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Research Article

Theoretical Investigations of Gyrating Ion Beam-Driven Ion-Acoustic Wave Instability in Plasma Using First Order Perturbation Theory

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A Gyrating ion beam traveling through a plasma cylinder comprising of electrons, positive ions (K^+), and heavy $(C_7F_{14}^-)$ negative ions, through Cerenkov interaction, drive IAWs (ion-acoustic waves) to unstable mode. In this process, two ion-acoustic (IA) unstable wave modes: a fast (K^+) mode and a slow $(C_7F_{14}^-)$ mode in the presence of positive and heavy negative ions are observed in the plasma. Numerical calculations of the sound wave phase velocity, unstable mode frequency, and the growth rates have been carried out. The calculations show that the growth rate and unstable wave frequencies for both the IA wave modes increase with the increase in the relative density of used heavy negative ions. Moreover, the growth rate is measured as $1/3^{\text{rd}}$ of ion beam density power and the phase velocity rises with the concentration of heavy negative ions. There is a considerable effect of an increase in the magnetic field on the growth rate. The growth rate of the fast (K^+) mode increases with the magnetic field more rapidly as compared to the slow ion mode $(C_7F_{14}^-)$.

1. Introduction

Ion-acoustic wave instability study in a collisional and collisionless plasma is growing very fast these days. IAWs are the low-frequency longitudinal plasma density oscillations. A simple theory of ionic and electronic oscillations in ionized gases was presented first by Tonks and Langmuir [1]. Plasma-electron oscillations were explained in detail in the study. Damping of IAWs was presented by Buti [2] for both the electron and ion species creating the plasma. The author observed that in a plasma which is not having any magnetic field and containing weak collisions, the characteristic frequency changes due to electron-ion collisions and it is independent of the temperature ratio of ions to electrons. Von Goeler et al. [3] observed that a reduction in the Landau damping (collisionless) of an IA wave may be found by adding negative ions. Here, the ion waves give two modes "fast and slow." However, a negative ion plasma was first reported by Wong et al. [4] produced with the help of electron attachment. The diffusion properties of negative ion plasma were well explained by the authors. Later, Hershkowitz and Intrator [5] have improved the concept given by Wong et al. [4] and studied the beam-plasma interactions in positive-negative ion plasmas. Intrator and Hershkowitz [6] presented the experimental observations of beam-plasma interaction in positive ion-negative ion plasma. A fluid description of the growing mode was also presented which was found consistent with their experimental data. Sheehan and Rynn [7] studied the different ion sources which can be used to study strongly turbulent plasma. Rosenberg [8] explored dust acoustic instabilities and dust ion-acoustic instabilities in a dusty plasma which was not magnetized by using Vlasov theory. Ma and Yu [9] have established the theory of IAWs and also performed the Langmuir wave instabilities in unmagnetized complex plasmas.

The formation of IA shocks in collisionless plasma containing light or heavy negative ions were observed by Takeuchi et al. [10]. Vladimirov et al. [11] presented the theory of linear IAWs in complex or dusty plasma comprising negative ions and dust grains. D'Angelo and Song

[12] presented the analysis of the hybrid-hybrid mode in negative ion plasmas. Abraham and Sebastian [13] have studied the instabilities of IAWs in a plasma comprising of electrons, hydrogen positive, and negatively charged oxygen ions. Akbari-Moghanjoughi [14] has studied the nonlinear growth of ion-acoustic waves which was periodically driven in plasma containing electrons and ions having adiabatic as well as isothermal ion fluids. Winske et al. [15] observed ion-acoustic wave instability [1–30] as well as two-stream ion-acoustic wave instabilities for quasiperpendicular subcritical shocks.

In the current paper, we study about the ion-acoustic wave instability by a gyrating ion beam in a plasma comprising heavy negative ions with the help of first-order perturbation theory. In Section 2, the instability investigation of an ion-acoustic wave is carried out. We obtain the expressions for phase velocity, unstable wave modes, and the growth rates for the two IA (fast and slow) modes in the presence of positive ions (K^+) and heavy negative ions ($C_7F_{14}^-$) where the effect of the increasing magnetic field on the growth rate of instability is also taken into account.

2. Analytical Model

Assume a plasma column of radius a_1 which comprises electrons, K^+ positive ions and $C_7F_{14}^-$ heavy negative ions with equilibrium electron, positive and negative ion densities as $n^{0e} = (1-\varepsilon)n^{0p}, n^{0+} = n^{0p}$ and $n^{0-} = \varepsilon n^{0p}$ immersed in a static magnetic field $B_s \parallel \hat{z}$, where $\varepsilon (= n_{C_7F_{14}^-}/n_{K^+})$ is the relative ion density of heavy negative ions and n^{0p} is the plasma density, where $n_{K^+}$and $n_{C_7F_{14}^-}$ represents the densities of potassium and perfluoro methylcyclohexane ions.

The mass, charge, and temperature of all the three species are given as $(m^e, -e, T^e)$, (m^+, e, T^+), and $(m^-, -e, T^-)$, respectively. The given plasma is taken as collisionless. A gyrating ion beam with velocity $\vec{v} = (v_\perp \hat{\theta} + v_{b0}\hat{z})$, mass m^b, density n^{0b} and radius $r_{b0} = 1.5\,cm$ transmits over the plasma cylinder.

The perturbed plasma potential is given by the following equation:

$$\phi_1 = \phi(r)e^{-i(\omega t - l\theta - k_z z)}, \tag{1}$$

where ω is frequency, θ is angle of propagation, k is wave vector and ℓ is azimuthal mode number.

Since we have taken $n^{0p} \gg n^{0b}$, so all the three species are treated as fluids.

$$m^e\left[\frac{\partial \vec{v}_e}{\partial t} + (\vec{v}_e.\nabla)\vec{v}_e\right] = -e\vec{E} - \frac{e}{c}(\vec{v}_e \times B_s) - T^e\frac{\nabla n^e}{n^{0e}}, \tag{2}$$

$$m^+\left[\frac{\partial \vec{v}_+}{\partial t} + (\vec{v}_+.\nabla)\vec{v}_+\right] = e\vec{E} - \frac{e}{c}(\vec{v}_+ \times B_s) - T^+\frac{\nabla n^+}{n^{0+}}, \tag{3}$$

$$m^-\left[\frac{\partial \vec{v}_-}{\partial t} + (\vec{v}_-.\nabla)\vec{v}_-\right] = -e\vec{E} - \frac{e}{c}(\vec{v}_- \times B_s) - T^-\frac{\nabla n^-}{n^{0-}}, \tag{4}$$

$$\left[\frac{\partial n^e}{\partial t} + \nabla.(n^e\vec{v}_e)\right] = 0, \tag{5}$$

$$\left[\frac{\partial n^+}{\partial t} + \nabla.(n^+\vec{v}_+)\right] = 0, \tag{6}$$

$$\left[\frac{\partial n^-}{\partial t} + \nabla.(n^-\vec{v}_-)\right] = 0. \tag{7}$$

We get the value of perturbed electron density from equation (2) as follows:

$$n^{1e} = \frac{(1-\varepsilon)n^{op}e\phi_1}{T^e}. \tag{8}$$

Similarly, the axial and perpendicular perturbed velocities can be obtained from equation (3) as follows:

$$\begin{aligned} v^{z1+} &= \frac{-k_z e\phi_1}{m^+\omega}, \\ v^{\perp 1+} &= \frac{e}{m^+}\left[\frac{{}_\perp\phi_1 \times \vec{\omega}_{c+} + i\omega\nabla_\perp\phi_1}{(\omega^2 - \omega_{c+}^2)}\right] - \frac{T^+}{m^+n^{0p}}\frac{(i\omega\nabla_\perp n^{1+} + \nabla_\perp n^{1+} \times \vec{\omega}_{c+})}{(\omega^2 - \omega_{c+}^2)}. \end{aligned} \tag{9}$$

Solving the equation of continuity, we obtain a perturbed density of positive ions as follows:

$$n^{1+} = \frac{n^{op}e}{m^+}\left[\frac{\nabla_\perp^2\phi_1}{\omega_{c+}^2} + \frac{k_z^2\phi_1}{\omega^2}\right], \tag{10}$$

and perturbed negative ion density is given as follows:

$$n^{1-} = -\frac{n^{op}\varepsilon c_-^2 e}{T^e}\left[\frac{\nabla_\perp^2\phi_1}{\omega_{c-}^2} + \frac{k_z^2\phi_1}{\omega^2}\right]. \tag{11}$$

Perturbed gyrating ion beam density is

$$n^{1b} = -\frac{N_0 e\delta(r - r_{b0})(l^2/r^2 + k_z^2)\phi_1}{(\varpi - l_{\omega_{cb}})^2 m^b 2\pi r_{b0}}. \tag{12}$$

Using Poisson's equation

$$\nabla^2\phi_1 = \left[n^{1e} - n^{1+} + n^{1-} - n^{1b}\right]4\pi e, \tag{13}$$

and putting the values given below in Poisson's equation.

$$\omega_{pe}^2=\frac{4\pi n^{0e}e^2}{m^e},\omega_{p-}^2=\frac{4\pi n^{0p}\varepsilon e^2}{m^-},\omega_{p+}^2=\frac{4\pi n^{0p}e^2}{m^+}\ \text{and}\ \varpi=\omega-k_z v_{b0}. \tag{14}$$

Equation (13) for the axially symmetric case can be rewritten as follows:

$$\frac{\partial^2\phi_1}{\partial r^2}+\frac{1}{r}\frac{\partial\phi_1}{\partial r}+Q^2\phi_1=-\frac{2N_0 e^2\delta(r-r_{b0})\left(l^2/r^2+k_z^2\right)\phi_1}{N\left(\varpi-l_{\omega_{cb}}\right)^2 m^b r_{b0}}, \tag{15}$$

where $N=(1+\omega_{p+}^2/\omega_{c+}^2+\omega_{p-}^2/\omega_{c-}^2)$ and $Q^2=(-\omega_{pe}^2 m^e/T^e+\omega_{p+}^2 k_z^2/\omega^2-k_z^2+\omega_{p-}^2 k_z^2/\omega^2/1+\omega_{p+}^2/\omega_{c+}^2+\omega_{p-}^2/\omega_{c-}^2)$.

Therefore, equation (15) can be rewritten as follows:

$$\begin{aligned}\nabla_\perp^2\phi_1+Q^2\phi_1&=-\frac{2N_0e^2\delta(r-r_{b0})\left(l^2/r^2+k_z^2\right)}{N\left(\varpi-l_{\omega_{cb}}\right)^2 m^b r_{b0}},\\ Q^2-p_m^2&=-\frac{4N_0e^2\left(l^2/r^2+k_z^2\right)J_l^2(p_m r_{b0})}{a_1^2N\left(\varpi-l_{\omega_{cb}}\right)^2 m^b J_{l+1}^2(p_m a_1)},\end{aligned} \tag{16}$$

and using this value $\omega_{pb}^2=4N_0 e^2/a_1^2 m^b$, we obtain equation (16) as follows:

$$Q^2-p_m^2=-\frac{\omega_{pb}^2\left(l^2/r_{b0}^2+k_z^2\right)J_l^2(p_m r_{b0})}{N\left(\varpi-l_{\omega_{cb}}\right)^2 J_{l+1}^2(p_m a_1)}. \tag{17}$$

After putting all the values together in equation (17), we obtain the following equation:

$$-\left(k_z^2+p_m^2\right)-\frac{\omega_{pe}^2 m^e}{T^e}+\frac{\omega_{p+}^2k_z^2}{\omega^2}+\frac{\omega_{p-}^2k_z^2}{\omega^2}-\frac{\omega_{p+}^2p_m^2}{\omega_{c+}^2}+\frac{\omega_{p-}^2p_m^2}{\omega_{c-}^2}=-\frac{\omega_{pb}^2\left(l^2/r_{b0}^2+k_z^2\right)J_l^2(p_m r_{b0})}{\left(\varpi-l_{\omega_{cb}}\right)^2 J_{l+1}^2(p_m a_1)}. \tag{18}$$

Assume, $\alpha=(k_z^2+p_m^2)/(1-\varepsilon)p_m^2+\omega_{pe}^2 m^e/T^e p_m^2$, equation (18) in its simpler form can be put as follows:

$$1-\frac{\omega_{p+}^2k_z^2}{\alpha(1-\varepsilon)\omega^2p_m^2}-\frac{\omega_{p-}^2k_z^2}{\alpha(1-\varepsilon)\omega^2p_m^2}-\frac{\omega_{p+}^2k_z^2}{\alpha(1-\varepsilon)\omega_{c+}^2p_m^2}+\frac{\omega_{p-}^2k_z^2}{\alpha(1-\varepsilon)\omega_{c-}^2p_m^2}=-\frac{\omega_{pb}^2\left(l^2/r_{b0}^2+k_z^2\right)J_l^2(p_m r_{b0})}{\alpha(1-\varepsilon)\left(\varpi-l_{\omega_{cb}}\right)^2 p_m^2J_{l+1}^2(p_m a_1)}. \tag{19}$$

Now, equation (19) can be evaluated for two fast and slow-growing modes in the existence of gyrating ion beam which is given as follows:

2.1. *Plasma Cylinder without Negative Ions.*

In the lack of negative ions, equation (19) is

$$1-\frac{\omega_{p+}^2k_z^2}{\alpha(1-\varepsilon)\omega^2p_m^2}+\frac{\omega_{p+}^2k_z^2}{\alpha(1-\varepsilon)\omega_{c+}^2p_m^2}=-\frac{\omega_{pb}^2\left(l^2/r_{b0}^2+k_z^2\right)J_l^2(p_m r_{b0})}{\alpha(1-\varepsilon)\left(\varpi-l_{\omega_{cb}}\right)^2 p_m^2J_{l+1}^2(p_m a_1)}. \tag{20}$$

We obtain equation (21) after simplifying equation (20)

$$\omega^4\left(1+\frac{\omega_{p+}^2}{\alpha(1-\varepsilon)\omega_{c+}^2}\right)-\omega^2\left(\omega_{c+}^2+\frac{\omega_{p+}^2}{\alpha(1-\varepsilon)}+\frac{\omega_{p+}^2k_z^2}{\alpha(1-\varepsilon)p_m^2}\right)+\frac{\omega_{p+}^2k_z^2\omega_{c+}^2}{\alpha(1-\varepsilon)p_m^2}=\frac{\omega_{pb}^2\left(l^2/r_{b0}^2+k_z^2\right)\omega^2\left(\omega^2-\omega_{c+}^2\right)J_l^2(p_m r_{b0})}{\alpha(1-\varepsilon)\left(\varpi-l_{\omega_{cb}}\right)^2 p_m^2J_{l+1}^2(p_m a_1)}, \tag{21}$$

where
$\alpha = (k_z^2 + p_m^2)/(1-\varepsilon)p_m^2 + \omega_{pe}^2 m^e/T^e p_m^2 = k^2/(1-\varepsilon)p_m^2 + \omega_{pe}^2 m^e/T^e p_m^2 \approx \omega_{pi}^2/c_s^2\, p_m^2$.

Equation (21) can be rewritten as follows:

$$\left(\omega^2 - c_1^2\right)\left(\varpi - l_{\omega_{cb}}\right)^2 = -\frac{\omega_{pb}^2\left(l^2/r_{b0}^2 + k_z^2\right)\left(\omega^2 - \omega_{c+}^2\right)J_l^2\left(p_m r_{b0}\right)}{\alpha(1-\varepsilon)p_m^2\,\alpha_1 J_{l+1}^2\left(p_m a_1\right)}, \quad (22)$$

where

$$c_1^2 = \frac{1}{(1-\varepsilon)}\,\frac{k^2 c_s^2}{\left[1 + \omega_{p+}^2 c_s^2\, p_m^2/\omega_{pi}^2\,\omega_{c+}^2(1-\varepsilon)\right]}, \quad (23)$$

or $c_1^2 = 1/(1-\varepsilon)k^2 c_s^2/\alpha_1$, where $\alpha_1 = 1 + \omega_{\mathrm{p}+}^2 c_{\mathrm{s}}^2\, \mathrm{p}_{\mathrm{m}}^2/(1-\varepsilon)\omega_{\mathrm{pi}}^2\,\omega_{\mathrm{c}+}^2$.

Here, $\omega \approx c_1$ resembles to the fast ion mode and $\omega \approx k_z v_{0b} + l_{\omega_{cb}}$ conforming to the beam mode.

When $\omega \approx k_z v_{0b} + l_{\omega_{cb}}$, we get the solutions and on solving equation (22), we obtained the growth rate as follows:

$$\gamma = \mathrm{Im}\delta_1 = \left[\frac{c_1}{2}\,\frac{\omega_{pb}^2\left(l^2/r_{b0}^2 + k_z^2\right)J_l^2\left(p_m r_{b0}\right)}{(1-\varepsilon)p_m^2\alpha\,\alpha_1 J_{l+1}^2\left(p_m a_1\right)}\right]^{1/3}\frac{\sqrt{3}}{2}. \quad (24)$$

The unstable real wave frequency is given by the following equation:

$$\omega_r = k_z\left(\frac{2eV_b}{m^b}\right)^{1/2} - \frac{1}{2}\left[\frac{c_1}{2}\,\frac{\omega_{pb}^2\left(l^2/r_{b0}^2 + k_z^2\right)J_l^2\left(p_m r_{b0}\right)}{(1-\varepsilon)p_m^2\alpha\,\alpha_1 J_{l+1}^2\left(p_m a_1\right)}\right]^{1/3}. \quad (25)$$

From equation (25), the phase velocity $v(=\omega/k)$ can be written as follows:

$$v_{ph}^{\mathrm{posi}}(\varepsilon) = \frac{1}{(1-\varepsilon)^{1/2}}\,\frac{c_s}{\left(1 + 1/1-\varepsilon p_m^2\, c_s^2/\omega_{c+}^2\right)^{1/2}}, \quad (26)$$

where $v_{ph}^{\mathrm{posi}}(\varepsilon)$ is the sound wave phase velocity in the existence of positive ions.

2.2. Plasma Cylinder without Positive Ions. In the lack of positive ions, equation (19) is written as follows:

$$1 - \frac{\omega_{p-}^2 k_z^2}{\alpha(1-\varepsilon)\,\omega^2 p_m^2} + \frac{\omega_{p-}^2 k_z^2}{\alpha(1-\varepsilon)\omega_{c-}^2\, p_m^2} = -\frac{\omega_{pb}^2\left(l^2/r_{b0}^2 + k_z^2\right)J_l^2\left(p_m r_{b0}\right)}{\alpha(1-\varepsilon)\left(\varpi - l_{\omega_{cb}}\right)^2 p_m^2 J_{l+1}^2\left(p_m a_1\right)}. \quad (27)$$

As discussed in the previous case, we obtain the following equation:

$$\omega^4\left(1 + \frac{\omega_{p-}^2}{\alpha(1-\varepsilon)\,\omega_{c-}^2}\right) - \omega^2\left(\omega_{c-}^2 + \frac{\omega_{p-}^2}{\alpha(1-\varepsilon)} + \frac{\omega_{p-}^2 k_z^2}{\alpha(1-\varepsilon)\,p_m^2}\right) + \frac{\omega_{p-}^2 k_z^2\omega_{c-}^2}{\alpha(1-\varepsilon)p_m^2} = \frac{\omega_{pb}^2\left(l^2/r_{b0}^2 + k_z^2\right)\omega^2\left(\omega^2 - \omega_{c-}^2\right)J_l^2\left(p_m r_{b0}\right)}{\alpha(1-\varepsilon)\left(\varpi - l_{\omega_{cb}}\right)^2 p_m^2 J_{l+1}^2\left(p_m a_1\right)}, \quad (28)$$

where
$\alpha = (k_z^2 + p_m^2)/(1-\varepsilon)p_m^2 + \omega_{pe}^2 m^e/T^e p_m^2 = k^2/(1-\varepsilon)p_m^2 + \omega_{pe}^2 m^e/T^e p_m^2 \approx \omega_{pi}^2/c_s^2\, p_m^2$.

Equation (28) can be rewritten as follows:

$$\left(\omega^2 - d_1^2\right)\left(\varpi - l_{\omega_{cb}}\right)^2 = -\frac{\omega_{pb}^2\left(l^2/r_{b0}^2 + k_z^2\right)\left(\omega^2 - \omega_{c-}^2\right)J_l^2\left(p_m r_{b0}\right)}{\alpha(1-\varepsilon)p_m^2\,\alpha_2 J_{l+1}^2\left(p_m a_1\right)}, \quad (29)$$

where

$$d_1^2 = \frac{\varepsilon}{(1-\varepsilon)}\,\frac{k^2 c_s^2\left(m^i/m^-\right)}{\left[1 + \varepsilon/(1-\varepsilon)\omega_{p-}^2\, c_s^2\, p_m^2/\omega_{pi}^2\,\omega_{c-}^2\right]}. \quad (30)$$

If we assume $n^{0-} = n^{0p}$, so that $\varepsilon/(1-\varepsilon)$ is replaced by $1/(1-\varepsilon)$; therefore, equation (30) can be rewritten as follows:

$$d_1^2 = \frac{1}{(1-\varepsilon)}\,\frac{k^2 c_s^2\left(m^i/m^-\right)}{\left[1 + 1/(1-\varepsilon)\omega_{p-}^2\, c_s^2\, p_m^2/\omega_{pi}^2\,\omega_{c-}^2\right]},$$

$$d_1^2 = \frac{1}{(1-\varepsilon)}\,\frac{k^2 c_s^2\left(m^i/m^-\right)}{\alpha_2}, \quad (31)$$

where $\alpha_2 = 1 + \omega_{p-}^2\, c_s^2\, p_m^2/(1-\varepsilon)\omega_{pi}^2\,\omega_{c-}^2$.

Similarly, on solving equation (31) we get the expression of the growth rate

$$\gamma = \mathrm{Im}\delta_1 = \left[\frac{d_1}{2}\,\frac{\omega_{pb}^2\left(l^2/r_{b0}^2 + k_z^2\right)J_l^2\left(p_m r_{b0}\right)}{(1-\varepsilon)p_m^2\alpha\,\alpha_2 J_{l+1}^2\left(p_m a_1\right)}\right]^{1/3}\frac{\sqrt{3}}{2}. \quad (32)$$

The unstable real wave frequency is given by the following equation:

$$\omega_r = k_z\left(\frac{2eV_b}{m^b}\right)^{1/2} - \frac{1}{2}\left[\frac{d_1}{2}\frac{\omega_{pb}^2\left(l^2/r_{b0}^2 + k_z^2\right)J_l^2\left(p_m r_{b0}\right)}{(1-\varepsilon)p_m^2\alpha\,\alpha_2 J_{l+1}^2\left(p_m a_1\right)}\right]^{1/3}. \quad (33)$$

From equation (31), the phase velocity for negative ions can be written as follows:

$$v_{ph}^{\text{negt.}}(\varepsilon) = \frac{c_s}{(1-\varepsilon)^{1/2}}\frac{\left(m^i/m^-\right)^{1/2}}{\left(1 + 1/(1-\varepsilon)p_m^2 c_s^2\,\omega_{p-}^2/\omega_{c-}^2\,\omega_{pi}^2\right)^{1/2}}. \quad (34)$$

3. Results and Discussion

To solve the paper numerically, the parameters of the experimental paper of Song et al. [17] have been used. Using equation (24) for positive ions K^+ and equation (32) for heavy negative ions $C_7F_{14}^-$ [when $\varepsilon/(1-\varepsilon) = 1/(1-\varepsilon)$]. Figures 1 and 2 are plotted for the dispersion curves of ion-acoustic waves containing light positive and heavy negative ion plasmas for the below-given plasma parameters: electron plasma density $n^{0e} \approx 10^9 cm^{-3}$, the relative density $\varepsilon(= n_{C_7F_{14}^-}/n_{K^+}) = 0, 0.3, 0.5, 0.7, 0.9, 0.98$, temperature of electrons and ions $T^e \approx T^i \approx 0.2eV$ and magnetic field is taken as $B_s = 4\times 10^3$gauss. The unstable wave mode frequencies and the corresponding wavenumbers can be drawn by the point of intersections between the gyrating ion beam mode and the positive or heavy negative ion plasma modes.

Using equations (26) and (34), we have also plotted in Figure 3, the phase velocity $v_{ph}(\varepsilon)$ in the existence of positive and heavy negative ions as a heavy negative ion concentration function. Here, it can be realized that as the percentage of heavy negative ions is increased, the phase velocity of both the modes increases. However, the increase in phase velocity of K^+ ion mode is more than the increase in heavy negative ion mode. Moreover, the phase velocity increases by a factor of ~2.6 when ε changes from 0.7 to 0.98 for K^+ ions and by a factor ~2.1, when ε changes 0.7 to 0.98 for $(C_7\,F_{14}^-)$ ions. If we compare Figure 3 of the present paper with Figure 4 of Song et al. [17]. We can say that our theoretical plot of the phase velocity of the sound wave in the presence of positive and heavy negative ions is qualitatively similar to the experimental and theoretical observations of existing research work. Using equations (24) and (32), we have plotted in Figure 5, the growth rate γ (in $\sec^{-1}$) of unstable ion modes $K^+$and $C_7F_{14}^-$ as a function of relative ion density ε for the same parameters as taken in Figures 1 and 2. It is found that the growth rate γ (in $\sec^{-1}$) of both the modes increase with ε and it increases drastically for $\varepsilon \gg 0.8$.

In Figures 6 and 7, the frequency of heavy negative ion mode and light positive ion mode as a function of the magnetic field B_s for different value of relative density of negative ions ε is plotted. Here, we observed that the frequency increases almost linearly as the value of the applied magnetic field increases. Moreover, the frequency of both

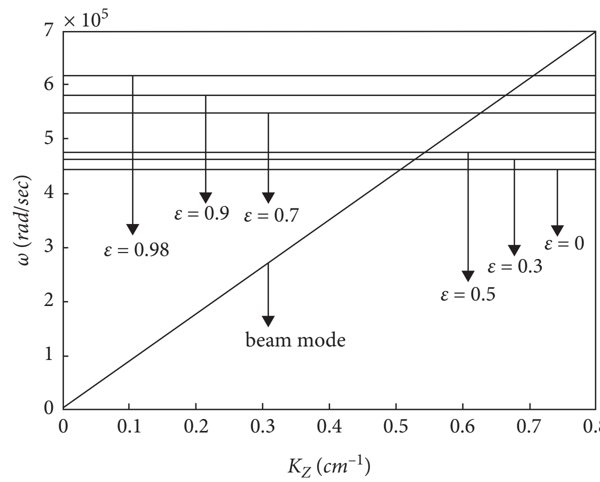

Figure 1: Dispersion curves of IAWs in the presence of positive ions and a beam mode.

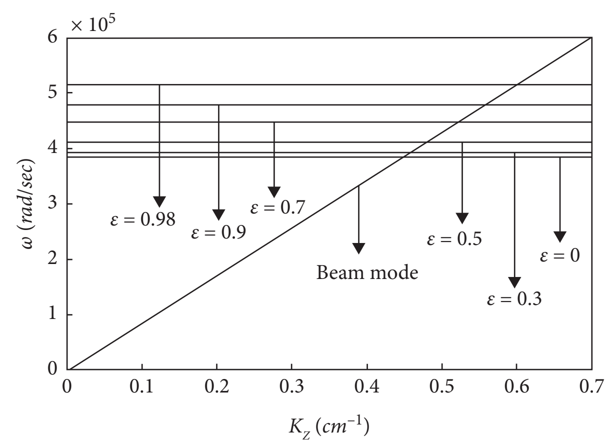

Figure 2: Dispersion curves of IAWs in the presence of heavy negative ions and a beam mode.

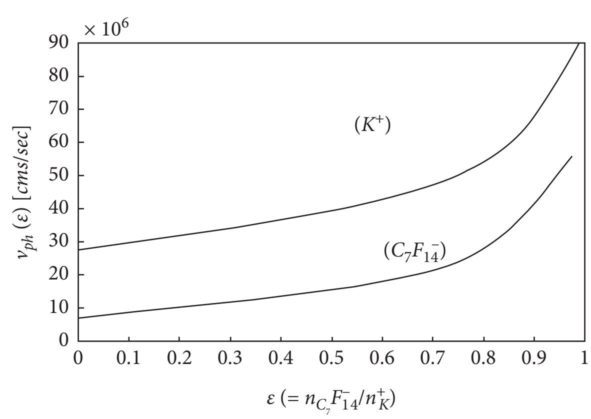

Figure 3: Phase velocity of sound waves $v_{ph}(\varepsilon)$ in the presence of positive and heavy negative ions as a function of relative density of heavy negative ion ε for the same parameters as taken in Figures 1 and 2, for beam density $n^{0b} = 2.5\times 10^8 cm^{-3}$.

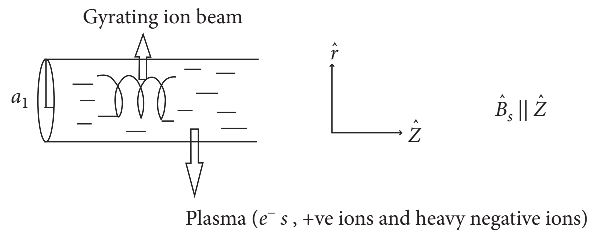

FIGURE 4: Diagram of gyrating ion beam and plasma cylinder (containing electrons, positive ions, and heavy negative ions).

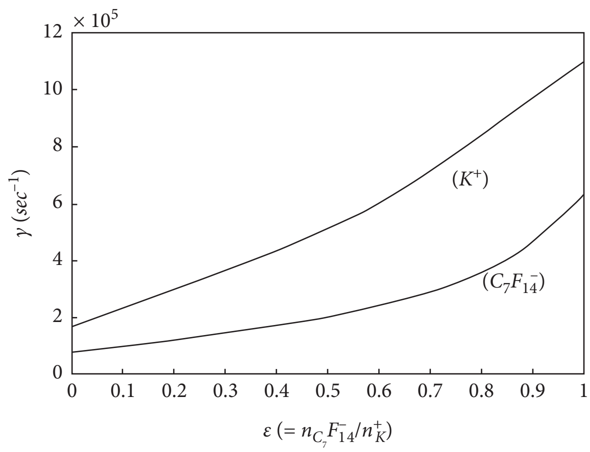

FIGURE 5: Growth rate γ of the unstable ion modes $K^+$and $C_7F_{14}^-$ versus relative ion density ε.

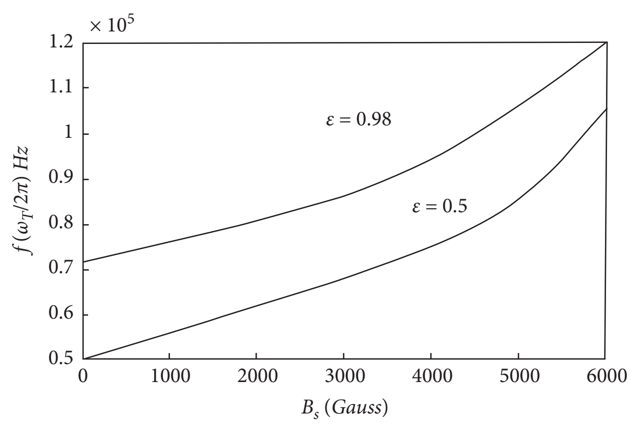

FIGURE 6: Frequency f (Hz) of heavy negative ion mode versus magnetic field B_s (Gauss) for different values of relative density of heavy negative ions ε.

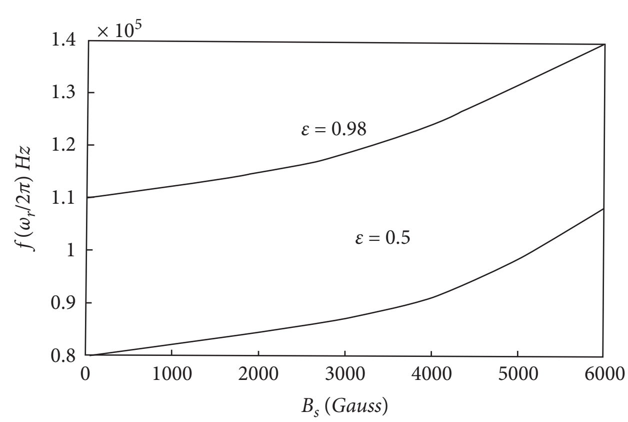

FIGURE 7: Frequency f (Hz) of light positive ion mode versus magnetic field B_s (Gauss) for different values of relative density of heavy negative ions ε.

the modes increases with the relative density of heavy negative ions. Here, it is clear that the frequency for $\varepsilon = 0.98$ is greater than the frequency for $\varepsilon = 0.5$. Moreover, the growth rate $\gamma(\sec^{-1})$ of the unstable modes in the presence of $C_7F_{14}^-$ and K^+ ions increase with the beam density and scales as the $1/3^{rd}$ power of the beam density [cf. equations (24) and (32)].

4. Conclusion

In conclusion, we may say that an increase in the relative density of heavy negative ions in the plasma results in the excitation of IAWs in a magnetized plasma and a gyrating ion beam transmitting through this collisionless magnetized plasma drives the ion-acoustic waves to instability via Cerenkov interactions. The growth rates and phase velocity of both the unstable, fast K^+, and slow $C_7F_{14}^-$ modes are evaluated based on the typical existing experimental plasma parameters. The mode frequencies, growth rates, and phase velocities for both modes are found to increase with the relative density of heavy negative ions. However, the increase in phase velocity of fast K^+ mode is more than the slow $C_7F_{14}^-$ mode which is in accordance with the experimental observations of Song et al. [17]. As the heavy negative ion concentration becomes quite appreciable ($\varepsilon > 0.5$), the phase velocity of the fast mode increases as predicted by Song et al. [17] and using Vlasov theory calculations. In the present work, the beam radius is much larger than the gyroradius of the ion beam ρ_i, hence the ion beam gyrates in the plasma. The frequency of fast K^+ and slow $C_7F_{14}^-$ modes increases with the azimuthal mode numbers and this increase is slightly more than linear. In the present work, we have observed the significant effect of an increase in the magnetic field as well as the azimuthal mode numbers on the growth rate of both the unstable fast and slow ion modes.

Data Availability

The data that supports the findings of this manuscript are available within the article from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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