4.3. Choices of Parameters. The parameters of the proposed $\text{HEWMA}_{\text{AIB}}^{(1)}$ and $\text{HEWMA}_{\text{AIB}}^{(2)}$ control charts are λ_1, λ_2, and ρ. Several settings of these parameters are used, and hence corresponding ARL, and standard deviation of RL (SDRL) are computed. The values of (λ_1, λ_2) are set as (0.05,0.05), (0.05,0.1), (0.05,0.2), (0.1,0.05), (0.1,0.1), (0.1,0.2), (0.2,0.05), (0.2,0.1), (0.2,0.2), (0.3,0.05), (0.3,0.1), and (0.3,0.2) and the values of $\text{L}_{\text{HEWMA}_{\text{AIB}}^{(1)}}$ and $\text{L}_{\text{HEWMA}_{\text{AIB}}^{(2)}}$ are determined to obtain $\text{ARL}_0 = 200$. The different settings of ρ and ρ^* are taken from the Haq [28] given as (ρ, ρ^*) = (0.25, 0.0563898), (0.50, 0.2293317), (0.75, 0.5313626), (0.90, 0.7870992), (0.95, 0.8880799). The numerical results of the proposed $\text{HEWMA}_{\text{AIB}}^{(1)}$ and $\text{HEWMA}_{\text{AIB}}^{(2)}$ control charts are presented in Tables 1–8.

5. Evaluation and Performance Comparison

This section provides extensive comparisons of the proposed $\text{HEWMA}_{\text{AIB}}^{(1)}$ and $\text{HEWMA}_{\text{AIB}}^{(2)}$ control charts with the HEWMA (Ali and Haq [16]), AEWMA (Haq [37]), HHW1 and HHW2 (Huwang, et al. [11]), and $\text{EWMA-I}_{\text{AIB}}$ and $\text{EWMA-II}_{\text{AIB}}$ (Haq [27]) control charts.

5.1. Proposed versus HEWMA Control Chart. The proposed $\text{HEWMA}_{\text{AIB}}^{(1)}$ and $\text{HEWMA}_{\text{AIB}}^{(2)}$ control charts are compared with the HEWMA control chart. The proposed control charts outperform the HEWMA control chart. For instance, at $\text{ARL}_0 = 200$, $\lambda_2 = 0.1$, $\lambda_1 = 0.5$, and $\tau = 1.1, 1.2$, the proposed $\text{HEWMA}_{\text{AIB}}^{(1)}$ and $\text{HEWMA}_{\text{AIB}}^{(2)}$ control charts

Table 1: Run-length profile of two-sided $HEWMA^{(1)}_{AIB}$ control chart at $ARL_0 = 200$.

ρ	0.25		0.50		0.75		0.90		0.95	
					$\lambda_1 = 0.05,\ \lambda_2 = 0.05$					
$L_{HEWMA^{(1)}_{AIB}}$	1.688		1.69		1.695		1.7026		1.705	
τ	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL
0.5	3.41	1.02	3.28	1.14	2.71	1.29	1.93	1.08	1.54	0.82
0.6	4.69	1.90	4.48	1.93	3.58	1.87	2.38	1.43	1.79	1.05
0.7	7.28	3.80	6.91	3.69	5.41	3.14	3.32	2.15	2.31	1.51
0.8	13.67	8.88	13.02	8.58	10.12	6.82	5.90	4.16	3.73	2.69
0.9	38.48	32.49	36.60	30.96	28.83	24.02	17.10	13.80	10.54	8.47
1.0	200.63	237.56	200.24	238.26	200.61	239.81	200.47	243.01	200.86	244.70
1.1	33.98	35.79	32.46	34.08	25.89	26.97	15.70	15.74	9.85	9.63
1.2	12.39	12.68	11.66	11.94	9.02	9.24	5.62	5.51	3.62	3.31
1.3	6.73	6.86	6.47	6.57	5.02	5.01	3.18	2.90	2.19	1.77
1.4	4.39	4.39	4.19	4.15	3.35	3.16	2.22	1.85	1.67	1.15
1.5	3.20	3.05	3.10	2.93	2.54	2.26	1.77	1.29	1.41	0.82
1.6	2.54	2.29	2.45	2.20	2.06	1.70	1.52	0.99	1.29	0.64
1.7	2.10	1.75	2.03	1.68	1.78	1.35	1.37	0.78	1.19	0.50
1.8	1.85	1.43	1.78	1.35	1.57	1.07	1.28	0.64	1.15	0.43
1.9	1.65	1.19	1.61	1.13	1.44	0.90	1.21	0.54	1.12	0.37
2.0	1.52	1.01	1.49	0.95	1.36	0.79	1.17	0.47	1.09	0.31
					$\lambda_1 = 0.05,\ \lambda_2 = 0.10$					
$L_{HEWMA^{(1)}_{AIB}}$	1.820		1.8249		1.8344		1.8416		1.8418	
0.5	3.885	1.032	3.717	1.183	3.040	1.385	2.102	1.168	1.635	0.896
0.6	5.265	1.917	5.046	1.976	4.025	1.964	2.604	1.524	1.914	1.129
0.7	8.083	3.819	7.679	3.742	6.018	3.239	3.649	2.257	2.511	1.603
0.8	15.005	8.964	14.389	8.633	11.073	6.894	6.490	4.264	4.082	2.801
0.9	42.482	34.768	40.294	32.985	31.962	25.478	18.565	14.202	11.323	8.625
1.0	200.27	226.37	200.23	227.38	200.24	227.47	200.27	226.37	200.96	231.61
1.1	35.302	36.257	33.737	34.720	26.894	27.012	35.302	36.257	10.437	9.720
1.2	12.822	12.651	12.198	12.004	9.484	9.208	12.822	12.651	3.822	3.362
1.3	6.991	6.860	6.679	6.563	5.296	5.067	6.991	6.860	2.305	1.851
1.4	4.584	4.408	4.371	4.208	3.522	3.225	4.584	4.408	1.719	1.185
1.5	3.321	3.099	3.220	2.973	2.661	2.339	3.321	3.099	1.470	0.875
1.6	2.614	2.296	2.530	2.203	2.143	1.750	2.614	2.296	1.309	0.666
1.7	2.197	1.820	2.121	1.737	1.828	1.378	2.197	1.820	1.219	0.542
1.8	1.896	1.467	1.844	1.390	1.617	1.107	1.896	1.467	1.167	0.456
1.9	1.700	1.236	1.656	1.169	1.485	0.963	1.700	1.236	1.125	0.386
2.0	1.558	1.049	1.525	1.013	1.385	0.786	1.558	1.049	1.095	0.327
					$\lambda_1 = 0.05,\ \lambda_2 = 0.20$					
$L_{HEWMA^{(1)}_{AIB}}$	1.952		1.957		1.967		1.974		1.976	
0.5	4.28	1.04	4.10	1.20	3.32	1.45	2.25	1.24	1.72	0.95
0.6	5.76	1.91	5.51	1.98	4.37	2.01	2.79	1.60	2.02	1.19
0.7	8.71	3.78	8.32	3.73	6.48	3.26	3.91	2.30	2.65	1.67
0.8	16.07	9.21	15.34	8.84	11.86	7.00	6.86	4.32	4.34	2.86
0.9	46.50	37.50	44.09	35.31	34.43	26.73	19.66	14.59	11.95	8.67
1.0	200.14	219.48	200.21	221.63	200.88	222.76	200.44	223.97	200.44	224.89
1.1	35.45	35.95	34.02	34.52	27.48	27.18	16.87	15.88	10.72	9.69
1.2	12.89	12.51	12.41	11.87	9.60	9.02	6.09	5.46	3.96	3.36
1.3	7.08	6.73	6.74	6.40	5.38	4.98	3.43	2.99	2.38	1.87
1.4	4.66	4.34	4.48	4.14	3.65	3.26	2.40	1.91	1.77	1.21
1.5	3.40	3.07	3.27	2.93	2.71	2.31	1.89	1.39	1.50	0.90
1.6	2.67	2.28	2.61	2.22	2.20	1.77	1.61	1.04	1.34	0.69
1.7	2.25	1.83	2.17	1.73	1.86	1.38	1.43	0.84	1.24	0.57
1.8	1.94	1.48	1.89	1.42	1.67	1.16	1.33	0.71	1.18	0.48
1.9	1.73	1.24	1.69	1.18	1.51	0.95	1.24	0.58	1.14	0.40
2.0	1.59	1.07	1.55	1.02	1.40	0.81	1.20	0.51	1.10	0.34

Table 1: Continued.

ρ	0.25		0.50		0.75		0.90		0.95	
					$\lambda_1 = 0.10,\ \lambda_2 = 0.05$					
$L_{HEWMA^{(1)}_{AIB}}$	1.8211		1.8255		1.835		1.8406		1.8405	
τ	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL
0.5	3.89	1.03	3.72	1.18	3.04	1.38	2.10	1.17	1.63	0.89
0.6	5.27	1.92	5.05	1.98	4.03	1.96	2.60	1.52	1.91	1.13
0.7	8.09	3.82	7.68	3.75	6.02	3.24	3.65	2.26	2.51	1.60
0.8	15.03	8.98	14.40	8.63	11.08	6.90	6.48	4.26	4.08	2.80
0.9	42.62	34.84	40.34	32.99	31.97	25.48	18.54	14.18	11.32	8.62
1.0	200.88	227.13	200.82	227.99	200.73	228.04	200.16	228.14	200.20	230.48
1.1	35.34	36.33	33.68	34.69	26.92	27.17	16.52	15.88	10.44	9.70
1.2	12.82	12.65	12.28	12.08	9.46	9.19	5.89	5.52	3.84	3.39
1.3	6.99	6.86	6.63	6.50	5.27	5.06	3.36	3.01	2.29	1.83
1.4	4.58	4.41	4.40	4.21	3.53	3.25	2.34	1.90	1.72	1.19
1.5	3.33	3.12	3.21	2.96	2.65	2.31	1.83	1.34	1.46	0.86
1.6	2.60	2.27	2.53	2.21	2.15	1.76	1.57	1.04	1.32	0.68
1.7	2.19	1.82	2.12	1.72	1.82	1.37	1.41	0.81	1.21	0.53
1.8	1.91	1.48	1.85	1.39	1.62	1.12	1.30	0.68	1.17	0.46
1.9	1.70	1.23	1.65	1.17	1.49	0.95	1.23	0.58	1.13	0.39
2.0	1.56	1.06	1.53	1.01	1.39	0.80	1.18	0.49	1.10	0.33
					$\lambda_1 = 0.10,\ \lambda_2 = 0.10$					
$L_{HEWMA^{(1)}_{AIB}}$	1.9664		1.970		1.983		1.991		1.993	
0.5	4.39	1.07	4.21	1.22	3.40	1.49	2.29	1.27	1.74	0.97
0.6	5.92	1.95	5.66	2.03	4.50	2.07	2.85	1.64	2.05	1.22
0.7	8.99	3.90	8.54	3.81	6.68	3.35	4.00	2.37	2.71	1.71
0.8	16.61	9.46	15.82	9.12	12.30	7.20	7.05	4.42	4.46	2.94
0.9	49.77	41.40	47.20	38.75	36.24	29.01	20.42	15.29	12.32	8.91
1.0	200.09	216.15	200.79	217.67	200.16	219.18	200.52	222.13	200.92	222.16
1.1	36.67	37.22	35.09	35.41	28.48	28.08	17.53	16.34	11.09	9.93
1.2	13.39	12.79	12.77	12.15	9.96	9.28	6.29	5.65	4.05	3.44
1.3	7.26	6.89	6.94	6.58	5.54	5.12	3.52	3.06	2.41	1.92
1.4	4.77	4.45	4.60	4.25	3.74	3.37	2.41	1.95	1.80	1.26
1.5	3.44	3.11	3.33	3.02	2.76	2.38	1.91	1.41	1.51	0.91
1.6	2.74	2.38	2.62	2.25	2.22	1.80	1.62	1.07	1.34	0.71
1.7	2.27	1.85	2.21	1.79	1.89	1.42	1.44	0.86	1.25	0.57
1.8	1.97	1.54	1.90	1.45	1.68	1.16	1.33	0.70	1.18	0.48
1.9	1.75	1.27	1.70	1.21	1.52	0.97	1.25	0.60	1.14	0.40
2.0	1.60	1.08	1.57	1.05	1.42	0.84	1.21	0.52	1.10	0.35
					$\lambda_1 = 0.10,\ \lambda_2 = 0.20$					
$L_{HEWMA^{(1)}_{AIB}}$	2.1109		2.1141		2.1314		2.1434		2.1448	
0.5	4.85	1.07	4.63	1.24	3.73	1.54	2.47	1.35	1.84	1.03
0.6	6.47	1.95	6.18	2.04	4.89	2.12	3.09	1.72	2.18	1.28
0.7	9.77	4.01	9.30	3.91	7.22	3.44	4.32	2.44	2.90	1.78
0.8	18.52	10.70	17.56	10.16	13.44	7.79	7.61	4.58	4.77	3.02
0.9	59.98	51.39	56.54	48.38	42.42	34.99	22.71	17.22	13.31	9.49
1.0	200.38	211.05	200.91	213.68	200.81	214.91	200.98	216.49	200.08	215.61
1.1	36.92	37.45	35.65	35.57	29.35	28.81	18.23	16.76	11.59	10.12
1.2	13.59	12.82	13.02	12.25	10.25	9.36	6.51	5.65	4.23	3.46
1.3	7.42	6.84	7.05	6.52	5.69	5.06	3.63	3.04	2.50	1.94
1.4	4.85	4.38	4.68	4.18	3.82	3.34	2.51	1.98	1.86	1.28
1.5	3.53	3.07	3.42	2.99	2.84	2.39	1.97	1.43	1.55	0.94
1.6	2.81	2.37	2.70	2.25	2.29	1.82	1.66	1.10	1.37	0.74
1.7	2.33	1.87	2.26	1.79	1.95	1.45	1.47	0.86	1.27	0.60
1.8	2.03	1.56	1.95	1.46	1.72	1.18	1.37	0.74	1.20	0.50
1.9	1.80	1.30	1.76	1.25	1.57	1.01	1.28	0.62	1.14	0.41
2.0	1.64	1.10	1.60	1.06	1.43	0.84	1.21	0.53	1.12	0.37

Table 2: Run-length profile of two-sided $HEWMA_{AIB}^{(1)}$ control chart at $ARL_0 = 200$.

ρ	0.25		0.50		0.75		0.90		0.95	
					$\lambda_1 = 0.20$, $\lambda_2 = 0.05$					
$L_{HEWMA_{AIB}^{(1)}}$	1.952		1.957		1.9672		1.9751		1.9755	
τ	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL
0.5	4.28	1.04	4.10	1.20	3.32	1.45	2.26	1.24	1.72	0.95
0.6	5.76	1.91	5.51	1.98	4.37	2.01	2.79	1.60	2.02	1.19
0.7	8.71	3.78	8.32	3.73	6.48	3.26	3.91	2.31	2.65	1.67
0.8	16.07	9.21	15.34	8.84	11.86	7.00	6.86	4.33	4.34	2.85
0.9	46.50	37.50	44.09	35.31	34.43	26.74	19.68	14.60	11.94	8.66
1.0	200.14	219.48	200.23	221.63	200.88	222.78	201.02	224.93	200.15	224.44
1.1	35.45	35.95	34.01	34.50	27.48	27.19	16.83	15.82	10.71	9.66
1.2	12.89	12.51	12.42	11.88	9.61	9.02	6.11	5.48	3.96	3.38
1.3	7.08	6.73	6.74	6.40	5.39	4.98	3.45	3.00	2.38	1.87
1.4	4.66	4.34	4.47	4.13	3.64	3.26	2.38	1.88	1.77	1.22
1.5	3.40	3.07	3.27	2.93	2.71	2.31	1.90	1.39	1.50	0.90
1.6	2.67	2.28	2.61	2.22	2.20	1.77	1.62	1.05	1.34	0.69
1.7	2.25	1.83	2.16	1.72	1.86	1.38	1.44	0.85	1.24	0.57
1.8	1.95	1.48	1.89	1.42	1.67	1.16	1.33	0.70	1.18	0.47
1.9	1.73	1.24	1.69	1.19	1.51	0.95	1.25	0.58	1.14	0.41
2.0	1.59	1.07	1.55	1.02	1.40	0.82	1.20	0.50	1.10	0.34
					$\lambda_1 = 0.20$, $\lambda_2 = 0.10$					
$L_{HEWMA_{AIB}^{(1)}}$	2.112		2.114		2.1312		2.143		2.145	
0.5	4.85	1.07	4.63	1.24	3.73	1.54	2.47	1.35	1.84	1.03
0.6	6.48	1.96	6.18	2.04	4.89	2.12	3.09	1.72	2.18	1.28
0.7	9.78	4.00	9.30	3.91	7.21	3.44	4.32	2.44	2.90	1.78
0.8	18.53	10.70	17.56	10.16	13.44	7.79	7.61	4.57	4.77	3.02
0.9	60.10	51.47	56.51	48.34	42.42	35.00	22.70	17.21	13.31	9.49
1.0	200.90	211.43	200.89	213.66	200.76	214.83	200.78	216.18	200.09	215.64
1.1	36.95	37.48	35.65	35.57	29.37	28.82	18.26	16.79	11.59	10.11
1.2	13.59	12.85	13.02	12.24	10.25	9.36	6.51	5.65	4.22	3.46
1.3	7.44	6.84	7.05	6.52	5.69	5.06	3.64	3.04	2.50	1.94
1.4	4.86	4.38	4.67	4.18	3.82	3.35	2.50	1.98	1.86	1.28
1.5	3.55	3.07	3.41	2.99	2.84	2.39	1.97	1.43	1.55	0.95
1.6	2.81	2.38	2.70	2.25	2.28	1.82	1.66	1.10	1.37	0.74
1.7	2.33	1.88	2.26	1.80	1.95	1.45	1.47	0.87	1.27	0.60
1.8	2.02	1.55	1.95	1.46	1.72	1.18	1.36	0.73	1.20	0.50
1.9	1.80	1.30	1.76	1.25	1.56	1.01	1.28	0.62	1.14	0.41
2.0	1.63	1.10	1.59	1.05	1.44	0.85	1.21	0.53	1.12	0.37
					$\lambda_1 = 0.20$, $\lambda_2 = 0.20$					
$L_{HEWMA_{AIB}^{(1)}}$	2.264		2.269		2.289		2.3029		2.308	
0.5	5.33	1.09	5.11	1.27	4.08	1.61	2.66	1.42	1.96	1.09
0.6	7.11	2.07	6.79	2.13	5.36	2.20	3.34	1.80	2.34	1.36
0.7	10.98	4.66	10.44	4.51	8.01	3.76	4.66	2.56	3.11	1.86
0.8	23.27	15.49	21.79	14.25	15.73	9.92	8.35	5.06	5.13	3.15
0.9	91.32	84.13	84.04	77.20	58.15	51.79	27.54	22.55	15.11	11.32
1.0	200.51	205.99	200.00	205.80	200.79	207.33	200.97	209.96	200.88	209.26
1.1	37.92	38.51	36.72	36.96	30.95	30.59	19.42	18.04	12.28	10.79
1.2	13.95	13.24	13.42	12.55	10.66	9.61	6.75	5.74	4.40	3.52
1.3	7.58	6.88	7.27	6.61	5.92	5.15	3.75	3.05	2.62	1.99
1.4	4.94	4.33	4.81	4.20	3.93	3.34	2.60	1.99	1.93	1.32
1.5	3.65	3.09	3.50	2.97	2.93	2.39	2.02	1.46	1.59	0.97
1.6	2.87	2.35	2.79	2.27	2.36	1.84	1.71	1.12	1.41	0.77
1.7	2.40	1.92	2.32	1.79	2.01	1.47	1.52	0.91	1.29	0.62
1.8	2.08	1.56	2.02	1.52	1.75	1.20	1.40	0.76	1.21	0.51
1.9	1.82	1.29	1.79	1.26	1.59	1.01	1.30	0.65	1.16	0.44
2.0	1.67	1.12	1.64	1.08	1.46	0.86	1.23	0.55	1.13	0.38

Table 2: Continued.

ρ	0.25		0.50		0.75		0.90		0.95	
					$\lambda_1 = 0.30,\ \lambda_2 = 0.05$					
$L_{HEWMA^{(1)}_{AIB}}$	2.029		2.030		2.042		2.051		2.053	
τ	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL
0.5	4.47	1.03	4.28	1.19	3.45	1.46	2.33	1.26	1.77	0.97
0.6	5.99	1.89	5.72	1.96	4.54	2.01	2.88	1.61	2.07	1.21
0.7	9.03	3.83	8.57	3.73	6.68	3.27	4.02	2.32	2.73	1.68
0.8	16.68	9.35	15.84	8.98	12.26	7.13	7.01	4.34	4.44	2.86
0.9	48.90	39.23	46.20	36.73	35.58	27.54	20.22	14.89	12.21	8.78
1.0	200.92	219.58	200.11	219.30	200.28	219.35	200.96	224.86	200.48	222.50
1.1	35.24	35.77	33.84	34.31	27.53	27.03	16.87	15.73	10.80	9.57
1.2	12.83	12.36	12.28	11.72	9.62	8.94	6.13	5.42	3.99	3.32
1.3	7.03	6.61	6.73	6.31	5.36	4.89	3.47	2.95	2.40	1.85
1.4	4.67	4.28	4.45	4.04	3.66	3.18	2.40	1.87	1.79	1.22
1.5	3.41	3.01	3.30	2.89	2.73	2.28	1.91	1.38	1.52	0.91
1.6	2.68	2.23	2.61	2.17	2.21	1.74	1.63	1.05	1.34	0.70
1.7	2.26	1.81	2.18	1.71	1.89	1.38	1.45	0.85	1.25	0.58
1.8	1.96	1.47	1.90	1.39	1.68	1.15	1.34	0.70	1.19	0.49
1.9	1.76	1.26	1.70	1.18	1.53	0.95	1.26	0.60	1.14	0.41
2.0	1.59	1.05	1.57	1.02	1.41	0.82	1.20	0.51	1.11	0.35
					$\lambda_1 = 0.30,\ \lambda_2 = 0.10$					
$L_{HEWMA^{(1)}_{AIB}}$	2.1939		2.198		2.217		2.2291		2.2348	
0.5	5.06	1.07	4.85	1.24	3.88	1.56	2.56	1.37	1.90	1.06
0.6	6.75	1.99	6.44	2.05	5.09	2.13	3.20	1.74	2.25	1.31
0.7	10.25	4.19	9.74	4.10	7.52	3.50	4.45	2.47	2.99	1.80
0.8	19.91	11.79	18.88	11.14	14.21	8.37	7.91	4.71	4.93	3.06
0.9	68.04	58.72	63.57	55.19	46.70	38.99	24.11	18.45	13.93	10.00
1.0	200.89	209.95	200.74	211.15	200.66	209.33	200.98	213.29	200.09	215.64
1.1	36.76	37.15	35.50	35.43	29.58	28.96	18.42	16.97	11.59	10.11
1.2	13.58	12.79	13.00	12.20	10.30	9.31	6.57	5.63	4.22	3.46
1.3	7.39	6.79	7.08	6.46	5.71	5.02	3.66	3.00	2.50	1.94
1.4	4.87	4.32	4.70	4.14	3.83	3.30	2.54	1.98	1.86	1.28
1.5	3.56	3.02	3.42	2.94	2.86	2.35	1.98	1.41	1.55	0.95
1.6	2.83	2.35	2.73	2.22	2.31	1.80	1.69	1.11	1.37	0.74
1.7	2.34	1.85	2.27	1.78	1.96	1.45	1.49	0.88	1.27	0.60
1.8	2.04	1.56	1.98	1.46	1.74	1.19	1.38	0.74	1.20	0.50
1.9	1.81	1.29	1.78	1.24	1.57	1.00	1.29	0.63	1.14	0.41
2.0	1.65	1.10	1.61	1.05	1.45	0.85	1.23	0.55	1.12	0.37
					$\lambda_1 = 0.30,\ \lambda_2 = 0.20$					
$L_{HEWMA^{(1)}_{AIB}}$	2.3616		2.369		2.3865		2.401		2.404	
0.5	5.64	1.13	5.40	1.32	4.30	1.66	2.77	1.45	2.02	1.12
0.6	7.62	2.28	7.27	2.33	5.66	2.31	3.48	1.84	2.41	1.39
0.7	12.34	5.78	11.66	5.50	8.63	4.26	4.89	2.66	3.21	1.88
0.8	29.20	21.21	26.94	19.61	18.12	12.29	8.95	5.58	5.35	3.30
0.9	131.49	124.30	118.63	112.08	73.59	68.04	31.73	27.25	16.45	12.73
1.0	200.27	204.34	200.85	204.42	200.11	204.41	200.88	206.78	200.36	204.54
1.1	38.11	38.63	37.06	37.32	31.51	31.32	20.09	18.82	12.72	11.20
1.2	14.13	13.44	13.60	12.82	10.85	9.80	6.87	5.81	4.47	3.55
1.3	7.65	6.90	7.38	6.63	5.97	5.18	3.78	3.03	2.66	1.99
1.4	4.98	4.32	4.85	4.18	3.97	3.33	2.63	1.99	1.95	1.32
1.5	3.68	3.06	3.55	2.93	2.97	2.38	2.06	1.47	1.62	0.98
1.6	2.92	2.35	2.81	2.25	2.40	1.84	1.74	1.13	1.42	0.78
1.7	2.44	1.92	2.35	1.81	2.02	1.47	1.55	0.94	1.30	0.63
1.8	2.10	1.56	2.04	1.50	1.78	1.20	1.40	0.76	1.22	0.52
1.9	1.85	1.29	1.83	1.27	1.61	1.01	1.31	0.66	1.17	0.45
2.0	1.70	1.13	1.64	1.07	1.48	0.86	1.24	0.55	1.14	0.39

Table 3: Run-length profile of one-sided $HEWMA^{(1)}_{AIB}$ control chart at $ARL_0 = 200$.

ρ	0.25		0.50		0.75		0.90		0.95	
					$\lambda_1 = 0.05$, $\lambda_2 = 0.05$					
$L_{HEWMA^{(1)}_{AIB}}$	1.2155		1.2161		1.2131		1.2153		1.213	
τ	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL
1.0	200.38	248.99	200.08	248.42	200.16	248.03	200.74	250.04	200.19	250.51
1.1	22.77	26.39	21.87	25.34	17.47	20.05	10.75	12.14	6.89	7.46
1.2	8.81	10.05	8.48	9.67	6.73	7.54	4.09	4.25	2.72	2.54
1.3	4.90	5.36	4.66	5.07	3.84	4.05	2.46	2.28	1.78	1.36
1.4	3.40	3.54	3.20	3.29	2.61	2.51	1.83	1.45	1.44	0.89
1.5	2.54	2.42	2.47	2.37	2.08	1.79	1.55	1.05	1.28	0.64
1.6	2.10	1.83	2.03	1.75	1.72	1.35	1.36	0.78	1.18	0.49
1.7	1.82	1.48	1.76	1.36	1.54	1.07	1.27	0.64	1.13	0.40
1.8	1.61	1.20	1.58	1.13	1.40	0.86	1.20	0.53	1.10	0.34
1.9	1.49	1.00	1.46	0.96	1.31	0.72	1.15	0.44	1.07	0.28
2.0	1.37	0.83	1.36	0.80	1.25	0.63	1.11	0.38	1.06	0.24
					$\lambda_1 = 0.05$, $\lambda_2 = 0.10$					
$L_{HEWMA^{(1)}_{AIB}}$	1.409		1.406		1.403		1.397		1.394	
1.0	200.84	237.00	200.61	236.04	200.25	233.53	200.31	236.04	200.61	236.36
1.1	24.80	27.24	23.69	25.98	19.00	20.58	11.80	12.41	7.57	7.74
1.2	9.51	10.17	9.05	9.75	7.39	7.84	4.46	4.43	2.99	2.73
1.3	5.39	5.64	5.09	5.29	4.13	4.16	2.66	2.44	1.91	1.46
1.4	3.64	3.62	3.51	3.50	2.84	2.68	1.97	1.59	1.51	0.96
1.5	2.79	2.64	2.65	2.42	2.18	1.89	1.61	1.10	1.32	0.70
1.6	2.24	1.99	2.19	1.92	1.83	1.44	1.43	0.87	1.22	0.55
1.7	1.90	1.55	1.85	1.48	1.62	1.16	1.30	0.69	1.15	0.44
1.8	1.70	1.27	1.66	1.23	1.46	0.93	1.22	0.56	1.11	0.36
1.9	1.54	1.06	1.49	0.99	1.36	0.80	1.17	0.48	1.09	0.32
2.0	1.44	0.90	1.40	0.86	1.27	0.64	1.13	0.41	1.07	0.28
					$\lambda_1 = 0.10$, $\lambda_2 = 0.05$					
$L_{HEWMA^{(1)}_{AIB}}$	1.4076		1.405		1.4042		1.398		1.394	
τ	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL
1.0	200.10	236.14	200.28	235.48	200.84	234.51	200.61	236.31	200.61	236.36
1.1	24.83	27.23	23.65	25.89	19.03	20.62	11.80	12.43	7.57	7.74
1.2	9.48	10.16	9.10	9.80	7.41	7.86	4.49	4.46	2.99	2.73
1.3	5.40	5.65	5.06	5.27	4.10	4.13	2.66	2.44	1.91	1.46
1.4	3.63	3.61	3.52	3.52	2.83	2.66	1.97	1.58	1.51	0.96
1.5	2.78	2.61	2.63	2.42	2.19	1.90	1.61	1.12	1.32	0.70
1.6	2.24	1.98	2.19	1.91	1.84	1.44	1.43	0.86	1.22	0.55
1.7	1.93	1.59	1.86	1.50	1.63	1.16	1.31	0.69	1.15	0.44
1.8	1.69	1.25	1.66	1.22	1.46	0.94	1.22	0.56	1.11	0.36
1.9	1.54	1.05	1.49	0.98	1.35	0.78	1.17	0.47	1.09	0.32
2.0	1.43	0.91	1.41	0.87	1.28	0.66	1.13	0.41	1.07	0.28
					$\lambda_1 = 0.10$, $\lambda_2 = 0.10$					
$L_{HEWMA^{(1)}_{AIB}}$	1.636		1.630		1.618		1.605		1.599	
1.0	200.79	225.52	200.76	225.46	200.00	222.81	200.03	223.34	200.70	225.34
1.1	27.45	28.95	26.27	27.44	21.21	21.79	13.10	13.08	8.50	8.15
1.2	10.49	10.56	9.94	10.05	8.11	8.09	4.95	4.67	3.26	2.88
1.3	5.96	5.90	5.69	5.65	4.51	4.35	2.94	2.63	2.06	1.60
1.4	4.06	3.94	3.89	3.70	3.07	2.82	2.11	1.68	1.58	1.04
1.5	3.01	2.79	2.90	2.66	2.39	2.07	1.72	1.22	1.38	0.78
1.6	2.41	2.10	2.32	2.01	1.97	1.56	1.48	0.91	1.25	0.59
1.7	2.06	1.69	1.97	1.59	1.70	1.22	1.35	0.74	1.18	0.48
1.8	1.79	1.36	1.75	1.32	1.53	0.99	1.26	0.61	1.14	0.41
1.9	1.63	1.15	1.59	1.08	1.42	0.86	1.20	0.52	1.10	0.34
2.0	1.49	0.97	1.46	0.92	1.33	0.73	1.16	0.45	1.08	0.29

Table 4: Run-length profile of one-sided $HEWMA_{AIB}^{(1)}$ control chart at $ARL_0 = 200$.

ρ	0.25		0.50		0.75		0.90		0.95	
					$\lambda_1 = 0.20$, $\lambda_2 = 0.05$					
$L_{HEWMA_{AIB}^{(1)}}$	1.5875		1.5825		1.572		1.5659		1.56	
τ	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL
1.0	200.88	230.95	200.05	230.21	200.10	228.49	200.96	228.86	200.63	229.12
1.1	25.79	27.51	24.87	26.28	19.87	20.93	12.38	12.57	8.06	7.82
1.2	9.90	10.13	9.49	9.68	7.74	7.84	4.74	4.48	3.16	2.78
1.3	5.71	5.71	5.41	5.39	4.29	4.14	2.84	2.53	2.01	1.53
1.4	3.86	3.67	3.66	3.47	2.99	2.73	2.06	1.62	1.57	1.00
1.5	2.92	2.70	2.82	2.53	2.28	1.94	1.69	1.18	1.36	0.75
1.6	2.34	2.02	2.26	1.93	1.93	1.51	1.47	0.90	1.25	0.58
1.7	1.99	1.60	1.95	1.55	1.67	1.18	1.33	0.71	1.17	0.47
1.8	1.76	1.33	1.71	1.24	1.51	0.98	1.25	0.59	1.13	0.40
1.9	1.61	1.10	1.55	1.03	1.39	0.79	1.19	0.50	1.10	0.34
2.0	1.47	0.92	1.45	0.92	1.32	0.70	1.16	0.44	1.08	0.29
					$\lambda_1 = 0.20$, $\lambda_2 = 0.10$					
$L_{HEWMA_{AIB}^{(1)}}$	1.8475		1.8449		1.821		1.799		1.789	
1.0	200.96	217.27	200.91	217.63	200.48	215.52	200.27	216.11	200.78	217.14
1.1	29.40	30.26	28.19	28.84	22.65	22.75	14.02	13.56	9.09	8.31
1.2	11.24	11.00	10.69	10.36	8.57	8.13	5.30	4.79	3.51	2.99
1.3	6.39	6.06	6.09	5.70	4.79	4.43	3.11	2.66	2.18	1.66
1.4	4.30	3.97	4.12	3.81	3.31	2.95	2.24	1.77	1.67	1.12
1.5	3.20	2.85	3.07	2.72	2.51	2.11	1.78	1.26	1.43	0.81
1.6	2.55	2.17	2.47	2.09	2.06	1.58	1.55	0.97	1.29	0.63
1.7	2.15	1.74	2.08	1.65	1.79	1.30	1.38	0.76	1.21	0.52
1.8	1.87	1.40	1.83	1.36	1.60	1.08	1.30	0.66	1.15	0.42
1.9	1.68	1.17	1.64	1.12	1.47	0.89	1.22	0.55	1.12	0.37
2.0	1.54	1.00	1.51	0.96	1.37	0.77	1.17	0.47	1.10	0.33
					$\lambda_1 = 0.30$, $\lambda_2 = 0.05$					
$L_{HEWMA_{AIB}^{(1)}}$	1.6875		1.6794		1.6672		1.655		1.648	
τ	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL
1.0	200.70	228.03	200.19	226.08	200.00	225.63	200.08	225.87	200.60	227.32
1.1	26.23	27.74	25.09	26.38	20.16	20.94	12.59	12.56	8.21	7.82
1.2	10.05	10.17	9.60	9.66	7.82	7.79	4.83	4.45	3.22	2.78
1.3	5.77	5.64	5.48	5.34	4.35	4.09	2.87	2.49	2.05	1.53
1.4	3.96	3.71	3.72	3.44	3.03	2.70	2.09	1.61	1.59	1.02
1.5	2.95	2.67	2.87	2.54	2.34	1.95	1.73	1.19	1.38	0.77
1.6	2.38	2.02	2.30	1.91	1.96	1.51	1.49	0.91	1.25	0.58
1.7	2.02	1.60	1.95	1.52	1.70	1.19	1.35	0.73	1.19	0.48
1.8	1.80	1.33	1.74	1.26	1.52	0.97	1.27	0.61	1.14	0.41
1.9	1.62	1.10	1.58	1.06	1.41	0.81	1.20	0.50	1.10	0.34
2.0	1.50	0.96	1.47	0.90	1.33	0.73	1.16	0.46	1.08	0.31
					$\lambda_1 = 0.30$, $\lambda_2 = 0.10$					
$L_{HEWMA_{AIB}^{(1)}}$	1.9708		1.9645		1.939		1.909		1.894	
1.0	200.10	213.90	200.52	214.69	200.16	213.50	200.37	213.25	200.49	214.33
1.1	30.30	30.96	28.91	29.30	23.29	23.31	14.45	13.84	9.30	8.41
1.2	11.54	11.15	11.00	10.57	8.76	8.18	5.43	4.83	3.60	3.00
1.3	6.54	6.09	6.26	5.78	4.89	4.43	3.19	2.67	2.23	1.67
1.4	4.41	3.98	4.19	3.79	3.37	2.96	2.28	1.78	1.70	1.13
1.5	3.26	2.86	3.13	2.72	2.56	2.10	1.83	1.28	1.45	0.83
1.6	2.61	2.18	2.52	2.10	2.13	1.62	1.57	0.98	1.31	0.65
1.7	2.19	1.73	2.12	1.66	1.83	1.33	1.41	0.80	1.22	0.54
1.8	1.91	1.43	1.87	1.38	1.63	1.07	1.31	0.66	1.16	0.43
1.9	1.71	1.17	1.68	1.13	1.49	0.91	1.24	0.55	1.13	0.38
2.0	1.57	1.01	1.54	0.97	1.38	0.77	1.18	0.48	1.10	0.34

($\rho = 0.5$) provides the ARL_1 values are (23.69, 9.05) and (25.05, 9.75), respectively, whereas the HEWMA control chart has the ARL_1 values equal to (25.79, 10.15) (see Tables 3, 7 versus 9). Similarly, the proposed control charts' superiority over the HEWMA control chart can also be visualized in Figure 1. Besides, the overall performance

Table 5: Run-length profile of two-sided $HEWMA^{(2)}_{AIB}$ control chart at $ARL_0 = 200$.

ρ	0.25		0.50		0.75		0.90		0.95	
					$\lambda_1 = 0.05, \lambda_2 = 0.05$					
$L_{HEWMA^{(2)}_{AIB}}$	1.722		1.718		1.715		1.710		1.710	
τ	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL
0.5	2.14	1.21	2.05	1.15	1.67	0.87	1.19	0.46	1.05	0.24
0.6	3.40	2.26	3.26	2.17	2.57	1.65	1.60	0.90	1.20	0.50
0.7	6.01	4.48	5.72	4.27	4.53	3.31	2.65	1.84	1.70	1.05
0.8	12.52	10.15	12.05	9.69	9.63	7.66	5.53	4.37	3.35	2.58
0.9	38.65	36.63	36.85	34.76	29.94	27.38	18.17	16.02	11.24	9.65
1.0	200.27	246.69	200.19	245.68	199.48	245.59	199.45	242.05	200.74	242.87
1.1	37.34	39.88	35.56	38.02	28.61	30.30	17.93	18.10	11.42	11.21
1.2	13.85	14.03	13.20	13.38	10.22	10.30	6.36	6.24	3.92	3.67
1.3	7.62	7.66	7.22	7.24	5.71	5.63	3.37	3.17	2.24	1.87
1.4	4.98	4.88	4.75	4.67	3.81	3.67	2.33	1.97	1.63	1.15
1.5	3.63	3.41	3.48	3.30	2.81	2.55	1.82	1.39	1.36	0.77
1.6	2.88	2.62	2.76	2.46	2.26	1.93	1.53	1.03	1.21	0.57
1.7	2.39	2.08	2.28	1.96	1.90	1.51	1.38	0.82	1.14	0.44
1.8	2.06	1.70	1.98	1.59	1.69	1.23	1.26	0.63	1.10	0.35
1.9	1.82	1.40	1.78	1.35	1.52	1.02	1.20	0.54	1.07	0.29
2.0	1.67	1.20	1.61	1.14	1.42	0.87	1.14	0.44	1.05	0.23
					$\lambda_1 = 0.05, \lambda_2 = 0.10$					
$L_{HEWMA^{(2)}_{AIB}}$	1.851		1.8493		1.8445		1.845		1.844	
0.5	2.35	1.30	2.25	1.23	1.82	0.95	1.24	0.51	1.06	0.26
0.6	3.71	2.35	3.56	2.25	2.81	1.74	1.72	0.97	1.25	0.55
0.7	6.49	4.54	6.22	4.36	4.92	3.40	2.90	1.94	1.82	1.12
0.8	13.48	10.31	12.88	9.79	10.31	7.74	6.00	4.47	3.65	2.68
0.9	41.46	38.28	39.95	36.67	32.05	28.54	19.58	16.32	12.11	9.78
1.0	200.21	232.77	201.25	232.48	199.96	230.16	200.13	225.69	200.18	231.85
1.1	39.54	40.33	37.78	38.64	30.69	30.75	20.17	18.45	12.27	11.25
1.2	14.70	14.04	14.07	13.44	11.02	10.40	7.14	6.19	4.25	3.78
1.3	8.08	7.65	7.73	7.30	6.17	5.73	3.86	3.26	2.40	1.95
1.4	5.32	4.91	5.08	4.70	4.06	3.73	2.62	2.10	1.72	1.21
1.5	3.88	3.52	3.75	3.38	2.99	2.65	2.01	1.49	1.40	0.83
1.6	3.07	2.73	2.95	2.58	2.39	1.99	1.66	1.12	1.25	0.62
1.7	2.53	2.14	2.43	2.06	2.03	1.59	1.47	0.89	1.17	0.48
1.8	2.18	1.77	2.09	1.67	1.75	1.27	1.33	0.70	1.11	0.38
1.9	1.92	1.48	1.86	1.40	1.58	1.07	1.24	0.59	1.08	0.30
2.0	1.72	1.22	1.69	1.20	1.46	0.90	1.18	0.49	1.06	0.25
					$\lambda_1 = 0.05, \lambda_2 = 0.20$					
$L_{HEWMA^{(2)}_{AIB}}$	1.9829		1.981		1.9769		1.978		1.975	
0.5	2.55	1.35	2.44	1.28	1.97	0.99	1.31	0.56	1.08	0.30
0.6	4.00	2.38	3.82	2.28	3.04	1.79	1.84	1.02	1.30	0.59
0.7	6.92	4.53	6.65	4.35	5.22	3.41	3.10	1.97	1.95	1.17
0.8	14.21	10.53	13.58	9.92	10.81	7.80	6.36	4.47	3.90	2.71
0.9	43.87	39.76	42.37	38.08	33.91	29.88	20.60	16.72	12.66	9.78
1.0	200.52	222.38	200.47	224.70	201.08	225.63	200.13	225.69	200.84	224.97
1.1	41.18	40.70	39.31	39.23	31.91	31.02	20.17	18.45	12.83	11.24
1.2	15.33	14.05	14.57	13.32	11.52	10.42	7.14	6.19	4.48	3.75
1.3	8.37	7.47	8.08	7.21	6.43	5.66	3.86	3.26	2.54	1.99
1.4	5.56	4.86	5.33	4.66	4.26	3.72	2.62	2.10	1.78	1.23
1.5	4.10	3.56	3.90	3.34	3.15	2.65	2.01	1.49	1.47	0.87
1.6	3.19	2.70	3.08	2.62	2.50	2.02	1.66	1.12	1.28	0.65
1.7	2.65	2.16	2.53	2.06	2.09	1.59	1.47	0.89	1.18	0.50
1.8	2.25	1.74	2.19	1.70	1.82	1.30	1.33	0.70	1.12	0.39
1.9	2.00	1.48	1.93	1.43	1.63	1.08	1.24	0.59	1.08	0.32
2.0	1.80	1.26	1.74	1.20	1.51	0.95	1.18	0.49	1.06	0.27

Table 5: Continued.

ρ	0.25		0.50		0.75		0.90		0.95	
					$\lambda_1 = 0.10$, $\lambda_2 = 0.05$					
$L_{HEWMA^{(2)}_{AIB}}$	1.851		1.849		1.846		1.846		1.845	
τ	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL
0.5	2.35	1.30	2.25	1.23	1.82	0.95	1.24	0.51	1.06	0.26
0.6	3.71	2.35	3.56	2.25	2.81	1.74	1.72	0.97	1.25	0.55
0.7	6.49	4.54	6.22	4.36	4.93	3.40	2.90	1.94	1.82	1.12
0.8	13.48	10.31	12.88	9.79	10.33	7.75	6.00	4.47	3.66	2.68
0.9	41.46	38.28	39.96	36.66	32.08	28.56	19.60	16.34	12.12	9.78
1.0	200.16	232.78	200.90	231.87	200.92	231.12	200.90	234.03	200.49	232.23
1.1	39.54	40.36	37.77	38.57	30.66	30.74	19.26	18.41	12.29	11.27
1.2	14.71	14.04	14.09	13.49	11.01	10.42	6.82	6.28	4.26	3.82
1.3	8.08	7.64	7.75	7.33	6.22	5.73	3.71	3.29	2.40	1.94
1.4	5.32	4.92	5.07	4.68	4.06	3.76	2.50	2.07	1.73	1.21
1.5	3.88	3.52	3.73	3.37	3.00	2.64	1.91	1.46	1.40	0.83
1.6	3.07	2.73	2.95	2.57	2.40	2.00	1.60	1.09	1.25	0.61
1.7	2.53	2.14	2.44	2.07	2.02	1.60	1.41	0.84	1.16	0.48
1.8	2.18	1.77	2.09	1.67	1.74	1.26	1.31	0.69	1.12	0.39
1.9	1.92	1.47	1.86	1.40	1.59	1.08	1.21	0.55	1.08	0.30
2.0	1.72	1.23	1.69	1.20	1.46	0.90	1.16	0.48	1.05	0.25
					$\lambda_1 = 0.10$, $\lambda_2 = 0.10$					
$L_{HEWMA^{(2)}_{AIB}}$	1.997		1.997		1.992		1.995		1.990	
0.5	2.59	1.38	2.49	1.32	2.00	1.02	1.32	0.58	1.08	0.30
0.6	4.11	2.45	3.92	2.34	3.11	1.84	1.87	1.04	1.31	0.60
0.7	7.12	4.66	6.86	4.46	5.38	3.51	3.16	2.03	1.98	1.20
0.8	14.70	10.79	14.08	10.28	11.20	7.99	6.57	4.60	4.01	2.79
0.9	46.68	42.94	44.84	40.90	35.77	31.78	21.35	17.37	13.09	10.04
1.0	200.14	218.75	200.50	221.26	199.78	220.54	200.48	222.59	200.44	221.85
1.1	43.02	43.02	41.35	41.25	33.39	32.63	21.01	19.21	13.34	11.56
1.2	15.96	14.56	15.23	13.82	11.96	10.75	7.36	6.34	4.61	3.87
1.3	8.68	7.62	8.30	7.33	6.64	5.86	4.00	3.38	2.60	2.06
1.4	5.72	5.04	5.48	4.80	4.37	3.81	2.67	2.16	1.80	1.27
1.5	4.17	3.65	4.02	3.52	3.24	2.74	2.04	1.54	1.48	0.90
1.6	3.29	2.79	3.15	2.66	2.55	2.07	1.70	1.17	1.29	0.65
1.7	2.67	2.19	2.60	2.13	2.13	1.65	1.47	0.89	1.19	0.50
1.8	2.31	1.82	2.22	1.73	1.83	1.32	1.34	0.75	1.13	0.40
1.9	2.01	1.51	1.96	1.45	1.66	1.12	1.25	0.58	1.09	0.33
2.0	1.82	1.31	1.76	1.24	1.51	0.96	1.19	0.51	1.07	0.28
					$\lambda_1 = 0.10$, $\lambda_2 = 0.20$					
$L_{HEWMA^{(2)}_{AIB}}$	2.1406		2.1404		2.1402		2.1399		2.1397	
0.5	2.83	1.41	2.71	1.37	2.18	1.07	1.40	0.63	1.11	0.34
0.6	4.44	2.48	4.25	2.37	3.38	1.89	2.02	1.09	1.39	0.65
0.7	7.63	4.72	7.30	4.52	5.77	3.55	3.42	2.07	2.14	1.25
0.8	15.87	11.57	15.17	11.04	11.92	8.29	7.01	4.64	4.30	2.80
0.9	51.95	47.78	49.71	45.71	39.70	36.01	23.14	18.91	14.05	10.57
1.0	200.52	212.02	201.39	215.11	200.50	213.54	199.04	212.09	200.08	212.69
1.1	46.11	45.54	44.09	43.42	36.04	34.53	22.53	20.12	14.17	11.92
1.2	16.62	14.85	15.94	14.10	12.75	10.98	7.80	6.34	4.91	3.88
1.3	9.07	7.67	8.65	7.33	6.96	5.85	4.23	3.37	2.75	2.08
1.4	5.97	5.03	5.76	4.82	4.63	3.79	2.81	2.18	1.91	1.32
1.5	4.39	3.61	4.21	3.48	3.40	2.74	2.16	1.57	1.54	0.94
1.6	3.44	2.79	3.29	2.65	2.68	2.08	1.77	1.20	1.33	0.69
1.7	2.82	2.23	2.72	2.12	2.22	1.66	1.53	0.94	1.21	0.53
1.8	2.39	1.85	2.30	1.75	1.94	1.39	1.38	0.76	1.14	0.42
1.9	2.10	1.55	2.04	1.51	1.73	1.17	1.28	0.63	1.10	0.36
2.0	1.90	1.34	1.83	1.27	1.58	1.00	1.21	0.55	1.07	0.29

Table 6: Run-length profile of two-sided $HEWMA_{AIB}^{(2)}$ control chart at $ARL_0 = 200$.

ρ	0.25		0.50		0.75		0.90		0.95	
					$\lambda_1 = 0.20$, $\lambda_2 = 0.05$					
$L_{HEWMA_{AIB}^{(2)}}$	1.981		1.979		1.976		1.976		1.975	
τ	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL
0.5	2.54	1.35	2.43	1.28	1.96	0.99	1.31	0.56	1.08	0.30
0.6	4.00	2.38	3.82	2.28	3.04	1.79	1.84	1.02	1.30	0.59
0.7	6.90	4.52	6.64	4.35	5.22	3.41	3.09	1.97	1.95	1.17
0.8	14.19	10.52	13.56	9.92	10.80	7.79	6.35	4.46	3.90	2.71
0.9	43.75	39.67	42.27	38.00	33.88	29.87	20.58	16.69	12.66	9.78
1.0	199.35	221.60	199.09	223.38	200.28	224.53	199.48	224.77	200.84	224.97
1.1	41.04	40.65	39.16	39.01	31.94	30.98	20.11	18.39	12.91	11.33
1.2	15.27	13.98	14.60	13.42	11.47	10.35	7.11	6.20	4.48	3.70
1.3	8.41	7.54	8.04	7.20	6.48	5.68	3.88	3.27	2.53	1.98
1.4	5.51	4.82	5.31	4.64	4.22	3.69	2.62	2.10	1.79	1.26
1.5	4.10	3.53	3.90	3.35	3.15	2.66	2.00	1.48	1.46	0.87
1.6	3.17	2.70	3.08	2.58	2.52	2.03	1.66	1.11	1.27	0.61
1.7	2.65	2.16	2.54	2.07	2.08	1.58	1.46	0.88	1.18	0.49
1.8	2.26	1.77	2.17	1.69	1.83	1.31	1.34	0.72	1.12	0.39
1.9	1.98	1.47	1.93	1.43	1.62	1.07	1.24	0.58	1.08	0.32
2.0	1.80	1.27	1.74	1.23	1.50	0.93	1.18	0.50	1.06	0.27
					$\lambda_1 = 0.20$, $\lambda_2 = 0.10$					
$L_{HEWMA_{AIB}^{(2)}}$	2.1398		2.139		2.1411		2.143		2.140	
0.5	2.83	1.41	2.71	1.37	2.18	1.07	1.40	0.63	1.11	0.34
0.6	4.44	2.47	4.25	2.37	3.38	1.89	2.02	1.10	1.39	0.65
0.7	7.63	4.72	7.30	4.52	5.78	3.55	3.43	2.07	2.14	1.25
0.8	15.86	11.57	15.15	11.03	11.93	8.29	7.01	4.64	4.30	2.80
0.9	51.91	47.75	49.62	45.61	39.73	36.01	23.21	18.96	14.06	10.57
1.0	200.01	211.54	201.23	214.87	201.02	214.16	200.84	213.71	200.19	212.79
1.1	46.05	45.43	44.02	43.35	35.99	34.47	22.62	20.18	14.17	11.92
1.2	16.65	14.82	15.93	14.11	12.77	11.00	7.80	6.32	4.91	3.88
1.3	9.05	7.66	8.66	7.32	6.98	5.85	4.23	3.35	2.75	2.09
1.4	5.98	5.02	5.76	4.83	4.61	3.77	2.83	2.20	1.92	1.33
1.5	4.40	3.64	4.20	3.47	3.40	2.75	2.16	1.58	1.53	0.93
1.6	3.43	2.77	3.30	2.66	2.68	2.08	1.77	1.20	1.33	0.69
1.7	2.81	2.22	2.72	2.12	2.23	1.67	1.52	0.91	1.22	0.53
1.8	2.40	1.85	2.30	1.75	1.93	1.38	1.40	0.78	1.14	0.42
1.9	2.11	1.56	2.04	1.50	1.73	1.17	1.28	0.64	1.11	0.36
2.0	1.89	1.32	1.83	1.27	1.57	1.00	1.22	0.55	1.07	0.29
					$\lambda_1 = 0.20$, $\lambda_2 = 0.20$					
$L_{HEWMA_{AIB}^{(2)}}$	2.296		2.291		2.298		2.299		2.300	
0.5	3.10	1.46	2.96	1.40	2.38	1.12	1.50	0.69	1.14	0.38
0.6	4.81	2.55	4.62	2.45	3.68	1.93	2.21	1.15	1.48	0.71
0.7	8.32	5.11	7.91	4.80	6.28	3.71	3.71	2.12	2.32	1.30
0.8	18.08	13.93	17.19	13.16	13.37	9.60	7.61	4.90	4.64	2.86
0.9	62.07	59.03	59.06	55.68	47.20	43.63	26.75	22.91	15.69	12.21
1.0	201.81	209.14	200.42	207.02	200.66	206.57	200.99	208.57	200.93	207.67
1.1	51.25	50.32	48.77	47.68	40.62	39.23	25.23	22.83	15.49	13.01
1.2	17.92	15.93	17.12	15.13	13.79	11.70	8.21	6.49	5.28	3.96
1.3	9.62	8.02	9.18	7.59	7.40	5.94	4.53	3.46	2.92	2.11
1.4	6.29	5.00	6.01	4.83	4.88	3.84	3.04	2.26	2.02	1.37
1.5	4.60	3.62	4.42	3.45	3.59	2.74	2.28	1.63	1.60	0.96
1.6	3.61	2.77	3.47	2.69	2.85	2.15	1.86	1.23	1.36	0.72
1.7	2.97	2.27	2.83	2.15	2.35	1.73	1.58	0.97	1.25	0.57
1.8	2.52	1.88	2.43	1.78	2.02	1.41	1.43	0.81	1.17	0.45
1.9	2.22	1.61	2.14	1.53	1.80	1.20	1.32	0.67	1.12	0.37
2.0	1.99	1.39	1.91	1.30	1.63	1.04	1.25	0.58	1.08	0.31

Table 6: Continued.

ρ	0.25		0.50		0.75		0.90		0.95	
	$\lambda_1 = 0.30$, $\lambda_2 = 0.05$									
$L_{HEWMA^{(2)}_{AIB}}$	2.0559		2.053		2.0494		2.054		2.050	
τ	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL
0.5	2.64	1.35	2.53	1.29	2.04	1.01	1.35	0.59	1.09	0.31
0.6	4.13	2.36	3.94	2.27	3.14	1.78	1.91	1.03	1.34	0.61
0.7	7.07	4.51	6.78	4.31	5.37	3.39	3.19	1.96	2.02	1.19
0.8	14.51	10.59	13.85	10.07	11.01	7.82	6.51	4.45	4.00	2.68
0.9	45.04	40.67	43.11	38.78	34.73	30.36	21.02	16.87	12.90	9.83
1.0	199.79	218.86	199.43	221.28	199.94	222.28	200.74	222.87	200.18	220.75
1.1	41.57	41.16	39.80	39.20	32.62	31.17	20.46	18.53	13.00	11.23
1.2	15.40	13.95	14.68	13.24	11.60	10.25	7.26	6.13	4.54	3.70
1.3	8.46	7.36	8.12	7.13	6.53	5.60	3.92	3.20	2.58	1.96
1.4	5.60	4.79	5.37	4.58	4.30	3.66	2.67	2.07	1.82	1.24
1.5	4.13	3.48	3.94	3.28	3.19	2.60	2.04	1.48	1.48	0.88
1.6	3.23	2.66	3.12	2.57	2.54	2.01	1.71	1.14	1.30	0.65
1.7	2.68	2.13	2.57	2.03	2.12	1.58	1.49	0.89	1.20	0.51
1.8	2.28	1.72	2.21	1.68	1.85	1.29	1.35	0.72	1.13	0.40
1.9	2.03	1.48	1.96	1.42	1.65	1.08	1.26	0.60	1.09	0.32
2.0	1.82	1.26	1.77	1.20	1.53	0.94	1.19	0.50	1.07	0.28
	$\lambda_1 = 0.30$, $\lambda_2 = 0.10$									
$L_{HEWMA^{(2)}_{AIB}}$	2.2257		2.2249		2.2259		2.229		2.227	
0.5	2.95	1.42	2.82	1.37	2.27	1.09	1.45	0.65	1.13	0.36
0.6	4.61	2.49	4.41	2.38	3.52	1.89	2.12	1.11	1.43	0.68
0.7	7.89	4.82	7.58	4.59	5.97	3.58	3.55	2.07	2.23	1.27
0.8	16.54	12.17	15.78	11.53	12.39	8.59	7.26	4.68	4.45	2.80
0.9	54.50	50.34	52.26	48.19	41.77	37.88	24.31	19.86	14.53	10.87
1.0	201.41	210.44	201.39	212.49	200.98	212.26	200.85	211.46	200.24	211.49
1.1	47.32	46.50	45.24	44.29	37.20	35.37	23.30	20.66	14.51	12.11
1.2	16.93	14.92	16.28	14.27	13.03	11.06	7.92	6.27	5.03	3.86
1.3	9.21	7.69	8.80	7.29	7.11	5.84	4.32	3.32	2.81	2.06
1.4	6.06	4.95	5.81	4.75	4.70	3.74	2.92	2.20	1.97	1.34
1.5	4.45	3.56	4.29	3.43	3.45	2.68	2.20	1.55	1.57	0.94
1.6	3.48	2.75	3.35	2.63	2.74	2.07	1.80	1.19	1.34	0.69
1.7	2.86	2.21	2.77	2.10	2.29	1.67	1.57	0.95	1.23	0.54
1.8	2.45	1.82	2.37	1.77	1.98	1.40	1.41	0.79	1.16	0.44
1.9	2.16	1.56	2.08	1.49	1.76	1.18	1.31	0.66	1.11	0.36
2.0	1.93	1.33	1.85	1.26	1.60	1.00	1.23	0.56	1.08	0.29
	$\lambda_1 = 0.30$, $\lambda_2 = 0.20$									
$L_{HEWMA^{(2)}_{AIB}}$	2.385		2.3846		2.384		2.387		2.390	
0.5	3.23	1.47	3.09	1.42	2.48	1.13	1.56	0.71	1.17	0.41
0.6	5.02	2.62	4.82	2.51	3.83	1.94	2.30	1.16	1.53	0.73
0.7	8.77	5.47	8.39	5.20	6.57	3.89	3.85	2.14	2.42	1.31
0.8	19.85	15.88	18.81	14.89	14.41	10.68	7.96	5.18	4.81	2.90
0.9	68.78	66.02	65.77	62.46	51.46	48.22	29.09	25.55	16.86	13.40
1.0	200.80	203.89	201.46	206.61	199.45	205.70	200.61	206.90	200.52	205.12
1.1	53.99	53.31	51.92	50.83	42.88	41.36	26.79	24.45	16.38	13.96
1.2	18.74	16.79	18.00	15.98	14.39	12.24	8.59	6.75	5.37	3.97
1.3	9.87	8.26	9.47	7.83	7.60	6.05	4.64	3.45	3.01	2.11
1.4	6.42	5.03	6.15	4.84	4.94	3.83	3.09	2.25	2.08	1.37
1.5	4.69	3.60	4.51	3.46	3.68	2.75	2.33	1.60	1.63	0.96
1.6	3.69	2.80	3.52	2.63	2.89	2.13	1.89	1.22	1.40	0.74
1.7	3.01	2.26	2.92	2.16	2.40	1.70	1.64	1.00	1.26	0.57
1.8	2.60	1.88	2.47	1.78	2.08	1.43	1.46	0.81	1.18	0.46
1.9	2.27	1.60	2.19	1.55	1.83	1.21	1.35	0.69	1.13	0.39
2.0	2.03	1.39	1.96	1.32	1.66	1.02	1.26	0.58	1.09	0.32

Table 7: Run-length profile of one-sided $\text{HEWMA}^{(2)}_{\text{AIB}}$ control chart at $\text{ARL}_0 = 200$.

ρ	0.25		0.50		0.75		0.90		0.95	
					$\lambda_1 = 0.05,\ \lambda_2 = 0.05$					
$L_{\text{HEWMA}^{(2)}_{\text{AIB}}}$	1.2333		1.2302		1.2296		1.229		1.225	
τ	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL
1.0	200.00	251.02	200.15	251.83	200.76	252.87	200.86	252.09	200.27	250.70
1.1	24.06	28.06	23.00	26.85	18.75	22.01	11.92	13.78	7.62	8.63
1.2	9.32	10.74	8.90	10.30	7.26	8.44	4.39	4.89	2.81	2.85
1.3	5.21	5.84	4.99	5.57	4.06	4.44	2.58	2.55	1.77	1.45
1.4	3.61	3.86	3.46	3.68	2.80	2.84	1.86	1.58	1.39	0.90
1.5	2.71	2.66	2.60	2.55	2.14	1.99	1.52	1.12	1.21	0.60
1.6	2.23	2.10	2.15	1.96	1.79	1.50	1.34	0.82	1.13	0.45
1.7	1.91	1.65	1.84	1.56	1.58	1.20	1.24	0.64	1.09	0.35
1.8	1.69	1.35	1.64	1.28	1.42	0.94	1.16	0.50	1.06	0.26
1.9	1.54	1.11	1.50	1.06	1.32	0.81	1.12	0.42	1.04	0.21
2.0	1.41	0.91	1.39	0.90	1.26	0.66	1.09	0.35	1.03	0.19
					$\lambda_1 = 0.05,\ \lambda_2 = 0.10$					
$L_{\text{HEWMA}^{(2)}_{\text{AIB}}}$	1.4001		1.3999		1.3936		1.3923		1.391	
1.0	200.00	235.98	200.63	239.37	200.48	236.35	200.94	237.48	200.50	237.16
1.1	26.02	28.52	25.05	27.49	20.58	22.42	13.14	13.98	8.52	8.92
1.2	10.16	10.83	9.75	10.35	7.96	8.51	4.89	5.03	3.12	2.99
1.3	5.82	6.12	5.58	5.85	4.44	4.54	2.82	2.70	1.90	1.55
1.4	3.95	3.94	3.79	3.78	3.02	2.96	2.00	1.69	1.45	0.96
1.5	2.99	2.92	2.89	2.79	2.35	2.13	1.62	1.20	1.26	0.67
1.6	2.40	2.20	2.33	2.11	1.92	1.60	1.40	0.88	1.16	0.48
1.7	2.04	1.74	1.98	1.67	1.67	1.28	1.28	0.69	1.10	0.37
1.8	1.79	1.44	1.73	1.34	1.50	1.01	1.19	0.54	1.07	0.30
1.9	1.63	1.19	1.58	1.16	1.40	0.87	1.14	0.46	1.05	0.24
2.0	1.49	1.00	1.46	0.96	1.30	0.74	1.10	0.38	1.03	0.20
					$\lambda_1 = 0.10,\ \lambda_2 = 0.05$					
$L_{\text{HEWMA}^{(2)}_{\text{AIB}}}$	1.401		1.3987		1.3941		1.3909		1.390	
τ	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL
1.0	200.43	236.48	200.01	238.74	200.61	236.51	200.34	236.69	200.19	236.95
1.1	25.99	28.45	25.03	27.41	20.60	22.46	13.13	13.99	8.46	8.86
1.2	10.16	10.83	9.77	10.39	7.97	8.50	4.87	5.01	3.12	3.00
1.3	5.85	6.12	5.57	5.83	4.45	4.56	2.83	2.71	1.90	1.55
1.4	3.98	3.99	3.77	3.75	3.01	2.95	2.00	1.70	1.45	0.95
1.5	2.99	2.91	2.90	2.79	2.36	2.15	1.62	1.19	1.26	0.67
1.6	2.40	2.21	2.33	2.11	1.91	1.59	1.40	0.88	1.16	0.49
1.7	2.03	1.73	1.97	1.66	1.67	1.27	1.27	0.68	1.10	0.37
1.8	1.80	1.44	1.75	1.37	1.50	1.01	1.19	0.55	1.07	0.30
1.9	1.63	1.19	1.57	1.13	1.40	0.89	1.14	0.46	1.05	0.25
2.0	1.50	1.01	1.47	0.96	1.30	0.74	1.11	0.39	1.03	0.20
					$\lambda_1 = 0.10,\ \lambda_2 = 0.10$					
$L_{\text{HEWMA}^{(2)}_{\text{AIB}}}$	1.5925		1.5979		1.5833		1.5799		1.5765	
1.0	200.34	222.30	200.15	222.82	200.34	222.72	200.11	221.85	200.00	221.71
1.1	28.92	29.87	27.88	28.77	22.88	23.27	14.68	14.53	9.51	9.15
1.2	11.26	11.21	10.78	10.66	8.79	8.69	5.50	5.28	3.50	3.17
1.3	6.55	6.33	6.23	6.04	4.98	4.76	3.12	2.85	2.07	1.67
1.4	4.41	4.23	4.26	4.06	3.40	3.19	2.19	1.85	1.55	1.06
1.5	3.29	3.05	3.15	2.93	2.56	2.24	1.73	1.28	1.31	0.73
1.6	2.64	2.35	2.54	2.24	2.10	1.72	1.47	0.94	1.19	0.52
1.7	2.23	1.87	2.14	1.77	1.80	1.39	1.32	0.75	1.12	0.41
1.8	1.92	1.52	1.88	1.47	1.60	1.12	1.23	0.60	1.08	0.32
1.9	1.73	1.27	1.68	1.21	1.47	0.95	1.17	0.49	1.06	0.27
2.0	1.58	1.07	1.54	1.03	1.36	0.80	1.13	0.42	1.04	0.22

Table 8: Run-length profile of one-sided $\text{HEWMA}^{(2)}_{\text{AIB}}$ control chart at $\text{ARL}_0 = 200$.

ρ	0.25		0.50		0.75		0.90		0.95	
					$\lambda_1 = 0.20, \lambda_2 = 0.05$					
$L_{\text{HEWMA}^{(2)}_{\text{AIB}}}$	1.5541		1.5511		1.5484		1.5419		1.541	
τ	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL
1.0	200.08	226.56	200.00	228.05	200.84	229.89	200.37	228.52	200.47	228.32
1.1	27.17	28.60	26.25	27.58	21.62	22.47	13.88	14.02	9.03	8.85
1.2	10.67	10.74	10.27	10.29	8.41	8.45	5.20	5.04	3.36	3.03
1.3	6.18	6.03	5.93	5.80	4.72	4.53	2.98	2.73	2.01	1.60
1.4	4.25	4.06	4.06	3.83	3.25	3.03	2.13	1.76	1.52	1.01
1.5	3.17	2.93	3.05	2.80	2.48	2.18	1.69	1.23	1.30	0.71
1.6	2.55	2.23	2.46	2.15	2.02	1.63	1.45	0.92	1.18	0.51
1.7	2.16	1.81	2.07	1.69	1.76	1.31	1.31	0.72	1.12	0.40
1.8	1.88	1.45	1.83	1.41	1.58	1.09	1.22	0.59	1.08	0.32
1.9	1.70	1.25	1.65	1.18	1.44	0.91	1.16	0.48	1.06	0.26
2.0	1.55	1.05	1.53	1.01	1.35	0.78	1.12	0.41	1.04	0.22
					$\lambda_1 = 0.20, \lambda_2 = 0.10$					
$L_{\text{HEWMA}^{(2)}_{\text{AIB}}}$	1.7707		1.7645		1.7545		1.7481		1.749	
1.0	200.84	215.56	200.52	214.67	200.28	214.27	200.70	214.46	200.43	214.08
1.1	31.02	31.08	29.91	29.79	24.65	24.08	15.74	14.87	10.26	9.34
1.2	12.15	11.46	11.63	10.88	9.32	8.65	5.90	5.31	3.80	3.26
1.3	7.06	6.47	6.73	6.12	5.36	4.89	3.38	2.92	2.22	1.75
1.4	4.74	4.26	4.52	4.07	3.61	3.16	2.34	1.87	1.64	1.10
1.5	3.54	3.10	3.40	2.98	2.78	2.33	1.81	1.31	1.37	0.77
1.6	2.82	2.39	2.73	2.28	2.25	1.80	1.54	1.00	1.23	0.58
1.7	2.34	1.89	2.27	1.83	1.90	1.43	1.38	0.78	1.14	0.43
1.8	2.05	1.56	1.98	1.50	1.68	1.19	1.26	0.63	1.10	0.36
1.9	1.83	1.34	1.76	1.25	1.52	0.98	1.20	0.54	1.07	0.29
2.0	1.66	1.13	1.62	1.07	1.41	0.85	1.15	0.44	1.05	0.24
					$\lambda_1 = 0.30, \lambda_2 = 0.05$					
$L_{\text{HEWMA}^{(2)}_{\text{AIB}}}$	1.6386		1.6363		1.6296		1.6229		1.622	
τ	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL
1.0	200.90	224.11	200.84	225.20	200.54	226.22	200.31	224.72	200.35	225.54
1.1	27.68	28.72	26.64	27.65	21.97	22.36	14.11	13.95	9.17	8.76
1.2	10.85	10.69	10.37	10.18	8.55	8.36	5.31	4.98	3.43	3.00
1.3	6.29	5.93	6.00	5.71	4.80	4.47	3.05	2.69	2.05	1.59
1.4	4.30	4.00	4.16	3.84	3.31	3.01	2.18	1.74	1.55	1.02
1.5	3.22	2.87	3.09	2.75	2.52	2.13	1.73	1.24	1.32	0.72
1.6	2.61	2.23	2.51	2.12	2.07	1.63	1.48	0.93	1.20	0.52
1.7	2.20	1.77	2.13	1.71	1.80	1.33	1.32	0.72	1.12	0.40
1.8	1.93	1.49	1.87	1.40	1.60	1.09	1.24	0.60	1.09	0.33
1.9	1.72	1.23	1.68	1.18	1.46	0.91	1.17	0.49	1.06	0.27
2.0	1.58	1.05	1.55	1.01	1.37	0.79	1.13	0.41	1.04	0.22
					$\lambda_1 = 0.30, \lambda_2 = 0.10$					
$L_{\text{HEWMA}^{(2)}_{\text{AIB}}}$	1.8667		1.859		1.8459		1.8398		1.842	
1.0	200.81	210.97	200.66	211.65	200.45	211.35	200.96	212.53	200.56	212.50
1.1	31.83	31.64	30.63	30.36	25.13	24.24	16.18	15.15	10.57	9.42
1.2	12.49	11.59	11.86	10.91	9.56	8.62	6.06	5.31	3.91	3.22
1.3	7.22	6.48	6.92	6.16	5.45	4.81	3.45	2.88	2.29	1.76
1.4	4.84	4.23	4.62	4.05	3.72	3.14	2.39	1.84	1.68	1.11
1.5	3.62	3.07	3.49	2.94	2.85	2.34	1.86	1.33	1.40	0.79
1.6	2.89	2.37	2.77	2.26	2.29	1.78	1.57	1.00	1.24	0.58
1.7	2.41	1.88	2.34	1.82	1.94	1.44	1.40	0.80	1.16	0.45
1.8	2.10	1.58	2.02	1.49	1.72	1.19	1.28	0.65	1.11	0.37
1.9	1.86	1.33	1.81	1.25	1.55	1.00	1.21	0.54	1.08	0.30
2.0	1.70	1.16	1.64	1.09	1.44	0.85	1.16	0.46	1.06	0.25

Table 9: Run-length profile of existing charts at $ARL_0 = 200$.

	HEWMA	AEWMA	HHW1	HHW2	EWMA-I_{AIB}	EWMA-II_{AIB}
					$\rho = 0.95$	$\rho = 0.95$
τ	$\lambda_1 = 0.05, \lambda_2 = 0.10$	$\lambda = 0.10$	$\lambda = 0.10$	$\lambda = 0.10$	$\lambda = 0.10$	$\lambda = 0.10$
1.0	201.71	200.76	199.69	200.12	200.42	200.3
1.1	25.79	26.04	34.52	32.11	9.73	10.96
1.2	10.15	10.35	14.09	12.72	3.72	4.03
1.3	5.79	5.71	8.2	7.22	2.34	2.37
1.4	3.99	3.78	5.64	4.9	1.79	1.76
1.5	2.96	2.83	4.28	3.67	1.51	1.45
1.6	2.39	2.29	3.46	2.96	1.36	1.28
1.7	2.03	1.97	2.91	2.48	1.26	1.19
1.8	1.78	1.75	2.53	2.16	1.2	1.13
1.9	1.62	1.58	2.25	1.93	1.15	1.09
2.0	1.49	1.47	2.05	1.77	1.12	1.07

Table 10: Overall performance measures of proposed charts vs existing charts.

	HEWMA	AEWMA	HHW1	HHW2	EWMA-I_{AIB}	EWMA-II_{AIB}	$HEWMA_{AIB}^{(1)}$	$HEWMA_{AIB}^{(2)}$
					$\rho = 0.95$	$\rho = 0.95$	$\rho = 0.95$	$\rho = 0.95$
	$\lambda_1 = 0.10, \lambda_2 = 0.05$	$\lambda = 0.10$	$\lambda = 0.10$	$\lambda = 0.10$	$\lambda = 0.10$	$\lambda = 0.10$	$\lambda_1 = 0.10, \lambda_2 = 0.05$	$\lambda_1 = 0.10, \lambda_2 = 0.05$
EQL	19.752	19.609	23.413	21.954	14.560	14.638	13.905	13.918
PCI	1.421	1.410	1.684	1.579	1.047	1.053	1.000	1.001
RARL	2.273	2.234	3.171	2.793	1.145	1.140	1.000	0.989

measures confirm the better performance of the proposed $HEWMA_{AIB}^{(1)}$ and $HEWMA_{AIB}^{(2)}$ control charts than the HEWMA control chart. For example, the proposed $HEWMA_{AIB}^{(1)}$ and $HEWMA_{AIB}^{(2)}$ control charts have EQL, PCI, and RARL values are 13.905,1,1 and 13.918, 1.001,0.989, respectively, whereas the HEWMA control chart has EQL, PCI, and RARL values are 19.752, 1.421,2.273 (see Table 10).

5.2. Proposed versus AEWMA Control Chart. The proposed $HEWMA_{AIB}^{(1)}$ and $HEWMA_{AIB}^{(2)}$ control charts are more efficient than the AEWMA control chart. For example, with $(\lambda, \lambda_1, \lambda_2) = 0.1$, $\rho = 0.75$, $\delta = 1.1$, and $ARL_0 = 200$, the proposed $HEWMA_{AIB}^{(1)}$ and $HEWMA_{AIB}^{(2)}$ control charts hold the ARL_1 values 21.21 and 22.88, respectively, while the ARL_1 value with the AEWMA control chart is 26.04 (see Tables 4, 7 versus 9, and Figure 1). Likewise, the $HEWMA_{AIB}^{(1)}$ and $HEWMA_{AIB}^{(2)}$ control charts have smaller EQL, PCI, and RARL values than the AEWMA control chart. For instance, the proposed $HEWMA_{AIB}^{(1)}$ and $HEWMA_{AIB}^{(2)}$ control charts provide EQL, PCI, and RARL values are 13.905, 1.000, 1.000 and 13.918, 1.001, 0.989, respectively, while the AEWMA control chart produces EQL, PCI, and RARL values are 19.609, 1.41, 2.234 (see Table 10).

5.3. Proposed versus HHW1 and HHW2 Control Charts. The proposed $HEWMA_{AIB}^{(1)}$ and $HEWMA_{AIB}^{(2)}$ control charts provide better performance against the HHW1 and HHW2 control charts. For example, at $(\lambda, \lambda_2) = 0.1$, $\lambda_1 = 0.05$, $\rho = 0.5$, and $\delta = 1.1, 1.2$, the proposed $HEWMA_{AIB}^{(1)}$ and $HEWMA_{AIB}^{(2)}$ control charts attained the ARL_1 values as (23.69, 9.05) and (25.05, 9.75), respectively, while the HHW1 and HHW2 control charts have the ARL_1 values as (34.52, 14.09) and (32.11, 12.72), respectively (see Tables 3, 7 versus 9). Furthermore, Figure 2 also shows the superiority of the proposed control charts over the HHW1 and HHW2 control charts. In terms of overall effectiveness (see Table 10), the $HEWMA_{AIB}^{(1)}$ and $HEWMA_{AIB}^{(2)}$ control charts are superior to the HHW1 and HHW2 charts. For instance, the proposed $HEWMA_{AIB}^{(1)}$ and $HEWMA_{AIB}^{(2)}$ control charts have EQL 13.905 and 13.918, whereas the HHW1 and HHW2 control charts have EQL values 23.413 and 21.954, respectively (see Table 10).

5.4. Proposed versus $EWMA - I_{AIB}$- and $EWMA - II_{AIB}$- Control Charts. The proposed $HEWMA_{AIB}^{(1)}$ and $HEWMA_{AIB}^{(2)}$ control charts attain outstanding performance against the EWMA-I_{AIB} and EWMA-II_{AIB} control charts. For example, when $(\lambda, \lambda_2) = 0.1$, $\lambda_2 = 0.05, \rho = 0.95$, and $\tau = 1.1$, the ARL_1 values of proposed $HEWMA_{AIB}^{(1)}$ and $HEWMA_{AIB}^{(2)}$ control charts are 7.57 and 8.52, whereas, the ARL_1 values of EWMA-I_{AIB} and EWMA-II_{AIB} control charts are 9.73 and 10.96, respectively (see Tables 4, 7 versus 9). Additionally, the dominance of the proposed $HEWMA_{AIB}^{(1)}$ and $HEWMA_{AIB}^{(2)}$ control charts over the EWMA-I_{AIB} and EWMA-II_{AIB} control charts can be seen in Figure 3. For overall performance, the proposed $HEWMA_{AIB}^{(1)}$ and

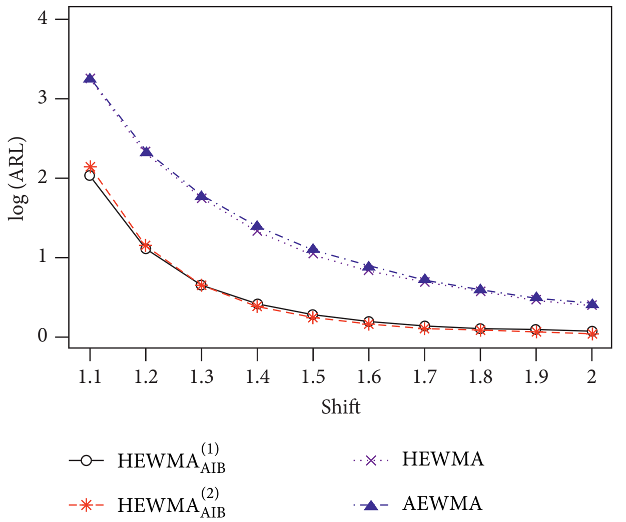

FIGURE 1: ARL comparison of the proposed HEWMA$_{AIB}^{(1)}$ and HEWMA$_{AIB}^{(2)}$ versus HEWMA, and AEWMA control charts at $(\lambda, \lambda_2) = 0.10$, $\lambda_1 = 0.05$ and $ARL_0 = 200$.

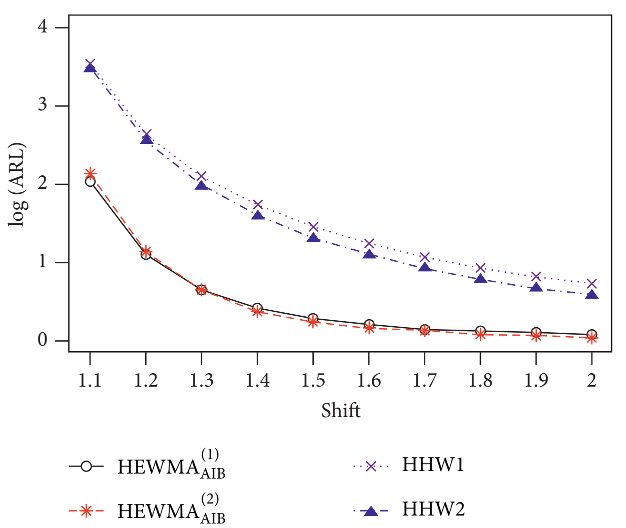

FIGURE 2: ARL comparison of the proposed HEWMA$_{AIB}^{(1)}$ and HEWMA$_{AIB}^{(2)}$ versus HHW1 and HHW2 control charts at $(\lambda, \lambda_2) = 0.10$, $\lambda_1 = 0.05$ and $ARL_0 = 200$.

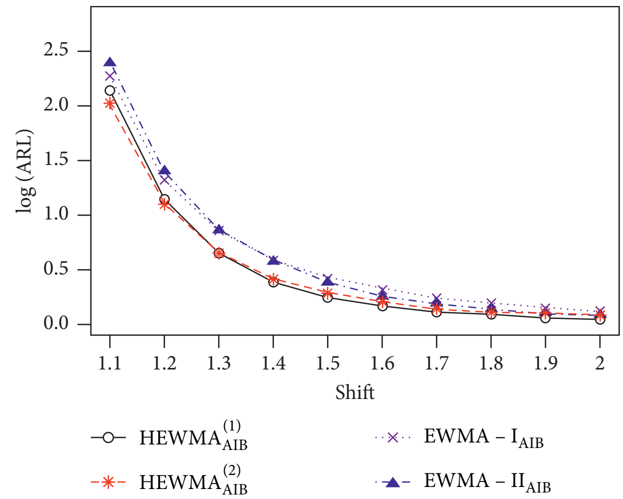

FIGURE 3: ARL comparison of the proposed HEWMA$_{AIB}^{(1)}$ and HEWMA$_{AIB}^{(2)}$ versus EWMA – I$_{AIB}$ and EWMA – II$_{AIB}$ control charts at $(\lambda, \lambda_2) = 0.10$, $\lambda_1 = 0.05, \rho = 0.95$ and $ARL_0 = 200$.

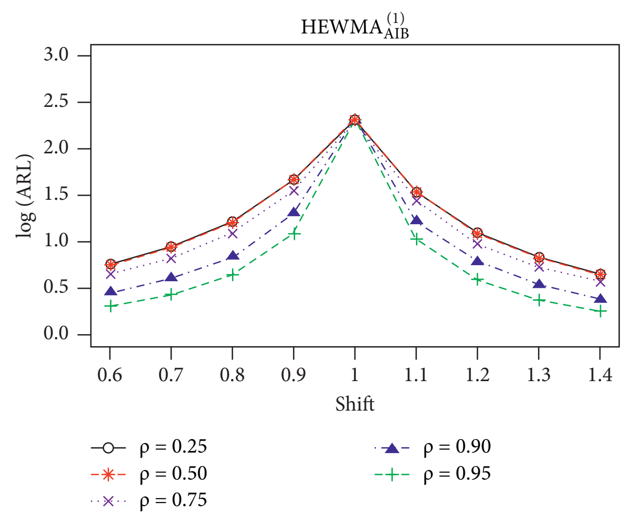

FIGURE 4: ARL behavior of the proposed HEWMA$_{AIB}^{(1)}$ control chart under different ρ for $(\lambda_1, \lambda_2) = 0.05$ at $ARL_0 = 200$.

HEWMA$_{AIB}^{(2)}$ control charts have smaller EQL, PCI, and RARL values than the EQL, PCI, and RARL values of the EWMA-I$_{AIB}$ and EWMA-II$_{AIB}$ control charts, respectively (see Table 10).

5.5. Main Outcomes of the Study. Some interesting outcomes of the proposed HEWMA$_{AIB}^{(1)}$ and HEWMA$_{AIB}^{(2)}$ control charts are listed as follows:

(i) The use of Hybrid EWMA statistic certainly boosts the detection ability of the proposed HEWMA$_{AIB}^{(1)}$ and HEWMA$_{AIB}^{(2)}$ control charts.

(ii) The performance of the proposed HEWMA$_{AIB}^{(1)}$ and HEWMA$_{AIB}^{(2)}$ control charts improve with the induction of suitable auxiliary information in the model.

(iii) The ARL_1 values of the proposed HEWMA$_{AIB}^{(1)}$ and HEWMA$_{AIB}^{(2)}$ control charts are smaller than HEWMA, AEWMA, HHW1, HHW2, EWMA-I$_{AIB}$, and EWMA-II$_{AIB}$ control charts.

(iv) The overall performance evaluation measures show the dominance of the HEWMA$_{AIB}^{(1)}$ and HEWMA$_{AIB}^{(2)}$ control charts against other control charts (see Subsections 5.1–5.4).

(v) The proposed HEWMA$_{AIB}^{(1)}$ and HEWMA$_{AIB}^{(2)}$ control charts provide the best performance for larger values of ρ (see Figures 4 and 5).

(vi) The ARL_1 performance of the proposed HEWMA$_{AIB}^{(1)}$ and HEWMA$_{AIB}^{(2)}$ control charts are increased for smaller values of λ_1 and λ_2 (see Tables 1–8).

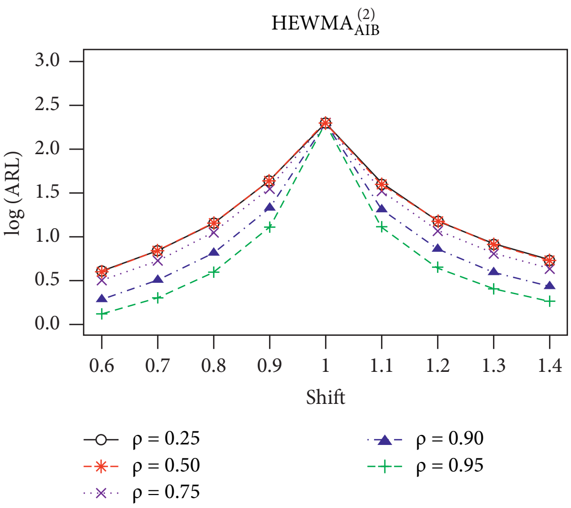

FIGURE 5: ARL behavior of the proposed HEWMA$^{(2)}_{AIB}$ control chart under different ρ for $(\lambda_1, \lambda_2) = 0.05$ at $ARL_0 = 200$.

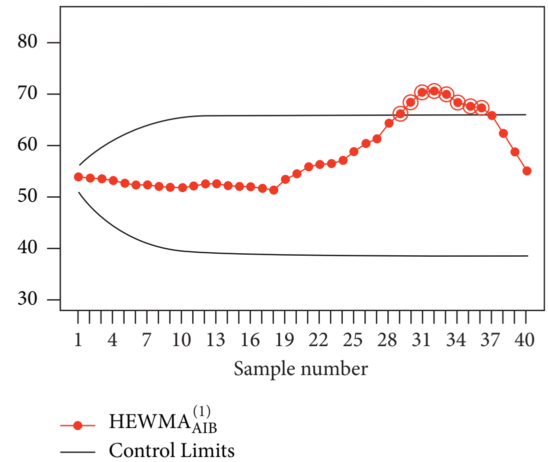

FIGURE 7: The proposed HEWMA$^{(1)}_{AIB}$ control chart for real-life data using $\rho = 0.905$, $\lambda_1 = 0.2$, $\lambda_2 = 0.2$, $L_{HEWMA^{(1)}_{AIB}} = 2.3029$ and $ARL_0 = 200$.

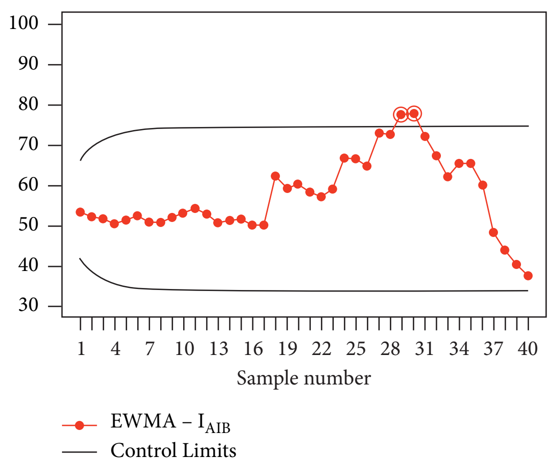

FIGURE 6: The existing EWMA-I$_{AIB}$ control chart for real-life data using $\rho = 0.905$, $\lambda = 0.2$, $L_{EWMA-I_{AIB}} = 2.785$ and $ARL_0 = 200$.

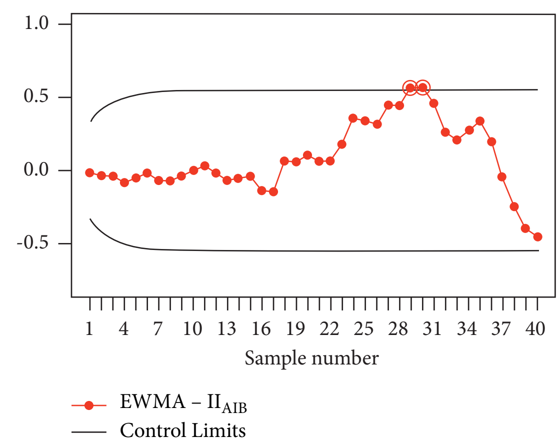

FIGURE 8: The existing EWMA-II$_{AIB}$ control chart for real-life data using $\rho = 0.905$, $\lambda = 0.2$ $L_{EWMA-II_{AIB}} = 2.677$ and $ARL_0 = 200$.

6. Real-Life Application

To demonstrate the practical implementation of the proposed HEWMA$^{(1)}_{AIB}$ and HEWMA$^{(2)}_{AIB}$ control charts, a real-life data set of glass thickness (X), and its impact on the stress strength (Y) of glass bottles is considered from Asadzadeh and Kiadaliry [38]. This data set contains 40 samples, each of size 5, of stress strength (kg/cm^2), thickness (cm). The proposed control charts are constructed under the assumption of known parameters. However, in real-life data application of the proposed control chart, the population parameters are not available. Therefore, for the practical implementation of the control charts, the estimated parameters are used for the empirical quantification of the quantities required to show the proposed control charts' implementation. The estimates of process parameters are given as: $\hat{\mu}_Y = 6.36$, $\hat{\sigma}_Y = 8.92$, $\hat{\mu}_X = 1.38$, $\hat{\sigma}_X = 0.62$, and $\hat{\rho} = 0.905$. In the data set, the first 20 samples are treated as IC, while the rest of the 20 samples are considered OOC. Following Anwar, et al. [39], the Y is multiplied by 1.3 for the OOC scenario.

The parameters of proposed HEWMA$^{(1)}_{AIB}$ and HEWMA$^{(2)}_{AIB}$ control charts are set on $\lambda_1 = 0.2$, $\lambda_2 = 0.2$, $L_{HEWMA^{(1)}_{AIB}} = 2.3029$, $L_{HEWMA^{(2)}_{AIB}} = 2.299$ and $\rho = 0.905$ with $ARL_0 = 200$. Similarly, the control chart parameters of the existing EWMA-I$_{AIB}$ and EWMA-II$_{AIB}$ control charts are $(\lambda, L_{EWMA-I_{AIB}}, L_{EWMA-II_{AIB}}) = (0.2, 2.785, 0.2, 2.677)$ at $\rho = 0.905$ and $ARL_0 = 200$.

The EWMA-I$_{AIB}$, EWMA-II$_{AIB}$, proposed HEWMA$^{(1)}_{AIB}$, and proposed HEWMA$^{(2)}_{AIB}$ control charts detect the first OOC signal at sample number 29 (see Figures 6–9). Overall, the existing EWMA-I$_{AIB}$ control chart detects 2 OOC signals while the HEWMA$^{(1)}_{AIB}$ control chart detects 8 OOC signals. In the same manner, the EWMA-II$_{AIB}$ control chart detects a total of 2 OOC signals, and the HEWMA$^{(2)}_{AIB}$ control chart detects 5 OOC signals. This indicates that the proposed HEWMA$^{(1)}_{AIB}$ and HEWMA$^{(2)}_{AIB}$ control charts are more efficient than the existing EWMA-I$_{AIB}$ and EWMA-II$_{AIB}$ control charts.

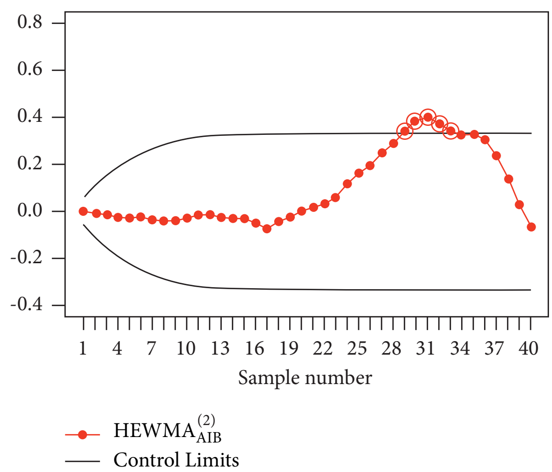

Figure 9: The proposed HEWMA$_{AIB}^{(2)}$ control chart for real-life data using $\rho = 0.905$, $\lambda_1 = 0.2$, $\lambda_2 = 0.2$, $L_{HEWMA_{AIB}^{(2)}} = 2.299$ and.$ARL_0 = 200$.

7. Concluding remarks

This study presented the two new auxiliary information-based hybrid EWMA control charts, named HEWMA$_{AIB}^{(1)}$ and HEWMA$_{AIB}^{(2)}$ control charts for process dispersion. The HEWMA$_{AIB}^{(1)}$ control chart used the auxiliary information through the regression estimator for the population variance, whereas the HEWMA$_{AIB}^{(2)}$ control chart used the auxiliary information through the difference estimator. The HEWMA$_{AIB}^{(1)}$ and HEWMA$_{AIB}^{(2)}$ control charts are constructed by combining the features of the AIB dispersion estimators with the HEWMA control chart. The proposed control charts' performance based on average run length, extra quadratic loss, relative average run length, and performance comparison index measures reveal the superiority over the competitive control charts. It is worth mentioning that the proposed HEWMA$_{AIB}^{(1)}$ and HEWMA$_{AIB}^{(2)}$ control charts performed very well to monitor small to moderate shifts in process dispersion, especially for large correlation coefficient values. Besides, a real-life application is also provided for users and practitioners to demonstrate the implementation of the proposed study from a practical perspective. This work can be extended to a non-normal process(s) and a multivariate case.

Data Availability

The data is available in the manuscript.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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