Fault-Tolerant Control of Wind Turbine System Using Linear Parameter-Varying Model

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This paper proposes a fault-tolerant control scheme for WTS with actuator faults and disturbance, based on a linear parameter-varying (LPV) model. First, a LPV model of WTS subject to disturbances and actuator fault is obtained. Using this model, unknown input observer (UIO) is developed to estimate the WTS fault and state variables. Then, an integral tracking controller for LPV systems is developed. Based on the Lyapunov and $H_{\infty}$ theory, the convergence of UIO and the stability of the closed-loop system are analyzed and ensured. The proposed fault-tolerant controller is applied to a wind turbine system with pitch and torque actuator faults, and its performances are compared with those of controller without integral action.

1. Introduction

Wind power is one of the main renewable energy sources and attracts researchers’ and governments’ attention around the world. The wind turbine systems (WTSs) are being used more and more widely. WTSs are complex nonlinear systems such that it is difficult to model and build a suitable controller [1].

Recently, linear parameter-varying (LPV) models have been used in the control design of different nonlinear systems due to the advantages of representation with highly nonlinear and uncertain models. The most widely used control frameworks for LPV models are the gain scheduled control [2] and $H_{\infty}$ control [3]. Such control approaches for WTS have been developed in [4, 5]. Also, using polytopic LPV models, robust gain scheduling control [6], switching control [7], and other linear control methods [8] have been proposed to regulate the WTS power or pitch angle. Because of the similarity of polytopic LPV and Takagi–Sugeno (TS) fuzzy models, the TS fuzzy control approach has also been applied for WTS [9]. Taking into account the advantages of the LPV models in nonlinear system control design, in this paper, we will use the LPV model to describe the dynamics of WTS.

The wind turbine system consists of a wind turbine, gearbox, generator, sensors, actuators, etc. It is inevitably subject to various sensors and actuators faults, such as rotor speed and pitch angle sensor faults and torque actuator and pitch actuator faults [10]. In order to improve WTS’s reliability and safety, fault-tolerant control (FTC) schemes have been proposed. There are passive and active fault-tolerant controllers. The passive fault-tolerant controllers are fixed in the operation period and need to be robust against a class of presumed faults [11, 12]. In turn, the active fault-tolerant controllers use algorithms to estimate and compensate actively the effect of faults or system component failures [13–15]. A number of fault-tolerant control (FTC) methods have been used for WTS: model-based methods [16, 17], data-driven methods [18, 19], etc. However, there exist difficulties in the application of these methods to WTS. In reference [20], the authors introduced a FTC approach based on a discrete LPV model using an interval observer. In [3], a full state robust $H_{\infty}$ controller was designed for WTS. In [20], the complexity of the estimation algorithm makes it hard to compute results online, while in [3], it is difficult to find a candidate Lyapunov function allowing to give theoretical guarantees.
To overcome these limitations, a new fault-tolerant control strategy for WTS is proposed in this paper. An unknown input observer (UIO) based on the polytopic LPV model is designed to estimate the state variables. In order to reduce tracking errors, an augmented system is built by adding an integrator. In addition, a residual-signal generation system is built for fault diagnosis detection. The proposed controller is applied to a model of WTS with actuator faults, and illustrative experiments are carried out. This paper’s primary contributions can be summarized as follows. (1) An unknown input observer based on the LPV model for estimating actuator fault is developed and sufficient conditions for the UIO are also given. (2) Different from the previous work in [21], a fault-tolerant control subject to actuator fault with integrator is designed and performs well.

The paper is organized as follows. In the next section, a LPV model of WTS is presented. Section 3 deals with the design and convergence analysis of UIO. In Section 4, an integral controller for LPV systems is designed and the stability of the closed-loop system is analyzed. The results of application of the proposed controller for 5 MW variable-pitch wind turbine benchmark are shown in Section 5. Conclusions are given in Section 6.

2. Problem Formulation and Preliminaries

The WTS is composed of aerodynamic subsystem, drivetrain subsystem, pitch subsystem, etc. In order to obtain an appropriate dynamical model, some components such as tower model are neglected. The output of the aerodynamic torque is considered as a nonlinear function of the wind speed $v$, rotor speed $\omega_r$, and pitch angle $\beta$. The conversion efficiency from wind energy to mechanical power is described by the power coefficient $C_p(\lambda, \beta)$. The aerodynamic torque is [22]

$$T_a = \frac{0.5 \cdot \rho \pi R^3 v^2 C_p(\lambda, \beta)}{\lambda},$$

(1)

where $v$ is the wind speed, $\beta$ is the pitch angle, $R$ is the rotor-plane radius, $\lambda = \omega_r R/v$ is the tip speed ratio, and $\rho$ is the air density. The aerodynamic torque can be linearized as

$$T_a (\varphi) = B_r (\varphi) \omega_r + k_{\beta r} (\varphi) \beta + k_{\beta r} (\varphi) v,$$

(2)

where $\varphi = (v, \omega_r, \beta)$ and $B_r (\varphi), k_{\beta r} (\varphi), k_{\beta r} (\varphi)$ are

$$B_r (\varphi) = \left. \frac{\partial T_a}{\partial \omega_r} \right|_{\varphi} = \frac{\partial C_p}{\partial \lambda} \frac{\partial \lambda}{\partial \omega_r},$$

$$k_{\beta r} (\varphi) = \left. \frac{\partial T_a}{\partial \beta} \right|_{\varphi} = \frac{T_a}{\beta} \left( 2 \frac{\partial C_q}{\partial \lambda} \frac{\partial \lambda}{\partial \beta} \right),$$

$$k_{\beta r} (\varphi) = \left. \frac{\partial T_a}{\partial \beta} \right|_{\varphi} = \frac{T_a}{\beta} \frac{C_q}{\lambda} \left( \frac{\partial \lambda}{\partial \beta} \right),$$

(3)

with $C_q(\lambda, \beta) = C_p(\lambda, \beta)/\lambda$.

Generally, the rotor turbine and the generator are connected by a gearbox and induction generator system. According to reference [22], the drive train model can be treated as a two-mass system:

$$\dot{\theta} = \frac{\omega_t - \omega_r}{ng},$$

$$I_g \dot{\omega_g} = \frac{(K_r \theta + B_r \omega_r - B_g \omega_g)}{ng - T_g},$$

$$I_r \dot{\omega_r} = T_a - K_r \theta - B_r \omega_r + B_g \omega_g,$$

(4)

where $n_g$ is the gearbox ratio, $K_r, B_r$ are the stiffness damping coefficients, respectively, $\omega_r$ is the generator speed, $I_r$ and $I_g$ are the rotor and generator inertia, respectively, and $\theta$ is the torsion angle of the drive train.

Usually, the electrical subsystem consists of generator and converter and is much faster than the mechanical subsystem. Hence, the electrical subsystem model can be simplified to the first-order equation

$$\dot{T}_g = -\frac{1}{\tau_2} \cdot T_g + \frac{1}{\tau_2} \cdot T_{gref},$$

(5)

where $T_{gref}$ is the reference generator torque signal and $\tau_2$ is the time constant of the electrical subsystem.

The pitch subsystem can be modeled as a first-order differential equation:

$$\dot{\beta} = -\frac{1}{\tau_1} \cdot \dot{\beta} + \frac{1}{\tau_1} \cdot \beta_{ref},$$

(6)

where $\tau_1$ is the time constant of the pitch subsystem and $\beta_{ref}$ is the reference of the pitch angle.

According to reference [22], the parameters $B_r (\varphi), k_{\beta r} (\varphi), k_{\beta r} (\varphi)$ can be evaluated over the entire range of wind speeds. Thus, they can be seen as functions that are only related to the wind speed, i.e., $B_r (\varphi) = B_r (v), k_{\beta r} (\varphi) = k_{\beta r} (v), k_{\beta r} (\varphi) = k_{\beta r} (v)$. Defining the state vector $\chi (t) = \left[ \theta \omega_r \omega_g \beta T_g \right]^T$, input vector $u = \left[ \beta_{ref} T_{gref} \right]^T$, and output vector $y = \left[ \beta T_g \right]^T$, a LPV model of the WTS can be given as

$$\dot{\chi}(t) = A(v)\chi(t) + Bu(t) + D(v)v(t),$$

$$y(t) = Cx(t),$$

(7)

where
\[ \dot{x}(t) = A(a)x(t) + B(a)u(t) + D(a)d(t) + F(a)f(t), \]
\[ y(t) = Cx(t). \]  
(13)

Assume that

(A1) The pair \((A_i, C)\) is observable, \(\forall i = 1, \ldots, r.\)

(A2) The matrix \(C_i\) is of full row rank and \(D_i\) is of full column rank for \(i = 1, 2, \ldots, r.\)

(A3) The disturbance \(d(t)\) and fault \(f(t)\) are functions with bounded first time derivative.

### 3. Unknown Input Observer Design for LPV Systems

#### 3.1. Unknown Input Observer for LPV System.

For system (13), an UIO can be designed as

\[ \dot{\hat{x}}(t) = \sum_{i=1}^{r} \rho_i(a) (A_i x(t) + B_i u(t) + D_i d(t) + F_i f(t)), \]
\[ y(t) = C_i x(t). \]  
(14)

where \(N_i, G_i, L_i, E_i\) are matrices with appropriate dimension and the matrices \(\Gamma_i, S_i\) will be chosen to guarantee the convergence of the fault estimation. The residual and the fault estimation error are defined as \(r(t) = y(t) - \hat{y}(t)\) and \(\tilde{f}(t) = \dot{\hat{x}}(t) - \dot{x}(t).\)

Defining the state estimation error \(e(t) = x(t) - \hat{x}(t)\) and the matrix \(M = EC + I,\) where \(I\) is a unit matrix with appropriate dimension, we obtain that

\[ e(t) = Mx(t) - \hat{x}(t). \]  
(15)

If the following conditions are satisfied:

\[ N_i = M A_i - K_i C_i, \]
\[ G_i = M B_i, \]
\[ L_i = K_i (I_1 + CE) - M A_i E_i, \]
\[ MD_i = 0, \]
\[ MF_i = T_i, \]

then the derivative of the state estimation error is

\[ \dot{e}(t) = \sum_{i=1}^{r} \rho_i(a) \left( N_i e(t) + T_i \tilde{f}(t) \right). \]  
(17)

If the fault estimation error \(\tilde{f}(t)\) tends to zero and the matrix \(N_i\) is a Hurwitz matrix, the state estimation error will converge to zero.

#### 3.2. Stability and Convergence Analysis for UIO

**Lemma 1** (see [23]). Given a scalar \(\mu\) and a symmetric positive definite matrix \(P_i,\) the following equality holds:
Theorem 1. If there exist symmetric positive definite matrices $P, \Gamma \in \mathbb{R}^{m \times m}$, such that the LMIs

\[
\begin{bmatrix}
N_i^T P + P N_i & * \\
\Gamma_i^T P N_i & \frac{\mu}{\sigma} P_1 - \frac{2}{\sigma} \Gamma_i^T P T_i
\end{bmatrix} < 0,
\tag{19}
\]

\[i = 1, \ldots, r, \text{ are satisfied,}
\]

\[
T_i^T P - S_i C = 0,
\tag{20}
\]

and the adaptation law for estimation of the fault $f(t)$ is chosen as

\[
\dot{\hat{f}}(t) = \sum_{i=1}^{r} \rho_i(\alpha) \Gamma_i S_i (\dot{\hat{x}}_i + \sigma r_i),
\tag{21}
\]

then the state estimation error and the fault estimation error converge to a small set.

Proof. Choose the candidate Lyapunov function

\[
V = e^T Pe + \frac{1}{\sigma} \Gamma^{-1} e^T \dot{\hat{f}}.
\tag{22}
\]

The time derivative of $V$ is

\[
\dot{V} = e^T \dot{P} e + e^T \dot{P} e + \frac{1}{\sigma} (\dot{\hat{f}}^T \Gamma^{-1} \dot{\hat{f}} + \dot{\hat{f}}^T \Gamma^{-1} \dot{\hat{f}}),
\]

\[
\dot{V} = \sum_{i=1}^{r} \rho_i(\alpha) \left( e^T \left( N_i^T P + P N_i \right) e - \frac{2}{\sigma} \Gamma_i S_i C N_i e \right)
\]

\[
+ \frac{2}{\sigma} \dot{\hat{f}}^T \Gamma^{-1} \dot{\hat{f}} + 2 \dot{\hat{f}}^T \left( T_i^T P - S_i C \right) e - \frac{2}{\sigma} \dot{\hat{f}}^T S_i C T_i \dot{\hat{f}}).
\tag{23}
\]

According to equation (21), we have

\[
\dot{\hat{f}}(t) = \sum_{i=1}^{r} \rho_i(\alpha) \left( \hat{f}(t) - \Gamma S_i (\dot{\hat{x}}_i + \sigma r_i) \right).
\tag{24}
\]

From (23) and (24), one obtains

\[
\dot{V} = \sum_{i=1}^{r} \rho_i(\alpha) \left( e^T \left( N_i^T P + P N_i \right) e - \frac{2}{\sigma} \Gamma_i S_i C N_i e \right)
\]

\[
+ \frac{2}{\sigma} \dot{\hat{f}}^T \Gamma^{-1} \dot{\hat{f}} + 2 \dot{\hat{f}}^T \left( T_i^T P - S_i C \right) e - \frac{2}{\sigma} \dot{\hat{f}}^T S_i C T_i \dot{\hat{f}} - \frac{2}{\sigma} \dot{\hat{f}}^T _1 (\Gamma^{-1} P)^{-1} \Gamma^{-1} f
\]

\[
\leq \frac{1}{\mu \sigma} P_1 \dot{\hat{f}}^T + \frac{\mu}{\sigma} \dot{\hat{f}}^T (\Gamma^{-1} P)^{-1} \Gamma^{-1} \dot{\hat{f}}
\]

\[
\leq \frac{1}{\mu \sigma} P_1 \dot{\hat{f}}^T + \frac{\mu}{\sigma} \lambda_{\max} \left( (\Gamma^{-1} P)^{-1} \Gamma^{-1} \right) \Gamma^{-1} f,
\tag{26}
\]

where $\epsilon_2 = \lambda_{\max} \left( (\Gamma^{-1} P)^{-1} \Gamma^{-1} \right)$ is the maximum eigenvalue of the matrix $(\Gamma^{-1} P)^{-1} \Gamma^{-1}$.

Note that if the fault signal is constant, i.e., $\dot{f}(t) = 0$, adaptive law (21) becomes an adaptive law for estimation of constant fault. In this case, a proportional integral observer is developed in [24] to estimate the fault by assuming that the fault is a step function. Furthermore, considering time-varying faults, an adaptive polytopic observer for singular delayed LPV systems is developed in [25].

Substituting equation (20) into (25) and using (26), the time derivative of $V$ becomes

\[
\dot{V} = \sum_{i=1}^{r} \rho_i(\alpha) \left( e^T \left( N_i^T P + P N_i \right) e + \frac{2}{\sigma} \Gamma^{-1} \dot{\hat{f}} \right)
\]

\[
- \frac{2}{\sigma} \dot{\hat{f}}^T T_i^T P N_i e - \frac{2}{\sigma} \dot{\hat{f}}^T T_i^T P T_i \dot{\hat{f}}
\]

\[
\leq \sum_{i=1}^{r} \rho_i(\alpha) \left( e^T \left( N_i^T P + P N_i \right) e - \frac{2}{\sigma} \Gamma_i^T P T_i \dot{\hat{f}} \right)
\]

\[
+ \frac{1}{\mu \sigma} P_1 \dot{\hat{f}} + \frac{\mu}{\sigma} \lambda_{\max} \left( (\Gamma^{-1} P)^{-1} \Gamma^{-1} \right) - \frac{2}{\sigma} \dot{\hat{f}}^T T_i^T P N_i e.
\tag{27}
\]

Setting the vector $\xi(t) \triangleq \left[ e^T (\Gamma^{-1} f(t)) \right]^T$, $V$ can be written as

\[
\dot{V} = \sum_{i=1}^{r} \rho_i(\alpha) \left( \xi^T \Phi_i \xi + \frac{\mu}{\sigma} \lambda_{\max} \left( (\Gamma^{-1} P)^{-1} \Gamma^{-1} \right) \right)
\]

\[
\dot{V} = \sum_{i=1}^{r} \rho_i(\alpha) \left( e^T \left( N_i^T P + P N_i \right) e - \frac{2}{\sigma} \Gamma_i^T P T_i \dot{\hat{f}} \right)
\]

\[
\leq \sum_{i=1}^{r} \rho_i(\alpha) \left( e^T \left( N_i^T P + P N_i \right) e - \frac{2}{\sigma} \Gamma_i^T P T_i \dot{\hat{f}} \right)
\]

\[
+ \frac{1}{\mu \sigma} P_1 \dot{\hat{f}} + \frac{\mu}{\sigma} \lambda_{\max} \left( (\Gamma^{-1} P)^{-1} \Gamma^{-1} \right) - \frac{2}{\sigma} \dot{\hat{f}}^T T_i^T P N_i e.
\tag{28}
\]

where $\epsilon_1 = \min(\lambda_{\min}(\Phi_i))$. Then, $\dot{V} < 0$, for $\sigma > \mu \lambda_{\max} \left( (\Gamma^{-1} P)^{-1} \Gamma^{-1} \right) \lambda_{\max} \left( (\Gamma^{-1} P)^{-1} \Gamma^{-1} \right)$. According to the Lyapunov stability theory [27], both the state estimation error $e(t)$ and the fault estimation error $\hat{f}$ converge to a small set.

From foregoing condition (16), one has

\[
M = EC + I,
\tag{30}
\]

\[
MD_i = 0.
\]

The development of equation (30) gives

\[
\left[ \begin{array}{c}
M & E
\end{array} \right] \left[ \begin{array}{ccc}
I & D_1 & \ldots & D_r \\
C & 0 & \ldots & 0
\end{array} \right] = \left[ \begin{array}{ccc}
I & 0 & \ldots & 0
\end{array} \right].
\tag{31}
\]

Denoting $H = \left[ \begin{array}{c} M & E \end{array} \right]$, $Y = \left[ \begin{array}{ccc}
I & D_1 & \ldots & D_r \\
C & 0 & \ldots & 0
\end{array} \right]$, and $Q = \left[ \begin{array}{ccc}
I & 0 & \ldots & 0
\end{array} \right]$, we have

\[
H = Q Y^t,
\tag{32}
\]

where $Y^t$ is a pseudo-inverse of $Y$. The matrices $M$ and $E$ can be obtained from the block matrix $H$. According to (16), we have
\[ N_i = MA_i - K_i C, \]
\[ T_i = MF_i, \]
\[ L_i = K_i (I_1 + CE) - MA_i E. \]  \hspace{1cm} (33)

**Corollary 1.** If there exist a symmetric positive definite matrix \( P \) and matrices \( \mathcal{K}_i, i = 1, \ldots, r \), such that
\[
\begin{bmatrix}
\mathcal{T}_{11} & \mathcal{T}_{12} \\
\mathcal{T}_{21} & \mathcal{T}_{22}
\end{bmatrix} < 0,
\]  \hspace{1cm} (34)
where
\[
\mathcal{T}_{11} = (MA_i)^T P + PMA_i - C^T \mathcal{K}_{i} - \mathcal{K}_{i} C, \quad \mathcal{T}_{21} = -1/\sigma T_f PMA_i - 1/\sigma T_f^2 \mathcal{K}_{i} C, \quad \mathcal{T}_{12} = \mathcal{T}_{21}^T, \quad \mathcal{T}_{22} = 1/\mu \chi P_1 - 2/\sigma T_f^2 PT_f,\]
then \( \text{UO} (14) \) exists and the estimation errors converge to zero.

**Proof.** Replacing \( \mathcal{K}_i \) by \( PK_i \) in LMI (34), Corollary 1 can be easily proved according to Theorem 1.

By solving LMI (34), the matrices \( P \) and \( \mathcal{K}_i \) can be obtained. Then, the remaining matrices are calculated in turn.

### 4. Integral Controller Design for LPV Systems

In this section, in order to track the reference \( y_r(t) \), an integrator is added to the standard state feedback controller. Then, the system input is
\[
u(t) = -K_i x(t) + K_{if} \int e_j(t) dt + B_i^T \tilde{f}(t).
\]  \hspace{1cm} (35)

Define auxiliary variable \( \zeta(t) \) such that
\[
\dot{\zeta}(t) = -\gamma_3 \cdot \zeta(t) + y_r(t) - Cx(t),
\]  \hspace{1cm} (36)
where \( \gamma_3 > 0 \).

The closed-loop system can be written as
\[
\begin{align*}
\dot{x}_{ci}(t) &= \sum_{i=1}^{r} \rho_i(\alpha) \left( (A_{ci} - B_{ci} K_{i}) x_r(t) + D_{ci} d_i(t) \right) + F_{ci} \tilde{f}(t), \\
y(t) &= \begin{bmatrix} C & 0 \end{bmatrix} x_r(t),
\end{align*}
\]  \hspace{1cm} (37)

where
\[
\begin{align*}
A_{ci} &= \begin{bmatrix} A_i & 0 \\ -C & -\gamma_3 I \end{bmatrix}, \\
B_{ci} &= \begin{bmatrix} B_i \\ 0 \end{bmatrix}, \\
D_{ci} &= \begin{bmatrix} D_i & 0 \\ 0 & I \end{bmatrix}, \\
F_{ci} &= \begin{bmatrix} F_i \\ 0 \end{bmatrix},
\end{align*}
\]
and \( K_{i} = \begin{bmatrix} -K_i & K_y \end{bmatrix}^T \).

The parameter \( \gamma_3 \) can be tuned to satisfy the controllability condition for calculating the parameters of the controller.

For \( H_\infty \) performance criterion \([27]\), the condition for convergence of tracking error to zero is
\[
\| x_r(t) \|_2 \leq \gamma_3 \| d(t) \|_2 + \gamma_4 \| \tilde{f}(t) \|_2, \\
\forall \tilde{f}(t) \neq 0, \text{ and } \neq 0.
\]  \hspace{1cm} (38)

**Theorem 2.** For system (37), if condition (38) is satisfied and there exist positive definite matrix \( P \) and matrix \( K_{ci} \) such that for \( i = 1, \ldots, r \) the following LMIs hold:
\[
\begin{bmatrix}
P_c D_{ci} & P_c F_{ci} \\
* & -\gamma_4^2 I
\end{bmatrix} < 0, 
\]  \hspace{1cm} (39)
where
\[
P_c = (A_{ci} - B_{ci} K_{i})^T P_c + P_c (A_{ci} - B_{ci} K_{i}) + I,
\]
then the closed-loop system is asymptotically stable with \( H_\infty \) performance.

**Proof.** Consider the Lyapunov function \( V_c = x_r^T P_c x_c \) and the function
\[
\psi_c = \dot{V}_c + x_r^T \tilde{f} - y_r^T \tilde{f} - \gamma_4^2 \| \tilde{d} \|_2^2 < 0.
\]  \hspace{1cm} (40)

Integrating function \( \psi_c \), one obtains
\[
\psi_c = \int \left( \dot{V}_c + x_r^T \tilde{f} - y_r^T \tilde{f} - \gamma_4^2 \| \tilde{d} \|_2^2 \right) dt,
\]  \hspace{1cm} (41)

Since \( V_c > 0 \), in order to ensure \( \psi_c < 0 \), the condition
\[
\| x_r \|_2 \leq \gamma_3 \| d(t) \|_2 + \gamma_4 \| \tilde{f}(t) \|_2
\]
should be satisfied. Note that equation (38) is a sufficient condition for ensuring the stability of closed-loop system.

The function \( \psi_c \) can be rewritten as
\[
\begin{align*}
\psi_c &= \dot{x}_r^T P_c x_r + x_r^T P_c \dot{x}_r + x_r^T \tilde{f} - y_r^T \tilde{f} - \gamma_4^2 \| \tilde{d} \|_2^2, \\
&= \sum_{i=1}^{r} \rho_i(\alpha) \left( (A_{ci} - B_{ci} K_{i})^T P_c + (P_c A_{ci} - B_{ci} K_{i}) \right) \\
&\quad + (D_{ci} d_i + F_{ci} \tilde{f})^T P_c x_r + x_r^T P_c (D_{ci} d_i + F_{ci} \tilde{f}) \\
&\quad - \gamma_4^2 \| \tilde{f} \|_2^2 - \gamma_4^2 \| \tilde{d} \|_2^2, \\
&= \sum_{i=1}^{r} \rho_i(\alpha) \begin{bmatrix} x_r^T \\ \tilde{f} \end{bmatrix} \begin{bmatrix} S_{ci} & P_c D_{ci} & P_c F_{ci} \\ * & -\gamma_4^2 I & 0 \\ * & * & -\gamma_4^2 I \end{bmatrix} \begin{bmatrix} x_r \\ \tilde{f} \end{bmatrix}
\end{align*}
\]  \hspace{1cm} (42)

Pre and post-multiplying \( \begin{bmatrix} S_{ci} & P_c D_{ci} & P_c F_{ci} \\ * & -\gamma_4^2 I & 0 \\ * & * & -\gamma_4^2 I \end{bmatrix} \) by
\[
P_a = \begin{bmatrix} P_c^{-1} & 0 & 0 \\ * & I & 0 \\ * & * & I \end{bmatrix},
\]
one obtains
\[
\begin{bmatrix}
S_{ci} & D_{ci} & F_{ci} \\
* & -\gamma_4^2 I & 0 \\
* & * & -\gamma_4^2 I
\end{bmatrix} < 0,
\]  \hspace{1cm} (43)

where
\[
S_{ci} = P_c^{-1} A_{ci} + A_{ci} P_c^{-1} - \mathcal{K}_{ci} B_{ci} - B_{ci} \mathcal{K}_{ci} + P_c^{-1} P_c^{-1}
\]
and \( \mathcal{K}_{ci} = K_{ci} P_c^{-1} \).

The matrices \( P_a \) and \( \mathcal{K}_{ci} \) can be computed by solving LMI (43). Then, the matrices \( K_c \) and \( K_y \) can be obtained by partitioning the matrix \( \mathcal{K}_{ci} = \mathcal{K}_{ci} P_c \).
5. Fault-Tolerant Control for WTS

In this section, fault-tolerant control is proposed for WTS in which actuator fault occurs. The scheme of the proposed control is given in Figure 1.

UIO is first designed using the method proposed in Section 3. If there does not exist a fault, the norm of the residual \( \|r_e(t)\|_2 \) maintains under a threshold if the matrix \( N_i \) in equation (17) is a Hurwitz matrix. The WTS fault estimate is

\[
\hat{f}(t) = \begin{cases} 
\sum_{i=1}^{r} \rho_i(\alpha) \left( \Gamma S_i e_y(t) \right) dt, & r_e(t) > \text{threshold}, \\
0, & r_e(t) \leq \text{threshold}. 
\end{cases}
\]  

(44)

If a fault occurs, \( \|r_e(t)\|_2 \) will be over the threshold. Thus, comparing the norm of the residual with the threshold, the fault can be detected. The threshold can be obtained by experiment. Note that when the system is in transient process (the reference is rapidly changed) or in initial stage, the residual should be neglected because of the initial errors of UIO start-up.

If there is no fault, the norm of the residual will be less than certain threshold and the system will operate in normal mode. The fault estimation subsystem will do not work and the fault estimate is set to zero. When the real time residual is over the threshold, a fault is detected and the system goes to the fault mode.

From equation (44), it can be seen that the fault estimate is zero in normal mode. The compensation of the actuator fault will be also zero in this mode. In fault mode, the fault is estimated and compensated. In these two modes, the control law is

\[
u(t) = -K\hat{x}(t) + K_y \int e_y(t) dt + B^T \hat{f}(t),
\]

(45)

where \( B^T \) is a pseudo-inverse of \( B \).

The main steps of the proposed FTC design method can be summarized as follows.

Step 1. Compute the matrix \( H \) from equation (30) and obtain the matrices \( E, M \).

Step 2. Solve LMI (34) and obtain the matrices \( P \) and \( K_i \) and then calculate \( K_i = P^{-1}K_i \) and \( S_i = T_i^TPC^i \).

Step 3. Calculate the matrices \( G_i \) from equation (16).

Step 4. Calculate the matrices \( N_i, L_i, T_i \) from equation (33). Thus, all UIO parameters are obtained.

Step 5. Solve LMI (43) and obtain the matrices \( K_c, P_c \) and then calculate the matrices \( K, K_y \) from

\[
K_c = K_i P_c, \\
K_y = [K_x, K_y]^T.
\]

6. Simulation Results for FTC of WTS

This section presents the results obtained by applying the proposed FTC to a LPV model of the variable-pitch WTS described in [28].

Note that the time-varying parameter in the WTS model of type (9) is the wind speed \( v \). Assuming that \( v \in [v_{\text{min}}, v_{\text{max}}] \), the weights of LPV model can be calculated as

![ FTC scheme of wind turbine system.](image-url)
\begin{align*}
\rho_1(v) &= \frac{v_{\text{max}} - v}{v_{\text{max}} - v_{\text{min}}} \\
\rho_2(v) &= \frac{v - v_{\text{min}}}{v_{\text{max}} - v_{\text{min}}} \quad \text{(46)}
\end{align*}

In variable-pitch WTS, the pitch subsystem will start working above the rated wind speed. If the wind speed exceeds the cut-out wind speed, the pitch subsystem will be forced to stop working in order to protect itself. In this paper, the interval of wind speed is set as \([6,25]\).

Using the WTS parameters values given in \([28]\), one obtains a WTS polytopic model of type (13) with

\[
A_1 = \begin{bmatrix} 0 & 1.0 & -0.0103 & 0 & 0 \\ -22.39 & 0.00297 & 0.00165 & 0.0041 & 0 \\ 16744 & 119.8 & -7.272 & 0 & -0.00187 \\ 0 & 0 & 0 & -10.0 & 0 \\ 0 & 0 & 0 & 0 & -10.0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 1.0 & -0.0103 & 0 & 0 \\ -22.39 & -0.7017 & 0.00165 & -0.029 & 0 \\ 16744 & 119.8 & -7.272 & 0 & -0.00187 \\ 0 & 0 & 0 & -10.0 & 0 \\ 0 & 0 & 0 & 0 & -10.0 \end{bmatrix}
\]

\[
B = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad D_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad D_2 = \begin{bmatrix} -0.0100 \\ 3.1185 \\ -0.0847 \\ 624.8144 \\ 25000 \end{bmatrix}, \quad N_1 = \begin{bmatrix} 0 & 1 & -2.086 & 0.0007 & 0 \\ -22.34 & 0.0030 & 0.1243 & 0.0040 & 0 \\ 8372 & 59.92 & -179.0 & 0.0565 & 0.0009 \\ 0 & 0 & -0.0001 & -5 & 0 \\ 0 & 0 & -0.0001 & 0 & -10.0 \end{bmatrix}, \quad N_2 = \begin{bmatrix} 0 & 1 & -2.086 & 0.0007 & 0 \\ -22.34 & -0.7004 & 0.1243 & -0.0288 & 0 \\ 8372 & 59.92 & -179.0 & 0.0565 & 0.0009 \\ 0 & 0 & -0.0001 & -5 & 0 \\ 0 & 0 & -0.0001 & 0 & -10.0 \end{bmatrix}
\]

The corresponding UIO parameters can be calculated by using YALMIP toolbox. One obtains

\[
G_1 = G_2 = T_1 = T_2, \quad G_1 = T_1 = T_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad S_1 = S_2 = \begin{bmatrix} 0.9972 & 0.0017 \\ -0.0000 & 0.0000 \end{bmatrix}
\]
Using Theorem 1 and solving LMI (43), the following controller gain matrix is obtained:

\[
K_c = \begin{bmatrix}
0.0000 & 0.0000 & 0.0000 & 0.0500 & 0.0000 & 0.0000 & 0.0000 \\
0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\
0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\
0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\
\end{bmatrix}
\]  

(50)

The wind speed used in the simulations is shown in Figure 2. The initial states are set as \(x(0) = [0, 1, 100, 0, 0]\) and \(\bar{x}(0) = [0, 0, 0, 0, 0, 0]\). According to reference [29], the actuator faults can be caused by both the pitch and converter systems and result in either an offset or changed dynamics of pitch angle and torque. In this paper, the following pitch actuator fault signal \(f_1(t)\) and torque actuator fault signal \(f_2(t)\) are considered:

\[
f_1(t) = \begin{cases} 
3 + \sin(t), & 70 \leq t \leq 90, \\
0, & \text{otherwise}, 
\end{cases} 
\]

(51)

\[
f_2(t) = \begin{cases} 
0.2u_2(t), & 40 \leq t \leq 50, \\
0, & \text{otherwise}. 
\end{cases} 
\]

(52)

The obtained simulation results are shown in Figures 3–7. The simulated and estimated signals of pitch and torque actuator faults are displayed in Figure 3. The simulated and estimated values of the system states are
shown in Figure 4. The corresponding estimation errors are illustrated in Figure 5 where the actuator fault effect can be seen.

Figures 6 and 7 display the WTS pitch angle and generator speed values obtained applying the proposed FTC method (denoted as “CF”), control method without fault compensation (denoted as “NF”), and state feedback control without integrator (denoted as “SF”). The “NF” controller is obtained by removing the fault compensation in the “CF” controller. The “SF” controller is designed using the method presented in [21]. For the “SF” controller, the fault and state observers are the same as for the “CF” controller.

These figures show that the proposed fault-tolerant controller can ensure better performances in fault and fault-free situations than controller without fault compensation and state feedback controller without integrator.

7. Conclusions

In this paper, a FTC scheme with actuator fault diagnosis based on UIO for WTS is proposed. Considering WTS disturbance and pitch actuator and torque actuator faults, an UIO is first designed to estimate the system fault and state variables. Then, an integral controller for WTS is designed and the stability of the closed-loop system is analyzed. The proposed FTC approach is tested for a variable-pitch wind turbine benchmark. The obtained simulation results demonstrate the high efficiency of the proposed FTC with actuator fault diagnosis.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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References


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Figure 6: Pitch angle (CF: control with fault compensation, NF: control without fault compensation, and SF: control without integrator).

Figure 7: Generator speed (CF: control with fault compensation, NF: control without fault compensation, and SF: control without integrator).


