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Research Article

Fully Bipolar Single-Valued Neutrosophic Transportation Problems

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Transportation problem (TP) has its uses in real life because it has versatile applications. Real-life problems are often uncertain due to which it is difficult to find the accurate cost. The fuzzy set and intuitionistic fuzzy set are useful for handling the uncertainty, but these also have some limitations. For that reason, in this study, we worked on another set of values called bipolar single-valued neutrosophic set (BSNS) which is the generalization of crisp sets, fuzzy sets, and intuitionistic fuzzy sets to handle the uncertain, unpredictable, and insufficient information in real-life problems. In this study, we develop a new technique for solving transportation problems based on bipolar single-valued neutrosophic sets having nonnegative triangular bipolar single-valued neutrosophic numbers (TBSNNs). A score function is used to transform bipolar single-valued neutrosophic numbers (BSNNs) into crisp numbers. We compare our proposed model with fuzzy transportation and intuitionistic fuzzy transportation models and proved that bipolar single-valued neutrosophic transportation model is more admirable than the existing models. Furthermore, we apply the proposed technique to fully solve the bipolar single-valued neutrosophic transportation (FBSNT) model.

1. Introduction

Nowadays, there is great competition among organizations to find better methods to create and give better services to their customers. They need a cost-effective method to send the products to their customers. This is a more challenging task. The models for transportation provide an effective framework to encounter this challenge. They guarantee the effective movement of raw materials and find products.

1.1. Motivation. In the mathematical programming problem, the linear programming problem is very well-known and it has a wide scope to cover various fields. Among these, transportation problem is the commonly used field. The TP has its own importance in the field of logistics, supply chain management, supplier selection problem, etc. The TPs have great importance in many real-life applications. It works to maintain the supply from source to destination. It is generally considered that transversal costs of supply and demand are expressed in terms of crisp numbers. These values are often not precise. As a result, various researchers have been working on different TPs in fuzzy [1–3] and intuitionistic fuzzy [4, 5] environments, respectively. We proposed a new technique to solve TPs in the bipolar single-valued neutrosophic environment, which is a more generalized form.

1.2. Literature Review. Zadeh [6, 7] proposed the concept of fuzzy sets and fuzzy numbers to reduce uncertainty and incomplete information. Atanassove [8] gave the idea of intuitionistic fuzzy sets (IFSs), which is a generalization of fuzzy sets. In an IFS, we consider the problem in both angles' positive side and negative side to handle uncertainty. In an IFS, truth membership and falsity membership are independent, while indeterminacy-membership depends on their sum. Smarandache [9] proposed the notion of neutrosophic set theory. Smarandache [9] and Wang et al. [10] defined single-valued neutrosophic sets (SNSs), which are an extension of FSs and IFSs. In an SNS all membership functions truth, falsity, and indeterminacy are independent.

Deli et al. [11] suggested the idea of bipolar single-valued neutrosophic set, which is the generalization of single-valued neutrosophic sets [10].

The TPs are related to the transportation of raw material from different sources to different destinations in such a way that the total transportation cost is minimized. Hitchcock [12] was the first to develop a basic transportation problem. The transportation problem can be elaborated as a standard linear programming problem. This can be solved by the simplex method. It was found that the simplex method when applied to the transportation problem becomes more effective when evaluating the simplex method information. Basirzadeh [1] proposed a method to solve fuzzy transportation problems. Ladji et al. [13] proposed a two-step method for solving fuzzy transportation problems where all of the parameters are represented by triangular fuzzy numbers i.e., two interval transportation problems. Pratihar et al. [14] have modified the classical vogel's approximation method for solving the fuzzy transportation problem. They also worked on the interval type 2 fuzzy set and used it in a fuzzy transportation problem to represent the transportation cost, demand, and supply. Cam et al. [15] worked on the formulation of a linear programming model for the vehicle routing problem to minimize idle time. In fuzzy linear programming problems (FLPPs), many scholars have contributed. By using multiobjective function, Zimmerman [16] gave a technique to solve LP problem. After that, to solve the transportation problems, Zimmerman's fuzzy linear programming has developed into several fuzzy optimization methods. In a fuzzy environment, Bellman and Zadeh [6] introduced the different concepts of decision making. Lotfi et al. [17] considered FFLP problems in which all parameters and variables are triangular fuzzy numbers. Allahviranloo et al. [18] solved FFLP problem by using a kind of defuzzification approach. Behera et al. [19] suggested two new methods to solve fuzzy linear programming (FLP) problems. They solved two types of problems with two different methods. Kaur and Kumar [20] gave an introduction to FLP problems. Kaur and Kumar [21] suggested an approach to find the exact fuzzy optimal solution to FFLP problems by using unrestricted fuzzy variables. Kaur and Kumar [22] proposed a new method for finding the fuzzy optimal solution to fuzzy transportation problems in which the transportation cost are represented by generalized fuzzy numbers. Najafi and Edalatpanah [23] suggested a better technique to solve the FFLP problem than Kumar et al. [21].

Intuitionistic fuzzy linear programming problem is an extension of the fuzzy linear programming problem. Many researchers have worked on different methods to solve LP problems in an intuitionistic fuzzy environment by using intuitionistic fuzzy numbers (IFNs) and LR-type IFNs. Singh and Yadav [4, 5] suggested two different techniques to solve an intuitionistic fuzzy transportation problem (IFTPs) by using triangular intuitionistic fuzzy numbers (TIFNs). Abhishekh and Nishad [24] suggested a novel ranking function for finding an optimal solution to fully LR-intuitionistic fuzzy transportation problem. In an intuitionistic fuzzy environment, Edalatpanah [25] designed a model of data envelopment analysis with triangular intuitionistic fuzzy numbers (TIFNs) and established a strategy to solve it. Kabiraj et al. [26] solved IFLP problems by using a method based on a method suggested by Zimmerman [16]. Malathi and Umadevi [27] worked on IFLP problems in an intuitionistic fuzzy environment. Pythagorean fuzzy linear programming is an extension of intuitionistic fuzzy linear programming. Akram et al. [28, 29] suggested a technique to solve Pythagorean fuzzy linear programming problems by using Pythagorean fuzzy numbers and LR-type Pythagorean fuzzy numbers. Akram et al. [30] used two different techniques to solve Pythagorean fuzzy linear programming problems having mixed constraints.

In daily life routine, we meet a variety of situations depending on multiple factors like uncertainty in judgements. Often it becomes difficult to get relevant data for cost parameters. The data of this type cannot always be represented by random variables obtained from the probability distribution. These data may be represented by bipolar single-valued neutrosophic numbers. So, a bipolar single-valued neutrosophic method to make the decision is needed. Abdel-Basset et al. [31] suggested a technique to solve the fully neutrosophic linear programming (FNLP) problems. Hussian et al. [32] suggested a linear programming model based on neutrosophic environment. Bera and Mahapatra [33] developed the Big-M simplex method to solve the neutrosophic linear programming (NLP) problem. Das and Chakraborty [34] considered a pentagonal NLP problem to solve it. Das and Dash [35] solved NLP problems with mixed constraints. Edalatpanah [36] presented a direct algorithm to solve the linear programming problems. Khalifa et al. [37] solved the NLP problem with single-valued trapezoidal neutrosophic numbers. Ahmed [38] suggested a technique to solve LR-type single-valued neutrosophic linear programming problems by using unrestricted LR-type single-valued neutrosophic numbers. He proposed the ranking function to transform LR-type single-valued neutrosophic problems into crisp problems. Deli et al. [11] gave the idea of bipolar single-valued neutrosophic set. Akram et al. [39] suggested a technique to solve LR-bipolar fuzzy linear system. Mehmood et al. [40, 41] defined LR-type bipolar fuzzy numbers and their arithmetic operations. They also introduced the ranking for LR-type bipolar fuzzy numbers and solved numerical examples. Ahmed et al. [42] suggested a technique to solve bipolar single-valued neutrosophic linear programming problems in which all the coefficients, variables, and right-hand side are presented by bipolar single-valued neutrosophic numbers. Kumar [43] presented the cut of single-valued pentagonal neutrosophic numbers and also introduced the arithmetic operation of single-valued pentagonal neutrosophic numbers. By using two different objective functions, Singh et al. [44] formulated the journey of a vaccine from its manufacture to its delivery using bilevel transportation problems in a neutrosophic environment. Veeranani et al. [45] solved the multiobjective fractional transportation problem by using the neutrosophic goal programming approach [46–48].

1.3. Our Contribution. Considering all the available data, there are no methods in literature for TPs under the bipolar single-valued neutrosophic environment. So, there is a need to introduce a technique for BSNTPs. As a result of our facts, there are no optimization models available for TPs under bipolar single-valued neutrosophic environment. This has urged us to develop a new technique to solve TP with the bipolar single-valued neutrosophic environment, which is solved for the first time with the proposed technique. Bipolar single-valued neutrosophic set theory is a well-known technique to deal with uncertainty in the optimization problem. This study has been categorized as follows: In Section 2, basic concepts of BSNS, BSNN, TBSNNs, and their arithmetic operations are discussed. In Section 3, methodology for solving FBSNT problems are explained. In Section 4, a mathematical transportation model is solved. In Section 5, comparative analysis is discussed. In Section 6, the advantages of the proposed method are discussed. In Section 7, the limitations of the proposed method are given, and the conclusion is given in Section 8.

2. Preliminaries

Definition 1 (see [11]). Let Y be a nonempty set. A bipolar single-valued neutrosophic set (BSNS) $\widetilde{\lambda}$ in Y is an object having the form

$$\widetilde{\lambda} = \left\{ \prec y, T^{p}_{\widetilde{\lambda}}(y), I^{p}_{\widetilde{\lambda}}(y), F^{p}_{\widetilde{\lambda}}(y), T^{n}_{\widetilde{\lambda}}(y), I^{n}_{\widetilde{\lambda}}(y), F^{n}_{\widetilde{\lambda}}(y) \succ :\ y \in Y \right\}, \tag{1}$$

where $T^{p}_{\widetilde{\lambda}}(y), I^{p}_{\widetilde{\lambda}}(y), F^{p}_{\widetilde{\lambda}}(y)\colon Y \longrightarrow [0,1]$ and $T^{n}_{\widetilde{\lambda}}(y), I^{n}_{\widetilde{\lambda}}(y), F^{n}_{\widetilde{\lambda}}(y)\colon Y \longrightarrow [-1,0]$. The positive membership degree $T^{p}_{\widetilde{\lambda}}(y), I^{p}_{\widetilde{\lambda}}(y), F^{p}_{\widetilde{\lambda}}(y)$ denotes the truth membership, indeterminate membership, and falsity membership of an element $y \in Y$ corresponding to a bipolar neutrosophic set $\widetilde{\lambda}$ similarly negative membership degree $T^{n}_{\widetilde{\lambda}}(y), I^{n}_{\widetilde{\lambda}}(y), F^{n}_{\widetilde{\lambda}}(y)$ denotes the truth membership, indeterminate membership, and falsity membership of an element $y \in Y$ to some implicit counter-property corresponding to a bipolar neutrosophic set $\widetilde{\lambda}$.

Definition 2 (see [33]). A BSNN on $\mathbb{R}$ is a BSN set such that

$$\begin{aligned}\widetilde{\lambda} = \prec &\left([\mu_1, \nu_1, \pi_1, \theta_1]; \chi_p\right), \left([\mu_2, \nu_2, \pi_2, \theta_2]; \beta_p\right), \left([\mu_3, \nu_3, \pi_3, \theta_3]; \zeta_p\right), \left([\gamma_1, \eta_1, \iota_1, \kappa_1]; \alpha_n\right),\\ &\left([\gamma_2, \eta_2, \iota_2, \kappa_2]; \varphi_n\right), \left([\gamma_3, \eta_3, \iota_3, \kappa_3]; \nu_n\right)\succ\end{aligned} \tag{2}$$

where $\chi_p, \beta_p, \zeta_p \in [0,1]$ and $\alpha_n, \varphi_n, \nu_n \in [-1,0] \subset \mathbb{R}$

The truth membership values are given as

$$T^{p}_{\widetilde{\lambda}}(y) = \begin{cases} S^{L}_{T}(y), & \mu_1 \le y \le \nu_1,\\ \chi_p, & \nu_1 \le y \le \pi_1,\\ S^{R}_{T}(y), & \pi_1 \le y \le \theta_1,\\ 0, & \text{otherwise}, \end{cases} \quad T^{n}_{\widetilde{\lambda}}(y) = \begin{cases} U^{L}_{T}(y), & \gamma_1 \le y \le \eta_1,\\ \alpha_n, & \eta_1 \le y \le \iota_1,\\ U^{R}_{T}(y), & \iota_1 \le y \le \kappa_1,\\ 0, & \text{otherwise}. \end{cases} \tag{3}$$

$S^{L}_{T}(y)$ and $U^{R}_{T}(y)$ are continuous and increasing functions satisfying the following conditions: $S^{L}_{T}(\mu_1) = 0, S^{L}_{T}(\nu_1) = \chi_p$, and $U^{R}_{T}(\gamma_1) = \alpha_n, U^{R}_{T}(\eta_1) = 0$, while $S^{R}_{T}(y)$ and $U^{L}_{T}(v)$ are continuous and decreasing functions and satisfying the following condition:

$$S^{R}_{T}(\pi_1) = \chi_p, S^{R}_{T}(\theta_1) = 0, U^{L}_{T}(\iota_1) = 0, U^{L}_{T}(\kappa_1) = \alpha_n, \tag{4}$$

where $\chi_p \in [0,1], \alpha_n \in [-1,0]$.

The indeterminacy-membership functions are given as

$$I^{p}_{\widetilde{\lambda}}(y) = \begin{cases} S^{L}_{I}(y), & \mu_2 \le y \le \nu_2,\\ \beta_p, & \nu_2 \le y \le \pi_2,\\ S^{R}_{I}(y), & \pi_2 \le y \le \theta_2,\\ 1, & \text{otherwise}, \end{cases} \quad I^{n}_{\widetilde{\lambda}}(y) = \begin{cases} U^{L}_{I}(y), & \gamma_2 \le y \le \eta_2,\\ \varphi_n, & \eta_2 \le y \le \iota_2,\\ U^{R}_{I}(y), & \iota_2 \le y \le \kappa_2,\\ -1, & \text{otherwise}. \end{cases} \tag{5}$$

$S_I^R(y)$ and $U_I^L(y)$ are continuous and increasing functions satisfying the following conditions: $S_I^R(\pi_2) = \beta_p, S_I^R(\theta_2) = 1$, $U_I^L(\gamma_2) = -1, U_I^L(\eta_2) = \varphi_n$, while $S_I^L(y)$ and $U_I^R(y)$ are continuous and decreasing functions and are satisfying the following conditions:

$$S_I^L(\mu_2) = 1, S_I^L(\nu_2) = \beta_p, U_I^R(\iota_2) = \varphi_n, U_I^R(\kappa_2) = -1, \quad (6)$$

where $\beta_p \in [0,1], \varphi_n \in [-1,0]$.The falsity membership functions are given as

$$F_{\tilde{\lambda}}^p(y) = \begin{cases} S_F^L(y), & \mu_3 \le y \le \nu_3, \\ \zeta_p, & \nu_3 \le y \le \pi_3, \\ S_F^R(y), & \pi_3 \le y \le \theta_3, \\ 1, & \text{otherwise}, \end{cases} F_{\tilde{\lambda}}^n(y) = \begin{cases} U_F^L(y), & \gamma_3 \le y \le \eta_3, \\ \nu_n, & \eta_3 \le y \le \iota_3, \\ U_F^R(y), & \iota_3 \le y \le \kappa_3, \\ -1, & \text{otherwise}. \end{cases} \quad (7)$$

$S_F^R(y)$ and $U_F^L(y)$ are continuous and increasing functions satisfying the following conditions:

$$S_F^R(\pi_3) = \zeta_p, S_F^R(\theta_3) = 1, U_F^L(\gamma_3) = -1, U_F^L(\eta_3) = \nu_n, \quad (8)$$

$S_F^L(y)$ and $U_F^R(y)$ are continuous and decreasing functions and are satisfying the following conditions:

$$S_F^L(\mu_3) = 1, S_F^L(\nu_3) = \zeta_p, U_F^R(\iota_3) = \nu_n, U_F^R(\kappa_3) = -1, \quad (9)$$

where $\zeta_p \in [0,1], \nu_n \in [-1,0]$.

Based on [49], we define some basic definitions.

Definition 3. We define a TBSNN defined on $\mathbb{R}$

$$\tilde{\lambda} = \prec([\mu_i, \nu_i, \pi_i]; \chi_p, \beta_p, \zeta_p), ([\gamma_i, \eta_i, \iota_i]; \alpha_n, \varphi_n, \nu_n)\succ, \quad (10)$$

is said to be nonnegative TBSNN if and only if $\mu_i \ge 0$ and $\gamma_i \ge 0$, where $i = 1, 2, 3$ such that $\mu_i \le \nu_i \le \pi_i$ similarly $\gamma_i \le \eta_i \le \iota_i$ also $\chi_p, \beta_p, \zeta_p \in [0,1]$ and $\alpha_n, \varphi_n, \nu_n \in [-1,0] \subset \mathbb{R}$.

Definition 4. We define a T_rBNN defined on $\mathbb{R}$

$$\begin{aligned} \tilde{\lambda} &= \prec([\mu_i, \nu_i, \pi_i, \theta_i]; \chi_p, \beta_p, \zeta_p), \\ &\cdot ([\gamma_i, \eta_i, \iota_i, \kappa_i]; \alpha_n, \varphi_n, \nu_n)\succ, \end{aligned} \quad (11)$$

is said to be positive T_rBSNN if and only if $\mu_i \ge 0$ and $\gamma_i \ge 0$, where $i = 1, 2, 3$ and $\mu_i \le \nu_i \le \pi_i \le \theta_i$. Similarly, $\gamma_i \le \eta_i \le \iota_i \le \kappa_i$,also $\chi_p, \beta_p, \zeta_p \in [0,1]$ and $\alpha_n, \varphi_n, \nu_n \in [-1,0] \subset \mathbb{R}$.

Definition 5. A TBSNN

$$\tilde{\lambda}_1 = \prec([\mu_1, \nu_1, \pi_1]; \chi_p, \beta_p, \zeta_p), ([\gamma_1, \eta_1, \iota_1]; \alpha_n, \varphi_n, \nu_n)\succ, \quad (12)$$

is said to be zero if $\mu_1 = 0, \nu_1 = 0, \pi_1 = 0, \gamma_1 = 0, \eta_1 = 0, \iota_1 = 0, \chi_p = 0, \beta_p = 0, \zeta_p = 0, \alpha_n = 0, \varphi_n = 0$, and $\nu_n = 0$.

Definition 6. Two TBSNNs

$$\begin{aligned} \tilde{\lambda}_1 &= \prec([\mu_1, \nu_1, \pi_1]; \chi_p, \beta_p, \zeta_p), ([\gamma_1, \eta_1, \iota_1]; \alpha_n, \varphi_n, \nu_n)\succ, \\ \tilde{\lambda}_2 &= \prec([\mu_2, \nu_2, \pi_2]; \chi_p', \beta_p', \zeta_p'), ([\gamma_2, \eta_2, \iota_2]; \alpha_n', \varphi_n', \nu_n')\succ, \end{aligned} \quad (13)$$

are said to be equal if $\mu_1 = \mu_2, \nu_1 = \nu_2, \pi_1 = \pi_2, \gamma_1 = \gamma_2, \eta_1 = \eta_2, \iota_1 = \iota_2, \chi_p = \chi_p', \beta_p = \beta_p', \zeta_p = \zeta_p', \alpha_n = \alpha_n', \varphi_n = \varphi_n'$, and $\nu_n = \nu_n'$.

Definition 7. Based on [50], we define T_rBSNN defined $\mathbb{R}$ denoted by

$$\begin{aligned} \tilde{\lambda}(Y) &= \prec([\mu_1, \nu_1, \pi_1, \theta_1]; \chi_p), ([\mu_2, \nu_2, \pi_2, \theta_2]; \beta_p), \\ &\cdot ([\mu_3, \nu_3, \pi_3, \theta_3]; \zeta_p), ([\gamma_1, \eta_1, \iota_1, \kappa_1]; \alpha_n), \\ &([\gamma_2, \eta_2, \iota_2, \kappa_2]; \varphi_n), ([\gamma_3, \eta_3, \iota_3, \kappa_3]; \nu_n)\succ, \end{aligned} \quad (14)$$

whose truth, indeterminacy, and falsity membership functions are presented by

$$T_{\tilde{\lambda}}^p(y) = \begin{cases} \dfrac{y-\mu_1}{\nu_1-\mu_1}\chi_p, & \mu_1 \le y \le \nu_1, \\ \chi_p, & \nu_1 \le y \le \pi_1, \\ \dfrac{\theta_1-y}{\theta_1-\pi_1}\chi_p, & \pi_1 \le y \le \theta_1, \\ 0, & \text{otherwise}, \end{cases} T_{\tilde{\lambda}}^n(y) = \begin{cases} \dfrac{y-\gamma_1}{\eta_1-\gamma_1}\alpha_n, & \gamma_1 \le y \le \eta_1, \\ \alpha_n, & \eta_1 \le y \le \iota_1, \\ \dfrac{\kappa_1-y}{\kappa_1-\iota_1}\alpha_n, & \iota_1 \le y \le \kappa_1, \\ 0, & \text{otherwise}. \end{cases}$$

$$I_{\tilde{\lambda}}^{p}(y)=\begin{cases}\dfrac{(\nu_2-y)+\beta_p(x-\mu_2)}{\nu_2-\mu_2}, & \mu_2\le y\le \nu_2,\\ \beta_p, & \nu_2\le y\le \pi_2,\\ \dfrac{(y-\pi_2)+\beta_p(\theta_2-y)}{\theta_2-\pi_2}, & \pi_2\le y\le \theta_2,\\ 1, & \text{otherwise},\end{cases}\quad I_{\tilde{\lambda}}^{n}(y)=\begin{cases}\dfrac{(\eta_2-y)+\varphi_n(y-\gamma_2)}{\eta_2-\gamma_2}, & \gamma_2\le y\le \eta_2,\\ \varphi_n, & \eta_2\le y\le \iota_2,\\ \dfrac{(y-\iota_2)+\varphi_n(\kappa_2-y)}{\kappa_2-\iota_2}, & \iota_2\le y\le \kappa_2,\\ -1, & \text{otherwise},\end{cases}$$

$$F_{\tilde{\lambda}}^{p}(y)=\begin{cases}\dfrac{(\nu_3-y)+\zeta_p(y-\mu_3)}{\nu_3-\mu_3}, & \mu_3\le y\le \nu_3,\\ \zeta_p, & \nu_3\le y\le \pi_3,\\ \dfrac{(y-\pi_3)+\zeta_p(\theta_3-y)}{\theta_3-\pi_3}, & \pi_3\le y\le \theta_3,\\ 1, & \text{otherwise},\end{cases}\quad F_{\tilde{\lambda}}^{n}(y)=\begin{cases}\dfrac{(\eta_3-y)+\nu_p(y-\gamma_3)}{\eta_3-\gamma_3}, & \gamma_3\le y\le \eta_3,\\ \nu_n, & \eta_3\le y\le \iota_3,\\ \dfrac{(y-\iota_3)+\nu_n(\kappa_3-y)}{\kappa_3-\iota_3}, & \iota_3\le y\le \kappa_3,\\ -1, & \text{otherwise},\end{cases}\tag{15}$$

where $\chi_p, \beta_p, \zeta_p \in [0,1]$ and $\alpha_n, \varphi_n, \nu_n \in [-1,0] \subset \mathbb{R}$.

Remark 1. If we set $\nu_i = \pi_i$ and $\eta_i = \iota_i$ in Definition 7, the triangular bipolar single-valued neutrosophic number (TBSNN) is obtained, where $i = 1, 2, 3$.

Definition 8 (see [51]). Let $\tilde{\lambda} = \prec (T^p(y), I^p(y), F^p(y), T^n(y), I^n(y), F^n(y))\succ$ be a BSNN then the score function is presented by

$$S(\tilde{\lambda}) = \frac{(T^p(y) + 1 - I^p(y) + 1 - F^p(y) + 1 + T^n(y) - I^n(y) - F^n(y))}{6}. \tag{16}$$

Definition 9. Based on [52], let

$$\begin{aligned}\tilde{\lambda}_1 &= \prec([\mu_1,\nu_1,\pi_1,\theta_1];\chi_p^1),([\mu_2,\nu_2,\pi_2,\theta_2];\beta_p^1),([\mu_3,\nu_3,\pi_3,\theta_3];\zeta_p^1),([\mu_4,\nu_4,\pi_4,\theta_4];\alpha_n^1),([\mu_5,\nu_5,\pi_5,\theta_5];\varphi_n^1),\\ &\quad([\mu_6,\nu_6,\pi_6,\theta_6];\nu_n^1)\succ \text{ and}\\ \tilde{\lambda}_2 &= \prec([\gamma_1,\eta_1,\iota_1,\kappa_1];\chi_p^2),([\gamma_2,\eta_2,\iota_2,\kappa_2];\beta_p^2),([\gamma_3,\eta_3,\iota_3,\kappa_3];\zeta_p^2),((\gamma_4,\eta_4,\iota_4,\kappa_4);\alpha_n^2),([\gamma_5,\eta_5,\iota_5,\kappa_5];\varphi_n^2),\\ &\quad([\gamma_6,\eta_6,\iota_6,\kappa_6];\nu_n^2)\succ\end{aligned}\tag{17}$$

be two nonnegative T_rBSNNs, then,

(1) $\begin{array}{l}\tilde{\lambda}_1\oplus\tilde{\lambda}_2 = \prec([\mu_1+\gamma_1,\nu_1+\eta_1,\pi_1+\iota_1,\theta_1+\kappa_1];\chi_p^1\wedge\chi_p^2),([\mu_2+\gamma_2,\nu_2+\eta_2,\pi_2+\iota_2,\theta_2+\kappa_2];\beta_p^1\vee\beta_p^2),\\ ([\mu_3+\gamma_3,\nu_3+\eta_3,\pi_3+\iota_3,\theta_3+\kappa_3];\zeta_p^1\vee\zeta_p^2),([\mu_4+\gamma_4,\nu_4+\eta_4,\pi_4+\iota_4,\theta_4+\kappa_4];\alpha_n^1\vee\alpha_n^2),\\ ([\mu_5+\gamma_5,\nu_5+\eta_5,\pi_5+\iota_5,\theta_5+\kappa_5];\varphi_n^1\wedge\varphi_n^2),([\mu_6+\gamma_6,\nu_6+\eta_6,\pi_6+\iota_6,\theta_6+\kappa_6];\nu_n^1\wedge\nu_n^2)\succ\end{array}.$

(2) $\begin{array}{l}\tilde{\lambda}_1\ominus\tilde{\lambda}_2 = \prec([\mu_1-\kappa_1,\nu_1-\iota_1,\pi_1-\eta_1,\theta_1-\gamma_1];\chi_p^1\wedge\chi_p^2),([\mu_2-\kappa_2,\nu_2-\iota_2,\pi_2-\eta_2,\theta_2-\gamma_2];\beta_p^1\vee\beta_p^2),\\ ([\mu_3-\kappa_3,\nu_3-\iota_3,\pi_3-\eta_3,\theta_3-\gamma_3];\zeta_p^1\vee\zeta_p^2),([\mu_4-\kappa_4,\nu_4-\iota_4,\pi_4-\eta_4,\theta_1-\gamma_1];\alpha_n^1\vee\alpha_n^2),\\ ([\mu_5-\kappa_5,\nu_5-\iota_5,\pi_5-\eta_5,\theta_5-\gamma_5];\varphi_n^1\wedge\varphi_n^2),([\mu_6-\kappa_6,\nu_6-\iota_6,\pi_6-\eta_6,\theta_6-\gamma_6];\nu_n^1\wedge\nu_n^2)\succ\end{array}$

(3) $\tilde{\lambda}_1\otimes\tilde{\lambda}_2 = \begin{cases}\prec([\mu_1\gamma_1,\nu_1\eta_1,\pi_1\iota_1,\theta_1\kappa_1];\chi_p^1\wedge\chi_p^2),([\mu_2\gamma_2,\nu_2\eta_2,\pi_2\iota_2,\theta_2\kappa_2];\beta_p^1\vee\beta_p^2),\\ ([\mu_3\gamma_3,\nu_3\eta_3,\pi_3\iota_3,\theta_3\kappa_3];\zeta_p^1\vee\zeta_p^2),([\mu_4\gamma_4,\nu_4\eta_4,\pi_4\iota_4,\theta_4\kappa_4];\alpha_n^1\vee\alpha_n^2),\\ ([\mu_5\gamma_5,\nu_5\eta_5,\pi_5\iota_5,\theta_5\kappa_5];\varphi_n^1\wedge\varphi_n^2),([\mu_6\gamma_6,\nu_6\eta_6,\pi_6\iota_6,\theta_6\kappa_6];\nu_n^1\wedge\nu_n^2)\succ\end{cases}$

3. Methodology to Solve FBSNT Problems

In this section, we present a new method that is based on the formulation of FBSNLP to solve FBSNT problems. We discuss the steps to calculate the bipolar single-valued neutrosophic optimal solution of FBSNT problems with nonnegative TBSNNs:

$$\text{Minimize Z} = \sum_{i=1}^{m}\sum_{j=1}^{n}\widetilde{C_{ij}^{W}}\otimes\widetilde{X_{ij}^{W}}, \tag{18}$$

subject to

$$\begin{aligned}&\sum_{j=1}^{n}\widetilde{X_{ij}^{W}} = \tilde{E}_i^{W} = \text{Supply}, \forall i = 1,2,3,\ldots,m,\\ &\sum_{i=1}^{m}\widetilde{X_{ij}^{W}} = \tilde{F}_j^{W} = \text{Demand}, \forall j = 1,2,3,\ldots,n,\\ &\tilde{X}_{ij}^{W} \ge 0, \forall i = 1,2,3,\ldots,m, \forall j = 1,2,3,\ldots,n,\end{aligned}\tag{19}$$

where $\widetilde{C_{ij}^{W}}, \widetilde{X_{ij}^{W}}, \tilde{E}_i^{W}$, and $\tilde{F}_j^{W}$ are all TBSNNs.

Step 1. Determine total bipolar single-valued neutrosophic availability and total bipolar single-valued neutrosophic demand.

If

$$\sum_{j=1}^{n}\widetilde{F}_j^W=\sum_{i=1}^{m}\widetilde{E}_i^W, \tag{20}$$

$\Rightarrow$ a balanced bipolar single-valued neutrosophic transportation problem (BSNTP).

If,

$$\sum_{j=1}^{n}\widetilde{F}_j^W\neq\sum_{i=1}^{m}\widetilde{E}_i^W. \tag{21}$$

An unbalanced BSNTP.

That is,

$$\begin{aligned}&\prec\left(\left[s_k,t_k,u_k\right];\chi_p,\beta_p,\zeta_p\right),\left(\left[v_k,w_k,x_k\right];\alpha_n,\varphi_n,\nu_n\right)\succ\\&\quad\neq\prec\left(\left[s_k',t_k',u_k'\right];\chi_p',\beta_p',\zeta_p'\right),\left(\left[v_k',w_k',x_k'\right];\alpha_n',\varphi_n',\nu_n'\right)\succ\\&k=1,2,3.\end{aligned} \tag{22}$$

Then one of the following case arise.

Case 1. $s_k\le s_k'$, $t_k-s_k\le t_k'-s_k'$, $u_k-t_k\le u_k'-t_k'$, $v_k\le v_k'$, $w_k-v_k\le w_k'-v_k'$, $x_k-w_k\le x_k'-w_k'$.

Case 2. $s_k\ge s_k'$, $t_k-s_k\ge t_k'-s_k'$, $u_k-t_k\ge u_k'-t_k'$, $v_k\ge v_k'$, $w_k-v_k\ge w_k'-v_k'$, $x_k-w_k\ge x_k'-w_k'$.

Case 3. When the above two cases do not hold, then there may exist infinitely many TBSNNs,

$$\begin{cases}\prec\left(\left[s_k^1,t_k^1,u_k^1\right];\chi_p^1,\beta_p^1,\zeta_p^1\right),\left(\left[v_k^1,w_k^1,x_k^1\right];\alpha_n^1,\varphi_n^1,\nu_n^1\right)\succ\\ \prec\left(\left[s_k^{1'},t_k^{1'},u_k^{1'}\right];\chi_p^{1'},\beta_p^{1'},\zeta_p^{1'}\right),\left(\left[v_k^{1'},w_k^{1'},x_k^{1'}\right];\alpha_n^{1'},\varphi_n^{1'},\nu_n^{1'}\right)\succ,\\ k=1,2,3,\end{cases} \tag{23}$$

such that

$$\begin{aligned}&\prec\left(\left[s_k,t_k,u_k\right];\chi_p,\beta_p,\zeta_p\right),\left(\left[v_k,w_k,x_k\right];\alpha_n,\varphi_n,\nu_n\right)\succ\oplus\prec\left(\left[s_k^1,t_k^1,u_k^1\right];\chi_p^1,\beta_p^1,\zeta_p^1\right),\left(\left[v_k^1,w_k^1,x_k^1\right];\alpha_n^1,\varphi_n^1,\nu_n^1\right)\succ=\\&\prec\left(\left[s_k',t_k',u_k'\right];\chi_p',\beta_p',\zeta_p'\right),\left(\left[v_k',w_k',x_k'\right];\alpha_n',\varphi_n',\nu_n'\right)\succ\oplus\prec\left(\left[s_k^{1'},t_k^{1'},u_k^{1'}\right];\chi_p^{1'},\beta_p^{1'},\zeta_p^{1'}\right),\left(\left[v_k^{1'},w_k^{1'},x_k^{1'}\right];\alpha_n^{1'},\varphi_n^{1'},\nu_n^{1'}\right)\succ\end{aligned} \tag{24}$$

But we have to determine such TBSNNs

$$\begin{cases}\prec\left(\left[s_k^1,t_k^1,u_k^1\right];\chi_p^1,\beta_p^1,\zeta_p^1\right),\left(\left[v_k^1,w_k^1,x_k^1\right];\alpha_n^1,\varphi_n^1,\nu_n^1\right)\succ\\ \prec\left(\left[s_k^{1'},t_k^{1'},u_k^{1'}\right];\chi_p^{1'},\beta_p^{1'},\zeta_p^{1'}\right),\left(\left[v_k^{1'},w_k^{1'},x_k^{1'}\right];\alpha_n^{1'},\varphi_n^{1'},\nu_n^{1'}\right)\succ,\\ k=1,2,3,\end{cases} \tag{25}$$

which satisfy the following conditions.

(i) $\prec\left(\left[s_k^1,t_k^1,u_k^1\right];\chi_p^1,\beta_p^1,\zeta_p^1\right),\left(\left[v_k^1,w_k^1,x_k^1\right];\alpha_n^1,\varphi_n^1,\nu_n^1\right)\succ$ and $\prec\left(\left[s_k^{1'},t_k^{1'},u_k^{1'}\right];\chi_p^{1'},\beta_p^{1'},\zeta_p^{1'}\right),\left(\left[v_k^{1'},w_k^{1'},x_k^{1'}\right];\alpha_n^{1'},\varphi_n^{1'},\nu_n^{1'}\right)\succ$ are nonnegative TBSNNs.

(ii) It satisfies

$$\begin{aligned}&\prec\left(\left[s_k,t_k,u_k\right];\chi_p,\beta_p,\zeta_p\right),\left(\left[v_k,w_k,x_k\right];\alpha_n,\varphi_n,\nu_n\right)\succ\oplus\prec\left(\left[s_k^1,t_k^1,u_k^1\right];\chi_p^1,\beta_p^1,\zeta_p^1\right),\left(\left[v_k^1,w_k^1,x_k^1\right];\alpha_n^1,\varphi_n^1,\nu_n^1\right)\succ=\\&\prec\left(\left[s_k',t_k',u_k'\right];\chi_p',\beta_p',\zeta_p'\right),\left(\left[v_k',w_k',x_k'\right];\alpha_n',\varphi_n',\nu_n'\right)\succ\oplus\prec\left(\left[s_k^{1'},t_k^{1'},u_k^{1'}\right];\chi_p^{1'},\beta_p^{1'},\zeta_p^{1'}\right),\left(\left[v_k^{1'},w_k^{1'},x_k^{1'}\right];\alpha_n^{1'},\varphi_n^{1'},\nu_n^{1'}\right)\succ.\end{aligned} \tag{26}$$

(iii) Further, if there exists two nonnegative TBSNNs $\prec\left(\left[m_k,n_k,o_k\right];\mu_p,\nu_p,\pi_p\right),\left(\left[p_k,q_k,r_k\right];\eta_n,\zeta_n,\theta_n\right)\succ$ and $\prec\left(\left[m_k',n_k',o_k'\right];\mu_p',\nu_p',\pi_p'\right),\left(\left[p_k',q_k',r_k'\right];\eta_n',\zeta_n',\theta_n'\right)\succ$ such that

$$\begin{aligned}&\prec\left(\left[s_k,t_k,u_k\right];\chi_p,\beta_p,\zeta_p\right),\left(\left[v_k,w_k,x_k\right];\alpha_n,\varphi_n,\nu_n\right)\succ\oplus\prec\left(\left[m_k,n_k,o_k\right];\mu_p,\nu_p,\pi_p\right),\left(\left[p_k,q_k,r_k\right];\eta_n,\zeta_n,\theta_n\right)\succ=\\&\prec\left(\left[s_k',t_k',u_k'\right];\chi_p',\beta_p',\zeta_p'\right),\left(\left[v_k',w_k',x_k'\right];\alpha_n',\varphi_n',\nu_n'\right)\succ\oplus\prec\left(\left[m_k',n_k',o_k'\right];\mu_p',\nu_p',\pi_p'\right),\left(\left[p_k',q_k',r_k'\right];\eta_n',\zeta_n',\theta_n'\right)\succ,\end{aligned} \tag{27}$$

then

$$\Re\prec\left([m_k,n_k,o_k];\mu_p,\nu_p,\pi_p\right),\left([p_k,q_k,r_k];\eta_n,\zeta_n,\theta_n\right)\succ\geq\Re\prec\left(\left[s_k^1,t_k^1,u_k^1\right];\chi_p^1,\beta_p^1,\zeta_p^1\right),\left(\left[v_k^1,w_k^1,x_k^1\right];\alpha_n^1,\varphi_n^1,\nu_n^1\right)\succ$$
$$\Re\prec\left([m_k',n_k',o_k'];\mu_p',\nu_p',\pi_p'\right),\left([p_k',q_k',r_k'];\eta_n',\zeta_n',\theta_n'\right)\succ\geq\Re\prec\left(\left[s_k^{1'},t_k^{1'},u_k^{1'}\right];\chi_p^{1'},\beta_p^{1'},\zeta_p^{1'}\right),\left(\left[v_k^{1'},w_k^{1'},x_k^{1'}\right];\alpha_n^{1'},\varphi_n^{1'},\nu_n^{1'}\right)\succ. \tag{28}$$

Step 2. Suppose $\widetilde{C_{ij}}^W=\prec([c_{ij}^1,c_{ij}^2,c_{ij}^3];\xi_{ij},\psi_{ij},\omega_{ij}),([c_{ij}^4,c_{ij}^5,c_{ij}^6];\xi_{ij}',\psi_{ij}',\omega_{ij}')\succ$,

$$\begin{aligned}\widetilde{X_{ij}^W}&=\prec\left(\left[x_{ij}^1,x_{ij}^2,x_{ij}^3\right];\sigma_{ij},\tau_{ij},\upsilon_{ij}\right),\left(\left[x_{ij}^4,x_{ij}^5,x_{ij}^6\right];\sigma_{ij}',\tau_{ij}',\upsilon_{ij}'\right)\succ\\ \widetilde{E}_i^W&=\prec\left(\left[e_i^1,e_i^2,e_i^3\right];\kappa_i,\lambda_i,\theta_i\right),\left(\left[e_i^4,e_i^5,e_i^6\right];\kappa_i',\lambda_i',\theta_i'\right)\succ\text{ and }\widetilde{F}_j^W=\prec\left(\left[f_j^1,f_j^2,f_j^3\right];\eta_j,\epsilon_j,\chi_j\right),\left(\left[f_j^4,f_j^5,f_j^6\right];\eta_j',\epsilon_j',\chi_j'\right)\succ.\end{aligned} \tag{29}$$

Then FBSNTP (18) can be transformed as

$$\text{Min Z}=\sum_{i=1}^m\sum_{j=1}^n\prec\left(\left[c_{ij}^1,c_{ij}^2,c_{ij}^3\right];\xi_{ij},\psi_{ij},\omega_{ij}\right),\left(\left[c_{ij}^4,c_{ij}^5,c_{ij}^6\right];\xi_{ij}',\psi_{ij}',\omega_{ij}'\right)\succ\otimes\prec\left(\left[x_{ij}^1,x_{ij}^2,x_{ij}^3\right];\sigma_{ij},\tau_{ij},\upsilon_{ij}\right),\left(\left[x_{ij}^4,x_{ij}^5,x_{ij}^6\right];\sigma_{ij}',\tau_{ij}',\upsilon_{ij}'\right)\succ, \tag{30}$$

subject to

$$\begin{aligned}&\sum_{j=1}^n\prec\left(\left[x_{ij}^1,x_{ij}^2,x_{ij}^3\right];\sigma_{ij},\tau_{ij},\upsilon_{ij}\right),\left(\left[x_{ij}^4,x_{ij}^5,x_{ij}^6\right];\sigma_{ij}',\tau_{ij}',\upsilon_{ij}'\right)\succ=\prec\left(\left[e_i^1,e_i^2,e_i^3\right];\kappa_i,\lambda_i,\theta_i\right),\left(\left[e_i^4,e_i^5,e_i^6\right];\kappa_i',\lambda_i',\theta_i'\right)\succ,\\&\sum_{i=1}^m\prec\left(\left[x_{ij}^1,x_{ij}^2,x_{ij}^3\right];\sigma_{ij},\tau_{ij},\upsilon_{ij}\right),\left(\left[x_{ij}^4,x_{ij}^5,x_{ij}^6\right];\sigma_{ij}',\tau_{ij}',\upsilon_{ij}'\right)\succ=\prec\left(\left[f_j^1,f_j^2,f_j^3\right];\eta_j,\epsilon_j,\chi_j\right),\left(\left[f_j^4,f_j^5,f_j^6\right];\eta_j',\epsilon_j',\chi_j'\right)\succ,\end{aligned} \tag{31}$$

where $\prec([x_{ij}^1,x_{ij}^2,x_{ij}^3];\sigma_{ij},\tau_{ij},\upsilon_{ij}),([x_{ij}^4,x_{ij}^5,x_{ij}^6];\sigma_{ij}',\tau_{ij}',\upsilon_{ij}')\succ$ are nonnegative TBSNNs.

Step 3. By applying arithmetic operations as described in Definition 9 and putting $\prec([c_{ij}^1,c_{ij}^2,c_{ij}^3];\xi_{ij},\psi_{ij},\omega_{ij}),([c_{ij}^4,c_{ij}^5,c_{ij}^6];\xi_{ij}',\psi_{ij}',\omega_{ij}')\succ\otimes\prec([x_{ij}^1,x_{ij}^2,x_{ij}^3];\sigma_{ij},\tau_{ij},\upsilon_{ij}),([x_{ij}^4,x_{ij}^5,x_{ij}^6];\sigma_{ij}',\tau_{ij}',\upsilon_{ij}')\succ=\prec([d_{ij}^1,d_{ij}^2,d_{ij}^3];\mu_{ij},\delta_{ij},\eta_{ij}),([d_{ij}^4,d_{ij}^5,d_{ij}^6];\mu_{ij}',\delta_{ij}',\eta_{ij}')\succ$, then the FBSNTP (30) can be transformed as

$$\begin{aligned}\text{Min Z}=\sum_{i=1}^m\sum_{j=1}^n&\prec\left(\left[d_{ij}^1,d_{ij}^2,d_{ij}^3\right];\mu_{ij},\delta_{ij},\eta_{ij}\right),\\&\left(\left[d_{ij}^4,d_{ij}^5,d_{ij}^6\right];\mu_{ij}',\delta_{ij}',\eta_{ij}'\right)\succ,\end{aligned} \tag{32}$$

subject to

$$\begin{aligned}&\sum_{j=1}^n\prec\left(\left[x_{ij}^1,x_{ij}^2,x_{ij}^3\right];\sigma_{ij},\tau_{ij},\upsilon_{ij}\right),\left(\left[x_{ij}^4,x_{ij}^5,x_{ij}^6\right];\sigma_{ij}',\tau_{ij}',\upsilon_{ij}'\right)\succ=\prec\left(\left[e_i^1,e_i^2,e_i^3\right];\kappa_i,\lambda_i,\theta_i\right),\left(\left[e_i^4,e_i^5,e_i^6\right];\kappa_i',\lambda_i',\theta_i'\right)\succ,\\&\sum_{i=1}^m\prec\left(\left[x_{ij}^1,x_{ij}^2,x_{ij}^3\right];\sigma_{ij},\tau_{ij},\upsilon_{ij}\right),\left(\left[x_{ij}^4,x_{ij}^5,x_{ij}^6\right];\sigma_{ij}',\tau_{ij}',\upsilon_{ij}'\right)\succ=\prec\left(\left[f_j^1,f_j^2,f_j^3\right];\eta_j,\epsilon_j,\chi_j\right),\left(\left[f_j^4,f_j^5,f_j^6\right];\eta_j',\epsilon_j',\chi_j'\right)\succ,\end{aligned} \tag{33}$$

where $\prec([x_{ij}^1,x_{ij}^2,x_{ij}^3];\sigma_{ij},\tau_{ij},\upsilon_{ij}),([x_{ij}^4,x_{ij}^5,x_{ij}^6];\sigma_{ij}',\tau_{ij}',\upsilon_{ij}')\succ$ are nonnegative TBSNNs.

Step 4. Now applying the score function, the FBSNTP (32) can be transformed as

$$\begin{aligned}\text{Min Z}=\sum_{i=1}^m\sum_{j=1}^n S&\prec\left(\left[d_{ij}^1,d_{ij}^2,d_{ij}^3\right];\mu_{ij},\delta_{ij},\eta_{ij}\right),\\&\left(\left[d_{ij}^4,d_{ij}^5,d_{ij}^6\right];\mu_{ij}',\delta_{ij}',\eta_{ij}'\right)\succ,\end{aligned} \tag{34}$$

subject to

$$\begin{gathered}
\sum_{j=1}^{n} x_{ij}^{1} = e_i^1, \forall i = 1,2,3,\dots,m, \\
\sum_{j=1}^{n} x_{ij}^{2} = e_i^2, \forall i = 1,2,3,\dots,m, \\
\sum_{j=1}^{n} x_{ij}^{3} = e_i^3, \forall i = 1,2,3,\dots,m, \\
\sum_{j=1}^{n} x_{ij}^{4} = e_i^4, \forall i = 1,2,3,\dots,m, \\
\sum_{j=1}^{n} x_{ij}^{5} = e_i^5, \forall i = 1,2,3,\dots,m, \\
\sum_{j=1}^{n} x_{ij}^{6} = e_i^6, \forall i = 1,2,3,\dots,m, \\
\sum_{j=1}^{n} \sigma_{ij} \wedge \kappa_i, \forall i = 1,2,3,\dots,m, \\
\sum_{j=1}^{n} \tau_{ij} \vee \lambda_i, \forall i = 1,2,3,\dots,m, \\
\sum_{j=1}^{n} \upsilon_{ij} \vee \theta_i, \forall i = 1,2,3,\dots,m, \\
\sum_{i=1}^{m} x_{ij}^{1} = f_j^1, \forall j = 1,2,3,\dots,n, \\
\sum_{i=1}^{m} x_{ij}^{2} = f_j^2, \forall j = 1,2,3,\dots,n, \\
\sum_{i=1}^{m} x_{ij}^{3} = f_j^3, \forall j = 1,2,3,\dots,n, \\
\sum_{i=1}^{m} x_{ij}^{4} = f_j^4, \forall j = 1,2,3,\dots,n, \\
\sum_{i=1}^{m} x_{ij}^{5} = f_j^5, \forall j = 1,2,3,\dots,n, \\
\sum_{i=1}^{m} x_{ij}^{6} = f_j^6, \forall j = 1,2,3,\dots,n, \\
\sum_{j=1}^{n} \sigma_{ij}' \vee \kappa_i', \forall j = 1,2,3,\dots,n, \\
\sum_{j=1}^{n} \tau_{ij}' \wedge \lambda_i', \forall j = 1,2,3,\dots,n, \\
\sum_{j=1}^{n} \upsilon_{ij}' \wedge \theta_i', \forall j = 1,2,3,\dots,n,
\end{gathered} \tag{35}$$

where $\prec([x_{ij}^1, x_{ij}^2, x_{ij}^3]; \sigma_{ij}, \tau_{ij}, \upsilon_{ij}), ([x_{ij}^4, x_{ij}^5, x_{ij}^6]; \sigma_{ij}', \tau_{ij}', \upsilon_{ij}')\succ$ are nonnegative TBSNNs.

Step 5. To obtain fuzzy optimal solution, the following crisp linear/nonlinear programming problem is solved:

$$\text{Min Z} = \sum_{i=1}^{m} \sum_{j=1}^{n} \frac{1}{6} \left(3 + d_{ij}^1 - d_{ij}^2 - d_{ij}^3 + d_{ij}^4 - d_{ij}^5 - d_{ij}^6\right), \tag{36}$$

subject to

$$\begin{gathered}
\sum_{j=1}^{n} x_{ij}^{1} = e_i^1, \forall i = 1,2,3,\dots,m, \\
\sum_{j=1}^{n} x_{ij}^{2} = e_i^2, \forall i = 1,2,3,\dots,m, \\
\sum_{j=1}^{n} x_{ij}^{3} = e_i^3, \forall i = 1,2,3,\dots,m, \\
\sum_{j=1}^{n} x_{ij}^{4} = e_i^4, \forall i = 1,2,3,\dots,m, \\
\sum_{j=1}^{n} x_{ij}^{5} = e_i^5, \forall i = 1,2,3,\dots,m, \\
\sum_{j=1}^{n} x_{ij}^{6} = e_i^6, \forall i = 1,2,3,\dots,m, \\
\sum_{j=1}^{n} \sigma_{ij} \wedge \kappa_i, \forall i = 1,2,3,\dots,m, \\
\sum_{j=1}^{n} \tau_{ij} \vee \lambda_i, \forall i = 1,2,3,\dots,m, \\
\sum_{j=1}^{n} \upsilon_{ij} \vee \theta_i, \forall i = 1,2,3,\dots,m, \\
\sum_{i=1}^{m} x_{ij}^{1} = f_j^1, \forall j = 1,2,3,\dots,n, \\
\sum_{i=1}^{m} x_{ij}^{2} = f_j^2, \forall j = 1,2,3,\dots,n, \\
\sum_{i=1}^{m} x_{ij}^{3} = f_j^3, \forall j = 1,2,3,\dots,n, \\
\sum_{i=1}^{m} x_{ij}^{4} = f_j^4, \forall j = 1,2,3,\dots,n, \\
\sum_{i=1}^{m} x_{ij}^{5} = f_j^5, \forall j = 1,2,3,\dots,n, \\
\sum_{i=1}^{m} x_{ij}^{6} = f_j^6, \forall j = 1,2,3,\dots,n, \\
\sum_{j=1}^{n} \sigma_{ij}' \vee \kappa_i', \forall j = 1,2,3,\dots,n, \\
\sum_{j=1}^{n} \tau_{ij}' \wedge \lambda_i', \forall j = 1,2,3,\dots,n, \\
\sum_{j=1}^{n} \upsilon_{ij}' \wedge \theta_i', \forall j = 1,2,3,\dots,n,
\end{gathered} \tag{37}$$

$x_{ij}^1 \geq 0,\ x_{ij}^2 - x_{ij}^1 \geq 0,\ x_{ij}^3 - x_{ij}^2 \geq 0,\ x_{ij}^4 \geq 0,\ x_{ij}^5 - x_{ij}^4 \geq 0,\ x_{ij}^6 - x_{ij}^5 \geq 0,\ \sigma_{ij},\ \tau_{ij},\ \upsilon_{ij} \in [0,1]$ and $\sigma_{,ij},\ \tau_{ij}',\ \upsilon_{ij}' \in [-1,0]$ $\forall i = 1,2,\ldots,m, \forall j = 1,2,\ldots,n.$

Step 6. An optimal solution is obtained by solving a crisp linear/nonlinear programming problem: $\{x_{ij}^1, x_{ij}^2, x_{ij}^3, x_{ij}^4, x_{ij}^5, x_{ij}^6, \sigma_{ij}, \tau_{ij}, \upsilon *_{ij}, \sigma_{ij}', \tau_{ij}', \upsilon_{ij}'\}$.

Step 7. Find the bipolar single-valued neutrosophic optimal solution $\widetilde{X}_{ij}^W$ of the FBSNTP (18) by putting the values of $x_{ij}^1, x_{ij}^2, x_{ij}^3, x_{ij}^4, x_{ij}^5, x_{ij}^6, \sigma_{ij}, \tau_{ij}, \upsilon_{ij}, \sigma_{ij}', \tau_{ij}'$ and υ_{ij}' in $\widetilde{X}_{ij}^W = \prec([x_{ij}^1, x_{ij}^2, x_{ij}^3]; \sigma_{ij}, \tau_{ij}, \upsilon_{ij}), ([x_{ij}^4, x_{ij}^5, x_{ij}^6]; \sigma_{ij}', \tau_{ij}', \upsilon_{ij}')\succ$.

Step 8. The minimum bipolar single-valued neutrosophic transportation cost/bipolar single-valued neutrosophic optimal value of the FBSNTP (18) are found by setting the values of $\widetilde{X}_{ij}^W$, as obtained in Step 7, in $\sum_{i=1}^m \sum_{j=1}^n \widetilde{C_{ij}^W} \otimes \widetilde{X_{ij}^W}$.

4. Numerical Example

Example 1. FFC Transportation Model

Fauji Fertilizer Company (FFC) has two plants in Gujranwala and Karachi and two main centers in Lahore and Peshawar. The capacities of producing urea at plants are $\prec([70,100,130];1,0,0.4),([55,100,155];-1,0,-0.3)\succ$ and $\prec([50,70,90];1,0,0.3),([35,70,115];-1,0,-0.2)\succ$ and the demands at the two delivery centers of urea for the same time are $\prec([50,60,70];1,0,0.2),([30,60,100];-1,0,-0.1)\succ$ and $\prec([40,70,100];1,0,0.1),([40,70,100];-1,0,-0.4)\succ$, respectively. The trucking association in charge of transporting the urea charge $\prec([5,6,7];1,0,0),([3,6,10];-1,0,-0.1)\succ$ Rs. per ton urea. Thus, the transporting costs per ton urea on different routes are given in Table 1.subject to

$$\begin{aligned}
\tilde{x}_{11} \oplus \tilde{x}_{12} &= \prec([70,100,130];1,0,0.4),([55,100,155];-1,0,-0.3)\succ,\\
\tilde{x}_{21} \oplus \tilde{x}_{22} &= \prec([50,70,90];1,0,0.3),([35,70,115];-1,0,-0.2)\succ,\\
\tilde{x}_{11} \oplus \tilde{x}_{21} &= \prec([50,60,70];1,0,0.2),([30,60,100];-1,0,-0.1)\succ,\\
\tilde{x}_{12} \oplus \tilde{x}_{22} &= \prec([40,70,100];1,0,0.1),([40,70,100];-1,0,-0.4)\succ,
\end{aligned} \tag{38}$$

where $\tilde{x}_{11}$, $\tilde{x}_{12}$, $\tilde{x}_{21}$, and $\tilde{x}_{22}$ are nonnegative TBSNNs.

$$\text{Minimize}\begin{pmatrix}
\prec([15,17,18];1,0,0.2),([16,17,19];-1,0,-0.3)\succ \otimes \tilde{x}_{11} \oplus\\
\prec([20,22,24];1,0,0.3),([21,22,23];-1,0,-0.4)\succ \otimes \tilde{x}_{12} \oplus\\
\prec([25,28,30];1,0,0.4),([27,28,29];-1,0,-0.5)\succ \otimes \tilde{x}_{21} \oplus\\
\prec([30,40,50];1,0,0.1),([35,40,45];-1,0,-0.1)\succ \otimes \tilde{x}_{22}
\end{pmatrix}, \tag{39}$$

4.1. Step 1. Now as,

total supply $= \prec([120,170,220];1,0,0.3),([90,170,270];-1,0,-0.3)\succ$

Total demand $= \prec([90,130,170];1,0,0.2),([70,130,200];-1,0,-0.4)\succ$

$\Rightarrow$ an unbalanced FBSNTP. So, we add dummy rows and dummy columns to make a balanced TP.

Dummy row $= \prec([10,30,30];1,0,0.1),([0,30,40];-1,0,-0.2)\succ$

Dummy column $= \prec([40,70,80];1,0,0.2),([20,70,110];-1,0,-0.3)\succ$

So, by assuming bipolar single-valued neutrosophic transportation cost of unit quantity of product from dummy source to all destinations and from all sources to dummy destination to be zero bipolar single-valued neutrosophic numbers, then FBSNTP (38) can be transformed as follows.

Table 1: FFC transportation model.

Plants	Lahore	Peshawar	Supply
Gujranwala	$\prec([15,16,17];1,0,0.2),([16,17,19];-1,0,-0.3)\succ$	$\prec([20,22,24];1,0,0.3),([21,22,23];-1,0,-0.4)\succ$	$\prec([70,100,130];1,0,0.4),([55,100,155];-1,0,-0.3)\succ$
Karachi	$\prec([25,28,30];1,0,0.4),([27,28,29];-1,0,-0.5)\succ$	$\prec([30,40,50];1,0,0.1),([35,40,45];-1,0,-0.1)\succ$	$\prec([50,70,90];1,0,0.3),([35,70,115];-1,0,-0.2)\succ$
Demand	$\prec([50,60,70];1,0,0.2),([30,60,100];-1,0,-0.1)\succ$	$\prec([40,70,100];1,0,0.1),([40,70,100];-1,0,-0.4)\succ$	

4.2. Step 2.

$$\text{Minimize}\begin{pmatrix} \prec([15,17,18];1,0,0.2),([16,17,19];-1,0,-0.3)\succ\otimes\tilde{x}_{11}\oplus \\ \prec([20,22,24];1,0,0.3),([21,22,23];-1,0,-0.4)\succ\otimes\tilde{x}_{12}\oplus \\ \prec([0,0,0];1,0,0),([0,0,0];-1,0,0)\succ\otimes\tilde{x}_{13}\oplus \\ \prec([25,28,30];1,0,0.4),([27,28,29];-1,0,-0.5)\succ\otimes\tilde{x}_{21}\oplus \\ \prec([30,40,50];1,0,0.1),([35,40,45];-1,0,-0.1)\succ\otimes\tilde{x}_{22}\oplus \\ \prec([0,0,0];1,0,0),([0,0,0];-1,0,0)\succ\otimes\tilde{x}_{23}\oplus \\ \prec([0,0,0];1,0,0),([0,0,0];-1,0,0)\succ\otimes\tilde{x}_{31}\oplus \\ \prec([0,0,0];1,0,0),([0,0,0];-1,0,0)\succ\otimes\tilde{x}_{32}\oplus \\ \prec([0,0,0];1,0,0),([0,0,0];-1,0,0)\succ\otimes\tilde{x}_{33} \end{pmatrix}, \tag{40}$$

subject to

$$\begin{aligned} \tilde{x}_{11}\oplus\tilde{x}_{12}\oplus\tilde{x}_{13} &= \prec([70,100,130];1,0,0.4),([55,100,155];-1,0,-0.3)\succ, \\ \tilde{x}_{21}\oplus\tilde{x}_{22}\oplus\tilde{x}_{23} &= \prec([50,70,90];1,0,0.3),([35,70,115];-1,0,-0.2)\succ, \\ \tilde{x}_{31}\oplus\tilde{x}_{32}\oplus\tilde{x}_{33} &= \prec([10,30,30];1,0,0.1),([0,30,40];-1,0,-0.2)\succ, \\ \tilde{x}_{11}\oplus\tilde{x}_{21}\oplus\tilde{x}_{31} &= \prec([50,60,70];1,0,0.2),([30,60,100];-1,0,-0.1)\succ, \\ \tilde{x}_{12}\oplus\tilde{x}_{22}\oplus\tilde{x}_{32} &= \prec([40,70,100];1,0,0.1),([40,70,100];-1,0,-0.4)\succ, \\ \tilde{x}_{13}\oplus\tilde{x}_{23}\oplus\tilde{x}_{33} &= \prec([40,70,80];1,0,0.2),([20,70,110];-1,0,-0.3)\succ, \end{aligned} \tag{41}$$

where $\tilde{x}_{11}$, $\tilde{x}_{12}$, $\tilde{x}_{13}$, $\tilde{x}_{21}$, $\tilde{x}_{22}$, $\tilde{x}_{23}$, $\tilde{x}_{31}$, $\tilde{x}_{32}$, and $\tilde{x}_{33}$, are nonnegative TBSNNs.

4.3. Step 3. By assuming

$$\begin{aligned} \tilde{x}_{11} &= \prec([l_{11},m_{11},n_{11}];\pi_{11},\beta_{11},\phi_{11}),([l_{11}',m_{11}',n_{11}'];\pi_{11}',\beta_{11}',\phi_{11}')\succ, \\ \tilde{x}_{12} &= \prec([l_{12},m_{12},n_{12}];\pi_{12},\beta_{12},\phi_{12}),([l_{12}',m_{12}',n_{12}'];\pi_{12}',\beta_{12}',\phi_{12}')\succ, \\ \tilde{x}_{13} &= \prec([l_{13},m_{13},n_{13}];\pi_{13},\beta_{13},\phi_{13}),([l_{13}',m_{13}',n_{13}'];\pi_{13}',\beta_{13}',\phi_{13}')\succ, \\ \tilde{x}_{21} &= \prec([l_{21},m_{21},n_{21}];\pi_{21},\beta_{21},\phi_{21}),([l_{21}',m_{21}',n_{21}'];\pi_{21}',\beta_{21}',\phi_{21}')\succ, \\ \tilde{x}_{22} &= \prec([l_{22},m_{22},n_{22}];\pi_{22},\beta_{22},\phi_{22}),([l_{22}',m_{22}',n_{22}'];\pi_{22}',\beta_{22}',\phi_{22}')\succ, \\ \tilde{x}_{23} &= \prec([l_{23},m_{23},n_{23}];\pi_{23},\beta_{23},\phi_{23}),([l_{23}',m_{23}',n_{23}'];\pi_{23}',\beta_{23}',\phi_{23}')\succ, \\ \tilde{x}_{31} &= \prec([l_{31},m_{31},n_{31}];\pi_{31},\beta_{31},\phi_{31}),([l_{31}',m_{31}',n_{31}'];\pi_{31}',\beta_{31}',\phi_{31}')\succ, \\ \tilde{x}_{32} &= \prec([l_{32},m_{32},n_{32}];\pi_{32},\beta_{32},\phi_{32}),([l_{32}',m_{32}',n_{32}'];\pi_{32}',\beta_{32}',\phi_{32}')\succ, \\ \tilde{x}_{33} &= \prec([l_{33},m_{33},n_{33}];\pi_{33},\beta_{33},\phi_{33}),([l_{33}',m_{33}',n_{33}'];\pi_{33}',\beta_{33}',\phi_{33}')\succ, \end{aligned} \tag{42}$$

where $\tilde{x}_{11}$, $\tilde{x}_{12}$, $\tilde{x}_{13}$, $\tilde{x}_{21}$, $\tilde{x}_{22}$, $\tilde{x}_{23}$, $\tilde{x}_{31}$, $\tilde{x}_{32}$, and $\tilde{x}_{33}$ are nonnegative TBSNNs, then the FBSNPP (40) can be transformed as follows.

$$\text{Minimize}\left(\begin{array}{c}
\prec([15,17,18];1,0,0.2),([16,17,19];-1,0,-0.3)\succ\otimes\\
\prec([l_{11},m_{11},n_{11}];\pi_{11},\beta_{11},\phi_{11}),([l_{11}',m_{11}',n_{11}'];\pi_{11}',\beta_{11}',\phi_{11}')\succ\oplus\\
\prec([20,22,24];1,0,0.3),([21,22,23];-1,0,-0.4)\succ\otimes\\
\prec([l_{12},m_{12},n_{12}];\pi_{12},\beta_{12},\phi_{12}),([l_{12}',m_{12}',n_{12}'];\pi_{12}',\beta_{12}',\phi_{12}')\succ\oplus\\
\prec([0,0,0];1,0,0),([0,0,0];-1,0,0)\succ\otimes\\
\prec([l_{13},m_{13},n_{13}];\pi_{13},\beta_{13},\phi_{13}),([l_{13}',m_{13}',n_{13}'];\pi_{13}',\beta_{13}',\phi_{13}')\succ\oplus\\
\prec([25,28,30];1,0,0.4),([27,28,29];-1,0,-0.5)\succ\otimes\\
\prec([l_{21},m_{21},n_{21}];\pi_{21},\beta_{21},\phi_{21}),([l_{21}',m_{21}',n_{21}'];\pi_{21}',\beta_{21}',\phi_{21}')\succ\oplus\\
\prec([30,40,50];1,0,0.1),([35,40,45];-1,0,-0.1)\succ\otimes\\
\prec([l_{22},m_{22},n_{22}];\pi_{22},\beta_{22},\phi_{22}),([l_{22}',m_{22}',n_{22}'];\pi_{22}',\beta_{22}',\phi_{22}')\succ\oplus\\
\prec([0,0,0];1,0,0),([0,0,0];-1,0,0)\succ\otimes\\
\prec([l_{23},m_{23},n_{23}];\pi_{23},\beta_{23},\phi_{23}),([l_{23}',m_{23}',n_{23}'];\pi_{23}',\beta_{23}',\phi_{23}')\succ\oplus\\
\prec([0,0,0];1,0,0),([0,0,0];-1,0,0)\succ\otimes\\
\prec([l_{31},m_{31},n_{31}];\pi_{31},\beta_{31},\phi_{31}),([l_{31}',m_{31}',n_{31}'];\pi_{31}',\beta_{31}',\phi_{31}')\succ\oplus\\
\prec([0,0,0];1,0,0),([0,0,0];-1,0,0)\succ\otimes\\
\prec([l_{32},m_{32},n_{32}];\pi_{32},\beta_{32},\phi_{32}),([l_{32}',m_{32}',n_{32}'];\pi_{32}',\beta_{32}',\phi_{32}')\succ\oplus\\
\prec([0,0,0];1,0,0),([0,0,0];-1,0,0)\succ\otimes\\
\prec([l_{33},m_{33},n_{33}];\pi_{33},\beta_{33},\phi_{33}),([l_{33}',m_{33}',n_{33}'];\pi_{33}',\beta_{33}',\phi_{33}')\succ
\end{array}\right), \tag{43}$$

subject to

$$\begin{aligned}
&\prec([l_{11},m_{11},n_{11}];\pi_{11},\beta_{11},\phi_{11}),([l_{11}',m_{11}',n_{11}'];\pi_{11}',\beta_{11}',\phi_{11}')\succ\oplus\prec([l_{12},m_{12},n_{12}];\pi_{12},\beta_{12},\phi_{12}),\\
&([l_{12}',m_{12}',n_{12}'];\pi_{12}',\beta_{12}',\phi_{12}')\succ\oplus\prec([l_{13},m_{13},n_{13}];\pi_{13},\beta_{13},\phi_{13}),([l_{13}',m_{13}',n_{13}'];\pi_{13}',\beta_{13}',\phi_{13}')\succ\\
&\quad=\prec([70,100,130];1,0,0.4),([55,100,155];-1,0,-0.3)\succ\\
&\prec([l_{21},m_{21},n_{21}];\pi_{21},\beta_{21},\phi_{21}),([l_{21}',m_{21}',n_{21}'];\pi_{21}',\beta_{21}',\phi_{21}')\succ\oplus\prec([l_{22},m_{22},n_{22}];\pi_{22},\beta_{22},\phi_{22}),\\
&([l_{22}',m_{22}',n_{22}'];\pi_{22}',\beta_{22}',\phi_{22}')\succ\oplus\prec([l_{23},m_{23},n_{23}];\pi_{23},\beta_{23},\phi_{23}),([l_{23}',m_{23}',n_{23}'];\pi_{23}',\beta_{23}',\phi_{23}')\succ\\
&\quad=\prec([50,70,90];1,0,0.3),([35,70,115];-1,0,-0.2)\succ\\
&\prec([l_{31},m_{31},n_{31}];\pi_{31},\beta_{31},\phi_{31}),([l_{31}',m_{31}',n_{31}'];\pi_{31}',\beta_{31}',\phi_{31}')\succ\oplus\prec([l_{32},m_{32},n_{32}];\pi_{32},\beta_{32},\phi_{32}),\\
&([l_{32}',m_{32}',n_{32}'];\pi_{32}',\beta_{32}',\phi_{32}')\succ\oplus\prec([l_{33},m_{33},n_{33}];\pi_{33},\beta_{33},\phi_{33}),([l_{33}',m_{33}',n_{33}'];\pi_{33}',\beta_{33}',\phi_{33}')\succ\\
&\quad=\prec([10,30,30];1,0,0.1),([0,30,40];-1,0,-0.2)\succ\\
&\prec([l_{11},m_{11},n_{11}];\pi_{11},\beta_{11},\phi_{11}),([l_{11}',m_{11}',n_{11}'];\pi_{11}',\beta_{11}',\phi_{11}')\succ\oplus\prec([l_{21},m_{21},n_{21}];\pi_{21},\beta_{21},\phi_{21}),\\
&([l_{21}',m_{21}',n_{21}'];\pi_{21}',\beta_{21}',\phi_{21}')\succ\oplus\prec([l_{31},m_{31},n_{31}];\pi_{31},\beta_{31},\phi_{31}),([l_{31}',m_{31}',n_{31}'];\pi_{31}',\beta_{31}',\phi_{31}')\succ\\
&\quad=\prec([50,60,70];1,0,0.2),([30,60,100];-1,0,-0.1)\succ\\
&\prec([l_{12},m_{12},n_{12}];\pi_{12},\beta_{12},\phi_{12}),([l_{12}',m_{12}',n_{12}'];\pi_{12}',\beta_{12}',\phi_{12}')\succ\oplus\prec([l_{22},m_{22},n_{22}];\pi_{22},\beta_{22},\phi_{22}),\\
&([l_{22}',m_{22}',n_{22}'];\pi_{22}',\beta_{22}',\phi_{22}')\succ\oplus\prec([l_{32},m_{32},n_{32}];\pi_{32},\beta_{32},\phi_{32}),([l_{32}',m_{32}',n_{32}'];\pi_{32}',\beta_{32}',\phi_{32}')\succ\\
&\quad=\prec([40,70,100];1,0,0.1),([40,70,100];-1,0,-0.4)\succ\\
&\prec([l_{13},m_{13},n_{13}];\pi_{13},\beta_{13},\phi_{13}),([l_{13}',m_{13}',n_{13}'];\pi_{13}',\beta_{13}',\phi_{13}')\succ\oplus\prec([l_{23},m_{23},n_{23}];\pi_{23},\beta_{23},\phi_{23}),\\
&([l_{23}',m_{23}',n_{23}'];\pi_{23}',\beta_{23}',\phi_{23}')\succ\oplus\prec([l_{33},m_{33},n_{33}];\pi_{33},\beta_{33},\phi_{33}),([l_{33}',m_{33}',n_{33}'];\pi_{33}',\beta_{33}',\phi_{33}')\succ\\
&\quad=\prec([40,70,80];1,0,0.2),([20,70,110];-1,0,-0.3)\succ.
\end{aligned} \tag{44}$$

4.4. Step 4.

4.5. Step 5. By applying arithmetic operations, the FBSNTP (43) can be transformed as

$$
\text{Minimize}\left(\begin{array}{c}
\prec\left(\left[15l_{11},17m_{11},18n_{11}\right];1\wedge\pi_{11},0\vee\beta_{11},0.2\vee\phi_{11}\right),\left(\left[16l_{11}',17m_{11}',19n_{11}'\right];\right.\\
\left.-1\vee\pi_{11}',0\wedge\beta_{11}',-0.3\wedge\phi_{11}'\right)\succ\oplus\prec\left(\left[20l_{12},22m_{12},24n_{12}\right];1\wedge\pi_{12},0\vee\beta_{12},0.3\vee\phi_{12}\right),\\
\left(\left[21l_{12}',22m_{12}',23n_{12}'\right];-1\vee\pi_{12}',0\wedge\beta_{12}',-0.4\wedge\phi_{12}'\right)\succ\oplus\prec\left(\begin{array}{c}\left[0l_{13},0m_{13},0n_{13}\right];\\ 1\wedge\pi_{13},0\vee\beta_{13},0\vee\phi_{13}\end{array}\right),\left(\left[0l_{13}',0m_{13}',0n_{13}'\right];-1\vee\pi_{13}',0\wedge\beta_{13}',0\wedge\phi_{13}'\right)\succ\oplus\\
\prec\left(\left[25l_{21},28m_{21},30n_{21}\right];1\wedge\pi_{21},0\vee\beta_{21},0.4\vee\phi_{21}\right),\left(\left[27l_{21}',28m_{21}',29n_{21}'\right];\right.\\
\left.-1\vee\pi_{21}',0\wedge\beta_{21}',-0.5\wedge\phi_{21}'\right)\succ\oplus\prec\left(\left[30l_{22},40m_{22},50n_{22}\right];1\wedge\pi_{22},0\vee\beta_{22},0.1\vee\phi_{22}\right),\\
\left(\left[35l_{22}',40m_{22}',45n_{22}'\right];-1\vee\pi_{22}',0\wedge\beta_{22}',-0.1\wedge\phi_{22}'\right)\succ\oplus\prec\left(\left[0l_{23},0m_{23},0n_{23}\right];\right.\\
\left.1\wedge\pi_{23},0\vee\beta_{23},0\vee\phi_{23}\right),\left(\left[0l_{23}',0m_{23}',0n_{23}'\right];-1\vee\pi_{23}',0\wedge\beta_{23}',0\wedge\phi_{23}'\right)\succ\oplus\\
\prec\left(\left[0l_{31},0m_{31},0n_{31}\right];1\wedge\pi_{31},0\vee\beta_{31},0\vee\phi_{31}\right),\left(\left[0l_{31}',0m_{31}',0n_{31}'\right];\right.\\
\left.-1\vee\pi_{31}',0\wedge\beta_{31}',0\wedge\phi_{31}'\right)\succ\oplus\prec\left(\left[0l_{32},0m_{32},0n_{32}\right];1\wedge\pi_{32},0\vee\beta_{32},0\vee\phi_{32}\right),\\
\left(\left[0l_{32}',0m_{32}',0n_{32}'\right];-1\vee\pi_{32}',0\wedge\beta_{32}',0\wedge\phi_{32}'\right)\succ\oplus\prec\left(\left[0l_{33},0m_{33},0n_{33}\right];\right.\\
\left.1\wedge\pi_{33},0\vee\beta_{33},0\vee\phi_{33}\right),\left(\left[0l_{33}',0m_{33}',0n_{33}'\right];-1\vee\pi_{33}',0\wedge\beta_{33}',0\wedge\phi_{33}'\right)\succ
\end{array}\right), \tag{45}
$$

subject to

$$
\begin{array}{l}
\left[\left(\sum_{j=1}^{3}l_{1j}=70,\sum_{j=1}^{3}m_{1j}=100,\sum_{j=1}^{3}n_{1j}=130,\ \sum_{j=1}^{3}l_{1j}'=55,\sum_{j=1}^{3}m_{1j}'=100,\sum_{j=1}^{3}n_{1j}'=155,\right);\left(\begin{array}{c}\wedge\left[\pi_{11}\wedge\pi_{12}\wedge\pi_{13}\right]=1\\ \vee\left[\beta_{11}\vee\beta_{12}\vee\beta_{13}\right]=0\\ \vee\left[\phi_{11}\vee\phi_{12}\vee\phi_{13}\right]=0.4\\ \vee\left[\pi_{11}'\vee\pi_{12}'\vee\pi_{13}'\right]=-1\\ \wedge\left[\beta_{11}'\wedge\beta_{12}'\wedge\beta_{13}'\right]=0\\ \wedge\left[\phi_{11}'\wedge\phi_{12}'\wedge\phi_{13}'\right]=-0.3\end{array}\right)\right]\\
\left[\left(\sum_{j=1}^{3}l_{2j}=50,\sum_{j=1}^{3}m_{2j}=70,\sum_{j=1}^{3}n_{2j}=90,\ \sum_{j=1}^{3}l_{2j}'=35,\sum_{j=1}^{3}m_{2j}'=70,\sum_{j=1}^{3}n_{2j}'=115,\right);\left(\begin{array}{c}\wedge\left[\pi_{21}\wedge\pi_{22}\wedge\pi_{23}\right]=1\\ \vee\left[\beta_{21}\vee\beta_{22}\vee\beta_{23}\right]=0\\ \vee\left[\phi_{21}\vee\phi_{22}\vee\phi_{23}\right]=0.3\\ \vee\left[\pi_{21}'\vee\pi_{22}'\vee\pi_{23}'\right]=-1\\ \wedge\left[\beta_{21}'\wedge\beta_{22}'\wedge\beta_{23}'\right]=0\\ \wedge\left[\phi_{21}'\wedge\phi_{22}'\wedge\phi_{23}'\right]=-0.2\end{array}\right)\right]\\
\left[\left(\sum_{j=1}^{3}l_{3j}=10,\sum_{j=1}^{3}m_{3j}=30,\sum_{j=1}^{3}n_{3j}=30,\ \sum_{j=1}^{3}l_{3j}'=0,\sum_{j=1}^{3}m_{3j}'=30,\sum_{j=1}^{3}n_{3j}'=40,\right);\left(\begin{array}{c}\wedge\left[\pi_{31}\wedge\pi_{32}\wedge\pi_{33}\right]=1\\ \vee\left[\beta_{31}\vee\beta_{32}\vee\beta_{33}\right]=0\\ \vee\left[\phi_{31}\vee\phi_{32}\vee\phi_{33}\right]=0.1\\ \vee\left[\pi_{31}'\vee\pi_{32}'\vee\pi_{33}'\right]=-1\\ \wedge\left[\beta_{31}'\wedge\beta_{32}'\wedge\beta_{33}'\right]=0\\ \wedge\left[\phi_{31}'\wedge\phi_{32}'\wedge\phi_{33}'\right]=-0.2\end{array}\right)\right],
\end{array}
$$

$$\begin{bmatrix}\left(\sum_{k=1}^{3} l_{k1}=50, \sum_{k=1}^{3} m_{k1}=60, \sum_{k=1}^{3} n_{k1}=70,\ \sum_{k=1}^{3} l_{k1}'=30, \sum_{k=1}^{3} m_{k1}'=60, \sum_{k=1}^{3} n_{k1}'=100,\right); \begin{pmatrix}\wedge[\pi_{11}\wedge\pi_{21}\wedge\pi_{31}]=1\\ \vee[\beta_{11}\vee\beta_{21}\vee\beta_{31}]=0\\ \vee[\phi_{11}\vee\phi_{21}\vee\phi_{31}]=0.2\\ \vee[\pi_{11}'\vee\pi_{21}'\vee\pi_{31}']=-1\\ \wedge[\beta_{11}'\wedge\beta_{21}'\wedge\beta_{31}']=0\\ \wedge[\phi_{11}'\wedge\phi_{21}'\wedge\phi_{31}']=-0.1\end{pmatrix}\end{bmatrix}$$
$$\begin{bmatrix}\left(\sum_{k=1}^{3} l_{k2}=40, \sum_{k=1}^{3} m_{k2}=70, \sum_{k=1}^{3} n_{k2}=100,\ \sum_{k=1}^{3} l_{k2}'=40, \sum_{k=1}^{3} m_{k2}'=70, \sum_{k=1}^{3} n_{k2}'=100,\right); \begin{pmatrix}\wedge[\pi_{12}\wedge\pi_{22}\wedge\pi_{32}]=1\\ \vee[\beta_{12}\vee\beta_{22}\vee\beta_{32}]=0\\ \vee[\phi_{12}\vee\phi_{22}\vee\phi_{32}]=0.1\\ \vee[\pi_{12}'\vee\pi_{22}'\vee\pi_{32}']=-1\\ \wedge[\beta_{12}'\wedge\beta_{22}'\wedge\beta_{32}']=0\\ \wedge[\phi_{12}'\wedge\phi_{22}'\wedge\phi_{32}']=-0.4\end{pmatrix}\end{bmatrix}$$
$$\begin{bmatrix}\left(\sum_{k=1}^{3} l_{k3}=40, \sum_{k=1}^{3} m_{k3}=70, \sum_{k=1}^{3} n_{k3}=80,\ \sum_{k=1}^{3} l_{k3}'=20, \sum_{k=1}^{3} m_{k3}'=70, \sum_{k=1}^{3} n_{k3}'=110,\right); \begin{pmatrix}\wedge[\pi_{13}\wedge\pi_{23}\wedge\pi_{33}]=1\\ \vee[\beta_{13}\vee\beta_{23}\vee\beta_{33}]=0\\ \vee[\phi_{13}\vee\phi_{23}\vee\phi_{33}]=0.2\\ \vee[\pi_{13}'\vee\pi_{23}'\vee\pi_{33}']=-1\\ \wedge[\beta_{13}'\wedge\beta_{23}'\wedge\beta_{33}']=0\\ \wedge[\phi_{13}'\wedge\phi_{23}'\wedge\phi_{33}']=-0.3\end{pmatrix}\end{bmatrix}, \quad (46)$$

where $\pi_{kj}, \beta_{kj}, \phi_{kj} \in [0,1]$, $\pi_{kj}', \beta_{kj}', \phi_{kj}' \in [-1,0]$ and l_{kj}, $m_{kj} - l_{kj}$, $n_{kj} - m_{kj} \geq 0, l_{kj}'$, $m_{kj}' - l_{kj}'$, $n_{kj}' - m_{kj}' \geq 0$; $k = 1,2,3$; $j = 1,2,3$.

4.6. Step 6. By applying score function Definition 8, the FBSNTP (45) can be transformed as

$$\text{Minimize } S\left[\begin{pmatrix} 15l_{11}+20l_{12}+0l_{13}+25l_{21}+30l_{22}+0l_{23}+0l_{31}+0l_{32}+0l_{33}\\ +17m_{11}+22m_{12}+0m_{13}+28m_{21}+40m_{22}+0m_{23}+0m_{31}+0m_{32}+0m_{33}\\ +18n_{11}+24n_{12}+0n_{13}+30n_{21}+50n_{22}+0n_{23}+0n_{31}+0n_{32}+0n_{33}\\ +16l_{11}'+21l_{12}'+0l_{13}'+27l_{21}'+35l_{22}'+0l_{23}'+0l_{31}'+0l_{32}'+0l_{33}'+\\ 17m_{11}'+22m_{12}'+0m_{13}'+28m_{21}'+40m_{22}'+0m_{23}'+0m_{31}'+0m_{32}'+0m_{33}'+\\ 19n_{11}'+23n_{12}'+0n_{13}'+29n_{21}'+45n_{22}'+0n_{23}'+0n_{31}'+0n_{32}'+0n_{33}'\end{pmatrix}\right], \quad (47)$$

subject to

$$\begin{bmatrix}\left(\sum_{j=1}^{3} l_{1j}=70, \sum_{j=1}^{3} m_{1j}=100, \sum_{j=1}^{3} n_{1j}=130,\ \sum_{j=1}^{3} l_{1j}'=55, \sum_{j=1}^{3} m_{1j}'=100, \sum_{j=1}^{3} n_{1j}'=155,\right); \begin{pmatrix}\wedge[\pi_{11}\wedge\pi_{12}\wedge\pi_{13}]=1\\ \vee[\beta_{11}\vee\beta_{12}\vee\beta_{13}]=0\\ \vee[\phi_{11}\vee\phi_{12}\vee\phi_{13}]=0.4\\ \vee[\pi_{11}'\vee\pi_{12}'\vee\pi_{13}']=-1\\ \wedge[\beta_{11}'\wedge\beta_{12}'\wedge\beta_{13}']=0\\ \wedge[\phi_{11}'\wedge\phi_{12}'\wedge\phi_{13}']=-0.3\end{pmatrix}\end{bmatrix},$$

$$\left[\left(\sum_{j=1}^{3} l_{2j} = 50, \sum_{j=1}^{3} m_{2j} = 70, \sum_{j=1}^{3} n_{2j} = 90,\ \sum_{j=1}^{3} l_{2j}' = 35, \sum_{j=1}^{3} m_{2j}' = 70, \sum_{j=1}^{3} n_{2j}' = 115,\right); \begin{pmatrix} \wedge[\pi_{21}\wedge\pi_{22}\wedge\pi_{23}] = 1 \\ \vee[\beta_{21}\vee\beta_{22}\vee\beta_{23}] = 0 \\ \vee[\phi_{21}\vee\phi_{22}\vee\phi_{23}] = 0.3 \\ \vee[\pi_{21}'\vee\pi_{22}'\vee\pi_{23}'] = -1 \\ \wedge[\beta_{21}'\wedge\beta_{22}'\wedge\beta_{23}'] = 0 \\ \wedge[\phi_{21}'\wedge\phi_{22}'\wedge\phi_{23}'] = -0.2 \end{pmatrix}\right]$$

$$\left[\left(\sum_{j=1}^{3} l_{3j} = 10, \sum_{j=1}^{3} m_{3j} = 30, \sum_{j=1}^{3} n_{3j} = 30,\ \sum_{j=1}^{3} l_{3j}' = 0, \sum_{j=1}^{3} m_{3j}' = 30, \sum_{j=1}^{3} n_{3j}' = 40,\right); \begin{pmatrix} \wedge[\pi_{31}\wedge\pi_{32}\wedge\pi_{33}] = 1 \\ \vee[\beta_{31}\vee\beta_{32}\vee\beta_{33}] = 0 \\ \vee[\phi_{31}\vee\phi_{32}\vee\phi_{33}] = 0.1 \\ \vee[\pi_{31}'\vee\pi_{32}'\vee\pi_{33}'] = -1 \\ \wedge[\beta_{31}'\wedge\beta_{32}'\wedge\beta_{33}'] = 0 \\ \wedge[\phi_{31}'\wedge\phi_{32}'\wedge\phi_{33}'] = -0.2 \end{pmatrix}\right]$$

$$\left[\left(\sum_{k=1}^{3} l_{k1} = 50, \sum_{k=1}^{3} m_{k1} = 60, \sum_{k=1}^{3} n_{k1} = 70,\ \sum_{k=1}^{3} l_{k1}' = 30, \sum_{k=1}^{3} m_{k1}' = 60, \sum_{k=1}^{3} n_{k1}' = 100,\right); \begin{pmatrix} \wedge[\pi_{11}\wedge\pi_{21}\wedge\pi_{31}] = 1 \\ \vee[\beta_{11}\vee\beta_{21}\vee\beta_{31}] = 0 \\ \vee[\phi_{11}\vee\phi_{21}\vee\phi_{31}] = 0.2 \\ \vee[\pi_{11}'\vee\pi_{21}'\vee\pi_{31}'] = -1 \\ \wedge[\beta_{11}'\wedge\beta_{21}'\wedge\beta_{31}'] = 0 \\ \wedge[\phi_{11}'\wedge\phi_{21}'\wedge\phi_{31}'] = -0.1 \end{pmatrix}\right]$$

$$\left[\left(\sum_{k=1}^{3} l_{k2} = 40, \sum_{k=1}^{3} m_{k2} = 70, \sum_{k=1}^{3} n_{k2} = 100,\ \sum_{k=1}^{3} l_{k2}' = 40, \sum_{k=1}^{3} m_{k2}' = 70, \sum_{k=1}^{3} n_{k2}' = 100,\right); \begin{pmatrix} \wedge[\pi_{12}\wedge\pi_{22}\wedge\pi_{32}] = 1 \\ \vee[\beta_{12}\vee\beta_{22}\vee\beta_{32}] = 0 \\ \vee[\phi_{12}\vee\phi_{22}\vee\phi_{32}] = 0.1 \\ \vee[\pi_{12}'\vee\pi_{22}'\vee\pi_{32}'] = -1 \\ \wedge[\beta_{12}'\wedge\beta_{22}'\wedge\beta_{32}'] = 0 \\ \wedge[\phi_{12}'\wedge\phi_{22}'\wedge\phi_{32}'] = -0.4 \end{pmatrix}\right]$$

$$\left[\left(\sum_{k=1}^{3} l_{k3} = 40, \sum_{k=1}^{3} m_{k3} = 70, \sum_{k=1}^{3} n_{k3} = 80,\ \sum_{k=1}^{3} l_{k3}' = 20, \sum_{k=1}^{3} m_{k3}' = 70, \sum_{k=1}^{3} n_{k3}' = 110,\right); \begin{pmatrix} \wedge[\pi_{13}\wedge\pi_{23}\wedge\pi_{33}] = 1 \\ \vee[\beta_{13}\vee\beta_{23}\vee\beta_{33}] = 0 \\ \vee[\phi_{13}\vee\phi_{23}\vee\phi_{33}] = 0.2 \\ \vee[\pi_{13}'\vee\pi_{23}'\vee\pi_{33}'] = -1 \\ \wedge[\beta_{13}'\wedge\beta_{23}'\wedge\beta_{33}'] = 0 \\ \wedge[\phi_{13}'\wedge\phi_{23}'\wedge\phi_{33}'] = -0.3 \end{pmatrix}\right], \tag{48}$$

where $\pi_{kj}, \beta_{kj}, \phi_{kj} \in [0,1]$, $\pi_{kj}', \beta_{kj}', \phi_{kj}' \in [-1,0]$, and l_{kj}, $m_{kj} - l_{kj}$, $n_{kj} - m_{kj} \geq 0$, l_{kj}', $m_{kj}' - l_{kj}'$, $n_{kj}' - m_{kj}' \geq 0$; $k = 1,2,3$; $j = 1,2,3$.

4.7. Step 7. Now, we solve the following crisp linear programming problem as

$$\text{Minimize}\,\frac{1}{6}\begin{pmatrix} 15l_{11} + 20l_{12} + 0l_{13} + 25l_{21} + 30l_{22} + 0l_{23} + 0l_{31} + 0l_{32} + 0l_{33} \\ + 17m_{11} + 22m_{12} + 0m_{13} + 28m_{21} + 40m_{22} + 0m_{23} + 0m_{31} + 0m_{32} + 0m_{33} \\ + 18n_{11} + 24n_{12} + 0n_{13} + 30n_{21} + 50n_{22} + 0n_{23} + 0n_{31} + 0n_{32} + 0n_{33} \\ + 16l_{11}' + 21l_{12}' + 0l_{13}' + 27l_{21}' + 35l_{22}' + 0l_{23}' + 0l_{31}' + 0l_{32}' + 0l_{33}' + \\ 17m_{11}' + 22m_{12}' + 0m_{13}' + 28m_{21}' + 40m_{22}' + 0m_{23}' + 0m_{31}' + 0m_{32}' + 0m_{33}' + \\ 19n_{11}' + 23n_{12}' + 0n_{13}' + 29n_{21}' + 45n_{22}' + 0n_{23}' + 0n_{31}' + 0n_{32}' + 0n_{33}' \end{pmatrix}, \tag{49}$$

subject to

$$\begin{aligned}
&\left[\left(\sum_{j=1}^{3} l_{1j} = 70, \sum_{j=1}^{3} m_{1j} = 100, \sum_{j=1}^{3} n_{1j} = 130,\ \sum_{j=1}^{3} l_{1j}' = 55, \sum_{j=1}^{3} m_{1j}' = 100, \sum_{j=1}^{3} n_{1j}' = 155,\right); \begin{pmatrix} \wedge[\pi_{11}\wedge\pi_{12}\wedge\pi_{13}] = 1 \\ \vee[\beta_{11}\vee\beta_{12}\vee\beta_{13}] = 0 \\ \vee[\phi_{11}\vee\phi_{12}\vee\phi_{13}] = 0.4 \\ \vee[\pi_{11}'\vee\pi_{12}'\vee\pi_{13}'] = -1 \\ \wedge[\beta_{11}'\wedge\beta_{12}'\wedge\beta_{13}'] = 0 \\ \wedge[\phi_{11}'\wedge\phi_{12}'\wedge\phi_{13}'] = -0.3 \end{pmatrix}\right] \\
&\left[\left(\sum_{j=1}^{3} l_{2j} = 50, \sum_{j=1}^{3} m_{2j} = 70, \sum_{j=1}^{3} n_{2j} = 90,\ \sum_{j=1}^{3} l_{2j}' = 35, \sum_{j=1}^{3} m_{2j}' = 70, \sum_{j=1}^{3} n_{2j}' = 115,\right); \begin{pmatrix} \wedge[\pi_{21}\wedge\pi_{22}\wedge\pi_{23}] = 1 \\ \vee[\beta_{21}\vee\beta_{22}\vee\beta_{23}] = 0 \\ \vee[\phi_{21}\vee\phi_{22}\vee\phi_{23}] = 0.3 \\ \vee[\pi_{21}'\vee\pi_{22}'\vee\pi_{23}'] = -1 \\ \wedge[\beta_{21}'\wedge\beta_{22}'\wedge\beta_{23}'] = 0 \\ \wedge[\phi_{21}'\wedge\phi_{22}'\wedge\phi_{23}'] = -0.2 \end{pmatrix}\right] \\
&\left[\left(\sum_{j=1}^{3} l_{3j} = 10, \sum_{j=1}^{3} m_{3j} = 30, \sum_{j=1}^{3} n_{3j} = 30,\ \sum_{j=1}^{3} l_{3j}' = 0, \sum_{j=1}^{3} m_{3j}' = 30, \sum_{j=1}^{3} n_{3j}' = 40,\right); \begin{pmatrix} \wedge[\pi_{31}\wedge\pi_{32}\wedge\pi_{33}] = 1 \\ \vee[\beta_{31}\vee\beta_{32}\vee\beta_{33}] = 0 \\ \vee[\phi_{31}\vee\phi_{32}\vee\phi_{33}] = 0.1 \\ \vee[\pi_{31}'\vee\pi_{32}'\vee\pi_{33}'] = -1 \\ \wedge[\beta_{31}'\wedge\beta_{32}'\wedge\beta_{33}'] = 0 \\ \wedge[\phi_{31}'\wedge\phi_{32}'\wedge\phi_{33}'] = -0.2 \end{pmatrix}\right] \\
&\left[\left(\sum_{k=1}^{3} l_{k1} = 50, \sum_{k=1}^{3} m_{k1} = 60, \sum_{k=1}^{3} n_{k1} = 70,\ \sum_{k=1}^{3} l_{k1}' = 30, \sum_{k=1}^{3} m_{k1}' = 60, \sum_{k=1}^{3} n_{k1}' = 100,\right); \begin{pmatrix} \wedge[\pi_{11}\wedge\pi_{21}\wedge\pi_{31}] = 1 \\ \vee[\beta_{11}\vee\beta_{21}\vee\beta_{31}] = 0 \\ \vee[\phi_{11}\vee\phi_{21}\vee\phi_{31}] = 0.2 \\ \vee[\pi_{11}'\vee\pi_{21}'\vee\pi_{31}'] = -1 \\ \wedge[\beta_{11}'\wedge\beta_{21}'\wedge\beta_{31}'] = 0 \\ \wedge[\phi_{11}'\wedge\phi_{21}'\wedge\phi_{31}'] = -0.1 \end{pmatrix}\right], \\
&\left[\left(\sum_{k=1}^{3} l_{k2} = 40, \sum_{k=1}^{3} m_{k2} = 70, \sum_{k=1}^{3} n_{k2} = 100,\ \sum_{k=1}^{3} l_{k2}' = 40, \sum_{k=1}^{3} m_{k2}' = 70, \sum_{k=1}^{3} n_{k2}' = 100,\right); \begin{pmatrix} \wedge[\pi_{12}\wedge\pi_{22}\wedge\pi_{32}] = 1 \\ \vee[\beta_{12}\vee\beta_{22}\vee\beta_{32}] = 0 \\ \vee[\phi_{12}\vee\phi_{22}\vee\phi_{32}] = 0.1 \\ \vee[\pi_{12}'\vee\pi_{22}'\vee\pi_{32}'] = -1 \\ \wedge[\beta_{12}'\wedge\beta_{22}'\wedge\beta_{32}'] = 0 \\ \wedge[\phi_{12}'\wedge\phi_{22}'\wedge\phi_{32}'] = -0.4 \end{pmatrix}\right] \\
&\left[\left(\sum_{k=1}^{3} l_{k3} = 40, \sum_{k=1}^{3} m_{k3} = 70, \sum_{k=1}^{3} n_{k3} = 80,\ \sum_{k=1}^{3} l_{k3}' = 20, \sum_{k=1}^{3} m_{k3}' = 70, \sum_{k=1}^{3} n_{k3}' = 110,\right); \begin{pmatrix} \wedge[\pi_{13}\wedge\pi_{23}\wedge\pi_{33}] = 1 \\ \vee[\beta_{13}\vee\beta_{23}\vee\beta_{33}] = 0 \\ \vee[\phi_{13}\vee\phi_{23}\vee\phi_{33}] = 0.2 \\ \vee[\pi_{13}'\vee\pi_{23}'\vee\pi_{33}'] = -1 \\ \wedge[\beta_{13}'\wedge\beta_{23}'\wedge\beta_{33}'] = 0 \\ \wedge[\phi_{13}'\wedge\phi_{23}'\wedge\phi_{33}'] = -0.3 \end{pmatrix}\right]
\end{aligned} \tag{50}$$

Table 2: Comparison of optimal values.

FBSNTP	$\prec([2050, 3470, 5300]; 1, 0, 0.4), ([1810, 3915, 6490]; -1, 0, -0.5)\succ$
FIFTP [5]	(2050, 3470, 5300); (2050, 3470, 5300)
FFTP [1]	(2050, 3470, 5300)

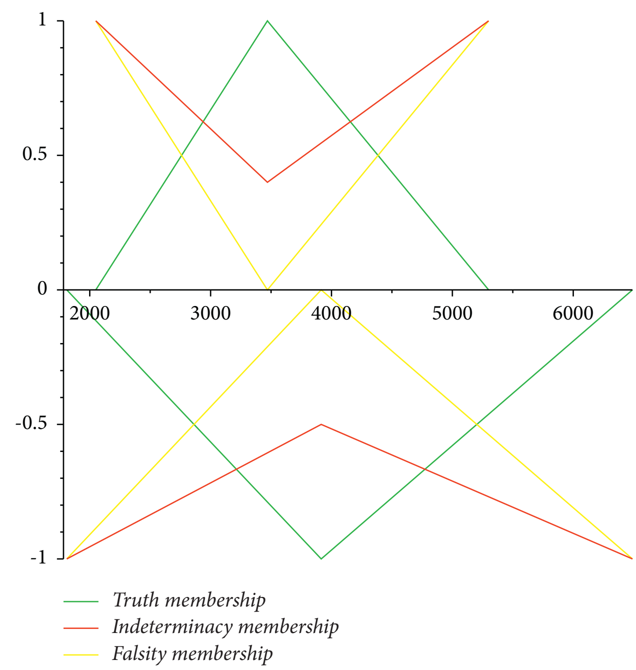

Figure 1: Graphical representation of bipolar single-valued neutrosophic transportation cost discussed in Example 1.

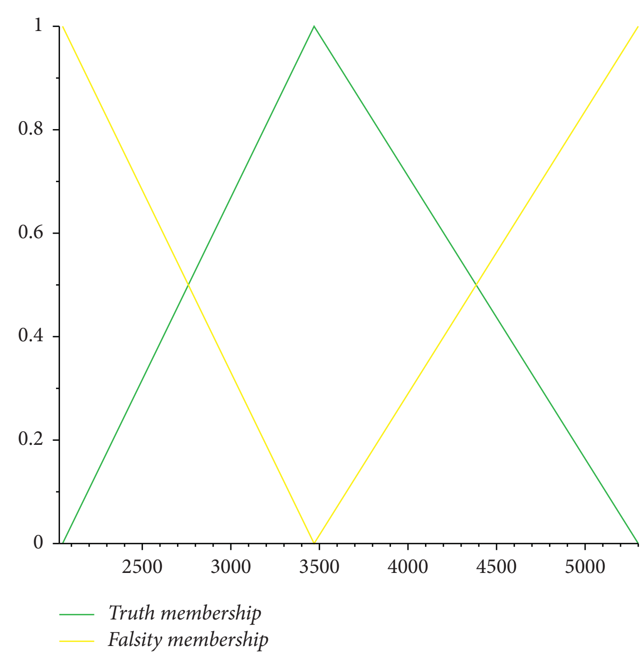

Figure 2: Graphical representation of intuitionistic transportation cost [5].

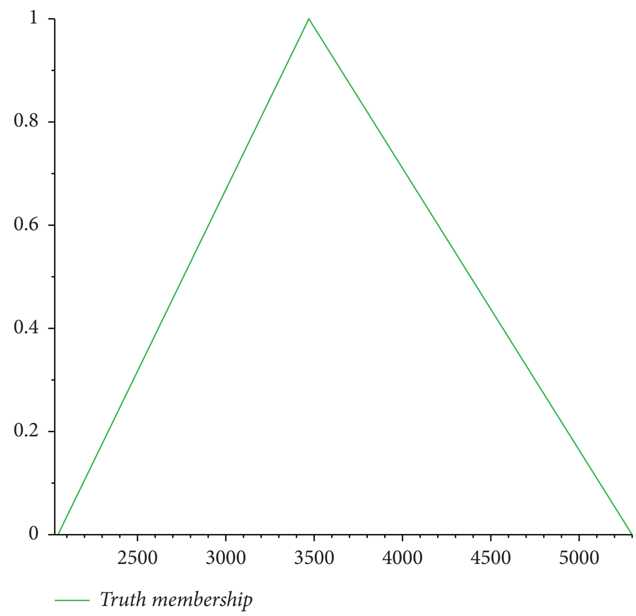

FIGURE 3: Graphical representation of fuzzy transportation cost [1].

where $\pi_{kj}, \beta_{kj}, \phi_{kj} \in [0,1]$, $\pi'_{kj}, \beta'_{kj}, \phi'_{kj} \in [-1,0]$, and l_{kj}, $m_{kj} - l_{kj}$, $n_{kj} - m_{kj} \geq 0$, l'_{kj}, $m'_{kj} - l'_{kj}$, $n'_{kj} - m'_{kj} \geq 0$; $k = 1,2,3$; $j = 1,2,3$.

4.8. *Step 8.*

By using the software MAPLE, we get optimal solution

$$\begin{aligned}
&l_{11} = 0,\ l_{12} = 40,\ l_{13} = 30, l_{21} = 50,\ l_{22} = 0,\ l_{23} = 0,\ l_{31} = 0,\ l_{32} = 0,\ l_{33} = 10,\ m_{11} = 10,\\
&m_{12} = 50,\ m_{13} = 40,\ m_{21} = 50,\ m_{22} = 20,\ m_{23} = 0,\ m_{31} = 0,\ m_{32} = 0,\ m_{33} = 30,\ n_{11} = 20,\\
&n_{12} = 60,\ n_{13} = 50,\ n_{21} = 50,\ n_{22} = 40,\ n_{23} = 0,\ n_{31} = 0,\ n_{32} = 0,\ n_{33} = 30,\ l'_{11} = 30,\ l'_{12} = 5,\\
&l'_{13} = 20,\ l'_{21} = 0,\ l'_{22} = 35,\ l'_{23} = 0,\ l'_{31} = 0,\ l'_{32} = 0,\ l'_{33} = 0,\ m'_{11} = 55,\ m'_{12} = 5,\ m'_{13} = 40,\\
&m'_{21} = 5,\ m'_{22} = 65,\ m'_{23} = 0,\ m'_{31} = 0,\ m'_{32} = 0,\ m'_{33} = 30,\ n'_{11} = 80,\ n'_{12} = 5,\ n'_{13} = 70,\ n'_{21} = 20,\\
&n'_{22} = 95,\ n'_{23} = 0,\ n'_{31} = 0,\ n'_{32} = 0,\ n'_{33} = 40.
\end{aligned} \tag{51}$$

4.9. *Step 9.*

The bipolar single-valued neutrosophic optimal solution is

$$\begin{aligned}
\tilde{x}_{11} &= \prec([0,10,20];1,0,0),\ ([30,55,80];-1,0,0),\\
\tilde{x}_{12} &= \prec([40,50,60];1,0,0),\ ([5,5,5];-1,0,0)\succ,\\
\tilde{x}_{13} &= \prec([30,40,50];1,0,0),\ ([20,40,70];-1,0,0)\succ,\\
\tilde{x}_{21} &= \prec([50,50,50];1,0,0),\ ([0,5,20];-1,0,0)\succ,\\
\tilde{x}_{22} &= \prec([0,20,40];1,0,0),\ ([35,65,95];-1,0,0)\succ,\\
\tilde{x}_{23} &= \prec([0,0,0];1,0,0),\ ([0,0,0];-1,0,0)\succ,\\
\tilde{x}_{31} &= \prec([0,0,0];1,0,0),\ ([0,0,0];-1,0,0)\succ,\\
\tilde{x}_{32} &= \prec([0,0,0];1,0,0),\ ([0,0,0];-1,0,0)\succ,\\
\tilde{x}_{33} &= \prec([10,30,30];1,0,0),\ ([0,30,40];-1,0,0)\succ.
\end{aligned} \tag{52}$$

4.10. *Step 10.*

The minimum bipolar single-valued neutrosophic optimal value of FBSNTP is

$$\begin{aligned}
&\prec([2050,3470,5300];1,0,0.4),\\
&\quad([1810,3915,6490];-1,0,-0.5)\succ.
\end{aligned} \tag{53}$$

5. Comparison with Existing Transportation Model

Singh et al. [5] and Basirzadeh [1] suggested different techniques to solve intuitionistic fuzzy transportation problems and fuzzy transportation problems, respectively. We have proposed a method to solve an unbalanced FBSNTP. By using our proposed method to Example 1, which is discussed in Section 3, we have obtained the minimum total single-valued neutrosophic transportation cost $\prec([2050,3470,5300];1,0,0.4)$,

Table 3: FFC transportation model.

Plants	Lahore	Peshawar	Supply
Gujranwala	$\prec([15,16,17];1,0,0.2),([16,17,19];-1,0,-0.3)\succ$	$\prec([20,22,24];1,0,0.3),([21,22,23];-1,0,-0.4)\succ$	$\prec([70,100,130];1,0,0.4),([55,100,155];-1,0,-0.3)\succ$
Karachi	$\prec([25,28,30];1,0,0.4),([27,28,29];-1,0,-0.5)\succ$	$\prec([30,40,50];1,0,0.1),([35,40,45];-1,0,-0.1)\succ$	$\prec([50,70,90];1,0,0.3),([35,70,115];-1,0,-0.2)\succ$
Demand	$\prec([50,60,70];1,0,0.2),([30,60,100];-1,0,-0.1)\succ$	$\prec([40,70,100];1,0,0.1),([40,70,100];-1,0,-0.4)\succ$	

$([1810, 3915, 6490]; \quad -1, 0, -0.5)\succ$, which can be interpreted as follows:

(i) The smallest amount of the minimum total transportation cost is 2050 units for positive membership and 1810 units for negative membership

(ii) The achievable amount of the minimum total transportation cost is 3470 units for positive and 3915 units for negative membership respectively

(iii) The largest amount of the minimum total transportation cost is 5300 units for positive membership and 6490 units for negative membership

Thus, the minimum total transportation cost for positive and negative memberships will always be larger than 2050, 1810 units, and smaller than 5300, 6490 units respectively, while for both memberships most probably the minimum total transportation cost will be 3470, 3915 units.

Results of Example 1 and existing models [1, 5] are given in Table 2 and are shown graphically in Figures 1–3.

From Figures 1–3 it is proved that single-valued neutrosophic transportation model is the most generalized model.

6. Advantages of Proposed Method

The proposed transportation model is based on a bipolar single-valued neutrosophic environment. This method is comparatively better than other methods in terms of advantages.

(i) In literature there is no method to solve an unbalanced FBSNTP. So, this is a new and helpful approach for the decision makers.

(ii) A BSNT model is more powerful than an intuitionistic fuzzy model [5] and a fuzzy model [1]. Thus, this technique is more general than fuzzy and intuitionistic fuzzy environments respectively.

7. Limitations of the Proposed Method

In this section, the limitations of proposed method 3 are pointed out.

(i) The proposed method 3 can be used to find the minimum bipolar single-valued neutrosophic optimal value of the balanced and unbalanced FBSNTP by using nonnegative triangular and trapezoidal bipolar single-valued neutrosophic numbers

(ii) The unbalanced transportation problem 4.1 is given in Table 3

8. Conclusion

In this study, we have suggested a new technique to solve an unbalanced FBSNT problem by using nonnegative triangular bipolar single-valued neutrosophic numbers. A score function has been applied to transform TBSNNs into crisp numbers. We have solved the FBSNTP on the basis of bipolar single-valued neutrosophic linear programming formulation. Furthermore, we have compared our method with the existing fully intutionistic fuzzy transportation models [5] and fully fuzzy transportation models [1].

We aim to extend our study to include the following topic:

(1) bipolar single-valued neutrosophic rough transportation models.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare no conflicts of interest.

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