

## Research Article

# Stochastic-Deterministic Assessment of Electric Field Radiated by Base Station Antenna above a Two-Layered Ground

Marin Galić,<sup>1</sup> Anna Šušnjara ,<sup>2</sup> and Dragan Poljak<sup>2</sup>

<sup>1</sup>Environmental Measurement Center (CMO), Zagreb, Croatia

<sup>2</sup>University of Split, FESB, Department of Electronics and Computer Science, Split, Croatia

Correspondence should be addressed to Anna Šušnjara; [ansusnja@fesb.hr](mailto:ansusnja@fesb.hr)

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The paper deals with stochastic-deterministic modelling of radiated electric field by base station antennas (BSAs) operating in GSM frequency range. Within the framework of deterministic analysis, the total electric field above a two-layered lossy ground is obtained by considering the incident and reflected ray in the far field zone. The influence of nonhomogeneous lower medium is taken into account via two approaches: Fresnel plane wave reflection coefficient (FRM) and simplified reflection coefficient stemming from Modified Image Theory (MIT). Antenna height, relative conductivity, and permittivity of each ground layer, as well as the thickness of the upper ground layer, are considered as input parameters with inherent uncertainty. To quantify the uncertainty of output electric field, deterministic models are treated as black boxes by two stochastic methods, Monte Carlo (MC) and Stochastic Collocation (SC), respectively. Stochastic mean and standard deviation of the output are computed, and sensitivity analysis is carried out in order to analyze the impact of input parameters' variations on the resulting electric field variance. The presented results expose weakness and strength of the two stochastic methods, particularly identifying the cases when the preferred SC method fails to converge. Furthermore, sensitivity analysis reveals that despite the smallest variation around its respective average value, the antenna height has the highest impact on the output variance at observation points in the vicinity of the antenna. However, as the distance from the antenna increases, the 1st layer depth and its relative electric permittivity become the most significant parameters.

## 1. Introduction

In the last few decades, radiation from base station antennas (BSAs) has become a controversial topic regarding the possible adverse health effects. Namely, it is a well-known fact that the principal biological effect due to exposure of humans to high frequency (HF) radiation is tissue heating. Two main organizations proposing guidelines for limiting values of EM fields for human exposure, IEEE and ICNIRP, have published revised editions regarding the HF exposure in 2019 [1] and 2020 [2], respectively. Both documents agree that the transition frequency of 6 GHz is the frequency above which body surface heating becomes dominant over volume heating. Therefore, the electromagnetic measure for body heating due to HF radiation is the specific absorption rate (SAR) for frequencies below 6 GHz, while for frequencies above transition frequency both guidelines suggest the

absorbed power density ( $S_{ab}$ ) to be calculated. Since either SAR or  $S_{ab}$  are not likely to be readily assessed, the IEEE and ICNIRP guidelines provide exposure reference levels defined in terms of incident power density ( $S_{inc}$ ) or incident electric field strength ( $E_{inc}$ ).

Therefore, BSAs are required to satisfy certain exposure conditions according to regulations based on standards such as [3, 4], which are in consistence with the IEEE and ICNIRP exposure reference levels [1, 2]. However, in calculation procedures of radiated field levels, most of the present standards consider incident field ray only, thus neglecting the reflection from the ground or other objects [4–7]. Furthermore, in order to consider the worst-case exposure scenario, standards such as [8] provide possibility to use correction factors based on fixed reflection coefficients. On the other hand, some studies deal with BSA actual transmitted power, instead of considering maximal BSA

transmitted power with the intention to provide the most realistic scenario [9]. This approach could be of some use in 5G electric field calculations and exposure estimations [10].

However, despite the efforts of the presented regulations [3–7], deterministic computations are not sufficient for computation of electric field levels due to the inherent uncertainty of the input parameters. Namely, it is practically impossible to prescribe exact/fixed values for input parameters in computational modelling of many realistic exposure scenarios. On one hand, there is an inherent uncertain nature of body tissues' electric parameters since their direct measurements on human beings are not acceptable. On the other hand, the sources of radiation also exhibit random nature, e.g., the position and the orientation of antennas have different impact on the resulting radiated and thus the internal fields [11]. Finally, the environment and soil parameters may also be considered as random variables since their structure and electric parameters are not always straightforward to obtain [12]. Therefore, in order to consider variable nature of input parameters, stochastic approaches in radiated field calculation play an important role. Since purely stochastic modelling is either time consuming or impossible to obtain, the hybrid approach in terms of stochastic-deterministic modelling in HF dosimetry has been readily used for a decade [13]. Many examples of stochastic-deterministic modelling in internal EM and thermal dosimetry can be found in, e.g., [11, 14–17]. However, extensive stochastic dosimetry of incident fields, i.e., the radiated fields, is still scarce in the literature. Some recent examples include the work of Al Hajj et al. in [18], where statistical estimation of 5G massive MIMO networks' exposure using stochastic geometry in mmWave bands is carried out. Namely, in [18], the stochastic input parameters are exponential and gamma distribution for antenna and channel gain, respectively. Therefore, since stochastic determination of electric field radiated by a base station antenna above a multilayered ground and incorporating the ground parameters as random variables has not yet been reported to the best of the authors' knowledge, the analysis presented in this paper serves as an opener to the subject.

The motivation for this research arises from the practical *on-site* situations when engineers work in a realistic environment, outside the lab. In such environments some of the parameters, which are crucial for electric field calculation, are difficult to measure, e.g., antenna height, soil structure, thickness of soil characteristic layers, their relative electric permittivities, and conductivities. Usually, such input parameters are prescribed with some fixed expected/average values according to the *in-situ* measurements and delivered technical data. Consequently, the deterministic computations of radiated electric field lead to fixed output value without the information about the field distribution or confidence intervals, thus limiting the knowledge about the field level. Therefore, the goal of this analysis is to investigate the most likely electric field radiation level in case of maximum transmitted power and its confidence intervals when certain input parameters cannot be prescribed with fixed values. In other words, the aim is to obtain the stochastic mean and standard deviation of the output field level,

as well as the impact of input parameters on the total electric field.

The single electric field computation is based on deterministic model which considers the incident and reflected ray, respectively, in the far field zone above the two-layered half-space media. The influence of nonhomogeneous lower medium is taken into account via both Fresnel plane wave reflection coefficient and simplified reflection coefficient stemming from Modified Image Theory Method [19]. Based on the chosen analytical model, the following input parameters are represented as random variables: antenna height, thickness of the upper ground layer, relative conductivity, and permittivity of both ground layers. By means of numerical stochastic methods, Monte Carlo and Stochastic Collocation, the uncertainties are propagated from the input parameter set to the output electric field, thus computing its expectation and variance. Due to the nonintrusive nature of the chosen stochastic methods, the deterministic electric field computation is considered as a black box. Furthermore, sensitivity analysis is carried out to investigate the impact of the input parameters on output field variance.

The rest of the paper is organized as follows: deterministic field computation is described in Section 2. Stochastic Collocation and Monte Carlo methods are described in Section 3, while Section 4 presents the procedures for sensitivity analysis. Numerical results are presented in Section 5, followed by discussion in Section 6. Finally, concluding remarks are given in the last section.

## 2. Deterministic Modelling of Electric Field

Geometry of interest is a BSA system radiating over a multilayered ground. The ground consists of two layers where the thickness of upper layer is defined as  $d_1$  and the thickness of bottom layer,  $d_2$  is infinite. Each layer is prescribed with the corresponding electric conductivity ( $\sigma_1$  and  $\sigma_2$ ) and relative electric permittivity ( $\epsilon_1$  and  $\epsilon_2$ ) (Figure 1).

There are several approaches to calculate radiated electric field at high frequencies and the review of the most used techniques can be found elsewhere, e.g., in [20]. The simplest approaches to calculate the total field ( $E_{TOT}$ ) account for incident ray only ( $E_i$ ), while some techniques include reflections ( $E_r$ ) with different approximations. As an example, according to regulations in [4], approximation of antenna insulated in free space is officially used to obtain radiated electric field while reflected field components are neglected.

Considering the defined antenna height ( $A_h = 20$  m), and operating frequency, ( $f = 936.8$  MHz), as well as the fact that maximum antenna dimension is approximately 2 m, it is assumed that all calculation points in this research are in far field area [4, 6]. Therefore, the following relation for the electric field magnitude in free space can be used [8]:

$$E = \frac{\sqrt{30 \cdot N \cdot EIRP \cdot 10^{G/10}}}{r}, \quad (1)$$

where  $EIRP$  stands for effective isotropic radiated power,  $G$  is antenna gain expressed in dB,  $N$  is the number of active

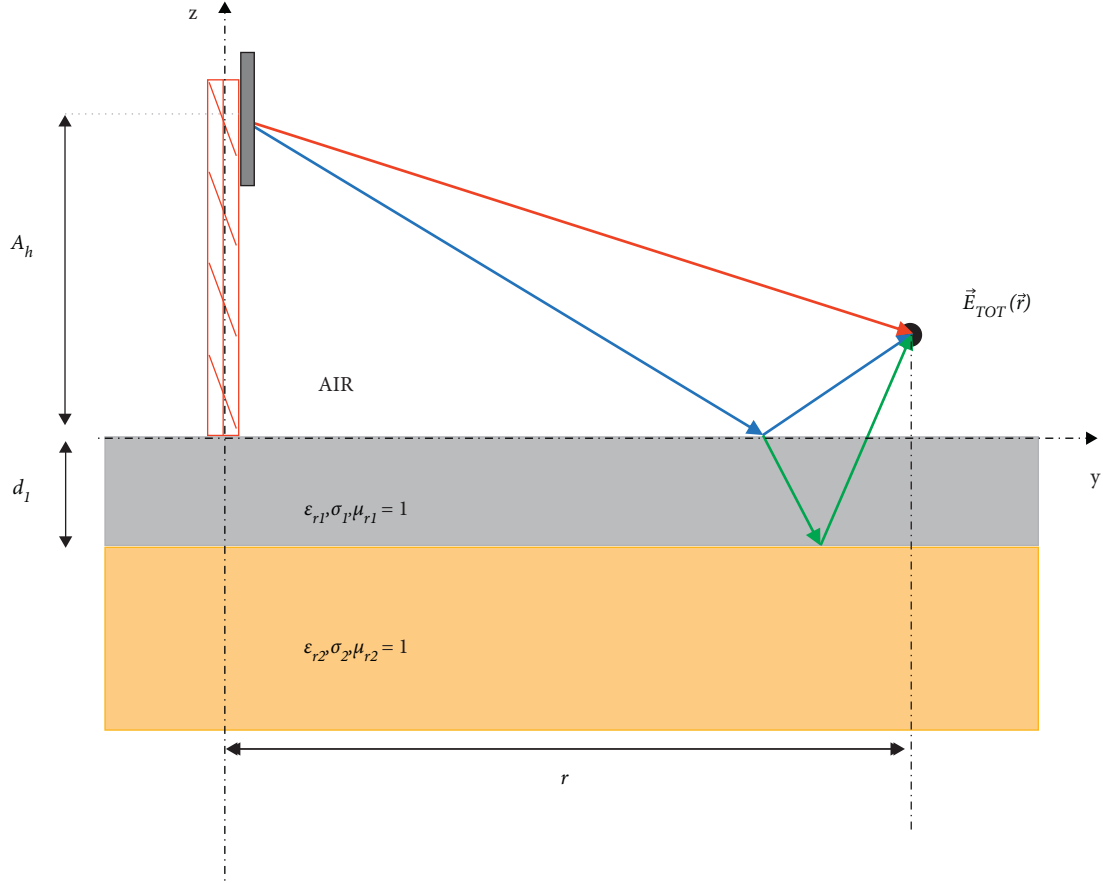


FIGURE 1: Base station antenna above a two-layered lossy ground. The red arrow represents the incident ray, blue arrows represent the ray reflected from the 1<sup>st</sup> ground layer, and green arrows represent the ray reflected from the 2<sup>nd</sup> ground layer.

channels ( $N=1$ ), and  $r$  is the distance from the antenna. Note that this relation is the official relation used from far field computation in practical engineering studies [21].

Since in this paper both the incident and the reflected rays are taken into consideration for the total field computation, the final expression assumes the summation of the incident and reflected field as follows:

$$E_{TOT} = \sqrt{E_i^2 + E_r^2}. \quad (2)$$

Note that both incident and reflected field components  $E_i$  and  $E_r$  are computed by means of an approximate formula (1) and the field reflected from the interface is determined by using the corresponding reflection coefficient, as reported elsewhere, e.g., in [19, 22].

It is worth emphasizing that the multilayered lossy half-space is taken into account by two approaches: Fresnel reflection coefficient model (FRM) and a more simplified reflection coefficient arising from Modified Image Theory method (MIT) [19].

The reflection coefficient for two-layered media  $\Gamma_r$  is given by the following expression [23]:

$$\Gamma_r = \frac{R_{02} + R_{12} \cdot e^{-j2\beta d_1 \cdot \cos(\vartheta_{i-01})}}{1 + R_{02} \cdot R_{12} \cdot e^{-j2\beta d_1 \cdot \cos(\vartheta_{i-01})}}, \quad (3)$$

where  $R_{mn}$  stands for the reflection coefficient between the two media (indexed by  $m$  and  $n$ ) computed either with FRM or MIT model. Indices 0, 1, and 2 stand for air, 1<sup>st</sup> ground layer, and 2<sup>nd</sup> ground layer, respectively. The incidence angle between the air and 1<sup>st</sup> layer is denoted by  $\theta_{i-01}$  while  $\beta$  is the phase propagation constant for air,  $\beta^2 = \omega^2 \mu_0 \epsilon_0$ . The angular frequency is denoted by  $\omega$ .

In particular, the reflection coefficient  $R_{mn}$  between the two media according to Fresnel approximation for transverse magnetic polarization is given by [19, 24]

$$R_{mn}^{FRM} = \frac{Z_m \cdot \cos(\theta_{i-mn}) - Z_n \cdot \cos(\theta_{t-mn})}{Z_m \cdot \cos(\theta_{i-mn}) + Z_n \cdot \cos(\theta_{t-mn})}, \quad (4)$$

where  $Z_m$  and  $Z_n$  stand for the impedance of upper and lower medium, respectively.  $\theta_{i-mn}$  and  $\theta_{t-mn}$  are the incidence and transmission angles with respect to the normal of the interface between the media  $m$  and  $n$ .

The general expression for the media impedance is defined as follows:

$$Z_m = \sqrt{\frac{j\mu_m \omega}{\sigma_m + j\omega\epsilon_m}}. \quad (5)$$

Unlike FRM, the model stemming from the Modified Image Theory neglects the angle of incidence and angle of reflectance, i.e., the incident wave is perpendicular with

respect to the media surface ( $\theta_i = 0$ ). Thus, the reflection coefficient  $R_{mn}$  between the two media according to MIT is given by [23]

$$R_{mn}^{MIT} = \frac{\varepsilon_{eff}^{(m)} - \varepsilon_{eff}^{(n)}}{\varepsilon_{eff}^{(m)} + \varepsilon_{eff}^{(n)}}, \quad (6)$$

where  $\varepsilon_{eff}$  is the refraction index of the medium:

$$\varepsilon_{eff}^{(m)} = \varepsilon_r^{(m)} - j \frac{\sigma_m}{\omega \varepsilon_0}. \quad (7)$$

Hence, the total reflection coefficient  $\Gamma_r$  stemming from MIT is analog to (3).

As it is visible from (1)–(7), the deterministic model is rather simple, i.e., it is based on the extension of the well-known ray-tracing algorithm (mostly dealing with free space, or perfectly conducting half-space, or half-space, at best) with reflection coefficient approximation (RCA) arising from Fresnel plane wave approach and Modified Image Theory (MIT), respectively, to account for the reflecting effects of a layered medium. As such a deterministic model is entirely analytical in nature, the total computational cost is negligible when compared to highly sophisticated full-wave models based on numerical methods.

### 3. Methods for Uncertainty Quantification (UQ)

The first step in the stochastic part of the analysis is to determine the input parameters exhibiting the random nature which are, in this case, antenna height, the thickness of the first layer, relative permittivities, and electric conductivities of both ground layers. These input parameters are grouped in the vector of random input parameters  $\mathbf{X} = [A_h, d_1, \varepsilon_1, \sigma_1, \varepsilon_2, \sigma_2]$ . Each input parameter is prescribed with the corresponding uniform distribution.

Uncertainty from the input parameter set is propagated to the output of interest and uncertainty quantification (UQ) of electric field is carried out. Stochastic moments (e.g., expectation, variance, and standard deviation) of the electric field are computed via two stochastic methods: Monte Carlo (MC) and Stochastic Collocation (SC).

**3.1. Monte Carlo.** MC is a simple and robust method based on statistical analysis of a large number of deterministic realizations of the output of interest. The mean and the variance of the output electric field are thus obtained by using the following formulas [25]:

MC electric field expectation:

$$E_{MC} \approx \frac{1}{N_{MC}} \sum_{i=1}^{N_{MC}} E_i. \quad (8)$$

MC electric field variance:

$$V(E)_{MC} \approx \frac{1}{N_{MC} - 1} \sum_{i=1}^{N_{MC}} (E_i - E_{MC})^2, \quad (9)$$

where  $E_i$  is the  $i$ -th out of the  $N_{MC}$  deterministic realizations of the model defined by equation (3).

MC convergence does not depend on the stochastic dimensionality of the model (which is 6 in this case) but only on total number of deterministic simulations ( $N_{MC}$ ). However,  $N_{MC}$  needs to be quite large, very often up to the order of  $10^4$  and higher, which presents a computational burden when underlying deterministic models require long execution time. Therefore, some other stochastic methods are being investigated in order to increase the computational efficiency.

**3.2. Stochastic Collocation.** SC is a sampling-based method where the focus is to utilize a polynomial expansion of the model output in the domain defined by probability density functions of stochastic input parameters [26]. The approximation for the output electric field is thus defined as follows:

$$\widehat{E}(A_h, d_1, \varepsilon_1, \sigma_1, \varepsilon_2, \sigma_2) = \sum_{i=1}^{N_{SC}} L_i(A_h, d_1, \varepsilon_1, \sigma_1, \varepsilon_2, \sigma_2) \cdot E_i, \quad (10)$$

where  $L_i(A_h, d_1, \varepsilon_1, \sigma_1, \varepsilon_2, \sigma_2) = L_i(\mathbf{X})$  is the multivariate basis function,  $E_i$  is the  $i$ -th output of deterministic model defined by eq. (3), and  $N_{SC}$  is the total number of deterministic simulations. The multivariate basis function  $L_i(\mathbf{X})$  is built by using the sparse grid algorithm with Lagrange basis functions in 1-dimensional case [26].

When the polynomial expression from equation (10) is plugged into a well-known formula for stochastic moments (mean, variance, etc.) [27], the final expressions for the electric field stochastic moments are as follows [26].

SC expectation of electric field:

$$E_{SC} \approx \sum_i^n E_i \cdot \omega_i. \quad (11)$$

SC variance:

$$V(E)_{SC} \approx \sum_i^n (E_i)^2 \cdot \omega_i - (E_{SC})^2. \quad (12)$$

Here,  $\omega_i$  denotes the weight of the  $i$ -th multidimensional simulation computed as [26]

$$\omega_i = \int_{\mathbf{X}} L_i(\mathbf{X}) \cdot \text{pdf}(\mathbf{X}) d\mathbf{X}, \quad (13)$$

where  $\text{pdf}(\mathbf{X}) = \prod_k \text{pdf}(X_k)$  is the joint probability density function. This integral is evaluated numerically. In this paper, the numerical integration is done according to the Clenshaw–Curtis integration rule [16].

Stochastic Collocation can be regarded as a “smart Monte Carlo” since it combines MC’s nonintrusive nature with a functional approximation of the output of interest thus enabling a rather small number of deterministic realizations of the model for accurate UQ. More details about the mathematical background can be found elsewhere, e.g., in [26].

3.3. *Standard Deviation and Confidence Intervals Computation.* Regardless of the method of choice for the UQ, the standard deviation is computed as the square root of variance [27]:

$$\text{Std}(E) = \sqrt{V(E)}. \quad (14)$$

The standard deviation is important for the crude estimation of confidence intervals. Therefore, the confidence intervals used in this paper (CI) are defined as

$$\text{CI} = E \pm 2 \cdot \text{Std}(E), \quad (15a)$$

$$\text{CI} = E \pm 3 \cdot \text{Std}(E), \quad (15b)$$

for 95% or 99% level of confidence, respectively.

Confidence intervals play an important role in the comparison with reference levels and basic restrictions defined by ICNIRP and IEEE since they define the range in which the output value is expected with a certain level of confidence. Also, confidence intervals defined as in equations (15.a) and (15.b) are of particular importance since calculation results in technical reports are mostly presented with the precision of 95% or 99.7%, respectively [10].

#### 4. Sensitivity Analysis

Sensitivity analysis (SA) is a process of ranking the input parameters from the least to the most significant ones with respect to the impact that each parameter's variability has on the output variance [28]. Two approaches are described in this section, the one-at-a-time (OAT) and Analysis of Variance (ANOVA). It is worth noting that both ANOVA and OAT SA are independent on the underlying UQ method; namely, the conditional variances, expectations as well as one-dimensional variances used in the computation can be obtained with either MC or SC method.

4.1. *“One-at-a-Time” (OAT) Approach.* “One-at-a-time” (OAT) approach is a simple SA approach that compares the variances stemming from univariate stochastic problems,  $V$

$[Y|X_k], k = 1, 2, \dots, n, (n = 6)$  [16]. Univariate stochastic model is 1-dimensional, i.e., only one input parameter exhibits random nature ( $n = 1$ ) while others are kept constant. Instead of variances, standard deviations of univariate cases may be compared, too. The main advantage of this approach is that it enables rather quick parameter ranking. When stochastic collocation is used as a method for variance computation, the OAT is computationally inexpensive when compared to MC method. Namely, SC for 1-dimensional problem requires only few deterministic simulations while MC always assumes large  $N_{MC}$ , regardless of the stochastic dimensionality.

4.2. *Analysis of Variance (ANOVA) Approach.* The drawback of the OAT approach is a possibility of masking out the impact that parameter's interactions have on the output variance. Therefore, the ANOVA based on Sobol's indices computation is often used in parallel [29]. The Sobol indices are based on Hoeffding's function decomposition of total variance into terms pertaining to variances of 1-dimensional, 2-dimensional, up to  $(n-1)$ -dimensional cases.

Hence, the first-order sensitivity index  $S_1(X_k)$  gives information about the impact of the  $k$ -th input parameter on the total variance [28]:

$$S_1(X_k) = \frac{V_{X_k}[E_{X \sim k}(E|X_k)]}{V(E)} \quad k = 1, 2, \dots, n, \quad (16)$$

where  $\langle E \rangle_{X \sim k}(E|X_k)$  is the conditional expectation for the output electric field. Namely, there are  $n-1$  such expectations: the  $k$ -th input parameter is kept at its constant value while the expected  $E$  field is computed for  $(n-1)$ -dimensional stochastic model. The tilde sign “ $\sim$ ” stands for “all except”. After the computation of  $(n-1)$  conditional expectations, their variance is computed, i.e.,  $V_{X_k}(\cdot)$ . The  $V(E)$  stands for the electric field variance in  $n$ -dimensional case (total variance).

The second-order sensitivity index gives information about the impact of the interaction between the  $i$ -th and  $j$ -th input parameters on the total variance:

$$S_2(X_i, X_j) = \frac{V_{X_i X_j}[E_{X \sim i, j}(E|X_i, X_j)] - V_{X_i}[E_{X \sim i}(E|X_i)] - V_{X_j}[E_{X \sim j}(E|X_j)]}{V(E)} \quad (17)$$

$$i = 1, 2, \dots, n,$$

$$j = 1, 2, \dots, n, i \neq j.$$

Since the computation of higher order indices (2<sup>nd</sup>, 3<sup>rd</sup>, 4<sup>th</sup>, ...) may present a certain computational burden for the postprocessing, the total effect sensitivity index  $S_T(X_k)$  is computed instead, giving the information about the impact of the  $k$ -th input parameter along with its interaction with all other input parameters [28]:

$$S_T(X_k) = 1 - \frac{V_{X \sim k}[E_{X \sim k}(E|X_k)]}{V(E)} \quad k = 1, 2, \dots, n. \quad (18)$$

If  $S_1(X_k)$  and  $S_T(X_k)$  have the same value, then the interaction between input parameters does not impact the output variance at all.

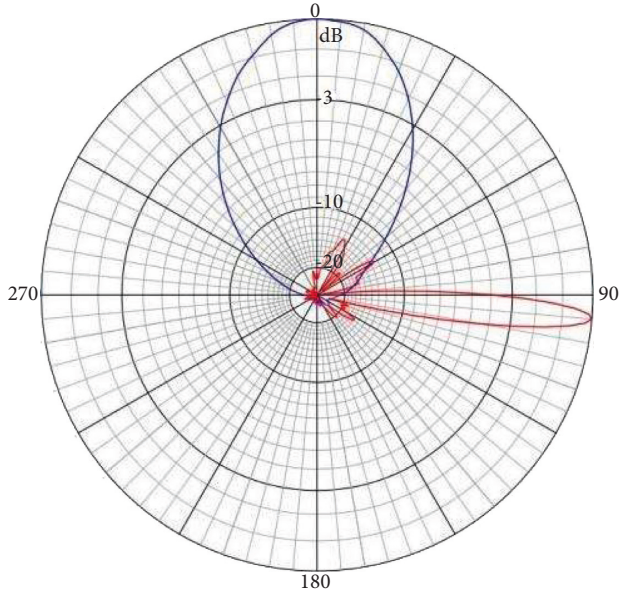


FIGURE 2: Horizontal (blue) and vertical (red) antenna patterns.

## 5. Numerical Results

Base station antenna system radiating over a multilayered ground depicted in Figure 1 is considered. The output antenna power ( $EIRP$ ) is set to 58.45 dBmW and operating frequency is 936.8 MHz which is the common frequency for 2G and 3G technology (GSM and UMTS). Antenna is modelled as a panel antenna whose horizontal and vertical radiation pattern are obtained by using the software package NEC (Numerical Electromagnetic Code) [30] (Figure 2).

In practice it is difficult to exactly determine all input parameters since some values are measured *in-situ*, therefore, specific uncertainties of measured parameters are expected. Therefore, for the purpose, the main antenna as well as ground layers' parameters are modelled as random variables (RVs) uniformly distributed around their respective averages. The values are presented in Table 1.

The rest of the section is organized as follows: firstly, the convergence of MC and SC methods is presented, and the two methods are compared for the chosen scenario. After that, the UQ of the electric field is presented, including the estimation of confidence intervals. Finally, the results of the sensitivity analysis are given.

**5.1. Convergence of SC and MC Methods.** The convergence of MC and SC methods is tested by changing the size  $N$  of input sets also referred to as the Design of Experiment (DoE), where  $N = N_{MC}$  or  $N_{SC}$  for MC or SC method, respectively. Input simulation points are  $n$ -dimensional, e.g.,  $n = 6$  for full stochastic model and  $n = 1$  for one-dimensional stochastic model (univariate cases, i.e., the case when only one input parameter is random variable, and the rest are fixed at their respective average values).

Generally, the convergence of MC method depends only on  $N_{MC}$ , hence, DoEs for 6-dimensional and 1-dimensional stochastic models are with  $N_{MC} = 1.000, 10.000,$  and  $100.000$

simulation points in either case. It is assumed that stochastic mean and standard deviation obtained with  $N_{MC} = 100.000$  represent the true stochastic mean and standard deviation values.

On the other hand,  $N_{SC}$  depends both on dimensionality of the stochastic model and on number of points used for the numerical evaluation of the integral in equation (13). For 1-dimensional cases, the size of the DoE is set to 3, 5, 9, and 17 while for 6-dimensional stochastic model  $N_{SC} = 13, 85,$  and  $1457$ .

The convergence of MC and SC methods for 6-dimensional stochastic model is depicted in Figure 5, Figure 3 and 4 for both Fresnel reflection coefficient and Modified Image Theory approaches.

The convergence of MC method is rather satisfactory for computation of both stochastic mean and standard deviation in FRM and MIT models. However, SC method exhibits problems with convergence when computing the standard deviation of the electric field. The convergence is not accomplished for chosen DoEs. To improve the convergence,  $N_{SC}$  should be increased over 1457 which is no longer efficient with respect to MC which accomplishes convergence for  $N_{MC} = 1.000$ .

It is worth noting that SC convergence for standard deviation tends to improve when observation points are moved further away from the antenna. Namely, at distances over appx. 650 m away from the antenna, standard deviation obtained by SC method converges. However, these values overestimate or underestimate the true standard deviation with relative absolute error up to 22% for FRM model. In case of MIT model, the standard deviation obtained by SC method overestimates the true standard deviation up to 35%.

In order to investigate the reason for poor SC convergence, the convergence of standard deviation for six 1-dimensional cases is depicted in Figure 5 for FRM model. The MC method, again, shows good results at  $N_{MC} = 1000$ .

The standard deviation of electric field obtained by SC method for univariate cases when  $A_h, \sigma_1, \epsilon_{r2}$  and  $\sigma_2$  (antenna height, 1<sup>st</sup> layer conductivity, 2<sup>nd</sup> layer relative permittivity, and 2<sup>nd</sup> layer conductivity, respectively), are set as random variables one-at-a-time, exhibits good convergence even at lowest number of simulation points ( $N_{SC} = 3$ ). It can be concluded that the variation of these input parameters does not have any effect on the convergence of the total standard deviation, i.e., the standard deviation of 6-dimensional stochastic model.

When only third input parameter, i.e., the relative permittivity of 1<sup>st</sup> ground layer ( $\epsilon_{r1}$ ), is RV, the SC method converges to correct value at  $N_{SC} = 9$  which is relatively small number of simulation points. However, if 9 collocation points are used in each of 6 dimensions, then for 6-dimensional stochastic model, the total  $N_{SC}$  is 1457 which is more than  $N_{MC} = 1000$ .

Finally, when the 1<sup>st</sup> layer thickness ( $d_1$ ) is considered as the only RV in the input parameter set, the convergence is accomplished at  $N_{SC} = 17$  collocation points which is efficient just for univariate stochastic model.

The same analysis is valid for MIT model of field computation, too.

TABLE 1: Input parameters.

		[min, avg, max]	Avg $\pm$ CV (%)
Antenna height	$A_h$ (m)	[19, 20, 21]	20 m $\pm$ 5%
1 <sup>st</sup> layer thickness	$d_1$ (cm)	[15, 20, 25]	20 cm $\pm$ 25%
1 <sup>st</sup> layer relative permittivity	$\epsilon_{r1}$	[12, 15, 18]	15 $\pm$ 20%
1 <sup>st</sup> layer electric conductivity	$\sigma_1$ (mS/m)	[0.8, 1, 1.2]	1 mS/m $\pm$ 20%
2 <sup>nd</sup> layer relative permittivity	$\epsilon_{r2}$	[3.2, 4, 4.8]	4 $\pm$ 20%
2 <sup>nd</sup> layer electric conductivity	$\sigma_2$ (S/m)	[0.04, 0.05, 0.06]	50 mS/m $\pm$ 20%

min and max stand for the lower and upper boundaries, respectively, of the uniform distribution; avg and CV stand for the average value and coefficient of variation.

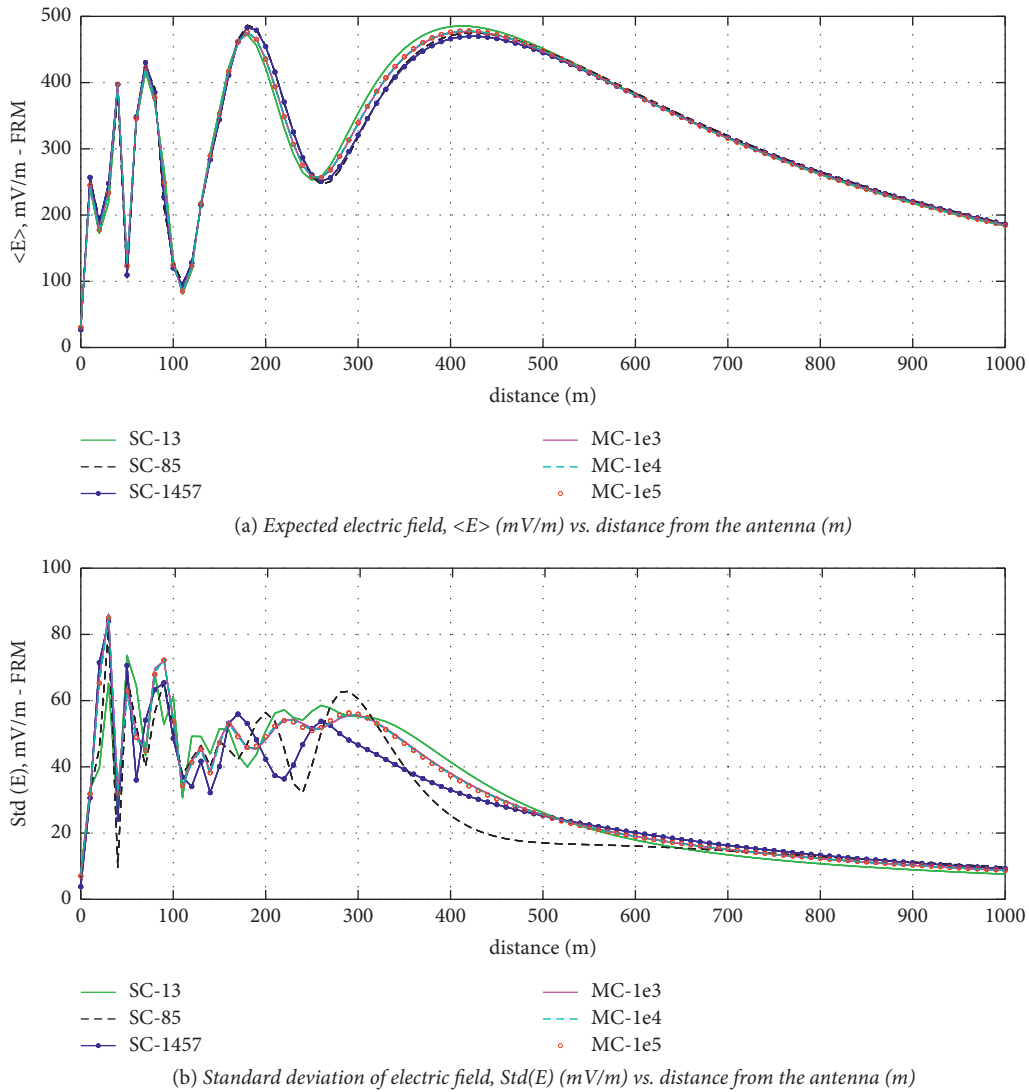


FIGURE 3: The convergence of MC and SC methods in computation of expected electric field (a) and its standard deviation (b). The deterministic computation is based on Fresnel reflection coefficient approach (FRM).

5.2. Uncertainty Quantification of Electric Field Strength.

The electric field expectation and confidence intervals are computed by using MC method only, since SC method did not show high reliability in computation of standard deviation at low  $N_{SC}$  level. As mentioned above, the size of  $N_{MC}$  (1,000, 10,000, or 100,000) has no significant impact on

electric field expectation and standard deviation, respectively. Therefore, the uncertainty quantification is carried out with 1,000 sets of input parameters (Figures 6 and 7).

Deterministic values of electric field obtained by using FRM and MIT models differ to a certain extent, and this has been documented elsewhere, e.g., in [22, 23]. Since input

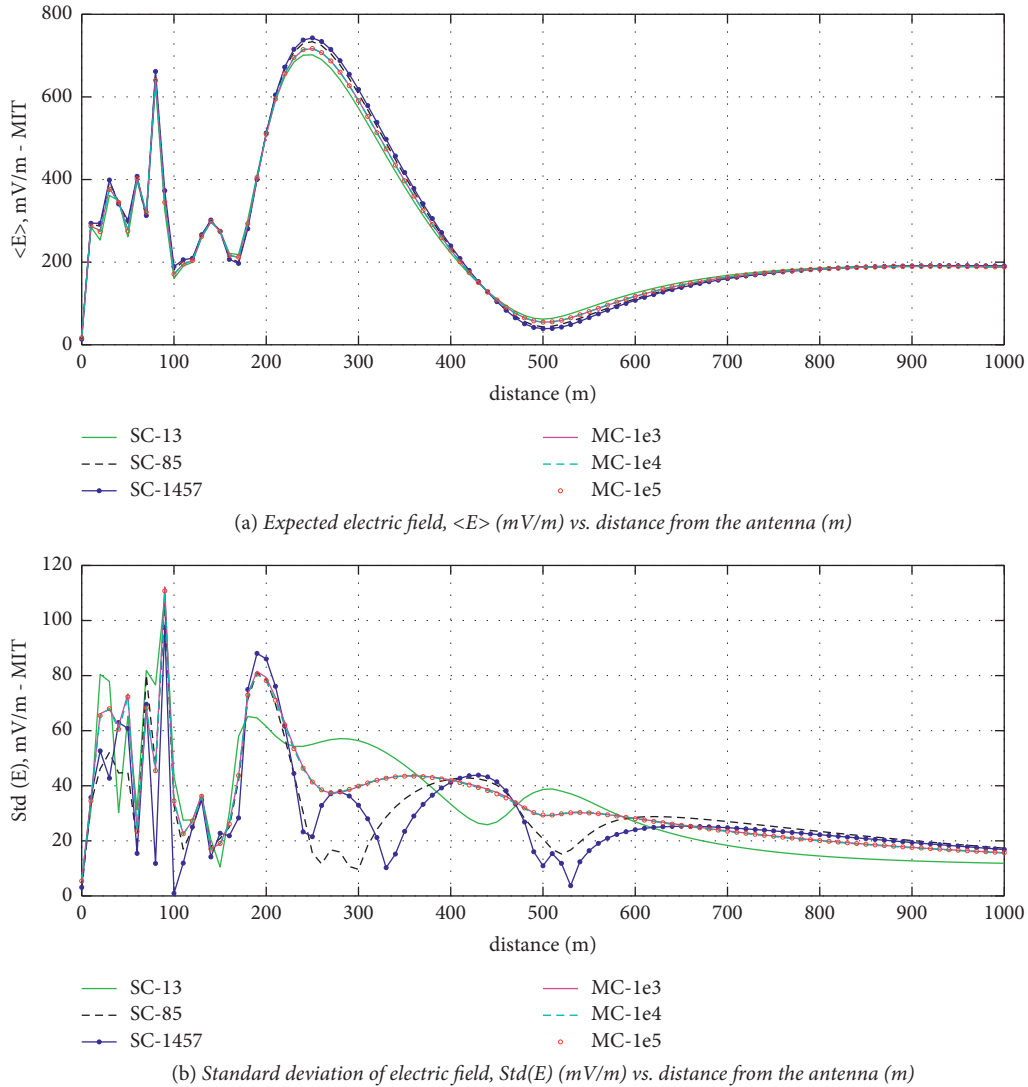


FIGURE 4: The convergence of MC and SC methods in computation of expected electric field (a) and its standard deviation (b). The deterministic computation is based on Modified Image Theory approach (MIT).

parameters are modelled as RVs, it is of particular interest to investigate to which extent the confidence intervals (CI) obtained with FRM and MIT models overlap.

At distances shorter than 200 m, CI stemming from FRM and MIT approaches overlap. However, as the distance increases up to 750–800 m, confidence intervals differ at some observation points. Finally, at distances above 750–800 m, confidence intervals overlap again, and electric field expectations obtained by FRM and MIT models approach to each other. It is worth noting that the width of CI does not change to a greater extent as the distance increases.

Although the width of CI is practically constant, the expectation for field level fluctuates between 100 mV/m and 500 mV/m for FRM model and 50 mV/m and 700 mV/m for MIT model. Therefore, it is useful to compare the half-width of the confidence intervals with the expected electric field value at each observation point of interest by computing the following metric:

$$\kappa_{\alpha}(\%) = \frac{\alpha \cdot Std(E)}{E} \cdot 100, \alpha = 2 \text{ or } 3. \quad (19)$$

The results for  $\kappa$  are depicted in Figure 8.

Considering the confidence interval with 95% level of precision i.e.,  $\pm 2Std(E)$ , the largest variability does not exceed 107% of expected field value no matter which calculation model is used, MIT or FRM, respectively.

In case of confidence level of 99.7% i.e.,  $\pm 3Std(E)$ , the variability does not exceed the 152% of expected field value, no matter which calculation model is used, MIT or FRM, respectively.

**5.3. Sensitivity Analysis of Input Parameters.** Using the Global Sensitivity Analysis Toolbox (GSAT) [31], ANOVA SA approach described in Section 4 is carried out. The SA analysis for FRM and MIT models is depicted in Figures 9 and 10, respectively.



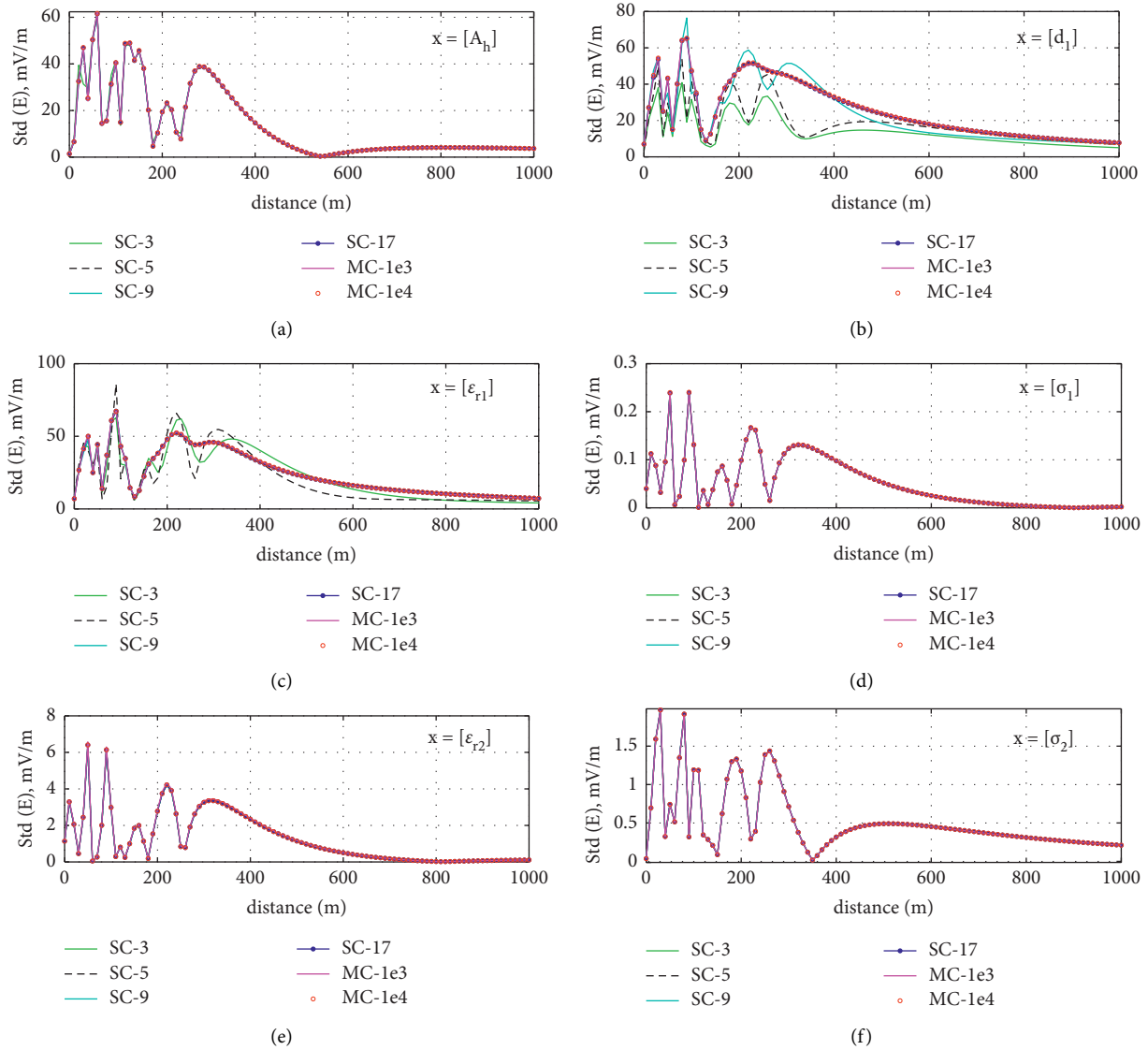


FIGURE 5: The convergence of MC and SC methods in computation of electric field standard deviation, Std(E) (V/m). Six univariate cases are depicted, i.e., when only one input parameter is random at a time: (a) antenna height– $A_h$ , (b) 1st layer thickness– $d_1$ , (c) 1st layer relative permittivity– $\epsilon_{r1}$ , (d) 1st layer conductivity– $\sigma_1$ , (e) 2nd layer relative permittivity– $\epsilon_{r2}$ , and (f) 2nd layer conductivity– $\sigma_2$ . The deterministic computation is based on Fresnel reflection coefficient approach (FRM).

Starting with the first-order sensitivity index for FRM model ( $S_1$  in Figure 9(a)), it can be concluded that antenna height is the parameter with the highest impact on the output electric field. This is a valid conclusion for most of the observation points of interest.

However, the summation of all indices (first, second, and higher, up to  $n-1=5$ ) must be equal to 1 or 100%, i.e.,  $\sum_i S_1 + \sum_j S_2 + \sum_k S_3 + \sum_{ii} S_4 + \sum_{jj} S_5 = 1$ . Since the summation of all six  $S_1$  indices is not equal to 1 at each observation point of interest, the first-order sensitivity indices are clearly not sufficient for SA. This indicates that the higher order sensitivity indices do have significant values, higher than 0. Therefore, a total effect sensitivity index (ST) is computed for each input parameter (Figure 9(b)).

Starting with the first input parameter, the 1<sup>st</sup> order sensitivity index  $S_1(X_1)$  and total effect sensitivity index  $ST(X_1)$  have approximately the same value. This means that the antenna height does not have any interactions with other input parameters which may have some significant impact on the output field variance.

On the other hand, the values of total effect sensitivity index  $ST(X_2)$  and  $ST(X_3)$  differ greatly from the corresponding first-order sensitivity indices  $S_1(X_2)$  and  $S_1(X_3)$ , respectively. Moreover,  $ST(X_2)$  and  $ST(X_3)$  have appx. the same value. This leads to the conclusion that, although input parameters 2 and 3 (1<sup>st</sup> layer thickness and 1<sup>st</sup> layer permittivity) do not impact directly on the output variance, their interactions in the mathematical model cause the

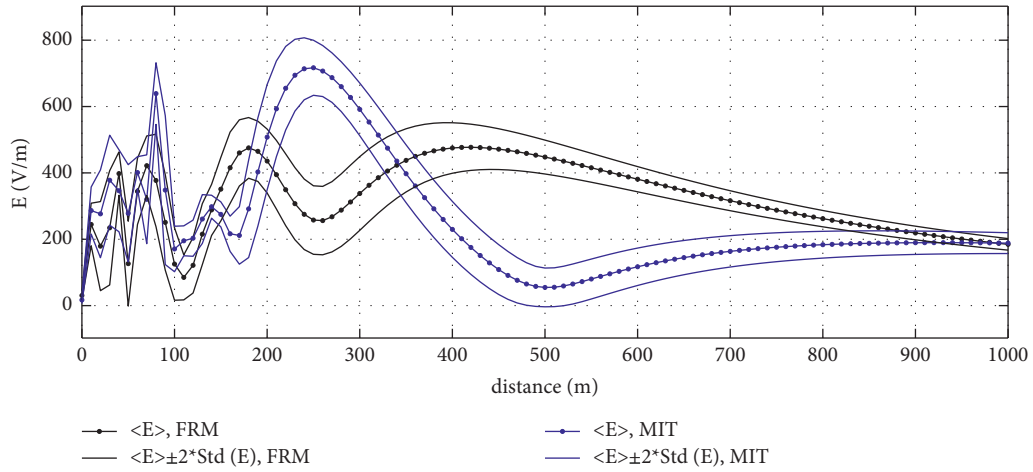


FIGURE 6: The expected electric field,  $\langle E \rangle$ , and confidence intervals (CI) with FRM and MIT calculation model, respectively. Confidence intervals of output electric field computed as  $CI = \langle E \rangle \pm 2\text{Std}(E)$ .

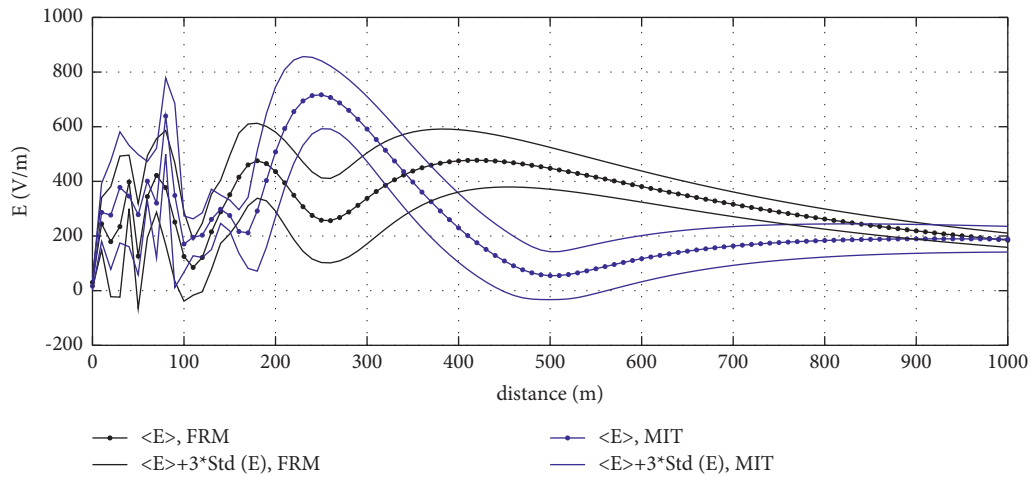


FIGURE 7: The expected electric field,  $\langle E \rangle$ , and confidence intervals (CI) with FRM and MIT calculation model, respectively. Confidence intervals of output electric field computed as  $CI = \langle E \rangle \pm 3\text{Std}(E)$ .

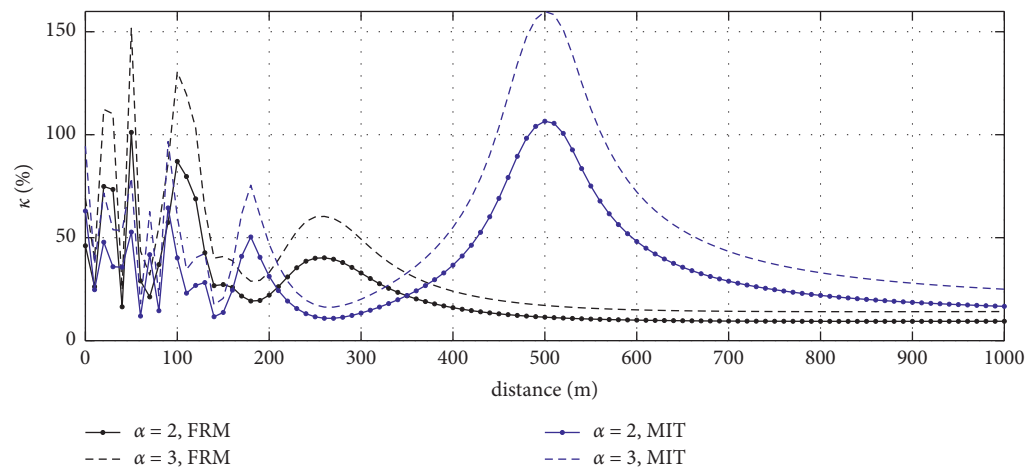


FIGURE 8: Ratio of  $\pm 2\text{Std}(E)$  and  $\pm 3\text{Std}(E)$  with respect to the expected electric field,  $\langle E \rangle$ .

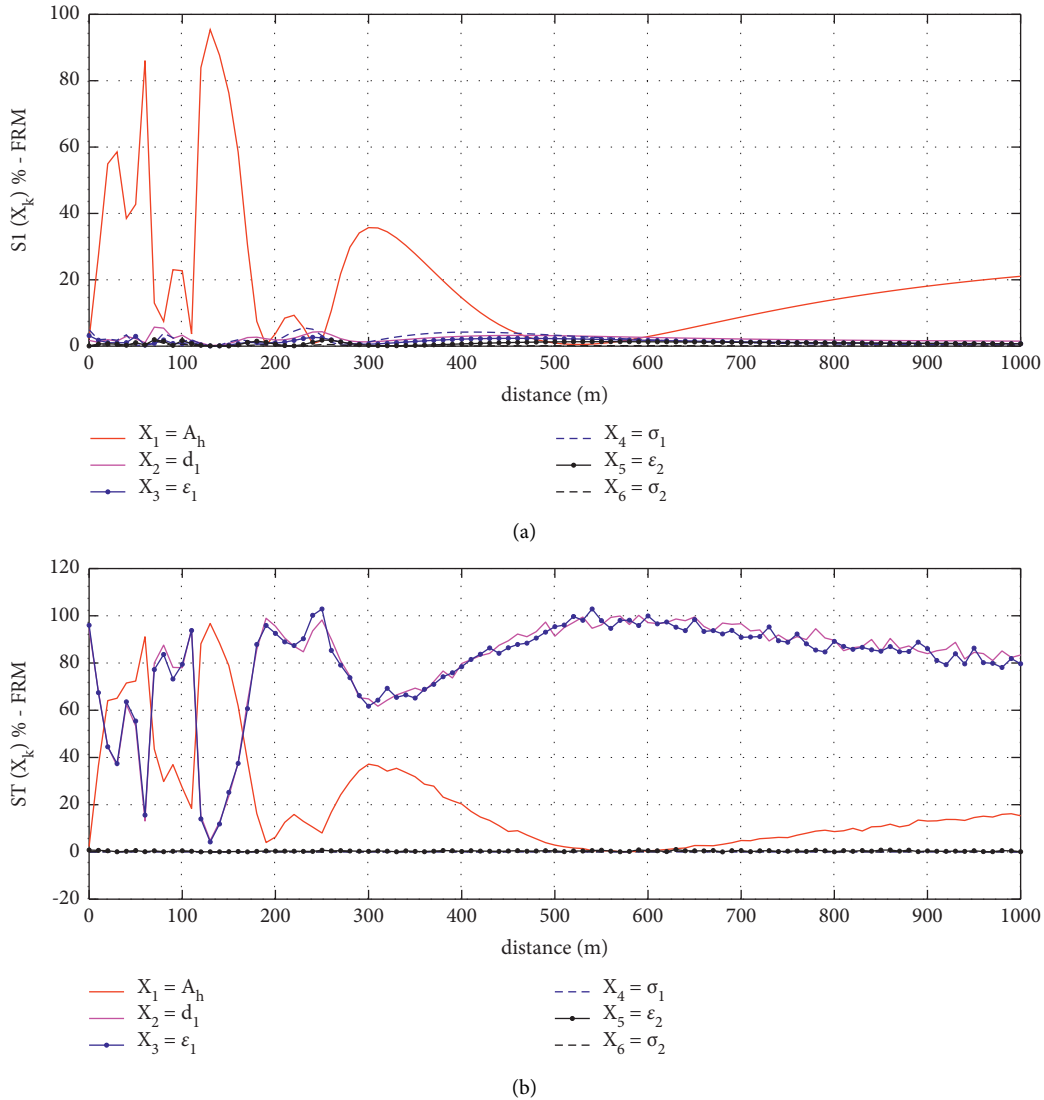


FIGURE 9: 1st order sensitivity index (a) and total effect sensitivity index (b) for six input parameters. The computation model is based on FRM approach.

variability in the electric field output, therefore, their uncertainties should not be neglected.

The rest of the input parameters do not impact the output variance significantly. Their impact is less than 5% for all observation points of interest.

It is worth noting that Figure 5, besides the convergence of the SC and MC methods, contains the information about the OAT sensitivity analysis for FRM model. Comparing the standard deviations in plots a) – f) in Figure 5, it can be concluded that antenna height, 1<sup>st</sup> layer depth, and 1<sup>st</sup> layer relative permittivity are the first three most significant input parameters. Their order of significance depends on the observation point of interest. However, OAT cannot give the information if the impact originates from the input parameter sole variability or its interaction with another parameter.

The SA analysis with both ST and S1 sensitivity indices for MIT model is depicted in Figure 10.

The conclusions are the same as for FRM model: the variability of antenna height has the highest impact on the total variance at certain observation points of interest, while at other points the mutual interaction between the 1<sup>st</sup> layer depth and 1<sup>st</sup> layer relative permittivity prevails. It is interesting how the impact of the antenna height,  $X_1$  alone and the impact originating from the interaction of the other two parameters,  $[X_2&X_3]$ , changes from one observation point to another. For smaller distances, the impact switches from  $X_1$  to  $[X_2&X_3]$ , but after approximately 500m the interaction between  $X_2$  and  $X_3$  is completely dominant and antenna height loses its significance.

## 6. Discussion

Stochastic-deterministic approach to computation of radiated electric field level is presented in this paper. The inherent uncertain nature of some input parameters is

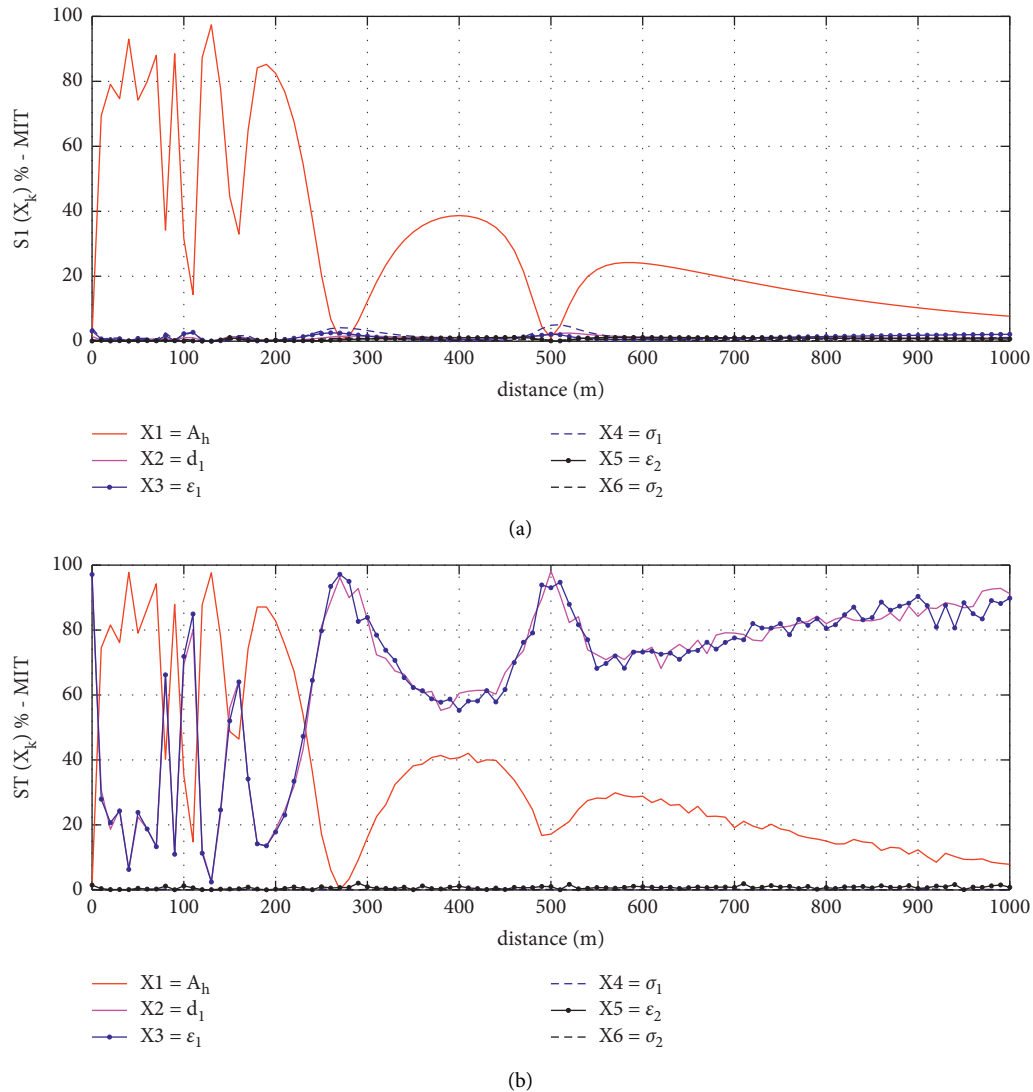


FIGURE 10: 1<sup>st</sup> order sensitivity index (a) and total effect sensitivity index (b) for six input parameters. The computation model is based on MIT approach.

considered, and the range of possible field levels is computed at different distances from the antenna. Two approaches for uncertainty quantification (UQ) of the output field are presented: one is a traditional and robust Monte Carlo method, and the second method is Stochastic Collocation stemming from spectral numerical approaches. Finally, the framework for sensitivity analysis based on the analysis of variance (ANOVA) is outlined, independent of the chosen UQ method. Beside the proposed methodology for the stochastic-deterministic modelling of field levels radiated by BSA, the results of stochastic and sensitivity analysis are depicted. The stochastic analysis is carried out as a pre- and postprocessing of the input and output samples, while a single electric field value is computed in a deterministic way.

Firstly, deterministic techniques for BSA far field computation include numerous formulations, e.g., a free space approximation, antenna above the perfectly conducting ground, ground modelled as a homogeneous half-space with finite conductivity, multilayered ground. Each

variant accounts for new parameters from the real exposure scenarios and thus different approximations. Interestingly, many international standards deal with free space approximation, or perfectly conducting (PEC) ground. Homogeneous half-space scenarios have still not been investigated to a greater extent, particularly layered half-space. Since the main feature of the paper is stochastic-deterministic analysis of the problem, the use of relatively simple analytical model of base station radiation in a deterministic sense is a plausible choice as an opener to the subject, while more sophisticated deterministic models are likely to be dealt with in future work.

Furthermore, to consider the presence of a two-layered ground, the deterministic model in this paper takes advantage of Fresnel reflection coefficient and another, simplified reflection coefficient stemming from the Modified Image Theory, FRM, and MIT models, respectively [19, 23]. Namely, the idea is to recognize the exposure scenarios in which the simplified reflection coefficient may be used

instead of FRM one. Results show that the two approaches may be considered as equivalent for distant observation points while for closer observation points there is a certain under- or overestimation of the referent values.

As for the stochastic part of the problem, the input parameters exhibiting the random nature are dictated by the deterministic model itself. Nevertheless, their choice is also based on experience from realistic scenarios in engineering practice. Hence, input parameters exhibiting random nature and their corresponding coefficients of variation (CV) are chosen as follows: antenna height (CV = 5%), thickness of the surface ground layer (CV = 25%), surface ground layer relative electric permittivity (20%) and conductivity (20%), 2<sup>nd</sup> ground layer relative electric permittivity (20%) and conductivity (20%). It is worth noting that additional input parameters whose inherent uncertainty may affect radiated field levels (e.g., the roughness of the surface layer) can be considered in future investigation, however, the deterministic model should be chosen accordingly. Once the random input parameters are determined, the proposed stochastic-deterministic model is used to explore of a complete input parameter space instead of only its average or extreme values (i.e., average, minimal, or maximal input parameter values) thus leading not only to computation of radiated field expectation but also its confidence intervals.

The results of stochastic-deterministic analysis presented in this paper can be summed up as follows. Based on the results for radiated field uncertainty quantification, it can be stated with 95% level of precision that the largest variability in field level does not exceed 107% of its expectation, no matter which calculation model is used, MIT or FRM. If the level of precision is increased to 99.7% the variability does not exceed the 152% of expected field value, no matter which deterministic calculation model is used. In addition, even with uncertainty in input parameters, the expected field levels, and corresponding confidence intervals from FRM and MIT model do not overlap at observation points positioned close to antenna, therefore, the stochastic-deterministic models cannot be considered as equivalent. As distance increases, however, expectations tend to approach to each other, therefore, the confidence intervals start to overlap, and the computation approaches can be considered as equivalent.

Furthermore, sensitivity analysis reveals that three input parameters have significant impact on the output CI width while other three can be neglected. Namely, up to approximately 180 m for FRM model and 275 m for MIT model, the domination changes between two groups of parameters, antenna height being the 1<sup>st</sup> group while the 2<sup>nd</sup> group pertains to 1<sup>st</sup> layer thickness and its permittivity. Namely, the variability of 5% in antenna height changes the relative elevation of the antenna with respect to the point of interest. This can lead to significant changes in correlation to vertical antenna diagram which results in enlarged changes in electric field strength. However, at distances higher than 180 m and 275 m for FRM and MIT models, respectively, the 2<sup>nd</sup> group becomes dominant and antenna height has no longer significant impact on field variance.

Hence, regardless of the model (FRM and MIT), the stochastic-deterministic approach to computing the field

level at larger distances is practically 2-dimensional as there are only 2 relevant RVs in the input parameter space, the first layer relative electric permittivity and its thickness. Uncertainty in the 1<sup>st</sup> layer thickness and relative electric permittivity may result in unreliable levels of electric field strength which imposes problems to reliable compliance to reference levels defined by standards such as ICNIRP and IEEE.

Additionally, beside the results obtained by uncertainty quantification and sensitivity analysis procedures, the aim of the paper was also to investigate if Monte Carlo method can be replaced by Stochastic Collocation. The convergence of the SC method is satisfactory for computation of electric field expectation both for 6-dimensional and 1-dimensional cases. However, the method does not converge fast enough for variance computation, and consequently, standard deviation computation. Namely, to accomplish the convergence, the number of simulations points in 6-dimensional case has to be larger than 1.000, which is, on the other hand, enough for MC method also. Nonetheless, given that SC method generally offers lower computation effort than MC method, this is worth of future investigation. However, beside the SC method, other methods exist whose efficiency is worth investigating which is part of a future work. On the other hand, future work is likely to deal with more sophisticated deterministic models, too.

## 7. Concluding Remarks

Stochastic-deterministic approach to computation of electric field radiated by a base station antenna is presented. The stochastic approaches in the form of two methods, Monte Carlo (MC) and Stochastic Collocation (SC), are used as a wrapper around the deterministic field computation. Within the framework of deterministic analysis, the total electric field above a two-layered lossy ground is obtained by considering the incident and reflected ray in the far field zone. The influence of nonhomogeneous lower medium is taken into account via two approaches: Fresnel plane wave reflection coefficient (FRM) and simplified reflection coefficient stemming from Modified Image Theory (MIT). Given that antenna height, relative permittivity, and conductivity of each ground layer, as well as the thickness of the upper ground layer are considered as input parameters with inherent uncertainty, the electric field mean along with confidence intervals is computed for observation points up to 1 km away from the antenna.

The results of the stochastic analysis can be summed up as follows:

- (i) The convergence of MC method is rather satisfactory for computation of both stochastic mean and confidence intervals for FRM and MIT models, requiring 1.000 input samples. On the other hand, to accomplish a satisfactory convergence, SC method requires more than 1.000 input samples for computation of confidence intervals which is no longer efficient with respect to MC method. SC method is generally more efficient in case of small

number of input random variables. However, this is not the case in the presented 6-dimensional stochastic-deterministic model. Therefore, the stochastic analysis of the electric field is carried out by using MC method with 1.000 samples for both FRM and MIT deterministic models.

- (ii) The width of confidence intervals does not change with distance from the antenna. As the distance increases the confidence intervals from FRM and MIT approaches start to overlap, therefore the two approaches can be considered as equivalent.
- (iii) Furthermore, sensitivity analysis of input parameters has been carried out via one-at-a-time (OAT) and analysis of variance (ANOVA) approaches. The impact of antenna height is dominant at some observation points up to 250 m distance. Although the 1<sup>st</sup> layer thickness and 1<sup>st</sup> layer relative permittivity do not impact directly on the output variance, their interactions in the mathematical model have impact on the variability in the electric field output and this influence is dominant at distances higher than 250 m. The impact of other input parameters is less than 5%. These results are valid for both FRM and MIT models.

## Data Availability

Authors can make data available on request. In this case, contact the corresponding author: ansusnja@fesb.hr.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

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