Hindawi
Mathematical Problems in Engineering
Volume 2022, Article ID 2039076, 7 pages
https://doi.org/10.1155/2022/2039076

Research Article

Sine Frèchet Model: Modeling of COVID-19 Death Cases in Kingdom of Saudi Arabia

Maha A. Aldahlan

University of Jeddah, College of Science, Department of Statistics, Jeddah, Saudi Arabia

Correspondence should be addressed to Maha A. Aldahlan; maal-dahlan@uj.edu.sa

Received 27 November 2021; Revised 19 January 2022; Accepted 27 January 2022; Published 22 February 2022

Academic Editor: Naeem Jan

In this research, we discuss a new lifespan model that extends the Frèchet (F) distribution by utilizing the sine-generated family of distributions, known as the sine Frèchet (SF) model. The sine Frèchet distribution, which contains two parameters, scale and shape, aims to give an SF model for data fitting. The sine Frèchet model is more adaptable than well-known models such as the Frèchet and inverse exponential models. The sine Frèchet distribution is used extensively in medicine, physics, and nanophysics. The SF model's statistical properties were computed, including the quantile function, moments, moment generating function (MGF), and order statistics. To estimate the model parameters for the SF distribution, the maximum likelihood (ML) estimation technique is applied. As a result of the simulation, the performance of the estimations may be compared. We use it to examine a current dataset of interest: COVID-19 death cases observed in the Kingdom of Saudi Arabia (KSA) from 14 April to 22 June 2020. In the future, the SF model could be useful for analyzing data on COVID-19 cases in a variety of nations for possible comparison studies. Finally, the numerical results are examined in order to assess the flexibility of the new model.

1. Introduction

Diverse academics have recently become interested in the families of distributions that have been produced [1]: the beta-G [2], Kumaraswamy-G [3], the suggested sine-generated (S-G) family [4], modified odd Weibull-G [5], new Kumaraswamy-G [6], xgamma G [7], the truncated Frèchet-G [8], truncated Cauchy power-G and the truncated Weibull-G in [9], odd generalized gamma in [10], odd Weibull-G in [11], exponentiated M-G in [12], odd Chen-Leone-G [13], Topp-Leone odd Fréchet-G [14], odd Burr-G [15], box Cox gamma-G [16], Burr X family [17], Marshall–Olkin exponentiated generalized-G [18], and recently, DUS transformation in [19] and KM-G family [20].

As a result, S-G has both a distribution function (CDF) and density function (PDF):

$$F(x;\ \xi) = \sin\left[\frac{\pi}{2} G(x;\xi)\right],\ x \in R, \tag{1}$$

$$f(x;\xi) = \frac{\pi}{2} g(x;\xi)\cos\left[\frac{\pi}{2} G(x;\xi)\right],\ x \in R, \tag{2}$$

where $g(x;\xi)$ is considered a PDF of baseline distribution.

The Frèchet (F) model is an important model which can be used to analyze the life time data with some monotone failure rates. The PDF and CDF of the F distribution are

$$g(x;\mu,\delta) = \frac{\delta\mu^{\delta}}{x^{\delta+1}} e^{-(\mu/x)^{\delta}},\ x,\mu,\delta > 0, \tag{3}$$

$$G(x;\alpha,\beta) = e^{-(\mu/x)^{\delta}}, x,\mu,\delta > 0. \tag{4}$$

This work tries to improve the flexibility of the F model by employing the S-G family. The new model has several

PDF shapes, including decreasing, right skewness, and unimodal shapes. The hazard rate function (HRF) can also be declining and J-shaped. These findings are depicted in Figures 1 and 2. Based on the S-G family of distributions, a unique two-parameter distribution is introduced and analyzed. The model that has been presented is known as the SF model. The SF model is more adaptable and applicable than the F model.

This article is organized as follows: in Section 2, a proposal is made for the SF distribution to be built. ML estimators are investigated in Sections 3 and 4, and simulation results of SF parameters are examined in Section 5. Using COVID-19 death data, Section 5 proposes a model that may be applied to real-world data. Section 6 deals with some basic properties of the SF model and the article ends with conclusions.

2. Construction of the SF Model

Here, the CDF, PDF, survival function, and HRF of the random variable (rv) X are calculated:

$$F(x) = \sin\left[\frac{\pi}{2}e^{-(\mu/x)^{\delta}}\right],\ x>0,\ \mu,\delta>0,$$

$$f(x) = \frac{\pi\delta\mu^{\delta}}{2x^{\delta+1}}e^{-(\mu/x)^{\delta}}\cos\left[\frac{\pi}{2}e^{-(x/\mu)^{\delta}}\right],\ x>0,\mu,\delta>0, \quad (5)$$

$$R(x) = 1-\sin\left[\frac{\pi}{2}e^{-(\mu/x)^{\delta}}\right],$$

$$h(x) = \frac{\left(\pi\delta\mu^{\delta}/2x^{\delta+1}\right)e^{-(\mu/x)^{\delta}}\cos\left[(\pi/2)e^{-(\mu/x)^{\delta}}\right]}{1-\sin\left[\pi/2e^{-(\mu/x)^{\delta}}\right]}, \quad (6)$$

where μ is a scale parameter and δ is the shape parameter.

Figures 1 and 2 show the plots of PDF and HRF for various parameter values. This model can have decreasing PDF, right-skewed, and unimodal HRF and decreasing PDF, right-skewed, and unimodal HRF.

3. Statistical Characteristics

Here, we will look at some of the statistical characteristics of the SF distribution.

3.1. Moments

Theorem 1. *Let X can be a rv. When using the SF model, its r^{th} moment*

$$\mu_r' = \sum_{i=0}^{\infty}\frac{\mu^{r-\delta}\varpi\Gamma(1-(r/\delta))}{\delta(2i+1)^{1-r/\delta}}. \quad (7)$$

Proof. Let X be a rv with PDF (6). The r^{th} moment of the SF distribution is calculated as

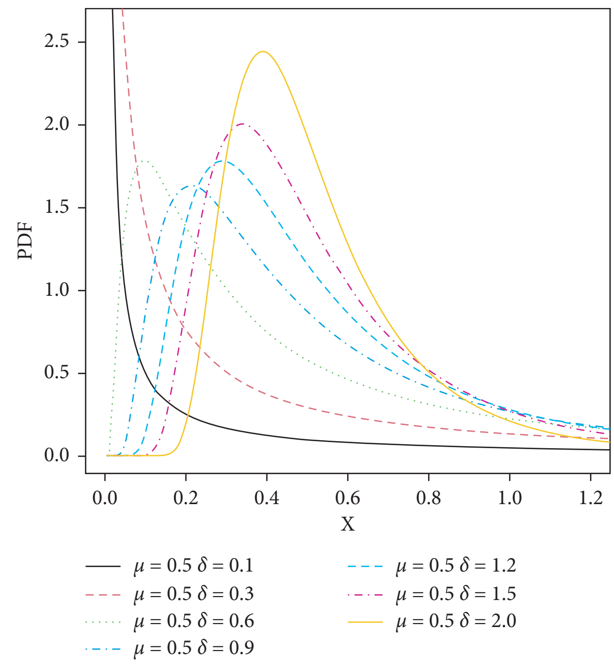

Figure 1: The PDF version of the SF model.

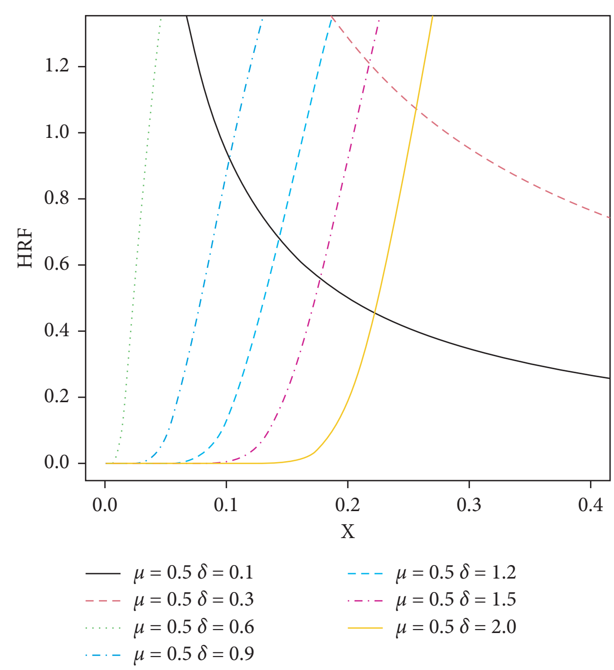

Figure 2: The HRF version of the SF model.

$$\mu_r' = \int_0^{\infty} x^r f(x)\mathrm{d}x$$
$$= \frac{\pi\delta\mu^{\delta}}{2}\int_0^{\infty} x^{r-\delta-1}e^{-(\mu/x)^{\delta}}\cos\left[\frac{\pi}{2}e^{-(\mu/x)^{\delta}}\right]\mathrm{d}x. \quad (8)$$

By inserting the expansion $\cos[G(x)] = \sum_{i=0}^{\infty}((-1)^i/(2i)!)G(x)^{2i}, n$ to the previous equation,

$$\mu_r' = \sum_{i=0}^{\infty} \frac{\delta\mu^{\delta}(-1)^i}{(2i)!}\left(\frac{\pi}{2}\right)^{2i+1} \int_0^{\infty} x^{r-\delta-1} e^{-(2i+1)(\mu/x)^{\delta}} \mathrm{d}x. \tag{9}$$

The last equation can be rewritten as

$$\mu_r' = \sum_{i=0}^{\infty} \varpi \int_0^{\infty} x^{r-\delta-1} e^{-(2i+1)(\mu/x)^{\delta}} \mathrm{d}x, \tag{10}$$

where $\varpi = \sum_{i=0}^{\infty} (\delta\mu^{\delta}(-1)^i/(2i)!)(\pi/2)^{2i+1}$.

Let $y = (\mu/x)^{\delta}$, then

$$\mu_r' = \sum_{i=0}^{\infty} \frac{\mu^{r-\delta}}{\delta} \varpi \int_0^{\infty} y^{-r/\delta} e^{-(2i+1)y} \mathrm{d}y. \tag{11}$$

Then,

$$\mu_r' = \sum_{i=0}^{\infty} \frac{\mu^{r-\delta}\varpi\Gamma(1-(r/\delta))}{\delta(2i+1)^{1-(r/\delta)}}. \tag{12}$$

The MGF of X is

$$M_X(t) = \sum_{r=0}^{\infty} \frac{t^r}{r!}\mu_r' = \sum_{r,i=0}^{\infty} \frac{t^r \mu^{r-\delta}\varpi\Gamma(1-(r/\delta))}{r!\delta(2i+1)^{1-(r/\delta)}}. \tag{13}$$

□

3.2. Quantile Function. If X ~ SF, then the quantile function of SF is

$$Q(u) = \mu\left[\ln\left(\frac{\pi}{2\mathrm{Arcsin}(u)}\right)\right]^{\frac{-1}{\delta}}, \tag{14}$$

and by substituing $u = 0.5$, we obtain the median (M) as

$$M = \mu[\ln(3)]^{-1/\delta}. \tag{15}$$

3.3. Order Statistics. Let $X_1, X_2, \ldots, X_n$ be r sample from the SF model with order statistics $X_{(1)}, X_{(2)}, \ldots, X_{(n)}$. The PDF of $X_{(k)}$ of order statistics is

$$f_{X_{(k)}}(x) = \frac{n!}{(k-1)!(n-k)!}F^{k-1}(x)f(x)(1-F(x))^{n-k}. \tag{16}$$

The PDF of $X_{(k)}$ can be expressed as

$$f_{X_{(k)}}(x) = \frac{n!\delta\mu^{\delta}\Xi}{x^{\delta+1}(k-1)!(n-k)!}(\sin[\Xi])^{k-1}\cos[\Xi](1-\sin[\Xi])^{n-k}, \tag{17}$$

where $(\pi/2)e^{-(\mu/x)^{\delta}} = \Xi$. We can obtain the PDF of the lowest and largest order statistics at $k = 1$ and $k = n$, respectively,

$$f_{X_{(1)}}(x) = \frac{n\delta\mu^{\delta}\Xi}{x^{\delta+1}}\cos[\Xi](1-\sin[\Xi])^{n-1}, \tag{18}$$

$$f_{X_{(n)}}(x) = \frac{n\delta\mu^{\delta}\Xi}{x^{\delta+1}}(\sin[\Xi])^{n-1}\cos[\Xi]. \tag{19}$$

4. ML Estimation

Let $x_1, x_2, \ldots, x_n$ be the observed values from the SF model with parameters $\varepsilon = (\mu, \delta)$. The total likelihood function corresponding to (6) is

$$\ln L = n\ln\frac{\pi\delta}{2} + \delta n\ln\mu - (\delta+1)\sum_{i=1}^{n}\ln(x_i) - \sum_{i=1}^{n}\left(\frac{\mu}{x_i}\right)^{\delta} + \sum_{i=1}^{n}\ln\left(\cos\left[\frac{\pi}{2}e^{-(\mu/x_i)^{\delta}}\right]\right). \tag{20}$$

The ML equations of the SF model are

$$\frac{\partial \ln L}{\partial\delta} = \frac{n}{\delta} + n\ln\mu - \sum_{i=1}^{n}\left(\frac{\mu}{x_i}\right)^{\delta} - \sum_{i=1}^{n}\left(\frac{\mu}{x_i}\right)^{\delta}\ln\left(\frac{\mu}{x_i}\right) + \frac{\pi}{2}\sum_{i=1}^{n}\left(\frac{\mu}{x_i}\right)^{\delta}\ln\left(\frac{\mu}{x_i}\right)e^{-(\mu/x_i)^{\delta}}\left(\tan\left[\frac{\pi}{2}e^{-(\mu/x_i)^{\delta}}\right]\right), \tag{21}$$

$$\frac{\partial \ln L}{\partial\mu} = \frac{n\delta}{\mu} - \frac{\delta}{\mu}\sum_{i=1}^{n}\left(\frac{\mu}{x_i}\right)^{\delta} + \frac{\delta\pi}{2\mu}\sum_{i=1}^{n}\left(\frac{\mu}{x_i}\right)^{\delta}e^{-(\mu/x_i)^{\delta}}\left(\tan\left[\frac{\pi}{2}e^{-(\mu/x_i)^{\delta}}\right]\right). \tag{22}$$

Then, the ML estimators of the parameters ε are calculated by substituting $U_{\varepsilon} = 0$ and solving it.

5. Numerical Results

The ML technique for estimating parameters is evaluated, and the modest numerical results are obtained. Mathematica 9 is used to do Monte Carlo simulations on an SF. The simulation points are arranged as follows:

(i) We generate random samples from SF distribution by using

$$x_q = \mu\left[\ln\left(\frac{\pi}{2\mathrm{Arcsin}(u)}\right)\right]^{-1/\delta}. \tag{23}$$

(ii) Each sample size of $n = 100$, 200, 300, and 500 was replicated 3000 times in order to obtain the data.

(iii) According to Tables 1–5, a variety of values are picked for the parameters.

(iv) Formulas used for investigating root mean square error (RMSE), lower bound (Z1), upper bound (Z2), and average length (Z3) of 90% and 95% are calculated.

Table 1: Numerical results for the SF model at ($\delta = 0.5$, $\mu = 0.5$).

n	MLE	RMSE	90% Z1	90% Z2	90% Z3	95% Z1	95% Z2	95% Z3
100	0.7453	0.2597	0.0436	1.4470	1.4034	−0.0908	1.5814	1.6722
	1.0047	1.0359	−0.4077	2.4171	2.8248	−0.6782	2.6875	3.3657
200	0.5344	0.0217	0.2899	0.7790	0.4891	0.2431	0.8258	0.5828
	0.5921	0.0994	0.1274	1.0568	0.9295	0.0384	1.1458	1.1074
300	0.4965	0.0111	0.3265	0.6664	0.3399	0.2940	0.6990	0.4050
	0.5181	0.0328	0.2002	0.8360	0.6357	0.1394	0.8968	0.7574
500	0.5364	0.0078	0.3889	0.6840	0.2951	0.3606	0.7122	0.3516
	0.5727	0.0316	0.2962	0.8493	0.5531	0.2432	0.9023	0.6591

Table 2: Numerical results for the SF model at ($\delta = 0.7$, $\mu = 0.5$).

n	MLE	RMSE	90% Z1	90% Z2	90% Z3	95% Z1	95% Z2	95% Z3
100	0.8246	0.1700	0.1571	1.4922	1.3351	0.0292	1.6200	1.5908
	0.7097	0.3171	−0.1866	1.6061	1.7927	−0.3583	1.7777	2.1360
200	0.7512	0.0433	0.3634	1.1391	0.7757	0.2892	1.2133	0.9242
	0.5275	0.0452	0.0677	0.9873	0.9196	−0.0203	1.0754	1.0957
300	0.7524	0.0432	0.4306	1.0742	0.6436	0.3690	1.1359	0.7669
	0.5272	0.0451	0.1541	0.9404	0.7862	0.0789	1.0156	0.9368
500	0.7109	0.0248	0.4864	0.9354	0.4490	0.4434	0.9784	0.5350
	0.5232	0.0417	0.2398	0.8065	0.5667	0.1856	0.8608	0.6752

Table 3: Numerical results for the SF model at ($\delta = 1.2$, $\mu = 0.5$).

n	MLE	RMSE	90% Z1	90% Z2	90% Z3	95% Z1	95% Z2	95% Z3
100	1.2990	0.0637	0.9283	1.6698	0.7415	0.8573	1.7408	0.8835
	0.5211	0.0083	0.3742	0.6680	0.2938	0.3461	0.6961	0.3501
200	1.2499	0.0241	1.0026	1.4973	0.4947	0.9552	1.5447	0.5894
	0.5145	0.0046	0.4114	0.6176	0.2062	0.3917	0.6373	0.2456
300	1.2220	0.0116	1.0533	1.3907	0.3374	1.0210	1.4230	0.4020
	0.5046	0.0015	0.4329	0.5763	0.1434	0.4192	0.5901	0.1709
500	1.2012	0.0034	1.0968	1.3056	0.2088	1.0768	1.3256	0.2488
	0.4999	0.0006	0.4547	0.5450	0.0902	0.4461	0.5536	0.1075

Table 4: Numerical results for the SF model at ($\delta = 1.8$, $\mu = 0.5$).

n	MLE	RMSE	90% Z1	90% Z2	90% Z3	95% Z1	95% Z2	95% Z3
100	1.9317	0.1452	1.3035	2.5598	1.2563	1.1833	2.6801	1.4969
	0.5283	0.0077	0.3903	0.6663	0.2760	0.3638	0.6927	0.3289
200	1.8633	0.1183	1.4420	2.2846	0.8426	1.3613	2.3652	1.0039
	0.5135	0.0062	0.4184	0.6087	0.1903	0.4001	0.6269	0.2268
300	1.7828	0.0313	1.5026	2.0629	0.5603	1.4490	2.1165	0.6675
	0.4992	0.0018	0.4332	0.5651	0.1319	0.4206	0.5778	0.1572
500	1.8049	0.0073	1.6259	1.9838	0.3578	1.5917	2.0181	0.4264
	0.5003	0.0004	0.4587	0.5419	0.0832	0.4507	0.5498	0.0991

Table 5: Numerical results for the SF model at ($\delta = 2.5$, $\boldsymbol{\mu} = 0.5$).

n	MLE	RMSE	90%			95%		
			Z1	Z2	Z3	Z1	Z2	Z3
100	2.7838	0.5311	1.7572	3.8104	2.0533	1.5606	4.0070	2.4465
	0.5301	0.0069	0.3988	0.6614	0.2626	0.3737	0.6866	0.3129
200	2.6603	0.3059	1.9912	3.3293	1.3382	1.8631	3.4575	1.5944
	0.5127	0.0043	0.4234	0.6021	0.1787	0.4063	0.6192	0.2129
300	2.5696	0.0795	2.1173	3.0220	0.9047	2.0306	3.1086	1.0780
	0.5092	0.0016	0.4460	0.5725	0.1265	0.4339	0.5846	0.1507
500	2.4760	0.0301	2.2034	2.7486	0.5451	2.1513	2.8008	0.6495
	0.4916	0.0005	0.4528	0.5304	0.0777	0.4453	0.5379	0.0925

Table 6: Some descriptive statistics for the data set.

n	Min	Max	Mean	Median	Mode	Variance	Skewness	Kurtosis
96	1	97	23.667	14	11	583.639	1.777	2.372

Table 7: The ML estimates and SEs for the data set.

Model	ML estimates and SEs	
SF (δ, μ)	0.915 (0.075)	13.452 (1.473)
F (δ, μ)	1.168 (0.094	8.679 (0.942
IE (μ)	9.247 (1.105)	

Table 8: The values of L1, L2, L3, L4, L5, and L6 the data set.

Model	Goodness-of-fit criteria					
	L1	L2	L4	L5	L3	L6
SF	534.841	538.841	538.531	540.627	539.02	0.132
F	543.894	547.894	547.584	549.68	548.073	0.1369
IE	547.221	549.221	549.066	550.114	549.279	0.1752

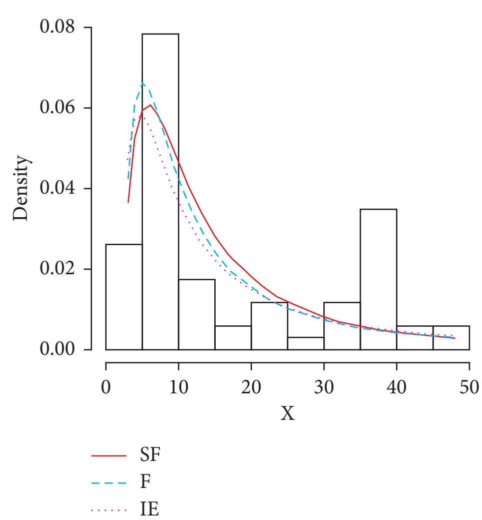

Figure 3: Estimated PDF for the data set.

6. Applications to Real Data

In this section, an actual data set is studied to demonstrate the benefits of the SF model over other known models such as the F and inverse exponential (IE) models.

We propose several information criteria (IC) to compare the competitive models, such as the minus of log-likelihood function (L1), Akaike IC (L2), the right Akaike IC (L3), Bayesian IC (L4), Hannan-Quinn IC (L5), and Kolmogorov–Smirnov IC (L6).

The data set proposes a specific application using a real-world data set to gauge interest in the SF model. The data evaluated were the daily fatality confirmed cases of COVID-19 in Saudi Arabia from 14 April to 22 June 2020. The data set was collected electronically from the following website: https://covid19.moh.gov.sa (6, 4, 4, 5, 5, 6, 6, 5, 7, 6, 9, 3, 5, 8, 5, 5, 7, 8, 7, 9, 9, 10, 10, 10, 7, 9, 9, 9, 10, 9, 10, 10, 8, 9, 10, 12, 13, 15, 11, 9, 12, 14, 16, 17, 22, 23, 22, 24, 30, 32, 31, 34, 36, 34, 37, 36, 38, 36, 39, 40, 39, 41, 39, 48, 45, 46, 37, 40, 39). Many studies investigated biomedical data sets, including those in [21–24].

Some descriptive statistics of the data are provided in Table 6. The ML estimates of all competitive models and their SEs are mentioned in Table 7. Values of measures of goodness of fit are provided in Table 8.

We find that the SF model provides a better fit than the other competitive models. It has the lowest value of L1, L2, L3, L4, L5, and L6 values among those considered here. Moreover, the plots of the estimated PDF of the data set for all competitive models are shown in Figure 3.

7. Conclusion

In this study, we introduced the SF distribution, a unique two-parameter model. In medicine, physics, and nano-physics, the sine Frèchet distribution is widely employed. The statistical features of the SF model, such as the quantile function, moments, moment generating function (MGF), and order statistics, were computed. The ML estimation approach is used to evaluate the estimate of the SF parameters. The simulation results are calculated to show the accuracy of the estimates. The modeling for COVID-19 in KSA from 14 April to 22 June 2020 real data set is used to explain the significance of the SF model in comparison to the other competitive models. Some basic SF model properties are proposed.

Data Availability

The numerical data set used to conduct the study is available from the corresponding author upon request.

Conflicts of Interest

The author declares no conflicts of interest.

References

[1] N. Eugene, C. Lee, and F. Famoye, "Beta-normal distribution and its applications," *Communications in Statistics - Theory and Methods*, vol. 31, no. 4, pp. 497–512, 2002.

[2] G. M. Cordeiro and M. de Castro, "A new family of generalized distributions," *Journal of Statistical Computation and Simulation*, vol. 81, no. 7, pp. 883–898, 2011.

[3] D. Kumar, U. Singh, and S. K. Singh, "A New distribution using Sine function- its application to bladder cancer patients data," *Journal of Statistics Applications & Probability*, vol. 4, no. 3, pp. 417–427, 2015.

[4] C. Chesneau and T. El Achi, "Modified odd Weibull family of distributions: properties and applications," *Journal of the Indian Society for Probability and Statistics*, vol. 21, no. 1, pp. 259–286, 2020.

[5] M. H. Tahir, M. A. Hussain, G. M. Cordeiro, M. El-Morshedy, and M. S. Eliwa, "A new Kumaraswamy generalized family of distributions with properties, applications, and bivariate extension," *Mathematics*, vol. 8, no. 11, p. 1989, 2020.

[6] G. M. Cordeiro, E. Altun, M. C. Korkmaz, R. R. Pescim, A. Z. Afify, and H. M. Yousof, "The xgamma family: censored regression modelling and applications," *REVSTAT-Statistical Journal*, vol. 18, no. 5, pp. 593–612, 2020.

[7] M. A. Aldahlan, "Type II truncated Fréchet generated family of distributions," *International Journal of Mathematics and Applications*, vol. 7, pp. 221–228, 2019, þ.

[8] M. A. Aldahlan, F. Jamal, Ch. Chesneau, M. Elgarhy, and I. Elbatal, "The truncated Cauchy power family of distributions with inference and applications," *Entropy*, vol. 22, pp. 1–25, 2020.

[9] H. Najarzadegan, M. H. Alamatsaz, and S. Hayati, "Truncated weibull-G more flexible and more reliable than beta-G distribution," *International Journal of Statistics and Probability*, vol. 6, no. 5, pp. 1–17, 2017.

[10] M. A. Nasir, M. H. Tahir, C. Chesneau, F. Jamal, and M. A. A. Shah, "The odd generalized gamma-G family of distributions: properties, regressions and applications," *Statistica*, vol. 80, no. 1, pp. 3–38, 2020.

[11] M. Bourguignon, R. B. Silva, and G. M. Cordeiro, "The Weibull-G family of probability distributions," *Journal of Data Science*, vol. 12, pp. 53–68, 2014.

[12] R. A. Bantan, Ch. Chesneau, F. Jamal, and M. Elgarhy, "On the analysis of new COVID-19 cases in Pakistan using an exponentiated version of the M family of distributions," *Mathematics*, vol. 8, pp. 1–20, 2020.

[13] M. El-Morshedy, M. S. Eliwa, and A. Z. Afify, "The odd Chen generator of distributions: properties and estimation methods withapplications in medicine and engineering," *Journal of the National Science Foundation of Sri Lanka*, vol. 48, pp. 113–130, 2020.

[14] S. Al-Marzouki, F. Jamal, C. Chesneau, and M. Elgarhy, "Topp-leone odd Frèhet generated family of distributions with applications to COVID-19 data sets," *Computer Modeling in Engineering and Sciences*, vol. 125, no. 1, pp. 437–458, 2020.

[15] M. Alizadeh, G. M. Cordeiro, A. D. C. Nascimento, M. d. C. S. Lima, and E. M. M. Ortega, "Odd-Burr generalized family of distributions with some applications," *Journal of Statistical Computation and Simulation*, vol. 87, no. 2, pp. 367–389, 2017.

[16] A. A. Al-Babtain, I. Elbatal, C. Chesneau, and F. Jamal, "Box-Cox gamma-G family of distributions: theory and applications," *Mathematics*, vol. 8, no. 10, p. 1801, 2020.
[17] H. M. Yousof, A. Z. Afify, G. G. Hamedani, and G. Aryal, "The Burr X generator of distributions for lifetime data," *Journal of Statistical Theory and Applications*, vol. 16, pp. 1–19, 2016.
[18] H. M. Yousof, M. Rasekhi, M. Alizadeh, and G. G. Hamedani, "The Marshall-Olkin exponentiated generalized G family of distributions: properties, applications and characterizations," *The Journal of Nonlinear Science and Applications*, vol. 13, pp. 34–52, 2020.
[19] S. K. Maurya, A. Kaushik, and S. K. Singh, "A new class of distribution having decreasing, increasing, and bathtub-shaped failure rate," *Communications in Statistics - Theory and Methods*, vol. 46, no. 20, Article ID 10359, 2017.
[20] P. Kavya and M. Manoharan, "Some parsimonious models for lifetimes and applications," *Journal of Statistical Compution and Simulation*, vol. 91, no. 18, pp. 3693–3708, 2021.
[21] M. B. Antor, A. H. M. S. Jamil, M. Mamtaz et al., "A comparative analysis of machine learning algorithms to predict alzheimer's disease," *Journal of Healthcare Engineering*, vol. 2021, Article ID 9917919, 12 pages, 2021.
[22] S. A. Alanazi, M. M. Kamruzzaman, M. Alruwaili, N. Alshammari, S. A. Alqahtani, and A. Karime, "Measuring and preventing COVID-19 using the SIR model and machine learning in smart health care," *Journal of Healthcare Engineering*, vol. 2020, Article ID 8857346, 12 pages, 2020.
[23] Abdul Latif, A. Latif, and A. Alam, "Artificial intelligence and medical internet of things framework for diagnosis of coronavirus suspected cases," *Journal of Healthcare Engineering*, vol. 2021, Article ID 3277988, 7 pages, 2021.
[24] M. Kaur, V. Kumar, V. Yadav, D. Singh, N. Kumar, and N. N. Das, "Metaheuristic-based deep COVID-19 screening model from chest X-ray images," *Journal of Healthcare Engineering*, vol. 2021, Article ID 8829829, 9 pages, 2021.