Hindawi
Mathematical Problems in Engineering
Volume 2022, Article ID 1463006, 22 pages
https://doi.org/10.1155/2022/1463006

Research Article

Consistency Improvement Method of Pairwise Matrix Based on Consistency Ratio Decreasing Rate and Attribute Weighting Method Considered Decision Makers' Levels in Analytic Hierarchy Process: Application to Hip Joint Prosthesis Material Selection

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Received 10 June 2022; Revised 30 August 2022; Accepted 21 September 2022; Published 17 October 2022

Academic Editor: Harish Garg

Analytic hierarchy process (AHP) is a well-known attribute weighting method in multiattribute decision-making. Its major requirement is to satisfy the consistency of pairwise matrix (PM). To solve this problem, we first propose a new consistency improvement method of PM based on consistency ratio (CR) decreasing rate. In this method, we calculate the CR decreasing rates of all the PMs reconstituted by replacing all elements of the PM with the lower and upper neighbouring 9-point scales and find the element with maximum CR decreasing rate, and then modify it to its lower or upper neighbouring scale. Second, we develop third-order approximate polynomial for random consistency index using least square method. It enables to determine the RI value according to the number of attributes without a numerical table. Third, we propose the final PM determining method and final attribute weighting method considered decision makers' levels based on the CR values of the individual PMs in case several decision makers perform their own pairwise comparisons. We test the performances of the proposed and some previous consistency improvement methods with two numerical examples. The results demonstrate that the proposed method improves the consistency of PM better and faster with smaller amount of modification than that of the previous methods, while it modify the elements of the PM to 9-point scales, necessarily. We apply the proposed method to hip joint prosthesis material selection. The proposed methods may be widely used in practical applications of AHP.

1. Introduction

Determining a reasonable attribute weights plays a vital role in multiattribute decision-making (MADM) and multi-objective optimization (MOO) because the decision-making and optimization results may differ according to the attribute weights. Analytic hierarchy process (AHP) is well-known attribute weighting method [1]. AHP determines the attribute weights based on pairwise comparison evaluation data for the pairs of attributes. The 9-point scales are used to transform the decision maker's judgments into numerical quantities [2]. The essential feature of AHP is the pairwise comparisons between the attributes instead of the direct allocation of the weights [3]. The details of AHP have been described in the literature.

AHP has been widely applied to calculate the subjective attribute weights in many practical MADM and MOO problems [4, 5]. Soni et al. [6] determined the criteria weights using AHP for material selection of reinforced sustainable composites by recycling waste plastics and agro-waste. Zhong et al. [7] constructed a cost evaluation system with 5 indices in the first level and 22 indices in the second level using AHP. Radulescu et al. [8] calculated the overall weights of the criteria as a linear combination of the individual weights obtained from the group AHP and extended entropy weighting method for evaluating the fourth wave of COVID-19 pandemic. Peng and Wu [9] determined the comprehensive weights of each index using AHP to score the index system for evaluating the benefit development of the offshore wind power after the cancellation of the public subsides. Wei et al. [10] constructed the evaluation model using AHP and fuzzy comprehensive evaluation. Mathew et al. [11] calculated the weights of the criteria using fuzzy AHP and determined the final ranking of the alternatives using spherical fuzzy TOPSIS. Rawa et al. [12] proposed an economical-technical-environmental operation for power networks with wind-solar-hydropower generation by using AHP and improved grey wolf algorithm. They used a weighted sum strategy using AHP to transform the multiobjective problem into a normalized single objective one. Okudan and Budayan [13] used fuzzy AHP for conducting the evaluation of project characteristics affecting risk occurrences in the construction projects. Dano [14] analysed the impacts of flash hazards using AHP and identified the most effective methods to reduce the flash flood impacts using expert's opinions in Jeddah. Chen et al. [15] dealt with the uncertainty of the wind power, the load demand, and the multiobjective function by using fuzzy chance constraint programming and improved AHP.

For group decision-making problems, hesitant fuzzy set, complex intuitionistic fuzzy set, and probabilistic hesitant fuzzy set have been introduced to handle and model the uncertainty and vagueness in decision-making very effectively, reflect the importance of different numerical values more clearly, elicit the decision makers' knowledge, and develop more effective decision-making model. [16–18] Rani and Garg [16] proposed a novel algorithm for multiattribute group decision-making using complex intuitionistic fuzzy values. Jin et al. [17] proposed a decision-making model using probabilistic hesitant fuzzy preference relations for reflecting clearly the importance of different numerical values and eliciting the decision makers' knowledge in the group decision-making problems. Liu et al. [18] calculated the probabilities of elements in the probabilistic hesitant fuzzy element and the probability of risk status by using two nonlinear programming models. Khan et al. [19] proposed a performance measure using an MADM method based on the complex T-spherical fuzzy power aggregation operators. Liu et al. [20] developed a novel correlation coefficient to measure the strength of the relationship between the hesitant fuzzy sets.

Although AHP is a useful tool for attribute weighting, it has some drawbacks. One drawback is that it is difficult to conduct pairwise comparison in practical applications. Yang et al. [5] proposed a simplest questionnaire to conduct the pairwise comparison, easily and conventionally. Another drawback is that it is difficult to satisfy the consistency of pairwise matrix (PM) in practical applications. To determine the reasonable attribute weights using AHP, the consistency of PM must be satisfied. When it does not satisfy the consistency, it is need to repair the primary PM. In order to satisfy the consistency of the PM, some researchers proposed the reconstitution methods of the inconsistent PM. Girsang et al. [21] proposed ANTAHP method using ant optimization algorithm to reconstitute the inconsistent PM by minimizing the distance between the primary and modified PMs. Wu et al. [22] improved the inconsistency of the PM using marginal optimization method. The method is based to increase (or decrease) all elements by a fixed value, and it calculates the marginal effect of each modification. Zeshui and Cuiping [23] proposed a consistency improvement method based on auto-adaptive process. In the method, the element a_{ij} of the inconsistent PM A is replaced by $b_{ij} = a_{ij}{}^{\alpha}$ $(w_i/w_j)^{1-\alpha}$, where $w = \ ((w_1, \ldots, w_i, \ldots, w_n)^T$ is the weight vector obtained from A. The generated matrix $B = [b_{ij}]$ has a reduced CR. This process is repeated until the consistency is satisfied. Cao et al. [24] proposed a heuristic method to modify inconsistent PM. They decomposed the primary PM as the Hadamard product of the consistent PM and a reciprocal deviation matrix. They constituted a modified PM by convex combination of the reciprocal zero deviation matrices. They proposed auto-adaptive modification algorithm using such convex combinations. Yang et al. [25] modified the inconsistent PM by combining the particle swarm optimization and Taguchi method. Benítez et al. [26] proposed a linearization method to provide the closest consistent PM to the inconsistent PM by using orthogonal projection in a linear space.

For consistency improvement, it needs to pay attention to guarantee a good balance between improvement of consistency and preservation of primary information. However, the previous methods are lacking in guaranteeing such balance. On the other hand, the elements of the PM are 9-point scales because the pairwise comparison is performed by means of 9-point scales {1/9, 1/8, . . ., 8, 9} in the conventional AHP. However, in the previous methods, the elements of the reconstituted PMs are no 9-point scales, and therefore, the PMs obtained from the previous methods are no inherent ones. To deal with this shortcoming, we propose a new consistency improvement method according to the following principles:

(i) The amount of consistency improvement of the PM should be as large as possible, and the deviation between the primary and reconstituted PMs and the number of the modified elements should be as small as possible.

(ii) The elements of the inconsistent PM should be replaced with the lower or upper neighbouring 9-point scales, and the elements of the reconstituted PM should be 9-point scales.

When two or more decision makers take part in the pairwise comparison between the attributes, the PMs and the attribute weights may differ according to their knowledge and opinions. Therefore, it is necessary to constitute a final PM by synthesizing the individual PMs obtained from each decision makers and determine the final attribute weighting from the final PM. It needs to consider the decision makers' levels to constitute the final PM. However, there is no reasonable objective method to determine the decision makers' levels, while the previous methods are generally subjective ones. To overcome this shortcoming, we propose a new objective method to determine the decision makers' levels based on the CR values of their PMs. The worse the consistency of the PM is, the more mistakes the decision maker's judgment has, and therefore, we can regard that the CR value reflects the decision maker's level.

We propose a new consistency improvement method of pairwise matrix based on consistency ratio decreasing rate and attribute weighting method considered decision makers' levels in analytic hierarchy process and apply the methods to hip joint prosthesis material selection.

The novelties and advantages of the proposed methods are as follows:

(i) In the CR decreasing rate-based consistency improvement method of inconsistent PM, the elements of the inconsistent PM are modified to the adjacent 9-point scales, and all the elements of the reconstituted PM are the 9-point scales, not real numbers. This method improves the consistency more, better, and faster with smaller number of elements and smaller amount of modification, and it guarantees a very good balance between consistency improvement and information preservation of the primary PM.

(ii) The approximate formula is used to determine the RI value according to the number of attributes, not the numerical table for RI. It enables to test the consistency of PM without a numerical table for RI.

(iii) In the final attribute weighting method considered decision makers' levels, the CR value is used as an objective measure that reflects the decision maker's level. It is possible to determine the decision makers' levels objectively, not subjectively. It enables to determine the attribute weights, more scientifically and reasonably. In this method, the elements of the final PM are also 9-point scales, and it enables to preserve the inherent characteristics of AHP.

The rest of this paper is organized as follows: In Section 2, we describe a new consistency improvement method of PM based on CR decreasing rate, a development method of approximate polynomial for RI according to number of attributes, and a constitution method of final PM and final attribute weighting method considered decision makers' levels. In Section 3, we describe the numerical test results of the proposed method and its application to hip joint prosthesis material selection. In Section 4, we present the conclusions. In Appendix section, we describe the attribute weighting method using AHP with the simplest questionnaire, development method of approximate polynomial by least square method, and three well-known MADM methods (MADMs) such as simple additive weighting (SAW) method, technique for order preference by similarity to ideal solution (TOPSIS) method, and grey relational analysis (GRA) method used in this work.

2. Methodology

2.1. CR Decreasing Rate-Based Consistency Improvement Method of Inconsistent PM. Let CR(A) be the CR value of the PM $A=(a_{ij})_{n\times n}$.

The main steps of a new consistency improvement method are as follows:

Step 1. For each element a_{lm} $(l=\overline{1,n-1}, m=\overline{l+1,n})$ in the upper triangular matrix of the PM $A=(a_{ij})_{n\times n}$, reconstitute the PM $A_{lm}^{-}=(a_{ij}^{-})_{n\times n}$ by replacing the element a_{lm} with the lower neighbouring scale h_{lm}^{-}, and then calculate its CR value $\mathrm{CR}(A_{lm}^{-})$ and CR decreasing rate $dC\ Rr_{lm}^{-}$ as follows:

$$dC\ Rr_{lm}^{-}=\frac{dC\ R_{lm}^{-}}{\left(\left|a_{lm}-h_{lm}^{-}\right|+\left|1/a_{lm}-1/h_{lm}^{-}\right|\right)}, \tag{1}$$

where

$$dC\ R_{lm}^{-}=\mathrm{CR}(A)-\mathrm{CR}(A_{lm}^{-}),$$
$$a_{lm}^{-}=h_{lm}^{-}, a_{ml}^{-}=\frac{1}{a_{lm}^{-}}, a_{ij}^{-}=a_{ij},\quad i,j=\overline{1,n}, (i,j)\neq(l,m). \tag{2}$$

The lower and upper neighbouring scales of the 9-point scales {1/9, 1/8, . . ., 8, 9} are shown in Table 1.

Step 2. Find the maximum value of the CR decreasing rate $dC\ Rr_{r_1s_1}^{-}$ from $\{dC\ Rr_{lm}^{-};\ l=\overline{1,n-1}, m=\overline{l+1,n}\}$ as follows:

$$dC\ Rr_{r_1s_1}^{-}=\max_{1\leq l<m\leq n}\{dC\ Rr_{lm}^{-}\}. \tag{3}$$

Step 3. For each element a_{lm} $(l=\overline{1,n-1}, m=\overline{l+1,n})$ in the upper triangular matrix of the PM $A=(a_{ij})_{n\times n}$, reconstitute the PM $A_{lm}^{+}=(a_{ij}^{+})_{n\times n}$ by replacing the element a_{lm} with the upper neighbouring scale h_{lm}^{+}, and then calculate its CR value $\mathrm{CR}(A_{lm}^{+})$ and CR decreasing rate $dC\ Rr_{lm}^{+}$ as follows:

$$dC\ Rr_{lm}^{+}=\frac{dC\ R_{lm}^{+}}{\left(\left|a_{lm}-h_{lm}^{+}\right|+\left|1/a_{lm}-1/h_{lm}^{+}\right|\right)}, \tag{4}$$

where

$$dC\ R_{lm}^{+}=\mathrm{CR}(A)-\mathrm{CR}(A_{lm}^{+}),$$
$$a_{lm}^{+}=h_{lm}^{+}, a_{ml}^{+}=\frac{1}{a_{lm}^{+}}, a_{ij}^{+}=a_{ij},\quad i,j=\overline{1,n}, (i,j)\neq(l,m). \tag{5}$$

Step 4. Find the maximum value of the CR decreasing rate $dC\ Rr_{r_2s_2}^{+}$ from $\{dC\ Rr_{lm}^{+};\ l=\overline{1,n-1}, m=\overline{l+1,n}\}$ as follows:

$$dC\ Rr_{r_2s_2}^{+}=\max_{1\leq l<m\leq n}\{dC\ Rr_{lm}^{+}\}. \tag{6}$$

Step 5. If $dC\ Rr_{r_1s_1}^{-}\geq dC\ Rr_{r_2s_2}^{+}$, then reconstitute the PM $B=(b_{ij})_{n\times n}$ by replacing the element $a_{r_1s_1}$ with the

Table 1: Lower and upper neighbouring scales of the 9-point scales.

9-Point scale	1/9	1/8	1/7	1/6	1/5	1/4	1/3	1/2	1	2	3	4	5	6	7	8	9
Lower neighbouring scale	1/9	1/9	1/8	1/7	1/6	1/5	1/4	1/3	1/2	1	2	3	4	5	6	7	8
Upper neighbouring scale	1/8	1/7	1/6	1/5	1/4	1/3	1/2	1	2	3	4	5	6	7	8	9	9

lower neighbouring scale $h^-_{r_1s_1}$ in the PM $A=(a_{ij})_{n\times n}$, where

$$b_{r_1s_1}=h^-_{r_1s_1},b_{s_1r_1}=\frac{1}{b_{r_1s_1}},b_{ij}=a_{ij},\quad i,j=\overline{1,n},\ (i,j)\neq(r_1,s_1). \tag{7}$$

If $dC\ Rr^-_{r_1s_1}<dC\ Rr^+_{r_2s_2}$, then reconstitute the PM $B=(b_{ij})_{n\times n}$ by replacing the element $a_{r_2s_2}$ with the upper neighbouring scale $h^+_{r_2s_2}$ in the PM $A=(a_{ij})_{n\times n}$, where

$$b_{r_2s_2}=h^+_{r_2s_2},b_{s_2r_2}=\frac{1}{b_{r_2s_2}},b_{ij}=a_{ij},\quad i,j=\overline{1,n},\ (i,j)\neq(r_2,s_2). \tag{8}$$

Step 6. Calculate the CR value CR(B) of the reconstituted PM $B=(b_{ij})_{n\times n}$.

Step 7. If CR(B) > CR0, then $A=B$ and go to Step 1.

Step 8. If CR(B) ≤ CR0, then calculate the principal eigenvector $(v=v_1,\ldots,v_j,\ldots,v_n)^T$ from the reconstituted PM $B=(b_{ij})_{n\times n}$.

Step 9. Calculate the attribute weights $(w_1,\ldots,w_j,\ldots,w_n)^T$ by normalizing $(v=v_1,\ldots,v_j,\ldots,v_n)^T$ as follows:

$$w_j=\frac{v_j}{\sum_{k=1}^n v_k},\quad j=\overline{1,n}. \tag{9}$$

We call this method CR decreasing rate-based method.

Let $A=(a_{ij})_{n\times n}$ and $B=(b_{ij})_{n\times n}$ be the primary inconsistent PM and reconstituted PM, respectively.

To evaluate the performance of the consistency improvement method of the inconsistent PM, we use the following three measures:

(i) CR decreasing amount,

$$dC\ R(A,B)=CR(B)-CR(A), \tag{10}$$

where CR(A) and CR(B) are the CR values of the primary PM A and the reconstituted PM B, respectively.

(ii) Matrix deviation,

$$dM(A,B)=\sum_{i=1}^n\sum_{j=1}^n\left|a_{ij}-b_{ij}\right|. \tag{11}$$

(iii) CR decreasing rate,

$$dC\ Rr(A,B)=\frac{dC\ R(A,B)}{dM(A,B)}. \tag{12}$$

The greater the values of $dCR(A,B)$ and $dCRr(A,B)$ are, the better the performance of the consistency improvement method is. The smaller the value of $dM(A,B)$ is, the better the reconstituted PM preserves the information of the primary PM. The CR decreasing rate $dCRr(A,B)$ becomes the major measure to evaluate the performance of the consistency improvement method of PM from among above three measures.

2.2. Development Method of Approximate Polynomial for RI According to the Number of Attributes. We develop m-th order approximate polynomial for RI as the following form:

$$RI=RI_m(n)=a_0+a_1n\ldots+a_{m-1}n^{m-1}+a_mn^m, \tag{13}$$

with the data set $\{(n, RI_n), n=\overline{3,15}.\}$, where n is the number of attributes and RI_n is the corresponding RI value (Table 2).

The MAE, MRE, and MSE of the m-th order approximate polynomial are as follows:

$$\begin{aligned}\text{MAE}_m&=\frac{1}{13}\sum_{n=3}^{15}\left|RI_n-RI_m(n)\right|,\\ \text{MRE}_m&=\frac{1}{13}\sum_{n=3}^{15}\left|\frac{RI_n-RI_m(n)}{RI_n}\right|\times 100\%,\\ \text{MSE}_m&=\frac{1}{13}\sum_{n=3}^{15}\left(RI_n-RI_m(n)\right)^2.\end{aligned} \tag{14}$$

The main steps to develop the approximate polynomial for RI value according to the number of attributes are as follows:

Step 1. Develop six approximate polynomials $RI=RI_m(n)$; $m=\overline{1,6}$ with the data $\{(n, RI_n), n=\overline{3,15}.\}$ (Table 2).

Step 2. Evaluate the MAE, MRE, and MSE of six approximate polynomials $RI=RI_m(n)$; $m=\overline{1,6}$.

Step 3. Select the suitable approximate polynomial $RI=RI_r(n)$ with acceptable MAE, MRE, and MSE from among six approximate polynomials $RI=RI_m(n)$; $m=\overline{1,6}$.

2.3. Constituting Method of Final PM Constituting Method and Final Attribute Weighting Method Considered Decision Makers' Levels. Let M be the number of the decision makers.

The main steps to constitute the final PM and determining the final attribute weights considered decision makers' levels are as follows:

Step 1. Constitute the simplest questionnaires by M decision makers and constitute M PMs

TABLE 2: RI value according to the number of attributes [28].

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
RI	0.0	0.0	0.58	0.90	1.12	1.24	1.32	1.41	1.45	1.49	1.51	1.53	1.56	1.57	1.59

$\{A^{(m)} = (a_{ij}^{(m)})_{n\times n};\ m = \overline{1,M}\ \}$ from the simplest questionnaires, where $A^{(m)} = (a_{ij}^{(m)})_{n\times n}$ is the PM constituted from m-th decision maker's questionnaire.

Step 2. Test the consistency of the PMs.

Step 3. Reconstitute the consistent PM by modifying the inconsistent PM using the CR decreasing rate-based consistency improvement method.

Denote the consistent PMs as $B^{(m)} = (b_{ij}^{(m)})_{n\times n};\ m = \overline{1,M}$.

Step 4. Calculate the CR value CR(m) of the PM $B^{(m)} = (b_{ij}^{(m)})_{n\times n};\ m = \overline{1,M}$.

Step 5. Calculate the inverse values of the CR values $\{\mathrm{ICR}(m) = 1/\mathrm{CR}(m);\ m = \overline{1,M}.\}$.

Step 6. Normalize the inverse values of the CR values $\{\mathrm{ICR}(m);\ m = \overline{1,M}\}$ and determine the normalized values as the decision makers' levels $\{h_m;\ m = \overline{1,M}\}$.

$$h_m = \frac{\mathrm{ICR}^{(m)}}{\sum_{j=1}^{M} \mathrm{ICR}^{(j)}}. \tag{15}$$

Commonly, the greater the CR value is, the worse the consistency of the PM is. The worse the consistency of the PM is, the more mistakes the decision maker's judgment has. Therefore, we can regard that the inverse value of the CR value of the PM reflects the decision maker's level. This is why we assign the normalized inverse values of CR values to the decision makers' levels.

Step 7. Constitute the composite PM $B^{(0)} = (b_{ij}^{(0)})_{n\times n}$ as the geometric mean of the individual PMs $\{B^{(m)} = (b_{ij}^{(m)})_{n\times n};\ m = \overline{1,M}\}$ as follows:

$$b_{ij}^{(0)} = \prod_{m=1}^{M} \left(b_{ij}^{(m)}\right)^{h_m}, \quad i,j = \overline{1,n}. \tag{16}$$

The elements of $B^{(0)} = (b_{ij}^{(0)})_{n\times n}$ may be no 9-point scales.

Step 8. Constitute the final PM $A^{(0)} = (a_{ij}^{(0)})_{n\times n}$ by transforming the elements of $B^{(0)} = (b_{ij}^{(0)})_{n\times n}$ to the nearest neighbouring 9-point scales.

The final PM $A^{(0)} = (a_{ij}^{(0)})_{n\times n}$ reflects not only the decision makers' pairwise judgments but also their levels.

Step 9. Determine the final attribute weights $(w_1, \ldots, w_j, \ldots, w_n)^T$ by normalizing the principal eigenvector $(v = v_1, \ldots, v_j, \ldots, v_n)^T$ of the final PM $A^{(0)} = (a_{ij}^{(0)})_{n\times n}$.

Figure 1 shows the flowchart of the proposed methods.

3. Results and Discussion

3.1. Numerical Test Results of the Proposed Consistency Improvement Method. We test the performance of proposed consistency improvement method of PM by applying it to two examples and compare with the previous methods.

Example 1. The primary PM [21, 22] is as follows:

$$\begin{pmatrix} 1 & 5 & 3 & 7 & 6 & 6 & \frac{1}{3} & \frac{1}{4} \\ \frac{1}{5} & 1 & \frac{1}{3} & 5 & 3 & 3 & \frac{1}{5} & 1/7 \\ \frac{1}{3} & 3 & 1 & 6 & 3 & 4 & 6 & \frac{1}{5} \\ \frac{1}{3} & \frac{1}{5} & \frac{1}{6} & 1 & \frac{1}{3} & \frac{1}{4} & \frac{1}{7} & \frac{1}{8} \\ \frac{1}{6} & \frac{1}{3} & \frac{1}{3} & 3 & 1 & \frac{1}{2} & \frac{1}{5} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{2} & \frac{1}{4} & 4 & 2 & 1 & \frac{1}{5} & \frac{1}{6} \\ 3 & 5 & \frac{1}{6} & 7 & 5 & 5 & 1 & \frac{1}{2} \\ 4 & 7 & 5 & 8 & 6 & 6 & 2 & 1 \end{pmatrix}, \mathrm{CR} = 0.169087. \tag{17}$$

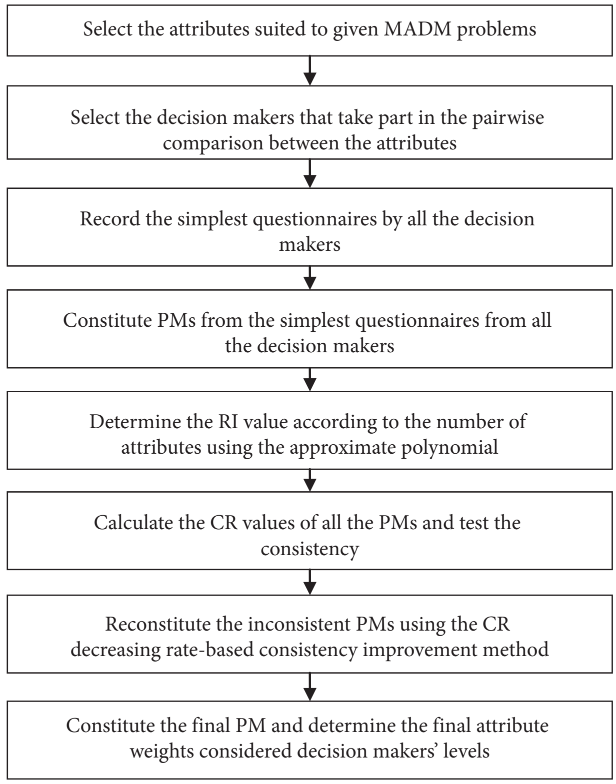

Figure 1: Flowchart of the proposed methods.

The reconstituted PM using the ANTAHP method [21] is as follows:

$$\begin{pmatrix} 1 & 4.2 & 2.4 & 7.4 & 5.8 & 5.4 & \frac{1}{2.4} & \frac{1}{3.4} \\ \frac{1}{4.2} & 1 & \frac{1}{3} & 4.4 & 2.6 & 2.4 & 1/4 & \frac{1}{6.8} \\ \frac{1}{2.4} & 3 & 1 & 6.8 & 3.4 & 4.2 & 4.4 & \frac{1}{4.2} \\ \frac{1}{7.4} & \frac{1}{4.4} & \frac{1}{6.8} & 1 & \frac{1}{2.8} & \frac{1}{3.4} & \frac{1}{7} & \frac{1}{9} \\ \frac{1}{5.8} & \frac{1}{2.6} & \frac{1}{3.4} & 2.8 & 1 & \frac{1}{1.8} & \frac{1}{5} & \frac{1}{6.8} \\ \frac{1}{5.4} & \frac{1}{2.4} & \frac{1}{4.2} & 3.4 & 1.8 & 1 & \frac{1}{5} & \frac{1}{6.8} \\ 2.4 & 4 & \frac{1}{4.4} & 7 & 5 & 5 & 1 & \frac{1}{2} \\ 3.4 & 6.8 & 4.2 & 9 & 6.8 & 6.8 & 2 & 1 \end{pmatrix}, \mathrm{CR} = 0.099838. \tag{18}$$

The reconstituted PMs using Wu method [22] are as follows:

In case of $d=0.1$ (where d is the fixed modifying amount),

$$\begin{pmatrix} 1 & 4.3 & 2.3 & 77 & 53 & 5.3 & 0.4545 & 0.303 \\ 023256 & 1 & 1 & 03333 & 43 & 2.3 & 0.2326 & 0.1587 \\ 043478 & 3.003 & 30003 & 1 & 67 & 4.3 & 5.3 & 0.2326 \\ 0.12987 & 0.23256 & 023256 & 014925 & 1 & 07692 & 0.1471 & 0.1149 \\ 0.18868 & 043478 & 043478 & 027027 & 22999 & 1 & 0.2326 & 0.1493 \\ 0.18868 & 043478 & 023256 & 33003 & 13001 & 1 & 0.2326 & 0.1493 \\ 2.2002 & 42992 & 018868 & 67981 & 42992 & 42992 & 1 & 0.3846 \\ 3.3003 & 63012 & 42992 & 87032 & 66979 & 66979 & 2.6001 & 1 \end{pmatrix}, CR = 0.099955. \tag{19}$$

In case of $d=0.01$,

$$\begin{pmatrix} 1 & 4.28 & 2.28 & 7.72 & 5.28 & 5.28 & 0.4386 & 0.3049 \\ 0.23364 & 1 & 0.3367 & 4.28 & 2.28 & 2.28 & 0.23360 & 0.1592 \\ 0.4386 & 2.97 & 1 & 6.72 & 3.72 & 4.32 & 5.28 & 0.2336 \\ 0.12953 & 0.23364 & 0.14881 & 1 & 0.4386 & 0.3049 & 0.1484 & 0.1147 \\ 0.18939 & 0.4386 & 0.26882 & 2.28 & 1 & 0.7812 & 0.2336 & 0.1488 \\ 0.18939 & 0.4386 & 0.23148 & 3.27981 & 1.2801 & 1 & 0.2336 & 0.1488 \\ 2.28 & 4.2808 & 0.18939 & 6.7385 & 4.2808 & 4.2808 & 1 & 0.3802 \\ 3.2798 & 6.2814 & 4.2808 & 8.7184 & 6.7204 & 6.7204 & 2.6302 & 1 \end{pmatrix}, \mathrm{CR} = 0.099825. \tag{20}$$

In case of $d=0.001$,

$$\begin{pmatrix} 1 & 4.282 & 2.282 & 7.718 & 5.282 & 5.282 & 0.4382 & 0.3047 \\ 0.23354 & 1 & 0.3364 & 4.282 & 2.282 & 2.282 & 0.2335 & 0.1592 \\ 0.43821 & 2.9727 & 1 & 6.718 & 3.718 & 4.328 & 5.282 & 0.2335 \\ 0.12957 & 0.23354 & 0.14885 & 1 & 0.4382 & 0.3047 & 0.1483 & 0.1147 \\ 0.18932 & 0.43821 & 0.26896 & 2.2821 & 1 & 0.78 & 0.2335 & 0.1489 \\ 0.18932 & 0.43821 & 0.23105 & 3.2819 & 1.2821 & 1 & 0.2335 & 0.1489 \\ 2.2821 & 4.2827 & 0.18932 & 6.7431 & 4.2827 & 4.2827 & 1 & 0.3802 \\ 3.2819 & 6.2814 & 4.2827 & 8.7184 & 6.7159 & 6.7159 & 2.6302 & 1 \end{pmatrix}, \mathrm{CR} = 0.099997. \tag{21}$$

The reconstituted PM using Xu and Wei method [23] is as follows:

$$\begin{pmatrix} 1 & 4.524 & 2.339 & 7.523 & 5.888 & 5.686 & 0.425 & 0.292 \\ 0.221 & 1 & 0.326 & 4.516 & 2.671 & 2.580 & 0.222 & 0.147 \\ 0.427 & 3.067 & 1 & 6.749 & 3.460 & 4.188 & 4.155 & 0.249 \\ 0.133 & 0.221 & 0.148 & 1 & 0.373 & 0.287 & 0.134 & 0.104 \\ 0.170 & 0.374 & 0.289 & 2.681 & 1 & 0.561 & 0.197 & 0.147 \\ 0.176 & 0.388 & 0.234 & 3.479 & 1.784 & 1 & 0.204 & 0.153 \\ 2.354 & 4.497 & 0.241 & 7.479 & 5.073 & 4.899 & 1 & 0.501 \\ 3.419 & 6.786 & 4.024 & 9.624 & 6.783 & 6.551 & 1.996 & 1 \end{pmatrix}, \mathrm{CR} = 0.096964. \tag{22}$$

The reconstituted PM using Cao method [24] is as follows:

$$\begin{pmatrix} 1 & 4.44412 & 2.3682 & 7.6743 & 5.8559 & 5.6079 & 0.4201 & 0.2968 \\ 0.2252 & 1 & 0.3210 & 4.4224 & 2.6175 & 3.5392 & 0.2268 & 0.1486 \\ 0.4223 & 3.1151 & 1 & 6.9149 & 3.5351 & 4.2774 & 4.5105 & 0.2487 \\ 0.1303 & 0.2261 & 0.1446 & 1 & 0.3805 & 0.2927 & 0.1313 & 0.1030 \\ 0.1708 & 0.3820 & 0.2829 & 2.6278 & 1 & 0.5722 & 0.1960 & 0.1444 \\ 0.1783 & 0.3938 & 0.2338 & 3.4166 & 1.7478 & 1 & 0.2055 & 0.1494 \\ 2.3804 & 4.4099 & 0.2217 & 7.6136 & 5.1012 & 4.8661 & 1 & 0.5004 \\ 3.3689 & 6.7293 & 4.0215 & 9.7130 & 6.9235 & 6.6949 & 1.9984 & 1 \end{pmatrix}, \mathrm{CR} = 0.107316. \tag{23}$$

The reconstituted PM using the proposed method is as follows ($\mathrm{CR}_0 = 0.1.$):

$$\begin{pmatrix} 1 & 5 & 3 & 7 & 6 & 6 & \frac{1}{3} & \frac{1}{4} \\ \frac{1}{5} & 1 & \frac{1}{3} & 5 & 3 & 3 & \frac{1}{5} & \frac{1}{7} \\ \frac{1}{3} & 3 & 1 & 6 & 3 & 4 & 1 & \frac{1}{5} \\ \frac{1}{7} & \frac{1}{5} & \frac{1}{6} & 1 & \frac{1}{3} & \frac{1}{4} & \frac{1}{7} & \frac{1}{8} \\ \frac{1}{6} & \frac{1}{3} & \frac{1}{3} & 3 & 1 & \frac{1}{2} & \frac{1}{5} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{3} & \frac{1}{4} & 4 & 2 & 1 & \frac{1}{5} & \frac{1}{6} \\ 3 & 5 & 1 & 7 & 5 & 5 & 1 & \frac{1}{2} \\ 4 & 7 & 5 & 8 & 6 & 6 & 2 & 1 \end{pmatrix}, \mathrm{CR} = 0.087973. \tag{24}$$

Table 3 shows the comparison results between some previous and proposed consistency improvement methods. The CR, *dCR,* and *dCRr* are, respectively, 0.087973, 0.081114, and 0.013905 by modifying only two elements with the *dM* of 5.833333 using the proposed method. The CR, *dCR,* and *dCRr* are, respectively, 0.096964, 0.072123, and 0.004792 by

Table 3: Comparison results between some consistency improvement methods about Example 1.

Performance measure	ANT-AHP [21]	Wu et al. [22]			Zeshui and Cuiping [23]	Cao et al. [24]	Proposed method
		$d = 0.1$	$d = 0.01$	$d = 0.001$			
CR	0.099838	0.099955	0.099825	0.099997	0.096964	0.107316	0.087973
dCR	0.069248	0.069130	0.069262	0.069090	0.072123	0.061771	0.081114
dM	14.842974	19.457000	20.009000	19.956000	15.050557	16.605557	5.833333
dCRr	0.004665	0.003553	0.0034615	0.003462	0.004792	0.003720	0.013905
Number of modified elements	46	54	56	56	56	56	2

modifying 56 elements with the *dM* of 15.050557 using the previous Cao's method.

Example 2. The primary inconsistent PM [26] is as follows:

$$\begin{pmatrix} 1 & \frac{1}{3} & \frac{1}{5} & 1 & \frac{1}{4} & 2 & 3 \\ 3 & 1 & \frac{1}{2} & 2 & \frac{1}{3} & 3 & 3 \\ 5 & 2 & 1 & 4 & 5 & 6 & 5 \\ 1 & \frac{1}{2} & \frac{1}{4} & 1 & \frac{1}{4} & 1 & 2 \\ 4 & 3 & \frac{1}{5} & 4 & 1 & 3 & 1 \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{6} & 1 & \frac{1}{3} & 1 & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{5} & \frac{1}{2} & 1 & 3 & 1 \end{pmatrix}, \mathrm{CR} = 0.112090. \tag{25}$$

The reconstituted PM using the Benítez method [26] is as follows:

$$\begin{pmatrix} 1 & 0.526 & 0.154 & 0.794 & 0.471 & 1.738 & 1.17 \\ 1.902 & 1 & 0.293 & 1.510 & 0.896 & 3.306 & 2.225 \\ 6.487 & 3.411 & 1 & 5.149 & 3.055 & 11.28 & 7.590 \\ 1.260 & 0.662 & 0.194 & 1 & 0.593 & 2.190 & 1.474 \\ 2.123 & 1.116 & 0.327 & 1.685 & 1 & 3.691 & 2.484 \\ 0.575 & 0.302 & 0.089 & 0.457 & 0.271 & 1 & 0.673 \\ 0.855 & 0.449 & 0.132 & 0.678 & 0.403 & 1.486 & 1 \end{pmatrix}, \quad \mathrm{CR} = 0.099677. \tag{26}$$

The reconstituted PMs using the proposed method are as follows:

In case of $\mathrm{CR}_0 = 0.01$,

$$\begin{pmatrix} 1 & \frac{1}{3} & \frac{1}{5} & 1 & \frac{1}{4} & 1 & 1 \\ 3 & 1 & \frac{1}{2} & 2 & 1 & 3 & 3 \\ 5 & 2 & 1 & 4 & 2 & 6 & 5 \\ 1 & \frac{1}{2} & \frac{1}{4} & 1 & \frac{1}{4} & 1 & 1 \\ 4 & 1 & \frac{1}{2} & 4 & 1 & 3 & 2 \\ 1 & \frac{1}{3} & \frac{1}{6} & 1 & \frac{1}{3} & 1 & 1 \\ 1 & \frac{1}{3} & \frac{1}{5} & 1 & \frac{1}{2} & 1 & 1 \end{pmatrix}, \mathrm{CR} = 0.009261. \tag{27}$$

In case of $\mathrm{CR}_0 = 0.005$,

$$\begin{pmatrix} 1 & \frac{1}{3} & \frac{1}{5} & 1 & \frac{1}{4} & 1 & 1 \\ 3 & 1 & \frac{1}{2} & 2 & 1 & 3 & 3 \\ 5 & 2 & 1 & 4 & 2 & 6 & 5 \\ 1 & \frac{1}{2} & \frac{1}{4} & 1 & \frac{1}{3} & 1 & 1 \\ 4 & 1 & \frac{1}{2} & 3 & 1 & 3 & 3 \\ 1 & \frac{1}{3} & \frac{1}{6} & 1 & \frac{1}{3} & 1 & 1 \\ 1 & \frac{1}{3} & \frac{1}{5} & 1 & \frac{1}{3} & 1 & 1 \end{pmatrix}, \mathrm{CR} = 0.0004667. \tag{28}$$

In case of $\mathrm{CR}_0 = 0.001$,

$$\begin{pmatrix} 1 & \frac{1}{3} & \frac{1}{5} & 1 & \frac{1}{3} & 1 & 1 \\ 3 & 1 & \frac{1}{2} & 3 & 1 & 3 & 3 \\ 5 & 2 & 1 & 5 & 2 & 6 & 5 \\ 1 & \frac{1}{3} & \frac{1}{5} & 1 & \frac{1}{3} & 1 & 1 \\ 3 & 1 & \frac{1}{2} & 3 & 1 & 3 & 3 \\ 1 & \frac{1}{3} & \frac{1}{6} & 1 & \frac{1}{3} & 1 & 1 \\ 1 & \frac{1}{3} & \frac{1}{5} & 1 & \frac{1}{3} & 1 & 1 \end{pmatrix}, \mathrm{CR} = 0.000772. \quad (29)$$

Table 4 shows the comparison result between Benítez method [26] and the proposed method.

The CR, *dCR,* and *dCRr* are, respectively, 0.000772, 0.111318, and 0.005214 by modifying 22 elements with the *dM* of 21.35 using the proposed method ($\mathrm{CR}_0 = 0.001$). The CR, *dCR,* and *dCRr* are, respectively, 0.000007693, 0.112082, and 0.003194 by modifying 42 elements with the *dM* of 35.096667 using the previous Benítez's method.

As can be seen in Tables 3 and 4, the proposed method is much better than the previous ones from the viewpoints of CR, *dCR*, *dM*, *dCRr,* and the number of modified elements.

3.2. Approximate Polynomial for Random Consistency Index (RI) According to the Number of Attributes. We develop approximate polynomial for *RI* according to the number of attributes by using the proposed method in Section 3.2.

Table 5 and Figures 2 and 3 show the performance test results of approximate polynomials for RI with different orders.

From Table 5 and Figures 2 and 3, we can know that third-order approximate polynomial is appropriate from the viewpoints of the performance and complexity.

Third-order approximate polynomial for RI is as follows:

$$RI = 0.001311n^3 - 0.045332n^2 + 0.533262n - 0.615824. \quad (30)$$

Table 6 shows the performance test result of the above third-order approximate polynomial.

As the result, the approximate formula of CR is as follows:

$$CR = \frac{CI}{RI} = \frac{\lambda_{\max} - n}{(n-1)RI} = \frac{(\lambda_{\max} - n)}{\left[(n-1)\left(0.00311n^3 - 0.045332n^2 + 0.533262n - 0.615824\right)\right]}. \quad (31)$$

By using (30) and (31), we can directly calculate the RI value and CR value without the numerical table for RI values according to the number of attributes and evaluate the consistency of the constituted PM rapidly.

On the other hand, we test the performance of the following previous approximate formula for RI: [4].

$$RI = \frac{1.98\,(n-2)}{n}. \quad (32)$$

Table 7 shows the performance test result of the previous approximate formula (32).

From Tables 6 and 7, we know that the proposed third-order approximate polynomial (30) has much better performance than the previous approximation formula (32).

3.3. Application to Hip Joint Prosthesis Material Selection. We use the proposed consistency improvement method of PM based on CR decreasing rate to hip joint prosthesis material selection [27].

The alternative materials for hip joint prosthesis are, respectively, stainless steels 316 (A1), stainless steels 317 (A2), stainless steels 321 (A3), stainless steels 347 (A4), Co-Cr alloys-cast alloy (A5), Co-Cr alloys-wrought alloy (A6), unalloyed titanium (A7), Ti-6Al-4V (A8), composites (fabric reinforced)-epoxy-70% glass (A9), composites (fabric reinforced)-epoxy-63% carbon (A10), and composites (fabric reinforced)-epoxy-62% aramid (A11). The material attributes are, respectively, tissue tolerance (TT), corrosion resistance (CR), tensile strength (TS) (MPa), fatigue strength (FS) (MPa), relative toughness (RT), relative wear resistance (RWR), elastic modulus (EM) (GPa), specific gravity (SG) (g/cc), and cost (C). Table 8 shows some hip joint prosthesis materials and their properties. [27].

First, we conduct the correlation analysis between the properties of hip joint prosthesis materials.

Table 9 shows the correlation coefficients between properties of hip joint prosthesis materials. As shown in Table 9, the elastic modulus and specific gravity have the high correlation coefficients with the tissue tolerance, fatigue strength, and relative wear resistance.

Therefore, we remove two properties such as EM (elastic modulus) and SG (specific gravity) from the consideration.

Consequently, we constitute the decision matrix with seven attributes: TT, CR, TS, FS, RT, RWR, and C. Table 10 shows the decision matrix.

Table 4: Comparison result between Benítez method and the proposed method about Example 2.

Performance measure	Benítez method [26]	Proposed method		
	$CR_0 = 0.005$	$CR_0 = 0.01$	$CR_0 = 0.005$	$CR_0 = 0.001$
CR	0.000007693	0.009261	0.004667	0.000772
dCR	0.112082	0.10283	0.10742	0.111318
dM	35.096667	15.467	18.05	21.35
dCRr	0.003194	0.006648	0.0059514	0.005214
Number of modified elements	42	14	16	22

Table 5: Performance test results of approximate polynomials for RI with different orders.

m	Mean absolute error	Maximum of absolute errors	Mean relative error (%)	Maximum of relative errors (%)	Mean-squared error (%)	Maximum of squared errors (%)
1	0.11101	0.33637	10.756	57.995	0.018312	0.11315
2	0.046806	0.11791	4.3837	20.33	0.0031264	0.013903
3	0.016633	0.038921	1.5316	5.4092	0.00040314	0.0015149
4	0.0078826	0.0228	0.6634	1.7273	$9.4729 \cdot 10^{-5}$	0.00051983
5	0.0056381	0.014733	0.42149	1.1161	$4.8904 \cdot 10^{-5}$	0.00021705
6	0.0049297	0.01272	0.3941	0.96365	$3.9756 \cdot 10^{-5}$	0.0001618

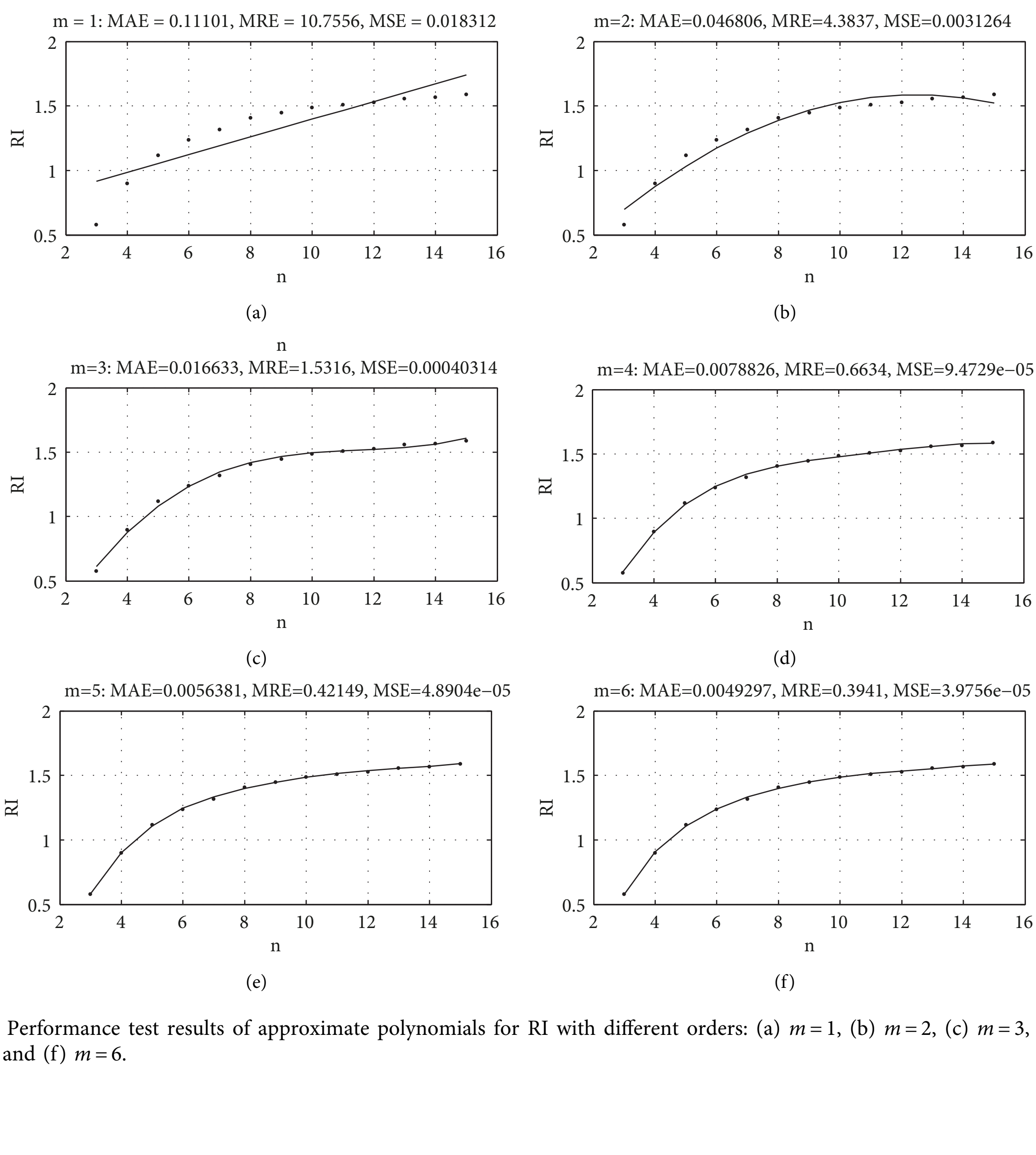

Figure 2: Performance test results of approximate polynomials for RI with different orders: (a) $m = 1$, (b) $m = 2$, (c) $m = 3$, (d) $m = 4$, (e) $m = 5$, and (f) $m = 6$.

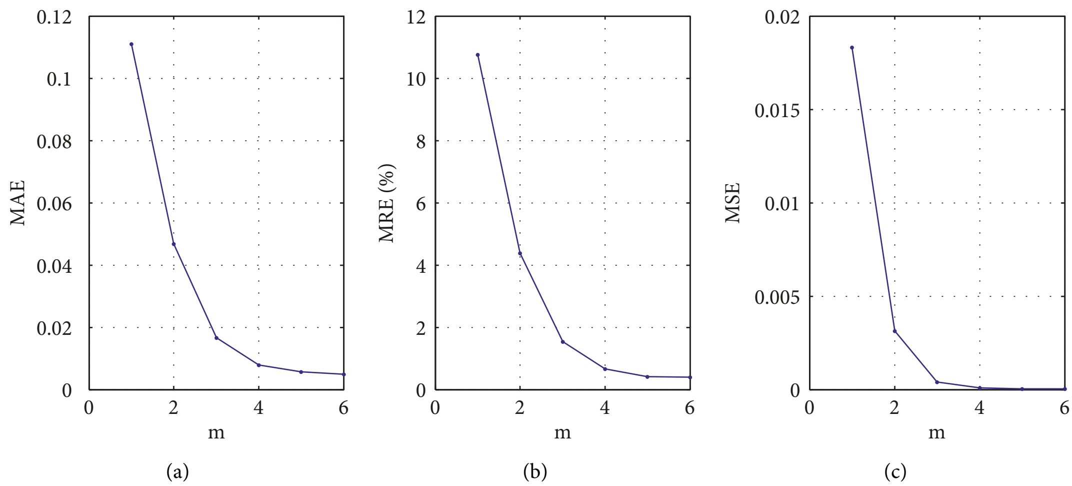

Figure 3: MAE, MRE, and MSE values of the approximate polynomials for RI according to different orders: (a) MAE, (b) MRE, and (c) MSE.

Table 6: Performance test result of the third-order approximate polynomial for RI (equation (21)).

n	RI	Calculated RI	Absolute error	Relative error (%)	Squared error
3	0.58	0.61137	0.031374	5.4092	0.0009843
4	0.9	0.87582	0.024176	2.6862	0.00058447
5	1.12	1.0811	0.038921	3.4751	0.0015149
6	1.24	1.235	0.004995	0.40282	$2.495{\cdot}10^{-5}$
7	1.32	1.3455	0.02547	1.9295	0.0006487
8	1.41	1.4203	0.01034	0.73331	0.00010691
9	1.45	1.4675	0.017483	1.2057	0.00030564
10	1.49	1.4948	0.0047652	0.31981	$2.2707{\cdot}10^{-5}$
11	1.51	1.5101	$5.4945{\cdot}10^{-5}$	0.0036387	$3.019{\cdot}10^{-9}$
12	1.53	1.5212	0.0087812	0.57394	$7.711{\cdot}10^{-5}$
13	1.56	1.5361	0.023876	1.5305	0.00057007
14	1.57	1.5626	0.0073626	0.46896	$5.4208{\cdot}10^{-5}$
15	1.59	1.6086	0.018626	1.1715	0.00034694
Mean			0.016633	1.5316	0.00040314
Max			0.038921	5.4092	0.0015149

Table 7: Performance test result of the previous approximate formula for RI (equation (23)).

n	RI	Calculated RI	Absolute error	Relative error (%)	Squared error
3	0.58	0.66	0.08	13.793	0.0064
4	0.9	0.99	0.09	10	0.0081
5	1.12	1.188	0.068	6.0714	0.004624
6	1.24	1.32	0.08	6.4516	0.0064
7	1.32	1.4143	0.094286	7.1429	0.0088898
8	1.41	1.485	0.075	5.3191	0.005625
9	1.45	1.54	0.09	6.2069	0.0081
10	1.49	1.584	0.094	6.3087	0.008836
11	1.51	1.62	0.11	7.2848	0.0121
12	1.53	1.65	0.12	7.8431	0.0144
13	1.56	1.6754	0.11538	7.3964	0.013314
14	1.57	1.6971	0.12714	8.0983	0.016165
15	1.59	1.716	0.126	7.9245	0.015876
Mean			0.097678	7.6801	0.00991
Max			0.12714	13.793	0.016165

Table 8: Alternative materials and their attribute values [27].

Alternative material	TT	CR	TS	FS	RT	RWR	EM	SG	C
A1	10	7	517	350	8	8	200	8	1
A2	9	7	630	415	10	8.5	200	8	1.1
A3	9	7	610	410	10	8	200	7.9	1.1
A4	9	7	650	430	10	8.4	200	8	1.2
A5	10	9	655	425	2	10	238	8.3	3.7
A6	10	9	896	600	10	10	242	9.1	4
A7	8	10	550	315	7	8	110	4.5	1.7
A8	8	10	985	490	7	8.3	124	4.4	1.9
A9	7	7	680	200	3	7	22	2.1	3
A10	7	7	560	170	3	7.5	56	1.6	10
A11	7	7	430	130	3	7.5	29	1.4	5

Table 9: Correlation coefficients between properties of hip joint prosthesis materials.

	TT	CR	TS	FS	RT	RWR	EM	SG	C
TT	1.000	0.161	0.229	0.780	0.530	0.792	**0.966**	**0.961**	−0.479
CR		1.000	0.570	0.476	−0.008	0.484	0.175	0.101	−0.127
TS			1.000	0.729	0.287	0.465	0.300	0.268	−0.157
FS				1.000	0.681	0.792	**0.848**	**0.832**	−0.492
RT					1.000	0.251	0.624	0.688	−0.624
RWR						1.000	**0.815**	0.744	−0.129
EM							1.000	**0.985**	−0.472
SG								1.000	−0.576
C									1.000

In this table, the bold values indicate higher correlation coefficients (ñ>0.8) than others.

Table 10: Decision matrix.

Alternative materials	TT	CR	TS	FS	RT	RWR	C
A1	10	7	517	350	8	8	1
A2	9	7	630	415	10	8.5	1.1
A3	9	7	610	410	10	8	1.1
A4	9	7	650	430	10	8.4	1.2
A5	10	9	655	425	2	10	3.7
A6	10	9	896	600	10	10	4
A7	8	10	550	315	7	8	1.7
A8	8	10	985	490	7	8.3	1.9
A9	7	7	680	200	3	7	3
A10	7	7	560	170	3	7.5	10
A11	7	7	430	130	3	7.5	5

As shown in Table 10, there are 11 alternative materials with 7 attributes. Excepting C (cost), six attributes are benefit attributes among seven attributes, while C (cost) is cost attribute.

The material attribute weights are determined by using the AHP with simplest questionnaire by three decision makers. Table 11 shows the simplest pairwise comparison questionnaire completed by the first decision maker.

From Table 11, the first decision maker's PM is as follows:

$$A^{(1)} = \begin{pmatrix} 1 & 1 & 5 & 5 & 3 & 5 & 5 \\ 1 & 1 & 5 & 5 & 3 & 5 & 5 \\ \frac{1}{5} & \frac{1}{5} & 1 & \frac{1}{5} & \frac{1}{5} & \frac{1}{3} & \frac{1}{5} \\ \frac{1}{5} & \frac{1}{5} & 5 & 1 & \frac{1}{3} & 3 & 5 \\ \frac{1}{3} & \frac{1}{3} & 5 & 3 & 1 & 3 & 5 \\ \frac{1}{5} & \frac{1}{5} & 3 & \frac{1}{3} & \frac{1}{3} & 1 & 3 \\ \frac{1}{5} & \frac{1}{5} & 5 & \frac{1}{5} & \frac{1}{5} & \frac{1}{3} & 1 \end{pmatrix}. \quad (33)$$

The consistency test result of the PM $A^{(1)}$ is as follows:

$$\lambda_{\max} = 7.913828, \mathrm{CI} = 0.152305, \mathrm{CR} = 0115382 > 01. \quad (34)$$

Therefore, we should modify the PM $A^{(1)}$ until $\mathrm{CR} < 0.1$ using the proposed consistency improvement method.

Table 11: Simplest pairwise comparison questionnaire completed by the first decision maker.

	Tissue tolerance	Corrosion resistance	Tensile strength	Fatigue strength	Relative toughness	Relative wear resistance	Cost
Tissue tolerance	1	1	5	5	3	5	5
Corrosion resistance		1	5	5	3	5	5
Tensile strength			1	5	5	3	5
Fatigue strength				1	3	3	5
Relative toughness					1	3	5
Relative wear resistance						1	3
Cost							1

It indicates that the value is the maximum. I have no objection to remove the Bold type.

In the first iteration, we first reconstitute the PM by replacing all elements of $A^{(1)}$ with the lower neighbouring scales and calculate CR values and CR decreasing rates (Table 12).

From Table 12, the maximum CR decreasing rate is 0.0077709 ($CR = 0.10632$) when we replace the element $a_{67} = 3$ in the primary PM with the lower neighbouring scale 2.

We next reconstitute the PM by replacing all elements of $A^{(1)}$ with the upper neighbouring scales and calculate the CR values and CR decreasing rates (Table 13).

From Table 13, the maximum CR decreasing rate is 0.010371 ($CR = 0.10449$) when we replace the element $a_{37} = 1/5$ in the primary PM with the upper neighbouring scale 1/4.

As $0.0077709 < 0.010371$, we decide to modify the element $a_{37} = 1/5$ of the primary PM to the upper neighbouring scale 1/4. The reconstituted PM is as follows:

$$B_1^{(1)} = \begin{pmatrix} 1 & 1 & 5 & 5 & 3 & 5 & 5 \\ 1 & 1 & 5 & 5 & 3 & 5 & 5 \\ \frac{1}{5} & \frac{1}{5} & 1 & \frac{1}{5} & \frac{1}{5} & \frac{1}{3} & \frac{1}{4} \\ \frac{1}{5} & \frac{1}{5} & 5 & 1 & \frac{1}{3} & 3 & 5 \\ \frac{1}{3} & \frac{1}{3} & 5 & 3 & 1 & 3 & 5 \\ \frac{1}{5} & \frac{1}{5} & 3 & \frac{1}{3} & \frac{1}{3} & 1 & 3 \\ \frac{1}{5} & \frac{1}{5} & 4 & \frac{1}{5} & \frac{1}{5} & \frac{1}{3} & 1 \end{pmatrix}, \mathrm{CI} = 010449 > 01. \tag{35}$$

As $CR = 0.10449 > 0.1$, we should modify the PM $B_1^{(1)}$ until $CR < 0.1$ again.

In the next iteration, we first reconstitute the PM $B_1^{(1)}$ by replacing all elements with the lower neighbouring scales and calculate the CR values and CR decreasing rates (Table 14).

From Table 14, the maximum CR decreasing rate is 0.0071198 ($CR = 0.096187$) when we replace the element $a_{67} = 3$ with the lower neighbouring scale 2.

We next reconstitute the PM by replacing all elements with the upper neighbouring scales and calculate the CR values and CR decreasing rates (Table 15).

From Table 15, the maximum CR decreasing rate is 0.010142 ($CR = 0.093506$) when we replace the element $a_{37} = 1/4$ with the upper neighbouring scale 1/3. As $0.0071198 < 0.010142$, we decide to modify the element $a_{37} = 1/4$ to the upper neighbouring scale 1/3. The reconstituted PM is as follows:

$$B^{(1)} = \begin{pmatrix} 1 & 1 & 5 & 5 & 3 & 5 & 5 \\ 1 & 1 & 5 & 5 & 3 & 5 & 5 \\ \frac{1}{5} & \frac{1}{5} & 1 & \frac{1}{5} & \frac{1}{5} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{5} & \frac{1}{5} & 5 & 1 & \frac{1}{3} & 3 & 5 \\ \frac{1}{3} & \frac{1}{3} & 5 & 3 & 1 & 3 & 5 \\ \frac{1}{5} & \frac{1}{5} & 3 & \frac{1}{3} & \frac{1}{3} & 1 & 3 \\ \frac{1}{5} & \frac{1}{5} & 3 & \frac{1}{5} & \frac{1}{5} & \frac{1}{3} & 1 \end{pmatrix}, \mathrm{CR}^{(1)} = 0.093506. < 0.1. \tag{36}$$

As $CR = 0.093506 < 0.1$, the reconstituted PM $B^{(1)}$ satisfies the consistency.

By the similar way, the reconstituted PMs based on the PMs constituted by the second and third decision makers are as follows:

Table 12: CR decreasing rates by replacing all elements with the lower neighbouring scales.

$dC\ Rr^{-}_{pq}$	2	3	4	5	6	7
1	−0.004171	−0.0077567	0.0038198	0.0042379	0.00012511	−0.0028697
2		−0.0077567	0.0038198	0.0042379	0.00012511	−0.0028697
3			−0.0022949	0.00080871	−0.0031613	−0.010273
4				−0.0077696	0.0061679	0.0062029
5					0.00035439	0.0018634
6						**0.0077709**

It indicates that the value is the maximum. I have no objection to remove the Bold type.

Table 13: CR decreasing rates by replacing all elements with the upper neighbouring scales.

$dC\ Rr^{+}_{pq}$	2	3	4	5		
1	−0.004171	0.0051298	−0.004292	−0.0058214	−0.001012	0.0013276
2		0.0051298	−0.004292	−0.0058214	−0.001012	0.0013276
3			0.0014299	−0.0021998	0.0007321	**0.010371**
4				0.0063837	−0.0075791	−0.0065044
5					−0.0025883	−0.0025898
6						−0.0090527

It indicates that the value is the maximum. I have no objection to remove the Bold type.

Table 14: CR decreasing rates by replacing all elements with the lower neighbouring scales.

$dC\ Rr^{-}_{pq}$	2	3	4	5	6	7
1	−0.0042129	−0.007257	0.0037917	0.0042349	$5.7868e-05$	−0.0032652
2		−0.007257	0.0037917	0.0042349	$5.7868e-05$	−0.0032652
3			−0.0026355	0.00050903	−0.0036426	−0.010371
4				−0.0078255	0.0062676	0.0057371
5					0.00031694	0.0014975
6						**0.0071198**

Table 15: CR decreasing rates by replacing all elements with the upper neighbouring scales.

$dC\ Rr^{+}_{pq}$	2	3	4	5	6	7
1	−0.0042129	0.0047588	−0.0042749	−0.0058397	−0.00096572	0.0016549
2		0.0047588	−0.0042749	−0.0058397	−0.00096572	0.0016549
3			0.0018419	−0.0018135	0.0013729	**0.010142**
4				0.0064268	−0.0076851	−0.0060728
5					−0.0025853	−0.0022606
6						−0.0084573

$$B^{(2)} = \begin{pmatrix} 1 & 1/3 & 7 & 5 & 3 & 5 & 5 \\ 3 & 1 & 5 & 5 & 3 & 5 & 7 \\ \frac{1}{7} & \frac{1}{7} & 1 & \frac{1}{3} & \frac{1}{5} & 1 & 1 \\ \frac{1}{5} & \frac{1}{5} & 3 & 1 & \frac{1}{3} & 3 & 5 \\ \frac{1}{3} & \frac{1}{3} & 5 & 3 & 1 & 3 & 5 \\ \frac{1}{5} & \frac{1}{5} & 1 & \frac{1}{3} & \frac{1}{3} & 1 & 3 \\ \frac{1}{5} & \frac{1}{7} & 1 & \frac{1}{5} & \frac{1}{5} & \frac{1}{3} & 1 \end{pmatrix}, \mathrm{CR}^{(2)} = 0.071796 < 0.1,$$

$$B^{(3)} = \begin{pmatrix} 1 & 3 & 7 & 5 & 5 & 7 & 5 \\ \frac{1}{3} & 1 & 7 & 5 & 5 & 7 & 5 \\ \frac{1}{7} & \frac{1}{7} & 1 & \frac{1}{3} & \frac{1}{5} & 1 & \frac{1}{3} \\ \frac{1}{5} & \frac{1}{5} & 3 & 1 & \frac{1}{3} & 3 & 3 \\ \frac{1}{5} & \frac{1}{5} & 5 & 3 & 1 & 5 & 3 \\ \frac{1}{7} & \frac{1}{7} & 1 & \frac{1}{3} & \frac{1}{5} & 1 & 1 \\ \frac{1}{5} & \frac{1}{5} & 3 & \frac{1}{3} & \frac{1}{3} & 1 & 1 \end{pmatrix}, \mathrm{CR}^{(3)} = 0.076189 < 0.1. \quad (37)$$

Based on the CR values {0.093506, 0.071796, 0.076189}, three decision makers' levels are determined as follows:

$$\begin{aligned} h1 &= 0.28331, \\ h2 &= 0.36898, \\ h3 &= 0.34771. \end{aligned} \quad (38)$$

By using (31), the composite PM is as follows:

$$B^{(0)} = \begin{pmatrix} 1 & 09769 & 63635 & 5 & 35831 & 56206 & 5 \\ 10236 & 1 & 56206 & 5 & 35831 & 56206 & 56609 \\ 015715 & 017792 & 1 & 028842 & 02 & 073253 & 049995 \\ 02 & 02 & 34672 & 1 & 033333 & 3 & 41863 \\ 027909 & 027909 & 5 & 3 & 1 & 35831 & 41863 \\ 017792 & 017792 & 13651 & 033333 & 027909 & 1 & 20475 \\ 02 & 017665 & 2.0002 & 0.23887 & 0.23887 & 0.4884 & 1 \end{pmatrix}. \quad (39)$$

By transforming the elements of $B^{(0)} = (b_{ij}^{(0)})_{n\times n}$ to the nearest neighbouring 9-point scales, the final PM is as follows:

$$A^{(0)} = \begin{pmatrix} 1 & 1 & 6 & 5 & 4 & 6 & 5 \\ 1 & 1 & 6 & 5 & 4 & 6 & 6 \\ \frac{1}{6} & \frac{1}{6} & 1 & \frac{1}{4} & \frac{1}{5} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{5} & \frac{1}{5} & 4 & 1 & \frac{1}{3} & 3 & 4 \\ \frac{1}{4} & \frac{1}{4} & 5 & 3 & 1 & 4 & 4 \\ \frac{1}{6} & \frac{1}{6} & 2 & \frac{1}{3} & \frac{1}{4} & 1 & 2 \\ \frac{1}{5} & \frac{1}{6} & 2 & \frac{1}{4} & \frac{1}{4} & \frac{1}{2} & 1 \end{pmatrix}. \quad (40)$$

Table 16: Comprehensive scores of the alternatives from three MADMs.

MADMs	Alternative materials										
	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	A11
SAW	0.532	0.491	0.480	0.493	0.680	0.874	0.617	0.680	0.078	0.042	0.049
TOPSIS	0.508	0.439	0.437	0.439	0.677	0.797	0.608	0.621	0.080	0.045	0.059
GRA	0.640	0.574	0.570	0.575	0.701	0.847	0.652	0.694	0.360	0.344	0.348

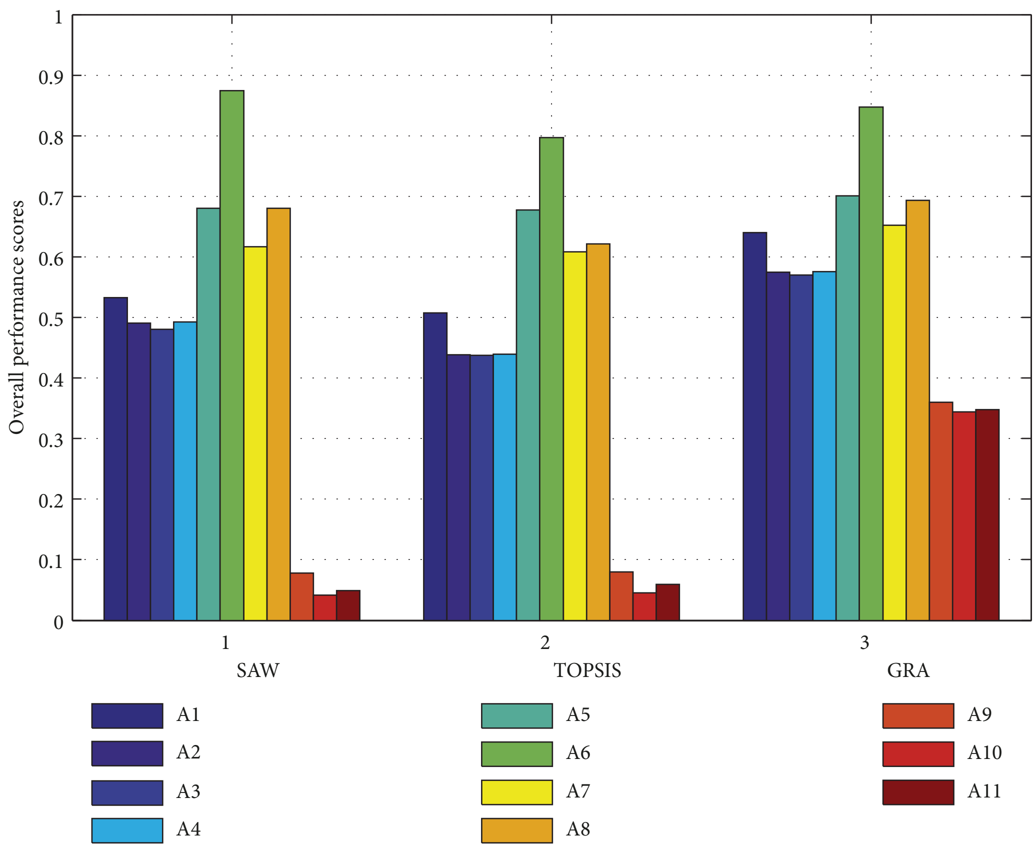

Figure 4: Comprehensive scores of the alternatives from three MADMs.

The CR value of the final PM $A^{(0)}$ is $CR^{(0)} = 0.064623$, and therefore the final PM satisfies the consistency.

The final attribute weights calculated from the final PM $A^{(0)}$ are as follows:

$$0.31665, 0.32205, 0.031323, 0.093804, 0.14673, 0.048864, 0.040575. \quad (41)$$

With these attribute weights, we can select the best hip joint prosthesis material using the well-known three MADMs such as SAW, TOPSIS, and GRA.

Table 16 and Figure 4 show the comprehensive scores of the alternatives from three MADMs.

Table 17 shows the ranking of the comprehensive scores of the alternatives from three MADMs.

From Table 17, we know that the final ranking of the alternative materials is as follows:

$$A6 > A5 > A8 > A7 > A1 > A4 > A2 > A3 > A9 > A11 > A10. \quad (42)$$

Therefore, we can select A6 (Co-Cr alloys-wrought alloy) as the best hip joint prosthesis material, and the next are A5 (Co-Cr alloys-cast alloy), A8 (Ti-6Al-4V), A7 (unalloyed titanium), and so on.

Table 17: Ranking of the alternatives using three MADMs.

MADMs	Alternative materials										
	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	A11
SAW	5	7	8	6	2	1	4	3	9	11	10
TOPSIS	5	7	8	6	2	1	4	3	9	11	10
GRA	5	7	8	6	2	1	4	3	9	11	10
Final ranks	5	7	8	6	2	1	4	3	9	11	10

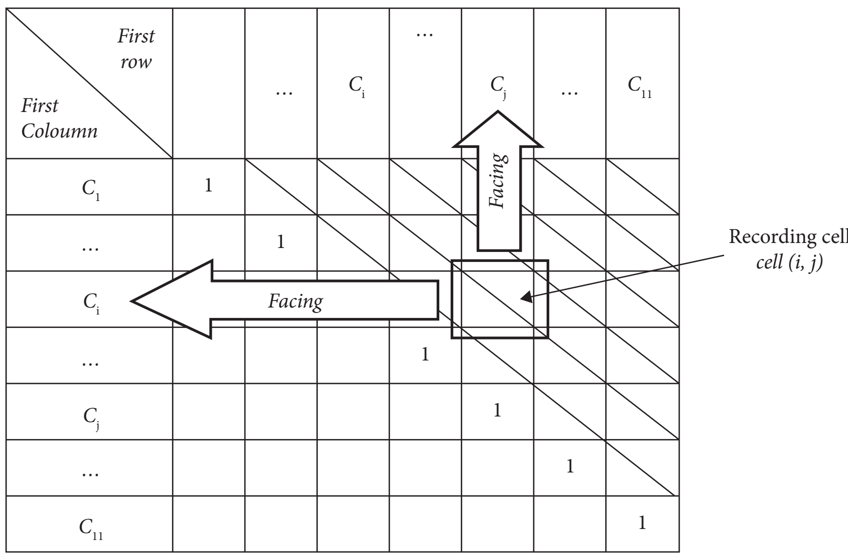

Figure 5: Recording method of the pairwise comparison judgment value on the recording cell cell(i, j) [5].

Table 18: Recording value according to verbal judgment.

Verbal judgment	Recording value
Equal importance	1
Weak importance	3
Strong importance	5
Demonstrated importance	7
Extreme importance	9
Intermediate values between two adjacent judgments	2, 4, 6, 8

4. Conclusions

We proposed a new consistency improvement method of inconsistent PM based on CR decreasing rate, developed an approximate polynomial for RI according to the number of attributes, and proposed a method to determine final PM and final attribute weights considered decision makers' levels. We compared the performances of the proposed and previous consistency improvement methods with two numerical examples, and then applied the proposed methods to determine material attribute weights in hip joint prosthesis material selection.

The main conclusions are as follows:

(i) The CR decreasing rate-based consistency improvement method of PM improves the consistency better with much smaller modification amounts and guarantees a very good balance between improvement of consistency and preservation of primary information.

(ii) The approximate polynomial for RI enables to calculate the RI value according to the number of attributes without the numerical table for RI.

(iii) The proposed method for constituting the final PM and determining the final attribute weights enables to reflect the decision makers' levels well, and it is useful for group AHP.

The limitation of this work is that we do not deal with the uncertainty of the pairwise comparison judgments. To handle the fuzziness and vagueness effectively, elicit the decision makers' knowledge and develop more effective decision-making model; future work needs to introduce the hesitant fuzzy set, complex intuitionistic fuzzy set, and so on.

Appendix

A. Preliminaries

A.1. Method to determine attribute weights using AHP with simplest questionnaire. Let considered attributes be noted $C_1, \ldots, C_j, \ldots, C_n$ and the weights of these attributes be noted $w_1, \ldots, w_j, \ldots, w_n$, where n is the number of the attributes and w_{jj} is the weight of j-th attribute C_j.

The main steps for determining the attribute weights using AHP with simplest questionnaire are as follows: [5]

Step 1. Record the pairwise comparison judgment values for the importance of the attributes in the simplest questionnaire.

In the simplest questionnaire, the attributes $C_1, \ldots, C_j, \ldots, C_n$ are listed in the same order at the first row and first column, respectively. The cell with slash "\" () is called recording cell. Figure 5 shows the recording method of the pairwise comparison judgment value. The recording cell cell(i, j) corresponds to i-th attribute C_i at the first column and j-th attribute C_j at the first row $(i = \overline{1, n-1}, j = \overline{i+1, n})$.

Compare i-th attribute C_i at the first column and j-th attribute C_j at the first row facing the recording cell *cell*(i,j). If i-th attribute C_i at the first column is more important over j-th attribute C_j at the first row, then record the judgment value a in the left area of the recording cell as . If j-th attribute C_j at the first row is more important over i-th attribute C_i at the first column, then record the judgment value a in the right area of the recording cell as . If two attributes C_i and C_j have an equal importance, then record the number 1 in the arbitrary area. Table 18shows the recording value according to the verbal judgment.

By the similar way, record all the recording cells in the simplest questionnaire.

Step 2. Constitute the PM $A = (a_{ij})_{n\times n}$ based on the complete questionnaire.

The recording cell *cell*(i,j) corresponds to the element a_{ij} of the upper triangular matrix in the PM A. If the state of *cell*(i,j) is , then a_{ij} is a $(a_{ij} = a)$. If the state of cell(i, j) is , then a_{ij} is $1/a$ $(a_{ij} = 1/a)$. From the states of all the recording cells, all the elements of the upper triangular matrix are determined in A. The lower triangular elements are always the positive reciprocal of the upper triangular elements $(a_{ji} = 1/a_{ij})$. All the principal diagonal elements are equal to 1. $(a_{ii} = 1;\ i = \overline{1, n})$

Step 3. Determine the principal eigenvector $v = (v_1, \ldots, v_j, \ldots, v_n)^T$ of $A = (a_{ij})_{n\times n}$ using the eigenvector method.

The principal eigenvector $v = (v_1, \ldots, v_j, \ldots, v_n)^T$ can be determined from the following equation:

$$A_{\mathbf{v}} = \lambda_{\max}\mathbf{v}, \tag{A.1}$$

where $\lambda_{\max}$ is the maximum eigenvalue of A and $\mathbf{v}$ is the corresponding eigenvector.

Step 4. Test the consistency of A using the consistency ratio (CR):

$$\mathrm{CR} = \frac{\mathrm{CI}}{\mathrm{RI}}, \tag{A.2}$$

where CI is the consistency index of A. It is calculated as follows:

$$\mathrm{CI} = \frac{(\lambda_{\max} - n)}{(n-1)}. \tag{A.3}$$

RI is the random consistency index (RI) (Table 2). If $\mathrm{CR} < \mathrm{CR}_0$, then the PM satisfies the consistency. Otherwise, the PM should be modified. CR_0 is the consistency threshold value. Usually, CR_0 is 0.1.

Step 5. Calculate the attribute weights $w_1 \ldots, w_j \ldots, w_n$ by normalizing the principal eigenvector $v = (v_1, \ldots, v_j, \ldots, v_n)^T$ of the consistent PM A as follows:

$$w_j = \frac{v_j}{\sum_{k=1}^{n} v_k}, \quad j = \overline{1, n}. \tag{A.4}$$

A.2. Development method of approximate polynomial by least square method. To develop approximate polynomial $P_m(x) = a_0 + a_1 x + \ldots + a_{m-1}x^{m-1} + a_m x^m$ with a set of data (x_n, y_n), $n = l, 2, \ldots, N$, least square method could be used [29, 30].

It requires to determine the coefficients $a_0, a_1, \ldots, a_{m-1}, a_m$ to minimize the following function:

$$E = \sum_{n=1}^{N} \left(y_n - P_m(x_n)\right)^2. \tag{A.5}$$

From $\partial E/\partial a_i = 0$; $i = \overline{0, m}$, we have a system of $m+1$ linear equations for $m+1$ unknowns a_i,

$$\begin{pmatrix} N & \sum_{n=1}^{N} x_n & \cdots & \sum_{n=1}^{N} x_n^m \\ \sum_{n=1}^{N} x_n & \sum_{n=1}^{N} x_n^2 & \cdots & \sum_{n=1}^{N} x_n^{m+1} \\ \cdots & \cdots & \cdots & \cdots \\ \sum_{n=1}^{N} x_n^m & \sum_{n=1}^{N} x_n^{m+1} & \cdots & \sum_{n=1}^{N} x_n^{2m} \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ \cdots \\ a_m \end{pmatrix} = \begin{pmatrix} \sum_{n=1}^{N} y_n \\ \sum_{n=1}^{N} x_n y_n \\ \cdots \\ \sum_{n=1}^{N} x_n^m y_n \end{pmatrix}. \tag{A.6}$$

This system will have a unique solution $a_0, a_1, \ldots, a_{m-1}, a_m$.

For evaluating the performance of the approximate polynomial, the mean absolute error (MAE), mean relative error (MRE), and mean squared error (MSE) are commonly used.

The MAE, MRE, and MSE of the approximate polynomial are as follows:

$$\mathrm{MAE}_m$$

$$\mathrm{MRE}_m = \frac{1}{N}\sum_{n=1}^{N}\left|\frac{y_n - P_m(x_n)}{y_n}\right| \times 100\%, \tag{A.7}$$

$$\mathrm{MSE}_m = \frac{1}{N}\sum_{n=1}^{N}\left[y_n - P_m(x_n)\right]^2.$$

The error may be reduced by increasing the polynomial order. However, for the high-order polynomial, the curve shows oscillatory behaviour. Therefore, the low-order polynomial with acceptable error is used commonly

A.3. Some well-known multiattribute decision-making methods. Multiattribute decision-making (MADM) method comprehensively evaluates the comprehensive scores of the alternatives based on multiple evaluating attributes and select the best one to have good performance from the alternatives. Simple additive weighting (SAW) method, technique for order preference by similarity to ideal solution (TOPSIS) method, and grey relational analysis (GRA) method are well-known MADMs [31–34].

Let the considered alternatives be $A_1, A_2, \ldots, A_n$ $(n \geq 2)$ and the evaluating attributes be $x_1, \ldots, x_k, \ldots, x_p$, where n and p are, respectively, the numbers of alternatives and attributes. The alternatives are evaluated on the basis of p attributes and their values constitute a decision-matrix $X = (x_{ik})_{n\times p}$, where x_{ik} is the performance value of k-th attribute for i-th alternative $(i = \overline{1,n}, k = \overline{1,p})$.

A.3.1. SAW. The main steps of the SAW method are as follows [33, 34]:

Step 1. Constitute a normalized decision-matrix $Z = (z_{ik})_{n\times p}$ from the decision-matrix $X = (x_{ik})_{n\times p}$.

There are some available normalization formulas such as vector normalization formula, linear sum-based normalization formula, linear ratio-based normalization formula, and linear min-max normalization formula [31, 34]. In this paper, the following linear min-max normalization formula is applied.

$$z_{ik} = \begin{cases} \dfrac{(x_{ik} - x_{k\min})}{(x_{k\max} - x_{k\min})}; k \in K^+, \\ \\ \dfrac{(x_{k\max} - x_{ik})}{(x_{k\max} - x_{k\min})}; k \in K^-, \end{cases} \tag{A.8}$$

where K^+ and K^- are the sets of the indices for the benefit and cost attributes and $x_{k\min}$ and $x_{k\max}$ are the minimum and maximum values of k-th attribute, respectively.

Step 2. Calculate the following simple weighted sums of all the alternatives:

$$S_i = \sum_{k=1}^{p} w_k z_{ik}; \quad i = \overline{1,n}, \tag{A.9}$$

where w_k denotes the *k-th* attribute weight. $(w_k \geq 0, k = \overline{1,p}, \sum_{k=1}^{p} w_k = 1.)$

Step 3. Rank the alternatives in the descending order based on $S_1, \ldots, S_i, \ldots, S_n$, and select the alternative with the maximum value as the best one.

A.3.2. TOPSIS. The main steps of the TOPSIS method are as follows [32, 34, 35]:

Step 1. Constitute a normalized decision-matrix $Z = (z_{ik})_{n\times p}$ from the decision-matrix $X = (x_{ik})_{n\times p}$ using equation (A.10).

Step 2. Constitute the weighted normalized decision-matrix $V = (v_{ik})_{n\times p}$ as follows:

$$v_{ik} = w_k \times z_{ik}, \quad i = \overline{1,n}, k = \overline{1,p}. \tag{A.10}$$

Step 3. Determine the positive ideal solution (PIS) $V^+ = (v_1^+, \cdots, v_k^+, \cdots, v_p^+)$ and the negative ideal solution (NIS) $V^- = (v_1^-, \cdots, v_k^-, \cdots, v_p^-)$ as follows:

$$v_k^+ = \max_{1\leq i\leq n} v_{ik}, v_k^- = \min_{1\leq i\leq n} v_{ik}. \tag{A.11}$$

Step 4. Calculate the distances from the alternatives to the PIS and NIS as follows:

$$D_i^+ = \sqrt{\sum_{k=1}^{p}\left(v_k^+ - v_{ik}\right)^2}, D_i^- = \sqrt{\sum_{j=1}^{p}\left(v_k^- - v_{ik}\right)^2}; \quad i = \overline{1,n}. \tag{A.12}$$

Step 5. Calculate the relative closeness values of the alternatives as follows:

$$C_i = \frac{D_i^-}{\left(D_i^+ + D_i^-\right)}; \quad i = \overline{1,n}. \tag{A.13}$$

Step 6. Rank the alternatives in the descending order based on $C_1, \ldots, C_i, \ldots, C_n$, and select the alternative with the maximum value as the best one.

A.3.3. GRA. The main steps of the GRA method are as follows [34]:

Step 1. Constitute a normalized decision-matrix $Z = (zik)n \times p$ from $X = (xij)n \times p$ using equation (A.10).

Step 2. Set $\{(zi1, \ldots, zik, \ldots, zip); i = \overline{1,n}\}$ as n comparative sequences.

Step 3. Determine the PIS $Z^+ = (z_1^+, \cdots, z_k^+, \cdots, z_p^+)$ and set it as a reference sequence, where $z_k^+ = \max_{1\leq i\leq n} z_{ik}$.

Step 4. Calculate the maximum and minimum values of the absolute deviations between the PIS (reference sequence) $Z^+ = (z_1^+, \cdots, z_k^+, \cdots, z_p^+)$ and n alternatives (comparative sequences) $\{(z_{i1},\ldots,z_{ik},\ldots,r_{ip}); i = \overline{1,n}\}$ as follows:

$$\Delta_{\min} = \min_{1\le i\le n}\min_{1\le k\le p}\{\Delta_{ik}\}, \Delta_{\max} = \min_{1\le i\le n}\min_{1\le k\le p}\{\Delta_{ik}\}, \quad (A.14)$$

where

$$\Delta_{ik} = \left|z_k^+ - z_{ik}\right|; \quad i = \overline{1,n}, k = \overline{1,p}. \quad (A.15)$$

Step 5. Calculate the grey relational coefficients between j-th attributes of i-th alternatives and PIS as follows:

$$\xi_{ik} = \frac{\Delta_{\min} + \rho\cdot\Delta_{\max}}{\Delta_{ik} + \rho\cdot\Delta_{\max}}, \quad i = \overline{1,n}, k = \overline{1,p}, \quad (A.16)$$

where ρ is the distinguishing coefficient and it is normally selected as 0.5.

Step 6. Calculate the grey relational grades of the alternatives as follows:

$$\gamma_i = \sum_{k=1}^{p} w_k \xi_{ik}. \quad (A.17)$$

Step 7. Rank the alternatives in the descending order based on $\gamma_1, \ldots, \gamma_i, \ldots, \gamma_n$, and select the alternative with the maximum value as the best one.

Data Availability

Data used to support the findings of this work are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

This research received no specific grant from any funding agency in the public, commercial, or not for-profit sectors.

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