Review Article

An Overview of Cable-Driven Parallel Robots: Workspace, Tension Distribution, and Cable Sagging

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The researching, designing, calculating, and controlling cable-driven parallel robots (CDPRs) are being promoted in recent years. The researches focus on optimizing the design of CDPRs configuration, computing workspace, calculating cable tension distribution, designing the mechanical structure, and developing controller. However, due to the complexity of the structure and unidirectional characteristic of cable, many computational methods have been applied to solve the above problems. To facilitate the performance of theoretical studies on important issues in the design and control of the CDPRs, a summary of computational methods is needed for important issues related to the functionality and accuracy of CDPRs such as defining the workspace, distributing the cable tensions, and calculating the sagging of the cable. This paper summarizes published studies on CDPRs and focuses on classifying and analyzing methods used to calculate important issues in the process of calculating and designing CDPRs. The efficiency of these calculation methods is also analyzed and evaluated based on the mathematical theories of kinematics and dynamics of CDPRs. The content of this study is an effective reference for studies on important problems in the process of designing and implementing CDPRs, helping researchers shorten the time to review related topics.

1. Introduction

Cable-driven parallel robot (CDPR) is a branch of parallel kinetic robot with the moving platform-driven by cables. CDPRs are also known by other names in published studies such as cable-driven parallel manipulators (CDPMs) [1, 2], or simply cable robots (CRs) [3]. As the actuators of CDPRs, cables are usually stored and delivered by sets of winches placed on the fixed frame and connected to the moving platform (MP) via anchor points and guide pulleys. With advantages such as simple structure, large working space, high precision, small volume of the actuator, small inertia force, and low manufacturing cost, CDPRs have been designed and developed for applications requiring large workspace, heavy load capacity, high speed, or for applications that require high flexibility [4, 5]. The world’s famous CDPR applications are shown in [4, 6, 7] such as FAST telescope-China with an aperture up to 500 m, SKYCAM—used to control a camera for recording live sports matches—with the size of a football field rock and cable-driven parallel robot prototype CoGiRo of dimensions 15 m × 11 m × 6 m. CDPRs are usually classified according to their structure, the number of degrees of freedom (DOFs), and the number of cables, where m is the number of DOFs and n is the number of drive cables, r = n − m is the number of redundant cables or the number of degrees of redundant (DORs). The basic configurations of CDPRs include the following [8]:

(a) r < 0, m ≤ 6: incompletely restrained positioning mechanism (IRPM) or underconstrained CDPRs, these types of CDPRs usually only work and reach equilibrium with gravity or a given force, and often cannot work with arbitrary external wrenches.

(b) r = 0: kinematically fully constrained CDPRs. The robot is completely limited in terms of kinematics, but the equilibrium equation still depends on gravity or other forces. Therefore, the robot can only work...
with a specific set of forces. Due to this property, some studies rank this robot in the same class as RRPM.

(c) \( r = 1 \): completely restrained positioning mechanisms (CRPM) or fully constrained CDPRs. The certain poses of the MP can be determined entirely through the cables. The limits of the MP movements and the wrenches acting on the MP depend on the cable tension limits.

(d) \( r > 1 \): redundantly restrained positioning mechanisms (RRPM). The robot is constrained redundancy and wrenches must be distributed by cables. An inverse kinematics result has many corresponding kinematic constraints because the number of kinematic constraints is more than the number of DOFs so the static equilibrium of CDPR can have many solutions.

(e) If the CDPRs are driven by cables mounted above the MP in the direction of gravity, they are called suspended configuration CDPRs. This configuration relies on the gravity of MP to achieve the equilibrium state [5]. The workspace of the suspended CDPRs is mostly located below the fixed cable anchor points. The above CDPR types are all configurable to operate in a suspended state. The working space of the suspended CDPRs depends greatly on the gravity of the MP, so the payload is one of the important parameters in the design process of the CDPRs with a given workspace. Depending on the design configuration, some CDPRs operate only in the suspended state, some CDPRs can operate in the fully restrained state, or may operate both in the fully restrained state and in the suspended state.

The research on CDPRs mainly focuses on the main contents such as optimizing the structure, determining the working space, developing the cable tension distribution algorithms, studying the models to calculate the sagging of the cable, and designing controller and motion trajectories [9–14]. Huajie Hong et al. [10] compiled CDPR case studies and presented studies on the basic theory and application of several CDPR configurations. The authors systematized and analyzed the studies on the topological architecture of CDPR. The optimal configuration depends on the relationship between the number of DOFs and the number of cables. From the optimized structure of CDPRs, the structure matrix (the transpose of the Jacobian matrix) is built to calculate the tension distribution and dynamic equilibrium. CDPRs have complex structures and constraints because of the unidirectional nature of transmission cables; therefore, the design, calculation, and implementation of an application of CDPRs are challenging. Sen Qian et al. [11] have summarized the development of CDPR and analyzed some recent successful application cases of CDPR. The authors have explored basic CDPR research such as mechanical design, performance analysis, and controller development. Hao Xiong et al. [12] have performed an overview analysis of cable-driven rehabilitation devices.

Many research works proposed methods to solve typical issues of CDPRs, which focus on calculating workspaces, configuring structure, building analytical methods cable tension, calculation of kinematics and dynamics, and calculation of transmission cable sagging, based on many different methods with complex constraints. For an overview of a large number of published CDPRs studies, this paper provides a systematic analysis of the methods used to solve the main issues in the design and development CDPRs. The study is divided into four main sections, the first one shows the kinematic and dynamic of CDPRs including the structure matrix, these are the mathematical model used for determining the main issues of CDPRs; the second analysis the articles that study determining workspace of CDPRs; the third section shows the articles study about tension distribution methods, this section analyzes many solutions built to calculate cable tension for fully of over constrain CDPRs; the fourth section calculates the sagging of cables for large workspace CDPRs.

2. Kinematic of CDPRs

The typical configuration of CDPRs (Figure 1) consists of a base frame (BF) with fixed anchor points and guiding pulleys used to guide the cables in different configurations, and the moving platform is connected to the cables via movable anchor points. Cables are stored and delivered by winches usually driven by servo motors so that the speed and position of the cables can be precisely controlled. Normally, cable robots work with large loads, so reduction gearboxes are used to reduce motor speed and increase torque. To facilitate fabrication and installation of cable distributors, winches are usually fixed to the fixed frame. However, there are some configurations where all the winches are placed on the moving platform, and the advantage of this structure is that it is convenient for the transmission of control signals between the motors and the main controller, especially for CDPRs with large workspaces [15, 16].

Figure 2 shows the general kinetics diagram of CDPRs. Where the global reference frame is denoted as B, the other one is the E frame attached to the MP, \( \mathbf{x} = (p, r) \in \mathbb{R}^m \) is the vector representing the pose including orientation and position of the E frame in the B frame, and \( m \) is the number of DOFs of CDPRs. According to the vector diagram in Figure 2, the equation for calculating the vector \( \mathbf{l}_i \) \((i = 0 \ldots n)\) corresponding to a configuration of MP can be obtained by the following equation:

\[
\mathbf{l}_i = \mathbf{a}_i - p - R\mathbf{b}_i. \tag{1}
\]

The length of cables can be obtained as in (2).

\[
|\mathbf{l}_i| = \sqrt{[\mathbf{a}_i - \mathbf{p} - R\mathbf{b}_i][\mathbf{a}_i - \mathbf{p} - R\mathbf{b}_i]^T}. \tag{2}
\]

In the general case, the vector \( \mathbf{p} = [P_x \ P_y \ P_z]^T \in \mathbb{R}^3 \) represents the origin \( P \) of the E frame in the B frame, the vector \( \mathbf{r} = [\alpha \ \beta \ \gamma]^T \in \mathbb{R}^3 \) represents the rotation angles of the E frame in the B frame (Roll–Pitch–Yaw representation or other orientation representation methods can be used),
the vector $\mathbf{a}_i = \begin{bmatrix} A_{ix} & A_{iy} & A_{iz} \end{bmatrix}^T \in \mathbb{R}^3$ is the coordinates of the fixed anchor points $A_i$ in the $B$ frame, the vector $\mathbf{b}_i = \begin{bmatrix} B_{ix} & B_{iy} & B_{iz} \end{bmatrix}^T \in \mathbb{R}^3$ is the coordinates of the movable anchor points $B_i$ in the $E$ frame and the unit vector $\mathbf{u}_i$ representing the direction of the cables $l_i$. (2) is rewritten as follows:

$$l_i = \mathbf{a}_i - \mathbf{p} - R \mathbf{b}_i = \begin{bmatrix} A_{ix} - P_x \\ A_{iy} - P_y \\ A_{iz} - P_y \end{bmatrix} - \begin{bmatrix} B_{ix} \\ B_{iy} \\ B_{iz} \end{bmatrix},$$

where $R$ is the matrix representing the direction of the $E$-axis system in the $B$-axis system according to Roll–Pitch–Yaw representation, the RPY consists of the rotation of $\gamma$ about the $z$-axis ($z$-axis of the $E$ frame) called Roll angle, the rotation of $\beta$ about the $y$-axis ($y$-axis of the $E$ frame) called Pitch angle, and the rotation of $\alpha$ about the $x$-axis ($x$-axis of the $E$ frame) called Yaw angle. The $c$ and $s$ denote the cosine and sine equation, respectively.

$$R = \begin{bmatrix} c\beta c\gamma - s\gamma c\alpha & c\beta s\gamma + s\alpha c\gamma & s\beta c\gamma \\ c\beta s\gamma - s\gamma s\alpha & c\beta c\gamma + c\alpha s\gamma & s\beta s\gamma \\ s\gamma c\alpha & -s\gamma s\alpha & c\gamma \end{bmatrix}. \quad (4)$$

The relationship between the external forces and the cable tensions acting on the MP for a given pose is represented by the equilibrium equation.

$$\mathbf{w} + \mathbf{A} \tau = 0,$$  

where $\tau = [\tau_1 \ \tau_2 \ \ldots \ \tau_n]^T \in \mathbb{R}_+^n$ is the positive cable tension vector acting on MP where $n$ is the number of cables. General structure matrix $\mathbf{A}$ is calculated by the following equation:

$$\mathbf{A} = \begin{bmatrix} \mathbf{u}_1 & \ldots & \mathbf{u}_n \\ \mathbf{R} \times \mathbf{u}_1 & \ldots & \mathbf{R} \times \mathbf{u}_n \end{bmatrix}. \quad (6)$$

Equations (1), (5), and (6) are important equations in determining the workspace, calculating cable tension, and designing the controller of CDPRs. In the following sections, the parameters defined in this section will be built,
developed, and analyzed to determine the design parameters of CDPRs, including methods of workspace determination, cable tension distribution, and calculation of sagging of cables.

3. Workspace and Design

The workspace of the robot is a 2D or 3D space that the robot’s end-effector can approach with different position and orientation constraints. This is an important parameter for designing, controlling, and applying the CDPRs. Determining the workspace of parallel robots is more complicated than serial robots because of the complexity of the constraints as well as the unidirectional character of the cables. The workspace of CDPRs is an important parameter in designing the robot structure, calculating the stiffness, as well as controlling the moving trajectory of the MP. With the same case size, CDPR’s workspace or moving platform mobility can vary depending on the size and structure of the moving platform, the payload, and the mounting locations of cables on fixed frames and moving platforms. Actuator capacity and load are also important parameters affecting the size of the workspace. The workspace definitions of CDPRs relate to constraints on external forces and limits of cable tensions. Based on the formulas (1–6), the conditions to define the workspace were presented by Vehoeven [17–20], mathematical models for workspace calculation are determined based on equilibrium equations, including general workspace, conditions of tension, stiffness, and singularity.

Wrench Closure Workspace (WCW)—Force Closure Workspace (FCW) [21, 22] is defined by the set of poses of the MP where there exists at least one set of cables tension balance to any external force with the upper limit of tension of cable being infinity.

$$\exists \tau \geq 0: A\tau + w d = 0, \forall w d \in \mathbb{R}^m.$$  

(7)

Total Orientation and Constant Orientation Wrench Closure Workspace (TOWCW-COWCW) [23, 24] is defined by the set of poses of the MP (with a set of orientation or one orientation) where there exists at least one set of tension balance to any external force with the upper limit of tension of cable being infinity.

Force Feasible Workspace (FFW)—Wrench Feasible Workspace (WFW) [2, 22, 25]: set of poses of the MP in which there exists at least one set of tension cables limited from min to max equal to a given external force or a given set of external forces:

$$\exists \tau \in [\tau_{\min}, \tau_{\max}], \quad \tau_{\min} \geq 0: A\tau + w d = 0, \forall w d \in W.$$  

(8)

Static Workspace (StW) is the set of poses of the MP for which there exists at least one positive cable tension system balance to the mass and load of MP.

Controllable workspace (CW) [2, 19]—A controllable workspace of postures where forces and torques at the MP can be controlled.

The workspace of CDPRs is a complex geometry due to the constraints of the design parameters. Therefore, it is difficult to determine the workspace of CDPRs corresponding to changes in key parameters. Due to the unidirectional characteristic of cables that only traction is applied to the moving platform, determining the equilibrium and workspace of CDPRs is a major challenge. The techniques and methods of calculating the workspace used for rigid-link parallel robots or serial manipulators cannot be used for CDPRs. The workspace of CDPRs is a set of poses of the moving platform that satisfies the constraints of geometry, force balance, and structural stiffness with boundary conditions such as force and impact moment, noise, engine power. There are three commonly used methods to define the CDPR workspace. The first method is the pointwise method that considers whether a finite set of discrete points satisfy the constraints of the workspace. The second method is the continuous method with the most commonly used algorithm being interval analysis. The third method is the analytical method that focuses on defining the boundary (geometrical shape) of the workspace, the advantage of this method is that it gives a visual look at the geometry of the workspace. However, this method is only applied individually for each robot configuration.

Structural matrix-based is a commonly used method to determine the workspace of CDPRs. Starting from the equilibrium condition (9), the general solution has the form:

$$\tau = A^T(AA^T)^{-1}w d + H\lambda = \tau + H\lambda,$$  

(9)

where $\tau$ is the minimum norm solution of, $H$ (9) is the null-space (kernel) of matrix $A$, and $\lambda$ is an n-m vector. The constraints for a pose to be included in WCW are rank($A$) = 6 and $\exists \tau_{\min} = H\lambda \in H: \tau_{\min} > 0$.

The pointwise method is often applied in determining the workspace of CDPRs. This method is based on considering the discrete points that satisfy the constraints of the poses of the MP to know whether these points belong to the workspace or not. The pointwise method has the advantage that the condition is clear and intuitive, but the accuracy at the boundary depends on the magnitude of the distance of the selected discrete points (resolution), the higher the resolution, the more accurate the results. However, this means increased computation time, especially for CDPRs with large workspace. An important issue of the pointwise workspace determination method is that it is not possible to determine the satisfaction of the points lying between the considered points according to a coordinate procedure such as the random method, radial method, or the equidistant method (Figure 3).

Verhoeven and Hiller [19] suggest a method for defining a controllable workspace based on the pointwise method. This method transforms the force equilibrium equation, then analyzes the rank and kernel of the structural matrix to check the existence of a possible solution for a given combination of forces. However, the study has not considered the moment equilibrium satisfaction for CDPRs with different types of moving platforms such as bars or frames. Verhoeven [20] has proposed a pointwise method to determine the controllable workspace for CDPRs. However, this method is only applicable to a limited number of CDPRs with a simple structure due to the complexity of the
expressions describing the system. Pusey et al. [23] analyzed the workspace of a CDPR 6 DOFs – 6 cables by pointwise method, a performance index was used to evaluate the influence of the robot’s structure (size of fixed and moving platforms) on the size of the robot’s workspace. These indices depend on the characteristics of the matrix structure. The results apply to the design calculations of CDPR 6 DOFs—6 cables and can be extended to other configurations.

Yang et al. [2] used the tension distribution configuration analysis method to combine the workspace for CDPRs, this method is based on the evaluation of the tension coefficient—the relative tension of the cables—to find the solution of tension distribution corresponds to the optimal workspace, where the determination of the tension coefficient is based on a linear optimization model. Diao and Ma [26] introduce a numerical method to determine FCW. This method checks the condition that a possible solution exists for any external force acting on the MP at a given position of the robot with 6 DOFs and 7 cables. Possible solutions are determined by analyzing the Jacobian matrix to see if there exists at least one all-positive cable tension solution. Lim et al. [27] have developed an algorithm to determine FCW for fully constrained CDPR, based on convex theory, the algorithm checks combinations from the structural matrix of the point under consideration. Ouyang and Shang [28] designed a method to determine the workspace of CDPRs based on the pointwise method. This method determines the satisfaction condition of a pose by computing the null-space of the structure matrix and solving the corresponding system of linear constraint inequalities. Cong Bang Pham et al. [29, 30] used a structure matrix size reduction algorithm based on convex theory to simplify the process of finding the force closure workspace of the fully and over constraint CDPR, the calculation results are applied on a planar CDPR 3 DOFs—4 cables and a spatial CPDR 6 DOFs—8 Cables.

To increase the accuracy when determining the boundary of the workspace compared to the pointwise method, analytical methods are used to determine the hull of the workspace. Marc Gouttefarde and Clement Gosselin [21] developed an algorithm to determine the COWCW of 3 DOFs planar CDPR driven by 4 cables. In this paper, the graphical representation is used to illustrate the workspace as a combination of quadratic curves. Gouttefarde et al. [24] have developed an analytical method to determine the boundary of the Wrench Closure Workspace of CDPRs having fewer DOFs than the cables. This method is based on the linear dependence between columns of the structure matrix to determine the WCW with fixed orientations of the MP, the boundaries of the WCW are defined as a set of cones formed by two redundant cables. The results show that the workspace boundary is composed of conic parts with different shapes. Based on the results of the model analysis in [24], Azizian et al. [31] offer a geometric method used to define the shape of conic sections forming a workspace with a fixed orientation to find the COWCW of planar CDPRs. However, due to the complexity of the calculation formulas, the application of this method to spatial CDPRs needs to be reconsidered. Pott [32, 33] proposed a method to estimate the hull of the translation workspace of CDPRs. This method is based on finding subsets with less dimension such as planes by the analytic method to reduce the computation process, then finding the intersection lines between planes and the workspace to build the hull of workspaces. This method can lead to incomplete estimation of the workspace, and the estimation results depend on the selection of the workspace center. Similar to [32, 33], Perreault et al. [34] have established a method for determining COWCW graphically, which uses lower-dimensional mathematical descriptions such as planes to define the hull of the workspace. The surface shape of the workspace is determined by the combination of curves that intersect the planes and the workspace. Rezaazadeh [35] has developed an analytical method for determining the workspace of the multibody CDPR, which builds on the null-space analysis and simplifies the force and moment constraints in the equilibrium equation. Based on [21, 24], Sheng et al. [36] have come up with a general algorithm to find the WCW of spatial CDPRs, this algorithm is based on calculating the kernel of the structural matrix of CDPRs with the Cramer’s rule method, thereby giving the satisfied condition of a pose of CPDRs 6 DOFs—7 cables.

Although analytical methods have improved the surface accuracy of the workspace, the volume of the workspace is not completely determined. Therefore, many methods of defining persistent workspaces have been developed to overcome this limitation. To identify continuous workspaces, interval analysis has been studied and applied to different types of CDPRs. This method is based on numerical analysis, in which interval vectors may fully satisfy, partially

![Figure 3: Pointwise workspace of suspended configuration CDPR 6 DOFs—8 cables; a-radial space, b-equidistant space, b-random space.](image-url)
satisfy, or not satisfy all the constraints of WFW or WCW. Partial satisfaction interval vectors will be subdivided into intersecting parts and continue testing until the size of the interval vectors is less than a preset threshold. This allows you to define workspaces that ensure continuity within a range and between adjacent intervals. However, the computation of this method is large, especially when the threshold is reduced to increase accuracy as well as smooth the boundary of the workspace. Bruckmann et al. [37] used the interval analysis algorithm to determine the continuous operation space of the CDPR robot. This method can be used to find the optimal configuration of the CDPR with additional criteria such as load, orientation angle, or stiffness of MP. Gouttefarde et al. [38] analyzed the WFW of CDPRs with m DOFs driven by the number of cables equal to or greater than m. An interval analysis algorithm is applied to determine WFW with reasonable computation time. Compared with [37], this method shortens the time to determine WFW by using an adding tool to check if the MP coordinates are completely within the feasible region or not. This method reduces the time to separate tension and external wrench. Khalilipour et al. [39] developed a systematic interval method to determine the workspace of planar CDPRs. Based on [38, 39], Loloei et al. [40] studied an interval algorithm based on the fundamental wrench to specify the controllable workspace of both planar and spatial CDPRs. Lamine et al. [41] also established the workspace for CDPRs during the design process based on equilibrium equation calculations with interval analysis, two Planar CDPR 3 DOFs—4 cables and spatial CDPR 6 DOFs—8 cables configurations were analyzed to calculate the minimum size of CDPRs with a given workspace.

Based on the numerical approach, the ray-based method was first studied by Merlet [42] to determine whether the design trajectory is in the workspace of the parallel robot, and this method determines the WCW by solving a combination of univariate polynomial equations. Based on triangulation of workspace hull analysis, Pott [32] developed a method to determine the hull of WFW of CDPR 6 DOFs and one redundant cable. This method defines a sphere with a predicted workspace center and extending in a-radial direction. This method has the disadvantage that the result depends on the estimation of the center of the workspace, but it has a simple process and can be applied to the quick determination of the workspace of common robot configurations. Based on the results of [32], Pott and Kraus [43] used the ray-based method of numerical analysis to combine WCW with the fixed orientation of MP. Here, a finite number of rays are used to form the hull of the workspace. Similar to [32, 43], Andreas Pott [33] proposes a workspace calculation scheme that works with the fixed or variable orientation of CDPRs. The triangulation representation allows for quick determination of the volume and boundary of the robot workspace. [44] Darwin Lau et al. proposed a technique that combines numerical and analytical methods to calculate WCW for fully constrained CDPR configurations. The method used is to reduce the size of the Jacobian matrix to improve the computation time. This method is applied to calculate WCW for CDPR with 6 DOFs and 7 cables. The results show that this method has improved calculation time and accuracy compared to the numerical method. Based on [43, 44], Ghasem Abbasnejad [45] introduces a ray-based method for determining WCWs for different CDPRs configurations, which allow for defining continuum generalized WCWs with any number of cables. This is a general analysis that can be applied to many different types of CDPRs configurations. In which, the two-dimensional representation of the graph can be applied to any number of DOFs of the CDPRs. Zeqing Zhang et al. [46] develop a new method to design trajectories for Spatial CDPRs 6DOFs—7 cables under interference-free (IFC) and wrench-closure (WCC) conditions. Based on the ray-based method, moving platform trajectories including position and orientation are represented as parametric equations with respect to time.

Riechel et al. [25] described the Force Feasible workspace, also known as a useful workspace. This workspace is determined based on the condition that the equilibrium equation is satisfied for a finite set of forces acting in the stationary state. This model is applied to build a workspace for a point-mass CDPR 3 DOFs—3 cables. Cong Bang Pham et al. [1] established a method to optimize the structure of CDPRs based on tension analysis and stiffness matrix. In this study, the connecting points in the fixed frame and moving platform are considered and analyzed to optimize the workspace. A planar CDPR 3 DOFs—4 cables is modeled to calculate the workspace based on structural stiffness parameters and cable tension limits. The results of the workspace calculations are analyzed and used for the design of an algorithm to optimize the robot structure. Giovanni Boschetti and Alberto Trevisani [47] have established a model to evaluate the overall performance of CDPRs with the Wrench Exertion Capability (WEC) performance index. The performance of a CDPR is considered an important parameter in the design and control process; it represents the relationship between the robot structure, the cable tension limits, the force, and torque limit so that the robot can act in a given direction of movement. This model is built based on the linear programming algorithm for the equilibrium equation of CDPRs. Zake et. al [14] introduced the concept of Control Stability Workspace (CSW), where the system stability criterion is an input parameter to determine the CSW. The CSW was significant in designing the controller for the CDPR, which means that for any MP pose in its CSW, the robot controller should be able to guide the MP to that target.

Through articles that have been reviewed, the methods of determining the workspace of CDPRs are studied focusing on the workspace types such as WFW, WCW, COWCW, COWFW, and controllable workspace. In these papers, the analysis method is used to determine the contour of the workspace, this method gives a visual view of the shape of the workspace, but does not determine the volume of the workspace. The disadvantage of this method is that the computational expressions are complex, and it is difficult to combine representation models for different types of CDPRs configurations. Numerical methods include the pointwise method, interval analysis, ray-based method, which are
useful in determining the surface and the volume of workspaces. The pointwise method results in a discontinuous workspace, the accuracy depends on how the coordinates are obtained and the resolution between the points in the workspace. The interval analysis method gives a more general result, ensuring the continuity of the workspace even at the intersections between intervals. However, the surface accuracy of the workspace depends on the threshold of the divisions. The ray-based method is a method that uses the number of rays to determine the surface of the workspace, and this method has better continuity than the pointwise method, but there are still gaps between rays that are not defined. Another problem is that the ray-based method is only applicable to the determination of the workspace for fully constrained CDPRs. However, there is an improvement in computation time compared to interval analysis. Through a summary of relevant research contents, the results of the analysis of the above studies have important implications in choosing suitable options to determine the workspace of CDPRs because each option has its advantages and disadvantages, so researchers need to consider carefully the constraints as well as the computation purposes to select the feasible calculation methods for each specific CDPR configuration. These contents can also help researchers shorten the time to synthesize relevant documents in the research process.

4. Tension Distribution

The calculation of cable tension distribution for CDPRs is very important in the process of calculating, designing, and controlling CDPRs. Cable tension involves important problems such as system equilibrium, workspace determination, actuator and load calculation, system stiffness, trajectory control, and controller design. There are many methods of calculating cable tension that has been studied and applied to the development of CDPRs. This section will analyze the mathematical basis and examine the published calculation methods, thereby giving a more systematic view of this issue.

The linear programming algorithm is often used to find the tension distribution solution with the optimal condition that the sum of the tension forces is minimal. From (5) and (10), the tension distribution for a pose of the MP is the solution of linear programming as follows:

$$\min \text{imimize } f(t) = c^T \tau.$$  \hspace{1cm} (10)

With the equality constraints

$$A\tau = -wd.$$ \hspace{1cm} (11)

Linear programming (p-norm = 1) is usually solved by Simplex Algorithm or Dual Simplex Algorithm, this algorithm will find the optimal point by detecting the vertices of the polytope constraint. Shiang et al. [48] used linear programming to find tension solutions for the Robotic crane based on its dynamics model. Pham et al. [49] used a linear programming optimization algorithm to calculate the distribution of cable tension. In this paper, the cable distribution model is reduced to the basic linear optimization form. Agrawal [50] used linear programming to calculate cable tension for planar CDPR, and the calculation results were simulated and compared with other methods. Based on this model, the authors also design a solution to control the robot with negative tension. Similarly, Borgström et al. [51] improved the linear programming algorithm to shorten the calculation time of cable tension distribution, and the active-set method is used to determine the optimal solution, the optimal criterion is also the safety tension. This method gives fast computation, but the continuity of cable tension and real-time response need to be further considered. Notash [52] provide an analytical solution to find the minimum cable tension for redundant planar CDPR applications in mechanical engineering.

Similar to linear programming, nonlinear programming was used to calculate the cable tension distribution. Compared with linear programming, this method gives continuous tension distribution results according to the joint trajectory. Quadratic programming (p-norm = 2) [50, 53] has a quadratic objective function, and this method gives continuous root forces because it limits the case those solutions are taken at the vertices of the convex polytope (set of solution). The tension distribution model of quadratic programming has the form:

$$\min \text{imimize } f(t) = c^T (\tau - \eta)^2.$$ \hspace{1cm} (12)

With the equality constraints

$$A\tau = -wd.$$ \hspace{1cm} (13)

Bruckmann et al. [54] introduced methods to define tension solutions for over-constraint CDPRs, including quadratic programming. In this paper, the quadratic objective function is used for better computation time than linear programming. Li et al. [55] used quadratic programming to determine the tension solution for the FAST telescope. In [56], the author used the nonlinear cost function for optimized cable to reduce power consumption. Cote et al. [57] applied quadratic programming to distribute the tension to the cables with the idea of obtaining a second optimal solution by adding a slack variable to the equilibrium equation, which is capable of returning the distribution of tension solution even at positions outside the workspace.

To reduce calculation time and ensure continuity of cable tensions along continuous motion trajectories, the Closed-Form Method is an algorithm developed to calculate cable tension distribution for CDPRs with the number of cables being more than the number of DOFs (r > 0). This method converts the problem of finding cable tension into basic numerical equations to shorten the calculation time. This method is suitable for real-time control requirements and minimized objective function is the distance from the solution to a reference vector (usually the mean value of the tension) with p-norm ≥ 2.

Richard Verhoeven [20] established an algorithm called Gradient-based optimization. This algorithm can be applied to different p-norm orders of the objective function, the two
are applied to the planar CDPR to evaluate the effectiveness of the analytical method. Bruckmann et al. [54] proposed a method to calculate the distribution of cable tension online for controlling redundant CDPRs. Based on the kinematic and equilibrium equation of CDPRs, the authors designed two cable tension calculation algorithms for CDPRs with 1 or more redundant cables. Two algorithms gradient-based optimizers and interval analysis are used to calculate cable tension with the requirement of continuous cable tensions along the trajectory of MP. Richard Verhoeven and Manfred Hiller [61] provide a method for determining the cable tension of CDPRs with more than one DORs, this method is based on convex polyhedron analysis of the solution space region for the equilibrium equation. The optimal result is a solution that minimizes the tension of cables but does not guarantee the continuity of the cable tensions along the joint trajectories as the moving platform moves along the given trajectories. Hassan and Khajepour [62] developed an iterative algorithm based on Dykstra’s Algorithm projection to find the tension distribution for a CDPR. This algorithm uses an iterative projection starting from the origin (zero) onto the solution space and back projecting the polyhedron onto the tension limit. The intersection (solution region) will be determined when the distance between the solution space and the polyhedron of the tension limit is zero. The results are applied to calculate the tension for a 3 DOFs robot driven by 3 cables and two redundant limbs to create cable tension with all applied forces.

Based on the geometrical characteristics of the convex polygon that represents the feasible region of cable tension—(17) with $w = 0$, Bruckmann et al. [63] proposed a new cable tension calculation algorithm—Safe Force Generation Method—with the requirement that the solutions must be feasible (located in the workspace) and continuous. The algorithm is based on the decomposition of the structural matrix, thereby selecting the solutions located in the safe area (priority to stay away from the boundary values of the cable tension). This allows the MP to be controlled at large speeds and accelerations without the transmission cables slacking. To calculate the real-time cable tensions for a CDPR 3 DOFs—4 cables for moving the camera, Su et al. [64] proposed a nonrepetitive solution based on convex theory, the simulation results show that the obtained cable tensions have a continuous form according to the control trajectory and the calculation time is suitable for real-time control applications. This solution can be modified to apply to other CDPR configurations. However, compute performance metrics and continuity metrics need to be considered for each specific type of configuration. Cui et al. [65] propose a nonrepetitive method to calculate cable tension distribution. This method defines the cable tension feasible region based on Graham’s scanning geometric method applicable to CDPR with 2 DORs. The paper also proposes a method to evaluate the safety of cable tension distribution results. Boumann and Bruckmann [66] develop a method to find solutions to tension distribution when CDPR operates outside the WFW. Based on geometric analysis, the Nearest Corner Method is used to find the closest solutions that satisfy the equilibrium equation based on detecting the
corners of the hypercube composed from the limit of cable tensions. Solution continuity is also considered in the case of robots in the WFW region.

Pott [67] improved the closed-form method to calculate the tension at points in undefined regions for over constraint CDPRs in real-time. Mikelsons et al. [68] designed a non-iterative method to calculate the distribution of cable tension with the optimal condition of safety tension. This method determines the vertices of the feasible convex polytope, then calculates the average value of the vertices found, the tension values are calculated close to the safe value (average of \( \tau_{\min} \) and \( \tau_{\max} \)), it is also called the Barycentric approach. However, the execution time is an issue to consider when applying this method. Based on Barycentric, Lamaury and Gouttefarde [69] propose a method “Fast Tension Distribution Algorithm” to improve the calculation of cable tension solutions for CDPRs with \( m \) DOFs and the number of cables \( n = m + 2 \). This method is based on determining the feasible convex polygon formed by the intersection between the set of cable tension systems satisfying the constraints in the equilibrium equation and the space bounded by the boundaries of the cable tension. By reducing the number of scanned vertices, the time to determine the feasible convex polygon is improved. The analysis and calculation method of the barycenter of the convex polygon above are applied on a CDPR 6 DOFs—8 cables. The simulation results show that the value of cable tension changes continuously along the design trajectory. Based on the results of [69], Marc Gouttefarde et al. [70] developed a cable tension calculation algorithm called Versatile Tension Distribution Algorithm; the algorithm is applied to CDPRs 6 DOFs driven by 8 cables. With this configuration, the set of possible cable tension solutions is a two-dimensional convex polygon corresponding to 2 DORs. The algorithm is designed based on determining the polygon’s vertices in a clockwise direction or vice versa. Calculation results are verified on two CDPR 6 DOFs—8 cables prototypes in CABLAR and CoGiRo projects. This method is effective with different requirements for determining the optimal tension distribution, as well as for establishing a maximum number of worst-case iterations. Also based on the algorithm of [69], Rashied et al. [71] designed an algorithm to find the optimal tension distribution for CDPR 6 DOFs—8 cables. This CDPR is reconfigurable since the 4 cable posts are movable with their wheels. Based on [63, 68], Song et al. [72] propose a method called convex analysis to find the optimal configuration for a CDPR 6 DOFs, where the direction of the moving platform is determined in the workspace. Cable mounting configurations are found by comparison and analysis of workspace and flexibility. The authors also propose an algorithm to find real-time cable tension solutions, which is based on calculating the equilibrium equations of a CDPR 6 DOFs driven by 8 cables. The results are compared with two methods, the minimum norm method and the safety tension method, the analysis results show that the tension solutions are continuous and within the limit of the required cable tension, similar to the safety tension method with fast computation time, suitable for real-time applications.

Another method developed based on the closed-form method to calculate the continuous minimum tension along the joint trajectory is the puncture method. This method is based on defining an initial solution in a feasible set, which can be computed using the closed-form method. Then combined with the solution close to the root from the null space of the structure matrix into a straight line, and the minimum solution will be determined on this line because of the linear continuity of the null space. Müller et al. [73] analyzed the results and limitations of the method of minimum tension distribution of the closed-form method, from this result, the author developed an improved puncture method, based on the combination of the improved closed-form method and puncture method. Where the cable tension is distributed continuously and tends to reach the lower limit of the cable tensions, the calculation results show that the improved puncture method gives the results of calculating the cable tension solution in a shorter time with a larger workspace, but the cable tension tends to distribute near the lower limit and has a high amplitude.

Pott [74] provided the analysis method to determine the limits of cable tension for CDPRs. For the maximum tension, the important parameters to be considered are the safety issue, the durability limit of the mechanical structure, the fatigue of the cable during operation, and the capacity of the actuator. Thus, the upper limit force of the transmission cable mainly depends on the mechanical structure and transmission mechanism, which are the main criteria affecting the cost of manufacturing and deploying CDPRs. For the minimum tension, the important criterion to calculate is the cable sagging, especially for the large length cables, the cable tension will be inversely proportional to the cable sagging. The stiffness and elasticity of cable also affect the accuracy of CPDRs, as it affects the operation of the cable distribution mechanism. The vibration of the cables is also analyzed when calculating the lower limit of the cable tension because it is possible to reduce the vibration of the system by increasing the tension in the cables. Notash [75] analyzes the properties of cable tension with the influence of unknown parameters on the structural matrix and the null space of CDPRs, the calculation results are applied to a planar CDPR to evaluate the efficiency of the model. Su et al. [76] propose an iterative method to calculate the cable tension distribution for a large-space camera CDPR3 DOFs—4 cables, the equilibrium equation is built based on the dynamic equation and the sag model of the transmission cable. The iterative algorithm is developed based on the kernel analysis of the structure matrix. The results show that the cable tension changes continuously along the motion trajectory of the CDPRs. Barroso and Saltaren [77] analyze the relationship between the maximum tensions and the external force acting on the MP, the goal is to determine the capacity of the actuator used to design the cable distribution mechanism, thereby determining the workspace of the CDPR based on the null space analysis of the structural matrix. Ueland et al. [78] analyze the problem of determining optimal cable tension distribution for over constraint CDPRs based on studies [58, 70, 77], thereby proposing a new optimal objective function that ensures the
continuity of the cable tension. Hussein et al. [79] provide a solution for determining the minimum value of the upper limits of cable tension for CDPRs with DOFs equal to or greater than 1. The constraint condition of this issue is that the cable tension limits must satisfy a required wrench set. This result can be applied to CDPRs that work with large weights and are driven with winches of different structures and capacities. The aim is to optimize the energy consumption and structure of the CDPR.

Through the analysis of the above studies, it is shown that many methods have been developed to calculate the tension distribution for fully and over constraint CDPRs. These methods focus on optimizing solutions according to different goals, such as minimizing tension for applications that require saving energy consumption, safe tension which is convenient for controlling, or maximum tension for increasing the stiffness of the structure. Three methods linear programming, Dykstra, and available wrench set methods give discontinuous tensions according to the joint trajectory, while quadratic programming, nonlinear programming, closed-form, improved closed-from, Barycentric, and puncture methods give continuous tensions following the motion trajectory. In which, closed-form, improved closed-from, Barycentric, puncture methods have fast calculation time, suitable for real-time control requirements. In addition, some methods such as linear programming, Dykstra, and improve-closed-form methods can also be used to determine the workspace according to the pointwise method. Studies [74, 77] also have given a way to determine the limits of cable tension. The lower limit and the upper limit of cable tensions have important significance in designing, determining the structure of the CDPR, calculating the tension distribution, and determining the workspace, and the stiffness of the CDPR.

5. Sagging

One of the important problems in the design and development of large CDPRs is the sagging of cables, as it directly affects the accuracy of the robot. The general procedure for determining cable sagging of CDPRs is complicated, due to the nonlinearity of the mathematical model. The sagging model of cables is a combination of kinematics, dynamics, cable tension distribution, and cable sagging model. The models for calculating the sagging of cables are mostly based on the catenary equation studied by Irvine [80], where the influence of cable mass, cable elasticity, and cable tension are taken into account in the sagging model.

Based on Irvine’s cable sagging equation, Kozak et al. [81] show the results of static model analysis of large-scale CDPR taking into account the mass of cables. Due to cable sagging, the static displacement of uniform elastic cable is also calculated in the postanalysis of the inverse kinematics and CDPR stiffness. This result can be used for the calculation of workspaces with sagging cables or the design process of large CDPRs. Korayem et al. [82] analyzed the kinematics and determined the workspace of the large CDPR taking into account the sagging of the elastic cable. Nicolas et al. [83] researched the influence of cable mass on the kinematics of CDPRs (the number of DOFs is equal to the number of cables). The forward and inverse kinematics of a large-size suspended CDPR with 3 DOFs-3 cables are calculated based on the elastic cable model of Irvine. The results show that the influence of cable mass on the sagging of cables and the accuracy of CDPR is significant when CDPRs operate with large workspaces or heavy payloads. Based on these results, the authors determined a large CDPR workspace that takes into account cable sagging [84].

FAST telescope is a famous project for the application of giant CDPR. Related to this project, Yao et al. [7] set up a complete model for CDPR 6 cables taking into account the mass and elasticity of the transmission cables, the model is set up as a simple structured equation, and the computation time is faster than catenary equation model. However, the accuracy of this model is not high and a calibrated model is needed to meet the required accuracy. This model is tested on a miniature version of FAST with an accuracy of less than 2 mm. Hui [85] calculated the inverse kinematics of the real version of FAST and planned the orientation of the cabin according to the given trajectory. The study shows that the orientation of the cabin depends on the position of the cabin and the sagging of the cables.

Based on the parabolic cable profile equation of Irvine [80], the sagging model of the large inelastic transmission cable is considered a parabolic curve. Gouttefarde et al. [86] proposed a new simplified static analysis of the CDPRs assuming that a cable with negligible mass and inelastic properties is used to drive the CDPRs. Based on the catenary equation (80), this cable sagging model is developed which assumes that the sagging of the cable is small. The model is simplified by linearizing the relationship between the two projections of the cable tension projected onto the analyzed axis system. This model cannot be applied to under-constrained CDPRs and elastic cables. Also based on the Irvine cable sagging model, Gouttefarde et al. [87] calculated cable sagging with the influence of guide pulley diameter. Nguyen et al. [6, 88] calculated the invert kinematic taking to account cable sagging for reconfigurable CDPRs, the paper proposes a simple cable sagging model and analyzes the remaining problems of this model. Dallej et al. [89] build a vision machine feedback controller. Where the cameras are designed to respond to the position and orientation of the MP to the controller, these data are used to directly correct the robot’s inverse kinematics with cable sagging. The article only stops at evaluating simulation results, not experimenting on the CDPR prototype. Sridhar [90] built and simulated a transmission cable compensation model for suspended CDPR in large outdoor spaces. This paper compares the straight-line cable model and the sagging cable model, thereby evaluating the relationship between the cable tension optimization and the error of the cable length calculation model.

Merlet [91–98] studied the inverse kinematic and forward kinematic, calculating cable sagging with different influencing parameters. In [91, 92], the forward kinematics for CDPRs were solved based on interval analysis taking into account the elasticity and mass of the cables, the cable sag
model is considered in static equilibrium. The results are tested on CDPR 6 DOFs—8 cables and give several interesting results. Besides, inverse kinematics for CDPR—6 DOFs—6 cables with cable sagging expressed as catenary equations was developed in [93], and interval analysis is also used to set up an algorithm to calculate the inverse kinematics of this robot. However, the relationship between the limit of MP and the workspace has not been clearly defined. In [94], the author analyzed the workspace of CDPRs for both straight cables and sagging cables. In the case of sagging cables, the boundary of the workspace is not clearly defined. The direct kinematic of CDPR in the case of cable sags [95] was calculated by sensors. The important properties of the static-elastic model based on Irvin’s cable sag equation were analyzed, and this result can be used to calculate the CDPR kinematics taking into account the sagging of cables [96]. In this study, a new form of the Irvin equation has been established that can reduce the calculation time of forward and inverse kinematics by interval analysis. Besides, the singularity of CDPR taking into account the sagging of the cables was analyzed, and the results show that CDPR’s singularity tends to appear near the edge of the workspace [97]. Calculation results only identify individual singularities, not yet identified singularities for both inverse and forward kinematic. In [98], the forward kinematics for the point-mass suspended CDPR with one redundant cable were calculated. The computational model is built based on the cable sagging equation and interval analysis. A system to measure the signals of cable lengths, cable angles and cable tensions were also built to evaluate the results of the forward kinematics with the influence of uncertainties. In [99], Fabritius and Pott built a new inverse kinematic code applied for CDPR that has more than one redundant cable. The results show that the new method for the workspace is nearly 20% larger than the other method. In [100], the authors also calculated the forward kinematics of CDPR taking into account the influence of cable sagging and guide pulleys. The calculation results show that there is a difference compared with the standard geometric model, in which the obtained WFW and stiffness are smaller than that of the standard model. These results show a significant influence of the cable sag model and pulley parameter on the calculation of forward and reverse kinematics of redundantly CDPRs. From point of view of Yuan et al. [101, 102] analyzed the relationship between cable sagging, stiffness, and dynamics for suspended CDPRs when the cable and MP vibrated during operation. These researches can be used to build vibration reduction solutions for CDPRs of large sizes or working outdoors. The authors also construct the dynamic stiffness matrix of a single cable to calculate the dynamic equations of CDPRs [103]. This computational model is tested on a spatial suspended CDPRs 6 DOFs—6 cables. The results show that there is an influence of dynamic cable on the dynamic model and the accuracy of CDPRs. Arsenault [3] has established a CDPR stiffness representation model that takes into account the sagging of the cables. The results show the influence of cable sagging on the workspace and the stiffness matrix of the robot.

Duan et al. [104] built a deployment and retrieval cable mathematical model based on the lumped mass method taking into account the cable mass. The simulation result for a suspended CDPR 6 DOFs - 6 cables (50 m scaled model) shows that the model is effective in analyzing the dynamic response of cables. Wei et al. [105] analyzed the influence of the cable’s inertia on the stability of the CDPR. That was a large size and high-speed cable robot used to move the camera. The dynamic model of the CDPR was determined based on the cable catenary equation and the finite element method, from these calculation results, a camera CDPR controller was designed and simulated. Phan Gia Luan et al. [106] have introduced a new model to calculate the inverse kinematic for CDPR with the influence of cable sagging in quasistatic based on both analytical and practical methods. Experimentally, this result is only valid for CDPRs operating at low speeds and small accelerations. Tho et al. [107] built a cable sagging calculation model using ANFIS, and this model is based on the results of Irvine’s catenary equation. The calculation results show that the accuracy of the model is quite good with small errors and short computation time. However, the model is only built for a given robot configuration and a specific method of cable tension distribution. The model can apply to real-time control requirements for large CDPRs.

6. Conclusion

This article summarized the studies, typical applications, and important issues in the research and implementation of CDPRs. The basic problems of CDPR are presented focusing on the design of the mechanical structure, analysis of workspace, distribution of cable tension, and determination of cable sag. The mathematical models for CDPRs are also presented in a general form to help readers save review time when synthesizing general knowledge about the design process and analyzing CDPRs. The special feature of CDPRs compared with parallel robots driven by rigid-link is that the position and orientation of the MP are controlled by flexible cables. This creates the outstanding advantages of CDPR such as high speed, high acceleration, and higher load-to-weight ratio. In particular, large workspaces can be achieved due to the flexibility in the storage and distribution of cables. However, a challenge when designing and modeling CDPRs is the unidirectional character of the cables, which mean that the cable only works with positive tension. This leads to a different approach in the study of CDPRs compared to rigid-link robots. The important issues of CDPRs such as kinematics, dynamics, and workspaces are different from those of rigid-link robots, and their mathematical basis is related to the tension of the cables. In recent years, many studies focusing on optimal design, kinetics, and controller systems of CDPRs, which facilitate the expansion of the applicability of CDPRs. Through the analysis of research works on CDPRs, it is shown that the structural optimization of CDPRs focuses on the constraints of the workspace, stiffness, payload, and geometric configuration. Especially studies on CDPRs are reconfigurable for more flexible applications.
Cable tension distribution studies focus on generating continuous tension distributions with constraints of dynamic stiffness, energy consumption, or safe tension. Many methods have been studied to reduce computation time, which is useful for applications requiring online control. Cable sagging studies are especially important for CDPRs with large workspace because cable sagging directly affects the CDPR kinematics accuracy. Most of the cable sagging calculation models are based on the famous catenary cable equation of Irvine. Based on the above analysis, further studies on CDPRs can be carried out in the following directions: developing controllers and tracking the moving trajectory of MP; improving high-load capacity with optimization of structure and materials; increasing the accuracy of workspace definition methods; shortening the time to calculate the tension distribution according to the given constraints; simplify the cable sagging calculation model for specific CDPRs configurations. In addition, the research direction on the reconfigured form of CDPRs promises to expand the applicability of CDPRs.

Data Availability

No data were used to support the findings of the study.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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