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Research Article

Multicriteria Decision-Making Problem via Weighted Cosine Similarity Measure and Several Characterizations of Hypergroup and (Weak) Polygroups under the Triplet Single-Valued Neutrosophic Structure

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Polygroups are an extended form of groups and a subclass of hypergroups that follow group-type axioms. In this paper, we define a triplet single-valued neutrosophic set, which is a generalization of fuzzy sets, intuitionistic fuzzy sets, and neutrosophic sets, and we combine this novel concept with hypergroups and polygroups. Firstly, the main goal of this paper is to introduce hypergroups, polygroups, and anti-polygroups under a triplet single-valued neutrosophic structure and then present various profound results. We also examine the interaction and properties of level sets of triplet single-valued neutrosophic polygroups and (normal) subpolygroups. Secondly, we rank the alternatives and select the best ones in a single-valued neutrosophic environment using the weighted cosine similarity measure between each alternative and the ideal alternative. Finally, we provide an example that clearly shows how the proposed decision-making method is applied.

1. Introduction

The classical methods of mathematical analysis are unable to make sense of the ambiguities that exist in the universe. As a consequence of this, these structures need to be rethought in order to take into account the possibility of uncertainty. In 1965, Zadeh [1] proposed a fuzzy set. A fuzzy set is a mathematical model of ambiguity in which things belong to a specific set to some degree. This degree is generally a number that falls within the unit range of $[0, 1]$.

In later years, as an extension of the fuzzy set, Sambuc [2] presented the notion of an interval-valued fuzzy set in 1975, Atanassov [3] provided the idea of an intuitionistic fuzzy set in 1984, Yager [4] initiated the concept of fuzzy multiset in 1986, Smarandache [5] presented the premise of a neutrosophic set (NS) in 1998, Molodstov [6] introduced the idea of soft sets in 1999, and Torra [7] developed a hesitant fuzzy set in 2010. Feng et al. [8] broadened soft sets by integrating them with fuzzy and rough sets, Aktas and Cagman [9] investigated soft groups, and Acar et al. [10] developed soft rings.

Marty [11] was the first to propose algebraic hyperstructures, which are an overarching concept of classical algebraic structures. He broadened the definition of a group to include the concept of a hypergroup. The resultant of two elements in a classical algebraic structure is an element.

However, the resultant of two elements in an algebraic hyperstructure is a set. Algebraic hyperstructures have been used in a wide range of subjects over the years, including hypergraphs, binary relations, cryptography, codes, median algebras, relation algebras, artificial intelligence, geometry, convexity, automata, combinatorial coloring problems, lattice theory, Boolean algebras, and logic probabilities. Hypergroups have mostly been used in the context of special subclasses.

Polygroups, which are spectacular subclasses of hypergroups, are developed by Ioulidis in [12] and employed to examine color algebras by Comer in [13, 14]. Comer showed the effectiveness of polygroups by exploring their connections to graphs, relations, Boolean, and cylindric algebras. The theory of algebraic hyperstructures has since been investigated and expanded by a number of scholars. Many scholars working in these domains have been drawn to the combination of fuzzy sets and algebraic hyperstructures, as well as neutrosophic sets and algebraic hyperstructures, resulting in the creation of new branches of research, namely fuzzy algebraic hyperstructures and neutrosophic algebraic hyperstructures.

Comer developed quasi-canonical hypergroups in [15] as an extension of canonical hypergroups, which were presented in [16]. In [17], Comer introduced a number of algebraic and combinatorial properties. In [18], Davvaz and Poursalavati introduced matrix representations of polygroups over hyperrings and the idea of a polygroup hyperring, which expanded the concept of a group ring. Davvaz devised permutation polygroups and topics connected to them, employing the notion of generalized permutation [19]. We refer to some important and recent innovative work relative to the fuzzy structures and polygroups in [20–42] for further information.

Neutrosophy is a new subfield of philosophy that investigates the origin, nature, and multitude of neutralities, as well as their interactions with other ideological spectrums, which was first proposed by Smarandache in 1995. In the neutrosophic set, indeterminacy is quantified explicitly and truth-membership, indeterminacy membership, and falsity-membership are independent. In a neutrosophic set, truth (T), indeterminacy (I), and falsity (F) are the three types of membership functions. In this work, we develop set theoretic operators on a special kind of the neutrosophic set known as the single-valued neutrosophic set. A single-valued neutrosophic set (SVNS) is a type of NS that may be employed to address intellectual and technical problems in the real world. As a result, the study of SVNSs and their attributes is essential in terms of applications as well as comprehending the principles of uncertainty.

In this article, first we define the generalized concept (η, ξ, φ)-SVNS and then apply this concept to hypergroups and polygroups. For decision-making problems, a weighted cosine similarity measure (WCSM) is applied to each alternative, and the ideal alternative is used to rank the alternatives and choose the best option. In addition, we compared our strategy to current approaches and demonstrated its superiority. In conclusion, an example scenario illustrates how the suggested D-M technique may be implemented. In comparison, existing fuzzy multicriteria decision-making (M-CDM) strategies are incapable of tackling the decision-making difficulty stated in this paper. The suggested single-valued neutrosophic (SVN) decision-making technique has the benefit of being able to cope with ambiguous and inconsistent information, both of which are typical in real-world circumstances.

The motivation of the proposed concept is explained as follows: to present a more generalized concept, i.e., (1) (η, ξ, φ)-single-valued neutrosophic hypergroups. (2) (η, ξ, φ)-single-valued neutrosophic polygroups. (3) (η, ξ, φ)-anti-single-valued neutrosophic polygroups. (4) Single-valued neutrosophic multicriteria decision-making method. Note that, clearly $\Upsilon^{\Omega} = \widetilde{\Omega}, \Upsilon^{\varnothing} = \widetilde{\varnothing}$, which shows that our proposed definition can be converted into a single-valued neutrosophic set. The purpose of this paper is to present the study of single-valued neutrosophic hypergroups and single-valued neutrosophic polygroups, and anti-single-valued neutrosophic polygroups under the triplet structure as a generalization of hypergroups, polygroups, and anti-polygroups as a powerful extension of single-valued neutrosophic sets.

This article is organized as follows: we offer some fundamental structure regarding single-valued neutrosophic sets, (η, ξ, φ)-single-valued neutrosophic hypergroup, and (weak) polygroups in Sections 2, 3, and 4, respectively. We present and analyze the idea of a (η, ξ, φ)-single-valued neutrosophic (weak) polygroup in Section 5. In Section 6, we explore the correlation between level sets of (η, ξ, φ)-single-valued neutrosophic polygroups ((η, ξ, φ)-SVNPs) and (normal) subpolygroups). Finally, in Section 7 we present the decision-making (D-M) procedure and for evaluation, we also offer an illustration example in Section 8.

2. Preliminaries

This section covers basic definitions related to SVNSs. In this section, we also present fundamental properties and relationships between SVNSs.

Definition 1 (see [44]). On the universe set Ω a SVNS Υ is stated as

$$\Omega = \{\langle u, \tau_{\Upsilon}(u), \iota_{\Upsilon}(u), \digamma_{\Upsilon}(u)\rangle, u \in \Omega\}, \quad (1)$$

where $\tau, \iota, \digamma\colon \Omega \longrightarrow [0,1]$, and $0 \le \tau_{\Upsilon}(u) + \iota_{\Upsilon}(u) + \digamma_{\Upsilon}(u) \le 3$, $\forall u \in \Omega, \tau_{\Upsilon}(u), \iota_{\Upsilon}(u), \digamma_{\Upsilon}(u) \in [0,1]$. $\tau_{\Upsilon}, \iota_{\Upsilon}, \digamma_{\Upsilon}$ indicates truth, indeterminacy, and falsity-membership function, in that order.

Definition 2 (see [44]). Let Ω be a set of objects, with u denoting a generic entity belong to Ω. A SVNS Υ on Ω is symbolized by truth τ_{Υ}, indeterminacy ι_{Υ}, and falsity-membership function $\digamma_{\Upsilon}$, in that order. $\forall u \in \Omega$, $\tau_{\Upsilon}(u), \iota_{\Upsilon}(u), \digamma_{\Upsilon}(u) \in [0,1]$. A SVNS Υ can be written accordingly as

$$\Upsilon = \sum_{i}^{n} \frac{\langle \tau(u_i), \iota(u_i), \digamma(u_i)\rangle}{u_i}, \quad u_i \in \Omega. \quad (2)$$

Definition 3 (see [44]). The complement of a SVNS Υ is indicated by $c(\Upsilon)$ and is characterized by

$$\begin{aligned} \tau_{c(\Upsilon)}(u) &= \mathsf{F}_{\Upsilon}(u), \\ \iota_{c(\Upsilon)}(u) &= 1 - \iota_{\Upsilon}(u), \\ \mathsf{F}_{c(\Upsilon)}(u) &= \tau_{\Upsilon}(u), \quad \forall u \in \Omega. \end{aligned} \tag{3}$$

Definition 4 (see [44]). Let Υ and Θ be two SVNSs on Ω. Then

(1)

$$\Upsilon \subseteq \Theta, \Leftrightarrow \Upsilon(u) \leq \Theta(u). \tag{4}$$

That is

$$\begin{aligned} \tau_{\Upsilon}(u) &\leq \tau_{\Theta}(u), \\ \iota_{\Upsilon}(u) &\geq \iota_{\Theta}(u), \\ \mathsf{F}_{\Upsilon}(u) &\geq \mathsf{F}_{\Theta}(u). \end{aligned} \tag{5}$$

Also

$$\begin{aligned} \Upsilon = \Theta \Leftrightarrow \Upsilon \subseteq \Theta \\ \Theta \subseteq \Upsilon. \end{aligned} \tag{6}$$

(2) $\Delta = \Upsilon \cup \Theta$ such that

$$\Delta(u) = \Upsilon(u) \vee \Theta(u), \tag{7}$$

such that

$$\Upsilon(u) \vee \Theta(u) = \left(\tau_{\Upsilon}(u) \vee \tau_{\Theta}(u), \iota_{\Upsilon}(u) \wedge \iota_{\Theta}(u), \mathsf{F}_{\Upsilon}(u) \wedge \mathsf{F}_{\Theta}(u)\right), \quad \forall u \in \Omega. \tag{8}$$

It means

$$\begin{aligned} \tau_{\Delta}(u) &= \max\{\tau_{\Upsilon}(u), \tau_{\Theta}(u)\}, \\ \iota_{\Delta}(u) &= \min\{\iota_{\Upsilon}(u), \iota_{\Theta}(u)\}, \\ \mathsf{F}_{\Delta}(u) &= \min\{\mathsf{F}_{\Upsilon}(u), \mathsf{F}_{\Theta}(u)\}. \end{aligned} \tag{9}$$

(3) $\Delta = \Upsilon \cap \Theta$ such that

$$\Delta(u) = \Upsilon(u) \wedge \Theta(u), \tag{10}$$

such that

$$\Upsilon(u) \wedge \Theta(u) = \left(\tau_{\Upsilon}(u) \wedge \tau_{\Theta}(u), \iota_{\Upsilon}(u) \vee \iota_{\Theta}(u), \mathsf{F}_{\Upsilon}(u) \vee \mathsf{F}_{\Theta}(u)\right), \quad \forall u \in \Omega. \tag{11}$$

It means

$$\begin{aligned} \tau_{\Delta}(u) &= \min\{\tau_{\Upsilon}(u), \tau_{\Theta}(u)\}, \\ \iota_{\Delta}(u) &= \max\{\iota_{\Upsilon}(u), \iota_{\Theta}(u)\}, \\ \mathsf{F}_{\Delta}(u) &= \max\{\mathsf{F}_{\Upsilon}(u), \mathsf{F}_{\Theta}(u)\}. \end{aligned} \tag{12}$$

Proposition 1 *(see [44]). Let the SVNSs on the common universe Ω be Υ, Θ, and Δ. Then the following conditions must hold the following:*

(1) $\Upsilon \cup \Theta = \Theta \cup \Upsilon, \Upsilon \cap \Theta = \Theta \cap \Upsilon$.

(2) $\Upsilon \cup (\Theta \cup \Delta) = (\Upsilon \cup \Theta) \cup \Delta, \Upsilon \cap (\Theta \cap \Delta) = (\Upsilon \cap \Theta) \cap \Delta$.

(3) $\Upsilon \cup (\Theta \cap \Delta) = (\Upsilon \cup \Theta) \cap (\Upsilon \cup \Delta), \Upsilon \cap (\Theta \cup \Delta) = (\Upsilon \cap \Theta) \cup (\Upsilon \cap \Delta)$.

(4) $\Upsilon \cap \widetilde{\varnothing} = \widetilde{\varnothing}, \Upsilon \cup \widetilde{\varnothing} = \Upsilon, \Upsilon \cup \widetilde{\Omega} = \widetilde{\Omega}, \Upsilon \cap \widetilde{\Omega} = \Upsilon$, *where*

$$\tau_{\widetilde{\varnothing}} = 0, \iota_{\widetilde{\varnothing}} = \mathsf{F}_{\widetilde{\varnothing}} = 1, \tau_{\widetilde{\Omega}} = 1, \iota_{\widetilde{\Omega}} = \mathsf{F}_{\widetilde{\Omega}} = 0. \tag{13}$$

(5) $c(\Upsilon \cup \Theta) = c(\Upsilon) \cap c(\Theta), c(\Upsilon \cap \Theta) = c(\Upsilon) \cup c(\Theta)$.

3. (η, ξ, φ)- Single-Valued Neutrosophic Hypergroup

We define and investigate the basic properties and characterizations of a single-valued neutrosophic set, single-valued neutrosophic hypergroup, and single-valued neutrosophic subhypergroup over hypergroup H under the triplet structure in this section. We basically start with some introductory (η, ξ, φ)-SVNS, then define (η, ξ, φ)-SVN hypergroup, the t-level set on (η, ξ, φ)-SVNS, important operations and properties of (η, ξ, φ)-SVN hypergroups, and then study crucial results, propositions, theorems and remarks related to SVN hypergroup and SVN subhypergroup under the triplet structure. In this section, we present a very important result, that is intersection of two (η, ξ, φ)-SVN hypergroups over H is again (η, ξ, φ)-SVN hypergroup in 3.18, which shows that (η, ξ, φ)-SVN hypergroups are closed under intersection, and union of two (η, ξ, φ)-SVN hypergroups over H need not be (η, ξ, φ)-SVN hypergroup over H.

Definition 5. If Υ be a single-valued neutrosophic (SVN) subset of Ω, then (η, ξ, φ)-SVN subset Υ of Ω is categorize as

$$\Upsilon^{(\eta,\xi,\varphi)} = \left\{\langle e, \tau_{\Upsilon}^{\eta}(u), \iota_{\Upsilon}^{\xi}(u), \mathsf{F}_{\Upsilon}^{\varphi}(u)\rangle | u \in \Omega\right\}, \tag{14}$$

where

$$\begin{aligned} \tau_{\Upsilon}^{\eta}(u) &= \wedge\{\tau_{\Upsilon}(u), \eta\}, \\ \iota_{\Upsilon}^{\xi}(u) &= \vee\{\iota_{\Upsilon}(u), \xi\}, \\ \mathsf{F}_{\Upsilon}^{\varphi}(u) &= \vee\{\mathsf{F}_{\Upsilon}(u), \varphi\}, \end{aligned} \tag{15}$$

such that

$$0 \leq \tau_{\Upsilon}^{\eta}(u) + \iota_{\Upsilon}^{\xi}(u) + \mathsf{F}_{\Upsilon}^{\varphi}(u) \leq 3, \tag{16}$$

where $\eta, \xi, \varphi \in [0, 1]$, also $\tau, \iota, \mathsf{F}\colon A \longrightarrow [0, 1]$, such that τ_{Υ}^{η}, ι_{Υ}^{ξ}, $\mathsf{F}_{\Upsilon}^{\varphi}$ represents the functions of truth, indeterminacy, and falsity-membership, respectively.

Definition 6. Let Ω be a space of objects, with u denoting a generic entity belong to Ω. A (η, ξ, φ)-SVNS Υ on Ω is symbolized by truth τ_{Υ}^{η}, indeterminacy ι_{Υ}^{ξ}, and falsity-membership function $\mathsf{F}_{\Upsilon}^{\varphi}$, respectively. For every u in Ω, $\tau_{\Upsilon}^{\eta}(u), \iota_{\Upsilon}^{\xi}(u), \mathsf{F}_{\Upsilon}^{\varphi}(u) \in [0, 1]$, a (η, ξ, φ)-SVNS Υ can be written accordingly as

$$\Upsilon^{(\eta,\xi,\varphi)} = \sum_{i}^{n} \frac{\langle \tau^{\eta}(u_i), \iota^{\xi}(u_i), \mathsf{F}^{\varphi}(u_i)\rangle}{u_i}, \quad u_i \in \Omega. \tag{17}$$

Definition 7. Let Υ and Θ be two (η, ξ, φ)-SVNSs on Ω. The followings must hold the following:

(1) $\Upsilon^{(\eta,\xi,\varphi)} \subseteq \Theta^{(\eta,\xi,\varphi)} \Leftrightarrow \Upsilon^{(\eta,\xi,\varphi)}(u) \leq \Theta^{(\eta,\xi,\varphi)}(u)$.

That is,

$$\begin{aligned} &\tau_{\Upsilon}^{\eta}(u) \leq \tau_{\Theta}^{\eta}(u),\\ &\iota_{\Upsilon}^{\xi}(u) \geq \iota_{\Theta}^{\xi}(u),\\ &\mathsf{F}_{\Upsilon}^{\varphi}(u) \geq \mathsf{F}_{\Theta}^{\varphi}(u), \end{aligned} \tag{18}$$

$$\Upsilon^{(\eta,\xi,\varphi)} = \Theta^{(\eta,\xi,\varphi)} \Leftrightarrow \Upsilon^{(\eta,\xi,\varphi)} \subseteq \Theta^{(\eta,\xi,\varphi)} \text{ and } \Theta^{(\eta,\xi,\varphi)} \subseteq \Upsilon^{(\eta,\xi,\varphi)}. \tag{19}$$

(2) The union of $\Upsilon^{(\eta,\xi,\varphi)}$ and $\Theta^{(\eta,\xi,\varphi)}$ is indicated by

$$\Delta^{(\eta,\xi,\varphi)} = \Upsilon^{(\eta,\xi,\varphi)} \cup \Theta^{(\eta,\xi,\varphi)}, \tag{20}$$

and defined as

$$\Delta^{(\eta,\xi,\varphi)}(u) = \Upsilon^{(\eta,\xi,\varphi)}(u) \vee \Theta^{(\eta,\xi,\varphi)}(u), \tag{21}$$

where

$$\begin{aligned} &\Upsilon^{(\eta,\xi,\varphi)}(u) \vee \Theta^{(\eta,\xi,\varphi)}(u)\\ &= \left(\tau_{\Upsilon}^{\eta}(u) \vee \tau_{\Theta}^{\eta}(u), \iota_{\Upsilon}^{\xi}(u) \wedge \iota_{\Theta}^{\xi}(u), \mathsf{F}_{\Upsilon}^{\varphi}(u) \wedge \mathsf{F}_{\Theta}^{\varphi}(u)\right), \quad \forall u \in \Omega. \end{aligned} \tag{22}$$

That is,

$$\begin{aligned} &\tau_{\Delta}^{\eta}(u) = \max\{\tau_{\Upsilon}^{\eta}(u), \tau_{\Theta}^{\eta}(u)\},\\ &\iota_{\Delta}^{\xi}(u) = \min\left\{\iota_{\Upsilon}^{\xi}(u), \iota_{\Theta}^{\xi}(u)\right\},\\ &\mathsf{F}_{\Delta}^{\varphi}(u) = \min\{\mathsf{F}_{\Upsilon}^{\varphi}(u), \mathsf{F}_{\Theta}^{\varphi}(u)\}. \end{aligned} \tag{23}$$

(3) The intersection of $\Upsilon^{(\eta,\xi,\varphi)}$ and $\Theta^{(\eta,\xi,\varphi)}$ is indicated by

$$\Delta^{(\eta,\xi,\varphi)} = \Upsilon^{(\eta,\xi,\varphi)} \cap \Theta^{(\eta,\xi,\varphi)}, \tag{24}$$

and defined as

$$\Delta^{(\eta,\xi,\varphi)}(u) = \Upsilon^{(\eta,\xi,\varphi)}(u) \wedge \Theta^{(\eta,\xi,\varphi)}(u), \tag{25}$$

where

$$\begin{aligned} &\Upsilon^{(\eta,\xi,\varphi)}(u) \wedge \Theta^{(\eta,\xi,\varphi)}(u)\\ &= \left(\tau_{\Upsilon}^{\eta}(u) \wedge \tau_{\Theta}^{\eta}(u), \iota_{\Upsilon}^{\xi}(u) \vee \iota_{\Theta}^{\xi}(u), \mathsf{F}_{\Upsilon}^{\varphi}(u) \vee \mathsf{F}_{\Theta}^{\varphi}(u)\right), \quad \forall u \in \Omega. \end{aligned} \tag{26}$$

That is,

$$\begin{aligned} &\tau_{\Delta}^{\eta}(u) = \min\{\tau_{\Upsilon}^{\eta}(u), \tau_{\Theta}^{\eta}(u)\},\\ &\iota_{\Delta}^{\xi}(u) = \max\left\{\iota_{\Upsilon}^{\xi}(u), \iota_{\Theta}^{\xi}(u)\right\},\\ &\mathsf{F}_{\Delta}^{\varphi}(u) = \max\{\mathsf{F}_{\Upsilon}^{\varphi}(u), \mathsf{F}_{\Theta}^{\varphi}(u)\}. \end{aligned} \tag{27}$$

Proposition 2. *Let Υ, Θ, and Δ be (η, ξ, φ)-SVNSs on the common universe ΩThen the following properties must hold the following:*

(1) $\Upsilon^{(\eta,\xi,\varphi)} \cup \Theta^{(\eta,\xi,\varphi)} = \Theta^{(\eta,\xi,\varphi)} \cup \Upsilon^{(\eta,\xi,\varphi)}$.
$\Upsilon^{(\eta,\xi,\varphi)} \cap \Theta^{(\eta,\xi,\varphi)} = \Theta^{(\eta,\xi,\varphi)} \cap \Upsilon^{(\eta,\xi,\varphi)}$.

(2) $\Upsilon^{(\eta,\xi,\varphi)} \cup (\Theta^{(\eta,\xi,\varphi)} \cup \Delta^{(\eta,\xi,\varphi)}) = (\Upsilon^{(\eta,\xi,\varphi)} \cup \Theta^{(\eta,\xi,\varphi)}) \cup \Delta^{(\eta,\xi,\varphi)}$. $\Upsilon^{(\eta,\xi,\varphi)} \cap (\Theta^{(\eta,\xi,\varphi)} \cap \Delta^{(\eta,\xi,\varphi)}) = (\Upsilon^{(\eta,\xi,\varphi)} \cap \Theta^{(\eta,\xi,\varphi)}) \cap \Delta^{(\eta,\xi,\varphi)}$.

(3) $\Upsilon^{(\eta,\xi,\varphi)} \cup (\Theta^{(\eta,\xi,\varphi)} \cap \Delta^{(\eta,\xi,\varphi)}) = (\Upsilon^{(\eta,\xi,\varphi)} \cup \Theta^{(\eta,\xi,\varphi)}) \cap (\Upsilon^{(\eta,\xi,\varphi)} \cup \Delta^{(\eta,\xi,\varphi)})$. $\Upsilon^{(\eta,\xi,\varphi)} \cap (\Theta^{(\eta,\xi,\varphi)} \cup \Delta^{(\eta,\xi,\varphi)}) = (\Upsilon^{(\eta,\xi,\varphi)} \cap \Theta^{(\eta,\xi,\varphi)}) \cup (\Upsilon^{(\eta,\xi,\varphi)} \cap \Delta^{(\eta,\xi,\varphi)})$.

(4) $\Upsilon^{(\eta,\xi,\varphi)} \cap \widetilde{\varnothing} = \widetilde{\varnothing}$, $\Upsilon^{(\eta,\xi,\varphi)} \cup \widetilde{\varnothing} = \Upsilon^{(\eta,\xi,\varphi)}$.
$\Upsilon^{(\eta,\xi,\varphi)} \cup \widetilde{\Omega} = \widetilde{\Omega}$, $\Upsilon^{(\eta,\xi,\varphi)} \cap \widetilde{\Omega} = \Upsilon^{(\eta,\xi,\varphi)}$,

where

$$\tau_{\widetilde{\varnothing}}^{\eta} = 0, \iota_{\widetilde{\varnothing}}^{\xi} = \mathsf{F}_{\widetilde{\varnothing}}^{\varphi} = 1, \tau_{\widetilde{\Omega}}^{\eta} = 1, \iota_{\widetilde{\Omega}}^{\xi} = \mathsf{F}_{\widetilde{\Omega}}^{\varphi} = 0. \tag{28}$$

(5) $c(\Upsilon^{(\eta,\xi,\varphi)} \cup \Theta^{(\eta,\xi,\varphi)}) = c(\Upsilon^{(\eta,\xi,\varphi)}) \cap c(\Theta^{(\eta,\xi,\varphi)})$. $c(\Upsilon^{(\eta,\xi,\varphi)} \cap \Theta^{(\eta,\xi,\varphi)}) = c(\Upsilon^{(\eta,\xi,\varphi)}) \cup c(\Theta^{(\eta,\xi,\varphi)})$? ?

Definition 8. The complement of a (η, ξ, φ)-SVNS Υ is denoted by $c(\Upsilon^{(\eta,\xi,\varphi)})$ and is defined by

$$c\left(\Upsilon^{(\eta,\xi,\varphi)}\right) = \langle u, \tau_{c(\Upsilon)}^{\eta}(u), \iota_{c(\Upsilon)}^{\xi}(u), \mathsf{F}_{c(\Upsilon)}^{\varphi}(u)\rangle, \tag{29}$$

where

$$\begin{aligned} &\tau_{c(\Upsilon)}^{\eta}(u) = \mathsf{F}_{\Upsilon}^{\varphi}(u),\\ &\iota_{c(\Upsilon)}^{\xi}(u) = 1 - \iota_{\Upsilon}^{\xi}(u),\\ &\mathsf{F}_{c(\Upsilon)}^{\varphi}(u) = \tau_{\Upsilon}^{\eta}(u), \forall u \in \Omega. \end{aligned} \tag{30}$$

Definition 9. The falsity-favorite of a (η, ξ, φ)-SVNS $\Theta^{(\eta,\xi,\varphi)}$ (i.e., $\Theta^{(\eta,\xi,\varphi)} = \triangledown\Upsilon^{(\eta,\xi,\varphi)}$) whose truth and falsity-membership functions are defined by

$$\tau_{\Theta}^{\eta}(x) = \tau_{\Upsilon}^{\eta}(u),$$
$$\iota_{\Theta}^{\xi}(u) = 0, \tag{31}$$
$$\digamma_{\Theta}^{\varphi}(u) = \min\left\{\digamma_{\Upsilon}^{\varphi}(u) + \iota_{\Upsilon}^{\xi}(u), 1\right\}.$$

Throughout this section H denotes the hypergroup $\langle H,^{\circ}\rangle$.

Definition 10 (see [45]). A set H is called hypergroup $\langle H,^{\circ}\rangle$ with an associative hyperoperation $(^{\circ})$: $H * H \longrightarrow P(H)$, which satisfies $x^{\circ} \pm H = H^{\circ}x = H$, $\forall x \in H$ (reproduction axiom).

Definition 11 (see [46]). If the following properties satisfy, a hyperstructure $\langle H,^{\circ}\rangle$ is called a H_v-group.

(1) $x^{\circ}(y^{\circ}z) \cap (x^{\circ}y^{\circ})z \neq \varnothing$, $\forall x, y, z \in H$, (H_v-semigroup).

(2) $x^{\circ}H = H^{\circ}x = H$, $\forall x \in H$.

Definition 12 (see [45]). A subset K of H is called as subhypergroup if $\langle K,^{\circ}\rangle$ is a hypergroup.

Definition 13. Let Υ be a (η, ξ, φ)-SVNS over H. Then Υ is called a (η, ξ, φ)-SVN hypergroup over H, if the following conditions are satisfied:

(i) $\forall u, v \in H$,

$$\min\{\tau_{\Upsilon}^{\eta}(u), \tau_{\Upsilon}^{\eta}(v)\} \leq \inf\{\tau_{\Upsilon}^{\eta}(w)\colon w \in u^{\circ}v\},$$
$$\max\left\{\iota_{\Upsilon}^{\xi}(u), \iota_{\Upsilon}^{\xi}(v)\right\} \geq \sup\left\{\iota_{\Upsilon}^{\xi}(w)\colon w \in u^{\circ}v\right\}, \tag{32}$$
$$\max\{\digamma_{\Upsilon}^{\varphi}(u), \digamma_{\Upsilon}^{\varphi}(v)\} \geq \sup\{\digamma_{\Upsilon}^{\varphi}(w)\colon w \in u^{\circ}v\}.$$

(ii) $\forall l, u \in H$, $\exists v \in H$ such that $u \in l^{\circ}v$ and

$$\min\{\tau_{\Upsilon}^{\eta}(l), \tau_{\Upsilon}^{\eta}(u)\} \leq \tau_{\Upsilon}^{\eta}(v),$$
$$\max\left\{\iota_{\Upsilon}^{\xi}(l), \iota_{\Upsilon}^{\xi}(u)\right\} \geq \iota_{\Upsilon}^{\xi}(v), \tag{33}$$
$$\max\{\digamma_{\Upsilon}^{\varphi}(l), \digamma_{\Upsilon}^{\varphi}(u)\} \geq \digamma_{\Upsilon}^{\varphi}(v).$$

(iii) $\forall l, u \in H$, $\exists w \in H$ such that $u \in w^{\circ}l$ and

$$\min\{\tau_{\Upsilon}^{\eta}(l), \tau_{\Upsilon}^{\eta}(u)\} \leq \tau_{\Upsilon}^{\eta}(w),$$
$$\max\left\{\iota_{\Upsilon}^{\xi}(l), \iota_{\Upsilon}^{\xi}(u)\right\} \geq \iota_{\Upsilon}^{\xi}(w), \tag{34}$$
$$\max\{\digamma_{\Upsilon}^{\varphi}(l), \digamma_{\Upsilon}^{\varphi}(u)\} \geq \digamma_{\Upsilon}^{\varphi}(w).$$

If $\Upsilon^{(\eta,\xi,\varphi)}$ satisfies condition (i) then Υ is a (η, ξ, φ)-SVN semihypergroup over H. Condition (ii) and (iii) represent the left and right reproduction axioms, respectively. Then Υ is a (η, ξ, φ)-SVN subhypergroup of H.

Example 1. If the family of t-level sets of (η, ξ, φ)-SVNS Υ over H.

$$\Upsilon_t^{(\eta,\xi,\varphi)} = \left\{u \in H | \tau_{\Upsilon}^{\eta}(u) \geq t, \iota_{\Upsilon}^{\xi}(u) \leq t \text{and} \digamma_{\Upsilon}^{\varphi}(u) \leq t\right\}, \tag{35}$$

is a subhypergroup of H. Then Υ is a (η, ξ, φ)-SVN hypergroup over H.

Theorem 1. *Let Υ be a (η, ξ, φ)-SVNS over H. Then Υ is a (η, ξ, φ)-SVN hypergroup over H if and only if Υ is a (η, ξ, φ)-SVN semihypergroup over H and also $\Upsilon^{(\eta,\xi,\varphi)}$ satisfies the left and right reproduction axioms.*

Proof 1. The proof is obvious from Definition 13. □

Theorem 2. *Let Υ be a (η, ξ, φ)-SVNS over H. If $\Upsilon^{(\eta,\xi,\varphi)}$ is a SVN hypergroup over H, then $\forall t \in [0, 1]\Upsilon_t^{(\eta,\xi,\varphi)} \neq \varnothing$ is a subhypergroup of H.*

Proof 2. Let Υ be a (η, ξ, φ)-SVN hypergroup over H and let $u, v \in \Upsilon_t^{(\eta,\xi,\varphi)}$, then

$$\tau_{\Upsilon}^{\eta}(u), \tau_{\Upsilon}^{\eta}(v) \geq t, \iota_{\Upsilon}^{\xi}(u), \iota_{\Upsilon}^{\xi}(v) \leq t \text{ and } \digamma_{\Upsilon}^{\varphi}(u), \digamma_{\Upsilon}^{\varphi}(v) \leq t. \tag{36}$$

Then we have

$$\inf\{\tau_{\Upsilon}^{\eta}(w)\colon w \in u^{\circ}v\} \geq \min\{\tau_{\Upsilon}^{\eta}(u), \tau_{\Upsilon}^{\eta}(v)\} \geq \min\{t, t\} = t,$$
$$\sup\left\{\iota_{\Upsilon}^{\xi}(w)\colon w \in u^{\circ}v\right\} \leq \max\{\tau_{\Upsilon}^{\eta}(u), \tau_{\Upsilon}^{\eta}(v)\} \leq \max\{t, t\} = t,$$
$$\sup\{\digamma_{\Upsilon}^{\varphi}(w)\colon w \in u^{\circ}v\} \leq \max\{\tau_{\Upsilon}^{\eta}(u), \tau_{\Upsilon}^{\eta}(v)\} \leq \max\{t, t\} = t. \tag{37}$$

This implies $w \in \Upsilon_t^{(\eta,\xi,\varphi)}$. Then $\forall w \in u^{\circ}v, u^{\circ}v \subseteq \Upsilon_t^{(\eta,\xi,\varphi)}$. Thus $\forall w \in \Upsilon_t^{(\eta,\xi,\varphi)}$, we obtain. $w^{\circ}\Upsilon_t^{(\eta,\xi,\varphi)} \subseteq \Upsilon_t^{(\eta,\xi,\varphi)}$

Now, Let $l, u \in \Upsilon_t^{(\eta,\xi,\varphi)}$, then there exist $v \in H$ such that $u \in l^{\circ}v$ and

$$\{\tau_{\Upsilon}^{\eta}(v)\} \geq \min\{\tau_{\Upsilon}^{\eta}(l), \tau_{\Upsilon}^{\eta}(u)\} \geq \min\{t, t\} = t,$$
$$\left\{\iota_{\Upsilon}^{\xi}(v)\right\} \leq \max\{\tau_{\Upsilon}^{\eta}(l), \tau_{\Upsilon}^{\eta}(u)\} \leq \max\{t, t\} = t, \tag{38}$$
$$\{\digamma_{\Upsilon}^{\varphi}(v)\} \leq \max\{\tau_{\Upsilon}^{\eta}(l), \tau_{\Upsilon}^{\eta}(u)\} \leq \max\{t, t\} = t.$$

This implies that $v \in \Upsilon_t^{(\eta,\xi,\varphi)}$. This proves that $\Upsilon_t^{(\eta,\xi,\varphi)} \subseteq w^{\circ}\Upsilon_t^{(\eta,\xi,\varphi)}$. As such. $\Upsilon_t^{(\eta,\xi,\varphi)} = w^{\circ}\Upsilon_t^{(\eta,\xi,\varphi)}$

which proves that $\Upsilon_t^{(\eta,\xi,\varphi)}$ is a subhypergroup of H. □

Theorem 3. *Let Υ be a (η, ξ, φ)-SVNS over HThen the following are equivalent:*

(i) Υ is a (η, ξ, φ)-SVN hypergroup over H.

(ii) $\forall t \in [0, 1]\Upsilon_t^{(\eta,\xi,\varphi)} \neq \varnothing$ is a subhypergroup of H.

Proof 3. (i) $\Rightarrow$ (ii) The proof is obvious from Theorem 2. (ii) $\Rightarrow$ (i) Now assume that $\Upsilon_t^{(\eta,\xi,\varphi)}$ is a subhypergroup of H. Let $u, v \in \Upsilon_{t_o}^{(\eta,\xi,\varphi)}$ and let. $\min\{\tau_{\Upsilon}^{\eta}(u), \tau_{\Upsilon}^{\eta}(v)\} = \max\left\{\iota_{\Upsilon}^{\xi}(u), \iota_{\Upsilon}^{\xi}(v)\right\} = \max\{\digamma_{\Upsilon}^{\varphi}(u), \digamma_{\Upsilon}^{\varphi}(v)\} = t_o$Since $u^{\circ}v \subseteq \Upsilon_{t_o}^{(\eta,\xi,\varphi)}$, then for every $w \in u^{\circ}v, \tau_{\Upsilon}^{\eta}(w) \geq t_o, \iota_{\Upsilon}^{\xi}(w) \leq t_o, \digamma_{\Upsilon}^{\varphi}(w) \leq t_o$.

$$\begin{aligned}
&\min\{\tau_{\Upsilon}^{\eta}(u),\tau_{\Upsilon}^{\eta}(u)\}\leq\inf\{\tau_{\Upsilon}^{\eta}(w)\colon w\in u^{\circ}v\},\\
&\max\left\{\iota_{\Upsilon}^{\xi}(u),\iota_{\Upsilon}^{\xi}(v)\right\}\geq\sup\left\{\iota_{\Upsilon}^{\xi}(w)\colon w\in u^{\circ}v\right\},\\
&\max\{\mathsf{F}_{\Upsilon}^{\varphi}(u),\mathsf{F}_{\Upsilon}^{\varphi}(v)\}\geq\sup\{\mathsf{F}_{\Upsilon}^{\varphi}(w)\colon w\in u^{\circ}v\}.
\end{aligned}\tag{39}$$

Condition (i) is verified.

Next, let $l,u\in\Upsilon_{t_1}^{(\eta,\xi,\varphi)}$, for every $t_1\in[0,1]$ and let $\min\{\tau_{\Upsilon}^{\eta}(l),\tau_{\Upsilon}^{\eta}(v)\}=\max\{\iota_{\Upsilon}^{\xi}(l),\iota_{\Upsilon}^{\xi}(v)\}=\max\{\mathsf{F}_{\Upsilon}^{\varphi}(l),\mathsf{F}_{\Upsilon}^{\varphi}(v)\}=t_1$.

Then there exist $v\in\Upsilon_{t_1}^{(\eta,\xi,\varphi)}$ such that $u\in l^{\circ}v\subseteq\Upsilon_{t_1}^{(\eta,\xi,\varphi)}$. Since $v\in\Upsilon_{t_1}^{(\eta,\xi,\varphi)}$, then

$$\begin{aligned}
&\tau_{\Upsilon}^{\eta}(v)\geq t_1=\min\{\tau_{\Upsilon}^{\eta}(l),\tau_{\Upsilon}^{\eta}(u)\},\\
&\iota_{\Upsilon}^{\xi}(v)\leq t_1=\max\left\{\iota_{\Upsilon}^{\xi}(l),\iota_{\Upsilon}^{\xi}(u)\right\},\\
&\mathsf{F}_{\Upsilon}^{\varphi}(v)\leq t_1=\max\{\mathsf{F}_{\Upsilon}^{\varphi}(l),\mathsf{F}_{\Upsilon}^{\varphi}(u)\}.
\end{aligned}\tag{40}$$

Condition (ii) is verified.

Next, let $l,u\in\Upsilon_{t_1}^{(\eta,\xi,\varphi)}$, for every $t_1\in[0,1]$ and let $\min\{\tau_{\Upsilon}^{\eta}(l),\tau_{\Upsilon}^{\eta}(l)\}=\max\{\iota_{\Upsilon}^{\xi}(l),\iota_{\Upsilon}^{\xi}(v)\}=\max\{\mathsf{F}_{\Upsilon}^{\varphi}(l),\mathsf{F}_{\Upsilon}^{\varphi}(v)\}=t_1$.

Then there exist $w\in\Upsilon_{t_1}^{(\eta,\xi,\varphi)}$ such that $u\in w^{\circ}l\subseteq\Upsilon_{t_1}^{(\eta,\xi,\varphi)}$. Since $w\in\Upsilon_{t_1}^{(\eta,\xi,\varphi)}$, then

$$\begin{aligned}
&\tau_{\Upsilon}^{\eta}(w)\geq t_1=\min\{\tau_{\Upsilon}^{\eta}(l),\tau_{\Upsilon}^{\eta}(u)\},\\
&\iota_{\Upsilon}^{\xi}(w)\leq t_1=\max\left\{\iota_{\Upsilon}^{\xi}(l),\iota_{\Upsilon}^{\xi}(u)\right\},\\
&\mathsf{F}_{\Upsilon}^{\varphi}(w)\leq t_1=\max\{\mathsf{F}_{\Upsilon}^{\varphi}(l),\mathsf{F}_{\Upsilon}^{\varphi}(u)\}.
\end{aligned}\tag{41}$$

Condition (iii) is verified. □

Theorem 4. *Let Υ be a (η,ξ,φ)-SVNS over H. Then Υ be a (η,ξ,φ)-SVN hypergroup over H if and only if $\forall\alpha,\beta,\gamma\in[0,1]$, $\Upsilon_{(\alpha,\beta,\gamma)}^{(\eta,\xi,\varphi)}$ is a subhypergroup of H.*

Proof 4. The proof is simple for readers. □

Theorem 5. *Let Υ be a (η,ξ,φ)-SVN hypergroup over H and $\forall t_1,t_2\in[0,1]$, $\Upsilon_{t_1}^{(\eta,\xi,\varphi)}$ and $\Upsilon_{t_2}^{(\eta,\xi,\varphi)}$ be the t-level sets of $\Upsilon^{(\eta,\xi,\varphi)}$ with $\Upsilon_{t_2}^{(\eta,\xi,\varphi)}$, then $\Upsilon_{t_1}^{(\eta,\xi,\varphi)}$ is a subhypergroup of $\Upsilon_{t_2}^{(\eta,\xi,\varphi)}$.*

Proof 5. $\forall t_1,t_2\in[0,1]$, $\Upsilon_{t_1}^{(\eta,\xi,\varphi)}$ and $\Upsilon_{t_2}^{(\eta,\xi,\varphi)}$ be the t-level sets of $\Upsilon^{(\eta,\xi,\varphi)}$ with $t_1\geq t_2$.This implies that $\Upsilon_{t_1}^{(\eta,\xi,\varphi)}\subseteq\Upsilon_{t_2}^{(\eta,\xi,\varphi)}$. By Theorem 2, $\Upsilon_{t_1}^{(\eta,\xi,\varphi)}$ is a subhypergroup of $\Upsilon_{t_2}^{(\eta,\xi,\varphi)}$. □

Proposition 3. *If Υ and Θ be two (η,ξ,φ)-SVN subset of hypergroup H, then*

$$(\Upsilon\cap\Theta)^{(\eta,\xi,\varphi)}=\Upsilon^{(\eta,\xi,\varphi)}\cap\Theta^{(\eta,\xi,\varphi)}.\tag{42}$$

Proof 6. Assume that Υ and Θ are two (η,ξ,φ)-SVN subset of hypergroup H.

$$\begin{aligned}
(\Upsilon\cap\Theta)^{(\eta,\xi,\varphi)}(u)&=\{\min\{\min\{\tau_{\Upsilon}(u),\tau_{\Theta}(u)\},\eta\},\max\{\max\{\iota_{\Upsilon}(u),\iota_{\Theta}(u)\},\xi\},\max\{\max\{\mathsf{F}_{\Upsilon}(u),\mathsf{F}_{\Theta}(u)\},\varphi\}\}\\
&=\{\wedge\{\wedge\{\tau_{\Upsilon}(u),\eta\},\wedge\{\tau_{\Theta}(u),\eta\}\},\vee\{\vee\{\iota_{\Upsilon}(u),\xi\},\vee\{\iota_{\Theta}(u),\xi\}\},\vee\{\vee\{\mathsf{F}_{\Upsilon}(u),\varphi\},\vee\{\mathsf{F}_{\Theta}(u),\varphi\}\}\\
&=\left\{\min\left(\{\tau_{\Upsilon}^{\eta}(u)\},\{\tau_{\Theta}^{\eta}(u)\}\right),\max\left(\left\{\iota_{\Upsilon}^{\xi}(u)\right\},\left\{\iota_{\Theta}^{\xi}(u)\right\}\right),\max\left(\{\mathsf{F}_{\Upsilon}^{\varphi}(u)\},\{\mathsf{F}_{\Theta}^{\varphi}(u)\}\right)\right\}\\
&=\Upsilon^{(\eta,\xi,\varphi)}(u)\cap\Theta^{(\eta,\xi,\varphi)}(u),\forall u\in H.
\end{aligned}\tag{43}$$

□

Theorem 6. *Let Υ and Θ be (η,ξ,φ)-SVN hypergroups over H. Then $\Upsilon\cap\Theta$ is a (η,ξ,φ)-SVN hypergroup over H if it is non-null.*

Proof 7. Let Υ and Θ be two (η,ξ,φ)-SVN hypergroups over H. Let $u\in H$ be any element,

$$\begin{aligned}
&(\Upsilon\cap\Theta)^{(\eta,\xi,\varphi)}(u)\\
&=\left\{(\tau_{\Upsilon}\wedge\tau_{\Theta})^{\eta}(u),(\iota_{\Upsilon}\vee\iota_{\Theta})^{\xi}(u)(\mathsf{F}_{\Upsilon}\vee\mathsf{F}_{\Theta})^{\varphi}(u)\right\}.
\end{aligned}\tag{44}$$

By using result of Proposition 3.,

$$(\Upsilon\cap\Theta)^{(\eta,\xi,\varphi)}(u)=\left(\Upsilon^{(\eta,\xi,\varphi)}\cap\Theta^{(\eta,\xi,\varphi)}\right)(u),\tag{45}$$

$$\begin{aligned}
&\left(\Upsilon^{(\eta,\xi,\varphi)}\cap\Theta^{(\eta,\xi,\varphi)}\right)(u)\\
&=\left\{\min\{\tau_{\Upsilon}^{\eta}(u)(u),\tau_{\Theta}^{\eta}(u)\},\max\left\{\iota_{\Upsilon}^{\xi}(u),\iota_{\Theta}^{\xi}(u)\right\},\right.\\
&\left.\max\{\mathsf{F}_{\Upsilon}^{\varphi}(u),\mathsf{F}_{\Theta}^{\varphi}(u)\})\right\},.
\end{aligned}\tag{46}$$

By using (44), (45), and (46), we get

$$\begin{aligned}
(\Upsilon\cap\Theta)^{(\eta,\xi,\varphi)}(u)&=\left\{(\tau_{\Upsilon}\wedge\tau_{\Theta})^{\eta}(u),(\iota_{\Upsilon}\vee\iota_{\Theta})^{\xi}(u)(\mathsf{F}_{\Upsilon}\vee\mathsf{F}_{\Theta})^{\varphi}(u)\right\}\\
&=\left\{\min\{\tau_{\Upsilon}^{\eta}(u),\tau_{\Theta}^{\eta}(u)\},\max\left\{\iota_{\Upsilon}^{\xi}(u),\iota_{\Theta}^{\xi}(u)\right\},\max\{\mathsf{F}_{\Upsilon}^{\varphi}(u),\mathsf{F}_{\Theta}^{\varphi}(u)\})\right\}.\\
&=\left\{(\tau_{\Upsilon}^{\eta}\wedge\tau_{\Theta}^{\eta})(u),\left(\iota_{\Upsilon}^{\xi}\vee\iota_{\Theta}^{\xi}\right)(u)\mathsf{F}_{\Upsilon}^{\varphi}(u)\vee\mathsf{F}_{\Theta}^{\varphi}(u)\right\}
\end{aligned}\tag{47}$$

Since, $(\Upsilon\cap\Theta)^{(\eta,\xi,\varphi)}(u) = \{\langle u, \tau^{\eta}_{\Upsilon\cap\Theta}(u), \iota^{\xi}_{\Upsilon\cap\Theta}(u), \mathsf{F}^{\varphi}_{\Upsilon\cap\Theta}(u)\rangle: u \in H\}$.

So by using (47), we get

$$\begin{aligned}\tau^{\eta}_{\Upsilon\cap\Theta}(u) &= \tau^{\eta}_{\Upsilon}(u)\wedge\tau^{\eta}_{\Theta}(u),\\ \iota^{\xi}_{\Upsilon\cap\Theta}(u) &= \iota^{\xi}_{\Upsilon}(u)\vee\iota^{\xi}_{\Theta}(u),\end{aligned} \tag{48}$$

$$\mathsf{F}^{\varphi}_{\Upsilon\cap\Theta}(u) = \mathsf{F}^{\varphi}_{\Upsilon}(u)\vee\mathsf{F}^{\varphi}_{\Theta}(u). \tag{49}$$

(i) For all $u, v \in H$,

$$\begin{aligned}&\min\{\tau^{\eta}_{\Upsilon\cap\Theta}(u), \tau^{\eta}_{\Upsilon\cap\Theta}(v)\}\\ &= \min\{\tau^{\eta}_{\Upsilon}(u)\wedge\tau^{\eta}_{\Theta}(u), \tau^{\eta}_{\Upsilon}(v)\wedge\tau^{\eta}_{\Theta}(v)\}\\ &= \min\{\tau^{\eta}_{\Upsilon}(u)\wedge\tau^{\eta}_{\Theta}(u), \tau^{\eta}_{\Upsilon}(v)\wedge\tau^{\eta}_{\Theta}(v)\}\\ &\leq \min\{\tau^{\eta}_{\Upsilon}(u), \tau^{\eta}_{\Upsilon}(v)\}\wedge\min\{\tau^{\eta}_{\Theta}(u), \tau^{\eta}_{\Theta}(v)\}\\ &\leq \inf\{\tau^{\eta}_{\Upsilon}(w): w \in u^{\circ}v\}\wedge\inf\{\tau^{\eta}_{\Theta}(w): w \in u^{\circ}v\}\\ &\leq \inf\{\tau^{\eta}_{\Upsilon}(w)\wedge\tau^{\eta}_{\Theta}(w): w \in u^{\circ}v\}\\ &= \inf\{\tau^{\eta}_{\Upsilon\cap\Theta}(w): w \in u^{\circ}v\}.\end{aligned} \tag{50}$$

This implies $\min\{\tau^{\eta}_{\Upsilon\cap\Theta}(u), \tau^{\eta}_{\Upsilon\cap\Theta}(v)\} \leq \inf\{\tau^{\eta}_{\Upsilon\cap\Theta}(w): w \in u^{\circ}v\}$.

Similarly for all $u, v \in H$, we get

$$\begin{aligned}&\max\left\{\iota^{\xi}_{\Upsilon\cap\Theta}(u), \iota^{\xi}_{\Upsilon\cap\Theta}(v)\right\}\\ &= \max\left\{\iota^{\xi}_{\Upsilon}(u)\vee\iota^{\xi}_{\Theta}(u), \iota^{\xi}_{\Upsilon}(v)\vee\iota^{\xi}_{\Theta}(v)\right\}\\ &\geq \max\left\{\iota^{\xi}_{\Upsilon}(u), \iota^{\xi}_{\Upsilon}(v)\right\}\vee\max\left\{\iota^{\xi}_{\Theta}(u), \iota^{\xi}_{\Theta}(v)\right\}\\ &\geq \sup\left\{\iota^{\xi}_{\Upsilon}(w): w \in u^{\circ}v\right\}\vee\sup\left\{\iota^{\xi}_{\Theta}(w): w \in u^{\circ}v\right\}\\ &\geq \sup\left\{\iota^{\xi}_{\Upsilon}(w)\vee\iota^{\xi}_{\Theta}(w): w \in u^{\circ}v\right\}\\ &= \sup\left\{\iota^{\xi}_{\Upsilon\cap\Theta}(w): w \in u^{\circ}v\right\}.\end{aligned} \tag{51}$$

This implies $\max\left\{\iota^{\xi}_{\Upsilon\cap\Theta}(u), \iota^{\xi}_{\Upsilon\cap\Theta}(v)\right\} \geq \sup\left\{\iota^{\xi}_{\Upsilon\cap\Theta}(w): w \in u^{\circ}v\right\}$,

Similarly we can show that

$$\max\{\mathsf{F}^{\varphi}_{\Upsilon\cap\Theta}(u), \mathsf{F}^{\varphi}_{\Upsilon\cap\Theta}(v)\} \geq \sup\{\mathsf{F}^{\varphi}_{\Upsilon\cap\Theta}(w): w \in u^{\circ}v\}. \tag{52}$$

(ii) $\forall l, u \in H$, $\exists\, v \in H$ such that $u \in l^{\circ}v$,

$$\begin{aligned}&\min\{\tau^{\eta}_{\Upsilon\cap\Theta}(l), \tau^{\eta}_{\Upsilon\cap\Theta}(u)\}\\ &= \min\{\tau^{\eta}_{\Upsilon}(l)\wedge\tau^{\eta}_{\Theta}(l), \tau^{\eta}_{\Upsilon}(u)\wedge\tau^{\eta}_{\Theta}(u)\}\\ &= \min\{\tau^{\eta}_{\Upsilon}(l), \tau^{\eta}_{\Upsilon}(u)\}\wedge\min\{\tau^{\eta}_{\Theta}(l), \tau^{\eta}_{\Theta}(u)\}\\ &\leq \{\tau^{\eta}_{\Upsilon}(v)\wedge\tau^{\eta}_{\Theta}(v)\}\\ &= \tau^{\eta}_{\Upsilon\cap\Theta}(v).\end{aligned} \tag{53}$$

This implies $\min\{\tau^{\eta}_{\Upsilon\cap\Theta}(l), \tau^{\eta}_{\Upsilon\cap\Theta}(u)\} \leq \tau^{\eta}_{\Upsilon\cap\Theta}(v)$.

Next, we get

$$\begin{aligned}&\max\left\{\iota^{\xi}_{\Upsilon\cap\Theta}(l), \iota^{\xi}_{\Upsilon\cap\Theta}(u)\right\}\\ &= \max\left\{\iota^{\xi}_{\Upsilon}(l)\vee\iota^{\xi}_{\Theta}(l), \iota^{\xi}_{\Upsilon}(u)\vee\iota^{\xi}_{\Theta}(u)\right\}\\ &= \max\left\{\iota^{\xi}_{\Upsilon}(l), \iota^{\xi}_{\Upsilon}(u)\right\}\vee\max\left\{\iota^{\xi}_{\Theta}(l), \iota^{\xi}_{\Theta}(u)\right\}\\ &\geq \left\{\iota^{\xi}_{\Upsilon}(v)\vee\iota^{\xi}_{\Theta}(v)\right\}\\ &= \iota^{\xi}_{\Upsilon\cap\Theta}(v).\end{aligned} \tag{54}$$

This implies $\max\left\{\iota^{\xi}_{\Upsilon\cap\Theta}(l), \iota^{\xi}_{\Upsilon\cap\Theta}(u)\right\} \geq \iota^{\xi}_{\Upsilon\cap\Theta}(v)$.

Similarly, we can show that $\max\{\mathsf{F}^{\varphi}_{\Upsilon\cap\Theta}(l), \mathsf{F}^{\varphi}_{\Upsilon\cap\Theta}(u)\} \geq \mathsf{F}^{\varphi}_{\Upsilon\cap\Theta}(v)$.

(iii) $\forall l, u \in H$, $\exists\, w \in H$ such that $u \in w^{\circ}u$,

$$\begin{aligned}&\min\{\tau^{\eta}_{\Upsilon\cap\Theta}(l), \tau^{\eta}_{\Upsilon\cap\Theta}(u)\}\\ &= \min\{\tau^{\eta}_{\Upsilon}(l), \tau^{\eta}_{\Upsilon}(u)\}\wedge\min\{\tau^{\eta}_{\Theta}(l), \tau^{\eta}_{\Theta}(u)\}\\ &\leq \{\tau^{\eta}_{\Upsilon}(w)\wedge\tau^{\eta}_{\Theta}(w)\}\\ &= \tau^{\eta}_{\Upsilon\cap\Theta}(w).\end{aligned} \tag{55}$$

This implies $\min\{\tau^{\eta}_{\Upsilon\cap\Theta}(l), \tau^{\eta}_{\Upsilon\cap\Theta}(u)\} \leq \tau^{\eta}_{\Upsilon\cap\Theta}(w)$.

Next, we get

$$\begin{aligned}&\max\left\{\iota^{\xi}_{\Upsilon\cap\Theta}(l), \iota^{\xi}_{\Upsilon\cap\Theta}(u)\right\}\\ &= \max\left\{\iota^{\xi}_{\Upsilon}(l), \iota^{\xi}_{\Upsilon}(u)\right\}\vee\max\left\{\iota^{\xi}_{\Theta}(l), \iota^{\xi}_{\Theta}(u)\right\}\\ &\geq \left\{\iota^{\xi}_{\Upsilon}(w)\vee\iota^{\xi}_{\Theta}(w)\right\}\\ &= \iota^{\xi}_{\Upsilon\cap\Theta}(w).\end{aligned} \tag{56}$$

This implies $\max\left\{\iota^{\xi}_{\Upsilon\cap\Theta}(l), \iota^{\xi}_{\Upsilon\cap\Theta}(u)\right\} \geq \iota^{\xi}_{\Upsilon\cap\Theta}(w)$.

Similarly, we can show that $\max\{\mathsf{F}^{\varphi}_{\Upsilon\cap\Theta}(l), \mathsf{F}^{\varphi}_{\Upsilon\cap\Theta}(u)\} \geq \mathsf{F}^{\varphi}_{\Upsilon\cap\Theta}(w)$.

Therefore, $\Upsilon\cap\Theta$ is a (η, ξ, φ)-SVN hypergroup over H. □

Remark 1. Union of two (η, ξ, φ)-SVN hypergroups over H need not be (η, ξ, φ)-SVN hypergroup over H.

Theorem 7. *Let Υ be a (η, ξ, φ)-SVN hypergroup over H. Then the falsity-favorite of $\Upsilon^{(\eta,\xi,\varphi)}$ (i.e., $\Theta = \nabla\Upsilon^{(\eta,\xi,\varphi)}$) is a SVN hypergroup over H.*

Proof 8. By definition, $\Theta = \nabla\Upsilon^{(\eta,\xi,\varphi)}$, where the membership values are $\tau^{\eta}_{\Theta}(u) = \tau^{\eta}_{\Upsilon}(u), \iota^{\xi}_{\Theta}(u) = 0$, and $\mathsf{F}^{\varphi}_{\Theta}(u) = \min\left\{\mathsf{F}^{\varphi}_{\Upsilon}(u) + \iota^{\xi}_{\Upsilon}(u), 1\right\}$,

(i) Then we have to prove for $\tau^{\eta}_{\Upsilon}, \mathsf{F}^{\varphi}_{\Theta}, \forall u, v \in H$.

$$\begin{aligned}\min\{\tau^{\eta}_{\Theta}(u), \tau^{\eta}_{\Theta}(v)\} &= \min\{\tau^{\eta}_{\Upsilon}(u), \tau^{\eta}_{\Upsilon}(v)\}\ (\text{by Definition})\\ &\leq \inf\{\tau^{\eta}_{\Upsilon}(w): w \in u^{\circ}v\}.\end{aligned} \tag{57}$$

And we get

$$\begin{aligned}
&\max\{\mathsf{F}^{\varphi}_{\Theta}(u), \mathsf{F}^{\varphi}_{\Theta}(v)\}\\
&\quad= \max\left\{\mathsf{F}^{\varphi}_{Y}(u) + \iota^{\xi}_{Y}(u)\wedge 1, \mathsf{F}^{\varphi}_{Y}(v) + \iota^{\xi}_{Y}(v)\wedge 1\right\}\\
&\quad= \max\left\{\mathsf{F}^{\varphi}_{Y}(u) + \iota^{\xi}_{Y}(u), \mathsf{F}^{\varphi}_{Y}(v) + \iota^{\xi}_{Y}(v)\right\}\wedge 1\\
&\quad\geq \left(\max\{\mathsf{F}^{\varphi}_{Y}(u), \mathsf{F}^{\varphi}_{Y}(v)\} + \max\left\{\iota^{\xi}_{Y}(u), \iota^{\xi}_{Y}(v)\right\}\right)\wedge 1\\
&\quad\geq \left(\sup\{\mathsf{F}^{\varphi}_{Y}(w)\colon w \in u^{\circ}v\} + \sup\left\{\iota^{\xi}_{Y}(w)\colon w \in u^{\circ}v\right\}\right)\wedge 1\\
&\quad= \sup\left\{\mathsf{F}^{\varphi}_{Y}(w) + \iota^{\xi}_{Y}(w)\wedge 1\colon w \in u^{\circ}v\right\}\\
&\quad= \sup\{\mathsf{F}_{\Theta}(w)\colon w \in u^{\circ}v\},\\
&\quad\Rightarrow \max\{\mathsf{F}^{\varphi}_{\Theta}(u), \mathsf{F}^{\varphi}_{\Theta}(v)\} \geq \sup\{\mathsf{F}_{\Theta}(w)\colon w \in u^{\circ}v\}.
\end{aligned} \tag{58}$$

Similarly we can show that $\max\left\{\iota^{\xi}_{\Theta}(u), \iota^{\xi}_{\Theta}(v)\right\} \geq \sup\{\iota_{\Theta}(w)\colon w \in u^{\circ}v\}$.

(ii) $\forall l, u \in H$, $\exists\, v \in H$ such that $u \in l^{\circ}v$,

$$\begin{aligned}
\min\{\tau^{\eta}_{\Theta}(l), \tau^{\eta}_{\Theta}(u)\} &= \min\{\tau^{\eta}_{Y}(l), \tau^{\eta}_{Y}(u)\}\,(\text{by Definition})\\
&\leq \{\tau^{\eta}_{Y}(v)\}.
\end{aligned} \tag{59}$$

And we get

$$\begin{aligned}
&\max\{\mathsf{F}^{\varphi}_{\Theta}(l), \mathsf{F}^{\varphi}_{\Theta}(u)\}\\
&\quad= \max\left\{\mathsf{F}^{\varphi}_{Y}(l) + \iota^{\xi}_{Y}(l)\wedge 1, \mathsf{F}^{\varphi}_{Y}(u) + \iota^{\xi}_{Y}(u)\wedge 1\right\}\\
&\quad= \max\left\{\mathsf{F}^{\varphi}_{Y}(l) + \iota^{\xi}_{Y}(l), \mathsf{F}^{\varphi}_{Y}(u) + \iota^{\xi}_{Y}(u)\right\}\wedge 1\\
&\quad\geq \left(\{\mathsf{F}^{\varphi}_{Y}(l), \mathsf{F}^{\varphi}_{Y}(u)\} + \left\{\iota^{\xi}_{Y}(l), \iota^{\xi}_{Y}(u)\right\}\right)\wedge 1\\
&\quad\geq \left(\{\mathsf{F}^{\varphi}_{Y}(v)\} + \left\{\iota^{\xi}_{Y}(v)\right\}\right)\wedge 1\\
&\quad= \left\{\mathsf{F}^{\varphi}_{Y}(v) + \iota^{\xi}_{Y}(v)\wedge 1\right\}\\
&\quad= \{\mathsf{F}_{\Theta}(v)\}.\\
&\quad\Rightarrow \max\{\mathsf{F}^{\varphi}_{\Theta}(l), \mathsf{F}^{\varphi}_{\Theta}(u)\} \geq \{\mathsf{F}_{\Theta}(v)\}.
\end{aligned} \tag{60}$$

Similarly we can show that $\max\left\{\iota^{\xi}_{\Theta}(l), \iota^{\xi}_{\Theta}(u)\right\} \geq \{\iota_{\Theta}(v)\}$.

(iii) $\forall l, u \in H$, $\exists\, w \in H$ such that $u \in w^{\circ}l$,

$$\begin{aligned}
\min\{\tau^{\eta}_{\Theta}(l), \tau^{\eta}_{\Theta}(u)\} &= \min\{\tau^{\eta}_{Y}(l), \tau^{\eta}_{Y}(u)\}\,(\text{by Definition})\\
&\leq \{\tau^{\eta}_{Y}(w)\}.
\end{aligned} \tag{61}$$

And we get

$$\begin{aligned}
&\max\{\mathsf{F}^{\varphi}_{\Theta}(l), \mathsf{F}^{\varphi}_{\Theta}(u)\}\\
&\quad= \max\left\{\mathsf{F}^{\varphi}_{Y}(l) + \iota^{\xi}_{Y}(l)\wedge 1, \mathsf{F}^{\varphi}_{Y}(u) + \iota^{\xi}_{Y}(u)\wedge 1\right\}\\
&\quad= \max\left\{\mathsf{F}^{\varphi}_{Y}(l) + \iota^{\xi}_{Y}(l), \mathsf{F}^{\varphi}_{Y}(u) + \iota^{\xi}_{Y}(u)\right\}\wedge 1\\
&\quad\geq \left(\{\mathsf{F}^{\varphi}_{Y}(l), \mathsf{F}^{\varphi}_{Y}(u)\} + \left\{\iota^{\xi}_{Y}(l), \iota^{\xi}_{Y}(u)\right\}\right)\wedge 1\\
&\quad\geq \left(\{\mathsf{F}^{\varphi}_{Y}(w)\} + \left\{\iota^{\xi}_{Y}(w)\right\}\right)\wedge 1\\
&\quad= \left\{\mathsf{F}^{\varphi}_{Y}(w) + \iota^{\xi}_{Y}(w)\wedge 1\right\}\\
&\quad= \{\mathsf{F}_{\Theta}(w)\}.\\
&\quad\Rightarrow \max\{\mathsf{F}^{\varphi}_{\Theta}(l), \mathsf{F}^{\varphi}_{\Theta}(u)\} \geq \{\mathsf{F}_{\Theta}(w)\}.
\end{aligned} \tag{62}$$

Similarly we can show that $\max\left\{\iota^{\xi}_{\Theta}(l), \iota^{\xi}_{\Theta}(u)\right\} \geq \{\iota_{\Theta}(w)\}$. $\Rightarrow \Theta = \nabla Y^{(\eta,\xi,\varphi)}$ is a SVN hypergroup over H. □

4. (Weak) Polygroups

This section contains basic definitions, remarks, propositions, and examples of (weak) polygroups (i.e., polygroup, commutative polygroup, and noncommutative polygroup).

Let H be a nonempty set, and $P^{*}(H)$ be the collection of all nonempty subsets of H. "$*$" should be formulated as follows:

$*\colon H \times H \longrightarrow P^{*}(H)(u, v)u * v$

Then $(H, *)$ becomes a hypergroupoid and "$*$" is a hyperoperation.

Definition 14 (see [13]). Let $(P, *)$ be a hypergroupoid. Then $(P, *)$ is a polygroup if the aforementioned conditions are fulfilled $\forall u, v, w \in P$.

(1) $u * (v * w) = (u * v) * w$,

(2) $\exists\, e$ in P with $e * u = u * e = u$, $\forall u \in P$,

(3) $u \in v * w$ implies $v \in u * w^{-1}$ and $w \in v^{-1} * u$.

Weak polygroups are generalization of polygroups and they are defined in the same way as polygroups but instead of (44) in Definition 14, we have $u * (v * w) \cap (u * v) * w \neq \varnothing$.

In a (weak) polygroup P, $(u^{-1})^{-1} = u$, $\forall u \in P$.

Remark 2. Every group is a (weak) polygroup.

We present examples on polygroups that are not groups.

Example 2. Let $P_1 = \{e, \vartheta_1, \vartheta_2\}$. Then $(P_1, .)$ defined in Table 1 is a polygroup with e serving as an identity.

Table 1: The polygroup $(P_1, .)$.

	e	ϑ_1	ϑ_2
e	e	ϑ_1	ϑ_2
ϑ_1	ϑ_1	$\{e, \vartheta_2\}$	$\{\vartheta_1, \vartheta_2\}$
ϑ_2	ϑ_2	$\{\vartheta_1, \vartheta_2\}$	$\{e, \vartheta_1\}$

Table 2: The polygroup $(P_2, .)$.

.	e	ϑ_1	ϑ_2	ϑ_3
e	e	ϑ_1	ϑ_2	ϑ_3
ϑ_1	ϑ_1	P_2	$\{\vartheta_1, \vartheta_2, \vartheta_3\}$	$\{\vartheta_1, \vartheta_2, \vartheta_3\}$
ϑ_2	ϑ_2	$\{\vartheta_1, \vartheta_2, \vartheta_3\}$	P_2	$\{\vartheta_1, \vartheta_2, \vartheta_3\}$
ϑ_3	ϑ_3	$\{\vartheta_1, \vartheta_2, \vartheta_3\}$	$\{\vartheta_1, \vartheta_2, \vartheta_3\}$	P_2

Table 3: The polygroup $(P_3, .)$.

.	e	ϑ_1	ϑ_2	ϑ_3
e	e	ϑ_1	ϑ_2	ϑ_3
ϑ_1	ϑ_1	ϑ_1	P_3	ϑ_3
ϑ_2	ϑ_2	$\{e, \vartheta_1, \vartheta_2\}$	ϑ_2	$\{\vartheta_2, \vartheta_3\}$
ϑ_3	ϑ_3	$\{\vartheta_1, \vartheta_3\}$	ϑ_3	P_3

Example 3 (see [47]). let $P_2 = \{e, \vartheta_1, \vartheta_2, \vartheta_3\}$. Then $(P_2, .)$ defined in Table 2 is a commutative polygroup with e serving as an identity.

Example 4 (see [47]). Let $P_3 = \{e, \vartheta_1, \vartheta_2, \vartheta_3\}$. Then $\{P_3, .\}$ defined in Table 3 is a noncommutative polygroup with e serving as an identity.

Definition 15 (see [47]). A subset Q of a polygroup $(P, *)$ is a subpolygroup of $P \Leftrightarrow (Q, *)$ is a polygroup.

Proposition 4 (see [47]). *A subset Q of P is a subpolygroup of polygroup $(P, *) \Leftrightarrow u * v \subseteq Q$ and $u^{-1} \in Q$, $\forall u, v \subseteq Q$.*

Definition 16 (see [47]). A subset subpolygroup Q of a polygroup $(P, *)$ is a normal subpolygroup of P if $u^{-1} * Q * u \subseteq P$, $\forall u \in P$.

5. (η, ξ, φ)- Single-Valued Neutrosophic (Weak) Polygroups

In this section, we present some fundamental definitions, characteristics, theorems, propositions, and examples in relation to the SVNPs, (η, ξ, φ)-SVNPs, (η, ξ, φ)-SVNWPs, and (η, ξ, φ)-ASVNPs. In addition to this, we provide an example of a (η, ξ, φ)-SVN subpolygroup that is not normal.

Definition 17 (see [48]). Let $(P, *)$ be a polygroup and Υ be a fuzzy set with a degree of membership m over P. Then, Υ is considered a fuzzy polygroup over P if the followings conditions are satisfied $\forall u, v \in P$.

(1) $m(w) \geq \min\{m(u), m(v)\}$, $\forall w \in u * v$,

(2) $m(u^{-1}) \geq m(u)$.

Remark 3 (see [44]). Intersection of fuzzy polygroups over P is a fuzzy polygroup.

Definition 18. If Υ be a single-valued neutrosophic (SVN) subset of Ω, then a (η, ξ, φ)-SVN subset Υ of Ω is categorize as

$$\Upsilon^{(\eta,\xi,\varphi)} = \left\{\langle e, \tau_\Upsilon^\eta(u), \iota_\Upsilon^\xi(u), \mathsf{F}_\Upsilon^\varphi(u)\rangle | u \in \Omega\right\}, \tag{63}$$

where

$$\begin{aligned} \tau_\Upsilon^\eta(u) &= \wedge\{\tau_\Upsilon(u), \eta\}, \\ \iota_\Upsilon^\xi(u) &= \wedge\{\iota_\Upsilon(u), \xi\}, \\ \mathsf{F}_\Upsilon^\varphi(u) &= \vee\{\mathsf{F}_\Upsilon(u), \varphi\}, \end{aligned} \tag{64}$$

such that

$$0 \leq \tau_\Upsilon^\eta(u) + \iota_\Upsilon^\xi(u) + \mathsf{F}_\Upsilon^\varphi(u) \leq 3. \tag{65}$$

Here, $\eta, \xi, \varphi \in [0, 1]$, also τ, ι, F: $A \longrightarrow [0, 1]$, such that τ_Υ^η, ι_Υ^ξ, $\mathsf{F}_\Upsilon^\varphi$ represents the functions of truth, indeterminacy, and falsity-membership, respectively.

Definition 19. Let Υ and Θ be two (η, ξ, φ)-SVNSs on Ω. The followings must hold the following:

(1) $\Upsilon^{(\eta,\xi,\varphi)} \subseteq \Theta^{(\eta,\xi,\varphi)} \Leftrightarrow \Upsilon^{(\eta,\xi,\varphi)}(u) \leq \Theta^{(\eta,\xi,\varphi)}(u)$.

That is,

$$\begin{aligned} \tau_\Upsilon^\eta(u) &\leq \tau_\Theta^\eta(u), \\ \iota_\Upsilon^\xi(u) &\leq \iota_\Theta^\xi(u), \\ \mathsf{F}_\Upsilon^\varphi(u) &\geq \mathsf{F}_\Theta^\varphi(u), \end{aligned} \tag{66}$$

and

$$\Upsilon^{(\eta,\xi,\varphi)} = \Theta^{(\eta,\xi,\varphi)} \Leftrightarrow \Upsilon^{(\eta,\xi,\varphi)} \subseteq \Theta^{(\eta,\xi,\varphi)} an\ d \Theta^{(\eta,\xi,\varphi)} \subseteq \Upsilon^{(\eta,\xi,\varphi)}. \tag{67}$$

(2) The union of $\Upsilon^{(\eta,\xi,\varphi)}$ and $\Theta^{(\eta,\xi,\varphi)}$ is indicated by

$$\Delta^{(\eta,\xi,\varphi)} = \Upsilon^{(\eta,\xi,\varphi)} \cup \Theta^{(\eta,\xi,\varphi)}, \tag{68}$$

and defined as

$$\Delta^{(\eta,\xi,\varphi)}(u) = \Upsilon^{(\eta,\xi,\varphi)}(u) \vee \Theta^{(\eta,\xi,\varphi)}(u), \tag{69}$$

where

$$\begin{aligned} &\Upsilon^{(\eta,\xi,\varphi)}(u) \vee \Theta^{(\eta,\xi,\varphi)}(u) \\ &= \left(\tau_\Upsilon^\eta(u) \vee \tau_\Theta^\eta(u), \iota_\Upsilon^\xi(u) \vee \iota_\Theta^\xi(u), \mathsf{F}_\Upsilon^\varphi(u) \wedge \mathsf{F}_\Theta^\varphi(u)\right), \forall u \in \Omega. \end{aligned} \tag{70}$$

That is,

$$\begin{aligned} \tau_\Delta^\eta(u) &= \max\{\tau_\Upsilon^\eta(u), \tau_\Theta^\eta(u)\}, \\ \iota_\Delta^\xi(u) &= \max\left\{\iota_\Upsilon^\xi(u), \iota_\Theta^\xi(u)\right\}, \\ \mathsf{F}_\Delta^\varphi(u) &= \min\{\mathsf{F}_\Upsilon^\varphi(u), \mathsf{F}_\Theta^\varphi(u)\}. \end{aligned} \tag{71}$$

(3) The intersection of $\Upsilon^{(\eta,\xi,\varphi)}$ and $\Theta^{(\eta,\xi,\varphi)}$ is indicated by

$$\Delta^{(\eta,\xi,\varphi)} = \Upsilon^{(\eta,\xi,\varphi)} \cap \Theta^{(\eta,\xi,\varphi)}, \tag{72}$$

and defined as

$$\Delta^{(\eta,\xi,\varphi)}(u) = \Upsilon^{(\eta,\xi,\varphi)}(u) \wedge \Theta^{(\eta,\xi,\varphi)}(u), \tag{73}$$

where

$$\begin{aligned}&\Upsilon^{(\eta,\xi,\varphi)}(u) \wedge \Theta^{(\eta,\xi,\varphi)}(u)\\ &= \left(\tau_{\Upsilon}^{\eta}(u) \wedge \tau_{\Theta}^{\eta}(u), \iota_{\Upsilon}^{\xi}(u) \wedge \iota_{\Theta}^{\xi}(u), \mathsf{F}_{\Upsilon}^{\varphi}(u) \vee \mathsf{F}_{\Theta}^{\varphi}(u)\right), \forall u \in \Omega.\end{aligned} \tag{74}$$

That is,

$$\begin{aligned}\tau_{\Delta}^{\eta}(u) &= \min\{\tau_{\Upsilon}^{\eta}(u), \tau_{\Theta}^{\eta}(u)\},\\ \iota_{\Delta}^{\xi}(u) &= \min\left\{\iota_{\Upsilon}^{\xi}(u), \iota_{\Theta}^{\xi}(u)\right\},\\ \mathsf{F}_{\Delta}^{\varphi}(u) &= \max\{\mathsf{F}_{\Upsilon}^{\varphi}(u), \mathsf{F}_{\Theta}^{\varphi}(u)\}.\end{aligned} \tag{75}$$

Definition 20. The complement of a (η, ξ, φ)-SVNS Υ is denoted by $c(\Upsilon^{(\eta,\xi,\varphi)})$ and is defined by

$$c\left(\Upsilon^{(\eta,\xi,\varphi)}\right) = \langle u, \tau_{c(\Upsilon)}^{\eta}(u), \iota_{c(\Upsilon)}^{\xi}(u), \mathsf{F}_{c(\Upsilon)}^{\varphi}(u)\rangle, \tag{76}$$

where

$$\begin{aligned}\tau_{c(\Upsilon)}^{\eta}(u) &= \mathsf{F}_{\Upsilon}^{\varphi}(u),\\ \iota_{c(\Upsilon)}^{\xi}(u) &= 1 - \iota_{\Upsilon}^{\xi}(u),\\ \mathsf{F}_{c(\Upsilon)}^{\varphi}(u) &= \tau_{\Upsilon}^{\eta}(u), \forall u \in \Omega.\end{aligned} \tag{77}$$

Definition 21. Let $(P, *)$ be a (weak) polygroup and Υ a (η, ξ, φ)-SVNS over P. Then Υ is called a (η, ξ, φ)-SVNP over P ((η, ξ, φ)-SVN weak polygroup ((η, ξ, φ)-SVNWP) over P) if for all $\forall u, v \in P$, the following conditions are satisfied.

(1) $\tau_{\Upsilon}^{\eta}(w) \geq \min\{\tau_{\Upsilon}^{\eta}(u), \tau_{\Upsilon}^{\eta}(v)\}$, $\iota_{\Upsilon}^{\xi}(w) \geq \min\left\{\iota_{\Upsilon}^{\xi}(u), \iota_{\Upsilon}^{\xi}(v)\right\}$ and $\mathsf{F}_{\Upsilon}^{\varphi}(w) \leq \max\{\mathsf{F}_{\Upsilon}^{\varphi}(u), \mathsf{F}_{\Upsilon}^{\varphi}(v)\}$

for all $z \in x * y$,

(2) $\tau_{\Upsilon}^{\eta}(u^{-1}) \geq \tau_{\Upsilon}^{\eta}(u)$, $\iota_{\Upsilon}^{\xi}(u^{-1}) \geq \iota_{\Upsilon}^{\xi}(u)$ and $\mathsf{F}_{\Upsilon}^{\varphi}(x^{-1}) \leq \mathsf{F}_{\Upsilon}^{\varphi}(u)$.

Example 5. Let $P_4 = \{0, 1\}$. Then $(P_4, *)$ defined in Table 4 is a polygroup with 0 serving as an identity.

Let

$$\Upsilon = \frac{\langle 0.6, 0.7, 0.2\rangle}{0} + \frac{\langle 0.1, 0.3, 0.7\rangle}{1}. \tag{78}$$

Consider $\eta = 0.4, \xi = 0.5, \varphi = 0.5$.

Then $\Upsilon^{(\eta,\xi,\varphi)} = \langle 0.4, 0.5, 0.5\rangle/0 + \langle 0.1, 0.3, 0.7\rangle/1$.

$\Rightarrow \Upsilon$ is a (η, ξ, φ)-SVNP over P_4.

Example 6. Let $P_5 = \{e, \vartheta_1, \vartheta_2, \vartheta_3\}$. Then $\{P_5, ^\circ\}$ defined in Table 5 is a weak polygroup with e serving as an identity.

Table 4: The polygroup $(P_4, *)$.

$*$	0	1
0	0	1
1	1	P_1

Table 5: The polygroup $(P_5, ^\circ)$.

$\circ$	e	ϑ_1	ϑ_2	ϑ_3
e	e	ϑ_1	ϑ_2	ϑ_3
ϑ_1	ϑ_1	$\{e, \vartheta_1\}$	ϑ_3	ϑ_2
ϑ_2	ϑ_2	ϑ_3	$\{e, \vartheta_2\}$	ϑ_1
ϑ_1	ϑ_1	ϑ_2	ϑ_1	$\{e, \vartheta_3\}$

Moreover, it is not a polygroup because $\circ$ is not associated, i.e.,

$$(\vartheta_1 \circ \vartheta_2) \circ \vartheta_3 = \vartheta_3 \circ \vartheta_3 = \{e, \vartheta_3\}, \vartheta_1 \circ (\vartheta_2 \circ \vartheta_3) = \vartheta_1 \circ \vartheta_1 = \{e, \vartheta_1\}. \tag{79}$$

Let

$$\begin{aligned}\Upsilon &= \langle 0.8, 0.7, 0.3\rangle/e + \langle 0.1, 0.3, 0.7\rangle/\vartheta_1\\ &\quad + \langle 0.1, 0.25, 0.9\rangle/\vartheta_2 + \langle 0.1, 0.25, 0.9\rangle/\vartheta_3.\end{aligned} \tag{80}$$

Consider $\eta = 0.4, \xi = 0.5, \varphi = 0.5$.

Then $\Upsilon^{(\eta,\xi,\varphi)} = \langle 0.4, 0.5, 0.5\rangle/e + \langle 0.1, 0.3, 0.7\rangle/\vartheta_1 + \langle 0.1, 0.25, 0.9\rangle/\vartheta_2 + \langle 0.1, 0.25, 0.9\rangle/\vartheta_3$.

$\Rightarrow \Upsilon$ is a (η, ξ, φ)-SVNWP over P_5.

Remark 4. All the theorems and results in this paper that are valid for (η, ξ, φ)-SVNP are also valid for (η, ξ, φ)-SVNWP. So, we restrict our results to (η, ξ, φ)-SVNP.

Proposition 5. *Let Υ a (η, ξ, φ)-SVNP over polygroup $(P, *)$. Then the preceding holds true $\forall u \in P$.*

(1) $\tau_{\Upsilon}^{\eta}(u^{-1}) = \tau_{\Upsilon}^{\eta}(u)$, $\iota_{\Upsilon}^{\xi}(u^{-1}) = \iota_{\Upsilon}^{\xi}(u)$, and $\mathsf{F}_{\Upsilon}^{\varphi}(u^{-1}) = \mathsf{F}_{\Upsilon}^{\varphi}(u)$;

(2) $\tau_{\Upsilon}^{\eta}(e) \geq \tau_{\Upsilon}^{\eta}(u)$, $\iota_{\Upsilon}^{\xi}(e) \geq \iota_{\Upsilon}^{\xi}(u)$, and $\mathsf{F}_{\Upsilon}^{\varphi}(e) \leq \mathsf{F}_{\Upsilon}^{\varphi}(u)$ where e is the identity in P.

Proof 9. Let $u \in P$.

(1) By Definition 21 implies that $\tau_{\Upsilon}^{\eta}(u^{-1}) \geq \tau_{\Upsilon}^{\eta}(u)$, $\iota_{\Upsilon}^{\xi}(u^{-1}) \geq \iota_{\Upsilon}^{\xi}(u)$, and $\mathsf{F}_{\Upsilon}^{\varphi}(u^{-1}) \leq \mathsf{F}_{\Upsilon}^{\varphi}(u)$. Also we have $(u^{-1})^{-1} = u$ implies that $\tau_{\Upsilon}^{\eta}(u) \geq \tau_{\Upsilon}^{\eta}(u^{-1})$, $\iota_{\Upsilon}^{\xi}(u) \geq \iota_{\Upsilon}^{\xi}(u^{-1})$, and $\mathsf{F}_{\Upsilon}^{\varphi}(u) \leq \mathsf{F}_{\Upsilon}^{\varphi}(u^{-1})$. Thus, $\tau_{\Upsilon}^{\eta}(u^{-1}) = \tau_{\Upsilon}^{\eta}(u)$, $\iota_{\Upsilon}^{\xi}(u^{-1}) = \iota_{\Upsilon}^{\xi}(u)$, and $\mathsf{F}_{\Upsilon}^{\varphi}(u^{-1}) = \mathsf{F}_{\Upsilon}^{\varphi}(u)$.

(2) Since $e \in u * u^{-1}$, it follows by Definition 21 (1) that $\tau_{\Upsilon}^{\eta}(e) \geq \min(\tau_{\Upsilon}^{\eta}(u), \tau_{\Upsilon}^{\eta}(u^{-1})) = \tau_{\Upsilon}^{\eta}(u)$, $\iota_{\Upsilon}^{\xi}(e) \geq \min(\iota_{\Upsilon}^{\xi}(u), \iota_{\Upsilon}^{\xi}(u^{-1})) = \iota_{\Upsilon}^{\xi}(u)$, and $\mathsf{F}_{\Upsilon}^{\varphi}(e) \leq \max(\mathsf{F}_{\Upsilon}^{\varphi}(u), \mathsf{F}_{\Upsilon}^{\varphi}(u^{-1})) = \mathsf{F}_{\Upsilon}^{\varphi}(u)$. □

Example 7. Let $P_6 = \{e, \vartheta_1, \vartheta_2, \vartheta_3\}$. Then $\{P_6, .\}$ defined in Table 6 is a polygroup with e serving as an identity.

Let

Table 6: The polygroup $(P_6, .)$.

.	e	ϑ_1	ϑ_2	ϑ_3
e	e	ϑ_1	ϑ_2	ϑ_3
ϑ_1	ϑ_1	e	ϑ_2	ϑ_3
ϑ_2	ϑ_2	ϑ_2	$\{e, \vartheta_1, \vartheta_3\}$	$\{\vartheta_2, \vartheta_3\}$
ϑ_3	ϑ_3	ϑ_3	$\{\vartheta_2, \vartheta_3\}$	$\{e, \vartheta_1, \vartheta_2\}$

$$\Upsilon = \frac{\langle 0.4, 0.6, 0.2\rangle}{e} + \frac{\langle 0.8, 0.3, 0.7\rangle}{\vartheta_1} + \frac{\langle 0.7, 0.3, 0.7\rangle}{\vartheta_2} + \frac{\langle 0.1, 0.2, 0.8\rangle}{\vartheta_3}. \tag{81}$$

Consider $\eta = 0.5, \xi = 0.5$, and $\varphi = 0.5$.

Then

$$\Upsilon^{(\eta,\xi,\varphi)} = \frac{\langle 0.4, 0.5, 0.5\rangle}{e} + \frac{\langle 0.8, 0.3, 0.7\rangle}{\vartheta_1} + \frac{\langle 0.5, 0.3, 0.7\rangle}{\vartheta_2} + \frac{\langle 0.1, 0.2, 0.8\rangle}{\vartheta_3}. \tag{82}$$

$\Rightarrow \Upsilon^{(\eta,\xi,\varphi)}$ is not a (η, ξ, φ)-SVNP over P_6 as $\tau_\Upsilon^\eta(e) \geq \tau_\Upsilon^\eta(\vartheta_1)$ does not hold.

Example 8. Let $(P_6, .)$ be the polygroup in example 7. Then $\{e\}$ and $\{e, \vartheta_1\}$ are subpolygroups of P_6 that are not normal.

Proposition 6. *Let Υ be a (η, ξ, φ)-SVNS over polygroup $(P, *)$, and $(\Upsilon^{(\eta,\xi,\varphi)})^{-1} = \{\langle \tau_\Upsilon^\eta(u^{-1}), \iota_\Upsilon^\xi(u^{-1}), \mathsf{F}_\Upsilon^\varphi(u^{-1})\rangle/u\colon u \in P\}$. If Υ is a (η, ξ, φ)-SVNP over P then $(\Upsilon^{(\eta,\xi,\varphi)})^{-1} = \Upsilon^{(\eta,\xi,\varphi)}$.*

Proof 10. It is simple by using Proposition 5. □

Proposition 7. *Let $(P, *)$ be a polygroup, τ_1, τ_2, τ_3 be numbers in the unit interval $[0, 1]$.If $\Upsilon^{(\eta,\xi,\varphi)} = \{\langle \tau_1, \tau_2, \tau_3\rangle/u\colon u \in P\}$. Then Υ is a (η, ξ, φ)-SVNP over P.*

Proof 11. The proof is simple for readers. □

Remark 5. The (η, ξ, φ)-SVNP present in Proposition 6 is called the constant (η, ξ, φ)-SNVP.

Theorem 8. *Let Υ a (η, ξ, φ)-SVNS over polygroup $(P, *)$. Then Υ and $c(\Upsilon)$ are (η, ξ, φ)-SVNP over P if and only if Υ is the constant (η, ξ, φ)-SVNP.*

Proof 12. If Υ is the constant (η, ξ, φ)-SVNP over P then $c(\Upsilon)$ is also the constant (η, ξ, φ)-SVNP over P. Let Υ and $c(\Upsilon)$ be (η, ξ, φ)-SVNP. Then $\forall u \in P$, we have

$$\tau_\Upsilon^\eta(e) \geq \tau_\Upsilon^\eta(u), \iota_\Upsilon^\xi(e) \geq \iota_\Upsilon^\xi(u), \mathsf{F}_\Upsilon^\varphi(e) \leq \mathsf{F}_\Upsilon^\varphi(u) \tag{83}$$

$$\begin{aligned}&\mathsf{F}_\Upsilon^\varphi(e) \geq \mathsf{F}_\Upsilon^\varphi(u), 1 - \iota_\Upsilon^\xi(e) \geq 1 - \iota_\Upsilon^\xi(u),\\ &\tau_\Upsilon^\eta(e) \leq \tau_\Upsilon^\eta(u)\end{aligned} \tag{84}$$

Equations (83) and (84) implies that $\tau_\Upsilon^\eta(e) = \tau_\Upsilon^\eta(u)$, $\iota_\Upsilon^\xi(e) = \iota_\Upsilon^\xi(u)$, $\mathsf{F}_\Upsilon^\varphi(e) = \mathsf{F}_\Upsilon^\varphi(u)$. Thus, Υ is the constant (η, ξ, φ)-SVNP over P. □

Definition 22 (see [48]). Let Υ be a fuzzy set over a polygroup $(P, *)$ with membership function m. Then Υ is called the anti-fuzzy polygroup over P if $\forall u, v \in P$, the following conditions are fulfilled.

(1) $m(w) \leq \max\{m(u), m(v)\}$, $\forall w \in u * v$,

(2) $m(u^{-1}) \leq m(u)$.

Remark 6 (see [48]). Union of anti-fuzzy polygroups over P is an anti-fuzzy polygroup.

Definition 23. Let Υ be a (η, ξ, φ)-SVNS over polygroup $(P, *)$. Then Υ is called an (η, ξ, φ)-anti SVNP ((η, ξ, φ)-ASVNP) over P if $\forall u, v \in P$, the following conditions are satisfied.

(1) $\tau_\Upsilon^\eta(w) \leq \max\{\tau_\Upsilon^\eta(u), \tau_\Upsilon^\eta(v)\}$, $\iota_\Upsilon^\xi(w) \leq \max\{\iota_\Upsilon^\xi(u), \iota_\Upsilon^\xi(v)\}$ and $\mathsf{F}_\Upsilon^\varphi(w) \geq \min\{\mathsf{F}_\Upsilon^\varphi(u), \mathsf{F}_\Upsilon^\varphi(v)\}$ for all $w \in u * v$,

(2) $\tau_\Upsilon^\eta(u^{-1}) \leq \tau_\Upsilon^\eta(u)$, $\iota_\Upsilon^\xi(u^{-1}) \leq \iota_\Upsilon^\xi(u)$, $\mathsf{F}_\Upsilon^\varphi(u^{-1}) \geq \mathsf{F}_\Upsilon^\varphi(u)$.

Proposition 8. *Let $(P, *)$ be a polygroup and Υ an (η, ξ, φ)-ASVNP over PThen following holds true:$\forall u \in P$.*

(1) $\tau_\Upsilon^\eta(u^{-1}) = \tau_\Upsilon^\eta(u)$, $\iota_\Upsilon^\xi(u^{-1}) = \iota_\Upsilon^\xi(u)$ and $\mathsf{F}_\Upsilon^\varphi(u^{-1}) = \mathsf{F}_\Upsilon^\varphi(u)$.

(2) $\tau_\Upsilon^\eta(e) \leq \tau_\Upsilon^\eta(u)$, $\iota_\Upsilon^\xi(e) \leq \iota_\Upsilon^\xi(u)$, and $\mathsf{F}_\Upsilon^\varphi(e) \geq \mathsf{F}_\Upsilon^\varphi(u)$ where e is the identity element in P.

Proof 13. The proof is similar to that of 5.10.

Example 9. Consider $(P_4, *)$ be the polygroup present in example 5.

Let

$$\Upsilon = \frac{\langle 0.4, 0.7, 0.9\rangle}{0} + \frac{\langle 0.6, 0.8, 0.2\rangle}{1}. \tag{85}$$

Consider $\eta = 0.5, \xi = 0.5, \varphi = 0.7$.

Then $\Upsilon^{(\eta,\xi,\varphi)} = \langle 0.4, 0.5, 0.9\rangle/0 + \langle 0.5, 0.5, 0.7\rangle/1$.

$\Rightarrow \Upsilon$ is a (η, ξ, φ)-ASVNP over P_4.

Theorem 9. *Let Υ be a (η, ξ, φ)-SVNS over polygroup $(P, *)$. Then Υ is a (η, ξ, φ)-SVNP over P if and only if τ_Υ^η and ι_Υ^ξ are fuzzy polygroups over P and $\mathsf{F}_\Upsilon^\varphi$ is an anti-fuzzy polygroup over P.*

Proof 14. It follows from the definition of (η, ξ, φ)-SVNP, fuzzy polygroups, and anti-fuzzy polygroups. □

Theorem 10. *Let Υ be a (η, ξ, φ)-SVNS over polygroup $(P, *)$. Then Υ is a $c(\Upsilon)$-ASVNP over P if and only if τ_Υ^η and ι_Υ^ξ are anti-fuzzy polygroups over P and $\mathsf{F}_\Upsilon^\varphi$ is an fuzzy polygroup over P.*

Proof 15. It follows from the definition of (η,ξ,φ)-ASVNP, fuzzy polygroups, and anti-fuzzy polygroups. □

Theorem 11. *Let Υ be a (η,ξ,φ)-SVNS over polygroup $(P,*)$. Then Υ is a (η,ξ,φ)-SVNP over P if and only if $c(\Upsilon)$ is an (η,ξ,φ)-ASVNP over P.*

Proof 16. Let Υ be a (η,ξ,φ)-SVNP. Theorem 9 asserts that τ_Υ^η and ι_Υ^ξ are fuzzy polygroups over P and $\mathrm{F}_\Upsilon^\varphi$ is an anti-fuzzy polygroup over P. We get now that $\tau_{c(\Upsilon)}^\eta = \mathrm{F}_\Upsilon^\varphi$ and $\iota_{c(\Upsilon)}^\xi = 1-\iota_\Upsilon^\xi$ are anti-fuzzy polygroups over P and $\mathrm{F}_{c(\Upsilon)}^\varphi = \tau_\Upsilon^\eta$ is a fuzzy polygroup over P. Using Theorem 10, it completes the proof. Similarly, we can prove that if $c(\Upsilon)$ is an (η,ξ,φ)-ASVNP over P then Υ is a (η,ξ,φ)-SVNP. □

Corollary 1. *Let Υ_λ be a (η,ξ,φ)-SVNS over polygroup $(P,*)$. If Υ_λ is a (η,ξ,φ)-SVNP over P then $\cap_{\lambda\in\varphi}\Upsilon_\lambda$ is (η,ξ,φ)-SVNP over P.*

Corollary 2. *Let Υ_λ be a (η,ξ,φ)-SVNS over polygroup $(P,*)$. If Υ_λ is a (η,ξ,φ)-ASVNP over P then $\cap_{\lambda\in\varphi}\Upsilon_\lambda$ is an (η,ξ,φ)-ASVNP over P.*

6. Level Sets of (η,ξ,φ)-Single-Valued Neutrosophic (Weak) Polygroups

This section defines level sets of (η,ξ,φ)-SVNPs and relate them with (normal) subpolygroups.

Definition 24. Let ξ be any set $t=(\tau_1,\tau_2,\tau_3)$, where $0\le\tau_1,\tau_2<1$ and $0<\tau_3\le 1$, and Υ be a (η,ξ,φ)-SVNS over ξ. Then $\Upsilon_t^{(\eta,\xi,\varphi)} = \{u\in\xi\colon \tau_\Upsilon^\eta(u)\ge\tau_1, \iota_\Upsilon^\xi(u)\ge\tau_2, \mathrm{F}_\Upsilon^\varphi(u)\le\tau_3\}$ is named a t-level set of $\Upsilon^{(\eta,\xi,\varphi)}$.

Theorem 12. *Let Υ be a (η,ξ,φ)-SVNS over polygroup $(P,*)$. Then Υ is a (η,ξ,φ)-SVNP over $P\Leftrightarrow\Upsilon_t^{(\eta,\xi,\varphi)}\ne\varnothing$ is a subpolygroup of (HTML translation failed) for every $t=(\tau_1,\tau_2,\tau_3)$, where $0\le\tau_1,\tau_2<1$ and $0<\tau_3\le 1$.*

Proof 17. Let Υ be a (η,ξ,φ)-SVNP over P and $u,v\in\Upsilon_t^{(\eta,\xi,\varphi)}\ne\varnothing$. $\forall\, w\in u*v$, we have

$$\begin{aligned}\tau_\Upsilon^\eta(w)&\ge\min\{\tau_\Upsilon^\eta(u),\tau_\Upsilon^\eta(v)\}\ge\tau_1,\\ \iota_\Upsilon^\xi(w)&\ge\min\{\iota_\Upsilon^\xi(u),\iota_\Upsilon^\xi(v)\}\ge\tau_2,\end{aligned}\tag{86}$$

$$\mathrm{F}_\Upsilon^\varphi(w)\le\max\{\mathrm{F}_\Upsilon^\varphi(u),\mathrm{F}_\Upsilon^\varphi(v)\}\le\tau_3.\tag{87}$$

Thus $u*v\subseteq\Upsilon_t^{(\eta,\xi,\varphi)}$.
Furthermore, we have

$$\begin{aligned}\tau_\Upsilon^\eta(u^{-1})&\ge\tau_\Upsilon^\eta(u)\ge\tau_1,\\ \iota_\Upsilon^\xi(u^{-1})&\ge\iota_\Upsilon^\xi(u)\end{aligned}\tag{88}$$

$$\mathrm{F}_\Upsilon^\varphi(u^{-1})\le\mathrm{F}_\Upsilon^\varphi(u)\le\tau_3.\tag{89}$$

This implies that $u^{-1}\in\Upsilon_t^{(\eta,\xi,\varphi)}$. Thus, $\Upsilon_t^{(\eta,\xi,\varphi)}$ is a subpolygroup of P.

Conversely, let $\Upsilon_t^{(\eta,\xi,\varphi)}\ne\varnothing$ be a subpolygroup of P and $u,v\in P$.

Set $\tau_1=\min\{\tau_\Upsilon^\eta(u),\tau_\Upsilon^\eta(v)\}$, $\tau_2=\min\{\iota_\Upsilon^\xi(u),\iota_\Upsilon^\xi(v)\}$, $\tau_3=\max\{\mathrm{F}_\Upsilon^\varphi(u),\mathrm{F}_\Upsilon^\varphi(v)\}$ and $t=(\tau_1,\tau_2,\tau_3)$.

So it illustrates that $u*v\subseteq\Upsilon_t^{(\eta,\xi,\varphi)}$ and $u^{-1}\in\Upsilon_t^{(\eta,\xi,\varphi)}$.

This indicates $\forall w\in u*v$,

$$\begin{aligned}\tau_\Upsilon^\eta(w)&\ge\tau_1=\min\{\tau_\Upsilon^\eta(u),\tau_\Upsilon^\eta(v)\},\\ \iota_\Upsilon^\xi(w)&\ge\tau_2=\min\{\iota_\Upsilon^\xi(u),\iota_\Upsilon^\xi(v)\},\end{aligned}\tag{90}$$

$$\mathrm{F}_\Upsilon^\varphi(w)\le\tau_3=\max\{\mathrm{F}_\Upsilon^\varphi(u),\mathrm{F}_\Upsilon^\varphi(v)\}.\tag{91}$$

As a result, condition (1) of Definition 17 is achieved. Moreover,

$$\begin{aligned}\tau_\Upsilon^\eta(u^{-1})&\ge\tau_1=\tau_\Upsilon^\eta(u),\\ \iota_\Upsilon^\xi(u^{-1})&\ge\tau_2=\iota_\Upsilon^\xi(u),\end{aligned}\tag{92}$$

$$\mathrm{F}_\Upsilon^\varphi(u^{-1})\le\tau_3=\mathrm{F}_\Upsilon^\varphi(u).\tag{93}$$

Thus, condition (2) of Definition 17 is satisfied. Therefore, Υ become (η,ξ,φ)-SVNP over P.

Corollary 3. *Let Υ be a (η,ξ,φ)-SVNP over polygroup $(P,*)$. Then P has no non-trivial proper subpolygroups if and only if the constant (η,ξ,φ)-SNVP and $\Upsilon^{(\eta,\xi,\varphi)}=\{\langle\tau_1,\tau_2,\tau_3\rangle/u+\langle\tau_1',\tau_2',\tau_3'\rangle/e\colon u\ne e\in P\}$, where $\tau_1\ge\tau_1',\tau_2\ge\tau_2'$ and $\tau_3\le\tau_3'$ are the only (η,ξ,φ)-SVNP over P.*

Example 10. Let $P_4=\{0,1\}$ and $(P_4,*)$ be the polygroup referred in example 5. Then the constant (η,ξ,φ)-SNVP and $\Upsilon^{(\eta,\xi,\varphi)}=\langle\tau_1,\tau_2,\tau_3\rangle/1+\langle\tau_1',\tau_2',\tau_3'\rangle/0$, where $\tau_1\le\tau_1'$, $\tau_2\le\tau_2'$, and $\tau_3\ge\tau_3'$ are the only (η,ξ,φ)-SVNP over P_4.

Notation 1. Let $t=(\tau_1,\tau_2,\tau_3)$ and let Υ is a (η,ξ,φ)-SVNS of P. Then by $A(u)=t$, we means that $\tau_\Upsilon^\eta(u)=\tau_1$, $\iota_\Upsilon^\xi(u)=\tau_2$ and $\mathrm{F}_\Upsilon^\varphi(u)=\tau_3$. And by $\Upsilon^{(\eta,\xi,\varphi)}(u)\le t$, we mean that $\tau_\Upsilon^\eta(u)\le\tau_1$, $\iota_\Upsilon^\xi(u)\le\tau_2$, and $\mathrm{F}_\Upsilon^\varphi(u)\ge\tau_3$.

Theorem 13. *Each subpolygroup of polygroup $(P,*)$ is a level set of a (η,ξ,φ)-SVNP over P.*

Proof 18. Let Q be a subpolygroup of P, consider $t=(\tau_1,\tau_2,\tau_3)$, where $0<\tau_1,\tau_2<1$, and $0<\tau_3<1$. Define the (η,ξ,φ)-SVNS over P as follows:

$$\Upsilon^{(\eta,\xi,\varphi)}(u)=\begin{cases}(\tau_1,\tau_2,\tau_3), & \text{if } x\in Q,\\ (0,0,1), & \text{otherwise.}\end{cases}\tag{94}$$

Let $t'=(\tau_1',\tau_2',\tau_3')$. Then

$$A_{t'}^{(\eta,\xi,\varphi)} = \begin{cases} Q & \text{if } \tau_1 \geq \tau_1', \tau_2 \geq \tau_2', \tau_3 \leq \tau_3', \\ P & \text{if } \tau_1' = 0, \tau_2' = 0, \quad \tau_3' = 1, \text{ is either } \varnothing \text{ or } a \text{ subpolygroup of } P. \\ \varnothing & \text{otherwise.} \end{cases} \tag{95}$$

Using Theorem 12, we get that Υ is a (η, ξ, φ)-SVNP over P. □

Definition 25. Let Υ be a (η, ξ, φ)-SVNP over polygroup $(P, *)$. Then Υ is said to be a normal (η, ξ, φ)-SVNP over P if $\Upsilon^{(\eta,\xi,\varphi)}(w) = \Upsilon^{(\eta,\xi,\varphi)}(w\prime)$, $\forall w \in u * v$, and $w\prime \in v * u$.

Example 11. Let Υ be a (η, ξ, φ)-SVNP over polygroup $(P, *)$. Then the constant (η, ξ, φ)-SNVP is a normal (η, ξ, φ)-SNVP over P.

Theorem 14. *Let Υ is a (η, ξ, φ)-SVNS over polygroup $(P, *)$. Then Υ is a normal (η, ξ, φ)-SVNP over $P \Leftrightarrow \Upsilon_t^{(\eta,\xi,\varphi)} \neq \varnothing$ is a normal subpolygroup of P for every $t = (\tau_1, \tau_2, \tau_3)$, where $0 \leq \tau_1, \tau_2 < 1$, and $0 < \tau_3 \leq 1$.*

Proof 19. Let Υ be a normal (η, ξ, φ)-SVNP over P and $u, v \in \Upsilon_t^{(\eta,\xi,\varphi)} \neq \varnothing$. Theorem 12 argues that $\Upsilon_t^{(\eta,\xi,\varphi)} \neq \varnothing$ is a subpolygroup of P. Let $u \in P$. We need to show that $u^{-1} * \Upsilon_t^{(\eta,\xi,\varphi)} * u \subseteq \Upsilon_t^{(\eta,\xi,\varphi)}$. Let $w \in u^{-1} * \Upsilon_t^{(\eta,\xi,\varphi)} * u$. Then $\exists\, v$ in $\Upsilon_t^{(\eta,\xi,\varphi)}$ such that $w \in u^{-1} * v * u$, hence $w \in u^{-1} * p$, where $p \in v * u$. The latter implies that $v \in p * u^{-1}$. And since Υ is a normal (η, ξ, φ)-SVNP over P. Accordingly, $\Upsilon^{(\eta,\xi,\varphi)}(w) = \Upsilon^{(\eta,\xi,\varphi)}(v)$. Hence, $w \in \Upsilon_t^{(\eta,\xi,\varphi)}$.

Conversely, suppose $\Upsilon_t^{(\eta,\xi,\varphi)} \neq \varnothing$ be a normal subpolygroup of P. Theorem 12 argues that Υ is a (η, ξ, φ)-SVNP over P. To show that Υ is a normal (η, ξ, φ)-SVNP over P, it is sufficient to enhance that $\Upsilon^{(\eta,\xi,\varphi)}(w) = \Upsilon^{(\eta,\xi,\varphi)}(w\prime) \forall w \in u * v$, $w\prime \in v * u$.

Let $w \in u * v$, $w\prime \in v * u$ with $\Upsilon^{(\eta,\xi,\varphi)}(w\prime) = t$. Having $w\prime \in v * u$ implies that $v \in w\prime * u^{-1}$. The latter reveals that let $w \in u * v\prime * u^{-1}$. Since $w\prime \in \Upsilon_t^{(\eta,\xi,\varphi)}$ and $\Upsilon_t^{(\eta,\xi,\varphi)} \neq \varnothing$ is a normal subpolygroup of P, it pursues that $w \in \Upsilon_t^{(\eta,\xi,\varphi)}$ and therefore, $\Upsilon^{(\eta,\xi,\varphi)}(w) \geq \Upsilon^{(\eta,\xi,\varphi)}(w\prime) = t$. Similarly, we get that $\Upsilon^{(\eta,\xi,\varphi)}(w\prime) \geq \Upsilon^{(\eta,\xi,\varphi)}(w)$. $\Rightarrow \Upsilon^{(\eta,\xi,\varphi)}(w\prime) = \Upsilon^{(\eta,\xi,\varphi)}(w)$. □

Corollary 4. *Let Υ be a (η, ξ, φ)-SVNP over polygroup $(P, *)$. Then P has no proper normal subpolygroups if and only if the constant (η, ξ, φ)-SNVP is the only normal (η, ξ, φ)-SNVP over P.*

Example 13. Let $P_6 = \{e, \vartheta_1, \vartheta_2, \vartheta_3\}$ and $(P_6, .)$ be the polygroup illustrated in example 7. Then the constant (η, ξ, φ)-SNVP is the only normal (η, ξ, φ)-SVNP over P_6.

Theorem 15. *Every normal subpolygroup of polygroup $(P, *)$ is a level set of a normal (η, ξ, φ)-SVNP over P.*

Proof 20. The result is identical to that of Theorem 13. □

Corollary 5. *Let Υ be a (η, ξ, φ)-SVNP over polygroup $(P, *)$. Then $(\Upsilon^{(\eta,\xi,\varphi)})^* = \{u \in P\colon \Upsilon^{(\eta,\xi,\varphi)}(u) = \Upsilon^{(\eta,\xi,\varphi)}(e)\}$ is a subpolygroup of P. Moreover, if Υ is a normal (η, ξ, φ)-SVNP over P, then $(\Upsilon^{(\eta,\xi,\varphi)})^*$ is a normal subpolygroup of P.*

Proof 21. Let $t = \Upsilon^{(\eta,\xi,\varphi)}(e)$. Then $\Upsilon_t^{(\eta,\xi,\varphi)} = \{u \in P\colon \tau_\Upsilon^\eta(u) \geq \tau_\Upsilon^\eta(e), \iota_\Upsilon^\xi(u) \geq \iota_\Upsilon^\xi(e), \mathrm{F}_\Upsilon^\varphi(u)\mathrm{F}_\Upsilon^\varphi(e)\}$. Proposition 5 and Proposition 6 asserts that

$$\begin{aligned} \Upsilon_t^{(\eta,\xi,\varphi)} &= \{u \in P\colon \tau_\Upsilon^\eta(u) = \tau_\Upsilon^\eta(e), \iota_\Upsilon^\xi(u) = \iota_\Upsilon^\xi(e), \mathrm{F}_\Upsilon^\varphi(u) \leq \mathrm{F}_\Upsilon^\varphi(e)\} \\ &= \left(\Upsilon^{(\eta,\xi,\varphi)}\right)^*. \end{aligned} \tag{96}$$

Theorem 12 and Theorem 14 complete the proof. □

7. Single-Valued Neutrosophic Multicriteria Decision-Making Method

Multiple-criteria decision-making is an operations research subdiscipline that explicitly assesses multiple competing criteria in decision-making (both in everyday life and in settings as well as in situations like as the business, government, and medicine). M-CDM offers a basis for choosing, categorizing, and ranking items and aids in the overall evaluation. M-CDM is a useful tool that may be used to a variety of complicated/sophisticated or when the materials are novel. It is especially beneficial in circumstances involving a decision between options. It helps us to focus on the real issues and it is logical and consistent and is easy to use; it has all the qualities of an excellent decision-making tool.

A SVNS is a stereotype of a classic set, a fuzzy set, a paraconsistent set, and an intuitionistic fuzzy set. It is more broad and can handle not only partial information but also equivocal and unreliable information, both of which are typical in real-world situations. As a result, SVN D-M is more suited for real-world scientific and technical applications.

In this section, we present strategies for resolving M-CDM issues in a SVN environment by using the WCSM between SVNSs.

Assume $\mathfrak{R}_1, \mathfrak{R}_2, \mathfrak{R}_3, \ldots, \mathfrak{R}_\mathfrak{r}$ resemble the alternatives and $\mathfrak{Y}_1, \mathfrak{Y}_2, \mathfrak{Y}_3, \ldots, \mathfrak{Y}_\mathfrak{y}$ represent the set of criteria. Consider the weight of the criterion $\mathfrak{Y}_\mathfrak{j}\ (\mathfrak{j} = 1, 2, \ldots, \mathfrak{y})$ enters by decision-makers is $\mathfrak{w}_\mathfrak{j}, \mathfrak{w}_\mathfrak{j} \in [0, 1]$ and

$\sum_{\mathfrak{j}=1}^{\mathfrak{y}} \mathfrak{w}_\mathfrak{j} = 1$. The preceding SVNS indicate the feature of alternative $\mathfrak{R}_\mathfrak{i}\ (\mathfrak{i} = 1, 2, \ldots, \mathfrak{r})$ in this case:

$$\mathfrak{R}_\mathfrak{i} = \sum_{\mathfrak{j}=1}^{\mathfrak{y}} \frac{\langle \tau_{\mathfrak{R}_\mathfrak{i}}(\mathfrak{Y}_\mathfrak{j}), \iota_{\mathfrak{R}_\mathfrak{i}}(\mathfrak{Y}_\mathfrak{j}), \mathrm{F}_{\mathfrak{R}_\mathfrak{i}}(\mathfrak{Y}_\mathfrak{j})\rangle}{\mathfrak{Y}_\mathfrak{j}}, \quad \mathfrak{Y}_\mathfrak{j} \in \mathfrak{Y}, \tag{97}$$

where $\tau_{\mathfrak{R}_\mathfrak{i}}(\mathfrak{Y}_\mathfrak{j}), \iota_{\mathfrak{R}_\mathfrak{i}}(\mathfrak{Y}_\mathfrak{j}), \mathrm{F}_{\mathfrak{R}_\mathfrak{i}}(\mathfrak{Y}_\mathfrak{j}) \in [0, 1]$, $\mathfrak{j} = 1, 2, \ldots, \mathfrak{y}$ and $\mathfrak{i} = 1, 2, \ldots, \mathfrak{r}$.

Table 7: Comparison between SVNS and some existing approaches.

	Set	Truth	Indeterminacy	Falsity	Attributes
Zadeh [1]	Fuzzy set	√	×	×	
Atanassov [3]	Intuitionistic fuzzy set	√	×	√	√
Yager [49]	Pythagorean fuzzy set	√	×	√	√
Chen et al. [50]	m-polar fuzzy set	√	×	×	√
Naeem et al. [51]	Pythagorean m-polar fuzzy set	√	×	√	√
Maji et al. [52]	Fuzzy soft set	√	×	×	√
Maji et al. [53]	Intuitionistic fuzzy soft set	√	×	√	√
Peng et al. [54]	Pythagorean fuzzy soft set	√	×	√	√
Zulqarnain et al. [55]	Intuitionistic fuzzy hyper soft set	√	×	√	√
Zulqarnain et al. [56]	Pythagorean fuzzy hyper soft set	√	×	√	√
Proposed technique	SVNS	√	√	√	√

We represent a SVNS by $\delta_{ij} = \langle \rho_{ij}, \varrho_{ij}, \sigma_{ij} \rangle$. An SVNS is often synthesized from the evaluation of an alternative $\mathfrak{R}_i$ with regard to a criteria $\mathfrak{Y}_j$ in implementation using a score law and data processing. As a result, we may derive a SVN decision matrix $\mathfrak{D} = (\delta_{ij})_{\mathfrak{r}\times\mathfrak{y}}$.

The notion of ideal point has been intended to assist discover the optimal option in a M-CDM scenario. Although the perfect alternative does not exist in the real world, it does give a valuable theoretical framework against which alternatives may be evaluated.

The notion of optimum point has been achieved by involving to discover the optimal option in a M-CDM context. Although the perfect alternative somehow does not persist in the everyday life, it does give a valuable theoretical framework against which alternatives may be evaluated.

As a reason, the ideal alternative $\mathfrak{R}^{@}$ is defined as the SVNS $\delta_j = \langle \rho_j^{@}, \varrho_j^{@}, \sigma_j^{@} \rangle = \langle 1, 0, 0 \rangle$ for $j = 1, 2, \ldots \mathfrak{y}$. The WCSM between an alternative $\mathfrak{R}_i$ and the ideal alternative $\mathfrak{R}^*$ represented by the SVNSs is defined by

$$\mathfrak{Q}_i\left(\mathfrak{R}_i, \mathfrak{R}^{@}\right) = \sum_{j=1}^{\mathfrak{y}} \mathfrak{w}_j \frac{\left[\rho_{ij}.\rho_j^* + \varrho_{ij}.\varrho_j^* + \sigma_{ij}.\sigma_j^*\right]}{\sqrt{\rho_{ij}^2 + \varrho_{ij}^2 + \sigma_{ij}^2}.\sqrt{\left(\rho_j^{@}\right)^2 + \left(\varrho_j^{@}\right)^2 + \left(\sigma_j^{@}\right)^2}} \tag{98}$$

Then, the higher the WCSM value, the better the option. The measure values can produce the ranking order of all alternatives and the best option by using (98).

8. Application

This section demonstrates an overview of a M-CDM issue with choices to exemplify the relevance and efficacy of the offered D-M strategy. Consider the paradox of D-M. There is an investment firm that wants to put money into the finest choice.There is a panel with four potential financing options:

(1) $\mathfrak{R}_1$ is a manufacturer of automobiles;

(2) $\mathfrak{R}_2$ is a manufacturer of electronics;

(3) $\mathfrak{R}_3$ is a vacation rentals; and

(4) $\mathfrak{R}_4$ is an industrial 3D printing builder company.

The investment firm must make a judgement based on the three criteria listed below:

(1) $\mathfrak{Y}_1$ is the financial, risk, and sensitivities;

(2) $\mathfrak{Y}_2$ is the progress assessment; and

(3) $\mathfrak{Y}_3$ is the environmental and location assessment.

The criteria's weight vector is hence specified by $\mathfrak{w} = (0.30, 0.25, 0.45)$.

The questionnaire of a professional expert is used to appraise an alternative $\mathfrak{R}_i\,(i = 1, 2, 3, 4)$ in relation to a criteria $\mathfrak{Y}_j\,(j = 1, 2, 3)$.

When asked to experts of their opinion on a potential alternative $\mathfrak{R}_1$ corresponding to $\mathfrak{Y}_1$, for instance, an expert might respond that there is a 0.6 chance that the statement is superb, a 0.2 chance that it is low, and a 0.1 chance that they are unsure. It may be written as $\delta_{11} = \langle 0.6, 0.2, 0.1 \rangle$ using the neutrosophic notation. The following SVN decision matrix $\mathfrak{D}$ may be obtained when the expert evaluates the four potential options in light of the aforementioned three criteria:

$$\mathfrak{D} = \begin{pmatrix} \langle 0.6, 0.2, 0.1 \rangle & \langle 0.5, 0.4, 0.3 \rangle & \langle 0.7, 0.5, 0.4 \rangle \\ \langle 0.3, 0.2, 0.3 \rangle & \langle 0.8, 0.3, 0.5 \rangle & \langle 0.5, 0.3, 0.2 \rangle \\ \langle 0.9, 0.5, 0.4 \rangle & \langle 0.7, 0.6, 0.5 \rangle & \langle 0.6, 0.5, 0.4 \rangle \\ \langle 0.8, 0.7, 0.3 \rangle & \langle 0.4, 0.1, 0.1 \rangle & \langle 0.9, 0.2, 0.2 \rangle \end{pmatrix}. \tag{99}$$

By employing (98), we can also give the following values of WCSM $\mathfrak{Q}_i\,(\mathfrak{R}_i, \mathfrak{R}^{@})\,(i = 1, 2, 3, 4)$ as

$$\begin{aligned} \mathfrak{Q}_1\left(\mathfrak{R}_1, \mathfrak{R}^{@}\right) &= 0.7899; \\ \mathfrak{Q}_2\left(\mathfrak{R}_2, \mathfrak{R}^{@}\right) &= 0.7589; \\ \mathfrak{Q}_3\left(\mathfrak{R}_3, \mathfrak{R}^{@}\right) &= 0.7190; \\ \mathfrak{Q}_4\left(\mathfrak{R}_4, \mathfrak{R}^{@}\right) &= 0.8823: \end{aligned} \tag{100}$$

The four options are thus ranked as follows: $\mathfrak{R}_4, \mathfrak{R}_1, \mathfrak{R}_2$, and $\mathfrak{R}_3$.

According to the order described by the rank matrix, industrial 3D printing builder company is turn out to be the best investment firm to put money into the finest choice whereas vacation rentals is the worst as per the criteria described.

8.1. Superiority of the Proposed Approach. Through this analysis and comparison, it was possible to conclude that the proposed procedure has produced more frequent results than either of the alternatives. In general, the D-M approach associated with prevalent D-M methods permits additional data to alleviate hesitancy. In the D-M process, it is thus acceptable to propagate false and unclear information. Therefore, the proposed method is reasonable, modest, and ahead of the fuzzy set's characteristic structures. The general information associated with the object could be stated precisely and analytically, as shown in Table 7.

9. Conclusion

This paper presented an algebraic hyperstructure of (η, ξ, φ)-SVNSs in the form of (η, ξ, φ)-SVN hypergroup, (η, ξ, φ)-SVNPs, and (η, ξ, φ)-ASVNPs. Several intriguing properties of the newly defined notions were discussed. The findings of this article can be thought of as a generalization of prior research on fuzzy hypergroups and fuzzy polygroups. We also discussed in this section a M-CDM system developed in an SVN environment using WCSM. WCSM between each option and the ideal alternative may be used to establish the ranking order of all alternatives and to readily identify the greatest alternative. Finally, an instructive example demonstrated how the new technique may be used. As a result, the proposed SVN M-CDM technique is more suited for real-world scientific and engineering applications since it can manage not only inadequate information but also indeterminate and inconsistent information, both of which are typical in real-world scenarios. The strategy suggested in this study enhances previous D-M methods and offers decision-makers with an useable method.

This work provided an algebraic hyperstructure of (η, ξ, φ)-SVNSs as (η, ξ, φ)-SVN hypergroup, (η, ξ, φ)-SVNPs, and (η, ξ, φ)-ASVNPs. Several remarkable characteristics of the newly formed concepts were addressed. The results of this article can be seen as a generalization of previous research on fuzzy hypergroups and fuzzy polygroups. In this part, we also described an M-CDM system constructed in an SVN environment utilizing WCSM. WCSM between each option and the best option may be used to define the ranking order of all options and quickly discover the best choice. Finally, an illustrative illustration explained how the new method may be implemented. Consequently, the suggested SVN M-CDM approach is more suitable for real-world scientific and engineering applications, since it can handle not only insufficient information but also indeterminate and inconsistent information, both of which are characteristic of real-world settings. This research proposes an approach that advances earlier D-M methods and provides decision-makers with a practical method.

(i) Researchers will continue to work on complex D-M issues with uncertain weights of criteria, as well as other disciplines such as expert systems, information fusion systems, biochemistry, epidemiology, geology, entomology, and biomedical engineering. In the realm of algebraic structure theory, it possesses a fantastic novel idea that has the potential to be utilized in the future for the solution of a variety of algebraic issues.

(ii) Using the algebraic structure of multi-polygroup in terms of intuitionistic fuzzy set theory, this method may be readily extended to the intuitionistic fuzzy multi-polygroups. Connecting intuitionistic fuzzy multiset theory, set theory, and polygroup theory may provide a novel notion of polygroup that may be used to illustrate the effect of intuitionistic fuzzy multisets on a polygroup's structure. Using this concept, researchers may study intuitionistic fuzzy normal multi-subpolygroups along with their characterizations and algebraic characteristics. Additionally, the homomorphisms of intuitionistic fuzzy multi-polygroups and some of their structural properties may be addressed. Additionally, this idea may be used to investigate intuitionistic fuzzy quotient multi-polygroups.

(iii) Researchers may expand this concept to include various neutrosophic multi-topological group structures. For this, they can introduce the definition of semi-open neutrosophic multiset, semi-closed neutrosophic multiset, neutrosophic multi-regularly open set, neutrosophic multi-regularly closed set, neutrosophic multi-continuous mapping. In addition, since the idea of the almost topological group is so novel, they may utilize the definition of neutrosophic multi almost topological group to define neutrosophic multi almost topological group.

(iv) This idea can be used to the development of the neutrosophic multi almost topological group of the neutrosophic multi-vector spaces, etc. This notion can be expanded to soft neutrosophic polygroups, weak soft neutrosophic polygroups, strong soft neutrosophic polygroups, soft neutrosophic polygroup homomorphism, and soft neutrosophic polygroup isomorphism. Furthermore, scholars might explore the homological properties of these polygroups.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that there are no conflicts of interest.

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