

# Research Article On the Nonlinear Mathematical Model Representing the Coriolis Effect

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In this study, the traveling wave solutions of a nonlinear partial differential equation (NPDE) called as Geophysics Korteweg-de Vries (GpKdV) equation are obtained using the modified exponential function method (MEFM). Coriolis effect is stated with the help of this model used in geophysics. The nonlinear model has a Coriolis coefficient representing this effect.

### 1. Introduction

Nonlinear partial differential equations are mathematical models that represent many natural phenomena. Such mathematical models also take an active role in various branches of science. For example, it is an effective model for analyzing events in many science subjects, such as plasma physics and geophysics. Researchers need to investigate the solutions of such nonlinear mathematical models. Because the obtained findings allow the analysis of the event, for this reason, there are various methods related to numerical or exact solutions of partial differential equations in the literature. Some of them are respectively the trial equation method [1], the extended trial equation method [2], the newfunction method [3-7], the improved Bernoulli subequation function method [8-10], Kudryashov method [11, 12], the sine-Gordon equation expansion method [13-16], generalized auxiliary equation method [17], first integral method [18], new extended direct algebraic method [19], Hirota bilinear method and the tanh-coth method [20, 21], the modified exponential function method [22], and Chebyshev-Tau method [23].

The GpKdv equation, an important model in geophysics, is analyzed in this study. This equation is especially preferred because it contains a coefficient representing a Coriolis effect, which is in the model and is very important for science. The Coriolis effect is the name given to the phenomenon that affects the scattering of fluids such as water or air in nature while moving on the Earth. For example, while storms move counterclockwise at the north pole, they move clockwise at the south pole. The direction of the circular airflow, which generally occurs in natural events such as storms, is from the high-pressure region to the low-pressure area. However, this orientation cannot move vertically because the factor that prevents this and causes the storm to be blown is the Coriolis effect. Therefore, it can be said that it has an active role in the occurrence of the difference in direction at the poles.

In order to analyze the Coriolis effect, the GpKdv equation in this study is [24–26]

$$u_t - \gamma \, u_x + \frac{3}{2} u u_x + \frac{1}{6} u_{xxx} = 0, \tag{1}$$

where u is a function representing the independent surface feed and  $\gamma$  is the Coriolis coefficient. The Coriolis constant  $\gamma$ may differ from region to region depending on the depth of the water. Also, it is of great importance to include the Coriolis term  $\gamma u_x$  in the KdV equation in order to be able to observe the effect of the Earth's rotation on the flows in tsunami waves.

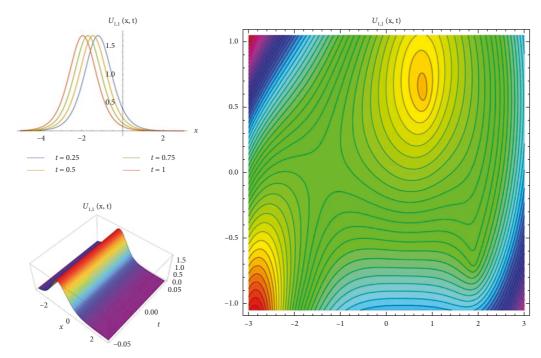


FIGURE 1: 2D and 3D graphs and graph simulating the coriolis effect of equation (16) with different t values.

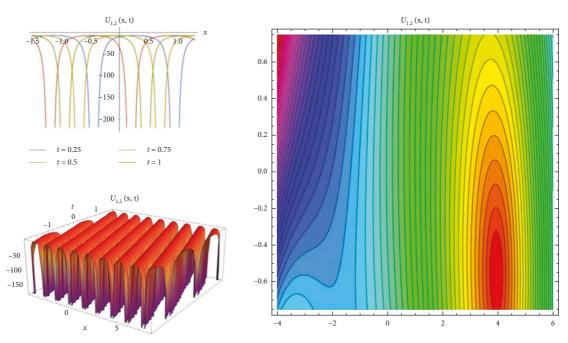


FIGURE 2: 2D and 3D graphs and graph simulating the coriolis effect of equation (17) with different t values.

# 2. Description of the Method

In

Let us take the general form according to the function derivative variables used in the GpKdV equation as follows:

$$P(u, u_x, u_t, u_{xx}, u_{xxx}, \ldots) = 0.$$
(2)

The wave transforms according to derivative variables in equation (2):

$$u(x,t) = U(\xi),$$
  

$$\xi = k(x - ct),$$
(3)

where k represents the height of the wave and c represents the frequency of the wave. The derivative terms in equation (2) are reduced to the following nonlinear ordinary differential equation form with a single derivative variable using the wave transform in equation (3):

$$N(U,U',U'',U''',\dots) = 0.$$
 (4)

The solution function sought according to the method in which the nonlinear partial differential equation and the nonlinear ordinary differential equation obtained by applying the wave transform to this partial equation are planned to be provided are as follows, respectively:

$$U(\xi) = \frac{\sum_{i=0}^{n} A_i [\exp(-\Omega(\xi))]^i}{\sum_{j=0}^{m} B_j [\exp(-\Omega(\xi))]^j}$$

$$= \frac{A_0 + A_1 \exp(-\Omega) + \dots + A_n \exp(n(-\Omega))}{B_0 + B_1 \exp(-\Omega) + \dots + B_m \exp(m(-\Omega))},$$
(5)

where  $A_i, B_j, (0 \le i \le n, 0 \le j \le m)$  are coefficients,  $A_n \ne 0, B_m \ne 0$ .

In the solution function assumed as equation (5), there are parameters that must be determined, respectively. The first of these is the omega function that is a solution of the nonlinear differential equation

$$\Omega'(\xi) = \exp(-\Omega(\xi)) + \mu \exp(\Omega(\xi)) + \lambda.$$
(6)

While integrating equation (6) according to  $\xi$ , various  $\Omega$  functions are obtained according to the states of  $\lambda$  and  $\mu$  in the omega function as follows [29]:

Family 1: when  $\mu \neq 0$ ,  $\lambda^2 - 4\mu > 0$ ,  $\Omega(\xi) = \ln\left(\frac{-\sqrt{\lambda^2 - 4\mu}}{2\mu} \tanh\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}(\xi + E)\right) - \frac{\lambda}{2\mu}\right),$ (7)

where E is an integration constant.

Family 2: when  $\mu \neq 0$ ,  $\lambda^2 - 4\mu < 0$ ,

$$\Omega\left(\xi\right) = \ln\left(\frac{\sqrt{-\lambda^2 + 4\mu}}{2\mu} \tan\left(\frac{\sqrt{-\lambda^2 + 4\mu}}{2}\left(\xi + E\right)\right) - \frac{\lambda}{2\mu}\right).$$
(8)

Family 3: when  $\mu = 0, \lambda \neq 0$  and  $\lambda^2 - 4\mu > 0$ ,

$$\Omega\left(\xi\right) = -\ln\left(\frac{\lambda}{\exp\left(\lambda\left(\xi + E\right)\right) - 1}\right).$$
(9)

Family 4: when  $\mu \neq 0, \lambda \neq 0$ , and  $\lambda^2 - 4\mu = 0$ ,

$$\Omega\left(\xi\right) = \ln\left(-\frac{2\lambda\left(\xi+E\right)+4}{\lambda^2\left(\xi+E\right)}\right).$$
(10)

Family 5: when 
$$\mu = 0, \lambda = 0$$
, and  $\lambda^2 - 4\mu = 0$ ,

$$\Omega(\xi) = \ln(\xi + E). \tag{11}$$

After obtaining the omega functions as above, a relation between m and n is found by applying the balancing principle to equation (4) according to the second operation of the method. Then, by determining a parameter suitable for m in this resulting relationship, the n parameter is found. In this way, the boundaries of the total symbols are determined. Then, the coefficients  $A_0, A_1, \ldots, A_n, B_0, B_1, \ldots, B_m$ in equation (5) will be determined. For this, when the necessary derivative terms of equation (4) are found from equation (5), and written in their places, an equation is obtained. The algebraic equation system consisting of the coefficients of the  $e^{\Omega(\xi)}$  function in the resulting equation is obtained.  $A_0, A_1, \ldots, A_n, B_0, B_1, \ldots, B_m$  coefficients are obtained by solving this system of equations with the help of the program.

### 3. Application

When the derivative terms in the nonlinear mathematical model (1) are substituted using the wave transform in equation (3), we obtain the following equation:

$$(-c - \gamma) u + \frac{3}{4} u^{2} + \frac{1}{6} k^{2} u'' = 0.$$
 (12)

If the balance procedure is applied to the term u'' with the highest order derivative in equation (12) and  $u^2$  of the highest order nonlinear term, the balance equation is obtained as follows:

$$n = m + 2. \tag{13}$$

Accordingly, if m = 1 in equation (13), it is obtained as n = 3. If these values are substituted in the solution function assumed to provide equation (1), the solution function and the derivative terms in the nonlinear ordinary differential equation are obtained as follows:

$$U(\xi) = \frac{\psi}{\varphi} = \frac{A_0 + A_1 e^{-\Omega(\xi)} + A_2 e^{-2\Omega(\xi)} + A_3 e^{-3\Omega(\xi)}}{B_0 + B_1 e^{-\Omega(\xi)}},$$
$$U'(\xi) = \frac{\psi' \varphi - \psi \varphi'}{\varphi^2},$$
$$U''(\xi) = \frac{\psi'' \varphi^3 - \varphi^2 \psi' \varphi' - (\psi \varphi'' + \psi' \varphi') \varphi^2 + 2(\psi')^2 \psi \varphi}{\varphi^4}.$$
(14)

After the terms in equation (14) are replaced in equation (12), the following coefficients are obtained when the system

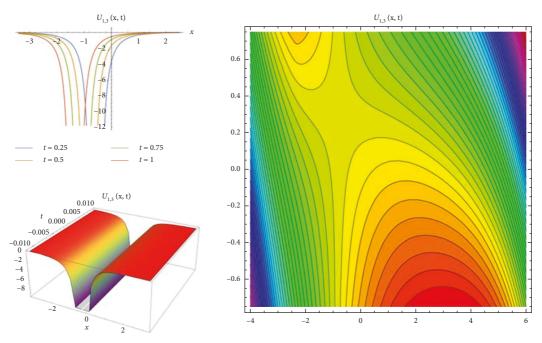


FIGURE 3: 2D and 3D graphs and graph simulating the coriolis effect of equation (18) with different values.

of algebraic equations consisting of coefficients is solved by classifying them according to the powers of  $e^{\Omega(\xi)}$ .

Case 1.

$$A_{0} = -\frac{4}{3}k^{2}\mu B_{0},$$

$$A_{1} = -\frac{4}{3}k^{2}(\lambda B_{0} + \mu B_{1}),$$

$$A_{2} = -\frac{4}{3}k^{2}(B_{0} + \lambda B_{1}),$$

$$A_{3} = -\frac{4}{3}k^{2}B_{1},$$

$$\gamma = -c + \frac{1}{6}k^{2}(\lambda^{2} - 4\mu).$$
(15)

These coefficients are replaced in the solution functions and derivatives in equation (14). Then, the omega functions in the following family cases are put into the solution function. Also, it is checked that the solution function satisfies the nonlinear ordinary differential equation, Figure 1 and then the nonlinear partial Figure 2 differential equation is performed with the help of the Mathematica.

Family 1:

$$u_{1,1}(x,t) = \frac{4k^2\mu(\lambda^2 - 4\mu)}{3(\lambda \cosh[1/2\Psi] + \sqrt{\lambda^2 - 4\mu}\sinh[1/2\Psi])^2},$$
(16)

where  $\Psi = (E + k(-ct + x))\sqrt{\lambda^2 - 4\mu}$ . Family 2:

$$u_{1,2}(x,t) = \frac{3k^{2}\mu(\lambda^{2} - 4\mu)}{3\left(\lambda \cos[1/2\Upsilon] - \sqrt{-\lambda^{2} + 4\mu} \sin[1/2\Upsilon]\right)^{2}},$$
(17)

where  $\Upsilon = (E + k(-ct + x))\sqrt{-\lambda^2 + 4\mu}$ . Family 3:

$$u_{1,3}(x,t) = -\frac{1}{3}k^2\lambda^2 \operatorname{csch}\left[\frac{1}{2}(E+k(x-ct))\lambda\right]^2.$$
 (18)

Family 4:

$$u_{1,4}(x,t) = \frac{k^2 ((E+\xi)\lambda (4+E\lambda+\xi)(\lambda^2-4\mu)-16\mu)}{3 (2+E\lambda+\xi\lambda)^2}.$$
(19)

Family 5:

$$u_{1,5}(x,t) = -\frac{4k^2}{3(E+\xi)^2}.$$
 (20)

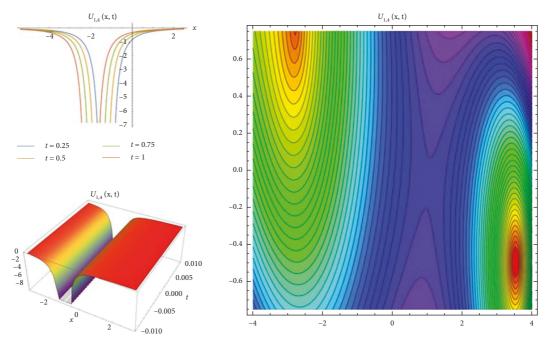


FIGURE 4: 2D and 3D graphs and graph simulating the coriolis effect of equation (19) with different t values.

Case 2.

$$A_{0} = \frac{1}{4}B_{0}\left(\frac{\lambda^{2}A_{3}}{B_{1}} + 8(c + \gamma)\right),$$

$$A_{1} = \frac{1}{4}\lambda A_{3}\left(\lambda + \frac{4B_{0}}{B_{1}}\right) + 2B_{1}(c + \gamma),$$

$$A_{2} = A_{3}\left(\lambda + \frac{B_{0}}{B_{1}}\right),$$

$$k = \frac{i\sqrt{3A_{3}}}{2\sqrt{B_{1}}},$$

$$\mu = \frac{\lambda^{2}}{4} + \frac{2B_{1}(c + \gamma)}{A_{3}}.$$
(21)

In this case, the following families of solutions are obtained by calculating the parameter k representing the height of the wave with the coefficients and the parameter  $\mu$ .

Family 1:

$$u_{2,1}(x,t) = \frac{\left(\operatorname{sech}\left[1/2\Psi\right]^{2}\left(\lambda^{2}-4\mu\right)\left(-2\mu+\left(\lambda^{2}-2\mu\right)\operatorname{cosh}\left[\Psi\right]+\lambda\sqrt{\lambda^{2}-4\mu}\operatorname{sinh}\left[\Psi\right]\right)A_{3}++8\left(2\mu+\left(\lambda^{2}-2\mu\right)\operatorname{cosh}\left[\Psi\right]+\lambda\sqrt{\lambda^{2}-4\mu}\operatorname{sinh}\left[\Psi\right]\right)B_{1}\left(c+\gamma\right)\right)}{\left(4B_{1}\left(\lambda+\sqrt{\lambda^{2}-4\mu}\operatorname{tanh}\left[1/2\Psi\right]\right)^{2}\right)}.$$
(22)

Family 2:

$$u_{2,2}(x,t) = \frac{\left(\sec\left[1/2Y\right]^{2} \left(\lambda^{2} - 4\mu\right) \left(-2\mu + \left(\lambda^{2} - 2\mu\right) \cos[Y] + \lambda \sqrt{-\lambda^{2} + 4\mu} \sin[Y]\right) A_{3} - 8\left(-2\mu - \left(\lambda^{2} - 2\mu\right) \cos[Y] + \lambda \sqrt{-\lambda^{2} + 4\mu} \sin[Y]\right) B_{1}(c+\gamma)\right)}{\left(4B_{1} \left(\lambda - \sqrt{-\lambda^{2} + 4\mu} \tan[1/2Y]\right)^{2}\right)}.$$
(23)

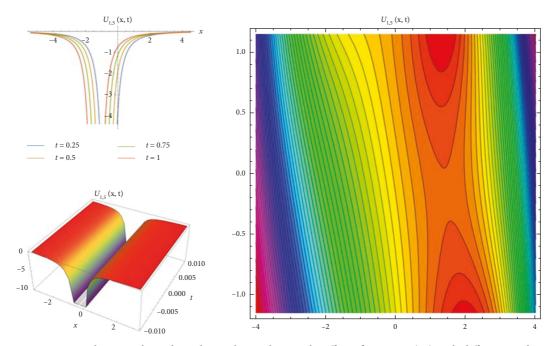


FIGURE 5: 2D and 3D graphs and graph simulating the coriolis effect of equation (20) with different t values.

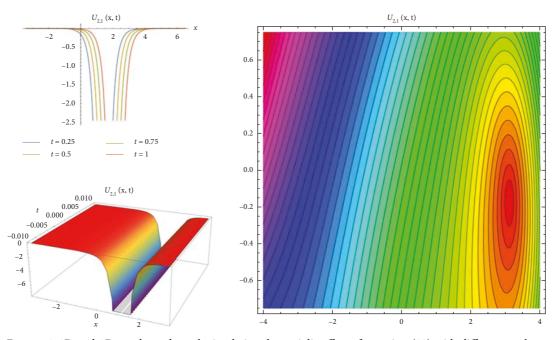


FIGURE 6: 2D and 3D graphs and graph simulating the coriolis effect of equation (22) with different t values.

Family 3:

$$u_{2,3}(x,t) = \frac{\lambda^2 A_3 \text{coth } [1/2(E+\xi)\lambda]^2}{4B_1}.$$
 (24)

Family 4:

$$u_{2,4}(x,t) = \frac{\lambda^2 A_3}{(2 + E\lambda + \xi\lambda)^2 B_1} + 2(c+\gamma).$$
(25)

In the case of Family 5, the solution function is determined as undefined since  $\lambda$  and  $\mu$  are zero and  $\lambda^2 - 4\mu = 0$  is zero. For this reason, no graphic drawings related to the mathematical model could be made.

Case 3.

$$A_0 = 0, A_1 = \frac{1}{6} \left( \lambda^2 + 2\mu \right) A_3, A_2 = \lambda A_3, B_0 = 0, c = -\frac{1}{6} k^2 \left( \lambda^2 - 4\mu \right) - \gamma.$$
(26)

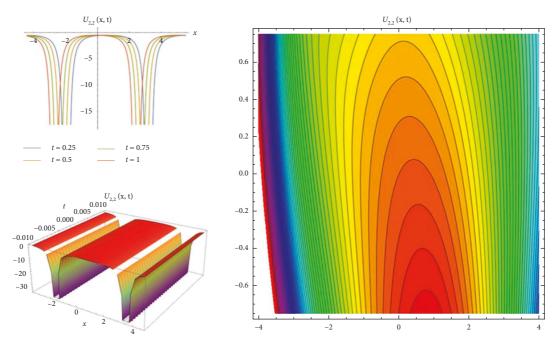


FIGURE 7: 2D and 3D graphs and graph simulating the coriolis effect of equation (23) with different t values.

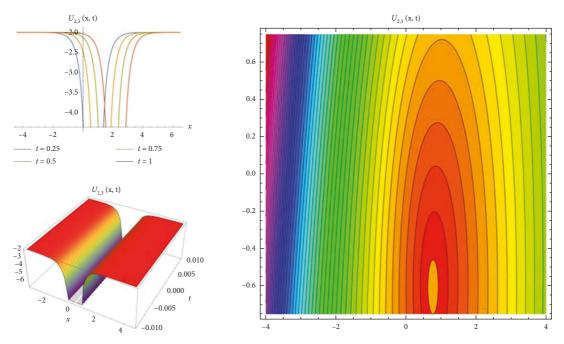


FIGURE 8: 2D and 3D graphs and graph simulating the coriolis effect of equation (24) with different t values.

Here, the following families of solutions are obtained by calculating the c parameters and coefficients representing the frequency of the wave.

Family 1:

$$u_{3,1}(x,t) = -\frac{\left(2k^{2}\operatorname{sech}[1/2\Psi]^{2}(\lambda^{2} - 4\mu)\left(-4\mu + (\lambda^{2} - 2\mu)\operatorname{cosh}[\Psi] + \lambda\sqrt{\lambda^{2} - 4\mu}\operatorname{sinh}[\Psi]\right)\right)}{\left(9\left(\lambda + \sqrt{\lambda^{2} - 4\mu}\operatorname{tanh}[1/2\Psi]\right)^{2}\right)}.$$
(27)

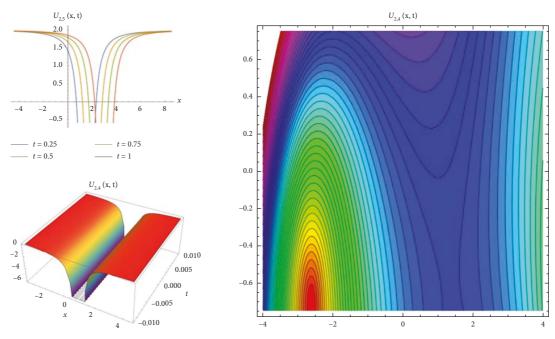


FIGURE 9: 2D and 3D graphs and graph simulating the coriolis effect of equation (25) with different t values.

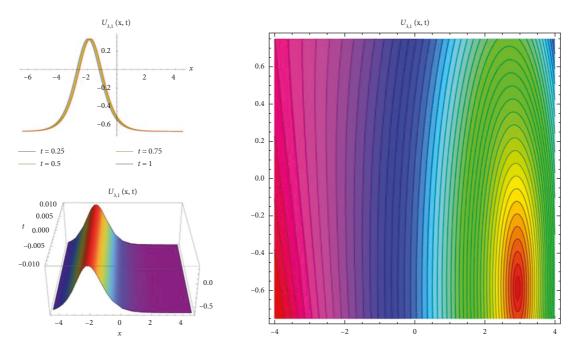


FIGURE 10: 2D and 3D graphs and graph simulating the coriolis effect of equation (27) with different t values.

Family 2:

$$u_{3,2}(x,t) = \frac{\left(2k^{2}\sec\left[1/2\Upsilon\right]^{2}\left(\lambda^{2} - 4\mu\right)\left(4\mu - \left(\lambda^{2} - 2\mu\right)\cos\left[\Upsilon\right] + \lambda\sqrt{-\lambda^{2} + 4\mu}\sin\left[\Upsilon\right]\right)\right)}{\left(9\left(\lambda - \sqrt{-\lambda^{2} + 4\mu}\tan\left[1/2\Upsilon\right]\right)^{2}\right)}.$$
(28)

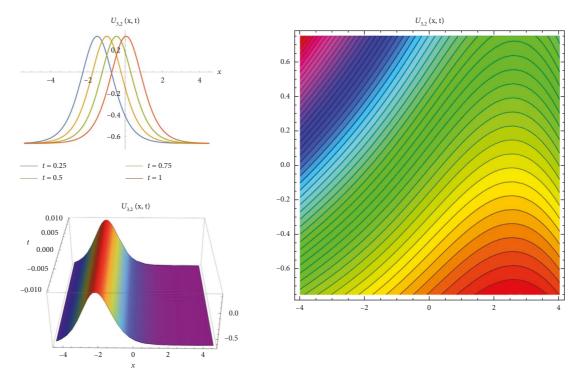


FIGURE 11: 2D and 3D graphs and graph simulating the coriolis effect of equation (28) with different t values.

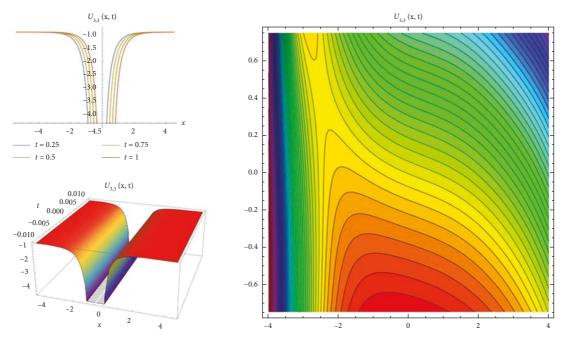


FIGURE 12: 2D and 3D graphs and graph simulating the coriolis effect of equation (29) with different t values.

Family 3:

$$u_{3,3}(x,t) = -\frac{1}{9}k^2\lambda^2 \left(2 + 3 \operatorname{csch}\left[\frac{1}{2}(E+\xi)\lambda\right]^2\right).$$
 (29)

$$u_{3,4}(x,t) = \frac{1}{9}k^{2} \left( \frac{\lambda^{2} \left( -8 + (E\lambda + \xi\lambda) \left( 4 + E\lambda + \xi\lambda \right) \right)}{\left( 2 + E\lambda + \xi\lambda \right)^{2}} - 4\mu \right).$$
(30)

Family 4:

Family 5:

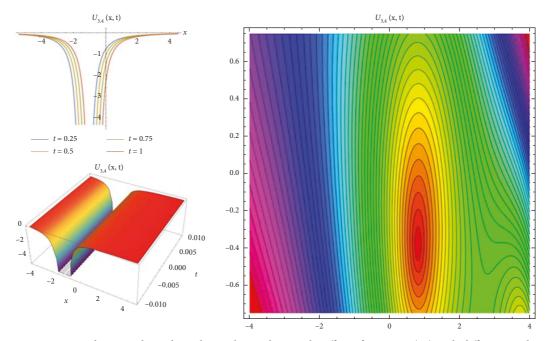


FIGURE 13: 2D and 3D graphs and graph simulating the coriolis effect of equation (30) with different t values.

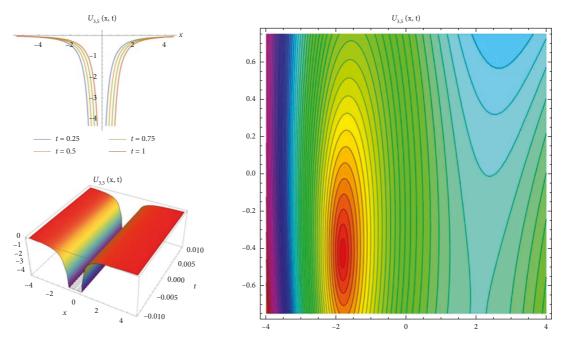


FIGURE 14: 2D and 3D graphs and graph simulating the coriolis effect of equation (31) with different t values.

$$u_{3,5}(x,t) = \frac{1}{9} \left( -\frac{4k^2}{3(E+\xi)^2} \right).$$
(31)

# 4. Conclusion

In this study, we successfully obtained the new traveling wave solutions of the GpKdv equation using the modified exponential function method. When the types of solution functions that provide the mathematical model are analyzed, it is seen that hyperbolic and trigonometric functions with periodic functions are singular solitons. We plotted the contour surfaces under appropriate constants, where all wave solutions are two and three-dimensional, and the Coriolis effect represented by the mathematical model is effectively seen. In Figures 1–14, the simulated graphs representing the mathematical model, the behavior of the solution function u representing the free surface progression, and the Coriolis effect accordingly are seen. The mentioned method is understood to be very effective in obtaining the wave solutions of such nonlinear differential equations. Because there is an exponential function in the solution function u, which is preferred as a hypothesis according to the method. In addition, the omega function, which is used as the power of the exponential function, is also a function that satisfies the Riccati equation. Considering all these situations, it allows obtaining solution functions with periodicity compared to other methods. This gives the researchers information about the behavior of the mathematical model used in a desired time interval. In addition, as far as we know, the obtained solution functions are included in the literature for the first time. When the literature is searched for this mathematical model, it is seen that various soliton solutions are also obtained using numerical methods, the homotopy perturbation method, finite element, and the Hirota bilinear method, which is expressed as an analytical method. Consequently, we believe the obtained solutions can effectively demonstrate the Coriolis effect in geophysics.

## **Data Availability**

No data were used to support this study.

# **Conflicts of Interest**

The authors declare that they have no conflicts of interest.

# **Authors' Contributions**

All authors read and approved the final manuscript.

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