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Research Article

Improved Estimators of Population Mean under Nonresponse in Successive Sampling

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It is well accepted that suitable additional information improves the efficiency of an estimator but at the same time, it faced the situation of nonresponse. The two-occasion successive sampling is useful to handle the absence of a full response from respondents. In this present study, we have developed exponential estimators for population mean using a subsampling nonrespondent procedure. To show the efficacy of the recommended estimators, several properties are derived, and respective optimum replacement strategies are inferred. To evaluate the performance of the recommended estimators empirically, a computational study is carried out as well. It was found that the recommended estimators outperform the existing ones under the nonresponse in successive sampling.

1. Introduction

Incomplete information is a well-known problem in sample surveys, especially in socio-economic surveys of households, in which individual data are collected. The reasons for missing information may be migration, refusal to respond, not being available at the time of surveys performed, etc.

Jessen [1] initially encountered the problem and suggested the method of estimation under the successive sampling with partial replacement of units utilizing the complete information at the previous occasion. Furthermore, Patterson [2]; Rao and Graham [3]; Feng and Zou [4] studied the properties of different estimators under successive sampling. Biradar and Singh [5] and Singh and Vishwakarma [6]; Singh and Pal [7]; Sanahulla et al. [8]; Javaid et al. [9] and Pal et al. [10] used additional information for estimation under successive sampling.

The concept used in this paper is nonresponse. First time during the data collection, Hansen and Hurwitz [11] realize the problem of nonresponse at the estimation stage. He took this problem forward and realizes that while taking the subsamples from the nonrespondents group effect of nonresponse can be reduced. Furthermore, he developed the estimators utilizing the information from response and nonresponse groups together which was well accepted in the sampling theory.

Later on, several researchers including Chaudhary et al. [12]; Singh and Priyanka [13], and Pal and Singh [14] combined the concept of successive sampling and nonresponse and used it for estimation of population mean on current occasion on two-occasion successive sampling. Under different practical situations using the auxiliary variable, the aforesaid authors suggested some estimators when nonresponse was observed on the current occasion in two-occasion successive sampling.

The remainder of this paper is organized as follows: in section 2, terminology and notations are presented further the resent estimators and proposed estimators along with their properties are in sections 3 and 4, respectively. Section 5 provides the computational study, and concluding remarks are offered in section 6.

2. Terminology and Notations

Let $\Omega = (\Omega_1, \Omega_2, \ldots, \Omega_N)$ be a finite population of N units, which has been sampled over two occasions. The character under study is denoted by $x\,(y)$ on the first (second) occasion. It is assumed that information on an ancillary variable z (with unknown population mean), which is positively correlated with the study variable, is readily available and almost stable over both the occasions. A simple random sample (without replacement) s_n of n units is drawn on the first occasion. A random subsample s_m of $m = n\lambda$ units is retained (matched) for its use on the second occasion. We assume that there is nonresponse at the current occasion, so that the population can be divided into two classes, those who will respond at the first attempt and those who will not respond. Let the sizes of these two classes be N_1 and N_2, respectively. At the current (second) occasion, a simple random sample (without replacement) s_v of $v = (n - m) = n\mu$ units is drawn afresh from the entire population so that the sample size on the current (second) occasion is also n. λ and μ, $(\lambda + \mu = 1)$ are the fractions of matched and fresh samples, respectively, at the current (second) occasion. We assume that in the unmatched portion of the sample on the current (second) occasion, v_1 units respond and v_2 units do not respond. Let $s_{v_1} \cap s_v$ and $s_{v2} \cap s_v$. Let v_{2h} denote the size of the subsample s_{v2h} (of s_{v2}) drawn from the nonresponding units in the unmatched (fresh) portion s_{v2} of the sample (s_v) on the current (second) occasion (i.e., from the v_2 nonrespondents, on SRSWOR of v_{2h} units is selected with the inverse sampling rates f_2, where $v_{2h} = (v_2/f_2)$, $f_2 > 1$.

The notations used in the research paper are shown in the Table 1.

3. Recent Developments of Estimators

For estimating the current population mean $\overline{Y}$, Singh et al. [15, 16] have given the estimators of set s_v as

$$T_{1v} = \overline{y}_v^* \exp\left[\frac{(\overline{Z} - \overline{z}_v^*)}{(\overline{Z} + \overline{z}_v)}\right], \qquad T_{2v} = \overline{y}_v^* \exp\left[\frac{(\overline{Z} - \overline{z}_v)}{(\overline{Z} + \overline{z}_v^*)}\right]. \tag{1}$$

Singh et al. [15] also gave the estimators of set s_m for estimating the current population mean $\overline{Y}$ as

$$T_m = \overline{y}_m\left(\frac{\overline{Z}}{\overline{z}_m}\right)\exp\left[\frac{(\overline{x}_n - \overline{x}_m)}{(\overline{x}_n + \overline{x}_m)}\right]. \tag{2}$$

Singh et al. [15] suggested the following estimators of population mean $\overline{Y}$ at the current (second) occasion by combining the estimators of sets s_v and s_m as

$$T_i = \phi_i T_{iu} + (1 - \phi_i) T_{im};\ (i = 1, 2), \tag{3}$$

where $\phi_i\,(0 \le \phi_i \le 1)$; $(i = 1, 2)$ are unknown constants (scalars) to be determined under certain criteria. For detailed properties of the estimators $T_i\,(i = 1, 2)$, readers are referred to Singh et al. [15].

4. New Development of Estimators

4.1. Estimators Based on the Unmatched Portion of the Ample s_u. Looking the formulation of the estimators T_{1v} given by (1) due to Singh et al. [15], it is clear that the estimator T_{1v} is defined in the situation, in which information on the auxiliary variable z is obtained for all the sample units v (drawn afresh from the entire population at the second occasion), and the population mean $\overline{Z}$ of the auxiliary variable z is known, but some sample units fail to supply information on the study variable y. We note that, when suggesting the estimators for the population mean $\overline{Y}$ on the current (second) occasion, Singh et al. [15] used only information on the sample mean $\overline{z}_v$ of the auxiliary variable z. However, one can also obtain the unbiased estimator $\overline{z}_v^*$ of the population mean $\overline{Z}$ (without any extra effort) while in the process of obtaining the unbiased estimator $\overline{y}_v^*$ of the population mean $\overline{Y}$. Thus, in this situation, where information on the auxiliary variable z is obtained for all the sample units v, we have two unbiased estimators $\overline{z}_v^*$ and $\overline{z}_v$, of the population mean $\overline{Z}$ of the auxiliary variable z (see, Singh and Kumar [17]). With this background, we suggest the following estimators (using $\overline{z}_v^*$ and $\overline{z}_v$ together) for the population mean $\overline{Y}$ on the current (second) occasion based on unmatched portion of the sample S_v of the size v.

$$P_{1v} = \overline{y}_v^* \exp\left(\frac{\overline{Z} - \overline{z}_v}{\overline{Z} + \overline{z}_v}\right)\exp\left(\frac{\overline{Z} - \overline{z}_v^*}{\overline{Z} + \overline{z}_v^*}\right),$$
$$P_{2v} = \overline{y}_v^* \exp\left(\frac{2\widehat{\beta}_{yz}}{\widehat{R}_z}\frac{(\overline{Z} - \overline{z}_v)}{(\overline{Z} + \overline{z}_v)}\right)\exp\left(\frac{4\widehat{\beta}_{yz(2)}}{\widehat{R}_z}\frac{\overline{Z}(\overline{z}_v - \overline{z}_v^*)}{(\overline{Z} + \overline{z}_v)(\overline{Z} + \overline{z}_v^*)}\right), \tag{4}$$

where $(\widehat{\beta}_{yz}^* = s_{yz}^*/s_z^{*\,2}, \widehat{\beta}_{yz(2)}^* = s_{yz(2)}^*/s_{z(2)}^{*2})$ are, respectively, the estimates of $(\beta_{yz(2)}^*, \beta_{yz(2)}^*)$,

$$s_{yz}^* = \left[\frac{1}{(v-1)}\right]\left(\sum_{s_{v1}} y_j z_j + f\sum_{s_{v2h}} y_j z_j - v\overline{y}_v^*\overline{z}_v^*\right),$$
$$s_z^{*\,2} = \left[\frac{1}{(v-1)}\right]\left(\sum_{s_{v1}} z_j^2 + f\sum_{s_{v2h}} z_j^2 - v\overline{z}_v^{*2}\right),$$
$$s_{yz(2)}^* = \left[\frac{1}{(v_{2h}-1)}\right]\left(\sum_{s_{v2h}}(y_j - \overline{y}_{v2h})(z_j - \overline{z}_{v2h})\right),$$
$$s_{z(2)}^{*2} = \left[\frac{1}{(v_{2h}-1)}\right]\left(\sum_{s_{v2h}}(z_j - \overline{z}_{v2h})^2\right), \tag{5}$$
$$\widehat{R}_z = \left(\frac{\overline{y}_v^*}{\overline{Z}}\right),$$
$$\overline{y}_{v2h} = \left[\frac{1}{(v_{2h})}\right]\left(\sum_{s_{v2h}} y_j\right),$$
$$\overline{z}_{v2h} = \left[\frac{1}{(v_{2h})}\right]\left(\sum_{s_{v2h}} z_j\right).$$

For the simplicity in calculation, Reddy (1974, [18]) and Ruiz Espejo [19] initially assumed that the coefficient of variation of study and auxiliary variables are equal. It is further assumed that under incomplete information, coefficient of variation of class is equal to coefficient of variation of population. Following this, we state that

Theorem 1. *The MSE of the estimator P_{1v} to the fda is given by*

$$\mathrm{MSE}(P_{1v}) = S_y^2\left[2\lambda_v\left(1-\rho_{yz}\right)+\theta_v\left(\left(\frac{5}{4}\right)-\rho_{yz(2)}\right)\right]. \tag{6}$$

Theorem 2. *The MSE of the estimator P_{2v} to the fda is given by*

$$\mathrm{MSE}(P_{2v}) = S_y^2\left[\lambda_v\left(1-\rho_{yz}^2\right)+\theta_v\left(1-\rho_{yz(2)}^2\right)\right]. \tag{7}$$

Remark 1. The *MSE* of the estimators P_{1v} and P_{2v} is given while considering the assumption made by Singh et al. [15, 16] that the population correlation coefficient is equal to the nonresponse class correlation coefficient $(\rho_{yz(2)} = \rho_{yz})$:

$$\begin{aligned}\mathrm{MSE}(P_{1v}) &= S_y^2\left[2\lambda_v\left(1-\rho_{yz}\right)+\theta_v\left(\left(\frac{5}{4}\right)-\rho_{yz}\right)\right],\\ \mathrm{MSE}(P_{2v}) &= S_y^2\left[\left(\lambda_v+\theta_v\right)\left(1-\rho_{yz}^2\right)\right].\end{aligned} \tag{8}$$

4.2. Estimators Based on the Matched Portion of the Sample s_m. It is assumed that there is no nonresponse on the first occasion as well on the matched portion of the sample. Under the above assumption, we consider the following estimators based on the matched sample s_m of size m:

$$P_{1m} = \left[\overline{y}_m + b_{yz(m)}\left(\overline{Z}-\overline{z}_m\right)\right]\exp\left[\frac{\left(\overline{x}_n-\overline{x}_m\right)}{\left(\overline{x}_n+\overline{x}_m\right)}\right], \tag{9}$$

where $b_{yz(m)}$ are the estimates of the population regression coefficients β_{yz}, respectively, based on the sample of size m.

To the *fda*, the *MSE* of the proposed estimators P_{1m} is given by

$$\mathrm{MSE}(P_{1m}) = \overline{Y}^2\left[\left(\frac{1}{m}-\frac{1}{N}\right)C_y^2\left(1-\rho_{yz}^2\right)+\left(\frac{1}{m}-\frac{1}{n}\right)\left(\frac{1}{4}C_x^2-\rho_{yx}C_yC_x+\rho_{yz}\rho_{xz}C_yC_x\right)\right]. \tag{10}$$

Under the assumption $C_x = C_z = C_y$, the MSEs (10) reduce to

$$\mathrm{MSE}(P_{1m}) = S_y^2\left[\left(\frac{1}{m}-\frac{1}{N}\right)\left(1-\rho_{yz}^2\right)+\left(\frac{1}{m}-\frac{1}{n}\right)\left(\frac{1}{4}-\rho_{yx}+\rho_{yz}\rho_{xz}\right)\right]. \tag{11}$$

4.3. Covariance between s_u and s_m. The covariance between (P_{1u}, P_{1m}) is given by

$$C_{11} = \mathrm{Cov}\left(P_{1u}, P_{1m}\right) = -\frac{S_y^2}{N}\left(1-\rho_{yz}^2\right). \tag{12}$$

It is to be noted that the expression (11) has been derived under the assumptions $C_x = C_z = C_y$, $\rho_{yz} = \rho_{yz(2)}$, and $C_y = C_{y(2)}$.

$$C_{21} = \mathrm{Cov}\left(P_{2u}, P_{1m}\right) = -\frac{S_y^2}{N}\left(1-\rho_{yz}^2\right). \tag{13}$$

4.4. Linear Combination of Estimators. We have suggested the following estimators for estimating the population mean $\overline{Y}$ at the current occasion by combining the different estimators at the matched and unmatched portions of the samples, respectively:

$$\begin{aligned}P_1 &= \phi_1 P_{1u} + \left(1-\phi_1\right)P_{1m},\\ P_2 &= \phi_2 P_{2u} + \left(1-\phi_2\right)P_{1m},\end{aligned} \tag{14}$$

where $\phi_i\,(1 \le \phi_i \le 1)$ is suitably chosen constant to be determined under certain assumptions.

4.5. MSEs of the Estimators P_1 and P_2. The *MSE*s of the estimators are derived up to *fda*, and under the assumption, $C_x = C_z = C_y$, $\rho_{yz} = \rho_{yz(2)}$, and $C_y = C_{y(2)}$.

Theorem 3. *MSEs of $P_i\,(i = 1, 2)$ to the fda are obtained as*

$$\begin{aligned}\mathrm{MSE}(P_1) = {}&\phi_1^2\mathrm{MSE}(P_{1u}) + \left(1-\phi_1\right)^2\mathrm{MSE}(P_{1m})\\ &+ 2\phi_1\left(1-\phi_1\right)C_{11},\end{aligned} \tag{15}$$

$$\begin{aligned}\mathrm{MSE}(P_2) = {}&\phi_2^2\mathrm{MSE}(P_{2u}) + \left(1-\phi_2\right)^2\mathrm{MSE}(P_{1m})\\ &+ 2\phi_2\left(1-\phi_2\right)C_{21}.\end{aligned} \tag{16}$$

4.6. MMSEs of the Estimators $P_i\,(i = 1, 2)$. Since the *MSE* of the estimators $P_i\,(i = 1,2)$ in (15) and (16) is functions of unknown constants ϕ_i's $(i = 1, 2)$, therefore, the *MSE*s are minimized with respect to $\phi_i\,(i = 1, 2)$, respectively, for

$$\begin{aligned}\phi_{1(\mathrm{opt})} &= \frac{\mathrm{MSE}(P_{1v}) - C_{11}}{\left(\mathrm{MSE}(P_{1v}) + \mathrm{MSE}(P_{1m}) - 2C_{11}\right)}\\ &= \frac{\left(1-\mu_1\right)\left[\hbar_{0(1)} - \mu_1 f\left(\kappa_5-\kappa_0\right)\right]}{\left[\hbar_{0(1)} + \mu_1\hbar_{1(1)} + \mu_1^2\hbar_{2(1)}\right]},\end{aligned} \tag{17}$$

$$\begin{aligned}\phi_{2(\mathrm{opt})} &= \frac{\mathrm{MSE}(P_{2v}) - C_{21}}{\left(\mathrm{MSE}(P_{2v}) + \mathrm{MSE}(P_{1m}) - 2C_{21}\right)},\\ &= \frac{\left(1-\mu_2\right)\hbar_{0(2)}}{\left[\hbar_{0(2)} + \mu_2\hbar_{1(2)} + \mu_1^2\hbar_{2(2)}\right]}.\end{aligned} \tag{18}$$

Table 1: Notations used and their meaning.

Notations	Meaning
$\overline{X}, \overline{Y}, \overline{Z}$	Population mean of the study variable y and two auxiliary variables x, z
$\overline{y}_m, \overline{y}_v, \overline{y}_{v_1}, \overline{y}_{v_{2h}}, \overline{y}_n, \overline{y}_{n_1}, \overline{y}_{n_{2h}}$	Sample means of the study variable under different situations
$\overline{x}_n, \overline{x}_m$	Sample mean of auxiliary variable x
$\overline{z}_m, \overline{z}_v, \overline{z}_{v_1}, \overline{z}_{v_{2h}}$	Sample mean of auxiliary variable z
$\rho_{yx}, \rho_{xz}, \rho_{yz}$	Population correlation coefficients
ρ_{2yz}	Population correlation coefficient under nonresponse
S_x^2, S_y^2, S_z^2	Population variances of the study and auxiliary variables
S_{2y}^2, S_{2z}^2	Population variances of y and z under nonresponse
$\Theta = (N_2/N)$	Proportion of nonresponding population to the total population at second occasion
$\overline{y}_v^* = (v_1\overline{y}_{v1} + u_2\overline{y}_{v2h})/v$	Hansen and Hurwitz estimator for study variable y
$\overline{z}_v^* = (v_1\overline{y}_{v1} + u_2\overline{y}_{v2h})/v$	Hansen and Hurwitz estimator for study variable x for the unmatched sample on the current occasion
$f_2 (= v_2/v_{2h})$	Correction factor under nonresponse

and thus, the resulting *MMSE*s of the estimators $P_i\,(i = 1, 2)$ are given as

$$\mathrm{MMSE}(P_1) = \frac{\mathrm{MSE}(P_{1v})\mathrm{MSE}(P_{1m}) - C_{11}^2}{(\mathrm{MSE}(P_{1v}) + \mathrm{MSE}(P_{1m}) - 2C_{11})} = \frac{\left[\bar{\lambda}_{0(1)} + \mu_2\bar{\lambda}_{1(1)} + \mu_2^2\bar{\lambda}_{2(1)}\right]}{\left[\hbar_{0(1)} + \mu_2\hbar_{1(1)} + \mu_2^2\hbar_{2(1)}\right]}\frac{S_y^2}{n}, \tag{19}$$

$$\mathrm{MMSE}(P_2) = \frac{\mathrm{MSE}(P_{2v})\mathrm{MSE}(P_{1m}) - C_{21}^2}{(\mathrm{MSE}(P_{2v}) + \mathrm{MSE}(t_{1m}) - 2C_{21})} = \frac{\left[\bar{\lambda}_{0(2)} + \mu_2\bar{\lambda}_{1(2)} + \mu_2^2\bar{\lambda}_{2(2)}\right]}{\left[\hbar_{0(2)} + \mu_2\hbar_{1(2)} + \mu_2^2\hbar_{2(2)}\right]}\frac{S_y^2}{n}. \tag{20}$$

where $\kappa_0 = 2(1-\rho_{yz})$, $\kappa_1 = \Theta(f_2 - 1)((5/4) - \rho_{yz})$, $\kappa_2 = ((1/4) - \rho_{yx} + \rho_{xz})$, $\kappa_3 = ((5/4) - \rho_{yz})$, $\kappa_4 = (1 - \rho_{yz}^2)$, $\kappa_5 = ((1/4) - \rho_{yz} + \rho_{yz}\rho_{xz})$, $\kappa_6 = [1 + \Theta(f_2 - 1)]$, $\hbar_{0(1)} = (\kappa_0 + \kappa_1)$, $\hbar_{1(1)} = [(1-f)(\kappa_4 - \kappa_0) - \kappa_1]$, $\hbar_{2(1)} = [\kappa_5 - f\kappa_4 + f\kappa_0]$, $\hbar_{0(2)} = \kappa_4\kappa_6$, $\hbar_{1(2)} = \kappa_4(1-\kappa_6)$, $\hbar_{2(2)} = \kappa_5$, $\bar{\lambda}_{0(1)} = (1-f)(\kappa_0 + \kappa_1)\kappa_4$, $\bar{\lambda}_{1(1)} = [(\kappa_0 + \kappa_1)\kappa_5 + f\kappa_1\kappa_4 - f^2\kappa_4(\kappa_4 - \kappa_0)]$, $\bar{\lambda}_{2(1)} = [f^2\kappa_4(\kappa_4 - \kappa_0) - f\kappa_0\kappa_5]$, $\bar{\lambda}_{0(2)} = (1-f)\kappa_4^2\kappa_6$, $\bar{\lambda}_{1(2)} = [\kappa_5\kappa_6 + f\kappa_4(\kappa_6 - 1)]\kappa_4$, $\bar{\lambda}_{2(2)} = -f\kappa_4\kappa_5$

4.7. Optimum Replacement Policy. It is observed from (18) and (19) that the MMSEs of the estimators $P_i\,(i = 1, 2)$ are the functions of $\mu_i\,(i = 1, 2)$ (functions of sample to be drawn afresh at the second occasion); therefore, the optimum values of μ_i are obtained to estimate the population mean $\overline{Y}$ with minimum precision and lowest cost. To obtain the optimum values of $\mu_i\,(i = 1, 2)$, we minimize the *MMSE*s of the proposed estimators $P_i\,(i = 1, 2)$ given in (18) to (19), respectively, with respect to $\mu_i\,(i = 1, 2)$ which result in quadratic equations in $\mu_i\ (i = 1, 2)$, and the respective salutations of $\mu_i\,(i = 1, 2)$ say $\widehat{\mu}_i\,(i = 1, 2)$ are given as follows:

$$\psi_{12(1)}\mu_1^2 + 2\psi_{02(1)}\mu_1 + \psi_{01(1)} = 0, \tag{21}$$

$$\widehat{\mu}_1 = \frac{-\psi_{02(1)} \pm \sqrt{\left(\psi_{12(1)}^2 - \psi_{01(1)}\psi_{12(1)}\right)}}{D_{12(1)}}, \tag{22}$$

$$\psi_{12(2)}\mu_2^2 + 2\psi_{02(2)}\mu_2 + \psi_{01(2)} = 0, \tag{23}$$

$$\widehat{\mu}_2 = \frac{-\psi_{02(2)} \pm \sqrt{\left(\psi_{12(2)}^2 - \psi_{01(2)}\psi_{12(2)}\right)}}{\psi_{12(2)}}, \tag{24}$$

where

$$\psi_{01(1)} = \left(\hbar_{0(1)}\bar{\lambda}_{1(1)} - \bar{\lambda}_{0(1)}\hbar_{1(1)}\right), \psi_{02(1)} = \left(\hbar_{0(1)}\bar{\lambda}_{2(1)} - \bar{\lambda}_{0(1)}\hbar_{2(1)}\right),$$
$$\psi_{12(1)} = \left(\hbar_{1(1)}\bar{\lambda}_{2(1)} - \bar{\lambda}_{1(1)}\hbar_{2(1)}\right), \psi_{01(2)} = \left(\hbar_{0(2)}\bar{\lambda}_{1(2)} - \bar{\lambda}_{0(2)}\hbar_{1(2)}\right),$$
$$\psi_{02(2)} = \left(\hbar_{0(2)}\bar{\lambda}_{2(2)} - \bar{\lambda}_{0(2)}\hbar_{2(2)}\right), \psi_{12(2)} = \left(\hbar_{1(2)}\bar{\lambda}_{2(2)} - \bar{\lambda}_{1(2)}\hbar_{2(2)}\right). \tag{25}$$

From (21) and (22), it is observed that real values of $\mu_i\,(i = 1, 2)$ exist, iff the quantities under square roots are greater than or equals to zero, and for any combinations ρ_{yx}, ρ_{yz}, and ρ_{xz}, which satisfy the conditions of real situations, two real values of $\mu_i\,(i = 1, 2)$ are possible, and hence, while choosing the values of $\widehat{\mu}_i\,(i = 1, 2)$, it should be remembered that $0 \le \mu_i \le 1\,(i = 1, 2)$. All other values of $\mu_i\,(i = 1, 2)$ are inadmissible. Substituting the admissible values of $\widehat{\mu}_i$ (say) $\widehat{\mu}_i^{(0)}\,(i = 1, 2)$ from (21) and (22) in (17) and (18), respectively, we have the optimum values of mean squared errors of $P_i\,(i = 1, 2)$ which are shown as follows:

$$\mathrm{MMSE}\left(P_1^{(0)}\right)_{\mathrm{opt}} = \frac{\left[\bar{\lambda}_{0(1)} + \mu_1^{(0)}\bar{\lambda}_{1(1)} + \mu_1^{(0)2}\bar{\lambda}_{2(1)}\right]}{\left[\hbar_{0(1)} + \mu_1^{(0)}\hbar_{1(1)} + \mu_1^{(0)2}\hbar_{2(1)}\right]}\frac{S_y^2}{n},$$
$$\mathrm{MMSE}\left(P_2^{(0)}\right)_{\mathrm{opt}} = \frac{\left[\bar{\lambda}_{0(2)} + \mu_2^{(0)}\bar{\lambda}_{1(2)} + \mu_2^{(0)2}\bar{\lambda}_{2(2)}\right]}{\left[\hbar_{0(2)} + \mu_2^{(0)}\hbar_{1(2)} + \mu_2^{(0)2}\hbar_{2(2)}\right]}\frac{S_y^2}{n}. \tag{26}$$

4.8. Efficiency Comparisons. The percent relative losses in efficiencies of the estimators $P_i\,(i = 1, 2)$ are obtained with respect to the similar estimator and natural successive sampling estimator when the nonresponse no observed on any occasion. The estimator $\Phi_1$is for complete information and under the similar assumption as estimator $P_i\,(i = 1, 2)$. Whereas the estimator Φ_2 is natural estimator under successive sampling, given by

Table 2: Percent relative losses L_{11} and L_{12} with respect to Φ_1 and Φ_2 for $f = 0.1$.

Θ	ρ_{yx}	ρ_{yz}	$f_2 = 1.5$			$f_2 = 2.0$		
			$\mu_1^{(0)}$	L_{11}	L_{12}	$\mu_1^{(0)}$	L_{11}	L_{12}
0.10	0.50	0.80	0.78	−32.63	−119.12	0.90	−26.42	−108.85
		0.85	0.63	−49.64	−176.93	0.72	−42.89	−164.44
		0.90	0.53	−79.78	−277.20	0.60	−70.70	−258.16
		0.95	0.43	−152.55	−510.50	0.49	−134.20	−466.16
	0.60	0.80	0.78	−28.02	−110.43	0.90	−22.03	−100.58
		0.85	0.63	−44.13	−165.96	0.72	−37.62	−153.96
		0.90	0.53	−72.77	−262.25	0.60	−64.05	−243.96
		0.95	0.43	−142.10	−486.31	0.49	−124.52	−443.73
	0.70	0.80	0.78	−23.16	−99.14	0.90	−17.39	−89.81
		0.85	0.63	−38.33	−151.69	0.72	−32.09	−140.33
		0.90	0.53	−65.44	−242.81	0.60	−57.09	−225.51
		0.95	0.43	−131.22	−454.85	0.49	−114.42	−414.55
	0.80	0.80	0.78	−18.01	−84.13	0.90	−12.48	−75.51
		0.85	0.63	−32.26	−132.71	0.72	−26.30	−122.22
		0.90	0.53	−57.83	−216.97	0.60	−49.86	−200.97
		0.95	0.43	−119.99	−413.02	0.49	−104.01	−375.76
0.15	0.50	0.80	0.84	−29.55	−114.03	1.02	−20.03	−98.30
		0.85	0.67	−46.26	−170.69	0.80	−36.15	−151.98
		0.90	0.56	−75.19	−267.58	0.66	−62.02	−239.96
		0.95	0.46	−143.08	−487.61	0.55	−118.03	−427.07
	0.60	0.80	0.84	−25.05	−105.55	1.02	−15.87	−90.45
		0.85	0.67	−40.88	−159.96	0.80	−31.14	−142.00
		0.90	0.56	−68.36	−253.01	0.66	−55.71	−226.48
		0.95	0.46	−133.02	−464.33	0.55	−109.01	−406.18
	0.70	0.80	0.84	−20.30	−94.52	1.02	−11.46	−80.23
		0.85	0.67	−35.22	−146.01	0.80	−25.87	−129.01
		0.90	0.56	−61.22	−234.07	0.66	−49.10	−208.96
		0.95	0.46	−122.55	−434.04	0.55	−99.62	−379.02
	0.80	0.80	0.84	−15.27	−79.86	1.02	−6.80	−66.64
		0.85	0.67	−29.28	−127.47	0.80	−20.35	−111.75
		0.90	0.56	−53.80	−208.89	0.66	−42.24	−185.67
		0.95	0.46	−111.74	−393.79	0.55	−89.92	−342.91
0.20	0.50	0.80	0.90	−26.42	−108.85	1.14	−13.56	−87.61
		0.85	0.72	−42.89	−164.44	0.89	−29.51	−139.68
		0.90	0.60	−70.70	−258.16	0.73	−53.76	−222.62
		0.95	0.49	−134.20	−466.16	0.60	−103.66	−392.32
	0.60	0.80	0.90	−22.03	−100.58	1.14	−9.62	−80.18
		0.85	0.72	−37.62	−153.96	0.89	−24.74	−130.18
		0.90	0.60	−64.05	−243.96	0.73	−47.77	−209.84
		0.95	0.49	−124.52	−443.73	0.60	−95.23	−372.81
	0.70	0.80	0.90	−17.39	−89.81	1.14	−5.46	−70.51
		0.85	0.72	−32.09	−140.33	0.89	−19.72	−117.83
		0.90	0.60	−57.09	−225.51	0.73	−41.50	−193.21
		0.95	0.49	−114.42	−414.55	0.60	−86.46	−347.44
	0.80	0.80	0.90	−12.48	−75.51	1.14	−1.04	−57.66
		0.85	0.72	−26.30	−122.22	0.89	−14.47	−101.41
		0.90	0.60	−49.86	−200.97	0.73	−34.99	−171.11
		0.95	0.49	−104.01	−375.76	0.60	−77.40	−313.71

Table 3: Percent relative losses L_{21} and L_{22} with respect to Φ_1 and Φ_2 for $f = 0.1$.

Θ	ρ_{yx}	ρ_{yz}	$f_2 = 1.5$			$f_2 = 2.0$		
			$\mu_2^{(0)}$	L_{21}	L_{22}	$\mu_2^{(0)}$	L_{21}	L_{22}
0.10	0.50	0.80	0.58	−41.77	−134.22	0.69	−37.11	−126.52
		0.85	0.52	−57.61	−191.68	0.59	−52.94	−183.04
		0.90	0.46	−88.47	−295.43	0.50	−83.28	−284.56
		0.95	0.38	−169.46	−551.39	0.40	−162.49	−534.54
	0.60	0.80	0.58	−36.85	−124.94	0.69	−32.35	−117.55
		0.85	0.52	−51.80	−180.12	0.59	−47.30	−171.82
		0.90	0.46	−81.12	−279.76	0.50	−76.14	−269.32
		0.95	0.38	−158.31	−525.57	0.40	−151.63	−509.39
	0.70	0.80	0.58	−31.65	−112.87	0.69	−27.33	−105.87
		0.85	0.52	−45.70	−165.09	0.59	−41.38	−157.24
		0.90	0.46	−73.43	−259.38	0.50	−68.66	−249.50
		0.95	0.38	−146.70	−492.00	0.40	−140.32	−476.69
	0.80	0.80	0.58	−26.15	−96.82	0.69	−22.00	−90.35
		0.85	0.52	−39.31	−145.11	0.59	−35.18	−137.84
		0.90	0.46	−65.46	−232.29	0.50	−60.91	−223.15
		0.95	0.38	−134.72	−447.37	0.40	−128.65	−433.22
0.15	0.50	0.80	0.64	−39.48	−130.44	0.80	−32.17	−118.36
		0.85	0.55	−55.28	−187.38	0.65	−48.20	−174.27
		0.90	0.48	−85.86	−289.97	0.54	−78.20	−273.88
		0.95	0.39	−165.93	−542.87	0.42	−155.82	−518.42
	0.60	0.80	0.64	−34.64	−121.31	0.80	−27.58	−109.71
		0.85	0.55	−49.56	−175.99	0.65	−42.74	−163.40
		0.90	0.48	−78.62	−274.52	0.54	−71.25	−259.07
		0.95	0.39	−154.93	−517.39	0.42	−145.24	−493.92
	0.70	0.80	0.64	−29.53	−109.43	0.80	−22.74	−98.46
		0.85	0.55	−43.55	−161.18	0.65	−37.00	−149.27
		0.90	0.48	−71.04	−254.42	0.54	−63.98	−239.80
		0.95	0.39	−143.47	−484.26	0.42	−134.22	−462.05
	0.80	0.80	0.64	−24.11	−93.64	0.80	−17.60	−83.50
		0.85	0.55	−37.25	−141.49	0.65	−30.99	−130.48
		0.90	0.48	−63.17	−227.70	0.54	−56.44	−214.18
		0.95	0.39	−131.65	−440.22	0.42	−122.84	−419.68
0.20	0.50	0.80	0.69	−37.11	−126.52	0.90	−27.04	−109.89
		0.85	0.59	−52.94	−183.04	0.71	−43.44	−165.46
		0.90	0.50	−83.28	−284.56	0.58	−73.22	−263.44
		0.95	0.40	−162.49	−534.54	0.44	−149.45	−503.01
	0.60	0.80	0.69	−32.35	−117.55	0.90	−22.63	−101.57
		0.85	0.59	−47.30	−171.82	0.71	−38.15	−154.93
		0.90	0.50	−76.14	−269.32	0.58	−66.47	−249.04
		0.95	0.40	−151.63	−509.39	0.44	−139.13	−479.11
	0.70	0.80	0.69	−27.33	−105.87	0.90	−17.98	−90.75
		0.85	0.59	−41.38	−157.24	0.71	−32.60	−141.25
		0.90	0.50	−68.66	−249.50	0.58	−59.40	−230.31
		0.95	0.40	−140.32	−476.69	0.44	−128.38	−448.03
	0.80	0.80	0.69	−22.00	−90.35	0.90	−13.04	−76.37
		0.85	0.59	−35.18	−137.84	0.71	−26.78	−123.07
		0.90	0.50	−60.91	−223.15	0.58	−52.07	−205.41
		0.95	0.40	−128.65	−433.22	0.44	−117.29	−406.72

$$\Phi_j = \psi_j \Phi_{ju} + \left(1 - \psi_j\right) T_{jm};\ (j = 1, 2), \tag{27}$$

where $\Phi_{1u} = \overline{y}_u \exp(\overline{Z} - \overline{z}_u / \overline{Z} + \overline{z}_u)$, $\Phi_{2u} = \overline{y}_u$,

$$T_{1m} = P_{1m} = \overline{y}_m \left(\frac{\overline{Z}}{\overline{z}_m}\right) \exp\left(\frac{\overline{x}_n - \overline{x}_m}{\overline{x}_n + \overline{x}_m}\right), T_{2m} = \overline{y}_m + b_{yx(m)} (\overline{x}_n - \overline{x}_m). \tag{28}$$

Proceeding in similar manner as discussed for the estimators $P_i\,(i = 1, 2)$, the optimum mean squared errors of the estimators Φ_j, $(j = 1, 2)$ are derived as

$$\Phi_j = \psi_j \Phi_{ju} + \left(1 - \psi_j\right) T_{jm},$$

$$\text{MMSE}\left(\Phi_1^{(0)}\right)_{\text{opt}} = \frac{\left[A_3 + \mu_1^{*\,(0)} A_2 + \mu_1^{*\,(0)2} A_1\right]}{\left[B_3 + \mu_1^{*\,(0)} B_2 + \mu_1^{*\,(0)2} B_1\right]} \frac{S_y^2}{n}, \tag{29}$$

$$\text{MMSE}\left(\Phi_2^{(0)}\right)_{\text{opt}} = \left[\frac{1}{2}\left\{1 + \sqrt{\left(1 - \rho_{xy}^2\right)}\right\} - f\right] \frac{S_y^2}{n},$$

where $\mu_1^{*\,(0)} = -Q_2 \pm \sqrt{Q_2^2 - Q_1 Q_3} / Q_1$ (fraction of the sample for the estimator Φ_1),

$$Q_1 = B_1 A_2 - A_1 B_2, Q_2 = B_1 A_3 - A_1 B_3, Q_3 = B_2 A_3 - A_2 B_3,$$

$$A_1 = \left[\left(\frac{9}{16}\right) f^2 \kappa_0^2 - f^2 \kappa_0 \kappa_3 - f \kappa_2 \kappa_3\right],$$

$$A_2 = \left[f \kappa_0 \kappa_3 + \kappa_2 \kappa_3 - f(1 - f) \kappa_0 \kappa_3 - \left(\frac{9}{16}\right) f^2 \kappa_0^2\right],$$

$$A_3 = (1 - f) \kappa_0 \kappa_3,$$

$$B_1 = \left[f \kappa_0 + \kappa_2 - \left(\frac{3}{2}\right) f \kappa_0 + f \kappa_3\right],$$

$$B_2 = \left[(1 - f) \kappa_0 - (1 + f) \kappa_3 + \left(\frac{3}{2}\right) f \kappa_0\right],$$

$$B_3 = \kappa_3. \tag{30}$$

Remark 2. For comparison of estimators P_i and Φ_j, it was advisable by Cochran [3] & Feng and Zou [4] to assume the intraclass correlation coefficient equal, that is, $\rho_{xz} = \rho_{yz}$.

5. Numerical Illustrations

For $N = 5000$, $n = 500$ and different choices of ρ_{yx} and ρ_{yz}, Tables 2 & 3 give the optimum values of $\mu_i^{(0)}$ and percent relative losses L_{ij}, $(i = 1, 2;\ j = 1, 2)$ in the precision of estimators $P_i\,(i = 1, 2)$ with respect to Φ_j, $(j = 1, 2)$.

The percent relative losses in the precision of estimators $P_i\,(i = 1, 2)$ with respect to Φ_j, $(j = 1, 2)$, under their respective optimality conditions are given by

$$L_{ij} = \frac{\text{MMSE}\left(P_i^{(0)}\right)_{\text{opt}} - \text{MMSE}\left(\Phi_i^{(0)}\right)_{\text{opt}}}{\text{MMSE}\left(P_i^{(0)}\right)_{\text{opt}}}, (i = 1, 2;\ j = 1). \tag{31}$$

The following inferences may draw from Tables 2 and 3. The above Table 2 depicts that

(i) Considering the constant value of Θ, f_2, and ρ_{yz}, we observe that values of $\mu_1^{(0)}$, L_{11}, and L_{12} decrease as ρ_{yx} increases. In other words, the intraclass correlation coefficient between y and z is inversely proportional to the new sample. That means the efficacy of the estimators under nonresponse depends positively on the auxiliary information. This attitude of the estimators is very important and extremely desirable.

(ii) Considering the constant value of Θ, f_2, and ρ_{yz}, we observe that values of $\mu_2^{(0)}$ remain the same whereas L_{11} and L_{12} increase as ρ_{yx} increases.

(iii) Considering the constant value of Θ, ρ_{yx}, and ρ_{yz}, we observe that values of $\mu_2^{(0)}$, L_{11}, and L_{12} increase as f_2 increases.

(iv) Considering the constant value of f_2, ρ_{yx}, and ρ_{yz}, we observe that values of $\mu_2^{(0)}$, L_{11}, and L_{12} increase as Θ increases. Table 3 depicts that nonresponse rate and sample size at current occasion are directly proportional and therefore the cost of the survey increases as well.

(v) Considering the constant value of Θ, f_2, and ρ_{yz}, we observe that values of $\mu_2^{(0)}$, L_{21}, and L_{22} decrease as ρ_{yx} increases. In other words, the intraclass correlation coefficient between y and z is inversely proportional to the new sample. That means the efficacy of the estimators under nonresponse depends positively on the auxiliary information. This attitude of the estimators is very important and extremely desirable.

(vi) Considering the constant value of Θ, f_2, and ρ_{yz}, we observe that values of $\mu_2^{(0)}$ remain the same whereas L_{21} and L_{22} increase as ρ_{yx} increases.

(vii) Considering the constant value of Θ, ρ_{yx}, and ρ_{yz}, we observe that values of $\mu_2^{(0)}$, L_{21}, and L_{22} increase as f_2 increases.

(viii) Considering the constant value of f_2, ρ_{yx}, and ρ_{yz}, we observe that values of $\mu_2^{(0)}$, L_{21}, and L_{22} increase as Θ increases. In other words, the nonresponse rate and sample size at the current occasion are directly proportional; consequently, the cost of the survey increases as well.

6. Conclusions

In this paper, we have proposed exponential type estimators for estimating the population mean under the unavailability of full response in two-occasion successive sampling using the additional information. An extensive theoretical and computational study led to the conclusion that both the proposed estimators are better than the usual estimators available in the literature, and moreover, the estimator P_2 is better than the estimator P_1 between the self-comparison. Therefore, the authors recommend the use of estimator P_2 over the other estimators considered in this paper in practice. Furthermore, the researchers can use the composition of the proposed estimators for their use [20, 21].

Data Availability

No external data were used to support this study.

Conflicts of Interest

The authors declare that there are no conflicts of interest.

References

[1] R. J. Jessen, *Statistical investigation of a sample survey for obtaining form facts*, vol. 304, Iowa Agricultural Experiment Station, Road Bulletin, Ames IA, 1942.

[2] H. D. Patterson, "Sampling on successive occasions with partial replacement of units," *Jour Roy, Statist. Assoc., B*, vol. 12, no. 2, pp. 241–255, 1950.

[3] J. N. K. Rao and J. E. Graham, "Rotation design for sampling on repeated occasions," *Journal of the American Statistical Association*, vol. 59, pp. 492–509, 1964.

[4] S. Feng and G. Zou, "Sampling rotation method with auxiliary variable," *Communications in Statistics - Theory and Methods*, vol. 26, no. 6, pp. 1497–1509, 1997.

[5] R. S. Biradar and H. P. Singh, "Successive sampling using additional information on both the occasions," *Calcutta. Statist. Assoc. Bull.*vol. 51, pp. 243–251, 2001.

[6] H. P. Singh and G. K. Vishwakarma, "A general procedure for estimating population mean in successive sampling," *Communications in Statistics - Theory and Methods*, vol. 38, pp. 293–308, 2009.

[7] H. P. Singh and S. K. Pal, "An efficient effective rotation pattern in successive sampling over two occasions," *Communications in Statistics - Theory and Methods*, vol. 47, no. 17, pp. 5017–5027, 2016.

[8] A. Sanaullah, M. Noor ul Amin, and M. Hanif, "Generalized exponential-type estimators for population mean taking two auxiliary variables for unknown means in stratified sampling with sub-sampling the non-respondents," *Int. J. Appl. Comput. Math*, vol. 4, p. 56, 2018.

[9] A. Javaid, M. Noor-ul-Amin, and M. Hanif, "Modified ratio estimator in systematic random sampling under non-response," *Proc. Natl. Acad. Sci., India, Sect. A Phys. Sci.*vol. 89, pp. 817–825, 2019.

[10] S. K. Pal, H. P. Singh, and V. Mehta, "Use of linear and power transformations for estimating the population mean in two occasion successive sampling," *Revista Invest. Opera.*vol. 40, no. 5, pp. 687–698, 2019.

[11] M. H. Hansen and W. N. Hurwitz, "The problem of non-response in sample surveys," *Journal of the American Statistical Association*, vol. 41, pp. 517–529, 1946.

[12] R. K. Chaudhary, H. V. L. Bathal, and U. C. Sud, "On non-response in sampling on two occasions," *Jour. Ind. Soc. Agril. Statist.*vol. 58, no. 3, pp. 331–343, 2004.

[13] G. Singh and K. Priyanka, "Effects of non-response on current occasion in search of good rotation patterns on successive occasions," *Statistics in Transition*, vol. 8, no. 2, pp. 273–292, 2007.

[14] S. K. Pal and H. P. Singh, "Investigation of competent estimation procedure for estimating the finite population mean at current occasion addressing non-response in two-occasion successive sampling," *JSTP*, vol. 15, no. 5, 2021.

[15] G. N. Singh, M. Khetan, and S. Maurya, "Assessment of exponential methods of estimation under non-response in two occasion successive sampling," *Jour. Statist. Theo. Pract.*vol. 9, no. 3, pp. 506–523, 2014.

[16] G. N. Singh, M. Khetan, and S. Maurya, "Estimation under non-response when it occurs on both occasions in two occasion successive sampling," *Communications in Statistics - Theory and Methods*, vol. 45, no. 13, pp. 3939–3951, 2016.

[17] H. P. Singh and S. Kumar, "A regression Approach to the estimation of the finite population mean in the presence of non-response," *Aust. N. Z. Jour. Stat.*vol. 50, no. 4, pp. 1–14, 2008.

[18] V. N. Reddy, "On a transformed ratio method of estimation," *Sankhya B*, vol. 36, pp. 59–70, 1974.

[19] H. P. Singh and M. R. Ruiz-Espejo, "On linear regression and ratio-product estimation of finite population mean," *The statistician*, vol. 52, no. 1, pp. 59–67, 2003.

[20] H. P. Singh, K. Sunil, and M. Kazak, "Improved estimation of finite population mean using sub-sampling to deal with non-response in two-phase sampling scheme," *Communications in Statistics - Theory and Methods*, vol. 39, no. 5, pp. 791–802, 2010.

[21] W. G. Cochran, *Sampling Technique 3rd Edition*, p. 448, Johan Wiley and Sons, New York, 1977.