Research on the Complexity of Oligopoly Game under Business Interruption Insurance of the Engineering Project

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1. Introduction

Affected by the new coronavirus pneumonia in early 2020, the supply chain of various industries has been hit hard. In addition, various emergencies and disasters (such as fire, earthquake, and flood) are hitting the industrial chain more and more frequently [19]. The globalization of the industrial chain further exacerbates the risk, especially the supply risk [10–15]. According to a survey of global risk management by the Yi'an group, the risk of business interruption has been one of the top ten risks in the world since 2007. In 2019, the risk of global business interruption has increased significantly. In China, companies also face a severe risk of business interruption [16, 17]. In 2000, according to the Allianz annual risk index survey, the risk of business interruption, including supply chain disruption, became the biggest risk for Chinese companies in the past two years, with 45% and 30% of votes obtained in 2019 and 2020, respectively. Therefore, it is of great significance to introduce business interruption insurance into supply chain risk management in view of the increasing supply chain risk [18–26].

Business interruption insurance refers to the enterprise being unable to produce and operate normally because of events within the scope of insurance [27–30]. The insurance company will compensate for the losses caused by the enterprise [10, 31]. The events in the insurance coverage are usually accidents or natural disasters, and the scope of compensation is generally the damage caused by the subject matter of insurance, the reduction of income caused by the breakage, and the increase in expenses [21, 32, 33]. As an important risk prevention and transfer measure, it has been widely popularized and applied in Europe and America, and many practical enterprise cases have proved its advantages and value in dealing with the risk [34–37]. However, the application of business interruption insurance in China is not very extensive. The main reason is that both supply and demand do not attach much importance to this insurance, and enterprises do not realize its importance. Insurance companies also do not improve the relevant types of insurance, and publicity is not enough.

Many scholars have made a detailed introduction to the related research on business interruption insurance [38–42]. The Business Interruption Insurance has introduced the business interruption insurance in detail, including the development course, the coverage, the exclusion liability, and the compensation condition. Domestic scholars have also carried out a lot of research, such as Theory of Business Insurance Theory by Changmei XU and Insurance Science by
Jing Tao and have carried on detailed introduction to the national business interruption insurer situation. Taking chemical enterprises as the background, Gao Wenjie and others put forward the problems that enterprises should pay attention to in the selection of insurance types by studying the selection of insurance types for related enterprises. In the field of application research, foreign scholars have more research, mostly through practical cases to verify the value of business interruption insurance to enterprises [43–47]. For example, Eser Durukal, Adam Rose, Robert Hartwig, and others analyzed the value of business interruption insurance to protect against major disasters or man-made events, such as Florida floods, Turkey earthquakes, and terrorist attacks on the Los Angeles power grid system.

2. Model Description

2.1. Benchmark Model. This paper considers the situation of the engineering project manufacturer facing the risk of business interruption and takes the risk of the manufacturer but does not take any measures as the benchmark model. Assume that only two manufacturers are competing in manufacturing industry, occupying the decisive market share of the industry. A duopoly competition model was established in which the two manufacturers produce homogeneous products, which are substitutes for each other, and consumers have no obvious preference for the products produced by the two manufacturers. Let manufacturer 1 and manufacturer 2 be the two manufacturers. There is only a competitive relationship between manufacturer 1 and manufacturer 2, no private collusion, no information transmission, and independent decision-making in each production cycle. Manufacturer i produce per unit product at $c_i$ cost and sell at $p_i$ price. Under normal circumstances, manufacturers i be able to produce finished products in $q_i$ units per production cycle, but due to market uncertainty, manufacturers 1 and 2 face the risk of production disruptions, which will lead to a decline in production. Because the production capacity and the risk management level of manufacturer 1 and manufacturer 2 are different, the risk size and the impact are different. Let the random delivery quantity by the manufacturer represent the interruption risk faced by the manufacturer i, let $\theta_i$ represent the random delivery factor by the manufacturer i, the greater the $\theta_i$, the smaller the risk faced by the manufacturer, conversely, if $\theta_i$ is smaller, The less finished product, the greater the risk impact. When $\theta_i = 1$, there is no interruption risk of the manufacturer i.

Based on the actual situation and the consideration of the model, this paper mainly makes the following basic assumptions:

1. Manufacturers conform to the rational human hypothesis that the risk is neutral, will not over-preference risk, and will not over-avoid risk, the pursuit of their profit maximization
2. Making decisions between manufacturers cannot know each other’s information, and decisions do not interfere with each other
3. For $\theta_i$, its probability density function is $g(\theta_i)$, and its cumulative probability distribution function is $G(\theta_i)$, its mean is $\mu_i$ and variance is $\sigma^2_i$

To facilitate recording and calculation, this chapter will use some parameters to represent some definitions in practice, and the specific model variable parameter settings are shown in Table 1.

The production of the manufacturer is uncertain in the case of the model. We use $i (i = 1/2)$ to represent the random delivery factor i by the manufacturer, and the quantity of the final finished product of the manufacturer can be expressed as $\theta_i q_i$. The demand functions of the manufacturer can be written as

$$q_i = a - b_i p_i + d_{2i} p_{2i}, \quad i = 1, 2. \quad (1)$$

Parameter $a (a > 0)$ represents the maximum demand of the product, parameter $b_i$ represents the price sensitivity of demand for the product $i$, and $d_i$ represents the price sensitivity of demand for the product $i$ to the prices of the competitive product.

To facilitate calculation and analysis, make $b_1 = b_2$, $d_1 = d_2$, and $b > d > 0$. Then, the demand functions of manufacturer 1 and manufacturer 2 can be written as

$$\begin{align*}
q_1 &= a - b_1 p_1 + d_2 p_2 \\
q_2 &= a - b_2 p_2 + d_1 p_1.
\end{align*} \quad (2)$$

The profit functions of manufacturer 1 and manufacturer 2 can be written as

$$\begin{align*}
\pi_1 &= \theta_1 p_1 q_1 - c_1 q_1 = (\theta_1 p_1 - c_1) (a - b_1 p_1 + d_2 p_2) \\
\pi_2 &= \theta_2 p_2 q_2 - c_2 q_2 = (\theta_2 p_2 - c_2) (a - b_2 p_2 + d_1 p_1).
\end{align*} \quad (3)$$

Proposition 1. There are optimal solution and are the only solution, $p_{1i}^* = (a + d_2 p_2) \theta_1 + b_2 c_1 / 2 b_1 \theta_1$, $p_{2i}^* = (a + d_1 p_1) \theta_2 + b c_2 / 2 b_2 \theta_2$, which make manufacturer 1 and manufacturer 2 the most profitable.

The optimal profit functions of manufacturer 1 and manufacturer 2 can be written as

$$\begin{align*}
\pi_1^* &= \frac{1}{4 b^2 \theta_1 \theta_2} (A - 2 b c_1) (2 a b \theta_1 \theta_2 - A b \theta_2 + B d \theta_2) \\
\pi_2^* &= \frac{1}{4 b^2 \theta_1 \theta_2} (B - 2 b c_2) (2 a b \theta_1 \theta_2 - B b \theta_1 + A d \theta_2).
\end{align*} \quad (4)$$

where $A = (a + d_2 p_2) \theta_1 + b_2 c_1$, $B = (a + d_1 p_1) \theta_2 + b c_2$.

2.2. Engineering Project Oligopoly Manufacturers Model considering Business Interruption Insurance. This section establishes a duopoly manufacturer model. In order to compare the differences between manufacturers before and after insurance, this section assumes that manufacturer 1 has purchased business interruption insurance, while manufacturer 2 has not purchased business interruption insurance. Manufacturer 1 under the risk buys business interruption insurance so that the insurance company can share the risk. A manufacturer and an insurance company sign an insurance contract, and the manufacturer decides the amount of insurance $L$ and pays the premium to the insurance company.
The insurance company does not participate in the supply chain decision as an exogenous variable, but the insurance company decides the insurance \( h \), so the premium that the supplier should pay is \( h_L \). The manufacturer’s actual loss of profit is written as \( \text{IL} = (wq - c.q - k\theta^2/2) - (w\theta q = c.q - k\theta^2/2) = (1 - \theta)wq \), because the amount of insurance \( L \) is the upper limit of the insurance company’s compensation, the insurance compensation is the \( \min(\text{IL}, L) \).

Based on the actual situation and the consideration of the model, this paper mainly makes the following basic assumptions:

1. Manufacturers conform to the rational human hypothesis that the risk is neutral, will not over-preference risk, will not over-avoid risk, the pursuit of their own profit maximization
2. Making decisions between manufacturers cannot know each other’s information, and decisions do not interfere with each other
3. For \( \theta_i \), its probability density function is \( g(y) \), and its cumulative probability distribution function is \( G(y) \), its mean is \( \mu \), and variance is \( \sigma^2 \)

\[
\begin{align*}
\pi_1^* &= \frac{1}{4} \left( \frac{a - dp_2}{2} + bH + d \left( \frac{(a + dp_1)\mu + bc}{2b} \right) \right) \left( (1 + \mu)(5 - 4h) \left( \frac{a + dp_2}{2b} + H \right) - c \right), \\
\pi_2^* &= \frac{(a + dp_1)\mu - bc}{2b} \left( \frac{(a - dp_1)\mu - bc}{2\mu} + d \left( \frac{a + dp_2}{2b} + H \right) \right),
\end{align*}
\]

where \( H = 2C/5(1 + \mu) \).

It is assumed that the players adjust the decision-making regularly according to the changes in the market. Moreover, the players’ decisions are restricted by the adjustment parameters. In this paper, we assume the decision-makers use limited rational expectations to make decisions, and the starting point of decision-making is mainly to consider their profits.

Let \( \nu_1 \) and \( \nu_2 \) represent the adjustment speed of suppliers and retailers in each decision cycle, respectively, which reflects their sensitivity to the change of their own profits. The dynamic decision system of decision-making can be described as follows:

1. Emergencies that lead to random supplier deliveries are covered by the insurance company.

To facilitate subsequent calculations and comparative analysis of manufacturer 1 and manufacturer 2, it is assumed that \( b_1 = b_2, c_1 = c_2, d_1 = d_2, \) and \( \theta_1 = \theta_2 \).

The profit functions of manufacturer 1 can be written as

\[
\pi_1^* = \theta p_1 q_1 - c q_1 + \min(\text{IL}, L) - eL,
\]

\[
0 \leq L \leq wq, 0 \leq h \leq 1,
\]

\[
E[\pi_1^*(p_1, L)] = (\mu p_1 - c)(a - bp_1 + dp_2) + \int_0^{1-\theta} L dG(y)
\]

\[
+ \int_1^{1-\theta/l_0} (1 - y) p_1 q_1 dG(y) - hL
\]

\[
= \frac{1}{4} (a - bp_1 + dp_2)[(1 + \mu)(5 - 4h) p_1 - c].
\]

The profit functions of manufacturer 2 can be written as

\[
\pi_2 = \theta p_2 q_2 - c q_2 = (\theta p_2 - c)(a - bp_2 + dp_1),
\]

\[
E[\pi_2(p_2)] = (\mu p_2 - c)(a - bp_2 + dp_1).
\]

**Proposition 2.** There are only optimal solution, \( p_1^* = a + dp_2/2b + 2C/5(1 + \mu), p_2^* = (a + dp_1)\mu + bc/2by \), which make manufacturer 1 and manufacturer 2 the most profitable.

The optimal profit functions of manufacturer 1 and manufacturer 2 can be written as

\[
\begin{align*}
p_1(t + 1) &= p_1(t) + v_1 p_1 \left[ \frac{5(1 + \mu)}{4} (a - 2bp_1 + dp_2) + bc \right], \\
p_2(t + 1) &= p_2(t) + v_2 p_2 \left[ (a - 2bp_2 + dp_1)\mu + bc \right]
\end{align*}
\]

System (9) is nonlinear, and when \( p1t = pit, 1 = 1/2, \) the four equilibrium points of the system can be obtained as \( E_0(0, 0), E_1(0, a\mu + bc/2by), E_2(5a(1 + \mu) + 4b/10b(1 + \mu), 0), E_3(a + dp_2/2b + H, (a + dp_1)\mu + bc/2by). \)
Through observing the four equilibrium points, it is found that they are the source of the system, $E_1$ and $E_2$ are the saddle points of their instability, and $E_3$ are the only equilibrium points of system (9). The local stability of the system can be judged by the Jacobian matrix.

$$J(E) = \begin{pmatrix}
1 + v_1 \left[ \frac{5(1 + \mu)}{4} (a - 4bp_1 + d) + bc \right] \\
\mu dv_2 p_2 \\
1 + v_2 \left[ \frac{5(1 + \mu)v_1 p_1}{4} - (a - 4bp_2 + dp_1) + bc \right]
\end{pmatrix}.$$ (9)

The Jacobian matrix of four equilibrium points is calculated, and its stability is judged according to the eigenvalue of the matrix. When the nonzero eigenvalue is greater than 1, the point is unstable. We can analyze that $E_0$, $E_1$, and $E_2$ are unstable points, and only $E_3$ is the Nash equilibrium point.

3. Numerical Simulation and Analysis

This section will analyze the complex characteristics of the system. The bifurcation and chaotic behaviors of system (9) are mainly analyzed. Figures 1 and 2 show the bifurcation diagram of the price of the manufacturer’s unit production cycle about the respective price adjustment parameters, where the blue dot line represents the bifurcation diagram of the price of manufacturer 1, and the red dot line represents the bifurcation diagram of the price of manufacturer 2.

Figures 1 and 2 describe the bifurcation diagram of the price of manufacturer 1 and manufacturer 2 when one manufacturer’s adjustment parameter takes a fixed value in the stable domain and another manufacturer’s adjustment parameter changes. Figure 1 depicts the bifurcation diagram of the manufacturer’s price when $v_1$ increases from 0 to 0.61. As $v_1$ grew to 0.45, the price decision system of duopoly manufacturers began to double the cycle bifurcation. $p_1$ is stable at 2.18 and $p_2$ is stable at 1.55. With the continuous growth of $v_1$, the price system enters a chaotic state. Figure 2 shows that when $v_1$ keeps a fixed value of 0.15, and $v_2$ growing from 0 to 3.1, the price bifurcation diagram of duopoly manufacturers. As $v_2$ grew to 2.18, the duopoly manufacturer’s price decision system begins to double the cycle bifurcation. At this point, $p_1$ is stable at 2.18 and $p_2$ is stable at 1.55. As $v_2$ grows, the price system enters a chaotic state. The optimal price values of duopoly manufacturers in this case are the same as those in Figure 1 ($p_1 = 2.18, p_2 = 1.55$).

It can be seen from Figures 3 and 4 that the largest Lyapunov exponent shows the same trend as the bifurcation diagram of Figures 1 and 2, so system (9) remains in the stable domain, periodic bifurcation, and chaotic state. Figures 3 and 4 show the largest Lyapunov exponent of a nonlinear system when one manufacturer’s adjustment parameter takes a fixed value in the stable domain and another manufacturer’s adjustment parameter changes. When the largest Lyapunov exponent is less than 0, the system is in a stable range or a double period state; when the largest Lyapunov exponent is greater than 0, the system will be chaotic; more importantly, the largest Lyapunov exponent equal to zero is the boundary of various system states.

In Figure 3, the price enters the double cycle state when the largest Lyapunov exponent is 0.45; the price enters the four-fold cycle state when the largest Lyapunov exponent is 0.50; the price enters the eight-fold cycle state when the largest Lyapunov exponent is 0.51; the price enters the chaotic state in Figure 4 when the largest Lyapunov exponent is 0.52. The largest Lyapunov exponent depicts the largest Lyapunov exponent of system (9) $v 2$, the price adjustment parameters of manufacturer 2. When the largest Lyapunov exponent is 2.18, the price enters the double cycle.
state; when the largest Lyapunov exponent is 2.45, the price enters the four-fold cycle state; when the largest Lyapunov exponent is 2.52, the price enters the eight-fold cycle state; when the largest Lyapunov exponent is 2.54, the price enters the chaotic state.

It can be seen that the boundaries of the different states of the system are the same in the largest Lyapunov exponent diagram and the bifurcation diagram. However, the period-doubling bifurcation points in the bifurcation diagram cannot be seen intuitively, while the bifurcation points can be clearly pointed out in the largest Lyapunov exponent diagram.

Next, consider the impact of manufacturer price adjustments on profits.

As shown in Figures 5, 6, and 7, when $\mu = 0.6$, the profit of manufacturer 1 (insured) is larger than that of manufacturer 2. There is a critical point that makes the profit of manufacturer 1 and manufacturer 2 go into chaos. Besides, by comparing the three figures, we can find that with the increase of the premium rate $h$, the profit of manufacturer 1 in a stable state decreases gradually, while that of manufacturer 2 increases gradually.

As shown in Figures 8, 9, and 10, when the premium rate $h$ is fixed at 0.6, the profit of manufacturer 1 is larger than that of manufacturer 2. With the increase of price adjustment parameters of manufacturer 1, there is a critical point, which makes the profit of manufacturer 1 and manufacturer
2 go into chaos. By comparing the three pictures, it can be found that with the increase of $\mu$, the profit of manufacturer 1 in a stable state increases gradually, and the profit of manufacturer 2 increases gradually.

4. Conclusion

This paper presents a duopoly model that includes two manufacturers to analyze the impact of the introduction of business interruption insurance on manufacturer profits. Manufacturer 1 is insured against business interruption insurance, and manufacturer 2 does not take any measures regarding the risk of business interruption. With the help of complex systems theory, we studied the stability of the dynamic system and illustrated the bifurcation diagram and the largest Lyapunov exponent diagram by numerical simulation.

The main findings are summarized as follows:

(1) When the manufacturer adjusts its decision variables too fast, it will make the system into chaos. At this time, the market is unstable, the manufacturer’s price and profit will become difficult to predict, so it is difficult for the manufacturer to obtain stable profit.

(2) Manufacturers should unite to adjust the speed of decision-making and reduce the market risk, which is more conducive to market stability and enable manufacturers to obtain stable profits.

(3) The introduction of business interruption insurance by manufacturers is conducive to improving their ability to resist risks and to improve their profit level.

The research in this study can provide support for decision-makers. Decision-makers can better specify competitive strategies by fully understanding the parameters in the dynamic model. Moreover, it has some limitations in our study. We will discuss the content of manufacturer insurance with more impact factors in future research work.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest regarding the publication of this paper.

References


