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Research Article

Research on the Cooperation of Emergency Response Subjects for the Shortage of Urban Natural Gas Energy

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This paper deals with the urgent problem of urban natural gas shortage. In order to improve emergency efficiency, enhance emergency efforts, and reduce urban disaster losses, the differential game theory is firstly applied for constructing a tripartite dynamic game model. Then, based on the Hamilton–Jacobi–Bellman (HJB) equation, the optimal effort degree, natural gas energy shortage, and maximum urban loss are obtained in three cases, i.e., spontaneous governance mode, superior dominant mode, and cooperative mode. The results show that the effort of provincial government, local government, and natural gas emergency enterprise is positively related to the emergency shadow, the impact of effort and natural gas energy attenuation coefficient of provincial government, local government, and enterprise, but it is negatively related to the emergency cost coefficient and the discount rate. From the perspective of emergency shortage and total urban loss, the government-enterprise cooperation mode turns out to be the best emergency mode, while the spontaneous governance mode remains the worst. At the same time, the government implements subsidies and incentives for nongovernmental organizations involved in emergency response, which is more conducive to the emergency of natural gas shortage in heavily suffering cities.

1. Introduction

The consumption of fossil fuels stimulates economic growth but inevitably causes serious pollution problems. Persistent environmental problems have aroused extensive public attention. Coal is the main component of energy consumption in China. The key to the adjustment and upgrade of China's energy structure is to transform its energy consumption structure. China's industrialization and urbanization have developed rapidly. In recent years, the country has been promoting the implementation of the "scientific development concept," the establishment of a "harmonious society," and the advocacy of scientific, balanced, and harmonious national economic development. Hence, the coverage of clean natural gas in China becomes increasingly extensive, and the urban gasification rate has increased significantly [1]. As a part of urban energy, natural gas energy plays an extremely important role in the sustainable development of cities, and urban natural gas energy development is of great significance for the improvement of life quality of urban residents, urban environment, and energy efficiency. With the increase in the length of transmission and distribution pipelines, coupled with the development of China's urbanization, the demand for gas continues to rise and the scale of its urban gas industry continues to expand [2].

The natural gas consumption in China keeps growing rapidly, which reached up to 276.6 billion cubic meters in 2018, with an annual increase of more than 39 billion cubic meters and an increase of 16.6%, accounting for 7.9% of total primary energy consumption [3]. By 2020, nonfossil energy took up 15% of primary energy consumption, while natural gas occupied more than 10% [4]. The acceleration of urbanization promotes the development of urban gas industry and becomes the main driving force for the growth of natural gas consumption. In the meantime, rapid increase in urban population, together with people's increased awareness of environmental protection and quality of life, will continue to increase the use of natural gas for urban energy consumption, thereby leading to the continuous increase of the supply and consumption [5].

Given the importance of natural gas energy in China, the problem of natural gas shortage has attracted public attention [6, 7]. Natural gas supply interruption caused by major natural disasters, such as wars, major production, and transportation safety accidents, is provided with the characteristics of suddenness, urgency, uncertainty of information, variability in development trends, and large scope of influence [8]. When the shortage of natural gas ceases, it may lead to the stagnation of industrial production, service industry supply, and social economic activities; people's daily life may be stopped from operating normally; at the same time, it may also cause social panic and stimulate potential consumer demand in a short period of time, thus leading to a sudden increase in consumption and prices and social instability [9, 10]. Although these articles have studied the importance of natural gas energy in China and analyzed some critical situations that might occur due to natural gas shortage; they are all qualitative analysis. There is no analysis of emergency measures after natural gas energy shortage or any in-depth analysis of the specific emergency evolution behavior of governments at all levels and natural gas enterprises. To this end, a mathematical model is hereby established to quantitatively analyze the emergency measures for the shortage of urban natural gas energy, and the specific evolutionary behavior of governments at all levels and energy enterprises in the emergency process is studied for the first time, providing a strong guiding significance for the government to guide the emergency response of urban energy shortage and a reference significance for other emergencies.

With the increase of natural gas consumption, sudden incidents of natural gas supply interruption have occurred frequently. For example, the snow disaster in southern China in 2008 led to gas shortage in many provinces; the abnormal weather triggered a large-scale natural gas shortage from the end of 2009 to the beginning of 2010; the gas pipeline cut-off by mistake caused the gas outage in a few areas for several days during the subway construction in 2011 in Kunming City, Yunnan Province; the road renovation of Change West Road in Xi'an City, Shaanxi Province, caused gas leakage in 2013 and stopped thousands of households from using gas [11], etc. In November 2017, "gas shortage" broke out in China, and most areas were strained with gas, which affected people's normal life. The same incidents happened in other countries. For example, the western Australia natural gas pipeline explosion in June 2008 cut off one-third of the region's natural gas supply; the natural gas dispute between Russia and Ukraine in January 2009 interrupted the supply of natural gas in some European countries; the U.S. hurricane in October 2018 named "Michael" caused a power outage, affecting approximately 400,000 local users, severely holding up the oil and gas production in the Gulf of Mexico. The frequent occurrence of sudden energy interruptions affects normal life of residents and national security and has attracted the attention of many scholars worldwide.

Scholars have conducted much research on energy emergency system, which can be described by two cases: one is considered against the context of China. Qing [12] revealed that in the process of responding to the energy crisis, European countries have formed a relatively complete energy emergency legal system and pinpointed its reference to China; Mastropietro [13] pointed out the energy legislation problems in China against the background of global climate change and proposed an energy emergency legal system suitable for China; Nancy [14] modeled an emergency management process of energy emergencies based on generalized stochastic Petri nets and obtained the state space reachability diagram based on the invariant judgment model. Najjar-Ghabel et al. [15, 16] established crisis early warning index systems for oil and natural gas safety; Shirley and Looi [17] conducted regional energy security research using system dynamics and provided guidance for regional energy security; Galvin [18] studied gas transmission pipelines and designed a basic framework of emergency rescue system for gas pipeline accidents; Liu [19] constructed a collaborative coal mine safety emergency management system for improving coal mine safety emergency management, which turned out more effective in responding to emergencies and provided references for other coal companies; Liu and Lv [20] designed an overall architecture and functions of the oil platform emergency decision support system and developed the oil platform emergency decision model network; Wang and Jing [11] reflected the dynamic characteristics of natural gas interruption emergency decision-making using the dynamics prediction method and conducted quantitative analysis of expert decision-making, using the fuzzy theory. Another case is given by some other countries. For example, Liu and Lv [21] constructed a framework model for energy emergency in response to the sudden interruption of energy shortage and proposed an innovative solution for load selection in the network in the case of limited or no main power supply (under the following conditions: emergency); Shen and Wang [22] established an exhaust model using Stonenell Pipe Simulator (SPS) and studied the influencing factors of exhaust time through SPS dynamic simulation; Kong and Zhu [23] claimed that the energy emergency system involves many stakeholders and emergency response requires the cooperation of governments at all levels concerning multiple levels of management, which makes it difficult to manage. They affirmed that the focus of energy emergency management is on local authorities, and they must be given priority for timely response; Vesa [24]put forward a hybrid electric-hydrogen energy storage system to fulfill a large-capacity independent emergency power supply, which can provide high-reliable and high-quality power in the case of large-scale natural disasters, when effective use of solar power will be utilized for generating electricity to deal with emergencies; Shirley and Xiaoyan [25] studied energy emergency using the linear programming theory and proposed that emergency preparedness information must be provided, so that the public are enabled to respond appropriately to potential energy interruption crises; Ray Galvin [20] proposed energy consumption trends and practices in emergency and offered suggestions about how to mitigate their damaging effects on the climate based on the actual situation in the United States. However, all these above studies mainly focus on qualitative research or only consider the problem from the single aspect of energy emergency and lack systematic research from the perspective of management. There is insufficient stakeholder research on emergency response, especially in multistakeholder situations.

In this case, from the perspective of game strategy, differential game was first used to the multiagent participation in emergency response, with HJB equation introduced into the game theory. It is noteworthy that HJB is the key to giving a solution in the game strategy. Thus, the authors in [26] and [27] proposed similar methods to solve the energy emergency problems, but the research is rather simple and their conclusions are insufficient to solve the practical problems. It is widely acknowledged that emergency is provided with rather important theoretical significance and practical value for the natural gas emergency treatment and safety early warning problems in China. However, the mentioned schemes [26, 27] on the emergency decision-making problem of natural gas supply interruption fail to consider the problem of multiagent government participation by the tool of model dynamics. Meanwhile, the change of state variables and the randomness in the game strategy are not considered. Until now, these problems are still full of challenges and remain unsettled.

The emergency main body plays a decisive role in the emergency action. Given the above reasons, a new emergency strategy is hereby put forward for dealing with the shortage of urban natural gas energy. In order to highlight the specific evolution behavior of the main body of emergency response, a mathematical model is established based on the actual situation to quantitatively study the degree of emergency response effort, input cost, urban loss, and emergency energy shortage of the main body of emergency response. Governments at all levels are incorporated into the main body of emergency decision-making, and a three-party dynamic game model composed of provincial government, local government, and natural gas supply chain company is established. The optimal effort degree, natural gas energy shortage, and maximum city loss under spontaneous governance mode, superior leadership mode, and intergovernmental cooperation mode are obtained, respectively, using the HJB equation. The optimal emergency strategy to reduce the city loss is thus proposed based on the comparative analysis of the results.

The research results of this paper are different from the conclusions of some major research; the study in [25] believed that the behavioral strategy selection of energy emergency response subjects is influenced by the income and government punishment and lacks the actual research background without considering emergency response efforts and urban loss; the study in [26] claimed that integrating the government, gas supply enterprises, and gas consumption enterprises for buildings and improving the emergency information platform of the natural gas industry are the focus and breakthrough of emergency response. One-sided emphasis on emergency information fusion does not take into account the effect and mode of emergency response and the specific evolution behavior of emergency response subjects; Kong Nana studied the novel coronavirus emergency response process in Wuhan, China, in 2020 from the qualitative aspect of public health, the conclusion of which is similar to the research results of this paper. The Wuhan municipal government and hospitals and other enterprises failed to control the spread of the virus quickly until the coordination of Hubei provincial government. This article studies from the quantitative aspect of the mathematical model and is considered more convincing.

The structure of this paper is as follows: Section 1 is the introduction narration; Section 2 is model establishment; Section 3 proposes the game strategy analysis on the main body of the emergency of urban natural gas energy shortage; Section 4 summarizes the comparative analysis of the results; Section 5 details the simulation of emergency decision-making; Section 6 puts forward the policy suggestions; Section 7 proposes the conclusion.

2. Model Establishment

The main system of the main decision-making system consists of provincial government, local government, and natural gas supply chain enterprises for urban natural gas energy shortage caused by emergencies. Given that the central government involves only in emergency and catastrophic disasters, it does not play a major role in this system. The degree of efforts made by the local government and natural gas supply chain companies in dealing with urban natural gas energy shortages is $E_1(t)$ and $E_2(t)$, respectively, while the provincial government's degree of efforts to local government in dealing with urban natural gas energy shortages is $E_3(t)$ and its degree of efforts to natural gas supply chain enterprises is $E_4(t)$. The investments of provincial governments, local governments, and natural gas supply chain companies to urban natural gas shortages are $C_3(t)$, $C_1(t)$, and $C_2(t)$, respectively:

$$C_3(t) = \frac{\varphi_3}{2}E_3^2(t) + \frac{\varphi_4}{2}E_4^2(t), \tag{1}$$

$$C_1(t) = \frac{\varphi_1}{2}E_1^2(t), \tag{2}$$

$$C_2(t) = \frac{\varphi_2}{2}E_2^2(t), \tag{3}$$

where φ_1, φ_2, φ_3, and φ_4 represent the emergency cost coefficients of the local government, natural gas supply chain companies, and provincial governments, respectively; $D_1(t)$ and $D_2(t)$ represent local governments and enterprises' emergency natural gas energy shortage at moment t, respectively. Emergency investment is directly proportional to the degree of effort of each subject. As the effort degree increases, the degree of further required effort will be greater. The reduction of local government's emergency natural gas energy shortage is attributed to the joint efforts of the provincial government and the local government; the reduction of the emergency natural gas energy shortage of enterprises comes from the joint efforts of provincial governments, local governments, and enterprises themselves. Emergency shortages will naturally decrease with the changes of the disaster situation. Considering the transboundary nature of urban natural gas energy shortages and the externalities of urban emergency decision-making, changes in emergency natural gas energy shortages are endowed with a certain spillover effect. The emergency natural gas energy shortage of enterprises is affected by the local government, and then, that of local governments and enterprises can be described by the following stochastic differential equation over time:

$$\frac{\mathrm{d}D_1(t)}{\mathrm{d}t} = D_1(t) - \alpha_1 E_3(t) - \beta_1 E_1(t) - k_1 D_1(t), \tag{4}$$

$$\begin{aligned}\frac{\mathrm{d}D_2(t)}{\mathrm{d}t} &= D_2(t) - \alpha_2 E_4(t) - \beta_2 E_2(t) - k_2 D_2(t) - \eta\frac{\mathrm{d}D_1(t)}{\mathrm{d}t}\\ &= D_2(t) - \alpha_2 E_4(t) - \beta_2 E_2(t) - k_2 D_2(t) - \eta\left[D_1(t) - \alpha_1 E_3(t) - \beta_1 E_1(t) - k_1 D_1(t)\right],\end{aligned} \tag{5}$$

where α_1 and β_1 indicate the degree of impact of the efforts of the provincial and local governments on the emergency natural gas energy shortage of the local government, respectively, that is, the efforts of the provincial government and the local government; α_2 and β_2 are the efforts of the provincial government and enterprises to deal with the emergency natural gas of the degree of influence of energy shortage, respectively; k_1 and $k_2 \geq 0$ are the attenuation degrees of the emergency shortage of natural gas energy, which means that natural gas energy emergencies are controlled, that the shortage situation is alleviated, and that the natural gas energy shortage is gradually reduced; and $\eta \geq 0$ is the impact coefficient of the emergency natural gas energy shortage of the local government on the company's emergency. It is assumed that the initial value of emergency shortage $D_1(0) = D_1 = D_2(0) = D_2 \geq 0$

Let $L_1(t)$, $L_2(t)$, and $L_3(t)$ represent the losses caused by the shortage of natural gas energy by local governments, enterprises, and provincial governments at time, $T_1(t)$, $T_2(t)$, and $T_3(t)$, respectively, when the loss of the local government $L_1(t)$ and that of the enterprise $L_2(t)$ can be expressed as

$$\begin{aligned}L_1(t) &= A(t) - \mu_1 E_3(t) - \sigma_1 E_1(t) - \theta_1 D_1(t),\\ L_2(t) &= B(t) - \mu_2 E_4(t) - \sigma_2 E_2(t) - \theta_2 D_2(t),\end{aligned} \tag{6}$$

where A and B are real numbers and $A > B$ indicates the maximum loss of local governments and enterprises; μ_1, σ_1 is a constant greater than zero, indicating the extent to which the efforts of the provincial government and the local government affect the city's losses due to the shortage of natural gas energy, namely, the execution ability of the provincial government and the local government; μ_2, σ_2 is also a constant greater than zero, indicating the extent to which the efforts of the provincial government and enterprises affect the city's losses due to the shortage of natural gas energy, i.e., the ability of the provincial government and enterprises to execute; and θ_1, θ_2 is the extent to which the government and enterprises respond to natural gas energy shortages in response to the loss of cities and enterprises.

3. Game Strategy of the Main Body in the Energy Shortage Emergency of Urban Natural Gas

3.1. The Spontaneous Management Model of the Energy Shortage Emergency Response to Urban Natural Gas. The spontaneous management mode of urban natural gas energy shortage emergency management exists on the basis of natural labor division. Local government operating in closed and decentralized operations is self-sufficient and has limited mutual communication and interaction. In this case, the provincial government has little supervision over the local government and it is common for neighboring local governments not to cooperate actively or even not to cooperate at all. Local governments and enterprises spontaneously carry out natural gas energy shortage emergency themselves without the intervention or funding of the central government and provincial governments, and the three parties engage in noncooperative games. Upholding the goal of minimizing the loss of the infinite time zone, all parties choose the optimal degree of effort and make rational emergency decisions. According to the current system, the losses caused are shared by both the provincial government and the local government, who have the same positive discount rate.

The objective function of the local government can be expressed as

$$T_1 = \int_0^\infty e^{-rt}\left\{(1-\tau)\left[A(t) - \mu_1 E_3(t) - \sigma_1 E_1(t) - \theta_1 D_1(t)\right] + \frac{\varphi_1}{2}E_1^2\right\}\mathrm{d}t. \tag{7}$$

The objective function of a natural gas supply chain company can be expressed as

$$T_2 = \int_0^\infty e^{-rt}\left\{\left[B(t) - \mu_2 E_4(t) - \sigma_2 E_2(t) - \theta_2 D_2(t)\right] + \frac{\varphi_2}{2}E_2^2\right\}\mathrm{d}t. \tag{8}$$

The objective function of the provincial government can be expressed as

$$T_3 = \int_0^{\infty} e^{-rt} \left\{ \begin{array}{c} \tau\left[A(t) - \mu_1 E_3(t) - \sigma_1 E_1(t) - \theta_1 D_1(t)\right] + \frac{\varphi_3}{2} E_3^2 \\ +\left[B(t) - \mu_2 E_4(t) - \sigma_2 E_2(t) - \theta_2 D_2(t)\right] + \frac{\varphi_4}{2} E_4^2 \end{array} \right\} \mathrm{d}t, \tag{9}$$

where $E_1(t)$, $E_2(t)$, $E_3(t)$, and $E_4(t)$ are control variables and state variables. All other parameters are constants greater than zero and are not related to time. Each game subject faces the same game in an infinite time zone, and its strategy is a static feedback equilibrium.

Proposition 1. *In the case of noncooperative game among provincial government, local government, and natural gas supply chain enterprises, the static feedback Nash equilibrium strategies of the three parties can be expressed as*

$$E_1^* = \frac{(1-\tau)}{\varphi_1}\left(\sigma_1 + \frac{\beta_1 \theta_1}{r - k_1 + 1}\right), \tag{10}$$

$$E_2^* = \frac{1}{\varphi_2}\left[\sigma_2 + \frac{\beta_2 \theta_2}{(r - k_2 + 1)}\right], \tag{11}$$

$$E_3^* = \frac{1}{\varphi_3}\left[\tau\mu_1 + \frac{\tau\alpha_1\theta_1}{(r - k_1 + 1)} - \frac{\alpha_1 \eta \theta_2 r}{(r - k_1 + 1)(r - k_2 + 1)}\right], \tag{12}$$

$$E_4^* = \frac{1}{\varphi_4}\left(\mu_2 + \frac{\alpha_2 \theta_2}{r - k_2 + 1}\right). \tag{13}$$

Proof. In order to obtain the Markov refined Nash equilibrium of the noncooperative game, suppose that there exists a continuous and bounded differential urban natural gas energy shortage loss function for all $D_1 \geq 0$ and $D_2 \geq 0$, which satisfies the Hamilton–Jacobi–Bellman equation:

$$r \cdot V_1(D_1, D_2) = \min_{E_1 \geq 0} \left\{ \begin{array}{c} (1-\tau)\left(A - \mu_1 E_3 - \sigma_1 E_1 - \theta_1 D_1\right) + \frac{\varphi_1}{2} E_1^2 \\ -\left(\frac{\partial V_1}{\partial D_1} - \eta \frac{\partial V_1}{\partial D_2}\right)\left(D_1 - \alpha_1 E_3 - \beta_1 E_1 - k_1 D_1\right) - \frac{\partial V_1}{\partial D_2}\left(D_2 - \alpha_2 E_4 - \beta_2 E_2 - k_2 D_2\right) \end{array} \right\}, \tag{14}$$

$$r \cdot V_2(D_1, D_2) = \min_{E_2 \geq 0} \left\{ \begin{array}{c} \left(B - \mu_2 E_4 - \sigma_2 E_2 - \theta_2 D_2\right) + \frac{\varphi_2}{2} E_2^2 \\ -\left(\frac{\partial V_2}{\partial D_1} - \eta \frac{\partial V_2}{\partial D_2}\right)\left(D_1 - \alpha_1 E_3 - \beta_1 E_1 - k_1 D_1\right) - \frac{\partial V_2}{\partial D_2}\left(D_2 - \alpha_2 E_4 - \beta_2 E_2 - k_2 D_2\right) \end{array} \right\}, \tag{15}$$

$$r \cdot V_3(D_1, D_2) = \min_{E_3 \geq 0, E_4 \geq 0} \left\{ \begin{array}{c} \tau\left(A - \mu_1 E_3 - \sigma_1 E_1 - \theta_1 D_1\right) + \left(B - \mu_2 E_4 - \sigma_2 E_2 - \theta_2 D_2\right) + \frac{\varphi_3}{2} E_3^2 + \frac{\varphi_4}{2} E_4^2 \\ -\left(\frac{\partial V_3}{\partial D_1} - \eta \frac{\partial V_3}{\partial D_2}\right)\left(D_1 - \alpha_1 E_3 - \beta_1 E_1 - k_1 D_1\right) - \frac{\partial V_3}{\partial D_2}\left(D_2 - \alpha_2 E_4 - \beta_2 E_2 - k_2 D_2\right) \end{array} \right\}. \tag{16}$$

We find the first-order partial derivatives of the parameter with respect to the right parts of formulas (15) and (16), respectively. Formula (17) sums the first-order partial derivatives and sets them equal to zero to obtain

$$E_1 = \frac{(1-\tau)\sigma_1 - \beta_1\left(\partial V_1/\partial D_1 - \eta \partial V_1/\partial D_2\right)}{\varphi_1}, \tag{17}$$

$$E_2 = \frac{\sigma_2 - \beta_2 \partial V_2/\partial D_2}{\varphi_2}, \tag{18}$$

$$E_3=\frac{\tau\mu_1-\alpha_1\left(\partial V_3/\partial D_1-\eta\partial V_3/\partial D_2\right)}{\varphi_3}, \tag{19}$$

$$E_4=\frac{\mu_2-\alpha_2\partial V_3/\partial D_2}{\varphi_4}, \tag{20}$$

$$rV_1=\left\{\begin{array}{c}[(\tau-1)\theta_1+m(k_1-1)]D_1+n(k_2-1)D_2-\dfrac{[(1-\tau)\sigma_1-\beta_1 m]^2}{2\varphi_1}\\ +\dfrac{n\beta_2}{\varphi_2}[\sigma_2-\beta_2 x]-\dfrac{[\tau\mu_1-\alpha_1 y]}{\varphi_3}[(1-\tau)\mu_1-m\alpha_1]+(1-\tau)A+\dfrac{n\alpha_2}{\varphi_4}[\mu_2-\alpha_2 z]\end{array}\right\}, \tag{21}$$

$$rV_2=\left\{\begin{array}{c}[l(k_1-1)]D_1+[x(k_2-1)-\theta_2]D_2+\dfrac{l\beta_1}{\varphi_1}[(1-\tau)\sigma_1-\beta_1 m]+B\\ +\dfrac{[\sigma_2-\beta_2 x]^2}{2\varphi_2}+\dfrac{l\alpha_1}{\varphi_3}[\tau\mu_1-\alpha_1 y]+\dfrac{[\alpha_2 x-\mu_2]}{\varphi_4}[\mu_2-\alpha_2 z]\end{array}\right\}, \tag{22}$$

$$rV_3=\left\{\begin{array}{c}[-\tau\theta_1+(k_1-1)y]D_1+[z(k_2-1)-\theta_2]D_2+\dfrac{[y\beta_1-\tau\sigma_1]}{\varphi_1}[(1-\tau)\sigma_1-\beta_1 m]\\ +\dfrac{[\sigma_2-\beta_2 x]}{\varphi_2}(z\beta_2-\sigma_2)-\dfrac{[\tau\mu_1-\alpha_1 y]^2}{2\varphi_3}-\dfrac{[\mu_2-\alpha_2 z]^2}{2\varphi_4}+\tau A+B\end{array}\right\}$$
$$\left(\frac{\partial V_1}{\partial D_1}-\eta\frac{\partial V_1}{\partial D_2}\right)=m,\frac{\partial V_1}{\partial D_2}=n,\frac{\partial V_2}{\partial D_1}-\eta\frac{\partial V_2}{\partial D_2}=l,\frac{\partial V_2}{\partial D_2}=x,\frac{\partial V_3}{\partial D_1}-\eta\frac{\partial V_3}{\partial D_2}=y,\frac{\partial V_3}{\partial D_2}=z. \tag{23}$$

It can be observed from equations (22)–(24) that the linear optimal function is the solution of the HJB equation. Let

$$V_1(D_1,D_2)=p_1D_1+q_1D_2+h_1, \tag{24}$$

$$V_2(D_1,D_2)=p_2D_1+q_2D_2+h_2, \tag{25}$$

$$V_3(D_1,D_2)=p_3D_1+q_3D_2+h_3, \tag{26}$$

$$\begin{aligned}r(p_1D_1+q_1D_2+h_1)&=[(\tau-1)\theta_1+(p_1-\eta q_1)(k_1-1)]D_1+q_1(k_2-1)D_2+(1-\tau)A\\&\quad+\frac{q_1\beta_2}{\varphi_2}[\sigma_2-\beta_2q_2]-\frac{[(1-\tau)\sigma_1-\beta_1(p_1-\eta q_1)]^2}{2\varphi_1}+\frac{q_1\alpha_2}{\varphi_4}[\mu_2-\alpha_2q_3]\\&\quad-\frac{[\tau\mu_1-\alpha_1(p_3-\eta q_3)]}{\varphi_3}[(1-\tau)\mu_1-(p_1-\eta q_1)\alpha_1],\end{aligned} \tag{27}$$

$$\begin{aligned}r(p_2D_1+q_2D_2+h_2)&=[(p_2-\eta q_2)(k_1-1)]D_1+[q_2(k_2-1)+\theta_2]D_2+\frac{[\sigma_2-\beta_2q_2]^2}{2\varphi_2}\\&\quad+\frac{(p_2-\eta q_2)\beta_1}{\varphi_1}[(1-\tau)\sigma_1-\beta_1(p_1-\eta q_1)]+B\\&\quad+\frac{(p_2-\eta q_2)\alpha_1}{\varphi_3}[\tau\mu_1-\alpha_1(p_3-\eta q_3)]+\frac{[\alpha_2q_2-\mu_2]}{\varphi_4}[\mu_2-\alpha_2q_3],\end{aligned} \tag{28}$$

$$\begin{aligned} r\left(p_3D_1+q_3D_2+h_3\right) = {} & \left[-\tau\theta_1+\left(k_1-1\right)\left(p_3-\eta q_3\right)\right]D_1+\left[q_3\left(k_2-1\right)+\theta_2\right]D_2+\tau A+B \\ & +\frac{\left[\sigma_2-\beta_2q_2\right]}{\varphi_2}\left(q_3\beta_2-\sigma_2\right)-\frac{\left[\tau\mu_1-\alpha_1\left(p_3-\eta q_3\right)\right]^2}{2\varphi_3}-\frac{\left[\mu_2-\alpha_2q_3\right]^2}{2\varphi_4} \\ & +\frac{\left[\left(p_3-\eta q_3\right)\beta_1-\tau\sigma_1\right]}{\varphi_1}\left[(1-\tau)\sigma_1-\beta_1\left(p_1-\eta q_1\right)\right]. \end{aligned} \tag{29}$$

If equations (28)–(30) satisfy the conditions $D_1 \geq 0$ and $D_2 \geq 0$, then

$$\begin{aligned} p_1 &= \frac{(\tau-1)\theta_1}{r-k_1+1}, q_1 = 0, \\ h_1 &= \frac{(1-\tau)A}{r}+\frac{\left[(1-\tau)\theta_1\beta_2\right]\left[\sigma_2\left(r-k_2+1\right)-\theta_2\beta_2\right]}{r\varphi_2\left(r-k_1+1\right)\left(r-k_2+1\right)}-\frac{\left[(1-\tau)\left(\sigma_1\left(r-k_1+1\right)-\beta_1\theta_1\right)\right]^2}{2r\varphi_1\left(r-k_1+1\right)^2} \\ &\quad +\frac{\left[(1-\tau)\theta_1\alpha_2\right]\left[\mu_2\left(r-k_2+1\right)-\alpha_2\theta_2\right]}{r\varphi_4\left(r-k_1+1\right)\left(r-k_2+1\right)} \\ &\quad -\frac{\tau\mu_1\left(r-k_2+1\right)^2-\alpha_1\theta_1\left(r-k_2+1\right)-\eta\theta_2 r}{r\varphi_3\left(r-k_2+1\right)\left(r-k_1+1\right)}\frac{\left[(1-\tau)\left(\mu_1\left(r-k_1+1\right)-\alpha_1\theta_1\right)\right]}{\left(r-k_1+1\right)}, \end{aligned} \tag{30}$$

$$\begin{aligned} p_2 &= \frac{-\eta\left(1-k_1\right)\theta_2}{\left(r-k_1+1\right)\left(r-k_2+1\right)}, q_2 = \frac{-\theta_2}{r-k_2+1}, h_2 = \frac{\left[\sigma_2\left(r-k_2+1\right)-\beta_2\theta_2\right]^2}{2\varphi_2 r\left(r-k_2+1\right)^2}+\frac{B}{r} \\ &\quad -\frac{\beta_1}{\varphi_1}\left[\frac{(1-\tau)\eta\theta_2}{\left(r-k_1+1\right)\left(r-k_2+1\right)}\right]\left[\frac{\sigma_1\left(r-k_1+1\right)-\beta_1\theta_1}{r-k_1+1}\right]-\frac{\left[\mu_2\left(r-k_2+1\right)-\varepsilon_2\theta_2\right]^2}{\varphi_4 r\left(r-k_2+1\right)^2} \\ &\quad -\frac{\alpha_1}{\varphi_3}\left[\frac{\left(r-k_2+1\right)\left[\tau\mu_1\left(r-k_2+1\right)-\alpha_1\theta_1-\eta\theta_2\right]-\alpha_1\eta\theta_2\left(1-k_2\right)}{\left(r-k_1+1\right)\left(r-k_2+1\right)}\right]\frac{\eta\theta_2}{\left(r-k_2+1\right)^2}, \end{aligned} \tag{31}$$

$$\begin{aligned} p_3 &= \frac{-\tau\theta_1\left(r-k_2+1\right)+\eta\left(k_1-1\right)\theta_2}{\left(r-k_2+1\right)\left(r-k_1+1\right)}, q_3 = \frac{-\theta_2}{r-k_2+1}, \\ h_3 &= \frac{\tau A+B}{r}-\frac{\left[\sigma_2\left(r-k_2+1\right)-\beta_2\theta_2\right]^2}{r\varphi_2\left(r-k_2+1\right)^2}-\frac{\left[\tau\mu_1\left(r-k_2+1\right)^2-\alpha_1\left(\theta_1\left(r-k_2+1\right)-\eta\theta_2 r\right)\right]}{2r\varphi_3\left(r-k_2+1\right)^2} \\ &\quad -\frac{\left[\mu_2\left(r-k_2+1\right)-\alpha_2\theta_2\right]^2}{2r\varphi_4\left(r-k_2+1\right)^2}+\frac{\left[\theta_1\left(r-k_2+1\right)-\eta\theta_2 r\right]\beta_1-\tau\theta_1\left(r-k_2+1\right)^2}{r\varphi_1\left(r-k_2+1\right)\left(r-k_1+1\right)}X \\ &\quad \cdot\frac{(1-\tau)\left[\sigma_1\left(r-k_1+1\right)-\beta_1\theta_1\right]}{\left(r-k_1+1\right)}. \end{aligned} \tag{32}$$

We substitute formulas (31)–(33) into equations (25)–(27), and the minimum loss function of natural gas energy shortage of local government, natural gas supply chain enterprises, and provincial government can be obtained as

$$V_1^* = \frac{(\tau-1)\theta_1}{r-k_1+1}D_1 + \frac{(1-\tau)A}{r} + \frac{[(\tau-1)\theta_1\beta_2][\sigma_2(r-k_2+1)+\theta_2\beta_2]}{r\varphi_2(r-k_1+1)(r-k_2+1)} - \frac{[(1-\tau)(\sigma_1(r-k_1+1)+\beta_1\theta_1)]^2}{2r\varphi_1(r-k_1+1)^2} + \frac{[(\tau-1)\theta_1\alpha_2][\mu_2(r-k_2+1)+\alpha_2\theta_2]}{r\varphi_4(r-k_1+1)(r-k_2+1)} - \frac{\tau\mu_1(r-k_2+1)^2+\alpha_1\theta_1(r-k_2+1)+\eta\theta_2 r}{r\varphi_3(r-k_2+1)^2}\frac{[(1-\tau)(\mu_1(r-k_1+1)+\alpha_1\theta_1)]}{(r-k_1+1)}, \tag{33}$$

$$V_2^* = \frac{-\eta(1-k_1)\theta_2}{(r-k_1+1)(r-k_2+1)}D_1 - \frac{\theta_2}{r-k_2+1}D_2 + \frac{[\sigma_2(r-k_2+1)+\beta_2\theta_2]^2}{2\varphi_2 r(r-k_2+1)^2} + \frac{B}{r} - \frac{\beta_1}{\varphi_1}\left[\frac{(\tau-1)\eta\theta_2}{(r-k_1+1)(r-k_2+1)}\right]\left[\frac{\sigma_1(r-k_1+1)+\beta_1\theta_1}{r-k_1+1}\right] - \frac{[\mu_2(r-k_2+1)+\beta_2\theta_2]^2}{\varphi_4 r(r-k_2+1)^2} + \frac{\alpha_1}{\varphi_3}\left[\frac{(r-k_2+1)[\tau\mu_1(r-k_2+1)+\alpha_1\theta_1+\eta\theta_2]+\alpha_1\eta\theta_2(1-k_2)}{(r-k_1+1)(r-k_2+1)}\right]\frac{\eta\theta_2}{(r-k_2+1)^2}, \tag{34}$$

$$V_3^* = \frac{\tau\theta_1(r-k_2+1)+\eta(k_1-1)\theta_2}{(r-k_2+1)(r-k_1+1)}D_1 - \frac{\theta_2}{r-k_2+1}D_2 + \frac{\tau A+B}{r} - \frac{[\sigma_2(r-k_2+1)+\beta_2\theta_2]^2}{r\varphi_2(r-k_2+1)^2} - \frac{[\tau\mu_1(r-k_2+1)^2+\alpha_1(\theta_1(r-k_2+1)+\eta\theta_2 r)]}{2r\varphi_3(r-k_2+1)^2} - \frac{[\mu_2(r-k_2+1)+\alpha_2\theta_2]^2}{2r\varphi_4(r-k_2+1)^2} + \frac{[-\theta_1(r-k_2+1)+\eta\theta_2 r]\beta_1+\tau\theta_1(r-k_2+1)^2}{r\varphi_1(r-k_2+1)^2}\frac{(1-\tau)[\sigma_1(r-k_1+1)+\beta_1\theta_1]}{(r-k_1+1)}, \tag{35}$$

where the total loss of the city caused by natural gas energy shortage is as follows:

$$V^*(D_1,D_2) = V_1^*+V_2^*+V_3^*$$

$$\left[\frac{-(1-\tau)\theta_1}{r-k_1+1} - \frac{\eta(1-k_1)\theta_2}{(r-k_1+1)(r-k_2+1)}\right]D_1\left(\frac{2\theta_2}{r-k_2+1}\right)D_2 + \frac{(1-\tau)A}{r} + \frac{B}{r} + \frac{[(\tau-1)\theta_1\beta_2][\sigma_2(r-k_2+1)+\theta_2\beta_2]}{r\varphi_2(r-k_1+1)(r-k_2+1)} - \frac{[(1-\tau)(\sigma_1(r-k_1+1)+\beta_1\theta_1)]^2}{2r\varphi_1(r-k_1+1)^2} + \frac{[(\tau-1)\theta_1\alpha_2][\mu_2(r-k_2+1)+\alpha_2\theta_2]}{r\varphi_4(r-k_1+1)(r-k_2+1)} + \frac{[\sigma_2(r-k_2+1)+\beta_2\theta_2]^2}{2\varphi_2 r(r-k_2+1)^2} - \frac{\tau\mu_1(r-k_2+1)^2+\alpha_1\theta_1(r-k_2+1)+\eta\theta_2 r}{r\varphi_3(r-k_2+1)^2}\frac{[(1-\tau)(\mu_1(r-k_1+1)+\alpha_1\theta_1)]}{(r-k_1+1)}$$

$$-\frac{\beta_1}{\varphi_1}\left[\frac{(\tau-1)\eta\theta_2}{(r-k_1+1)(r-k_2+1)}\right]\left[\frac{\sigma_1(r-k_1+1)+\beta_1\theta_1}{r-k_1+1}\right]-\frac{[\mu_2(r-k_2+1)+\beta_2\theta_2]^2}{\varphi_4 r(r-k_2+1)^2}$$
$$+\frac{\alpha_1}{\varphi_3}\left[\frac{(r-k_2+1)[\tau\mu_1(r-k_2+1)+\alpha_1\theta_1+\eta\theta_2]+\alpha_1\eta\theta_2(1-k_2)}{(r-k_1+1)(r-k_2+1)}\right]\frac{\eta\theta_2}{(r-k_2+1)^2}$$
$$-\frac{\left[\tau\mu_1(r-k_2+1)^2+\alpha_1(\theta_1(r-k_2+1)+\eta\theta_2 r)\right]}{2r\varphi_3(r-k_2+1)^2}-\frac{[\mu_2(r-k_2+1)+\alpha_2\theta_2]^2}{2r\varphi_4(r-k_2+1)^2} \tag{36}$$
$$+\frac{[-\theta_1(r-k_2+1)+\eta\theta_2 r]\beta_1+\tau\theta_1(r-k_2+1)^2}{r\varphi_1(r-k_2+1)^2}\frac{(1-\tau)[\sigma_1(r-k_1+1)-\beta_1\theta_1]}{(r-k_1+1)}$$
$$+\frac{\tau A+B}{r}-\frac{[\sigma_2(r-k_2+1)+\beta_2\theta_2]^2}{r\varphi_2(r-k_2+1)^2}.$$

The partial derivatives of (34)–(36) are about D_1 and D_2 and are then substituted into equations (18)–(21) to obtain (11)–(14).

Under the Nash noncooperative equilibrium state, the emergency shortage of natural gas energy of local government is as follows:

$$\begin{cases} D_1^*(t)=\dfrac{\alpha_1}{(k_1-1)\varphi_3}\left[\tau\mu_1+\dfrac{\tau\alpha_1\theta_1}{(r-k_1+1)}-\dfrac{\alpha_1\eta\theta_2 r}{(r-k_1+1)(r-k_2+1)}\right]+\dfrac{\beta_1(1-\tau)}{(k_1-1)\varphi_1}, \\ \left(\sigma_1+\dfrac{\beta_1\theta_1}{r-k_1+1}\right)-\dfrac{1}{(k_1-1)}e^{(1-k_1)t+d_1}, \\ D_1^*(0)=D_1. \end{cases} \tag{37}$$

The emergency shortage of natural gas energy in natural gas supply chain enterprises can be expressed as

$$\begin{cases} D_2^*(t)=\dfrac{\eta\alpha_1}{(k_2-1)\varphi_3}\left[\tau\mu_1+\dfrac{\tau\alpha_1\theta_1}{(r-k_1+1)}-\dfrac{\alpha_1\eta\theta_2 r}{(r-k_1+1)(r-k_2+1)}\right]+\dfrac{\eta\beta_1(1-\tau)}{(k_2-1)\varphi_1}\left(\sigma_1+\dfrac{\beta_1\theta_1}{r-k_1+1}\right), \\ -\dfrac{1}{(k_2-1)}\eta e^{(1-k_1)t+d_1}+\dfrac{\alpha_2}{(k_2-1)\varphi_4}\left(\mu_2+\dfrac{\alpha_2\theta_2}{r-k_2+1}\right)+\dfrac{\beta_2}{(k_2-1)\varphi_2}\left[\sigma_2+\dfrac{\beta_2\theta_2}{(r-k_2+1)}\right], \\ -\dfrac{1}{(k_2-1)}e^{(1-k_2)t+d_2}, \\ D_2^*(0)=D_2, \\ e^{d_1}=\dfrac{1}{\varphi_3}\left[\tau\mu_1+\dfrac{\tau\alpha_1\theta_1}{(r-k_1+1)}-\dfrac{\alpha_1\eta\theta_2 r}{(r-k_1+1)(r-k_2+1)}\right]-\dfrac{\beta_1(1-\tau)}{\varphi_1}\left(\sigma_1+\dfrac{\beta_1\theta_1}{r-k_1+1}\right)-(k_1-1)D_1. \end{cases} \tag{38}$$

It can be seen from equations (11) and (12) that under the spontaneous governance model of urban and natural gas supply chain enterprises that deals with shortages, the local government's effort level and cost coefficient, share ratio, natural gas energy shortage attenuation coefficient, and impact coefficient of local government's efforts to deal with natural gas are negatively correlated with the extent of the impact of natural gas shortage on city losses but are positively correlated with that of the local government's efforts on the city's losses caused by natural gas energy shortages and the discount rate. The effort of natural gas supply chain enterprises is negatively correlated with this coefficient, natural gas energy shortage attenuation coefficient, the influence coefficient of enterprise efforts on natural gas energy shortage, and the impact of natural gas shortage on the losses caused by natural gas shortage, while the impact of the loss is positively correlated with the discount rate. Local governments and enterprises should increase their efforts to affect the losses caused by natural gas shortages, increase capital utilization, coordinate and unify departments, increase their implementation capabilities and other related issues, and reduce losses caused by the shortages.

It can be seen from equations (13) and (14) that under the spontaneous governance model of urban and natural gas supply chain enterprises dealing with shortages, both the provincial government's effort and cost coefficient to the local government and the impact of natural gas shortages on the city's losses are negatively related to factors including the share ratio, the degree of the impact of the provincial government's efforts on the city's losses caused by natural gas energy shortages, the attenuation coefficient of natural gas energy shortages, the conversion rate, the degree of the impact of natural gas shortages on the losses of enterprises, and the provincial government's efforts to respond to local governments. However, the impact of natural gas energy shortage is positively correlated with the discount rate. The degree of provincial government's efforts on enterprises is negatively correlated with factors including the cost coefficient, natural gas energy shortage attenuation coefficient, the impact of natural gas shortages on enterprises' losses, and the degree of the provincial government's efforts on the enterprise's emergency natural gas energy shortages and also negatively correlated with the provincial government efforts but has a positive correlation with the discount rate on the impact of the city's losses caused by natural gas energy shortages. To this end, local governments and enterprises should increase their efforts to affect the losses caused by natural gas shortages, increase capital utilization, coordinate and unify the departments, increase their implementation capabilities and other related issues, and reduce the losses caused by shortages.

In this spontaneous emergency mode, provincial governments and local governments are relatively independent entities in terms of their respective interests and responsibilities. Local governments and enterprises choose the best effort level that minimizes their own losses, without considering the overall interests. Besides, emergency coordination will eventually move towards emergency response or even immobility without the intervention and guidance of the central government, thereby causing difficulties handling multiple emergency affairs. □

3.2. Emergency Superior Mode for Urban Natural Gas Energy Shortage. In order to improve the efficiency of resource allocation, the provincial government has made policy guidance through special fiscal expenditures, that is, the provincial government shares a certain proportion of the emergency cost of shortages for the local government and enterprises and the funding intensity improves. After local governments and enterprises receive funding and notice the actions of the provincial government, they choose the appropriate level of effort and follow the actions of the provincial government. In this way, the three parties start the Starberg master-slave game, with the provincial government as the leader.

The objective function of the local government can be expressed as

$$T_1 = \int_0^{\infty} e^{-rt}\left\{(1-\tau)\left[A(t) - \mu_1 E_3(t) - \sigma_1 E_1(t) - \theta_1 D_1(t)\right] + \frac{\varphi_1}{2}(1-\lambda_1)E_1^2\right\}\mathrm{d}t. \tag{39}$$

The objective function of natural gas supply chain enterprises can be expressed as

$$T_2 = \int_0^{\infty} e^{-rt}\left\{\left[B(t) - \mu_2 E_4(t) - \sigma_2 E_2(t) - \theta_2 D_2(t)\right] + \frac{\varphi_2}{2}(1-\lambda_2)E_2^2\right\}\mathrm{d}t. \tag{40}$$

The objective function of the provincial government can be expressed as

$$T_3 = \int_0^{\infty} e^{-rt} \left\{ \begin{aligned} &\tau\left[A(t) - \mu_1 E_3(t) - \sigma_1 E_1(t) - \theta_1 D_1(t)\right] + \frac{\varphi_3}{2}E_3^2 + \frac{1}{2}\varphi_1\lambda_1 E_1^2 \\ &+\left[B(t) - \mu_2 E_4(t) - \sigma_2 E_2(t) - \theta_2 D_2(t)\right] + \frac{\varphi_4}{2}E_4^2 + \frac{1}{2}\varphi_2\lambda_2 E_2^2 \end{aligned} \right\} \mathrm{d}t. \tag{41}$$

Proposition 2. *Under the superior mode, the three-party static feedback Nash equilibrium strategies of the provincial government, local government, and natural gas supply chain companies can be expressed as*

$$E_1^{**} = \frac{1}{2\varphi_1}\left[(\tau+1)\sigma_1 + \frac{\beta_1\theta_1(2-\tau)}{(r-k_1+1)} - \frac{2\eta\theta_2 r\beta_1}{(r-k_1+1)(r-k_2+1)}\right], \tag{42}$$

$$E_2^{**} = \frac{3}{2\varphi_2}\left(\sigma_2 + \frac{\beta_2\theta_2}{(r-k_2+1)}\right), \tag{43}$$

$$E_3^{**} = \frac{1}{\varphi_3}\left[\tau\mu_1 + \frac{\tau\alpha_1\theta_1}{(r-k_1+1)} - \frac{\alpha_1\eta\theta_2 r}{(r-k_1+1)(r-k_2+1)}\right], \tag{44}$$

$$E_4^{**} = \frac{1}{\varphi_4}\left(\mu_2 + \frac{\alpha_2\theta_2}{r-k_2+1}\right), \tag{45}$$

$$\lambda_1 = \frac{(3\tau-1)\sigma_1 - \beta_1\left[\theta_1(1+\tau)(r-k_2+1) + \eta\theta_2/(r-k_1+1)(r-k_2+1)\right]}{(\tau+1)\sigma_1 - \beta_1\left[\theta_1(3-\tau)(r-k_2+1) - \eta\theta_2/(r-k_1+1)(r-k_2+1)\right]}, \tag{46}$$

$$\lambda_2 = \frac{1}{3}. \tag{47}$$

Proof. The reverse induction method is used to obtain the Stackelberg equilibrium of this game. Suppose that there is a loss function for the continuous and bounded differential urban natural gas energy shortage $V_i(D_1, D_2)$, $i \in (1, 2, 3)$, for all $D_1 \geq 0$ and $D_2 \geq 0$, which satisfies the Hamilton–Jacobi–Bellman equation:

$$r\cdot V_1(D_1, D_2) = \min_{E_1\geq 0}\left\{ \begin{gathered} (1-\tau)(A - \mu_1E_3 - \sigma_1E_1 - \theta_1D_1) + \frac{\varphi_1}{2}(1-\lambda_1)E_1^2 \\ -\left(\frac{\partial V_1}{\partial D_1} - \eta\frac{\partial V_1}{\partial D_2}\right)(D_1 - \alpha_1E_3 - \beta_1E_1 - k_1D_1) - \frac{\partial V_1}{\partial D_2}(D_2 - \alpha_2E_4 - \beta_2E_2 - k_2D_2) \end{gathered} \right\}, \tag{48}$$

$$r\cdot V_2(D_1, D_2) = \min_{E_2\geq 0}\left\{ \begin{gathered} (B - \mu_2E_4 - \sigma_2E_2 - \theta_2D_2) + \frac{\varphi_2}{2}(1-\lambda_2)E_2^2 \\ -\left(\frac{\partial V_2}{\partial D_1} - \eta\frac{\partial V_2}{\partial D_2}\right)(D_1 - \alpha_1E_3 - \beta_1E_1 - k_1D_1) - \frac{\partial V_2}{\partial D_2}(D_2 - \alpha_2E_4 - \beta_2E_2 - k_2D_2) \end{gathered} \right\}. \tag{49}$$

For the right end of (49) and (50), the first-order partial derivatives of E_1 and E_2 with respect to the degree of effort are obtained. We set them equal to zero, and it can be obtained as

$$E_1 = \frac{(1-\tau)\sigma_1 - \beta_1(\partial V_1/\partial D_1 - \eta\partial V_1/\partial D_2)}{\varphi_1(1-\lambda_1)}, \tag{50}$$

$$E_2 = \frac{\sigma_2 - \beta_2\partial V_2/\partial D_2}{\varphi_2(1-\lambda_2)}. \tag{51}$$

The provincial government can rationally predict that the local government and enterprises will choose their effort functions E_1 and E_2, according to the previous formula. Therefore, the provincial government should determine its own effort strategy and funding based on the rational response of the local government and enterprises. The HJB equation can be expressed as

$$r\cdot V_3(D_1,D_2)=\min_{E_3\geq 0,E_4\geq 0}\left\{\begin{array}{c}\tau(A-\mu_1E_3-\sigma_1E_1+\theta_1D_1)+(B-\mu_2E_4-\sigma_2E_2+\theta_2D_2)+\frac{\varphi_3}{2}E_3^2+\frac{\varphi_4}{2}E_4^2+\frac{1}{2}\lambda_1\varphi_1E_1^2\\ +\frac{1}{2}\lambda_2\varphi_2E_2^2-\left(\frac{\partial V_3}{\partial D_1}-\eta\frac{\partial V_3}{\partial D_2}\right)(D_1-\alpha_1E_3-\beta_1E_1-k_1D_1)-\frac{\partial V_3}{\partial D_2}(D_2-\alpha_2E_4-\beta_2E_2-k_2D_2)\end{array}\right\}. \tag{52}$$

By substituting (51) and (52) into equation (53), the first-order partial derivatives of E_3, $E_4\lambda_1$, and λ_2 are obtained. We set them equal to zero, and it can be obtained as

$$E_3=\frac{\tau\mu_1-\alpha_1(\partial V_3/\partial D_1-\eta\partial V_3/\partial D_2)}{\varphi_3}, \tag{53}$$

$$E_4=\frac{\mu_2-\alpha_2\partial V_3/\partial D_2}{\varphi_4}, \tag{54}$$

$$\lambda_1=\frac{(3\tau-1)\sigma_1+\beta_1[(\partial V_1/\partial D_1-\eta\partial V_1/\partial D_2)-2(\partial V_3/\partial D_1-\eta\partial V_3/\partial D_2)]}{(\tau+1)\sigma_1-\beta_1[(\partial V_1/\partial D_1-\eta\partial V_1/\partial D_2)+2(\partial V_3/\partial D_1-\eta\partial V_3/\partial D_2)]}, \tag{55}$$

$$\lambda_2=\frac{\sigma_2+\beta_2(\partial V_2/\partial D_2-2\partial V_3/\partial D_2)}{3\sigma_2-\beta_2(\partial V_2/\partial D_2+2\partial V_3/\partial D_2)}. \tag{56}$$

Based on (49)–(53) and (54)–(57), the following can be obtained:

$$rV_1=\left\{\begin{array}{c}[(\tau-1)\theta_1+m(k_1-1)]D_1+n(k_2-1)D_2+\left[(\tau-1)\sigma_1+\beta_1m+\frac{\varphi_1}{2}(1-\lambda_1)E_1\right]E_1\\ +\beta_2nE_2-[(1-\tau)\mu_1-\alpha_1m]E_3+n\alpha_2E_4+(1-\tau)A\end{array}\right\}, \tag{57}$$

$$rV_2=\left\{\begin{array}{c}(k_1-1)lD_1+[-\theta_2-x(1-k_2)]D_2+\left[\beta_2x-\sigma_2+\frac{\varphi_2}{2}(1-\lambda_2)E_2\right]E_2\\ +\beta_1lE_1+\alpha_1lE_3-(\mu_2-\alpha_2x)E_4+B\end{array}\right\}, \tag{58}$$

$$rV_3=\left\{\begin{array}{c}[-\tau\theta_1-y(1-k_1)]D_1+[-\theta_2-z(1-k_2)]D_2+\left[\beta_1y-\sigma_1\tau+\frac{\varphi_1}{2}\lambda_1E_1\right]E_1+\tau A+B\\ +\left[\beta_2z-\sigma_2+\frac{\varphi_2}{2}\lambda_2E_2\right]E_2+\left[\alpha_1y-\tau\mu_1+\frac{\varphi_3}{2}E_3\right]E_3+\left[\alpha_2z-\mu_2+\frac{\varphi_4}{2}E_4\right]E_4\end{array}\right\}. \tag{59}$$

According to (58)–(60), the linear optimal functions of D_1 and D_2 are the solutions to the HJB equation. Set

$$V_1(D_1,D_2)=p_1D_1+q_1D_2+h_1, \tag{60}$$

$$V_2(D_1,D_2)=p_2D_1+q_2D_2+h_2, \tag{61}$$

$$V_3(D_1, D_2) = p_3 D_1 + q_3 D_2 + h_3, \quad (62)$$

where p_1, q_1, p_2, q_2, p_3, and q_3 are all the constants. Find the derivatives of D_1 and D_2 for (61)–(63), and substitute them into formulas (58)–(60); it can be obtained as

$$\begin{aligned} r(p_1D_1 + q_1D_2 + h_1) = {} & [(\tau-1)\theta_1 + (p_1-\eta q_1)(k_1-1)]D_1 + q_1(k_2-1)D_2 + (1-\tau)A \\ & + \frac{1}{4\varphi_1}[(\tau-1)\sigma_1 + (p_1-\eta q_1)][(\tau+1)\sigma_1 - \beta_1(p_1-\eta q_1) - 2\beta_1(p_3-\eta q_3)] \\ & + \frac{\beta_2 q_1}{\varphi_2}\frac{(\sigma_2-\beta_2q_2)[3\sigma_2-\beta_2(q_2+2q_3)]}{4\sigma_2-2\beta_2(q_2+2q_3)} + \frac{\alpha_2 q_1}{\varphi_4}(\mu_2-\alpha_2q_3) \\ & - \frac{1}{\varphi_3}[(1-\tau)\mu_1 + (p_1-\eta q_1)][\tau\mu_1 - \alpha_1(p_3-\eta q_3)], \end{aligned} \quad (63)$$

$$\begin{aligned} r(p_2D_1 + q_2D_2 + h_2) = {} & [(p_2-\eta q_2)(k_1-1)]D_1 + [\theta_2 - q_2(1-k_2)]D_2 + B \\ & + \frac{\beta_1(p_2-\eta q_2)}{2\varphi_1}[(\tau+1)\sigma_1 - \beta_1(p_1-\eta q_1) - 2\beta_1(p_3-\eta q_3)] \\ & + \frac{(\beta_2q_2-\sigma_2)}{2\varphi_2}\frac{[3\sigma_2-\beta_2(q_2+2q_3)]}{4\sigma_2-2\beta_2q_2}(\sigma_2-\beta_2q_2) + \frac{(\alpha_2q_2-\mu_2)}{\varphi_4}[\mu_2-\alpha_2q_3] \\ & + \frac{\alpha_1(p_2-\eta q_2)}{\varphi_3}[\tau\mu_1 - \alpha_1(p_3-\eta q_3)], \end{aligned} \quad (64)$$

$$\begin{aligned} r(p_3D_1 + q_3D_2 + h_3) = {} & [-\tau\theta_1 - (p_3-\eta q_3)(1-k_1)]D_1 + [-\theta_2 - q_3(1-k_2)]D_2 + \tau A + B \\ & + \frac{\beta_2q_3-\sigma_2}{\varphi_2(4\sigma_2-2\beta_2q_2)}(\sigma_2-\beta_2q_2)[3\sigma_2-\beta_2(q_2+2q_3)]\frac{[\beta_2(q_2-2q_3)-\sigma_2]}{2(4\sigma_2-2\beta_2q_2)}(\sigma_2-\beta_2q_2) \\ & + \frac{1}{4\varphi_1}\left[\beta_1(p_3-\eta q_3) - \frac{1}{2}\sigma_1(\tau+1) + \frac{1}{2}(p_1-\eta q_1)\right] \\ & \times\{(\tau+1)\sigma_1 - \beta_1[(p_1-\eta q_1) + 2(p_3-\eta q_3)]\} - \frac{1}{2\varphi_4}[\alpha_2q_3-\mu_2]^2 \\ & - \frac{1}{2\varphi_3}[\alpha_1(p_3-\eta q_3) - \tau\mu_1]^2. \end{aligned} \quad (65)$$

Suppose that (64)–(66) satisfy $D_1 \geq 0$ and $D_2 \geq 0$; then,

$$\begin{aligned} & p_1 = \frac{(\tau-1)\theta_1}{r-k_1+1}, q_1 = 0, \\ & h_1 = \frac{(1-\tau)A}{r} - \frac{1}{r\varphi_3}\left[(1-\tau)\mu_1 + \frac{(1-\tau)\theta_1}{r-k_1+1}\right]\left[\tau\mu_1 - \alpha_1\frac{\theta_1(r-k_2+1)-\eta\theta_2 r}{(r-k_1+1)(r-k_2+1)}\right] + \\ & \quad \frac{1}{4r\varphi_1}\left[(\tau-1)\sigma_1 + \frac{(1-\tau)\theta_1}{r-k_1+1}\right]\left[(\tau+1)\sigma_1 - \frac{\beta_1(1-\tau)\theta_1}{r-k_1+1} - 2\beta_1\frac{\theta_1(r-k_2+1)-\eta\theta_2 r}{(r-k_1+1)(r-k_2+1)}\right], \\ & p_2 = \frac{\eta\theta_2(k_1-1)}{(r-k_1+1)(r-k_2+1)}, q_2 = \frac{-\theta_2}{r-k_2+1}, \end{aligned} \quad (66)$$

$$\begin{aligned}
h_2 &= \frac{B}{r} + \left[\frac{\beta_2\theta_2 - \sigma_2(r-k_2+1)}{2r(r-k_2+1)\varphi_2} \cdot \frac{3\sigma_2(r-k_2+1) - 3\beta_2\theta_2}{4\sigma_2(r-k_2+1) - 2\beta_2\theta_2} \cdot \frac{\sigma_2(r-k_2+1) - \beta_2\theta_2}{(r-k_2+1)}\right] \\
&\quad - \frac{\beta_1\eta\theta_2}{2\varphi_1(r-k_1+1)(r-k_2+1)}\left[(\tau+1)\sigma_1 - \beta_1\frac{(1-\tau)\theta_1}{r-k_1+1} - 2\beta_1\frac{\theta_1(r-k_2+1) - \eta\theta_2 r}{(r-k_1+1)(r-k_2+1)}\right] \\
&\quad - \frac{\alpha_1\eta\theta_2}{(r-k_1+1)(r-k_2+1)\varphi_3}\left[\tau\mu_1 - \alpha_1\frac{\theta_1(r-k_2+1) - \eta\theta_2 r}{(r-k_1+1)(r-k_2+1)}\right] - \frac{1}{r}\left[\frac{\alpha_2\theta_2 - \mu_2(r-k_2+1)}{r-k_2+1}\right]^2,
\end{aligned} \tag{67}$$

$$\begin{aligned}
p_3 &= \frac{\theta_1(r-k_2+1) + \eta\theta_2(1-k_1)}{(r-k_1+1)(r-k_2+1)}, q_3 = \frac{\theta_2}{r-k_2+1}, \\
h_3 &= \frac{\tau A + B}{r}, -\frac{1}{2r}\frac{\beta_2\theta_2 + \sigma_2(r-k_2+1)}{[4\sigma_2(r-k_2+1) - 2\beta_2\theta_2]} \cdot \frac{\sigma_2(r-k_2+1) - \beta_2\theta_2}{(r-k_2+1)} \\
&\quad + \frac{\beta_2\theta_2 - \sigma_2}{r\varphi_2[4\sigma_2(r-k_2+1) - 2\beta_2\theta_2]} \cdot \frac{\sigma_2(r-k_2+1) - \beta_2\theta_2}{(r-k_2+1)} \cdot \frac{3\sigma_2(r-k_2+1) - 3\beta_2\theta_2}{(r-k_2+1)} \\
&\quad + \frac{1}{4r\varphi_1}\left[\beta_1\frac{\theta_1(r-k_2+1) - \eta\theta_2 r}{(r-k_2+1)(r-k_1+1)} - \frac{1}{2}\sigma_1(1+\tau) + \frac{1}{2}\frac{(1-\tau)\theta_1}{(r-k_1+1)}\right] \cdot \left[(1+\tau)\sigma_1 - \beta_1\frac{\theta_1(r-k_2+1)(3-\tau) - 2\eta\theta_2 r}{(r-k_2+1)(r-k_1+1)}\right] \\
&\quad - \frac{1}{2r\varphi_3}\left[\alpha_1\frac{\theta_1(r-k_2+1) - \eta\theta_2 r}{(r-k_2+1)(r-k_1+1)} - \tau\mu_1\right]^2 - \frac{1}{2r\varphi_4}\left[\frac{\alpha_2\theta_2}{(r-k_2+1)} - \mu_2\right]^2.
\end{aligned} \tag{68}$$

By substituting formulas (67)–(69) into equations (61)–(63), the minimum loss function of natural gas energy shortage of local government, natural gas supply chain enterprises, and provincial government can be obtained as

$$\begin{aligned}
V_1^{**} &= \frac{(\tau-1)\theta_1}{r-k_1+1}D_1 - \frac{1}{r\varphi_3}\left[(1-\tau)\mu_1 - \frac{(1-\tau)\theta_1}{r-k_1+1}\right]\left[\tau\mu_1 + \alpha_1\frac{\theta_1(r-k_2+1) - \eta\theta_2 r}{(r-k_1+1)(r-k_2+1)}\right] \\
&\quad + \frac{1}{4r\varphi_1}\left[(\tau-1)\sigma_1 - \frac{(1-\tau)\theta_1}{r-k_1+1}\right]\left[(\tau+1)\sigma_1 + \frac{\beta_1(1-\tau)\theta_1}{r-k_1+1} + 2\beta_1\frac{\theta_1(r-k_2+1) - \eta\theta_2 r}{(r-k_1+1)(r-k_2+1)}\right], \\
&\quad + \frac{(1-\tau)A}{r},
\end{aligned} \tag{69}$$

$$\begin{aligned}
V_2^{**} &= \frac{\eta\theta_2(k_1-1)}{(r-k_1+1)(r-k_2+1)}D_1 - \frac{\theta_2}{r-k_2+1}D_2 + \frac{B}{r} - \frac{1}{r}\left[\frac{-\alpha_2\theta_2 - \mu_2(r-k_2+1)}{r-k_2+1}\right]^2 \\
&\quad + \frac{\beta_1\eta\theta_2}{2\varphi_1(r-k_1+1)(r-k_2+1)}\left[(\tau+1)\sigma_1 + \beta_1\frac{(1-\tau)\theta_1}{r-k_1+1} + 2\beta_1\frac{\theta_1(r-k_2+1) - \eta\theta_2 r}{(r-k_1+1)(r-k_2+1)}\right] \\
&\quad + \left[\frac{-\beta_2\theta_2 - \sigma_2(r-k_2+1)}{2r(r-k_2+1)\varphi_2} \cdot \frac{3\sigma_2(r-k_2+1) + 3\beta_2\theta_2}{4\sigma_2(r-k_2+1) + 2\beta_2\theta_2} \cdot \frac{\sigma_2(r-k_2+1) + \beta_2\theta_2}{(r-k_2+1)}\right] \\
&\quad + + \frac{\alpha_1\eta\theta_2}{(r-k_1+1)(r-k_2+1)\varphi_3}\left[\tau\mu_1 + \alpha_1\frac{\theta_1(r-k_2+1) - \eta\theta_2 r}{(r-k_1+1)(r-k_2+1)}\right],
\end{aligned} \tag{70}$$

$$\begin{aligned}V_3^{**} ={}& \frac{-\theta_1(r-k_2+1)+\eta\theta_2(k_1-1)}{(r-k_1+1)(r-k_2+1)}D_1-\frac{\theta_2}{r-k_2+1}D_2-\frac{1}{2r\varphi_4}\left[\frac{-\alpha_2\theta_2}{(r-k_2+1)}-\mu_2\right]^2\\&+\frac{\tau A+B}{r}-\frac{1}{2r}\frac{-\beta_2\theta_2+\sigma_2(r-k_2+1)}{[4\sigma_2(r-k_2+1)+2\beta_2\theta_2]}\cdot\frac{\sigma_2(r-k_2+1)+\beta_2\theta_2}{(r-k_2+1)}\\&+\frac{-\beta_2\theta_2-\sigma_2}{r\varphi_2[4\sigma_2(r-k_2+1)+2\beta_2\theta_2]}\cdot\frac{\sigma_2(r-k_2+1)+\beta_2\theta_2}{(r-k_2+1)}\cdot\frac{3\sigma_2(r-k_2+1)+3\beta_2\theta_2}{(r-k_2+1)}\\&+\frac{1}{4r\varphi_1}\left[\beta_1\frac{-\theta_1(r-k_2+1)+\eta\theta_2 r}{(r-k_2+1)(r-k_1+1)}-\frac{1}{2}\sigma_1(1+\tau)-\frac{1}{2}\frac{(1-\tau)\theta_1}{(r-k_1+1)}\right]\\&\cdot\left[(1+\tau)\sigma_1+\beta_1\frac{\theta_1(r-k_2+1)(3-\tau)-2\eta\theta_2 r}{(r-k_2+1)(r-k_1+1)}\right]-\frac{1}{2r\varphi_3}\left[\alpha_1\frac{\theta_1(r-k_2+1)-\eta\theta_2 r}{(r-k_2+1)(r-k_1+1)}-\tau\mu_1\right]^2,\end{aligned}\tag{71}$$

when the total loss of the city due to natural gas energy shortage is as follows:

$$\begin{aligned}V^{**}(D_1,D_2) ={}& V_1^{**}+V_2^{**}+V_3^{**},\\ ={}& \frac{-\theta_1(2-\tau)(r-k_2+1)-2\eta\theta_2(1-k_1)}{(r-k_1+1)(r-k_2+1)}D_1-\frac{2\theta_2}{r-k_2+1}D_2\frac{A+2B}{r}\\&-\frac{1}{r\varphi_3}\left[(1-\tau)\mu_1-\frac{(1-\tau)\theta_1}{r-k_1+1}\right]\left[\tau\mu_1+\alpha_1\frac{\theta_1(r-k_2+1)-\eta\theta_2 r}{(r-k_1+1)(r-k_2+1)}\right]\\&\frac{1}{4r\varphi_1}\left[(\tau-1)\sigma_1-\frac{(1-\tau)\theta_1}{r-k_1+1}\right]\left[-(\tau+1)\sigma_1+\frac{\beta_1(1-\tau)\theta_1}{r-k_1+1}+2\beta_1\frac{\theta_1(r-k_2+1)-\eta\theta_2 r}{(r-k_1+1)(r-k_2+1)}\right]\\&-\frac{1}{r}\left[\frac{-\alpha_2\theta_2-\mu_2(r-k_2+1)}{r-k_2+1}\right]^2-\frac{1}{2r\varphi_4}\left[\frac{-\alpha_2\theta_2}{(r-k_2+1)}-\mu_2\right]^2\\&+\frac{\beta_1\eta\theta_2}{2\varphi_1(r-k_1+1)(r-k_2+1)}\left[(\tau+1)\sigma_1+\beta_1\frac{(1-\tau)\theta_1}{r-k_1+1}+2\beta_1\frac{\theta_1(r-k_2+1)-\eta\theta_2 r}{(r-k_1+1)(r-k_2+1)}\right]\\&+\left[\frac{-\beta_2\theta_2-\sigma_2(r-k_2+1)}{2r(r-k_2+1)\varphi_2}\cdot\frac{3\sigma_2(r-k_2+1)+3\beta_2\theta_2}{4\sigma_2(r-k_2+1)+2\beta_2\theta_2}\cdot\frac{\sigma_2(r-k_2+1)+\beta_2\theta_2}{(r-k_2+1)}\right]\\&+\frac{\alpha_1\eta\theta_2}{(r-k_1+1)(r-k_2+1)\varphi_3}\left[\tau\mu_1+\alpha_1\frac{\theta_1(r-k_2+1)-\eta\theta_2 r}{(r-k_1+1)(r-k_2+1)}\right]\\&+\frac{\tau A+B}{r}-\frac{1}{2r}\frac{-\beta_2\theta_2+\sigma_2(r-k_2+1)}{[4\sigma_2(r-k_2+1)+2\beta_2\theta_2]}\cdot\frac{\sigma_2(r-k_2+1)+\beta_2\theta_2}{(r-k_2+1)}\\&+\frac{-\beta_2\theta_2-\sigma_2}{r\varphi_2[4\sigma_2(r-k_2+1)+2\beta_2\theta_2]}\cdot\frac{\sigma_2(r-k_2+1)+\beta_2\theta_2}{(r-k_2+1)}\cdot\frac{3\sigma_2(r-k_2+1)+3\beta_2\theta_2}{(r-k_2+1)}\\&+\frac{1}{4r\varphi_1}\left[\beta_1\frac{-\theta_1(r-k_2+1)+\eta\theta_2 r}{(r-k_2+1)(r-k_1+1)}-\frac{1}{2}\sigma_1(1+\tau)-\frac{1}{2}\frac{(1-\tau)\theta_1}{(r-k_1+1)}\right]\\&\cdot\left[(1+\tau)\sigma_1-\beta_1\frac{-\theta_1(r-k_2+1)(3-\tau)+2\eta\theta_2 r}{(r-k_2+1)(r-k_1+1)}\right]-\frac{1}{2r\varphi_3}\left[\alpha_1\frac{-\theta_1(r-k_2+1)+\eta\theta_2 r}{(r-k_2+1)(r-k_1+1)}-\tau\mu_1\right]^2.\end{aligned}\tag{72}$$

The partial derivatives of formulas (69)–(71) with respect to D_1 and D_2 are obtained and substituted into equations (51)–(54) to obtain (44)–(48).

Under the Nash noncooperative equilibrium state, the emergency shortage of natural gas energy of local government is as follows:

$$\begin{cases} D_1^{**}(t) = \dfrac{\alpha_1}{(k_1-1)\varphi_3}\left[\left[\tau\mu_1 + \dfrac{\tau\alpha_1\theta_1}{(r-k_1+1)} - \dfrac{\alpha_1\eta\theta_2 r}{(r-k_1+1)(r-k_2+1)}\right] + \dfrac{\beta_1}{2(k_1-1)\varphi_1}\right], \\ \left[(\tau+1)\sigma_1 + \dfrac{\beta_1\theta_1(2-\tau)}{(r-k_1+1)} - \dfrac{2\eta\theta_2 r\beta_1}{(r-k_1+1)(r-k_2+1)}\right]\Bigg] - \dfrac{1}{(k_1-1)}e^{(1-k_1)t+d_1}, \\ D_1^{**}(0) = D_1. \end{cases} \tag{73}$$

The emergency shortage of natural gas energy in natural gas supply chain enterprises is as follows:

$$\begin{cases} D_2^{**}(t) = \dfrac{\eta\alpha_1}{(k_2-1)\varphi_3}\left[\tau\mu_1 + \dfrac{\tau\alpha_1\theta_1}{(r-k_1+1)} - \dfrac{\alpha_1\eta\theta_2 r}{(r-k_1+1)(r-k_2+1)}\right] + \dfrac{\eta\beta_1}{2(k_2-1)\varphi_1}, \\ \left[(\tau+1)\sigma_1 + \dfrac{\beta_1\theta_1(2-\tau)}{(r-k_1+1)} - \dfrac{2\eta\theta_2 r\beta_1}{(r-k_1+1)(r-k_2+1)}\right] - \eta e^{(1-k_1)t+d_1} + \dfrac{\alpha_2}{(k_2-1)\varphi_4}, \\ \left(\mu_2 + \dfrac{\alpha_2\theta_2}{r-k_2+1}\right) + \dfrac{3\beta_2}{2(k_2-1)\varphi_2}\left[\left(\sigma_2 + \dfrac{\beta_2\theta_2}{(r-k_2+1)}\right)\right], \\ -\dfrac{1}{(k_2-1)}e^{(1-k_2)t+d_2}, \\ D_2^{**}(0) = D_2, \\ e^{d_1} = \dfrac{1}{\varphi_3}\left[\tau\mu_1 + \dfrac{\tau\alpha_1\theta_1}{(r-k_1+1)} - \dfrac{\alpha_1\eta\theta_2 r}{(r-k_1+1)(r-k_2+1)}\right] + \dfrac{\beta_1}{2\varphi_1}, \\ \left[(\tau+1)\sigma_1 + \dfrac{\beta_1\theta_1(2-\tau)}{(r-k_1+1)} - \dfrac{2\eta\theta_2 r\beta_1}{(r-k_1+1)(r-k_2+1)}\right]\Bigg] - (k_1-1)D_1. \end{cases} \tag{74}$$

It can be seen from equation (43) that under the government's leading emergency response model for natural gas energy shortages, the local government's emergency effort E_1 is proportional to the share ratio τ and the impact coefficient of the local government's emergency natural gas energy shortage on the enterprise's emergency response and natural gas shortage. The influence coefficient η of the enterprise is directly proportional to θ_2, while other influence factors are consistent with those under the spontaneous governance model. Equation (44) shows that under the government-led model, the degree of influence of the enterprise's effort E_2 is less than that under the noncooperative model, revealing the positive role of the government-led model. Equations (45) and (46) indicate that under the government emergency-led model, the provincial government's effort and related influencing factors are the same as those under the spontaneous governance model, indicating that the government must comprehensively consider the actual situation, from the utilization of funds, the executing ability, and other relevant details to reasonable emergency decision-making.

Equation (47) shows that the provincial government's choice of emergency investment funding ratio to the local government depends on the local urban disaster loss bearing ratio τ. The provincial government bearing smaller urban disaster losses $0 \le \tau \le 1/3$ will not share the cost through the funding mechanism. If the provincial government bears more disaster losses, however, it will turn to the central government and ask for disaster emergency funds. Besides, special subsidies from the provincial government will be considered to improve the emergency response efficiency of the city, reflecting the "economic man" characteristics of the government [22]. It can be seen from equation (48) that the proportion of provincial government funding to enterprises is positively correlated; that is, subsidies are positively correlated with the impact of enterprises' efforts on the city's losses caused by natural gas energy shortages, indicating that the government encourages enterprises to actively participate in the emergency process.

When natural gas energy is in short supply due to emergencies and the government adopts the dominant mode of emergency response, the intervention of the provincial government can effectively solve the problem of market failure caused by emergency externalities. The provincial government is the direct manager of the local government and functions more effectively in the overall planning. The determined emergency financial subsidy mechanism can optimize the intergovernmental relationship in emergency decision-making, reduce the uncertainty and confusion of intergovernmental behavior, strengthen the trust of the local government and the enterprise in the provincial government and the cooperative relationship between each other for emergency response, and forge the foundation for their linkage. It can also strengthen the participation of the central government, strengthen its supervision and guidance, and provide better decisions for improving the emergency response efficiency. □

3.3. Intergovernment Coordination Mode for the Energy Shortage Emergency of Urban Natural Gas. In the case of a shortage of natural gas energy and other emergency events, governments and enterprises at all levels desire for the optimal configuration during the emergency process, which, according to the analysis of the Coase Theorem, is possible between governments and between governments and enterprises to realize economies of scale through cooperation and obtain policy spillover effects under suitable emergency situations. For the shortage of natural gas energy in a city, the emergency process needs to integrate various information and coordinate multidepartmental cooperation. The traditional order and obedience style is being gradually transformed into a negotiation and cooperation style. To this end, it is imperative to build an emergency intergovernmental coordination model. In order to better improve the emergency response efficiency, provincial governments, local governments, and enterprises should collaborate with each other to determine the best emergency response strategy and reduce urban emergency disaster losses.

Proposition 3. *In the case of full communication and collaboration between the provincial government, local government, and natural gas emergency companies, the best effort strategies for them are*

$$E_1^{***} = \frac{1}{\varphi_1}\left[\sigma_1 + \frac{\beta_1\theta_1}{r+1-k_1} - \frac{\beta_1\theta_2\eta r}{(r+1-k_1)(r+1-k_2)}\right], \tag{75}$$

$$E_2^{***} = \frac{1}{\varphi_2}\left[\sigma_2 + \frac{\beta_2\theta_2}{r+1-k_2}\right], \tag{76}$$

$$E_3^{***} = \frac{1}{\varphi_3}\left[\mu_1 + \frac{\alpha_1\theta_1}{r+1-k_1} - \frac{\alpha_1\theta_2\eta r}{(r+1-k_1)(r+1-k_2)}\right], \tag{77}$$

$$E_4^{***} = \frac{1}{\varphi_4}\left[\mu_2 + \frac{\alpha_2\theta_2}{r+1-k_2}\right]. \tag{78}$$

Proof. When the emergency relationship of the provincial government, local government, and enterprises is led by superiors to coordinate and cooperate, all parties will aim at minimizing urban losses and jointly determine the optimal values of E_1, E_2, E_3, and E_4, when the loss caused by natural gas energy shortage disasters can be expressed as

$$T = \int_0^\infty e^{-rt}\left\{\begin{array}{l} A(t) - \mu_1 E_3(t) - \sigma_1 E_1(t) - \theta_1 D_1(t) + \frac{1}{2}\varphi_1 E_1^2 + \frac{1}{2}\varphi_2 E_2^2 \\ +B(t) - \mu_2 E_4(t) - \sigma_2 E_2(t) - \theta_2 D_2(t) + \frac{1}{2}\varphi_3 E_3^2 + \frac{1}{2}\varphi_4 E_4^2 \end{array}\right\} \mathrm{d}t. \tag{79}$$

Suppose that there is a continuous and bounded differential urban disaster loss function $V(D_1, D_2)$ for all $D_1 \geq 0$ and $D_2 \geq 0$, which satisfies the HJB equation:

$$r \cdot V(D_1, D_2) = \min_{\substack{E_1 \geq 0, E_2 \geq 0 \\ E_3 \geq 0, E_4 \geq 0}} \left\{ \begin{array}{c} (A - \mu_1 E_3 - \sigma_1 E_1 - \theta_1 D_1) + (B - \mu_2 E_4 - \sigma_2 E_2 - \theta_2 D_2) \\ + \frac{1}{2}\varphi_1 E_1^2 + \frac{1}{2}\varphi_2 E_2^2 + \frac{\varphi_3}{2} E_3^2 + \frac{\varphi_4}{2} E_4^2 \\ -\left(\frac{\partial V}{\partial D_1} - \eta \frac{\partial V}{\partial D_2}\right)(D_1 - \alpha_1 E_3 - \beta_1 E_1 - k_1 D_1) \\ -\frac{\partial V}{\partial D_2}(D_2 - \alpha_2 E_4 - \beta_2 E_2 - k_2 D_2) \end{array} \right\}. \tag{80}$$

We find the first-order partial derivatives of E_1, E_2, E_3, and E_4, respectively, and set them to zero to obtain

$$E_1 = \frac{\sigma_1 - \beta_1 (\partial V / \partial D_1 - \eta \partial V / \partial D_2)}{\varphi_1}, \tag{81}$$

$$E_2 = \frac{\sigma_2 - \beta_2 \partial V / \partial D_2}{\varphi_2}, \tag{82}$$

$$E_3 = \frac{\mu_1 - \alpha_1 (\partial V / \partial D_1 - \eta \partial V / \partial D_2)}{\varphi_3}, \tag{83}$$

$$E_4 = \frac{\mu_2 - \alpha_2 \partial V / \partial D_2}{\varphi_4}. \tag{84}$$

Substituting formulas (82)–(85) into formula (81), it can be obtained as

$$\begin{aligned} rV = {} & \left[\theta_1 - (1 - k_1)\left(\frac{\partial V}{\partial D_1} - \eta \frac{\partial V}{\partial D_2}\right)\right] D_1 + \left[\theta_2 - (1 - k_2)\frac{\partial V}{\partial D_2}\right] D_2 \\ & - \frac{[\sigma_1 - \beta_1 (\partial V / \partial D_1 - \eta \partial V / \partial D_2)]^2}{2\varphi_1} - \frac{[\sigma_2 - \beta_2 \partial V / \partial D_2]^2}{2\varphi_2} - \frac{[\mu_1 - \alpha_1 (\partial V / \partial D_1 - \eta \partial V / \partial D_2)]^2}{2\varphi_3} \\ & - \frac{[\mu_2 - \alpha_2 \partial V / \partial D_2]^2}{2\varphi_4} + A + B. \end{aligned} \tag{85}$$

According to equation (86), the linear minimum disaster loss function for D_1 and D_2 is the solution to the HJB equation. Set

$$V(D_1, D_2) = pD_1 + qD_2 + h, \tag{86}$$

where p, q, and h are constants. The first derivative of equation (87) is obtained and substituted into (86):

$$\begin{aligned} r(pD_1 + qD_2 + h) = {} & [-\theta_1 - (1 - k_1)(p - \eta q)]D_1 + [-\theta_2 - (1 - k_2)q]D_2 + A + B \\ & - \frac{[\sigma_1 - \beta_1 (p - \eta q)]^2}{2\varphi_1} - \frac{[\sigma_2 - \beta_2 q]^2}{2\varphi_2} - \frac{[\mu_1 - \alpha_1 (p - \eta q)]^2}{2\varphi_3} - \frac{[\mu_2 - \alpha_2 q]^2}{2\varphi_4}. \end{aligned} \tag{87}$$

According to formula (88), the following results can be obtained:

$$p = \frac{-\theta_1 + (1-k_1)\eta q}{r+1-k_1}, q =, \frac{-\theta_2}{r+1-k_2}, h = \frac{A+B}{r}, -\frac{[\sigma_1 - \beta_1 (p-\eta q)]^2}{2r\varphi_1}, -\frac{[\sigma_2 - \beta_2 q]^2}{2r\varphi_2} - \frac{[\mu_1 - \alpha_1 (p-\eta q)]^2}{2r\varphi_3} - \frac{[\mu_2 - \alpha_2 q]^2}{2r\varphi_4}. \tag{88}$$

Substituting (89) into equation (87), the minimum loss function for natural gas energy shortage can be expressed as

$$\begin{aligned} V^{***}(D_1, D_2) = {} & \frac{A+B}{r} + \left[\frac{-\theta_1 (r+1-k_2) + (k_1-1)\eta\theta_2}{(r+1-k_1)(r+1-k_2)}\right] D_1 - \frac{\theta_2}{r+1-k_2} D_2 \\ & - \frac{[\sigma_1 (r+1-k_2)(1-k_1) - \beta_1\theta_1 (r+1-k_2) + \beta_1\eta\theta_2 r]^2}{2r\varphi_1 [(r+1-k_1)(r+1-k_2)]^2} - \frac{[\sigma_2 (r+1-k_2) - \beta_2\theta_2]^2}{2r\varphi_2 (r+1-k_2)^2} \\ & - \frac{[\mu_1 (r+1-k_2)(1-k_1) - \alpha_1\theta_1 (r+1-k_2) + \alpha_1\eta\theta_2 r]^2}{2r\varphi_3 [(r+1-k_1)(r+1-k_2)]^2} - \frac{[\mu_2 (r+1-k_2) - \alpha_2\theta_2]^2}{2r\varphi_4 (r+1-k_2)^2}. \end{aligned} \tag{89}$$

We find the partial derivatives of D_1 and D_2 from equation (90) and substitute them into equations (82)–(85) to obtain equations (76)–(79), when the emergency shortage of natural gas energy by the local government is

$$\begin{cases} D_1^{***}(t) = \dfrac{\alpha_1}{(k_1-1)\varphi_3}\left[\left[\mu_1 + \dfrac{\alpha_1\theta_1}{r+1-k_1} - \dfrac{\alpha_1\theta_2\eta r}{(r+1-k_1)(r+1-k_2)}\right] + \dfrac{\beta_1}{(k_1-1)\varphi_1},\right. \\ \left.\left[\sigma_1 + \dfrac{\beta_1\theta_1}{r+1-k_1} - \dfrac{\beta_1\theta_2\eta r}{(r+1-k_1)(r+1-k_2)}\right]\right] - e^{(1-k_1)t+d_1}, \\ D_1^{***}(0) = D_1. \end{cases} \tag{90}$$

The emergency shortage of natural gas energy supply chain enterprises is

$$\begin{cases} D_2^*(t) = \dfrac{\eta\alpha_1}{(k_2-1)\varphi_3}\left[\mu_1 + \dfrac{\alpha_1\theta_1}{r+1-k_1} - \dfrac{\alpha_1\theta_2\eta r}{(r+1-k_1)(r+1-k_2)}\right] + \dfrac{\eta\beta_1}{(k_2-1)\varphi_1}\left[\begin{array}{c}\sigma_1 + \dfrac{\beta_1\theta_1}{r+1-k_1} - \\ \\ \dfrac{\beta_1\theta_2\eta r}{(r+1-k_1)(r+1-k_2)}\end{array}\right] \\ -\dfrac{\eta}{(k_2-1)}e^{(1-k_1)t+d_1} + \dfrac{\alpha_2}{(k_2-1)\varphi_4}\left[\mu_2 + \dfrac{\alpha_2\theta_2}{r+1-k_2}\right] + \dfrac{\beta_2}{(k_2-1)\varphi_2}\left[\sigma_2 + \dfrac{\beta_2\theta_2}{r+1-k_2}\right] - \dfrac{1}{(k_2-1)}e^{(1-k_2)t+d_2}, \\ D_2^*(0) = D_2, \\ e^{d_1} = \dfrac{1}{\varphi_3}\left[\mu_1 + \dfrac{\alpha_1\theta_1}{r+1-k_1} - \dfrac{\alpha_1\theta_2\eta r}{(r+1-k_1)(r+1-k_2)}\right] + \dfrac{\beta_1}{\varphi_1}\left[\begin{array}{c}\sigma_1 + \dfrac{\beta_1\theta_1}{r+1-k_1} - \\ \\ \dfrac{\beta_1\theta_2\eta r}{(r+1-k_1)(r+1-k_2)}\end{array}\right] - (k_1-1)D_1. \end{cases} \tag{91}$$

It can be seen from (76)–(79) that in the case of emergency intergovernment cooperation, the optimal effort of provincial and local governments no longer involves the proportion of disaster losses, while other influencing factors are related to the spontaneous governance model and the superior leadership. The pattern remains the same, where it can be observed that the degree of effort has increased compared with the dominant loss and this model is proven better. □

4. Comparative Analysis of the Equilibrium Results

In the following section, the equilibrium results, natural gas energy shortages, and disaster losses are compared under the proposed three models to analyze which method is more conducive to emergency construction investment, emergency response efficiency improvement, and urban natural gas energy shortage disaster losses reduction.

4.1. Comparative Analysis of the Optimal Effort

Proposition 4. *(1) The optimal emergency effort level of the local government $E_1^* < E_1^{**} \le E_1^{***}$; (2) the optimal emergency effort level of natural gas supply chain enterprises $E_2^* = E_2^{***} < E_2^{**}$; (3) the optimal emergency effort level of the provincial government to the local government $E_3^* = E_3^{**} \le E_3^{***}$; (4) the optimal degree of emergency response efforts of the provincial government to the local government $E_4^* = E_4^{**} = E_4^{***}$; (5) the optimal funding ratios $\lambda_1 = E_1^{**} - E_1^*/E_1^{**}$ and $\lambda_2 = 1/3$.*

Proof

(1) According to formulas (11), (43), and (76), since $\tau > 1/3$, the simplification results can be obtained as

$$\begin{aligned} E_1^{**} - E_1^* &= \frac{1}{2\varphi_1}(3\tau - 1) + \frac{1}{2\varphi_1}\beta_1\theta_1(1+\tau) \ge 0. \\ E_1^{***} - E_1^{**} &= \frac{1}{2\varphi_1}\left[(1-\tau)\sigma_1 + \frac{\tau\beta_1\theta_1}{r-k_1+1}\right] \ge 0. \end{aligned} \tag{92}$$

Consequently, $E_1^* < E_1^{**} \le E_1^{***}$.

(2) According to (12), (44), and (77), it can be obtained as

$$E_2^{**} - E_2^{***} = \frac{1}{2\varphi_2}\left(\sigma_2 + \frac{\beta_2\theta_2}{(r-k_2+1)}\right) \ge 0. \tag{93}$$

Consequently, $E_2^* = E_2^{***} < E_2^{**}$.

(3) According to Equations (13), (45), and (78), it can be obtained as

$$E_3^{***} - E_3^{**} = \frac{1}{\varphi_3}\left((1-\tau)\mu_1 + \frac{(1-\tau)\alpha_1\theta_1}{(r-k_1+1)}\right) \ge 0. \tag{94}$$

(4) From formulas (14), (46), and (79), the provincial government's optimal emergency response level $E_4^* = E_4^{**} = E_4^{***}$ to the local government is the same under the three modes when the enterprise does not bear the proportion of natural gas energy shortage loss.

(5) According to Equations (11), (43), and (47), Equations (12), (44), and (48) are calculated and simplified to obtain

$$\lambda_1 = \frac{E_1^{**} - E_1^*}{E_1^{**}}, \lambda_2 = \frac{E_2^{**} - E_2^*}{E_2^{**}} = \frac{1}{3}. \tag{95}$$

This proposition shows that compared with spontaneous governance models, the superior-led model for local governments and enterprises can more effectively solve the problem of urban natural gas shortages, reflecting the improvement of the efficiency of government-led emergency decision-making. With the maximization of the efforts of provincial and local governments, their coordination and cooperation have minimized the shortage of natural gas energy, indicating that intergovernmental cooperation is the most effective mechanism for urban emergency response and that effective cooperation between governments can achieve economies of scale and obtain policy spillover effects. □

4.2. Comparative Analysis of the Emergency Shortage of Natural Gas

Proposition 5. *(1) Comparison of emergency natural gas energy shortages by local governments $D_1^* \geq D_1^{**} \geq D_1^{***}$; (2) comparison of emergency natural gas energy shortages by enterprises $D_2^* \geq D_2^{***} \geq D_2^{**}$.*

Proof. According to (38), (75), and (92), it can be obtained as

$$\begin{cases} D_1^{**} - D_1^* = \dfrac{\left(1 - e^{(k_1 - 1)t}\right)\beta_1}{(k_1 - 1)\varphi_1}\left[\dfrac{1}{2\varphi_1}(3\tau - 1) + \dfrac{1}{2\varphi_1}\beta_1\theta_1(1 + \tau)\right]\Bigg] \leq 0, \\ D_1^*(0) = D_1^{**}(0), \end{cases}$$
$$\begin{cases} D_1^{***} - D_1^{**} = \dfrac{\left(1 - e^{(1-k_1)t}\right)\alpha_1}{(k_1 - 1)\varphi_3}\left[(1 - \tau)\mu_1\right] + \dfrac{\left(1 - e^{(1-k_1)t}\right)\beta_1}{(k_1 - 1)\varphi_1}\left[\dfrac{(1 - \tau)\alpha_1\theta_1}{(r - k_1 + 1)}\right]\Bigg] \leq 0, \\ D_1^{***}(0) = D_1^{**}(0). \end{cases} \tag{96}$$

Consequently, $D_1^* \geq D_1^{**} \geq D_1^{***}$.

Similar principles also prove that

$$\begin{cases} D_2^{**} - D_2^* \leq 0, \\ D_1^{**}(0) = D_1^*(0), \end{cases} \begin{cases} D_2^{***} - D_2^{**} \leq 0, \\ D_1^{***}(0) = D_1^{**}(0). \end{cases} \tag{97}$$

Consequently, $D_2^* = D_2^{***} \geq D_2^{**}$.

This proposition shows that compared with the spontaneous governance mode, the superior-led mode for local governments and enterprises can more effectively solve the problem of urban natural gas shortage, reflecting the improvement of the efficiency of government-led emergency decision-making. With the maximum efforts of the provincial government and the local government, the collaborative cooperation can minimize the shortage of natural gas energy, indicating that intergovernmental cooperation is the most effective mechanism for urban emergency response and that effective cooperation between governments can realize economies of scale and obtain the policy spill-over effect. □

4.3. Comparative Analysis of Disaster Losses in Natural Gas Shortage Cities

Proposition 6. *There exist local government disaster losses for any $D_1 \geq 0$ and $D_2 \geq 0$: $V_1^*(D_1, D_2) \geq V_1^{**}(D_1, D_2)$; enterprise disaster loss comparison: $V_2^*(D_1, D_2) \geq V_2^{**}(D_1, D_2)$; and provincial government disaster losses: $V^*(D_1, D_2) \geq V^{**}(D_1, D_2) \geq V^{***}(D_1, D_2)$.*

Proof. According to (34) and (37), the following results are obtained:

$$\begin{aligned} V_1^*(D_1, D_2) - V_1^{**}(D_1, D_2) \geq{}& \frac{1}{4r\varphi_1}\left[(1 - \tau)\sigma_1 + \frac{(1 - \tau)\theta_1}{r - k_1 + 1}\right]\left[\frac{2\beta_1\eta\theta_2 r}{(r - k_1 + 1)(r - k_2 + 1)}\right] \\ &+ \frac{[(1 - \tau)\theta_1\beta_2][\sigma_2(r - k_2 + 1)]}{r\varphi_2(r - k_1 + 1)(r - k_2 + 1)} + \frac{[(1 - \tau)\theta_1\alpha_2][\mu_2(r - k_2 + 1)]}{r\varphi_4(r - k_1 + 1)(r - k_2 + 1)} \geq 0, \\ V_2^*(D_1, D_2) - V_2^{**}(D_1, D_2) \geq{}& \frac{\beta_1}{2\varphi_1}\left[\frac{(1 - \tau)\eta\theta_2}{(r - k_1 + 1)(r - k_2 + 1)}\right]\left[2\beta_1\frac{2\beta_1\eta\theta_2 r}{(r - k_1 + 1)(r - k_2 + 1)}\right], \\ &+ \frac{\alpha_1\eta\theta_2}{(r - k_1 + 1)(r - k_2 + 1)\varphi_3}\left[\frac{\alpha_1\eta\theta_2 r}{(r - k_1 + 1)(r - k_2 + 1)}\right] \geq 0. \end{aligned} \tag{98}$$

The application of similar principles can prove $V_3^*(D_1, D_2) \geq V_3^{**}(D_1, D_2)$ and $V^*(D_1, D_2) \geq V^{**}(D_1, D_2) \geq V^{***}(D_1, D_2)$.

Proposition 7 shows that the superior-led model for provincial governments and local governments is superior to the spontaneous governance model in reducing urban disaster losses and that the Stalberg game is superior to the Nash noncooperative game. From the overall perspective, the intergovernment coordination and cooperation model is the best choice for the governance model if a reasonable urban disaster loss bearing plan can be formulated.

In the emergency response spontaneous road model of natural gas shortage, the main characteristics of the local government and enterprises are obvious and the enthusiasm for infrastructure construction and disaster emergency management in the region is high, but there is almost no emergency linkage. Under the superior mode, the intervention of the provincial government has optimized the allocation of resources and considerably improved the efficiency of departmental emergency decision-making. Comparative analysis has found that the intergovernment coordination model is the best choice for urban emergency response, which improves emergency response efficiency and is a continuous process that maximizes both the integration of the advantages of various decision-making entities and the decision-making efficiency. □

5. Emergency Decision Simulation

Based on existing studies and similar studies, as well as the actual scenario of energy emergency and the model given in literature [21, 22], model parameters are selected as follows to study the optimal effort level, natural gas energy emergency shortage, and urban disaster loss under the three modes. Suppose we select $\varphi_1 = 7$, $\varphi_2 = 5$, $\varphi_3 = 8$, $\varphi_4 = 8$, $\alpha_1 = 5$, $\alpha_2 = 4$, $\beta_1 = 4$, $\beta_2 = 3$, $k_1 = 0.8$, $k_2 = 0.6$, $\eta = 0.2$, $\mu_1 = \mu_2 = 3$, $\sigma_1 = 2$, $\sigma_2 = 1$, $\theta_1 = 2$, $\theta_2 = 1$, $A = 60$, $B = 60$, $\tau = 0.6$, and $r = 0.05$.

5.1. Comparative Analysis of Emergency Effort. We introduce these parameters and calculate the degree of emergency effort as

$$\begin{aligned} &E_1^* = 1.945, E_2^* = 1.095, E_3^* = 3.28, E_4^* = 1.49 \\ &E_1^{**} = 2.92, E_2^{**} = 1.64, E_3^{**} = 3.28, E_4^{**} = 1.49 \\ &E_1^{***} = 4.81, E_2^{***} = 1.095, E_3^{***} = 5.32, E_4^{***} = 1.49. \end{aligned} \tag{99}$$

In the spontaneous governance model, the provincial government, local government, and enterprises present the lowest emergency response efforts. During the emergency response process led by the superior, the efforts of the provincial government have not changed, which is consistent with the results of the spontaneous governance model and the emergency efforts of local governments and enterprises. In the collaborative cooperation model, the emergency efforts of the local government have reached the highest level, while those of the enterprises have declined, returning to the level of the spontaneous governance model, which lacks incentives and subsidies, and the enterprise's efforts have declined. Provincial governments have reached the highest level of efforts to local governments, while the level of efforts to enterprises has not changed and remains consistent.

5.2. Comparative Analysis of Emergency Shortage of Urban Natural Gas Energy. In the spontaneous governance model, the emergency shortage of natural gas energy of the local government is $D_1^*(t) = -118.11 + 178.135e^{-0.2t}$, while that of the enterprise is $D_2^*(t) = -41.497 + 178.135e^{-0.2t} + 89.06e^{-0.4t}$; in the emergency superior-led model, the emergency shortage of natural gas energy of the local government is $D_1^{**}(t) = -146.81 + 206.78e^{-0.2t}$, while that of the enterprise is $D_2^{**}(t) = -53.71 + 103.39e^{-0.2t} + 92.113e^{-0.4t}$; and under the emergency coordination and cooperation, the emergency shortage of natural gas energy of the local government is $D_1^{***}(t) = -179.12 + 239.11e^{-0.2t}$, while that of the enterprise is $D_2^{***}(t) = -37.56 + 119.56e^{-0.2t} + 118.2e^{-0.4t}$.

The change trend of emergency natural gas energy shortages of local governments and enterprises over time is shown in Figures 1 and 2. Under the three models, natural gas energy shortages decrease over time, the decline rate of the spontaneous governance model is slower than that of the superior-led model, and that of the superior-led model is less than that of the collaborative cooperation model. This result is consistent with the calculated result, indicating that the collaborative cooperation model is the most effective one in the case of energy shortage in the city.

5.3. Comparative Analysis of Disaster Losses. In the urban emergency spontaneous governance model, the minimum disaster loss of the local government is Figure 3, disaster loss of the enterprise is $V_1^* = 1479.15 - 570.1e^{-0.2t}$ and that of the provincial government is $V_2^* = 1340.32 - 855.03e^{-0.2t} - 395.82e^{-0.4t}$. In the urban emergency superior-led model, the minimum disaster loss of the local government is $V_3^* = 885.61 - 387.94e^{-0.2t} - 197.91e^{-0.4t}$, that of the industry is $V_1^{**} = 921.78 - 661.7e^{-0.2t}$, and that of the provincial government is $V_3^{**} = 1189.7 - 457.9e^{-0.2t} - 204.7e^{-0.4t}$. Meanwhile, the total loss under the spontaneous governance model is $V^* = 2253.2 - 1813e^{-0.2t} - 593.3e^{-0.4t}$, that under the superior-dominated model is $V^{**} = 1128.6 - 3152.2e^{-0.2t} - 614.1e^{-0.4t}$, and that under the collaborative cooperation model is $V^{***} = 140.86 - 1307.13e^{-0.2t} - 262.67e^{-0.4t}$.

Under the three models, the disaster losses of provincial governments, local governments, and enterprises are shown in Figures 3–6, where it can be observed that the losses under the superior leadership model are lower than those under the spontaneous governance model and those under the collaborative cooperation model are lower than those under the superior leadership model. The same is true for total losses. The results are also consistent with Proposition 3 to Proposition 6.

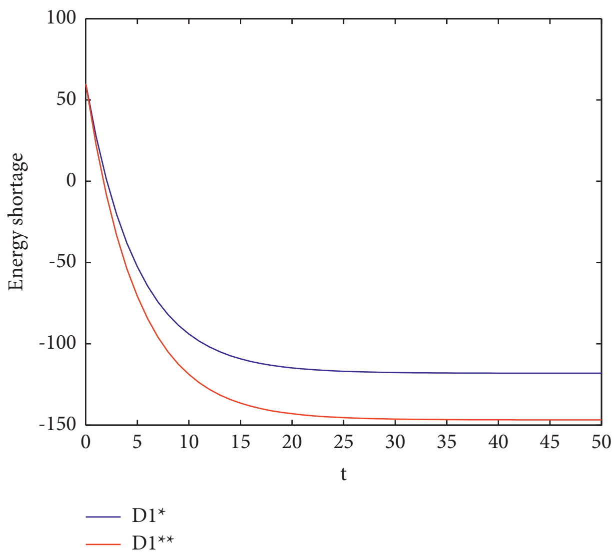

Figure 1: Volume of local government emergency natural gas.

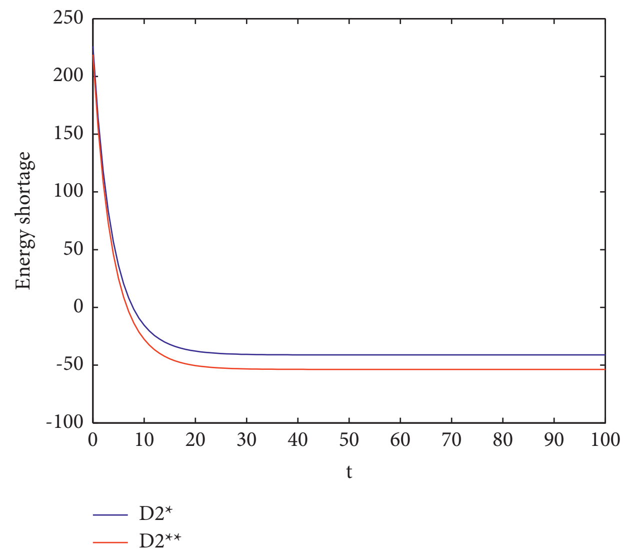

Figure 2: Volume of enterprises emergency natural gas shortage.

6. Policy Recommendations

It can be found through the above analysis that first of all, under the three emergency modes, the degree of effort has a negative relationship with the cost coefficient and the discount rate. The stronger the government's ability to influence and execute is, the more efforts should be invested to urban disaster emergency response; second, the emergency effort level of the local government reaches the highest under the intergovernmental collaborative governance model, followed by that under the superior-led model and the lowest under the spontaneous governance model; that of enterprises reaches the highest under superior-led model and spontaneous governance model, which is consistent with the results under the intergovernment coordination model; and that of the provincial government reaches the highest under the intergovernment coordination model; third, from the perspective of emergency natural gas energy shortages, the intergovernment coordination and cooperation model is superior to the spontaneous governance model and superior-led model in terms of achieving Pareto optimum of the system; fourth, emergency response efforts of enterprises are optimal under the superior-led model where government subsidies play a major incentive role, and government special subsidies can effectively inspire local governments and enterprises to make more emergency efforts and reduce urban losses.

According to the hereby drawn conclusion, the emergency decision-making of urban natural gas energy shortage emphasizes the characteristics of coordination, linkage, and information sharing, especially under the construction of ecological civilization cities, and promoting the emergency construction of natural gas energy shortage is provided with strong practical significance. Emergency measures should be formulated scientifically based on the influence, execution, and supervision capabilities of governments at all levels and the actual situation of local cities. (1) After the occurrence of urban natural gas energy shortage, provincial governments should timely organize and coordinate local governments and natural gas emergency enterprises to participate in emergency response as soon as possible, focus on coordination, and give full play to emergency response efforts, while local governments and emergency enterprises should be strictly prohibited for the independent spontaneous management emergency mode. (2) In the case of a huge regional or interprovincial shortage of natural gas energy, the central government should also play its organizational and coordination roles, and exert maximum emergency efforts to minimize the shortage of natural gas energy under the coordination and command of the central government. (3) The local government is supposed to establish special emergency funds to subsidize the natural gas enterprises and other enterprises involved in emergency response, so as to give full play to the enthusiasm and maximum efforts of

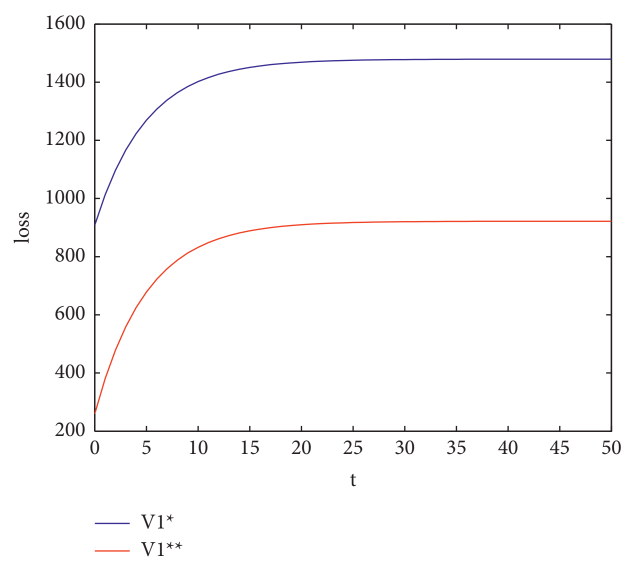

Figure 3: Local government disaster loss.

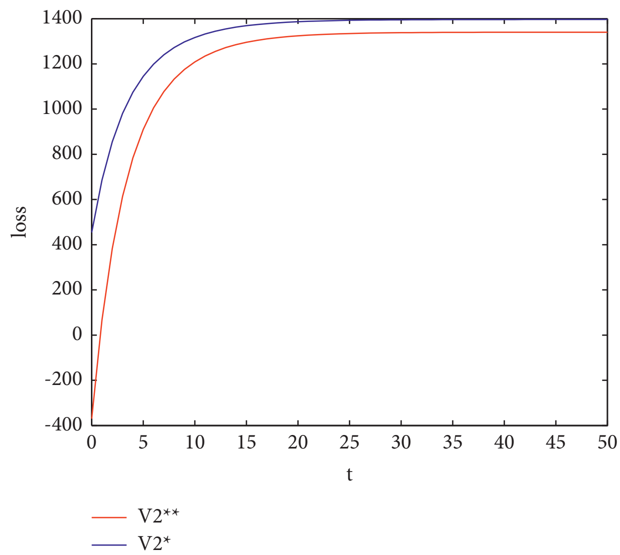

Figure 4: Enterprises disaster loss.

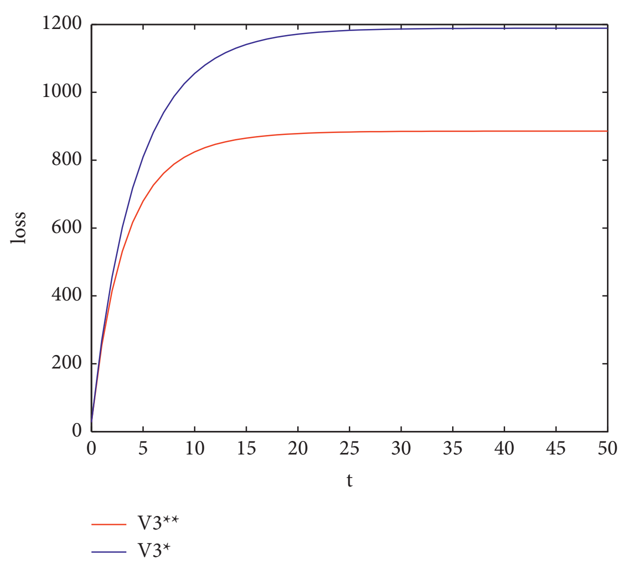

Figure 5: Provincial government disaster loss.

enterprises in emergency response and reduce the loss of urban emergency disasters. (4) The local government should also improve the utilization rate of emergency funds and reduce the cost coefficient of emergency process. The above conclusion shows that under the three emergency modes, the degree of emergency effort is negatively related to the cost coefficient and the discount rate. The government optimizes the use of emergency funds, rationally allocates resources with modern means, and reduces the emergency cost. Emergency enterprises can make full use of the financial subsidies from the government to maximize their efforts. (5) The government should make reasonable plans for the disaster relief materials, funds, and information that cities can use for emergency response to ensure the orderly mobilization of resources and low-cost requirements. The

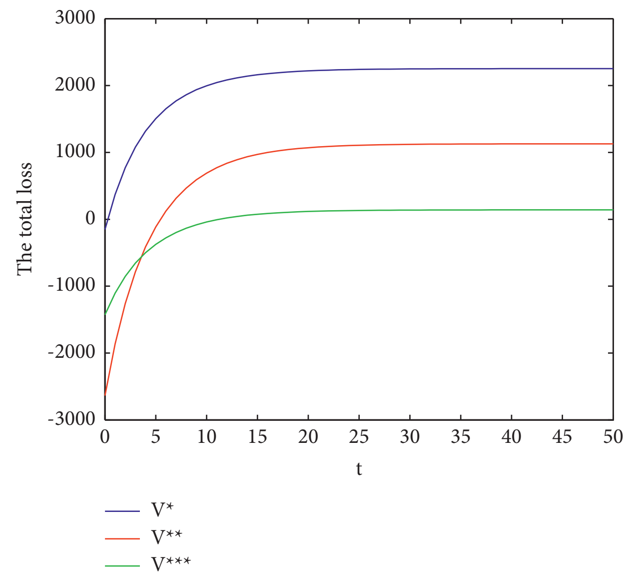

Figure 6: Total disaster loss.

emergency information center should be constructed to ensure the information compatibility and mutual sharing of different departments, thereby achieving rapid response and optimal resource scheduling. (6) Reasonable subsidy mechanism and sharing mechanism should be established. Strengthening emergency subsidies to enterprises and other nongovernmental organizations is conducive to the reduction of disaster losses and disaster relief. To this end, it is advisable to increase emergency investment, adjust fiscal mechanisms, set up special emergency funds, establish and improve emergency social insurance and social assistance, strengthen the depth, breadth, and intensity of exchanges and cooperation between governments and enterprises using the intelligent information technology, and deal with urban disaster losses properly. (7) Measures should also be taken to improve the government's capacity for influence and enforcement, train professional emergency management personnel, enhance the theoretical knowledge of management and the practical ability, and publicize basic knowledge of emergency response, basic knowledge of emergency response, and psychological counseling to reduce psychological panic among the masses.

7. Conclusion

The shortage emergency response of urban natural gas energy is a dynamic and complex process. In this paper, urban natural gas energy shortages are studied in the process of the provincial government, local government, and gas emergency enterprise cooperation strategies using the differential game theory, and a tripartite game model is constructed based on the HJB equation obtained by the spontaneous governance model and superior dominant situation of the optimal effort level, gas loss to the energy shortage, and the largest city. It is found that the efforts of the government and natural gas emergency enterprises are positively correlated with their emergency impact degree, effort impact degree, and natural gas energy attenuation coefficient but are negatively correlated with their emergency cost coefficient and discount rate. From the perspective of emergency shortage and total loss of cities, the intergovernment cooperation mode is the best emergency management mode. At the same time, the government implements subsidy incentive measures for non-governmental organizations involved in emergency response, with one-third of the optimal subsidy proportion, which can stimulate their enthusiasm and improve the level of emergency response efforts, thereby reducing urban disaster losses.

Data Availability

The datasets used and/or analyzed during the current study are available from the corresponding author on reasonable request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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