Hindawi
Mathematical Problems in Engineering
Volume 2022, Article ID 2484838, 17 pages
https://doi.org/10.1155/2022/2484838

Research Article

Sports Town Development Strategy Driven by the Sports Industry and Multi-Industry Integration

Kui Deng

School of Physical Education, University of South China, Hengyang, China

Correspondence should be addressed to Kui Deng; 3628322745@qq.com

Received 22 October 2021; Revised 26 March 2022; Accepted 25 April 2022; Published 25 May 2022

Academic Editor: Andras Szekrenyes

Copyright © 2022 Kui Deng. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Many sports towns have emerged in China in recent years, and the prerequisite for their healthy development requires a suitable investment strategy. Therefore, this article constructs an optimal decision model in which the sports industry is the primary industry, promoting the development of other industries. The results show that subsidies from higher levels of government can incentivize sports towns to invest in sports capital; there is a complementary relationship between the sports capital and other capital investments and sports product marketing efforts; as the subsidy rate increases, profits of the towns and the social benefits created show an inverted U-shape. Choosing the correct subsidy rate is essential for both the towns and their parent government. The research framework and results provide a valuable tool for sports towns and their parent governments to make decisions and adapt their strategies to changing circumstances timely.

1. Introduction

The development of the regional economy needs to select industries that meet the needs of the times according to the region's characteristics [1]. Since the reform and opening-up, China's industrial structure has been upgrading, and the regional economy also develops rapidly [2]. Especially in recent years, the development strategies of rural revitalization, industrial poverty alleviation, and new urbanization have led to the development of the characteristic town movement in the vast rural areas of China [3]. In 2016, the urban and rural housing construction department, the finance department, and the National Development and Reform Commission of the People's Republic of China jointly issued the Notice on the Cultivation of Characteristic Towns, and the release of this document received positive attention and response from the governments of provinces and autonomous regions. In 2017, the Ministry of Housing and Urban-Rural Development, the National Development and Reform Commission, the Ministry of Environmental Protection, and the Ministry of State-owned Assets jointly issued "Several Opinions on Regulating and Promoting the Construction of Characteristic Towns and Small Towns." Since sports towns or sports and leisure towns belong to the category of characteristic towns, and at the same time, it fits the spirit of the National Fitness Plan (2016–2020) issued by the Chinese State Council in 2016, government departments at all levels highly value the construction of sports characteristic towns. At the central government level, in May 2017, the State General Administration of Sports issued the Notice on Promoting the Construction of Sports and Leisure Characteristic Towns, officially launching the construction of sports characteristic towns. On August 9, 2017, the State General Administration of Sports announced the pilot list of the first 96 sports characteristic towns, kicking the construction of sports characteristic towns into high gear. The construction of these characteristic towns can meet the needs of the masses for leisure and entertainment, promote urbanization, and improve the ecological environment. The number of sports towns nationwide has reached about 1,000 [4]. The investment amount of these towns is as little as billions of yuan and tens of billions of yuan, which shows that all circles highly expect sports towns [5]. In order to improve investment efficiency and achieve high-quality development, the construction of sports towns needs to be integrated with other industries suitable for local

development to achieve multi-industry integration driven by the sports industry.

The construction of sports towns has taken off in China, and at the same time, it has gained much attention from the academic community. Among the many research pieces of literature that have emerged, two of the research streams are closely related to this article.

The first research stream relevant to this article is about the concept, development history, and impact of sports towns on the local economy. Zhang [6] and Song et al. [7] analyze sports towns' concept, connotation, and development path. Wilson and Hat [8], Beyazit and Koc [9], Cubizolles [10], Barbara [11], and Psczobut [12] study the development of sports-related activities in some small towns in Western countries such as the United States and the United Kingdom. Pillay and Bass [13] and some other scholars have analyzed the positive impact of sports industry development on the local economy; for example, Pillay and Bass [13] study the performance of the city of Indianapolis, USA, in using sports to revitalize its declining downtown (downtown). Rosentraub [14] analyzed the success of sports-dependent economic development strategies in real growth through six classic cases, including Indianapolis, San Diego, and Los Angeles. Chapin [15] compares the transformation of Camden Yards and Gateway from 1980 to 2000 based on planning and GIS data and found that sports played a role in it. Shonk and Chelladurai [16] argue that hosting sporting events can increase a destination's image development and exposure, thus bringing multiple benefits to the location. Similarly, Russo and Zarick [17] believe that developing a sports industry has economic and social benefits to the local area. Desai and Vahed [18] find that the sports + tourism development model can significantly grow revenue from tourists and tourism. Yu [19] shows that sports industry development has a vital role in poverty alleviation.

The second research stream relevant to our article is the study on the development strategies of sports towns. Xu and Yin [4] argue that Chinese sports towns should establish a complete industrial chain and reasonably choose "sports + residence," "sports + culture," "sports + technology," "sports + IP" and other development modes. Xian and Cheng [20] believe that sports towns should fully use the characteristic local resources and avoid homogenous development with other sports towns. Wang et al. [1] show that the construction of sports towns should pay attention to industrial integration and urban-rural coordination. Si and Wang [21] find that power and capital are the main influencing factors for the development of sports towns. The construction of small towns should follow the logic of capital circulation and should strengthen the investment in the public service area of the town. Zhu et al. [5] take the sports town of Neishui Town as an example to study the problems in the development of the current sports town and propose solutions. Ma et al. [22] compare the construction of sports towns in China and other countries to provide foreign experience for the development of sports towns in China. Pouder et al. [23] provide strategies for developing the sports tourism market by interviewing relevant personnel from destination marketing organizations. Hao [24] analyzes the problems in developing sports towns in China, taking the development of sports towns in Shanxi Province, China, as an example, and propose strategies to solve these problems.

In summary, the rise of sports towns has attracted the attention of the academic community, and scholars have conducted much research on various aspects such as the concept and role of sports towns, development goals, problems, and countermeasures, which have had a positive impact on the construction of sports towns. As investment decisions have a great relationship with the success or failure of sports town construction, the timing of investment, the scale of investment, and the type of investment all directly impact the future development of the investment town. In particular, the investment in a sports town is enormous, and if the investment fails, it will have a significant negative impact on the town's economy. For example, the Chengdu Longtan Water Town in China officially opened for business in April 2013, with a total investment of RMB 2 billion and a conservative estimate of 130,000 visitors in the first three days of opening. However, today, the prosperity was short-lived, with only a few businesses operating, mostly ordinary snacks, and most shops closed down. As a result of the failed investment, the town was not only built without producing the expected returns but also left the local government and a large number of owners in debt (https://www.People.com. Why the Chengdu Longtan Water Town, an antique town with an investment of about 2 billion yuan, was short-lived. 11 February 2019. https://travel.people.com.cn/n1/2019/0211/c41570-30616737.html). It shows that investment decisions are of great importance to constructing sports towns. Therefore, this article takes the investment decision of a sports town as the subject of research.

The analysis of economic dynamics has always been of great interest in mainstream economics, and we encounter many dynamic optimization problems in the economic field. It has become increasingly important in solving economic problems. Given that investment is a dynamic process, capital is formed through the continuous accumulation of investment. This article uses a dynamic optimization approach to study sports towns' investment decision-making. This approach has been widely used in economic management decision-making and has achieved significant results. For example, Yu et al. [25] used this approach to study the joint optimization and coordination of charging facility investment and pricing in an automotive retail supply chain and provided an optimal joint optimization solution for charging facility investment and pricing in an automotive supply chain. Zhang and Zhang [26] provide an optimal portfolio strategy for a power company in a renewable energy multipolicy scenario using a dynamic optimization approach. Hou and Yao [27] proposed a dynamic investment decision model for highway projects, which is widely used in the decision-making of highway project investment.

The remainder of the article is as follows. First, the model raises in Section 2. Then, in Section 3, we solve the steady-state equilibrium results without and with upper-level government subsidies. Next, we analyze and discuss the results in Section 4. Finally, Section 5 summarizes the article.

2. Basic Model

Consider a small town that wants to develop itself into a sports town. It invests in selected sports and sports-related projects according to its resource endowment in the construction process. The selected sports projects include (1) sports events, such as the Wimbledon Tennis and Sports Town in the United Kingdom and the Kunming Hot Spring Peninsula International Tennis Center in Kunming, China; (2) outdoor activities, such as the sports and leisure town of Shengtian Lake, Shanxi, China; and (3) sports and leisure, such as Hunan Qianlong Lake International Leisure Sports Town, China. Suppose the number of sports products that the town can offer at time t is $q(t)$. One can interpret this figure as the number of sports consumers that the town can accommodate, and the town can increase it by offering investments in sports products.

The demand for sports products in the town is assumed to obey the following function:

$$q(t) = a - p(t) + \alpha_1 k_1(t) + \alpha_2 k_2(t) + \beta m(t), \quad (1)$$

where a denotes the potential market size. $p(t)$ describes the price of sports products, for example, the amount of money required per person per day. $k_1(t)$ represents the amount of sports capital, and $k_2(t)$ is the amount of capital other than sports capital (other capital for short). Capital other than sports capital is the capital provided by the sports town to support sports capital and the capital that relies on sports capital to develop other industries, including tourism industry, cultural industry, education, information industry, health industry, and real estate industry. The synergistic investment of sports capital and other capital is in line with the current development model of sports towns such as "sports + residence," "sports + culture," and "sports + technology." $m(t)$ stands for the marketing efforts for sports products, including activities such as advertising sports products and other facilities and soliciting support from higher levels of government. α_1, α_2, and β represent the marginal impact of sports capital, other capital, and marketing effort on demand for sports products, respectively. They are related to the town's resource endowment. That is, if there is a high degree of integration between sports capital and other capital choices and the town's resource endowment, the marginal contribution of capital will be significant, and the marginal contribution of marketing effort will be correspondingly higher.

Equation (1) holds for the following reasons. First, consistent with the general demand function, quantity demanded negatively correlates with price. Second, the amount of sports capital determines both the amount of sports products supplied, positively affecting the demand for sports products. After all, the more extensive and more diverse the sports product offered, the more popular it will be with consumers. For example, consumers prefer a supermarket to a small one. For example, with the increase in investment scale and volume, the annual number of visitors to the international leisure and sports town of Qianlong Lake in Hunan, China, has increased year by year, from 800,000 in 2016 to 1 million in 2017 and 2 million in 2020 (Qianlonghu official website: https://www.qianlonghu.com/). Third, the development of sports supporting industries such as culture, tourism, health, information, and real estate also positively impacts the consumption of sports products. For example, Queenstown in New Zealand takes advantage of the terrain to carry out different types of racetrack skiing, mountaintop parachuting, canyon swing, bungee jumping, and other thrilling and exciting projects. At the same time, the town hotel has luxurious service facilities, top-class service, and luxury service products. In the thrilling adventure sports, tourists and explorers can enjoy a comfortable and perfect experience, so as to linger. The development of a top-tier service sector has a considerable impact on the consumption of sports products in Queenstown [22]. In addition, there is a consensus that marketing efforts are a positive factor in driving consumption.

Sports capital and other capitals have the commonality of capital. Investments can increase them, and losses in the process of use can decrease them. Assume that they obey the following dynamic processes, respectively:

$$\dot{k}_1(t) = u_1(t) - \delta_1 k_1(t), \quad (2)$$

$$\dot{k}_2(t) = u_2(t) - \delta_2 k_2(t), \quad (3)$$

where $u_1(t)$ and $u_2(t)$ are the town's investments in sports capital and other capital, respectively. Meanwhile, δ_1 and δ_2 represent their depreciation rates.

Consistent with the widely used investment cost functions such as Yi et al. [28] and Wei et al. [29], we assume that the investment costs of sports and other capital are incremental quadratic cost functions and have the following forms $u_1^2(t)/2$ and $u_2^2(t)/2$, respectively. Furthermore, for simplicity, we assume that the marginal cost of providing the sports product is zero, or the price of the sports product is the net price after deducting the marginal cost.

Next, we analyze the revenue of the sports town. The first part of the revenue is the revenue from the sports products $p(t)q(t)$. Then, the second part of the revenue is the revenue from other sports-related industries such as culture, recreation, tourism, information, and real estate. The amount of it is related to the consumption of sports products and the size of other capital. Because first, if the more people come to consume sports products, the greater the consumption of other products, and second, if the more people consume sports products combined with the more significant the size of other capital, the town can multiply the net income obtained from other industries. The larger the size of the capital in other industries, the more the town may offer consumers other consumption items than sports. Therefore, it is reasonable to assume that the town's net benefits in other industries have the following multiplier form $\phi q(t) k_2(t)$, where ϕ is a constant, and it specifies the marginal impact of sports product consumption and other capital on the net returns of other industries.

In summary, the objective of the sports town is to maximize the discounted value of future net benefits over the continuous-time, $t \in [0, \infty)$, by choosing its sports product

prices, investment amount in sports capital and other capital investments, and sports product marketing efforts. Furthermore, if the higher government does not subsidize the sports investment in the sports town, it has the following objective function:

$$\max_{p,u_1,u_2,m} \pi(t) = \int_0^{\infty} e^{-\rho t}\left[p(t)q(t) + \phi q(t)k_2(t) - \frac{u_1^2(t)}{2} - \frac{u_2^2(t)}{2} - \frac{m^2(t)}{2}\right]\mathrm{d}t,$$
$$\text{s.t.} \begin{cases} \dot{k}_1(t) = u_1(t) - \delta_1 k_1(t), \\ \dot{k}_2(t) = u_2(t) - \delta_2 k_2(t), \end{cases} \tag{4}$$

where $\rho > 0$ denotes the risk-free interest rate.

In order to promote the development of sports towns, the Chinese government is currently taking a series of support measures for selected sports towns, of which subsidizing investment in sports products is its essential measure. Therefore, this article will analyze the role of government (mainly the central and provincial governments) subsidies to examine the optimal level of support from the higher level of government while analyzing the optimal strategy for sports towns.

If the subsidy rate of the higher-level government for sports investment in this sports town is τ, then the actual investment cost of sports capital decreases as $(1-\tau)u_1^2(t)$. Then, the objective function of the town becomes

$$\max_{p,u_1,u_2,m} \pi(t) = \int_0^{\infty} e^{-\rho t}\left[p(t)q(t) + \phi q(t)k_2(t) - \frac{(1-\tau)u_1^2(t)}{2} - \frac{u_2^2(t)}{2} - \frac{m^2(t)}{2}\right]\mathrm{d}t,$$
$$\text{s.t.} \begin{cases} \dot{k}_1(t) = u_1(t) - \delta_1 k_1(t), \\ \dot{k}_2(t) = u_2(t) - \delta_2 k_2(t). \end{cases} \tag{5}$$

Next, we discuss the investment strategy of this sports town in two scenarios: without and with higher government subsidies.

3. Game Equilibrium and Policy Implications

In this section, we first investigate the investment strategies of sports towns without higher-level government subsidies to get a benchmark for comparison and then analyze the investment strategies with higher-level government subsidies.

3.1. Investment Strategies without Subsidies. Without subsidies from the higher-level government, from equation (4), the current-value Hamiltonian of the town is

$$\begin{aligned} H(p,m,u_1,u_2,k_1,k_2) &= \left[p(t)q(t) + \phi q(t)k_2(t) - \frac{u_1^2(t)}{2} - \frac{u_2^2(t)}{2} - \frac{m^2(t)}{2}\right], \\ &\quad + \lambda_1(t)\left[u_1(t) - \delta_1 k_1(t)\right] + \lambda_2(t)\left[u_2(t) - \delta_2 k_2(t)\right] \\ &= \left[\mathrm{ap}(t) - p^2(t) + \alpha_1 p(t)k_1(t) + \alpha_2 p(t)k_2(t) + \beta m(t)p(t) + a\phi k_2(t) - \phi p(t)k_2(t)\right. \\ &\quad \left. + \alpha_1\phi k_1(t)k_2(t) + \alpha_2\phi k_2^2(t) + \beta\phi m(t)k_2(t) - \frac{u_1^2(t)}{2} - \frac{u_2^2(t)}{2} - \frac{m^2(t)}{2}\right] \\ &\quad + \lambda_1(t)\left[u_1(t) - \delta_1 k_1(t)\right] + \lambda_2(t)\left[u_2(t) - \delta_2 k_2(t)\right], \end{aligned} \tag{6}$$

where $\lambda_1(t)$ and $\lambda_2(t)$ are dynamic costate variables associated with state variables $k_1(t)$ and $k_2(t)$.

From the first-order conditions and costate conditions of current-value Hamiltonian (6), we get

$$\frac{\partial H}{\partial u_1(t)} = \lambda_1(t) - u_1(t) = 0, \tag{7}$$

$$\frac{\partial H}{\partial u_2(t)} = \lambda_2(t) - u_2(t) = 0, \tag{8}$$

$$\frac{\partial H}{\partial p(t)} = a - 2p(t) + \alpha_1 k_1(t) + (\alpha_2 - \phi)k_2(t) + \beta m(t) = 0, \tag{9}$$

$$\frac{\partial H}{\partial m(t)} = \beta p(t) + \beta\phi k_2(t) - m(t) = 0, \tag{10}$$

$$\dot{\lambda}_1(t) = \rho\lambda_1(t) - \frac{\partial H}{\partial k_1(t)} = (\rho + \delta_1)\lambda_1(t) - \alpha_1 p(t) - \alpha_1\phi k_2(t), \tag{11}$$

$$\begin{aligned}\dot{\lambda}_2(t) = \rho\lambda_2(t) - \frac{\partial H}{\partial k_2(t)} &= (\rho + \delta_2)\lambda_2(t) \\ &- [\alpha_2 p(t) + a\phi - \phi p(t) + \alpha_1\phi k_1(t) \\ &+ 2\alpha_2\phi k_2(t) + \beta\phi m(t)].\end{aligned} \tag{12}$$

Rectifying equations (7)–(12), obtain the following dynamic system:

$$\begin{cases} \dot{k}_1(t) = u_1(t) - \delta_1 k_1(t), \\ \dot{k}_2(t) = u_2(t) - \delta_2 k_2(t), \\ \dot{\lambda}_1(t) = (\rho + \delta_1)\lambda_1(t) - \alpha_1 p(t) - \alpha_1\phi k_2(t), \\ \dot{\lambda}_2(t) = (\rho + \delta_2)\lambda_2(t) - [\alpha_2 p(t) + a\phi \\ \qquad -\phi p(t) + \alpha_1\phi k_1(t) + 2\alpha_2\phi k_2(t) + \beta\phi m(t)], \\ \dot{u}_1(t) = \dot{\lambda}_1(t), \\ \dot{u}_2(t) = \dot{\lambda}_2(t), \\ \dot{p}(t) = \dfrac{\alpha_1}{(2-\beta^2)}u_1(t) + \dfrac{a\delta_1}{(2-\beta^2)} - \dfrac{2\delta_1}{(2-\beta^2)}p(t) \\ \qquad + \left[\dfrac{\delta_1(\alpha_2 - \phi)}{(2-\beta^2)} - \dfrac{\delta_2(\alpha_2 - \phi + \beta^2\phi)}{(2-\beta^2)}\right]k_2(t) \\ \qquad + \dfrac{\beta\delta_1}{(2-\beta^2)}m(t) + \dfrac{(\alpha_2 - \phi + \beta^2\phi)}{(2-\beta^2)}u_2(t), \\ \dot{m}(t) = \dfrac{\alpha_1\beta}{(2-\beta^2)}u_1(t) - \dfrac{\alpha_1\beta\delta_1}{(2-\beta^2)}k_1(t) \\ \qquad + \dfrac{\beta(\alpha_2 + \phi)}{(2-\beta^2)}u_2(t) + \dfrac{\beta\delta_2(\alpha_2 + \phi)}{\phi(2-\beta^2)}p(t) \\ \qquad - \dfrac{\delta_2(\alpha_2 + \phi)}{\phi(2-\beta^2)}m(t). \end{cases} \tag{13}$$

Under the steady-state equilibrium conditions of the dynamic system (13), $\dot{k}_1(t) = \dot{k}_2(t) = \dot{\lambda}_1(t) = \dot{\lambda}_2(t) = \dot{u}_2(t) = \dot{u}_1(t) = \dot{p}(t) = \dot{m}(t) = 0$, we obtain the steady-state solution identified by the superscript "NC" for the sports town in the absence of higher-level government subsidies as follows:

$$\begin{aligned} p^{NC} &= \frac{a\phi}{(\alpha_2 + \phi)} + \frac{a\delta_2(\rho + \delta_2)\left[\alpha_1^2\phi + \delta_1(\rho + \delta_1)(\alpha_2 - \phi + \beta^2\phi)\right]}{A_1(\alpha_2 + \phi)}, \\ k_1^{NC} &= \frac{a\alpha_1\delta_2(\rho + \delta_2)}{A_1}, \\ k_2^{NC} &= -\frac{a}{(\alpha_2 + \phi)} + \frac{a\delta_2(\rho + \delta_2)\left[\delta_1(\rho + \delta_1)(2 - \beta^2) + \alpha_1^2\right]}{A_1(\alpha_2 + \phi)}, \\ u_1^{NC} &= \frac{a\alpha_1\delta_1\delta_2(\rho + \delta_2)}{A_1}, \\ u_2^{NC} &= -\frac{a\delta_2}{(\alpha_2 + \phi)} + \frac{a\delta_2^2(\rho + \delta_2)\left[\delta_1(\rho + \delta_1)(2 - \beta^2) - \alpha_1^2\right]}{A_1(\alpha_2 + \phi)}, \\ m^{NC} &= \frac{a\beta\delta_1\delta_2(\rho + \delta_1)(\rho + \delta_2)}{A_1}, \end{aligned} \tag{14}$$

where $A_1 = \delta_2\delta_1(\rho + \delta_1)(\rho + \delta_2)(2 - \beta^2) - \delta_1(\rho + \delta_1)(\alpha_2 + \phi)^2 - \alpha_1^2\delta_2(\rho + \delta_2)$.

We have obtained the optimal strategy for the town and the corresponding optimal capital level in the absence of higher government subsidies. Next, we investigate the optimal strategy when government subsidies are available.

3.2. Investment Strategies with Subsidies. Since the construction of sports and leisure towns is in line with the concept of health for all, it can promote the structural reform of the sports supply-side and implement the new urbanization strategy. Therefore, in recent years, it has gained the attention of the Chinese government, and the successfully declared town can get financial support from the higher-level government. For example, during the 13th Five-Year Plan, the Zhejiang Province of China will cultivate 15 sports towns with a total investment of 75.17 billion RMB. For the "Sports and Leisure Town of Zhejiang Province," the government of Zhejiang Province gives each 3 million yuan in funding subsidies; for the town that accumulates sports fixed asset investment of not less than 1 billion yuan within five years, increase the funding subsidies to 5 million yuan each (China Business Industry Research Institute. Characteristic towns: A list of the first sports and leisure towns in Zhejiang Province. 2019-03-25. https://f.qianzhan.com/tesexiaozhen/detail/200117-e59d153f.html). In 2017, the Ministry of Finance of the People's Republic of China and the General Administration of Sports of the People's Republic of China will provide financial support to each eligible declared project at the following rates: 3 million yuan for sports and leisure characteristic town facilities, 1 million yuan for sports park facilities, 1 million yuan for skateboarding facilities, 3 million yuan for rock climbing facilities, and 1 million yuan for mountaineering fitness trails (Sohu news. Supplementary report of Ministry of Finance and General Administration of

Sports: sports and leisure featured town facilities subsidy of 3 million, sports park facilities subsidy of 1 million, etc.2017-12-04. https://www.sohu.com/a/208460792_783770).

Thus, the above example shows that the towns that are successful in the declaration can receive subsidies from the higher level of government. From equation (5), the present value Hamiltonian function of this sports town is

$$\begin{aligned} H(p,m,u_1,u_2,k_1,k_2) &= \left[p(t)q(t)+\phi q(t)k_2(t)-\frac{(1-\tau)u_1^2(t)}{2}-\frac{u_2^2(t)}{2}-\frac{m^2(t)}{2}\right] \\ &\quad +\lambda_1(t)\left[u_1(t)-\delta_1 k_1(t)\right]+\lambda_2(t)\left[u_2(t)-\delta_2 k_2(t)\right] \\ &= \left[\mathrm{ap}(t)-p^2(t)+\alpha_1 p(t)k_1(t)+(\alpha_2-\phi)p(t)k_2(t)\right. \\ &\quad +a\phi k_2(t)+\alpha_1\phi k_1(t)k_2(t)+\alpha_2\phi k_2^2(t) \\ &\quad \left.+\beta m(t)\left(p(t)+\phi k_2(t)\right)-\frac{(1-\tau)u_1^2(t)}{2}-\frac{u_2^2(t)}{2}-\frac{m^2(t)}{2}\right] \\ &\quad +\lambda_1(t)\left[u_1(t)-\delta_1 k_1(t)\right]+\lambda_2(t)\left[u_2(t)-\delta_2 k_2(t)\right], \end{aligned} \tag{15}$$

where $\lambda_1(t)$ and $\lambda_2(t)$ are dynamic costate variables associated with state variables $k_1(t)$ and $k_2(t)$.

From the first-order conditions and costate conditions of current-value Hamiltonian (15), we get

$$\frac{\partial H}{\partial u_1(t)}=\lambda_1(t)-(1-\tau)u_1(t)=0, \tag{16}$$

$$\frac{\partial H}{\partial u_2(t)}=\lambda_2(t)-u_2(t)=0, \tag{17}$$

$$\frac{\partial H}{\partial p(t)}=a-2p(t)+\alpha_1 k_1(t)+(\alpha_2-\phi)k_2(t)+\beta m(t)=0, \tag{18}$$

$$\frac{\partial H}{\partial m(t)}=\beta p(t)+\beta\phi k_2(t)-m(t)=0, \tag{19}$$

$$\dot{\lambda}_1(t)=\rho\lambda_1(t)-\frac{\partial H}{\partial k_1(t)}=(\rho+\delta_1)\lambda_1(t)-\alpha_1 p(t)-\alpha_1\phi k_2(t), \tag{20}$$

$$\begin{aligned} \dot{\lambda}_2(t) &= \rho\lambda_2(t)-\frac{\partial H}{\partial k_2(t)}=(\rho+\delta_2)\lambda_2(t)-\left[\alpha_2 p(t)+a\phi\right. \\ &\quad \left.+\phi p(t)+\alpha_1\phi k_1(t)+2\alpha_2\phi k_2(t)+\beta\phi m(t)\right]. \end{aligned} \tag{21}$$

According to equations (16)–(21), one can obtain the following dynamic system:

$$\begin{cases} \dot{k}_1(t)=u_1(t)-\delta_1 k_1(t), \\ \dot{k}_2(t)=u_2(t)-\delta_2 k_2(t), \\ \dot{\lambda}_1(t)=(\rho+\delta_1)\lambda_1(t)-\alpha_1 p(t)-\alpha_1\phi k_2(t), \\ \dot{\lambda}_2(t)=(\rho+\delta_2)\lambda_2(t)-\left[\alpha_2 p(t)+a\phi-\phi p(t)\right. \\ \left.+\alpha_1\phi k_1(t)+2\alpha_2\phi k_2(t)+\beta\phi m(t)\right], \\ \dot{u}_1(t)=(\rho+\delta_1)u_1(t)-\dfrac{\alpha_1}{(1-\tau)}p(t)-\dfrac{\alpha_1\phi}{(1-\tau)}k_2(t), \\ \dot{u}_2(t)=\dot{\lambda}_2(t), \\ \dot{p}(t)=\dfrac{\alpha_1}{\left(2-\beta^2\right)}u_1(t)+\dfrac{a\delta_1}{\left(2-\beta^2\right)}-\dfrac{2\delta_1}{\left(2-\beta^2\right)}p(t) \\ +\left[\dfrac{\delta_1(\alpha_2-\phi)}{\left(2-\beta^2\right)}-\dfrac{\delta_2\left(\alpha_2-\phi+\beta^2\phi\right)}{\left(2-\beta^2\right)}\right]k_2(t)+\dfrac{\beta\delta_1}{\left(2-\beta^2\right)}m(t) \\ +\dfrac{\left(\alpha_2-\phi+\beta^2\phi\right)}{\left(2-\beta^2\right)}u_2(t), \\ \dot{m}(t)=\dfrac{\alpha_1\beta}{\left(2-\beta^2\right)}u_1(t)-\dfrac{\alpha_1\beta\delta_1}{\left(2-\beta^2\right)}k_1(t)+\dfrac{\beta(\alpha_2+\phi)}{\left(2-\beta^2\right)}u_2(t) \\ +\dfrac{\beta\delta_2(\alpha_2+\phi)}{\phi\left(2-\beta^2\right)}p(t)-\dfrac{\delta_2(\alpha_2+\phi)}{\phi\left(2-\beta^2\right)}m(t). \end{cases} \tag{22}$$

Table 1: The basic parameters used in the numerical example.

a	δ_1	δ_2	ϕ	ρ	α_1	α_2	τ
100	0.08	0.08	0.05	0.04	0.3	0.1	0.3

Solving dynamic system (22), we obtain the investment strategy and the optimal capital level of the sports town characterized by superscript "C" when it can get subsidies from the higher government:

$$
\begin{aligned}
p^C &= \frac{a\phi}{(\alpha_2+\phi)} \\
&\quad + \frac{a\delta_2(\rho+\delta_2)\left[\alpha_1^2\phi + \delta_1(\rho+\delta_1)\left((1-\tau)(\alpha_2-\phi)+\beta^2\phi\right)\right]}{A_2(\alpha_2+\phi)}, \\
k_1^C &= \frac{a\alpha_1\delta_2(\rho+\delta_2)}{A_2}, \\
k_2^C &= -\frac{a}{(\alpha_2+\phi)} + \frac{a\delta_2(\rho+\delta_2)\left[\delta_1(\rho+\delta_1)(2(1-\tau)-\beta)^2 - \alpha_1^2\right]}{A_2(\alpha_2+\phi)}, \\
u_1^C &= \frac{a\alpha_1\delta_1\delta_2(\rho+\delta_2)}{A_2}, \\
u_2^C &= -\frac{a\delta_2}{(\alpha_2+\phi)} + \frac{a\delta_2^2(\rho+\delta_2)\left[\delta_1(\rho+\delta_1)\left(2(1-\tau)-\beta^2\right) - \alpha_1^2\right]}{A_2(\alpha_2+\phi)}, \\
m^C &= \frac{a\beta\delta_1\delta_2(\rho+\delta_1)(\rho+\delta_2)}{A_2},
\end{aligned}
\tag{23}
$$

where
$A_2 = A_1 - 2\tau\delta_1\delta_2(\rho+\delta_1)(\rho+\delta_2) + \tau\delta_1(\rho+\delta_1)(\alpha_2+\phi)^2$.

4. Numerical Examples and Policy Implications

We have obtained the calculation results of the optimal strategy of the sports town with and without the government subsidy. In this part, we use data examples to analyze the sports town's optimal strategy and get necessary policy implications.

For data example analysis, we refer to the current situation of many sports towns in China for initial parameter assignment. For example, the depreciation rate of sports capital and other industrial capital is 5%. It is based on the relevant provisions of the Regulations of the People's Republic of China on the Implementation of the Enterprise Income Tax Law, which stipulate that the minimum period for calculating the depreciation of buildings is 20 years and the depreciation rate is 5%. The potential market size is 1 million. It is reported that the world-famous sports town of Queenstown, New Zealand, can receive up to 3 million people, but China's sports towns are still in their infancy, and a medium-sized sports town such as Shichuan Sports and Leisure Town in Gaolan County, Lanzhou City, has a potential reception size of 1 million (Xiaoxiang Morning Post. Gaolan County Shichuan sports and leisure town broadens win-win channels. 2022-02-14). Therefore, this article assigns a value of 1 million to the potential market size of sports towns; the risk-free rate is 4%. The 10-year treasury bond or 5-year deposit rate is generally taken as the risk-free rate, around 4%, so this article uses 4% to represent the risk-free rate; the sports investment subsidy rate of the superior government to the sports town is 30%. The higher-level governments that subsidize sports towns include the central, provincial, and municipal governments. For example, in 2017, the subsidy for sports and leisure characteristic town facilities was 3 million. The corresponding subsidy rates in Zhejiang Province and Anhui Province were 3 million and 5 million, respectively. However, the amount of subsidy from different municipal finances varied widely, such as the implementation opinions of the Office of the People's Government of Tongling City on the cultivation of characteristic towns, which stipulated that the municipal finances would give a 50% subsidy of the award funds after passing the annual assessment (Sohu News. How Anhui, the first sports town in the country to issue guidance, is making the town more distinctive. 2017–05. https://www.sohu.com/a/140385285_534292). Therefore, based on local circumstances, this article assumes an initial subsidy rate of 30%; the effect of sports capital on demand for sports products is greater than the effect of other industrial capital on demand for sports products, so they are assigned values of 0.4 and 0.1. These data were obtained from our survey of several Hunan, Zhejiang, and Anhui sports towns. We summarize all parameter assignments in Table 1.

To ensure the robustness of the simulation results, we first perform sensitivity analysis on the calculated results and the assigned parameters. Referring to a related study such as Yi et al. [30], we assign the first class of critical parameters including δ_1, δ_2, and a to vary −50%, −25%, 0, +25%, and +50%. In the meantime, we set the second category of critical parameters including α_1, α_2, and ϕ as "low," "medium," and "high" in terms of assigned values. Then, we calculate the results for each of the main variables, as shown in Tables 2–4.

As can be seen in Table 2, for all "low," "medium," and "high" α_1, the optimal sports capital and other capital investments and marketing efforts in sports towns raise with α_1 no matter how δ_1, δ_2, and a change. Moreover, both capital investments and marketing efforts are higher when subsidized by higher levels of government than in the absence of subsidies, independent of parameter changes, for all parameter configurations. These scenarios illustrate the robustness of the model analysis results to withstand parameter perturbations. In addition, the trends of the individual changes shown in Table 2 are consistent with the

Table 2: Impacts of the first class of critical parameters and α_1 on the three categories of investments.

		$a_{1\text{low}} = 0.15$			$\alpha_{1\text{medium}} = 0.25$			$\alpha_{1\text{high}} = 0.35$		
		k_1^{NC}/k_1^C	k_2^{NC}/k_2^C	m^{NC}/m^C	k_1^{NC}/k_1^C	k_2^{NC}/k_2^C	m^{NC}/m^C	k_1^{NC}/k_1^C	k_2^{NC}/k_2^C	m^{NC}/m^C
δ_1	0.04	129.85/203.04	99.30/108.68	6.75/10.55	331.06/737.98	151.89/237.02	10.32/23.02	2256.01/1339.76	739.36/307.35	50.27/29.85
	0.06	110.2623/170.0512	96.8578/104.5648	6.5863/10.1577	260.3101/518.5785	137.1987/191.3250	9.3295/18.5859	971.1622/1296.29	365.6140/482.2322	24.86/45.98
	0.08	94.94/144.88	94.94/101.41	6.45/9.85	211.90/393.51	127.14/165.27	8.64/16.05	603.65/915.01	558.70/274.50	17.59/28.95
	0.1	82.71/125.16	93.42/98.95	6.35/9.61	176.88/313.12	119.86/148.53	8.15/14.42	430.31/1164.13	208.28/197.73	14.16/38.06
	0.12	72.77/109.37	92.18/96.98	6.26/9.42	150.50/257.36	114.38/136.91	7.77/13.30	329.85/942.43	179.06/158.12	12.17/34.78
δ_2	0.04	99.30/152.00	129.85/139.13	6.75/10.33	225.10/426.01	176.62/233.97	9.18/17.38	685.50/6592.6	384.18/432.49	19.97/54.98
	0.06	96.85/148.00	110.26/117.93	6.58/10.06	217.64/407.49	148.65/194.83	8.87/16.62	637.91/25919.92	311.22/8852.11	18.59/755.38
	0.08	94.94/144.88	94.94/101.41	6.45/9.85	211.90/393.51	127.14/165.27	8.64/16.05	603.65/9915.01	258.70/2974.50	17.59/288.95
	0.1	93.42/142.40	82.71/88.26	6.35/9.68	207.36/382.64	110.16/142.29	8.46/15.61	577.95/6560.83	219.31/1742.72	16.84/191.20
	0.12	92.18/140.38	72.77/77.58	6.26/9.546	203.72/373.99	96.49/124.00	8.31/15.25	558.06/5112.63	188.81/1210.88	16.26/148.99
a	50	47.47/72.44	47.47/50.709	3.22/4.92	105.95/196.75	63.57/82.63	4.32/8.02	301.82/4957.50	129.35/1487.25	8.79/144.47
	75	71.21/108.66	71.21/76.06	4.84/7.38	158.92/295.13	95.35/123.95	6.48/12.04	452.73/7436.26	194.03/2230.87	13.19/216.71
	100	94.94/144.88	94.94/101.41	6.45/9.85	211.90/393.51	127.14/165.27	8.64/16.05	603.65/9915.01	258.70/2974.50	17.59/288.95
	125	118.68/181.10	118.68/126.77	8.07/12.31	264.87/491.89	158.92/206.59	10.80/20.06	754.57/12393.76	323.38/3718.13	21.99/361.18
	150	142.42/217.32	142.42/152.12	9.68/14.77	317.85/590.27	190.71/247.91	12.96/24.08	905.48/14872.52	388.06/4461.75	26.38/433.42

results of the subsequent graphical analysis of the simulations.

Tables 3 and 4 examine the changes in the three types of investment in sports towns for the combinations of "low," "medium," and "high" α_2 with the first category of parameters and "low," "medium," and "high" ϕ with the first category of parameters, respectively. From Tables 3 and 4, we find that the three types of investment in sports towns raise with α_1 no matter how δ_1, δ_2, and a change. Moreover, both capital investments and marketing efforts are higher when subsidized by higher levels of government than in the absence of subsidies. Therefore, these show the robustness of the model analysis results to withstand parameter perturbations.

The analysis of Tables 2–4 shows that the results of the model calculations are extremely robust around the assigned parameters. Therefore, we can apply the assigned parameters for the data example analysis.

Figures 1–5 show the optimal decision for the sports town, and its decision outcome varies with several vital parameters and the subsidy rate. In particular, the five simulations also depict the comparative results for the two scenarios without and with upper-level government subsidies.

From Figures 1–4, when the higher-level government subsidizes the sports town, the sports town has higher sports industry investment, other industry investment, level of marketing effort, number of sports products, and the town's profit and corresponding level of social welfare than when there is no subsidy. Furthermore, if the higher-level government subsidizes the town's sports investment, the cost of its sports product investment decreases, causing the town to increase its sports industry investment. At the same time, the investment in the sports industry makes the demand for sports products increase, which brings opportunities for the development of other industries and makes the investment in other industries increase. Furthermore, to promote the consumption of sports products, the town's marketing efforts should also be increased accordingly. All these changes lead to higher profit levels for the town and bring about higher consumer surplus and social welfare.

Figures 1–3 show the changes in the optimal decision of the sports town and the corresponding outcome when the marginal impact of sports industry capital, other industry capital,

Table 3: Impacts of the first class of critical parameters and α_2 on the three categories of investments.

		$a_{2\text{low}} = 0.05$			$\alpha_{2\text{medium}} = 0.1$			$\alpha_{2\text{high}} = 0.15$		
		k_1^{NC}/k_1^C	k_2^{NC}/k_2^C	m^{NC}/m^C	k_1^{NC}/k_1^C	k_2^{NC}/k_2^C	m^{NC}/m^C	k_1^{NC}/k_1^C	k_2^{NC}/k_2^C	m^{NC}/m^C
δ_1	0.04	521.07/2295.84	132.82/409.65	13.54/59.69	624.81/4705.23	238.90/1259.34	16.24/122.33	866.25/10027.52	441.62/3578.45	22.52/260.71
	0.06	377.26/1056.52	110.46/216.55	11.26/31.55	437.70/1448.66	192.24/445.39	13.09/43.26	564.26/1615.70	330.44/1236.24	16.85/90.06
	0.08	291.31/669.94	97.10/156.31	9.90/22.77	331.56/832.63	165.78/291.42	11.27/28.30	411.07/1261.56	274.04/588.73	13.97/42.89
	0.1	234.45/482.14	88.26/127.05	9.00/18.51	263.53/573.17	148.82/226.57	10.11/22.01	318.90/779.13	240.11/410.64	12.24/29.91
	0.12	194.24/371.64	82.01/109.84	8.36/16.00	216.43/430.79	137.07/190.98	9.32/18.55	257.64/554.31	217.56/327.65	11.09/23.87
δ_2	0.04	521.07/2295.84	132.82/409.65	13.54/59.69	624.81/4705.23	238.90/1259.34	16.24/32.70	866.25/10027.52	441.62/3578.4	22.52/67.26
	0.06	377.26/1056.52	110.46/216.55	11.26/31.55	437.70/1448.66	192.24/445.39	13.07/30.13	564.26/3015.70	330.44/1236.24	16.85/51.24
	0.08	291.31/669.94	97.10/156.31	9.90/22.77	331.56/832.63	165.78/291.42	11.27/28.30	411.07/1261.56	274.04/588.73	13.97/42.89
	0.1	234.45/482.14	88.26/127.05	9.00/22.37	263.53/573.17	148.82/226.57	10.11/26.95	318.90/779.13	240.11/410.64	12.24/37.79
	0.12	194.24/371.64	82.01/109.84	8.36/22.05	216.43/430.79	137.07/190.98	9.32/25.92	257.64/554.31	217.56/327.65	11.09/34.37
a	50	145.65/334.97	48.55/78.15	4.95/11.38	165.78/416.31	82.89/145.71	5.63/14.15	205.53/630.78	137.02/294.36	6.98/21.44
	75	218.48/502.45	72.82/117.23	7.42/17.08	248.67/624.47	124.33/218.56	8.45/21.23	308.30/946.17	205.53/441.54	10.48/32.16
	100	291.31/669.94	97.10/156.31	9.90/22.7	331.56/832.63	165.78/291.42	11.27/28.30	411.07/1261.56	274.04/588.73	13.97/42.89
	125	364.14/837.42	121.38/195.39	12.3/28.47	414.45/1040.79	207.22/364.27	14.09/35.38	513.83/1576.95	342.55/735.91	17.47/53.61
	150	436.97/1004.91	145.65/234.47	14.85/34.12	497.34/1248.95	248.67/437.13	16.90/42.46	616.60/1892.34	411.07/883.09	20.96/64.33

and marketing effort on demand for sports products rises. From Figures 1–3, we find that when the three marginal impacts of sports capital on demand for sports products rise, the town's optimal investments in sports capital and other capital, marketing effort, and the number of sports products demanded rise. Furthermore, the town's profits and social welfare rise. Figures 1–3 illustrate that the existence of complementary relationships between sports capital investment, other industrial capital investment, and marketing efforts; that is, when one of the three types of investments, including sports capital, other industrial capital, and marketing, increases, the other two investments should also increase.

Figure 4 depicts the changes in the town's optimal strategies and the strategies outcomes when the marginal effect of other industrial capital on the net benefits of other industries in the sports town changes. From this, one can see that the marginal impact of other industrial capital on the net income of other industries in the town is positive, with one exception; that is, without higher government subsidies, the town's profits decline with the marginal impact of capital in other industries on the net income of other industries. This phenomenon has two implications. On the one hand, it shows the importance of higher government subsidies for the development of sports towns, and on the other hand, it shows that if the town has advantages in other industries, it should not force the development of sports towns, but should concentrate on the industries that have advantages. For example, according to Xu and Yin [4], there are more than 1000 sports towns in China so far, most of them are developing well, but there are still some sports towns that are not developing as well as they should. Xu and Yin [4] point out that many sports towns in China currently lack core competitive factors. Although a small number of sports towns are unique in terms of the development scale, operation mode, and core elements, more than half of the sports towns imitate and follow the trend, which makes them marginalized in the competition and restricts the progress and development of sports towns. As a result, it has limited the progress and development of sports towns. Therefore, it is essential to consider whether these towns have not yet refined their core competitive resources or lack the natural conditions to build a sports town.

Figure 5 depicts the changes in optimal strategies and welfare of a sports town when the subsidy rate from the higher-level government rises. Obviously, without considering subsidies from higher levels of government, increases

Table 4: Impacts of the first class of critical parameters and ϕ on the three categories of investments.

		$\phi_{low} = 0.01$			$\phi_{medium} = 0.05$			$\phi_{high} = 0.1$		
		k_1^{NC}/k_1^C	k_2^{NC}/k_2^C	m^{NC}/m^C	k_1^{NC}/k_1^C	k_2^{NC}/k_2^C	m^{NC}/m^C	k_1^{NC}/k_1^C	k_2^{NC}/k_2^C	m^{NC}/m^C
δ_1	0.04	536.02/2511.9	150.29/493.03	13.93/65.31	624.81/4705.23	238.90/1259.34	16.24/122.3	866.25/4002.75	441.62/2078.45	22.52/160.71
	0.06	386.22/1106.86	124.39/249.55	11.53/33.05	437.70/1448.66	192.2/445.39	13.07/43.26	564.26/3015.70	330.44/1236.24	16.85/90.06
	0.08	297.38/692.68	109.04/177.78	10.11/23.55	331.51/832.63	165.78/291.42	11.27/28.30	411.07/1261.56	274.04/588.73	13.97/42.89
	0.1	238.88/495.36	98.92/143.59	9.17/19.02	263.53/573.17	148.82/226.57	10.1198/22.01	318.90/779.13	240.11/410.64	12.2/29.91
	0.12	197.65/380.42	91.79/123.68	8.51/16.38	216.43/430.79	137.07/190.98	9.32/18.55	257.64/554.31	217.56/327.65	11.09/23.87
δ_2	0.04	308.77/737.03	148.05/247.37	10.49/25.05	359.03/962.03	234.75/440.31	12.20/32.70	494.45/1978.28	431.06/1207.29	16.81/67.26
	0.06	302.40/711.94	126.22/208.02	10.28/24.20	343.38/886.25	195.4/353.11	11.675/30.1327	444.81/1507.15	337.58/800.67	15.12/51.24
	0.08	297.38/692.68	109.04/177.78	10.11/23.55	331.56/832.63	165.78/291.42	11.27/28.30	411.07/1261.56	274.07/588.73	13.97/42.89
	0.1	293.35/677.49	95.23/153.96	9.973/23.03	322.32/792.92	142.71/245.72	10.96/26.95	386.78/1111.59	228.30/459.30	13.15/37.79
	0.12	290.05/665.28	83.96/134.80	9.86/22.61	315.07/762.47	124.36/210.68	10.71/25.92	368.54/1010.96	193.97/372.45	12.53/34.37
a	50	148.69/346.34	54.52/88.89	5.05/11.77	165.78/416.31	82.89/145.71	5.63/14.15	205.53/630.78	137.02/294.36	6.98/21.44
	75	223.03/519.51	81.78/133.34	7.58/17.66	248.67/624.47	124.33/218.56	8.45/21.23	308.30/946.17	205.53/441.54	10.48/32.16
	100	297.38/692.68	109.04/177.78	10.1/23.55	331.56/832.63	165.78/291.42	11.27/28.30	411.07/1261.56	274.04/588.73	13.97/42.89
	125	371.72/865.85	136.30/222.22	12.63/29.43	414.45/1040.79	207.22/364.27	14.09/35.38	513.83/1576.95	342.55/735.91	17.47/53.61
	150	446.07/1039.02	163.56/266.68	15.16/35.32	497.34/1248.95	248.67/437.13	16.90/42.46	616.60/1892.34	411.07/883.09	20.96/64.33

in subsidy rates do not affect changes in decisions and outcomes in sports towns, so they are a horizontal straight line. However, if we consider the subsidies from higher levels of government, the town's investment in sports capital and other industrial capital, and impact efforts all rise, as discussed earlier. However, subsidies do not always positively affect the town's profit level and social welfare; they both show a tendency to rise and then fall.

Considering the profit side of the town, it is likely that most people believe that more subsidies from higher levels of government are better for the town. However, Figure 5 shows a different result from this view. It is because the general view sees only the beneficial effects of subsidies on the town. However, there is also a negative side of the effect of subsidies on the town. Receiving subsidies from higher levels of government can reduce the cost of sports capital investment, promote sports capital investment, and drive other industrial capital and marketing efforts, resulting in higher levels of profitability for the town, which is a positive effect of higher levels of government subsidies. Nevertheless, as subsidies increase, sports capital investment and the supply of sports products increase, which results in a decline in the price of sports products, negatively affecting sports towns' profit creation. Therefore, sports towns should actively seek subsidies from higher governments, but based on their efforts mainly, supplemented by seeking subsidies from higher governments. Only in this way, can we embark on the road to healthy development.

Considered in terms of social welfare, Figure 5 also brings significant management implications to higher levels of government. Increasing subsidies to sports towns can undoubtedly promote the development of sports towns. However, providing too many subsidies may trigger an oversupply of sports town products, reducing resource allocation and decreasing social welfare instead. Therefore, the central and provincial governments' subsidies to sports towns should be coordinated and kept in a moderate range. The key is to enhance the blood-making function of sports towns and never to subsidize sports capital investment in sports towns excessively.

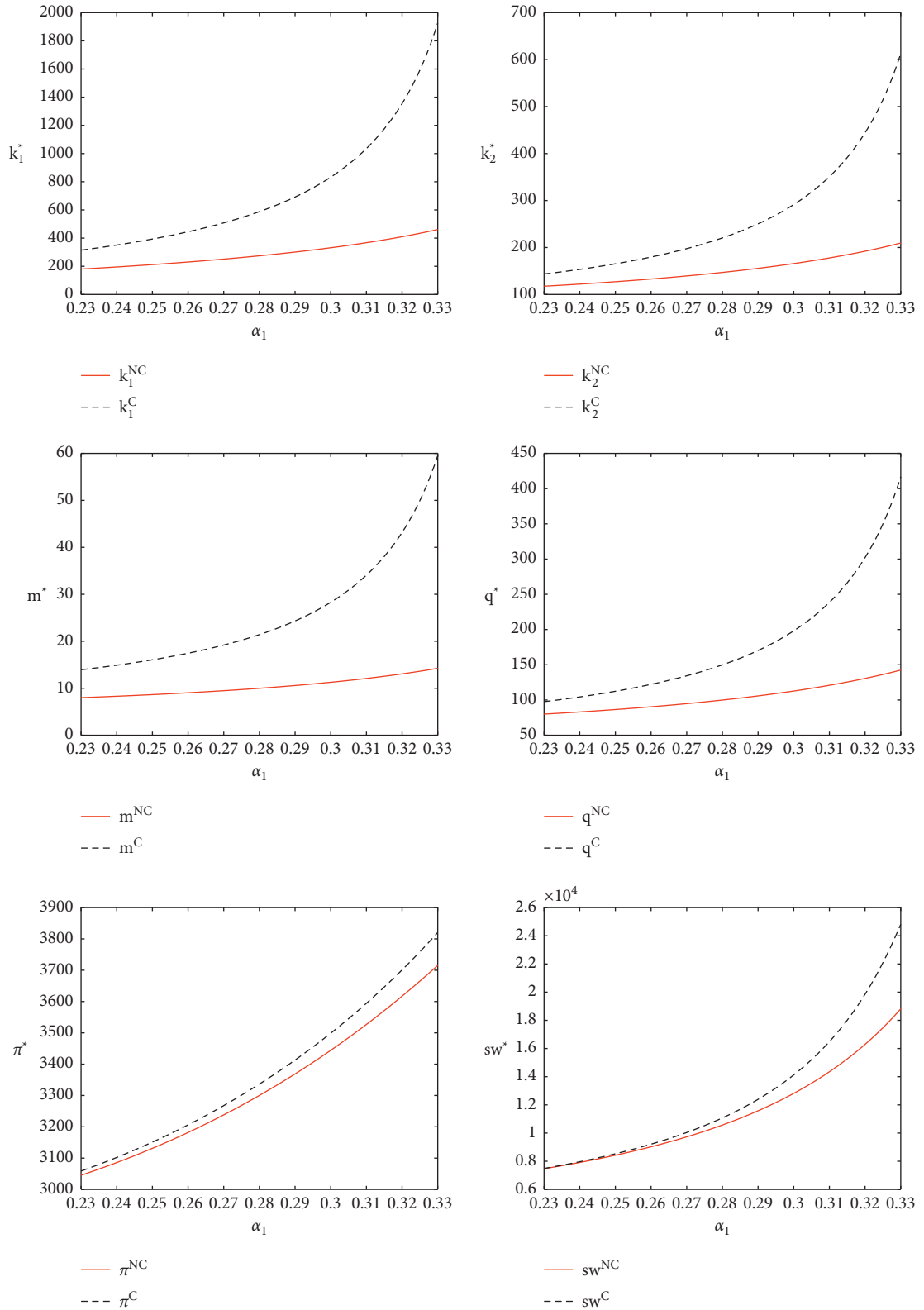

Figure 1: Changes in optimal decision and results with the marginal contribution of sports capital to the demand.

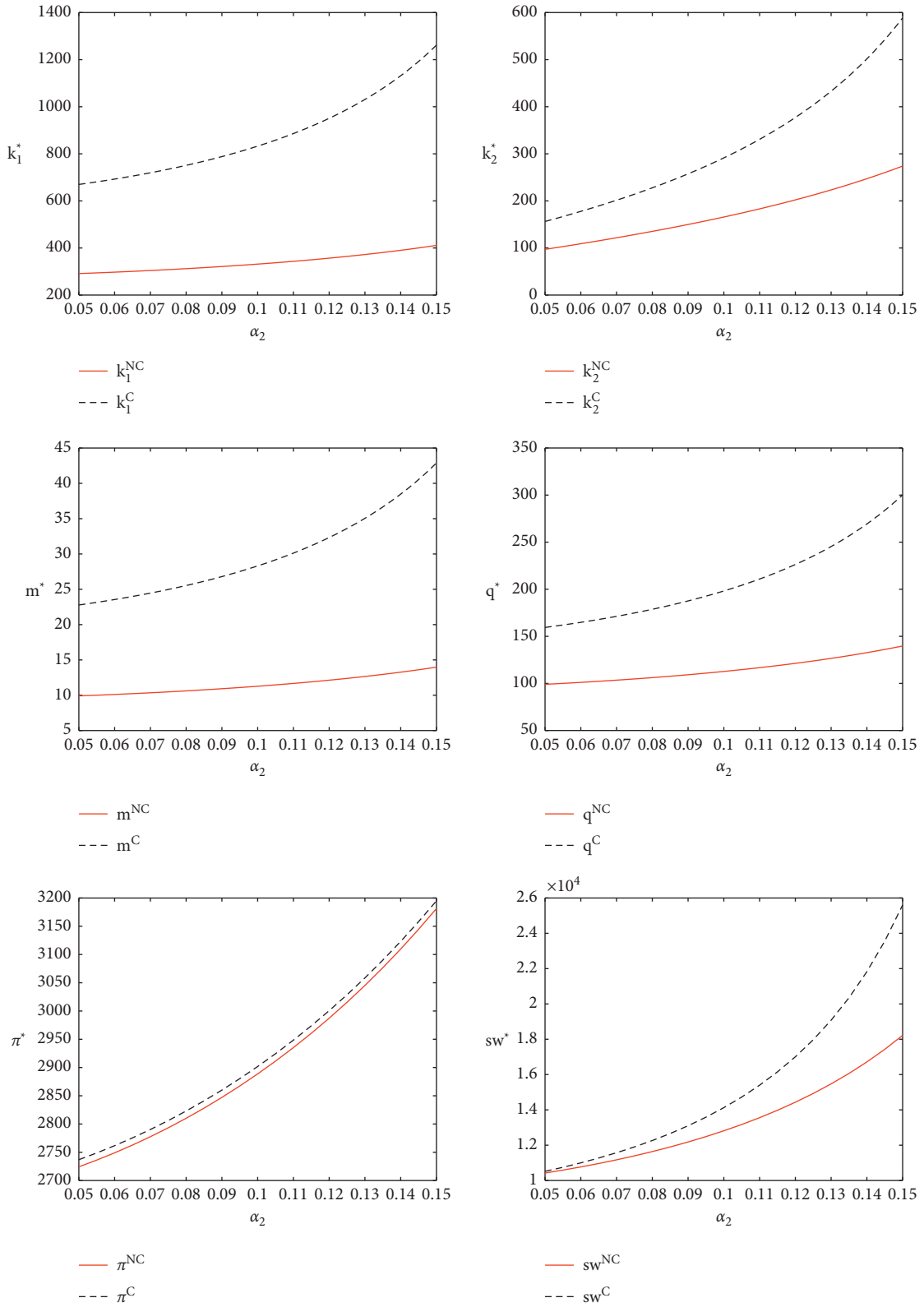

Figure 2: Changes in optimal decision and results with the marginal contribution of notsports capital to the demand.

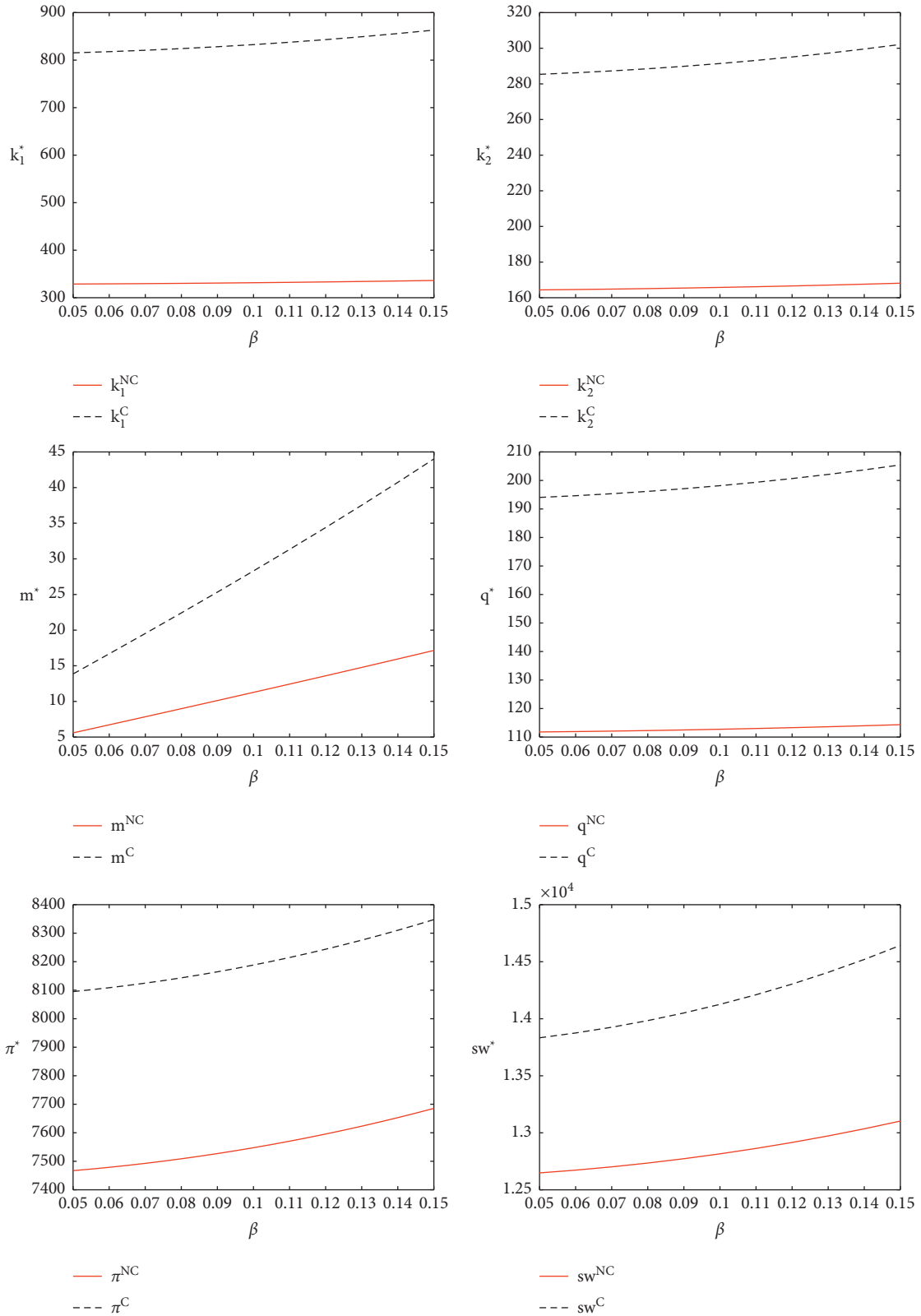

Figure 3: Changes in optimal decision and results with the marginal contribution of marketing effort to the demand.

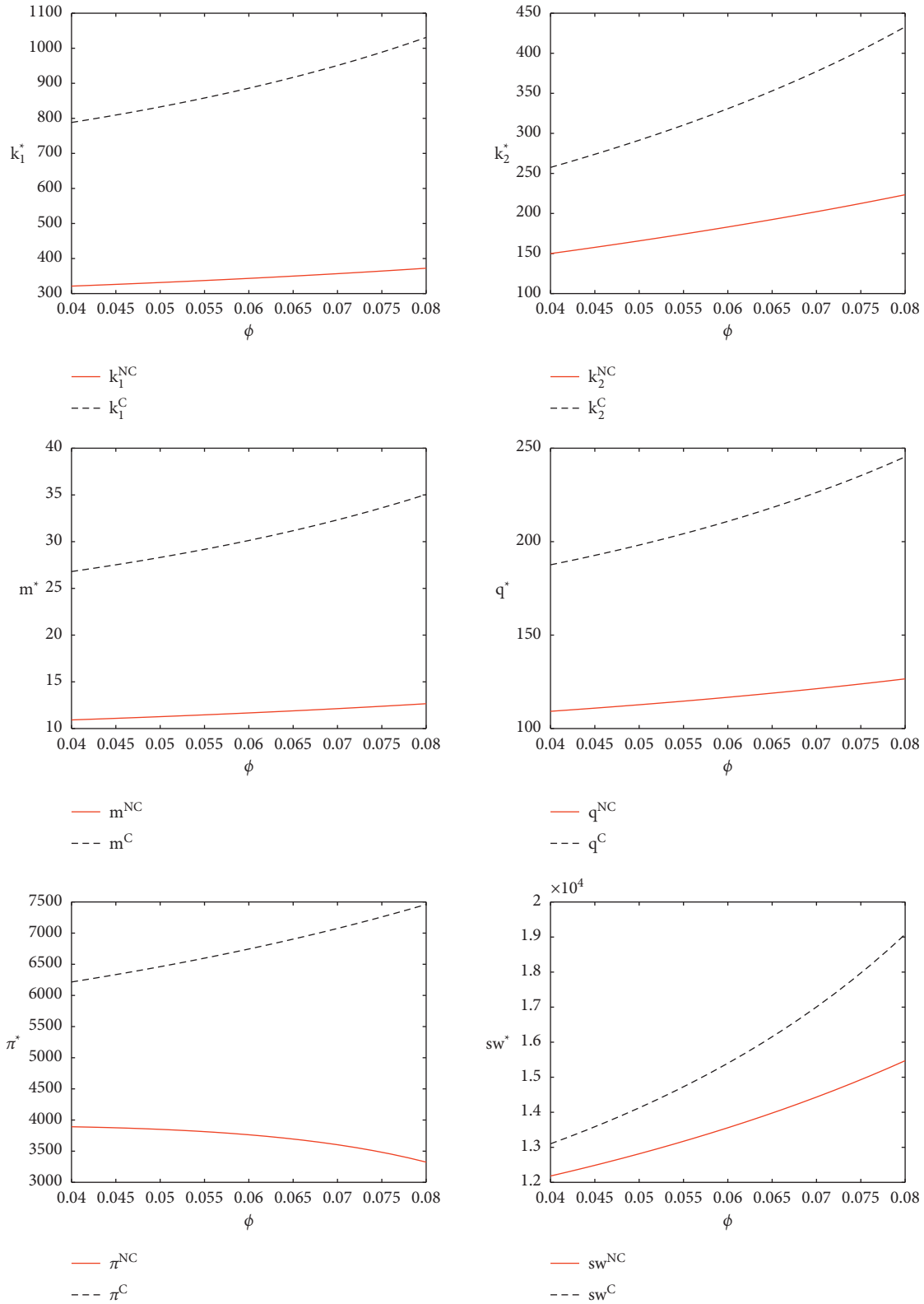

Figure 4: Changes in optimal decision and results with the marginal contribution of notsports capital to nonsports industry net income.

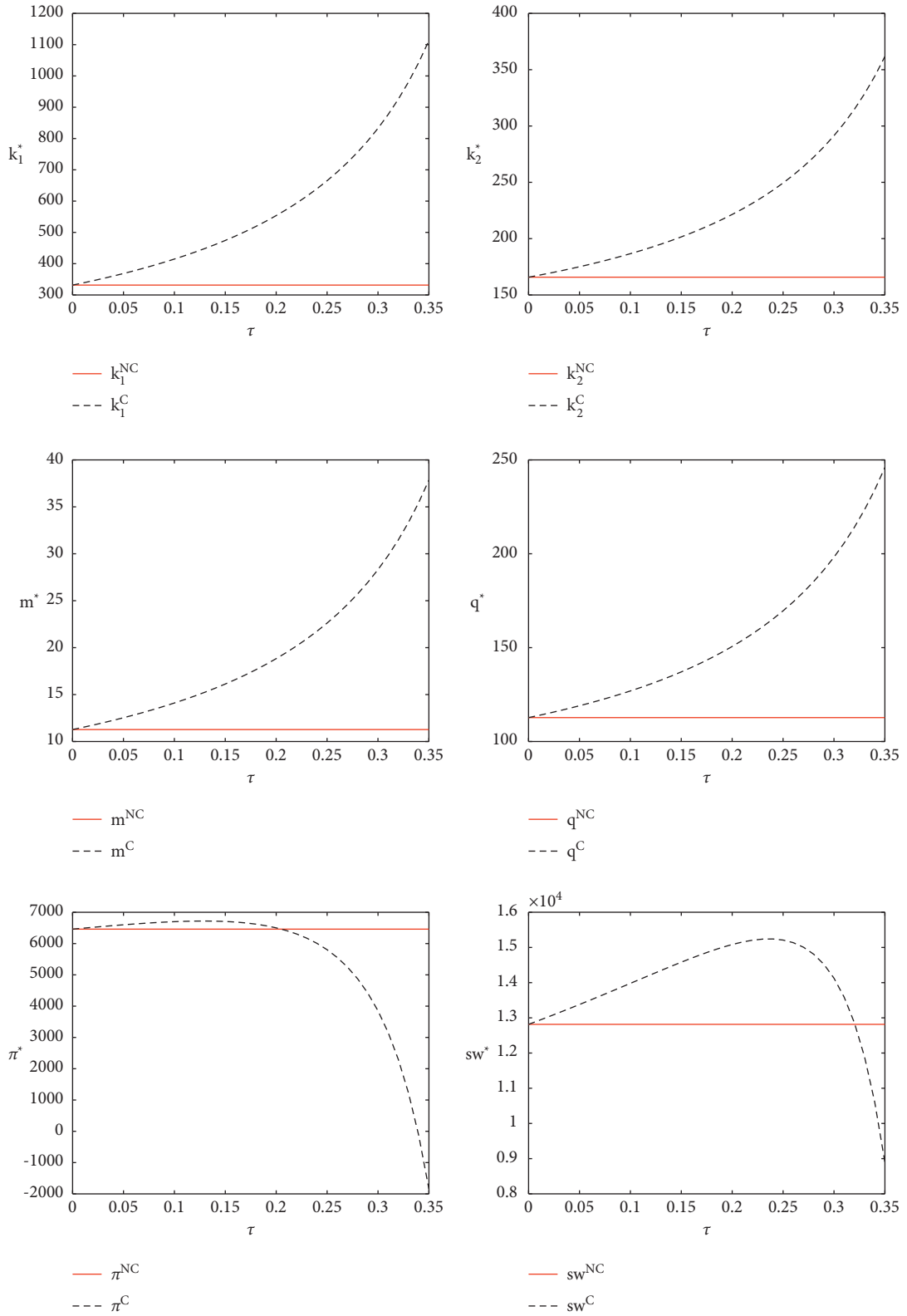

Figure 5: Changes in optimal decision and results with the subsidy rate from the higher-level government.

5. Conclusion

This article divides the industries of a sports town into the sports industry and other industries. We construct an optimal decision model for the town's economic development in which the sports industry is the primary industry and by which to promote the development of other industries. Applying this new model, we analyze the optimal strategies for the town's sports industry investment, other industry investment, and sports and its related product marketing efforts in two cases—without and with higher government subsidies for sports investment, and obtain the corresponding optimal strategy results. This article has the following main innovations compared to existing studies: firstly, in contrast, existing literature considers the investment in sports towns from total investment. This article differentiates between investment in the sports industry and investment in other industries concerning their different characteristics and impacts on the development of sports towns and provides optimal investment strategies for both types of investment. Secondly, few studies on the rate of subsidies to sports towns by higher-level governments have been found from the existing literature. This article examines this issue and provides an optimal subsidy rate from the higher-level government to the sports town. Thirdly, most of the existing studies are static. However, sports capital and other industrial capital investment have prominent dynamic characteristics, so this article takes a dynamic research approach through which it can provide an optimal dynamic strategy for sports towns and their higher-level government departments. Summarizing the research of the article, we reach some conclusions with important managerial insights:

First, sports towns require investment in three areas: sports industry capital, other industry capital, and sports and related product marketing efforts. We find that there is a complementary relationship between the three of them. If the investment in one of them increases, the sports towns should also raise the investment level in the other two categories.

Second, if the higher government subsidizes the sports capital investment in a sports town, it increases its sports capital investment and drives it to invest in other industries. Therefore, the Chinese central government's strategy of increasing subsidies or incentives to promote the development of sports towns is effective.

Thirdly, the sports town should adopt a development model of self-investment as the main focus, supplemented by seeking subsidies from higher levels of government. A rising subsidy rate results in an increase in the profit level of the town if the subsidy rate is in a specific range. However, if the sports investment subsidies are too high, it may cause the oversupply of the sports industry, and the profit level of the town is decreasing instead.

Fourth, the central and provincial governments should grasp the strength of the subsidies for sports towns. It is because too high a subsidy rate tends to cause vicious competition in sports products, leading to a decline in social welfare.

In a word, we have provided insight with important significance for sports towns and their upper-level government. Moreover, we have provided a quantitative computational framework of the optimal strategy for both of them. It is needed but not available before our article. Apply our model, sports towns can obtain the optimal investment rate and subsidy rate, and the central and provincial governments can quantitatively calculate the optimal level of support for sports towns. At the same time, they can adjust the respective optimal strategies according to the variation of the parameters.

It is also important to note that there are still some shortcomings that need to strengthen. For example, the article assumes a linear market demand curve. Nevertheless, empirical research has demonstrated that the demand function is nonlinear in most markets. In addition, we did not take into account the competition between sports towns. This model assumption may be valid in China's current situation of the relative scarcity of sports products. However, with the increase of sports product supplying agents in the future, the competition of sports products will undoubtedly become more and more intense. Therefore, future research should take note of the competition among sports towns.

Data Availability

All data generated or analyzed during this study are included in this published article.

Conflicts of Interest

The author declares that there are no conflicts of interest.

Acknowledgments

This work was supported by the Major Project of Hunan Philosophy and Social Science Foundation (Grant no. 18ZDB033). Project name: Research on the construction path and institutional innovation of Hunan sports and leisure characteristic towns.

References

[1] T. Wang, S. Xiao, J. Yan, and P. Zhang, "Regional and sectoral structures of the Chinese economy: a network perspective from multi-regional input-output tables," *Physica A: Statistical Mechanics and Its Applications*, vol. 581, no. 1, Article ID 126196, 2021.

[2] S. He, F. Haifeng Liao, and G. Li, "A spatiotemporal analysis of county economy and the multi-mechanism process of regional inequality in rural China," *Applied Geography*, vol. 111, Article ID 102073, 2019.

[3] W. X. Zhao and Y. H. Zou, "Creating a makerspace in a characteristic town: the case of Dream Town in Hangzhou," *Habitat International*, vol. 114, Article ID 102399, 2021.

[4] C. F. Xu and T. L. Yin, "Examination and thinking on the development of sports distinctive towns in China," *Sichuan Sports Science*, vol. 40, no. 4, pp. 92–95, 2021.

[5] L. J. Zhu, C. N. Fna, and T. Xiao, "The staged practical experience, practical dilemma and optimization path of the first

generation sports town," *Journal of Sports Science*, vol. 42, no. 1, pp. 106–113, 2021.

[6] L. Zhang, "Sports and leisure characteristic town: concept, category and development path," *China Sport Science*, vol. 38, no. 1, pp. 18–26, 2018.

[7] Y. H. Song, B. Li, S. L. Li, J. An, and L. Y. Ma, "Research on concept, connotation and type of sports town in China," *Journal of Harbin Sport University*, vol. 36, no. 2, pp. 57–61, 2018.

[8] L. J. R. Wilson and M. Hat, "Medicine Hat—"the sporting town," 1883-1905," *Canadian Journal of History of Sport*, vol. 16, no. 2, pp. 15–32, 1985.

[9] M. F. Beyazit and E. Koc, "An analysis of snow for Ski resort stablishments," *Tourism Management*, vol. 31, no. 5, pp. 878–683, 2010.

[10] S. Cubizolles, "Integrating a popular sport into the patrimony in a South African provincial town: the case of football in stellenbosch," *African Studies*, vol. 71, no. 1, pp. 108–126, 2012.

[11] N. Barbara, *For the Love of the Game: Amateur Sport in Small-Town ontario, 1838-1895*, Mcgill-queens Press, Montreal, Canada, 2016.

[12] J. Psczobut, "Revitalization of sopot-the seaside spa town in Poland," *Procedia Engineering*, vol. 161, pp. 2026–2039, 2016.

[13] U. Pillay and O. Bass, "Mega-events as a responds to poverty reduction: the 2010 fifa world cup and its urban development implications," *Urban Forum*, vol. 19, no. 3, pp. 328–346, 2008.

[14] M. S. Rosentraub, *Using Sports and Cultural Centers as Tools for Economis Development*, CRC Press Taylor and Francis Group, New York, USA, 2010.

[15] T. S. Chapin, "Sports facilities as urban redevelopment catalysts: baltimore's camden Yards and cleveland's Gateway," *Journal of the American Planning Association*, vol. 70, no. 2, pp. 193–209, 2004.

[16] D. J. Shonk and P. Chelladurai, "Service quality, satisfaction, and intent to return in event sport tourism," *Journal of Sport Management*, vol. 22, no. 5, pp. 587–602, 2008.

[17] B. Russo and J. Zarick, "The trends in dining, lodging, entertainment, and recreation among Virginia youth travel soccer participants, their family, and friends when traveling for youth travel soccer tournaments," *Journal of Tourism Insights*, vol. 1, no. 1, pp. 91–97, 2010.

[18] A. Desai and G. Vahed, "World cup 2010: africa's turn or the turn on africa?" *Soccer and Society*, vol. 11, no. 1-2, pp. 154–167, 2010.

[19] S. W. Yu, "Sports help regional poverty alleviation: classic case in the United States and enlightenment of China," *Journal of Sports Science*, vol. 42, no. 3, pp. 7–15, 2021.

[20] Y. Xian and L. L. Cheng, "Study on the Choice of operational types in sports town based onthe theory of industrial aggregation and location," *Journal of Sports Science*, vol. 39, no. 3, pp. 60–68, 2018.

[21] L. Si and W. Wang, "Theoretical framework and practice approaches to production of space in sports town in China," *Journal of Shenyang Sport University*, vol. 36, no. 5, pp. 53–58, 2017.

[22] Y. L. Ma, Z. M. Mu, and Y. T. Wwang, "Comparative study on Chinese and foreign sports characteristic towns," *Liaoning Sports Science and Technology*, vol. 43, no. 4, pp. 20–23, 2021.

[23] R. W. Pouder, J. D. Clark, and G. G. Fenich, "An exploratory study of how destination marketing organizations pursue the sports tourism market," *Journal of Destination Marketing & Management*, vol. 9, pp. 148–193, 2018.

[24] L. Hao, "Research on the development path of sports characteristic towns—take Shanxi Province as an example," *Contemporary Sports Technology*, vol. 36, pp. 68–70, 2020.

[25] Y. Yu, D. Zhou, D. Zha, and Q. Wang, "Joint optimization of charging facility investment and pricing in automobile retail supply chain and coordination," *Computers & Industrial Engineering*, vol. 156, Article ID 107296, 2021.

[26] Y. Zhang and X. Zhang, "Robust optimization-based dynamic power generation mix evolution under the carbon-neutral target," *Resources, Conservation and Recycling*, vol. 178, Article ID 106103, 2022.

[27] Y. Hou and Z. Yao, "The development of investment decision model in highway engineering projects and its dynamic optimization," *China Journal of Highway and Transport*, vol. 1, no. 3, pp. 1–9, 1991.

[28] Y. Yi, Z. Wei, and C. Fu, "An optimal combination of emissions tax and green innovation subsidies for polluting oligopolies," *Journal of Cleaner Production*, vol. 284, no. 3, Article ID 124693, 2020.

[29] Z. J. Wei, Y. X. Yi, and C. Y. Fu, "Cournot competition and "green" innovation under efficiency-improving learning by doing," *Physica A*, vol. 531, no. 10, Article ID 121762, 2019.

[30] Y. Yi, S. Liu, C. Fu, and Y. Li, "Joint tax-subsidy for a manufacturing-recycling system under further extended producer responsibility," *Sustainable Production and Consumption*, vol. 28, pp. 610–623, 2021.