Research Article

A Study on Dynamic Behaviour of Thermally Distributed Exponentially Graded Rotor System with Induced Porosities

Vijayakumar Vaka,1 Prabhakar Sathujoda,1 Neelanchali Asija Bhalla,1 Y. V. Satish Kumar,2 and Roberto Citarella3

1Department of Mechanical Engineering, Bennett University, Greater Noida 201310, India
2ASquare Infotech Consultancy Pvt Ltd., Hyderabad 500072, India
3Department of Industrial Engineering, University of Salerno, Fisciano 84084, Italy

Correspondence should be addressed to Prabhakar Sathujoda; prab_sat@yahoo.com

Received 18 April 2022; Revised 8 May 2022; Accepted 11 May 2022; Published 2 June 2022

Academic Editor: Chiara Bedon

Copyright © 2022 Vijayakumar Vaka et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

The present work deals with the dynamic analysis of exponential law-based functionally graded (FG) rotor-bearing systems. The effect of thermal gradation and porosity on dynamic characteristics of FG rotor shafts has been studied first time, using exponential law with a novel two-nodded FG rotor element based on Timoshenko beam theory (TBT). Porous material properties are assorted using exponential law and thermal gradation across the cross section of the FG shaft using exponential temperature distribution (ETD). The effects of temperature and porosity on natural frequencies and whirl frequencies are studied. It has been observed that there is a significant reduction in natural frequencies and whirl frequencies with an increase in volume fraction of porosity and temperature. Attempts have been made to obtain suitable reasons for the behaviours based on the material properties. Furthermore, the steady-state and transient vibration responses have been simulated using the Houbolt time marching technique for the ceramic-based FG rotor shaft system. The result shows the maximum amplitude of the steady-state and transient vibration responses is increased, and the critical speed of the FG rotor system shifts towards the left with the increase in volume fraction of porosity and temperature.

1. Introduction

In modern applications, demand for advanced materials with improved dynamic characteristics in high-temperature environments has increased. Rotor shafts made of composites have shown several advantages when compared to traditional metallic rotor shafts due to high stiffness to weight ratio, specific strength, and specific stiffness. However, in higher temperature environments, composite materials have issues of delamination, de-bonding, and residual stresses. The development of functionally graded materials (FGMs) can overcome such problems. FGMs are inhomogeneous composites made of two or more constituents. Japanese scientists in the 1980s first introduced FGMs. Usually, FGMs are composed of ceramic and metal. These are capable of withstanding high-temperature conditions since the ceramic component alleviates thermal stress concentration, whereas the metallic constituents impart robust mechanical performance and diminish the risk of catastrophic failure. FGMs can be fabricated to use in the aerospace, automobile, sensor, and biomedical industries to sustain high-temperature environments. One of the critical parameters for the FGM is the porosity, which, in general, forms during the manufacturing process or is purposefully induced to reduce the thermal conductivity, increase stiffness to weight ratio and reduce the residual thermal stresses in a specific direction. Therefore, porosity is one of the critical parameters for investigating the behaviour of FG material properties in a high-temperature environment. This work is mainly focused on studying the impact of porosity and temperature on the dynamic characteristics of FG rotor shafts. Exponential law is used for porous FG material.
gradation and temperature distribution to understand the influence of temperature and porosity on dynamic behaviour.

As material properties can influence the dynamic properties of a rotor system, the FGMs have been considered in the present analysis to investigate the dynamic behaviour of rotating systems. Material gradation laws, such as power, exponential, and sigmoidal laws, vary the material constituents along a particular direction in an FGM. A few researchers have been modelling and analysing the FGMs [1–3]. Chakraborty et al. [4] investigated the thermoelastic behaviour of the power law- and exponential law-based FG beam using the first-order shear deformation theory. Zghal and Dammak [5] performed the vibrational analysis of the FG beams based on a mixed formulation theory. Simsek [6] has computed fundamental frequencies of a power law-based FG beam for various boundary conditions based on different shear deformation beam theories. Aydogdu and Taskin [7] calculated the free vibration frequencies of an FG beam by using the Navier-type solution method. Ziaee [8] carried out the steady-state analysis of the Euler–Bernoulli FG nanobeams resting on a viscous foundation. Aubad et al. [9] performed the transient and modal analyses of an axially FG beam using FEM. Few studies have also been reported on the dynamic behaviour of plates [10–13] and shells [14–16].

Since rotating machinery has a wide range of applications in the mechanical, aerospace, automotive, and mining industries, research into dynamic analysis of rotor-bearing systems has also been reported in the literature. Bose and Sathujoda [17] determined the natural frequencies of a power law-based FG rotor system by using ANSYS. Rao et al. [18] developed an FE model to carry out the dynamic analysis of a bidirectional FG rotor. Sui et al. [19] investigated the transverse vibration behaviour of an axially moving FG beam based on the Timoshenko beam theory. Since the FGMs can sustain harsh temperature environments, dynamic analysis of FG beams/rotors under a thermal environment has been reported in the literature. Mahi et al. [20] performed the eigenfrequency analysis of a temperature-dependent FG beam for various material distributions and boundary conditions. Nguyen and Bui [21] carried out the dynamic analysis of a higher-order finite FG beam element under a thermal environment. Obalareddy et al. [22] investigated the free vibration behaviour of an FG rotor-bearing system subjected to temperature gradients by employing the dynamic stiffness matrix approach. The effect of thermal gradients on the vibrational behaviour of the FG rotor system was analysed by Bose and Sathujoda [23].

Some researchers have been exploring the FG rotor systems having imperfections such as corrosion and cracks. Natural and whirl frequency analysis has been performed on the power law- and exponential law-based FG rotor-bearing system with corrosion [24, 25]. Gayen et al. [26] carried out the whirl frequency analysis of an FG rotor system with a transverse crack. Batchu and Sathujoda [27] studied the dynamic behaviour of a power law-based slant-cracked FG rotor-bearing system. Gayen et al. [28] analysed the dynamic behaviour of cracked functionally graded structural components. The imperfections mentioned earlier can affect the dynamic behaviour of the rotor systems. For that reason, inspecting the effect of porosity on the vibrational behaviour of the FG shafts is also equally important.

Porosity is materialised in the FGM during the manufacturing processes. There are several ways to fabricate an FGM-centrifugal casting, chemical and physical vapour deposition, additive manufacturing and powder sintering, etc. [29]. Primarily, FGMs are manufactured using the powder sintering process as it is cost-effective and consumes less energy to fabricate near-net shape products [30]. Due to the significant difference in solidification temperatures of metal and ceramic constituents, porosities are devised within inter-layers of the FGM. The major drawback of the sintering process is higher porosity induced in the components compared to other fabricating procedures. Therefore, a few scientists have explored the dynamic behaviour of the porous FG structures and rotors.

A review of the dynamic analysis of porous FG rotors has been carried out by Vaka et al. [31]. Bollf et al. [32] carried out a thermal buckling analysis on a porous FG Euler beam for different boundary conditions. The vibration behaviour of nonlinear FG beams with porosity is investigated by Heshmati and Daneshmand [33]. Askbas [34] studied the thermal effects on the free vibration behaviour of FG deep beams with porosity. Wattanasakulpong [35] computed the natural and whirl frequencies of a porous FG beam by using the differential transformation method. Batchu et al. [36] developed a finite element model of a power law-based porous FG rotor system. Sathujoda et al. [37] computed the natural and whirl frequencies of a power law-based functionally graded rotor-bearing system with porosities. However, there are studies related to the free vibration analysis of power law-based porous FG rotor systems in the literature. To the best of the authors' knowledge, no research has been reported on the dynamic behaviour of the exponential law-based FG rotor-bearing systems with porosities.

Since there is a substantial research gap in exploring the dynamic characteristics of porous FG rotor-bearing systems, the authors have addressed the vibrational behaviour of an exponential law-based FG rotor-bearing system with induced porosities. As the porosity aids to reduce the induced thermal stresses due to thermal gradients, it is crucial to investigate the dynamic behaviour of the porous FG rotor systems. The novelty of the present work is to explore and study the behaviour of the natural whirl frequencies and steady-state and transient vibration responses of a porous exponential law-based FG rotor-bearing system subjected to thermal loads. A unique two-nodded FG rotor element has been developed based on Timoshenko beam theory. The steady-state and transient time responses of a porous FG rotor-bearing system are simulated using the Houbolt time marching method. The corresponding frequency spectra are obtained using fast Fourier transform for various porosity indices, angular speeds, angular accelerations, and temperature gradients.

2. Materials and Methods

Material properties of the FGM depend on the gradation position, additive manufacturing and powder sintering, etc.

as it is cost-effective and consumes less energy to fabricate near-net shape products [30]. Due to the significant difference in solidification temperatures of metal and ceramic constituents, porosities are devised within inter-layers of the FGM. The major drawback of the sintering process is higher porosity induced in the components compared to other fabricating procedures. Therefore, a few scientists have explored the dynamic behaviour of the porous FG structures and rotors.

A review of the dynamic analysis of porous FG rotors has been carried out by Vaka et al. [31]. Bollf et al. [32] carried out a thermal buckling analysis on a porous FG Euler beam for different boundary conditions. The vibration behaviour of nonlinear FG beams with porosity is investigated by Heshmati and Daneshmand [33]. Askbas [34] studied the thermal effects on the free vibration behaviour of FG deep beams with porosity. Wattanasakulpong [35] computed the natural and whirl frequencies of a porous FG beam by using the differential transformation method. Batchu et al. [36] developed a finite element model of a power law-based porous FG rotor system. Sathujoda et al. [37] computed the natural and whirl frequencies of a power law-based functionally graded rotor-bearing system with porosities. However, there are studies related to the free vibration analysis of power law-based porous FG rotor systems in the literature. To the best of the authors' knowledge, no research has been reported on the dynamic behaviour of the exponential law-based FG rotor-bearing systems with porosities.

Since there is a substantial research gap in exploring the dynamic characteristics of porous FG rotor-bearing systems, the authors have addressed the vibrational behaviour of an exponential law-based FG rotor-bearing system with induced porosities. As the porosity aids to reduce the induced thermal stresses due to thermal gradients, it is crucial to investigate the dynamic behaviour of the porous FG rotor systems. The novelty of the present work is to explore and study the behaviour of the natural whirl frequencies and steady-state and transient vibration responses of a porous exponential law-based FG rotor-bearing system subjected to thermal loads. A unique two-nodded FG rotor element has been developed based on Timoshenko beam theory. The steady-state and transient time responses of a porous FG rotor-bearing system are simulated using the Houbolt time marching method. The corresponding frequency spectra are obtained using fast Fourier transform for various porosity indices, angular speeds, angular accelerations, and temperature gradients.

2. Materials and Methods

Material properties of the FGM depend on the gradation
is assumed as the gradation direction. The material gradation is controlled by gradation laws such as power law, exponential law, and sigmoid law.

The ceramic-metal FGM is comprised of ceramic—ZrO₂ on the outer layer of the FG shaft—and a metal—stainless steel (SS) in the inner core of the FG shaft—as shown in Figure 1(a). The detailed explanation of material property distribution in FGM, exponential law, modified exponential due to porosities, and exponential temperature distribution is discussed in the following subsections. A Python code has been developed to distribute the material properties and vary the temperature along the radial direction of the exponential law-based FG shaft.

2.1. Material Property Distribution in FGM. Generally, material properties depend on temperature. Since the metal and ceramic constituents are distributed along the gradation direction of the FG shaft, the material properties must also rely on the position. Therefore, the simple rule for mixtures of composites, the Voigt model [38], is used to accomplish the position-dependent material properties for a specific layer (Pₗ) of the FG shaft.

\[ Pₗ = P_mV_m + P_cV_c, \]  

(1)

V indicates volume fraction, P indicates material properties, and m and c represent metal and ceramic, respectively. Since there are only two components in the FG shaft, the sum of metal and ceramic volume fractions in a specific layer must be 1.

\[ V_m + V_c = 1. \]  

(2)

The nonlinear temperature distribution is assumed in the present work. This type of temperature distribution is addressed by Touloukian [39].

\[ P(T) = P_0(P_{r-1}T^{-1} + 1 + P_1T + P_2T^2 + P_3T^3). \]  

(3)

Reddy and Chin [40] listed the coefficients of the temperature, P₋₁, P₀, P₁, P₂, and P₃ in the literature. T is the temperature in Kelvin (K).

2.2. Material Distribution in the Exponential Law-Based FG Shaft. The accurate details of the volume fractions of constituents cannot be determined. Therefore, the material gradation laws have been used to assign the material properties to the FG shaft. Exponential law has been employed to distribute the material properties, such as Young’s modulus, thermal conductivity, Poisson’s ratio, etc., across the cross section of the FG shaft. The exponential law is mathematically controllable and has an easily estimated parameter (λ), so it is preferred over other gradation laws. The material properties of the exponential law-based FG shaft are expressed as follows:

\[ P^e(r, T) = P_m\exp[\lambda(r - R_i)], \]

\[ \lambda = \left[ \frac{\log(P_c/P_m)}{R_o - R_i} \right]. \]

(4)

where \( R_o \) is the outer radius and \( R_i \) is the inner radius of the exponential law-based FG shaft. \( P^e(r, T) \) is the position and temperature-dependent material property.

2.3. Modified Exponential Law due to Porosities. The porosity distributions are classified into even and uneven. In the present work, the porosities are assumed to be evenly distributed in the exponential law-based FG rotor, as shown in Figure 1(b). The simple rule for the mixture of composites is modified, as in equation (5), due to the presence of the porosities in the FG shaft.

\[ P_t = P_m\left(V_m - \frac{\alpha}{2}\right) + P_c\left(V_c - \frac{\alpha}{2}\right), \]  

(5)

where \( \alpha (\alpha \ll 1) \) is the volume fraction of porosity of a porous exponential law-based FG shaft. Therefore, the exponential law is modified into equation (6), as the material properties now depend on the volume fraction of porosity.

\[ P^e(r, \alpha, T) = P_m\exp[\lambda(r - R_i) - (P_c + P_m)\frac{\alpha}{2}], \]  

\[ \lambda = \left[ \frac{\log(P_c/P_m)}{R_o - R_i} \right]. \]

(6)

2.4. Exponential Temperature Distribution. Temperature is also varied along the radial direction of the exponential law-based FG shaft. Fourier heat conduction theory is used to calculate the temperature variation across the cross section of a circular exponential law-based FG shaft. Fourier heat conduction equation is expressed as follows:

\[ \frac{d}{dr}\left[ rK\frac{dT}{dr} \right] = 0. \]  

(7)

Heat generation is assumed to be zero in the present work. r is the radial coordinate, and K is the thermal conductivity. The boundary conditions are \( T = T_m \) when \( r = R_i \) and \( T = T_c \) when \( r = R_o \). Therefore, the temperature at any distance is expressed as follows:

\[ T = T_m + \frac{\Delta T(e^{-at} - 1)}{e^{-at} - 1}, \]  

(8)

where \( T_m \) is the temperature of metal and \( T_c \) is the temperature of ceramic. Temperature gradient \( \Delta T = T_m - T_c \), \( a = \ln(K_c/K_m) \) and \( t = [(r - R_i)/(R_o - R_i)] \). \( R_i = 0 \) for the solid shaft.

3. Formulation of an Exponential Law-Based FG Rotor-Bearing System with Porosities

A two-noded porous exponential law-based FG rotor element, shown in Figure 2(a), has been developed using the Timoshenko beam theory. The effects of transverse shear deformations and gyroscopic moments along with translational and rotational inertia are considered. Two translational and two rotational degrees of freedom are
considered on each node of the exponential law-based FG shaft. The FG shaft is discretised into 20 finite rotor elements. An exponential law-based FG rotor-bearing system is represented in Figure 2(b). The equations of motion of an exponential law-based FG rotor element, uniform steel disc, and linear isotropic bearings are discussed in the following subsections.

3.1. Equation of Motion of a Porous Exponential Law-Based FG Shaft Element. Nelson [41] developed an equation of motion of a uniform shaft using finite element formulation. The extended Hamilton’s principle is used to accompany the equations of kinetic energy, potential energy, and work functions to determine the equation of motion of a porous FG shaft element, which is as follows:

\[
\begin{bmatrix}
[M_{tr}^p] + [M_{ro}^p] & \tilde{q} \\
\tilde{q}^t & [q]
\end{bmatrix}
- \Omega [G^p] [q] + [K^p] [q] = [Q^p],
\]

where \([M_{tr}^p]\) is the translational mass element matrix, \([M_{ro}^p]\) is the rotational mass element matrix, \(\tilde{q}\) is a nodal acceleration vector, \(\Omega\) is the rotor speed in rad/s, \([G^p]\) is the gyroscopic moment element matrix, \([q]\) is a nodal velocity vector, \([K^p]\) is the stiffness element matrix, \([q]\) is a nodal displacement vector, and \([Q^p]\) is the external force vector of the shaft element. \([M_{tr}^p]\), \([M_{ro}^p]\), and \([K^p]\) are symmetric matrices, whereas \([G^p]\) is the skew-symmetric matrix. These elemental matrices are derived using the spatial constraint matrices associated with the translational and rotational shape functions similar to Nelson and McVaugh [42]. The expressions of all the elemental matrices are represented in Appendix (A.1).

3.2. Equation of Motion of a Uniform Steel Disc. The equation of motion of a uniform steel disc [42] is expressed as follows:

\[
\begin{bmatrix}
[M_{tr}^d] + [M_{ro}^d] & \tilde{q}^d \\
\tilde{q}^d & [q^d]
\end{bmatrix}
- \Omega [G^d] [q^d] + [K^d] [q^d] = [Q^d],
\]

where \([M_{tr}^d]\), \([M_{ro}^d]\), and \([G^d]\) are translational, rotational, and gyroscopic matrices of the uniform steel disc, respectively. \([Q^d]\) is the external loading vector.

3.3. Equation of Motion of Linear Bearings. The bearings used in the present work are isotropic. The equation of motion of linear isotropic bearings is expressed as follows:
3.4. Global Equation of Motion of an Exponential Law-Based FG Rotor-Bearing System with Porosities. As in the following equation, the global equation of the exponential law-based FG rotor-bearing system is formed by incorporating all the respective element matrices of the finite rotor element, uniform steel disc, and linear isotropic bearings for performing the dynamic analysis.

\[
[C_b] \{q_p\} + [K_b] \{q_b\} = \{Q_b\},
\]

where \([C_b]\) and \([K_b]\) are damping and stiffness matrix of the bearings, respectively. Since the bearings used in the present work are isotropic, \(c_{wb}^b = c_{rv}^b = 0\) and \(k_{wb}^b = k_{rv}^b = 0\).

4. Dynamic Response of an Exponential Law-Based FG Rotor-Bearing System with Porosities

The dynamic analysis of an exponential law-based porous FG rotor-bearing system has been performed by using steady-state and transient responses. Due to the unbalance eccentricity (ue) of the uniform steel disc, the unbalanced forces of the disc are exerted on the FG shaft. As the disc is located at the centre of the FG rotor, these unbalanced forces, in equations (13) and (14), of disc mass \((M_d)\) act on the mid-span of exponential law-based FG shaft along \(x\) and \(y\) directions at an angular displacement (\(\theta\)).

\[
\begin{align*}
F_x^d &= M_d \text{ue} \left\{ \ddot{\theta} \cos \theta + \dot{\theta} \sin \theta \right\}, \\
F_y^d &= M_d \text{ue} \left\{ \ddot{\theta} \sin \theta - \dot{\theta} \cos \theta \right\}.
\end{align*}
\]

The Houbolt time marching technique has been used to determine the steady-state and transient time responses, and the fast Fourier transform is used to obtain the corresponding frequency responses of an exponential law-based porous FG rotor-bearing system. The conditions for steady-state analysis of an exponential law-based FG rotor with porosities are as follows: angular displacement: \(\dot{\theta} = \Omega t\), angular velocity: \(\ddot{\theta} = \Omega\), and angular acceleration: \(\ddot{\theta} = 0\). The conditions for transient analysis of an exponential law-based FG rotor with porosities are as follows: angular displacement: \(\dot{\theta} = \Omega_0 t + 1/2 \alpha t^2\), angular velocity: \(\ddot{\theta} = \Omega_0 + \alpha t\), and angular acceleration: \(\ddot{\theta} = \alpha\).

5. Solution Procedure to Compute the Natural and Whirl Frequencies of an Exponential Law-Based FG Rotor-Bearing System with Porosities

The eigenfrequencies of an exponential law-based FG shaft with porosities are obtained by rearranging equation (12) into the following equation:

\[
Cb + Db = 0,
\]

\[
C = \begin{bmatrix} 0 & [M^p] \\ [M^p] & -\Omega[G^p] \end{bmatrix}, \quad D = \begin{bmatrix} -[M^p] & 0 \\ 0 & [K^p] \end{bmatrix}, \quad b = \begin{bmatrix} \dot{q} \\ q \end{bmatrix}.
\]

The solution of equation (15) can be assumed as

\[
b = b_0 e^{\lambda t}.
\]

After substituting equations (16) in (15), the eigenvalue problem can be expressed as

\[
(D^{-1}C + \lambda I)b_0 = 0.
\]

The computed eigenvalues are of the form

\[
\lambda_n(\Omega) = \xi_n(\Omega) \pm i\omega_n(\Omega).
\]

When \(\Omega = 0\), the natural frequencies are obtained by doing the inverse of the imaginary part, and when \(\Omega > 0\), the whirl frequencies are attained by the inverse of the imaginary part.

6. Validating the Developed FE Code to Perform the Dynamic Analysis

A finite element code has been developed using Python to perform free vibration analysis and steady-state and transient vibration analyses of an exponential law-based FG rotor-bearing system with porosities. A step-by-step code validation of exponential law material gradation, exponential temperature distribution, finite element formulation, porosity distribution, and the Houbolt method has been discussed in the following subsections. The whirl frequencies of an FG rotor system have been computed to validate the exponential law-based material gradation, exponential temperature distribution, and the finite element modelling of the FG rotor in subsection 6.1. Porosity validation and correctness of the Houbolt method have been ensured in subsections 6.2 and 6.3, respectively.
6.1. Whirl Frequencies of an Exponential Law-Based FG Rotor-Bearing System for Different Thermal Gradients. The whirl frequencies of a nonporous exponential law-based FG rotor-bearing system have been computed to validate the exponential law gradation, exponential temperature distribution, mass, stiffness, and gyroscopic matrices. The length and diameter of the SS-ZrO₂ shaft are 0.5 m and 0.02 m, respectively. The inner metallic core (stainless steel) temperature is considered at 300 K. The rotor speed is 4000 rpm. The computed results are tabulated in Table 1 and found to be in good agreement with the literature [24] as the percentage of the error is minimal. Therefore, it can be ensured that the FE code used in the present work is accurate.

6.2. Fundamental Frequencies of a Simply Supported FG Beam with Porosities. Since there is no work reported on exponential law-based porous FG beams/rotors, the nondimensional free vibration frequencies of a porous power law-based FG square cross-sectional beam have been validated with the literature [43] to ensure the correctness of the porosity formulation used in the present work. The metal used in the FG beam is SS, and Si₃N₄ is considered the ceramic in the FG beam. The slenderness ratio (L/h) of the FG beam is 20. The natural frequencies are tabulated in Table 2 and are computed for different volume fractions of porosity and power law indices. The calculated results are in accordance with the literature as the error percentage is meagre.

6.3. Dynamic Response of a Uniform Steel Rotor-Bearing System. The dynamic response of a uniform steel rotor-bearing system has been computed to validate the Houbolt method used in the present work. The steel disc is located at the mid-span of the rotor system, and the linear isotropic bearings support the steel shaft’s ends. The steady-state response has been validated with the literature [44], and the transient response has been validated with the literature [45]. Since the plotted graphs in Figures 3 and 4 are almost identical to the literature, it can be ensured that the Houbolt method used in the present work is accurate.

7. Results and Discussions

An exponential law-based FG rotor-bearing system with porosities is considered for dynamic vibration analysis. The inner metallic core and outer ceramic layer are comprised of SS and zirconia (ZrO₂), respectively. The exponential law-based porous FG shaft is discretised into 20 elements. An FE code has been developed in Python to perform the dynamic analysis. The natural and whirl frequencies are computed for different parameters such as volume fraction of porosity and temperature gradients. Steady-state and transient responses are developed to understand how porosity affects the dynamic behaviour of the exponential law-based FG rotor-bearing system. The steady-state and transient time responses are obtained using Houbolt time marching techniques, and the steady-state and transient frequency responses are simulated using fast Fourier transform. Rotor-bearing data used in the present work are represented in Table 3.

7.1. Effect of Volume Fraction of Porosity on Natural Frequencies for Different Temperatures. The natural frequencies of an exponential law-based FG rotor-bearing system with porosities are plotted for different thermal gradients in Figure 5. It has been observed that the natural frequencies of the rotor system decrease with an increase in the volume fraction of porosity. The reason behind this phenomenon is that porosities affect Young’s modulus and density of the FG shaft. Consequently, the stiffness and mass get reduced. Therefore, the ratio of stiffness and mass decreases with the increase in porosity.

The material properties are also affected due to the temperature rise in the FG shaft. Therefore, Young’s modulus is furthermore reduced with the rise in a temperature gradient. So, it can be observed from Figure 5 that the natural frequencies at ΔT = 300 K are less than the natural frequencies at ΔT = 0 K, and natural frequencies at 600 K and 900 K are even lesser than the natural frequencies at ΔT = 300 K.

7.2. Effect of Volume Fraction of Porosity on Whirl Frequencies. Whirl frequencies of an exponential law-based porous FG rotor-bearing system have been calculated for different volume fractions of porosity and thermal gradients at rotor speed (Ω) = 4000 rpm. The inner metallic core of the FG shaft is considered at 300 K. However, the outer ceramic layer’s temperature can be 300 K, 600 K, and 1200 K, respectively. Whirl frequencies decrease with an increase in volume fractions of porosity and thermal gradients for the same reasons affirmed in subsection 7.1. The split between the first forward and backward whirls are undetectable in the plots. The backward whirl (BW) and forward whirl (FW) are set about to split with the increase in rotor speed. The Campbell diagrams are plotted in Figure 6 for α = 0 and 0.2 at different temperature gradients (ΔT). The first forward and backward whirl frequencies are decreased with an increase in volume fraction of porosity for the same reason stated in subsection 7.1.

7.3. Steady-State Responses of an Exponential Law-Based FG Rotor-Bearing System with Porosities. Steady-state time responses of an exponential law-based nonporous and porous FG rotor-bearing system have been simulated at a rotor speed of 60 rad/s (9.54 Hz) using the Houbolt method. The corresponding frequency responses have been obtained using fast Fourier transform (FFT). As it is difficult to comprehend the presence of porosity from steady-state time responses (Figures 7(a) and 7(c)), the frequency responses are computed to investigate the presence of porosity. The first and second peaks in Figures 7(b)–7(d) indicate rotor speed (9.54 Hz) and critical speed of the exponential law-based FG shaft, respectively. The maximum amplitude of the frequency response is increased when the porosity is present, as evident from Figures 7(b) and 7(d). Additionally, the
critical speed of the rotor-bearing system decreases with an increase in the volume fraction of porosity, and it is clearly visible that the frequency peaks at a critical speed shift towards the left, as shown in Figure 8, which confirms the presence of porosities in the FG shaft.

Frequency responses of an exponential law-based FG rotor-bearing system have been simulated for different temperature gradients at $\alpha = 0.2$. Since the rotor speed is the same in Figure 9, the first peak (maximum amplitude) of all the frequency responses is formed at 9.54Hz. However, the second peak (critical speed) is formed at different frequencies as the critical speed changes with an increase in temperature. The critical speed shifts towards the left on the steady-state responses with an increase in volume fraction of porosity as well as with the rise in a temperature gradient. Therefore, for investigating the presence of porosities in the thermally loaded FG rotor-bearing system, the temperature gradient of the rotor system must be considered.

### Table 1: Whirl frequencies of a nonporous FG rotor-bearing system.

<table>
<thead>
<tr>
<th>Modes</th>
<th>$\Delta T = 0$</th>
<th>$\Delta T = 600$</th>
<th>$\Delta T = 0$</th>
<th>$\Delta T = 600$</th>
<th>$\Delta T = 0$</th>
<th>$\Delta T = 600$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 BW</td>
<td>24.063</td>
<td>23.211</td>
<td>24.066</td>
<td>23.214</td>
<td>0.0125</td>
<td>0.0129</td>
</tr>
<tr>
<td>1 FW</td>
<td>24.125</td>
<td>23.292</td>
<td>24.128</td>
<td>23.295</td>
<td>0.0124</td>
<td>0.0129</td>
</tr>
<tr>
<td>2 BW</td>
<td>78.646</td>
<td>77.348</td>
<td>78.656</td>
<td>77.358</td>
<td>0.0127</td>
<td>0.0129</td>
</tr>
<tr>
<td>2 FW</td>
<td>128.869</td>
<td>127.913</td>
<td>128.886</td>
<td>127.930</td>
<td>0.0132</td>
<td>0.0133</td>
</tr>
<tr>
<td>3 BW</td>
<td>270.226</td>
<td>246.571</td>
<td>270.262</td>
<td>246.602</td>
<td>0.0133</td>
<td>0.0126</td>
</tr>
<tr>
<td>3 FW</td>
<td>293.155</td>
<td>266.928</td>
<td>293.193</td>
<td>266.962</td>
<td>0.0130</td>
<td>0.0127</td>
</tr>
</tbody>
</table>

### Table 2: Free vibration frequencies of a porous power law-based FG beam.

<table>
<thead>
<tr>
<th>$k$</th>
<th>Present</th>
<th>[43]</th>
<th>Error (%)</th>
<th>Present</th>
<th>[43]</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4.5283</td>
<td>4.5158</td>
<td>0.2</td>
<td>3.9695</td>
<td>3.9583</td>
<td>0.28</td>
</tr>
<tr>
<td>0.1</td>
<td>4.5947</td>
<td>4.5821</td>
<td>0.27</td>
<td>3.9621</td>
<td>3.9509</td>
<td>0.28</td>
</tr>
<tr>
<td>0.2</td>
<td>4.6806</td>
<td>4.6678</td>
<td>0.27</td>
<td>3.9516</td>
<td>3.9406</td>
<td>0.28</td>
</tr>
</tbody>
</table>

### Figure 3: Steady-state response of a uniform steel rotor-bearing system.

### Figure 4: Transient response of a uniform steel rotor-bearing system.

Critical speed of the rotor-bearing system decreases with an increase in the volume fraction of porosity, and it is clearly visible that the frequency peaks at a critical speed shift towards the left, as shown in Figure 8, which confirms the presence of porosities in the FG shaft.

Frequency responses of an exponential law-based FG rotor-bearing system have been simulated for different temperature gradients at $\alpha = 0.2$. Since the rotor speed is the same in Figure 9, the first peak (maximum amplitude) of all the frequency responses is formed at 9.54Hz. However, the second peak (critical speed) is formed at different frequencies as the critical speed changes with an increase in temperature. The critical speed shifts towards the left on the steady-state responses with an increase in volume fraction of porosity as well as with the rise in a temperature gradient. Therefore, for investigating the presence of porosities in the thermally loaded FG rotor-bearing system, the temperature gradient of the rotor system must be considered.

### 7.4. Transient Responses of an Exponential Law-Based FG Rotor-Bearing System with Porosities.

It is also equally important to analyse the dynamic behaviour of a porous FG rotor passing through the critical speed. In the previous subsection, porosity has been investigated when a rotor is operated at a uniform speed. In order to understand the way to study the porosity in an accelerated rotor-bearing system, the transient responses of an exponential law-based FG rotor-bearing system have been simulated. Transient time responses have been computed at an angular acceleration of 30 rad/s² using the Houbolt method, and the corresponding frequency responses are calculated using FFT. As it is complicated to investigate the presence of porosities from Figures 10(a) and 10(c), the corresponding frequency responses have been used to investigate the presence of...
Table 3: Rotor-bearing data.

<table>
<thead>
<tr>
<th>Shaft Properties</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length ($L$)</td>
<td>0.5 m</td>
</tr>
<tr>
<td>Diameter ($D$)</td>
<td>0.02 m</td>
</tr>
<tr>
<td>Disc Location</td>
<td>Mid-span</td>
</tr>
<tr>
<td>Mass ($m$)</td>
<td>5.5 kg</td>
</tr>
<tr>
<td>Polar moment of inertia ($I_p$)</td>
<td>0.01546 kg m$^2$</td>
</tr>
<tr>
<td>Diametral moment of inertia ($I_d$)</td>
<td>0.00773 kg m$^2$</td>
</tr>
<tr>
<td>Unbalance eccentricity ($e$)</td>
<td>0.1 mm</td>
</tr>
<tr>
<td>Bearing stiffness (rigid bearings)</td>
<td>$10^5$ N/m</td>
</tr>
<tr>
<td>Damping</td>
<td>100 Ns/m</td>
</tr>
<tr>
<td>Material Properties</td>
<td></td>
</tr>
<tr>
<td>Stainless steel (SS)</td>
<td>207.8 GPa</td>
</tr>
<tr>
<td>Zirconia (ZrO$_2$)</td>
<td>168 GPa</td>
</tr>
<tr>
<td>Density (kg/m$^3$)</td>
<td>8166</td>
</tr>
<tr>
<td>Zirconia (ZrO$_2$)</td>
<td>5700</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.3</td>
</tr>
<tr>
<td>Zirconia (ZrO$_2$)</td>
<td>0.24</td>
</tr>
</tbody>
</table>

Figure 5: Influence of temperature gradients on natural frequencies.

Figure 6: Continued.
porosity in an exponential law-based FG rotor-bearing system. The maximum amplitude of the porous FG rotor-bearing system (Figure 10(d)) is higher than the nonporous rotor system (Figure 10(b)). The critical speeds of the FG rotor-bearing system also decrease with an increase in volume fraction of porosity, as shown in Figure 11. Therefore, it is helpful to confirm the presence of porosity from the transient frequency responses. Unlike steady-state FFT, transient FFT has only one peak, as shown in Figures 10(b) and 10(d), that is the critical speed of the rotor. The critical speed, as shown in Figure 12, of the exponential law-based FG rotor-bearing system shifts towards the left with an increase in a temperature gradient. Therefore, to confirm the presence of porosity in a thermally loaded accelerated FG rotor-bearing system, the temperature gradient of the FG shaft must be considered.

<table>
<thead>
<tr>
<th>Spin speed Ω (rpm)</th>
<th>Frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>23.597</td>
</tr>
<tr>
<td>50</td>
<td>23.602</td>
</tr>
<tr>
<td>100</td>
<td>23.597</td>
</tr>
<tr>
<td>150</td>
<td>80.708</td>
</tr>
<tr>
<td>200</td>
<td>123.29</td>
</tr>
<tr>
<td>250</td>
<td>265.27</td>
</tr>
<tr>
<td>300</td>
<td>266.605</td>
</tr>
<tr>
<td>350</td>
<td>272.154</td>
</tr>
<tr>
<td>400</td>
<td>273.349</td>
</tr>
</tbody>
</table>

Figure 6: Campbell diagrams of an exponential law-based FG rotor-bearing system for (a) $\alpha = 0$ at $\Delta T = 0$ K; (b) $\alpha = 0.2$ at $\Delta T = 0$ K; (c) $\alpha = 0$ at $\Delta T = 300$ K; (d) $\alpha = 0.2$ at $\Delta T = 300$ K; (e) $\alpha = 0$ at $\Delta T = 900$ K; and (f) $\alpha = 0.2$ at $\Delta T = 900$ K.
Figure 7: Steady-state (a) time signal for $\alpha = 0$, (b) frequency response for $\alpha = 0$, (c) time signal for $\alpha = 0.3$, and (d) frequency response for $\alpha = 0.3$.

Figure 8: Steady-state frequency responses of an exponential law-based FG porous FG rotor-bearing system for different porosity volume fractions.

Figure 9: Steady-state frequency responses of an exponential law-based FG porous FG rotor-bearing system for $\alpha = 0.2$ at $\Delta T = 0$ K, $\Delta T = 300$ K, $\Delta T = 600$ K, and $\Delta T = 900$ K.
Figure 10: Transient (a) time signal for $\alpha = 0$, (b) frequency response for $\alpha = 0$, (c) time signal for $\alpha = 0.3$, and (d) frequency response for $\alpha = 0.3$.

Figure 11: Transient frequency responses of an exponential law-based FG porous FG rotor-bearing system for different porosity volume fractions.
8. Conclusions

The vibrational behaviour of an exponential law-based FG rotor-bearing system with porosities has been studied using the finite element method in the present work. Material properties are assigned along the radial direction of the shaft based on exponential law, and the exponential temperature distribution (ETD) is used for temperature variation. A two-noded exponential law-based porous FG shaft element has been developed to perform the dynamic analysis. The influence of porosity on natural and whirl frequencies and steady-state and transient responses has been investigated for various parameters, such as volume fraction of porosity, temperature gradient, etc., and essential conclusions from the study are mentioned below:

(i) Natural and whirl frequencies decrease with an increase in volume fraction of porosity and temperature gradient due to the reduction of material properties such as Young’s modulus and density.

(ii) The maximum amplitude of the steady-state frequency responses increases, and the critical speed of the porous exponential law-based FG rotor-bearing system shifts towards the left (decreases) with an increase in volume fraction of porosity. Therefore, the presence of porosities can be confirmed when a rotor is rotating at a constant speed.

(iii) Similar to the steady-state responses, when a rotor is in accelerated angular motion, that is when a rotor passes through the critical speed, the critical speed decreases, whereas the maximum amplitude increases with an increase in volume fraction of porosity.

(iv) The critical speeds of the exponential law-based FG rotor system shift towards the left with an increase in temperature gradients in both the steady-state and transient frequency responses, therefore, confirming the presence of porosity in the rotor system.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this article.

References


