Research Article

Reliability Analysis of Poll Data with Novel Entropy Information Measure in Multicriteria Decision-Making Based upon Picture Fuzzy Environment

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The polling system has a considerable role in the democratic nation. The uncertainty of the people's participation in polling generally affects the electoral-based system. Therefore, PFS (picture fuzzy set) is the furthestmost efficient and useful extension of IFS (intuitionistic fuzzy set) in a fuzzy system capable of precisely handling human perception in the decision-making system. The PFS structure involves the different degrees, i.e., membership, nonmembership, neutral, and hesitancy which are comprehensively applied to such types of complex practical problems in the real-life scenario. This advantage of PFS motivates the author to propose PFSs centred novel entropy measure via this communication, which is comparatively more generalized, reliable, and simplified in place of the existing uncertain measures. The practicability of the proposed present research work is to deal with real-world problems pertaining to the relative importance of the attributes. Therefore, certainly, the proposed novel entropy developed a different approach to handle the uncertainty more precisely as a part of the existing approaches. The validation proof of the proposed entropy measure is proved in an organized manner and practically employed in the perspective of the polling data outcomes about the people's opinions with the VIKOR-TOPSIS approach.

1. Introduction

In the modern era, fuzzy sets and their various generalized extensions are predominantly utilized by researchers and decision-makers in the multiple domains of the research field. For the useful life, the different researchers broadly used the concept of fuzzy logic in several applications of modern technology, for instance, washing machine [1], modern cars [2], and traffic signals control system [3]. The author in [4] was the pioneer to present the abstract of fuzzy theory.

Later, various authors presented the modified structure of F_{ss} and their efficient use to explore the vagueness and

uncertainty in many problems of multicriteria decision-making support systems. Initially, the fuzzy set may be possible to prevail only the knowledge about the membership degree (μ) of the element. It restricts the fuzzy set to the specific domain of M_{CDM} . Further, the author was inspired [5] with the concept of nonmembership function (ν) and reshaped the existing structure of F_{ss} and added on one more parameter π : $0 \leq \mu + \nu + \pi \leq 1$ and I_{FS} comes to light. I_{FS} attracts the great attentions of various researchers, authors, and decision-makers across the globe because of its feature of presenting π . It makes I_{FS} more reliable and concise towards the challenges in real-life scenario. In this perception, several authors around the globe contributed to

the research as per their point of view; for instance, the authors in [6] introduced the idea of homomorphism between the subgroup (cyclic) and I_{Fs} subgroup. Further, the authors in [7] developed complex I_{Fs} approach which relates with group theory and the authors in [8] proposed some important algebraic structures based upon $(\alpha\beta)$ complex fuzzy – based subgroup. First time, the author was able to validate various parameters of I_{Fs} in context of human perception and related various components of I_{Fs} as per the public opinion in the voting system, i.e., μ for *Support*, ν stands for *Unsupport*, and π implies neither *support nor unsupport*. As I_{Fs} handles the M_{CDM} problems more precisely and in efficient manner comparatively F_{ss} , however, the researchers analysed that I_{Fs} is not applicable comprehensively in all such fuzzy environment-based situations. For instance, in the voting system, the people who are in the mindset of the dilemma are not undertaken in the framework of I_{Fs} . To encounter such indeterminacy, the authors [9] were motivated with idea of neutral membership degree and incorporated the new component η to the present structure of I_{Fs} : $0 \leq \mu + \nu + \eta + \pi \leq 1$ which is known as P_{Fs} . Therefore, whenever I_{Fs} restricts its scope, P_{Fs} perfectly handles the uncertainty in a better way. Certainly, the parameter η has great importance to capture the logic of the human mind's ambiguity in the behavioural approach towards complex situations. Figure 1 shows the pretty presentation of various parameters from F_{ss} to P_{Fs} .

The researchers and decision-makers take advantage of this unique feature of P_{Fs} and used it tremendously in several applications of modern technologies and social issues. The researcher [10] applied the concept of P_{Fs} using the distance measure for the health issues. Nowadays, in various research areas, the researchers are using the framework of P_{Fs} in the hybrid form. Further, the authors [11] successively utilized P_{Fs} -based hybrid distance measure with the concept of rough set theory in selection criteria of the students. In this context, the authors [12] contributed some important correlation coefficients and their application in analysis of clustering and pattern recognition in computing research, and the authors in [13] presented the hybrid form of similarity measures and biparametric distance to handle the uncertainty in medical diagnosis. In the forecasting system, the author [14] significantly implied the P_{Fs} feature and proposed the novel method and created the time series model for forecasting. The decision-makers [15] extended the bidirectional projection method for P_{Fs} and applied to assess the safety features for construction design.

Recently, the researchers have implied the different picture fuzzy uncertain measures with the use of different methods or approaches to cope up the vagueness in M_{CDM} challenges. The authors [16] developed the algorithms based upon interaction aggregation operators defined on P_{Fs} for complex multivariable decision-making models. Thus, the researcher [17] contributed the concept of cross-entropy in P_{Fs} and its application in M_{CDM} support system. Continuing with the P_{Fs} theory, the authors submitted the entropy weighted picture fuzzy information-based clustering-algorithm for segmentation of images. The important entropy similarity measure was proposed by [18], and their

application in the decision-making system was developed. Further, the authors [19] worked upon and submitted the divergence information measure of Jensen Tasalli along with its application in decision problems for P_{Fs} . The concept of R - S norms was broadly used by the authors in [20, 21], and the information measure for the picture fuzzy situations was proposed. Moreover, the researchers [22] also proposed the entropy information measure based upon linguistic P_{Fs} with M_{CDM} application. The author [23] proposed the generalized model based upon the concept of maximum entropy negation to explore the information involved in the field of the artificial intelligent decision-making system. Further, the researcher [24] put forwarded the decision-making models to handle the issues like a cost-aware, fault-tolerant, and reliable strategy in complex problems under fuzzy environment. However, the literature findings help us in problem formulation which prevails across the different areas of the research; fuzzy logic and fuzzy sets are considerably applied to capture the ambiguity available in data as well as in practical problems of the real world.

1.1. Research Gap. The human mindsets seem too inefficient to meet the solution of complex multivariable problems in real-time scenarios. Therefore, fuzzy sets are quite proficient and reliable in handling ambiguity and the dilemma approach of human mindsets in complex decision-making issues. P_{Fs} can handle the neutral factors involved in the M_{CDM} problems more efficiently; for instance, in the outcomes of the exit poll in the electoral system, the neutral voters have a vital role. So far, several authors proposed a wide range of diverse information measures and approaches defined on picture fuzzy sets to cope with uncertainty as per their mindsets. However, there is still a scarcity of sophisticated, reliable, and simpler measures defined on P_{Fs} to deal with complicated M_{CDM} problems in a fuzzy environment. This inspires the authors to work upon the proposed research work. Therefore, the proposed research work certainly overcomes the research gap of the different existing approaches and uncertainty measures of P_{Fs} .

1.2. Methodology. The different methodologies are proposed to deal with M_{CDM} problems to select the best alternatives in accordance with the choice of the criteria. The purpose of approaches or techniques is to handle the available data with greater accuracy and to reduce uncertainty. The proposed novel entropy is reliable and more applicable with the complex M_{CDM} problems allied with the relative importance of the criteria. Therefore, the practicability of novel entropy is applied to the prediction of the polling data; however, the criteria involved in the present problem have the relative importance of the criteria. Therefore, such type of problems handles with the VIKOR approach which provides the compromise solutions. All through the TOPSIS approach was utilized to select the best alternatives irrespective of the relative importance of the different criteria. Therefore, the application part of the proposed entropy was validated with both approaches.

FIGURE 1: The pretty presentation of various parameters from F_{ss} to P_{Fs} .

Further, the presentation of the proposed work is classified in various parts as shown in Figure 2.

2. Preliminaries

Definition 1. The fuzzy set theory generalized the concept of classical sets, which it perceives as an extension of classical sets. The pioneer of the fuzzy set theory was [4] which defined elements of the fuzzy set (F_{ss}) in terms of the membership function (μ_{∇}) on finite universe of discourse, i.e., $Z = \{Z_1, Z_2, Z_3, \dots, Z_n\}$ and defined F_{ss}, ∇ as

$$\nabla = \{\langle Z_i \in Z \rangle\}, \quad (1)$$

where membership function of F_{ss} is $\mu_{\nabla}: Z \rightarrow [0, 1]$ and $\mu_{\nabla}(Z_i)$ signifies the membership degree $\mu_{\nabla}(Z_i)$ of the $Z_i \in Z$ in F_{ss} . The fuzzy set (F_{ss}) has only explained $\mu_{\nabla}(Z_i)$ and $1 - \mu_{\nabla}(Z_i)$, i.e., nonmembership function and no idea about hesitancy element. The idea of hesitancy element was further introduced by [5] to the existing framework of F_{ss} called the degree of hesitancy.

Definition 2. The fuzzy sets are restricted to employ in certain M_{CDM} problems, which are unable to handle the uncertainty factor precisely. The idea of (I_{Fs}) intuitionistic fuzzy set was presented by [5] which added on one more important factor, i.e., nonmembership function in the present structure of F_{ss} . Consequently, $I_{Fs}\Delta$ defined on Z (universe of discourse) is as follows:

$$\Delta = \{\langle Z_i \in Z \rangle\}, \quad (2)$$

and satisfies $0 \leq \mu_{\Delta}(Z_i) + \nu_{\Delta}(Z_i) \leq 1$, $\forall Z_i \in Z$, along with hesitancy degree $\pi_{\Delta}(Z_i) = 1 - \mu_{\Delta}(Z_i) - \nu_{\Delta}(Z_i)$. Apart of this, the functions $\mu_{\Delta}: Z \rightarrow [0, 1]$ and $\nu_{\Delta}: Z \rightarrow [0, 1]$

stated the membership and nonmembership function of I_{Fs} , and the terms $(\mu_{\Delta}(Z_i))$ and $(\nu_{\Delta}(Z_i))$ described the degree of membership and nonmembership of the element belongs to I_{Fs} . The author in [5] introduced I_{Fs} which extend the scope of fuzzy set in decision-making environment. In real-life scenario, the I_{Fs} membership, nonmembership, and uncertain degree make the sense of favour, against, and abstain in M_{CDM} problems but are unable to describe refusal degree which restricts its scope of application. Therefore, this inconsistency is vanquished in picture fuzzy set (P_{Fs}) to add on one more component "Neutral membership degree" in the present framework of I_{Fs} by [9].

Definition 3. Over the period, the decision-makers analysed that I_{Fss} is not feasible towards certain complex issues. The author in [9] presented the concept of a picture fuzzy set, which is an advanced generalized form of fuzzy sets that are more capable of dealing with complex issues in a fuzzy environment. The picture fuzzy set (P_{Fs}) ∇ on Z is defined as

$$\hat{\nabla} = \{\langle Z_i \in Z \rangle\}. \quad (3)$$

with inequality $0 \leq \mu_{\hat{\nabla}}(Z_i) + \eta_{\hat{\nabla}}(Z_i) + \nu_{\hat{\nabla}}(Z_i) \leq 1$, $\forall Z_i \in Z$.

This describes the positive membership degree $\mu_{\hat{\nabla}}(Z_i) \in [0, 1]$, neutral membership degree $\eta_{\hat{\nabla}}(Z_i) \in [0, 1]$, and negative membership degree $\nu_{\hat{\nabla}}(Z_i) \in [0, 1]$. The component $\pi_{\hat{\nabla}}(Z_i) = 1 - \mu_{\hat{\nabla}}(Z_i) - \eta_{\hat{\nabla}}(Z_i) - \nu_{\hat{\nabla}}(Z_i)$ is refusal membership degree for each $Z_i \in Z$. Therefore, the structure of P_{Fs} is more generalized and classified form as compared with F_{ss} and I_{Fs} to capture the vagueness in M_{CDM} practical problems.

FIGURE 2: Proposed workflow.

Definition 4. Let Q and R be P_{Fs} defined on Z .

$$\begin{aligned} Q &= \{\langle Z_i \in Z \rangle\}, \\ R &= \{\langle Z_i \in Z \rangle\}. \end{aligned} \quad (4)$$

The authors in [25] introduced the following important operations and relations for any $Q, R \in P_{Fs}(Z)$ defined as follows:

- (1) $Q \subseteq R \Leftrightarrow \mu_Q(Z_i) \leq \mu_R(Z_i), \eta_Q(Z_i) \leq \eta_R(Z_i), \nu_Q(Z_i) \leq \nu_R(Z_i) \quad \forall Z_i \in Z$
- (2) $Q = R \Leftrightarrow Q \subseteq R$ and $R \subseteq Q$
- (3) Q^C (the complement of Q) = $\{\langle Z_i \in Z \rangle\}$

$$\begin{aligned} (4) \quad Q \cap R &= \{\langle \mu_Q(Z_i) \wedge \mu_R(Z_i), \eta_Q(Z_i) \vee \eta_R(Z_i), \nu_Q(Z_i) \vee \nu_R(Z_i) \rangle | Z_i \in Z\} \\ (5) \quad Q \cup R &= \{\langle \mu_Q(Z_i) \vee \mu_R(Z_i), \eta_Q(Z_i) \wedge \eta_R(Z_i), \nu_Q(Z_i) \wedge \nu_R(Z_i) \rangle | Z_i \in Z\} \end{aligned}$$

Definition 5. Assume that Q and R are two $P_{Fs} \in Z$ such that

$$\begin{aligned} Q &= \{\langle Z_i \in Z \rangle\}, \\ R &= \{\langle Z_i \in Z \rangle\}. \end{aligned} \quad (5)$$

There are two distance measures, i.e., *Hamming distance measure* and *Euclidean distance* between the two $P_{Fs} Q$ and R defined as

$$H_D(Q, R) = 0.5 \sum_{i=1}^n (|\mu_Q(Z_i) - \mu_R(Z_i)| + |\eta_Q(Z_i) - \eta_R(Z_i)| + |\nu_Q(Z_i) - \nu_R(Z_i)| + |\pi_Q(Z_i) - \pi_R(Z_i)|), \quad (6)$$

$$E_D(Q, R) = \sqrt{\sum_{i=1}^n \left\{ (\mu_Q(Z_i) - \mu_R(Z_i))^2 + (\eta_Q(Z_i) - \eta_R(Z_i))^2 + (\nu_Q(Z_i) - \nu_R(Z_i))^2 + (\pi_Q(Z_i) - \pi_R(Z_i))^2 \right\}}. \quad (7)$$

3. Fuzzy Entropy Information Measure for P_{FSS}

The entropy information measure is the fuzziness degree of F_{ss} . The author [26] introduced the set of postulates based upon probability theory of Shannon for F_{ss}

Definition 6. The entropy is a real function $\theta: F_{ss}(Z) \rightarrow [0, \infty)$ which satisfies the following axioms:

- (1) $PF_{Sharpness}$. $\hat{\theta}(\hat{Q}) = 0 \Leftrightarrow \hat{Q}$ defined as the crisp set
- (2) $PF_{Maximality}$. $\hat{\theta}(\hat{Q})$ acquires maximum value $\Leftrightarrow \mu_Q^\wedge = 0.5 \forall \hat{Q} \in F_{ss}(Z)$
- (3) $PF_{Resolution}$. $\hat{Q}$ and $\hat{R} \in F_{ss}(Z) | \hat{Q}$ is crisper than $\hat{R}$, i.e., $\hat{\theta}(\hat{Q}) \leq \hat{\theta}(\hat{R}) \Rightarrow \mu_Q^\wedge \leq \mu_R^\wedge$ for $\mu_R^\wedge \leq 1/2$ and $\mu_Q^\wedge \geq \mu_R^\wedge$ if $\mu_R^\wedge \geq 0.5$
- (4) $PF_{Symmetry}$. $\hat{\theta}(\hat{Q}) = \hat{\theta}(\hat{Q})^c$ where $(\hat{Q})^c$ represents the compliment of $\hat{Q}$

Definition 7 (see [27]). The entropy is a real function $\theta: I_{Fs}(Z) \rightarrow [0, \infty)$ which satisfies the following properties:

- (1) $PF_{Sharpness}$. $\check{\theta}(\check{Q}) = 0 \Leftrightarrow \check{Q} \in I_{Fs}(Z)$ defined as the crisp set
- (2) $PF_{Maximality}$. $\check{\theta}(\check{Q})$ is maximum value if $\mu_{\check{Q}} = \nu_{\check{Q}} = \pi_{\check{Q}} = 1/3 \quad \forall \check{Q} \in I_{Fs}(Z)$
- (3) $PF_{Resolution}$. If $\check{Q}$ and $\check{R} \in I_{Fs}(Z) | \check{Q}$ is crisper than $\check{R}$, i.e., $\check{\theta}(\check{Q}) \leq \check{\theta}(\check{R}) \Rightarrow \mu_{\check{Q}} \leq \mu_{\check{R}}$ and $\nu_{\check{Q}} \leq \nu_{\check{R}}$ for maximum $(\mu_{\check{R}}, \nu_{\check{R}}) \leq 1/3$ and if $\mu_{\check{Q}} \geq \mu_{\check{R}}, \nu_{\check{Q}} \geq \nu_{\check{R}}$, then $\min(\mu_{\check{R}}, \nu_{\check{R}}) \geq 1/3$
- (4) $PF_{Symmetry}$. $\check{\theta}(\check{Q}) = \check{\theta}(\check{Q})^C$ where $(\check{Q})^C$ denotes the compliment of $\check{\theta}$

Moreover, P_{FSS} is the well-known generalized form of I_{Fs} with the four components, i.e., (μ, ν, η, π) which hold the condition such that $(\mu, \nu, \eta, \pi) \in [0, 1]$ and $\mu + \nu + \eta + \pi = 0$. These parameters may be presumed as probability measures

that characterized the picture fuzzy set. This indicates that entropy measure must have maximum value if all parameters uniformly have the value one-fourth, i.e., $\mu_Q = \eta_Q = \nu_Q = \pi_Q = 1/4$ and must be zero if all the components vanish except one such as

$$\begin{aligned}
 &\mu_Q = 1, \\
 &\eta_Q = \nu_Q = \pi_Q = 0, \\
 &\text{or,} \\
 &\eta_Q = 1, \\
 &\mu_Q = \nu_Q = \pi_Q = 0, \\
 &\text{or,} \\
 &\nu_Q = 1, \\
 &\mu_Q = 1, \\
 &\mu_Q = \eta_Q = \pi_Q = 0, \\
 &\text{or,} \\
 &\pi_Q = 1, \\
 &\mu_Q = \eta_Q = \nu_Q = 0.
 \end{aligned} \tag{8}$$

Definition 8. The axiomatic definition of entropy measure for P_{FS} proposed in view of the concept of entropy measure of I_{FS} introduced by [27] is as follows.

The entropy measure is a real function $\theta: P_{FS}(Z) \rightarrow [0, \infty)$ and agreed with the following axioms:

P_{FS1} (sharpness). $\theta(Q) = 0 \Leftrightarrow P_{FS}Q \in (Z)$ which holds Q is a crisp set

P_{FS2} (maximality). If $\mu_Q = \eta_Q = \nu_Q = \pi_Q = 1/4 \Rightarrow \theta(Q)$ have the maximum value then

P_{FS3} (resolution). This property holds if Q is crisper than R , i.e., $\theta(Q) \leq \theta(R)$ such that $\mu_Q \geq \mu_R, \eta_Q \geq \eta_R, \nu_Q \geq \nu_R$ for $(\mu_R, \eta_R, \nu_R) \geq (1/4)$, $\mu_Q \leq \mu_R, \eta_Q \leq \eta_R, \nu_Q \leq \nu_R$ for $(\mu_R, \eta_R, \nu_R) \leq (1/4)$

P_{FS4} (symmetry). If Q^c is the complement of Q , symmetry implies $\theta(Q) = \theta(Q^c)$

In fact, several authors proposed different uncertain measures on P_{FS} and utilized them to solve the complex multivariable problems. In this context, the novel entropy measure is proposed to enhance the preciseness and improve the reliability and handle the ambiguity of M_{CDM} problems in a simpler and more logical way. This would certainly overcome such shortcomings existing in uncertain measures defined on P_{FS} . In the next part, a novel entropy information measure has been proposed on P_{FSS} , keeping in view these important fundamental concepts.

4. A Novel Entropy Information Measure for P_{FSS}

Let us assume the finite set of comprehensive probability distribution signified as $L_n = \langle \bar{O} = \langle a_1, a_2, \dots, a_n \rangle; a_i \geq 0 \text{ and } \sum_{i=1}^n a_i = 1 \rangle; n \geq 2 \text{ for any } \bar{O} \in L_n$. Firstly, Shannon [28] initiated to define the information measure explained which is widely known as Shannon entropy

$$R(\bar{O}) = - \sum_{i=1}^n (a_i) \log(a_i). \tag{9}$$

In the passage of time, Zadeh (1965) put forward the concept of fuzzy set which develops the new approach to quantify the uncertainty/fuzziness. Further, in 1968, Zadeh prolonged the concept of Shannon entropy; however, this information was not capable to meet the purpose. The continued efforts of researchers with information measure in 1972 [26] were capable to extend the axioms of Shannon entropy and proposed the new fuzzy information postulates which are accepted widely across the world and set as criteria for fuzzy entropy measure mentioned in Definition 6. This also motivated the authors [26] to propose the fuzzy entropy measure consistent with Shannon entropy presented as

$$R_{LT}(\nabla) = -\frac{1}{n} \sum_{i=1}^n [\mu_{\Delta}(Z_i) \log \log(\mu_{\Delta}(Z_i)) + (1 - \mu_{\Delta}(Z_i)) \log \log(1 - \mu_{\Delta}(Z_i))], \tag{10}$$

where ∇ and $Z_i \in F_{ss}(Z)$.

This developed the author's insight more into F_{ss} and presented different information measures according to their

point of view. The authors [29] contributed exponential entropy measure for I_{FS} given as follows:

$$R_{RB}(\Delta) = \frac{1}{n\sqrt{\exp-1}} \sum_{i=1}^n \left(\frac{\mu_{\Delta}(Z_i) + 1 - \nu_{\Delta}(Z_i)}{2} \right) \exp\left(\frac{\mu_{\Delta}(Z_i) + 1 - \nu_{\Delta}(Z_i)}{2} \right) + \left(\frac{\mu_{\Delta}(Z_i) + 1 - \nu_{\Delta}(Z_i)}{2} \right) \exp\left(\frac{\mu_{\Delta}(Z_i) + 1 - \nu_{\Delta}(Z_i)}{2} \right) - 1, \quad (11)$$

where Δ and $Z_i \in I_{FS}$.

Further, the researchers [30] have explored the idea of entropy measure based on exponential function to P_{PFS} (Pythagorean fuzzy set) given by

$$R_{RG}(\hat{\Delta}) = \frac{1}{n(1 - (\sqrt[3]{\exp})^2)} \sum_{i=1}^n 1 - \exp\left| \mu_{\hat{\Delta}}^2(Z_i) + \nu_{\hat{\Delta}}^2(Z_i) - \pi_{\hat{\Delta}}^2(Z_i) \right| \text{ where } \hat{\Delta} \text{ and } Z_i \in P_{PFS}. \quad (12)$$

Over the period, several authors have introduced the different work on entropy information measure. The authors [31] contributed residual entropy information measure in the application of the group decision-making. The geometric aggregation operators were introduced [32] based upon I_{FS} , respectively, with undefined criteria weights and their application in M_{CDM} . Further, the researcher [33] presented information entropy measure with the generalized form of I_{FS} and utilized in decision-making problems.

With this information as background, in the next part, new entropy measure is proposed to the picture fuzzy set (P_{FS}) as the generalized form of I_{FS} .

Definition 9. The real function θ is defined on $Q \in P_{FS}(Z)$. The new probability type picture information measure is proposed as follows:

$$\theta(Q) = \frac{1.582}{n \exp} \sum_{i=1}^n \exp - \exp |\mu_Q(Z_i) + \eta_Q(Z_i) - \nu_Q(Z_i) - \pi_Q(Z_i)|; \quad Q \text{ and } Z_i \in P_{FS}(Z),$$

where $\mu_Q(Z_i) \rightarrow$ positive membership degree,

$\eta_Q(Z_i) \rightarrow$ neutral membership degree,

$\nu_Q(Z_i) \rightarrow$ negative membership degree,

$\pi_Q(Z_i) \rightarrow$ degree of the refusal membership

$$1.582 = \frac{\text{Exp}}{\text{Exp} - 1}.$$

The validity of information measure is proved if it satisfied the fundamental postulates of entropy measure listed in Definition 8.

4.1. Validation of Proposed Information Measure. Firstly, the following inequalities for any Q and $R \in P_{FS}(Z)$: Q crisper than R listed in axioms P_{FS3} are proved:

$$\left| \mu_Q(Z_i) - \frac{1}{4} \right| + \left| \eta_Q(Z_i) - \frac{1}{4} \right| + \left| \nu_Q(Z_i) - \frac{1}{4} \right| + \left| \pi_Q(Z_i) - \frac{1}{4} \right| \geq \left| \mu_R(Z_i) - \frac{1}{4} \right| + \left| \eta_R(Z_i) - \frac{1}{4} \right| + \left| \nu_R(Z_i) - \frac{1}{4} \right| + \left| \pi_R(Z_i) - \frac{1}{4} \right|, \quad (14)$$

and

$$\begin{aligned} & \left(\mu_Q(Z_i) - \frac{1}{4}\right)^2 + \left(\eta_Q(Z_i) - \frac{1}{4}\right)^2 + \left(\nu_Q(Z_i) - \frac{1}{4}\right)^2 + \left(\pi_Q(Z_i) - \frac{1}{4}\right)^2 \geq \left(\mu_R(Z_i) - \frac{1}{4}\right)^2 + \left(\eta_R(Z_i) - \frac{1}{4}\right)^2 \\ & + \left(\nu_R(Z_i) - \frac{1}{4}\right)^2 + \left(\pi_R(Z_i) - \frac{1}{4}\right)^2. \end{aligned} \quad (15)$$

Proof. If $\mu_Q(Z_i) \leq \mu_R(Z_i)$, $\eta_Q(Z_i) \leq \eta_R(Z_i)$, $\nu_Q(Z_i) \leq \nu_R(Z_i)$ and $\pi_Q(Z_i) \geq \pi_R(Z_i)$ with $\max\langle \mu_R(Z_i), \eta_R(Z_i), \nu_R(Z_i) \rangle \leq 1/4 \Rightarrow \mu_Q(Z_i) \leq \mu_R(Z_i) \leq 1/4$ and $\eta_Q(Z_i) \leq \eta_R(Z_i) \leq 1/4$, $\nu_Q(Z_i) \leq \nu_R(Z_i) \leq 1/4$ and $\pi_Q(Z_i) \geq \pi_R(Z_i) \geq 1/4$. This proves that equations (14) and (15) are correct.

If $\mu_Q(Z_i) \geq \mu_R(Z_i)$, $\eta_Q(Z_i) \geq \eta_R(Z_i)$, $\nu_Q(Z_i) \geq \nu_R(Z_i)$ and $\pi_Q(Z_i) \leq \pi_R(Z_i)$ with $\min\langle \mu_R(Z_i), \eta_R(Z_i), \nu_R(Z_i) \rangle \geq 1/4 \Rightarrow \mu_Q(Z_i) \geq \mu_R(Z_i) \geq 1/4$ and $\eta_Q(Z_i) \geq \eta_R(Z_i) \geq 1/4$, $\nu_Q(Z_i) \geq \nu_R(Z_i) \geq 1/4$ and $\pi_Q(Z_i) \leq \pi_R(Z_i) \leq 1/4$ which hold (16) and (20).

There are two distance measures, i.e., Hamming and Euclidean distance which are widely used to determine the distance between the two different I_{FSS} . The authors in [34] introduced the concept of distance between the two different

I_{FSS} specifically in terms of their parameters (μ, ν, π) . Subsequently, P_{FSS} is the generalized form of I_{FSS} which have four components, i.e., (μ, ν, η, π) . Therefore, this concept of distance measure is significantly used from I_{FSS} to P_{FSS} . Thus, inequalities (14) and (15) concluded that P_{FSR} is nearest to the maximum value $(1/4, 1/4, 1/4, 1/4)$ than Q . $\square$

Theorem 1. To validate the proof of the novel entropy information measure for P_{FS} .

Proof. The proof of the validity of the proposed measure will be established if it agreed with the axioms listed in Definition 8.

P_{FS1} (sharpness). Initially, we assume $\theta(Q) = 0$.

$$\theta(Q) = \frac{1}{n} \sum_{i=1}^n \frac{1.582}{\exp} (\exp - \exp |\mu_Q(Z_i) + \eta_Q(Z_i) - \nu_Q(Z_i) - \pi_Q(Z_i)|): Q \text{ and } Z_i \in P_{FS}(Z). \quad (16)$$

This will hold if following cases are possible:

If $\mu_Q(Z_i) = 1$,

$$\eta_Q(Z_i) = \nu_Q(Z_i) = \pi_Q(Z_i) = 0,$$

or,

$$\eta_Q = 1, \mu_Q(Z_i) = \nu_Q(Z_i) = \pi_Q(Z_i) = 0,$$

or,

$$\nu_Q(Z_i) = 1,$$

$$\mu_Q(Z_i) = \eta_Q(Z_i) = \pi_Q(Z_i) = 0,$$

or,

$$\pi_Q(Z_i) = 1,$$

$$\mu_Q(Z_i) = \eta_Q(Z_i) = \nu_Q(Z_i) = 0.$$

Hence, all the above cases show that $\theta(Q) = 0$. This implies that θ is the crisp set. Therefore conversely, assume that θ is the crisp set. This provides

$$\text{either } \mu_Q(Z_i) = 1$$

$$\eta_Q(Z_i) = \nu_Q(Z_i)$$

$$= \pi_Q(Z_i) = 0,$$

or

$$\eta_Q = 1, \mu_Q(Z_i) = \nu_Q(Z_i)$$

$$= \pi_Q(Z_i) = 0,$$

or

$$\nu_Q(Z_i) = 1$$

$$\mu_Q(Z_i) = \eta_Q(Z_i) = \pi_Q(Z_i) = 0,$$

or

$$\pi_Q(Z_i) = 1, \mu_Q(Z_i)$$

$$= \eta_Q(Z_i)$$

$$= \nu_Q(Z_i) = 0.$$

Thus, $\theta(Q) = 0 \Leftrightarrow Q$ is a crisp set.

P_{FS2} (maximality). Equation (16) has the maximum value for

$$\theta(Q) = \frac{1}{n} \sum_{i=1}^n \frac{1.582}{\exp} (\exp - \exp |\mu_Q(Z_i) + \eta_Q(Z_i) - \nu_Q(Z_i) - \pi_Q(Z_i)|): Q, \quad (19)$$

$$Z_i \in P_{FS}(Z).$$

It is obvious that if $\mu_Q = \eta_Q = \nu_Q = \pi_Q = 1/4$, this implies that entropy measure has the maximum value.

P_{FS3} (resolution). The property P_{FS2} shows that $\theta(Q)$ attains the maximum value if $\mu_Q = \eta_Q = \nu_Q = \pi_Q = 1/4$. Therefore, if $\max \langle \mu_Q(Z_i), \eta_Q(Z_i), \nu_Q(Z_i) \rangle \leq 1/4$, and, $\mu_Q \leq \mu_R, \eta_Q \leq \eta_R$, and $\nu_Q \leq \nu_R$. This concludes $\pi_Q(Z_i) \geq \pi_R(Z_i) \geq 1/4$.

Subsequently, if minimum $\langle \mu_Q(Z_i), \eta_Q(Z_i), \nu_Q(Z_i) \rangle \geq 1/4$, Then, $\mu_Q \geq \mu_R, \eta_Q \geq \eta_R$, and $\nu_Q \geq \nu_R$. Thus, $\pi_Q(Z_i) \leq \pi_R(Z_i) \leq 1/4$. Therefore, use of equations (14) and (15). This is concluded that $\theta(Q)$ holds the property of resolution.

P_{FS4} (symmetry). This property is satisfied directly from the definition, i.e., $\theta(Q) = \theta(Q^c)$.

This is the complete proof of the theorem. $\square$

Theorem 2. . If any two $Q, R \in P_{FS}(Z): Q \subseteq R$ or $R \subseteq Q$, then

$$\theta(Q \cup R) + \theta(Q \cap R) = \theta(Q) + \theta(R). \quad (20)$$

Proof. Let Z_1 and Z_2 be the two parts of $\in Z$ (universal set) such that

$$Z_1 = \langle Z_i \in Z: Q \subseteq R \rangle, \quad (21)$$

$$\langle Z_2 = Z_i \in Z: R \subseteq Q \rangle,$$

$$\mu_Q \leq \mu_R$$

$$\eta_Q \leq \eta_R,$$

$$\nu_Q \geq \nu_R \text{ for all } Z_i \in Z_1,$$

$$\mu_Q \geq \mu_R,$$

$$\eta_Q \geq \eta_R,$$

$$\nu_Q \leq \nu_R \text{ for all } Z_i \in Z_2, \quad (22)$$

$$\begin{aligned} \theta(Q \cup R) &= \frac{1.582}{n \exp} \sum_{i=1}^n (\exp - \exp |\mu_{(Q \cup R)}(Z_i) + \eta_{(Q \cup R)}(Z_i) - \nu_{(Q \cup R)}(Z_i) - \pi_{(Q \cup R)}(Z_i)|), \\ \theta(Q \cup R) &= \frac{1.582}{n \exp} \sum_{Z_1} (\exp - \exp |\mu_{(Q \cup R)}(Z_i) + \eta_{(Q \cup R)}(Z_i) - \nu_{(Q \cup R)}(Z_i) - \pi_{(Q \cup R)}(Z_i)|) \\ &\quad + \frac{1.582}{n \exp} \sum_{Z_2} (\exp - \exp |\mu_{(Q \cup R)}(Z_i) + \eta_{(Q \cup R)}(Z_i) - \nu_{(Q \cup R)}(Z_i) - \pi_{(Q \cup R)}(Z_i)|), \\ \theta(Q \cup R) &= \frac{1.582}{n \exp} \sum_{Z_1} (\exp - \exp |\mu_R(Z_i) + \eta_R(Z_i) - \nu_R(Z_i) - \pi_R(Z_i)|) \\ &\quad + \frac{1.582}{n \exp} \sum_{Z_2} (\exp - \exp |\mu_Q(Z_i) + \eta_Q(Z_i) - \nu_Q(Z_i) - \pi_Q(Z_i)|). \end{aligned} \quad (23)$$

As accordingly,

$$\begin{aligned}
\theta(Q \cap R) &= \frac{1.582}{n \exp} \sum_{Z_1} \left(\exp - \exp \left| \mu_{(Q \cap R)}(Z_i) + \eta_{(Q \cap R)}(Z_i) - \nu_{(Q \cap R)}(Z_i) - \pi_{((Q \cap R))}(Z_i) \right| \right) \\
&\quad + \frac{1.582}{n \exp} \sum_{Z_2} \left(\exp - \exp \left| \mu_{(Q \cap R)}(Z_i) + \eta_{(Q \cap R)}(Z_i) - \nu_{(Q \cap R)}(Z_i) - \pi_{(Q \cap R)}(Z_i) \right| \right), \\
\theta(Q \cap R) &= \frac{1.582}{n \exp} \sum_{Z_1} \left(\exp - \exp \left| \mu_Q(Z_i) + \eta_Q(Z_i) - \nu_Q(Z_i) - \pi_Q(Z_i) \right| \right) \\
&\quad + \frac{1.582}{n \exp} \sum_{Z_2} \left(\exp - \exp \left| \mu_R(Z_i) + \eta_R(Z_i) - \nu_R(Z_i) - \pi_R(Z_i) \right| \right).
\end{aligned} \tag{24}$$

Therefore, equations (23) and (24) prove that

$$\theta(Q \cup R) + \theta(Q \cap R) = \theta(Q) + \theta(R). \quad (25) \quad \square$$

5. The Proposed Picture Fuzzy Entropy Measure with Application in M_{CDM} Process

The M_{CDM} process involves the different techniques to solve the decision and planning problems, which involve multiple criteria. The objective is to support decision-maker in tackling such problems. To attain the unique optimal solution of such problems, the decision-makers fixed a set of alternatives and criteria. The fuzzy set and its various extended forms were included in M_{CDM} techniques because of the vagueness and ambiguity in the data analysis. Therefore, fuzzy sets have a great importance in the decision-making process. The M_{CDM} problems may be symbolized as a matrix say $(P_{FDM})_{\tilde{m} \times \tilde{n}}$ which represents the $\tilde{m}$ alternatives $\{ALT_1, ALT_2, \dots, ALT_m''\}$ and $\tilde{n}$ criteria $\{CR_1, CR_2, \dots, CR_n''\}$ in the shape of rows and columns or vice versa. The entries in $(P_{FDM})_{\tilde{m} \times \tilde{n}}$ refer the degrees in accordance with P_{FS} described in Definition 3. In practical problems where the human opinions are there, P_{FS} is comparatively useful. Let the group of decision-makers $[\tilde{N}]$ expressed in P_{FS} describe the (+) membership degree for support $[\tilde{n}_{pos}(ij)]$, neutral membership degree $[\tilde{n}_{neu}(ij)]$ signify the undecided or neutral, and (-) membership $[\tilde{n}_{neg}(ij)]$ represent unfavour of alternative for specific criteria. Consequently, the following statistical tools are used to express different degrees pertaining to P_{FS} .

$$\begin{aligned}
\check{\mu}_{ij} &= \left\{ \frac{\tilde{n}_{pos}(ij)}{\tilde{N}} \right\}, \\
\check{\eta}_{ij} &= \left\{ \frac{\tilde{n}_{neu}(ij)}{\tilde{N}} \right\}, \\
\check{\nu}_{ij} &= \left\{ \frac{\tilde{n}_{neg}(ij)}{\tilde{N}} \right\}.
\end{aligned} \tag{26}$$

The statistical tool mentioned in (26) is significantly used if the available data are in the form of numerical quantity. In decision-making analysis, the different degrees of P_{FS} are

entirely reliant upon the single opinion or choice of decision-makers apart of any kind of information to render a decision. Such practical problems in P_{FS} decision-making process are solved more effectively with greater accuracy.

5.1. Picture Fuzzy Sets with Proposed VIKOR Approach.

The author [35] was the pioneer of VIKOR approach to obtain the most desired alternatives based upon the closeness index to the (+) ideal solution and farthest from the (-) ideal solution in decision-making problems. The key salient feature of present approach helps to provide the compromise solution. Initially, the idea of compromise solution was developed by [36] in 1973. The following procedural steps are widely used to solve the fuzzy decision-making problems.

Step 1. Formation of picture fuzzy decision matrix $(P_{FDM})_{\tilde{m} \times \tilde{n}}$.

$(P_{FDM})_{\tilde{m} \times \tilde{n}}$ is constructed in such a way that entries of the matrix elements are computed in accordance with different membership degrees of P_{FS} briefed in equation (26).

Step 2. Normalization of $(P_{FDM})_{\tilde{m} \times \tilde{n}}$.

The fabricated matrix $(P_{FDM})_{\tilde{m} \times \tilde{n}}$ is normalized as shown in Table 1.

Where, $\{\check{\mu}_{ij}, \check{\eta}_{ij}, \check{\nu}_{ij}\}$

$$= \{\mu_{ij}, \eta_{ij}, \nu_{ij} \text{ for benefit criteria } \nu_{ij}, \mu_{ij} \text{ for cost criteria.} \quad (27)$$

Step 3. Assessment of criteria weights

The choice of selecting the weights of criteria or attributes has a substantial role in the way out of the solution of M_{CDM} problems. The appropriate and justified assessment of criteria weights might lead to the desired result of a suitable alternative. Considering the classified the criteria weights in two categories, i.e., *subjective and objective weights* [37], the objective weight apprehends the criteria information in a quantified manner; therefore, entropy practice is the best reliable approach to capture and handle such information. Sometimes, in M_{CDM} process, the decision-makers have the criteria weights information in separate manner, i.e., entirely known or unknown and may be partially known according to problem scenario.

TABLE 1: The fabricated matrix $(P_{FDM})_{\tilde{m} \times \tilde{n}}$.

	CR_1	CR_2	...	CR_n
ALT ₁	$\{\tilde{\mu}_{11}, \tilde{\eta}_{11}, \tilde{\nu}_{11}\}$	$\{\tilde{\mu}_{12}, \tilde{\eta}_{12}, \tilde{\nu}_{12}\}$	...	$\{\tilde{\mu}_{1n}, \tilde{\eta}_{1n}, \tilde{\nu}_{1n}\}$
ALT ₂	$\{\tilde{\mu}_{21}, \tilde{\eta}_{21}, \tilde{\nu}_{21}\}$	$\{\tilde{\mu}_{22}, \tilde{\eta}_{22}, \tilde{\nu}_{22}\}$	...	$\{\tilde{\mu}_{2n}, \tilde{\eta}_{2n}, \tilde{\nu}_{2n}\}$
...	...	...	...	...
ALT _m	$\{\tilde{\mu}_{m1}, \tilde{\eta}_{m1}, \tilde{\nu}_{m1}\}$	$\{\tilde{\mu}_{m2}, \tilde{\eta}_{m2}, \tilde{\nu}_{m2}\}$	...	$\{\tilde{\mu}_{mn}, \tilde{\eta}_{mn}, \tilde{\nu}_{mn}\}$

Note. $\{\tilde{\mu}_{ij}, \tilde{\eta}_{ij}, \tilde{\nu}_{ij}\} = \{\mu_{ij}, \eta_{ij}, \nu_{ij}\}$ for benefit criteria $\nu_{ij}, \eta_{ij}, \mu_{ij}$ for cost criteria.

As a result, the following approaches are used to compute the criteria weights for both the cases:

- (a) *Methodology for Unknown Criteria Weights.* To compute the unspecified criteria-weight, the author [37] proposed the method.

$$\hat{w}_j = \frac{I - \theta(Q)_j}{n - \sum_{j=1}^n \theta(Q)_j}; \quad j = 1, 2, 3, \dots, n, \quad (28)$$

where

$$\theta(Q) = \frac{1}{n} \sum_{i=1}^m \frac{1.582}{\exp} \left(\exp - \exp |\mu_{ij} + \eta_{ij} - \nu_{ij} - \pi_{ij}| \right). \quad (29)$$

- (b) *Methodology for Partially Known Criteria Weights.*

There are many complexities which are cited by the decision-makers to solve the problems in the decision-making system such as paucity of time, information, and lack of expertise around problem domain. Therefore, the decision-makers are enabled to give their judgment or decision precisely in the form of numeral value. They often express their opinions of the judgment in intervals to overcome such incongruities. In this framework, to explore the attribute weights from the set of partial information, the decision-makers generally use the method of minimum entropy proposed by [38]. Hence, the alternatives entropy across the distinct criteria CR_j is presented as follows:

$$\hat{\alpha}(ALT_i) = \sum_{j=1}^n \hat{\alpha}(\mu_{ij}, \eta_{ij}, \nu_{ij}, \pi_{ij}), \quad (30)$$

$$\text{such that } \hat{\alpha}(\mu_{ij}, \eta_{ij}, \nu_{ij}, \pi_{ij}) = \frac{1}{n} \sum_{i=1}^m \frac{1.582}{\exp} \left(\exp - \exp |\mu_{ij} + \eta_{ij} - \nu_{ij} - \pi_{ij}| \right).$$

Thus, the following programming model is frequently used by the decision-makers for the optimum criteria evaluation:

$$(\hat{E}) = \sum_{i=1}^M W_j \hat{\alpha}(ALT_i) = \frac{1.582}{n \exp} \sum_{i=1}^m \sum_{j=1}^n w_j \left(\exp - \exp |\mu_{ij} + \eta_{ij} - \nu_{ij} - \pi_{ij}| \right), \quad (31)$$

$$\text{where } \begin{cases} w \in \beta \sum_{j=1}^n w_j = 1 \\ w_j \geq 1: \quad j = 1, 2, \dots, n. \end{cases}$$

β denotes the set of partial known informations to evaluate the weights of different criteria.

Step 4. Determination of the relative ideal solution for P_{FS} .

In M_{CDM} problems, the ideal solution provides the best alternatives which maximize the most benefited attribute or criteria and minimize the least cost criteria. The objective of the VIKOR method is to opt the best alternative nearest to the (+) ideal solution along with farthest away from the (-)

ideal solution. Firstly, the picture fuzzy relative (+) ideal solution and (-) ideal solution are constructed to determine the alternatives.

For benefit criteria,

$$\begin{aligned} ALT^+ &= (ALT_1^+, ALT_2^+, \dots, ALT_n^+), \\ ALT^- &= (ALT_1^-, ALT_2^-, \dots, ALT_n^-), \end{aligned} \quad (32)$$

where

$$\begin{aligned}
ALT_j^+ &= (\mu_{ij}^+, \eta_{ij}^+, \nu_{ij}^+) = ((\check{\mu}_{ij}), (\check{\eta}_{ij}), (\check{\nu}_{ij})), \\
\pi_{ij}^+ &= 1 - \mu_{ij}^+ - \eta_{ij}^+ - \nu_{ij}^+; \quad j = 1, 2, \dots, n, \\
ALT_j^- &= (\mu_{ij}^-, \eta_{ij}^-, \nu_{ij}^-) = ((\check{\mu}_{ij}), (\check{\eta}_{ij}), (\check{\nu}_{ij})), \\
\pi_{ij}^- &= 1 - \check{\mu}_{ij} - \check{\eta}_{ij} - \check{\nu}_{ij}; \quad j = 1, 2, \dots, n.
\end{aligned} \quad (33)$$

For cost criteria,

$$\begin{aligned}
ALT^+ &= (ALT_1^+, ALT_2^+, \dots, ALT_n^+), \\
ALT^- &= (ALT_1^-, ALT_2^-, \dots, ALT_n^-).
\end{aligned} \quad (34)$$

where

$$\begin{aligned}
ALT_j^+ &= (\mu_{ij}^+, \eta_{ij}^+, \nu_{ij}^+) = ((\check{\mu}_{ij}), (\check{\eta}_{ij}), (\check{\nu}_{ij})), \\
\pi_{ij}^+ &= 1 - \mu_{ij}^+ - \eta_{ij}^+ - \nu_{ij}^+; \quad j = 1, 2, \dots, n, \\
ALT_j^- &= (\mu_{ij}^-, \eta_{ij}^-, \nu_{ij}^-) = ((\check{\mu}_{ij}), (\check{\eta}_{ij}), (\check{\nu}_{ij})), \\
\pi_{ij}^- &= 1 - \check{\mu}_{ij} - \check{\eta}_{ij} - \check{\nu}_{ij}; \quad j = 1, 2, \dots, n.
\end{aligned} \quad (35)$$

Step 5. Evaluation of compromise solution

In decision sciences, the VIKOR method is very useful for the compromise solution, when the preference of decision-makers about alternatives is conflicting from the set of criteria information. In the VIKOR method, the compromise solution is obtained with the use of (L_p) metric which was earlier introduced by [36]. The metric (L_p) is an aggregate function used as follows:

$$\begin{aligned}
\hat{L}_{p,i} &= \left[\sum_{j=1}^n \left(w_j \frac{(ALT^+ - \hat{\Phi}_{ij})}{(ALT^+ - ALT^-)} \right)^p \right]^{1/p}, \\
p &\in [1, \infty), i \in [1, 2, \dots, m],
\end{aligned} \quad (36)$$

where ALT^+ and ALT^- are described in equations (33) and (35).

$$\begin{aligned}
\hat{\Phi}_{ij} &= (\check{\mu}_{ij}, \check{\eta}_{ij}, \check{\nu}_{ij}), \\
w_j &\text{ are the weight of the criteria } j^{th}; \quad j = 1, \dots, n.
\end{aligned} \quad (37)$$

The parameter p has the distinct values varying between $[1, \infty)$ which imply the different meaning in perspective of group utility and individual regret. As the increased value of p , the individual regret is substantially increased and the group utility or efficacy is decreased. Hence, the VIKOR

method gives the compromise solution with maximum group efficacy ($L_{1,i}$) and reduces the individual regret ($L_{\infty,i}$) as follows:

$$\begin{aligned}
\hat{G}_{ui} &= \hat{L}_{1,i} \\
&= \sum_{j=1}^n \left\{ w_j \frac{(ALT^+ - \hat{\Phi}_{ij})}{(ALT^+ - ALT^-)} \right\},
\end{aligned} \quad (38)$$

$$\begin{aligned}
\hat{I}_{Ri} &= \hat{L}_{\infty,i} \\
&= \sum_{j=1}^n \left\{ w_j \frac{(ALT^+ - \hat{\Phi}_{ij})}{(ALT^+ - ALT^-)} \right\}.
\end{aligned} \quad (39)$$

Step 6. VIKOR index coefficient.

The index coefficient (I_c) in the VIKOR method is computed as

$$\begin{aligned}
\hat{I}_c &= \tilde{\Theta} \left\{ \frac{\hat{G}_{ui} - \hat{G}_{u*}}{\hat{G}_{u*} - \hat{G}_{u*}} \right\} + (1 - \tilde{\Theta}) \left\{ \frac{\hat{I}_{Ri} - \hat{I}_{R*}}{\hat{I}_{R*} - \hat{I}_{R*}} \right\}, \\
\text{where } \begin{cases} \hat{G}_{u*} = \hat{G}_{ui}, \\ \hat{G}_{u*} = \hat{G}_{ui}, \\ \hat{I}_{R*} = \hat{I}_{Ri}, \\ \hat{I}_{R*} = \hat{I}_{Ri}. \end{cases}
\end{aligned} \quad (40)$$

In addition, the parameters $(\tilde{\Theta})$ and $(1 - \tilde{\Theta})$ represent the group utility weight and individual regret as per the choice of its value by decision-makers as follows:

$$IF\tilde{\Theta} = \{0.5 \Rightarrow \text{consensus} > 0.5 \Rightarrow \text{voting by majority} < 0.5 \Rightarrow \text{Veto}\}. \quad (41)$$

Step 7. Finally, the ascending order of the determined values of G_{ui} , I_{Ri} , and I_c is arranged, and for the most preferred alternatives, the CONI and CONII are two respective conditions satisfied as

CONI $\Rightarrow$ Acceptable Advantage : IF the value,

$$\left[\hat{I}(ALT^2) - \hat{I}(ALT^1) \right] \geq \frac{1}{n-1}; \quad ALT^1 \text{ and } ALT^2 \text{ are cataloged at } 1^{st} \text{ and } 2^{nd} \text{ place in distinct value of } \hat{I}_c,$$

CONII $\Rightarrow$ Acceptable Stability : ALT_1 must be cataloged at 1^{st} place in the distinct value of either $\hat{G}_{ui}$ and $\hat{I}_{Ri}$.

(42)

If the stated condition does not satisfy together, then we look upon a compromise solution proceed as follows.

Case 1. ALT^1 and ALT^2 are the compromise solutions if *Acceptable Stability* condition does not satisfy.

$$\left[\hat{I}(ALT^m) - \hat{I}(ALT^1) \right] < \frac{1}{n-1}; m \Rightarrow \text{for some maximum and } n \text{ implies the number of alternatives.} \quad (43)$$

5.1.1. Picture Fuzzy Sets with Proposed TOPSIS Approach. The concept of TOSIS approach initiates and incorporates in M_{CDM} decision sciences by [39]. Since then, this method is prominently used to many complex practical problems in a fuzzy situation. The selection process of the best attributes in the proposed approach is to focus on the alternatives that one closest to the ideal solution and farthest, respectively, from the $(-)$ ideal solution.

Further, to complete the calculative part of the M_{CDM} problem, we carry forward the first four steps stated above in the first approach.

Step 8. This step involves the picture fuzzy discriminant measure E_D^+ and E_D^- for distinct attributes from ALT^+ and ALT^- , respectively, with use of equation (7).

$$\begin{aligned} E_D^+ &= \left[\sum_{j=1}^n w_j \left\{ \hat{\Phi}_{ij} - ALT^+ \right\}^2 \right]^{0.5}, \\ E_D^- &= \left[\sum_{j=1}^n w_j \left\{ \hat{\Phi}_{ij} - ALT^- \right\}^2 \right]^{0.5}. \end{aligned} \quad (44)$$

Step 9. The relative closeness coefficient (R_{cc}) is computed as follows:

$$R_{cc} = \left\{ \frac{E_D^-}{E_D^- + E_D^+} \right\}. \quad (45)$$

Step 10. At the end, alternatives ranking have been done according to the ordering the distinct values of R_{cc} . The larger the value is, the most suitable the alternatives are. Thus, we have successfully presented the procedural steps to solve the picture fuzzy M_{CDM} problems by employing the proposed picture fuzzy entropy measure with the modified form of VIKOR and TOPSIS and for the sake of understanding the flow chart shown in Figure 3.

6. Validation of Proposed Entropy Measure with Example

The application of the PFS is to solve the MCDM problems, where simultaneously several attributes are likely to exist. In such type of environment, the human mindsets are always in dilemma; therefore, P_{FSS} is considerably used. In that point of view, a national welfare society conducted the social

Case 2. If Acceptable Advantage is not accepted, then alternatives $ALT^1, ALT^2, \dots, ALT^m$ have the compromise solution along with ALT^m if it does hold the following condition:

survey to get the public opinion for the functioning and implementation of the government policies for the national importance. They fix the framework of six different criteria, i.e.,

- CR_1 : unemployment
- CR_2 : foreign relations
- CR_3 : national education policy
- CR_4 : financial services
- CR_5 : strategy for COVID – 19
- CR_6 : national policies for social development and empowerment

And the national parties are classified into five categories, i.e., " $ALT_1, ALT_2, ALT_3, ALT_4, ALT_5$ ". The survey was conducted on a sample of $1000 = N$ peoples. The results show 450 peoples in support with CR_1 , 320 peoples unsupport with CR_1 , 120 peoples are neutral, and 110 peoples are not participated, i.e., abstain. Then, with the use of equation (26), the entries of the P_{FS} matrix can be determined as follows:

$$\begin{aligned} \check{\mu}_{11} &= \frac{450}{1000}, \\ \check{\eta}_{11} &= \frac{120}{1000}, \\ \check{\nu}_{11} &= \frac{320}{1000}. \end{aligned} \quad (46)$$

Further, entries of $(P_{FDM})_{5 \times 6}$ are computed in the same manner.

VIKOR approach for proposed entropy measure is as follows:

- (1) The accumulated information as per opinion of peoples constitutes the normalized $(P_{FDM})_{5 \times 6}$ listed in Table 2.
- (2) The notion P_{FDMW} used for the calculated values of different criteria with information measure mentioned in (14) is listed in Table 3.
- (3) Utilizing equation (28), the weights are determined for the defined criteria throughout the alternatives with use of values P_{FDMW} catalogued in Table 4.
- (4) The values for ALT^+ and ALT^- are computed with use of equation (34) as listed in Table 5.

FIGURE 3: The procedural steps to solve the picture fuzzy M_{CDM} problems by employing the proposed picture fuzzy entropy measure with the modified form of VIKOR and TOPSIS.

TABLE 2: Opinion of peoples constitutes the normalized $(P_{FDM})_{5 \times 6}$.

CR_1	CR_2	CR_3	CR_4	CR_5	CR_6
ALT ₁ 0.45, 0.32, 0.12	0.34, 0.15, 0.07	0.69, 0.06, 0.17	0.78, 0.02, 0.12	0.84, 0.04, 0.08	0.13, 0.53, 0.05
ALT ₂ 0.46, 0.05, 0.12	0.18, 0.13, 0.31	0.81, 0.11, 0.04	0.69, 0.12, 0.11	0.36, 0.32, 0.31	0.05, 0.66, 0.16
ALT ₃ 0.24, 0.27, 0.12	0.57, 0.10, 0.27	0.81, 0.03, 0.10	0.48, 0.24, 0.23	0.17, 0.58, 0.16	0.77, 0.04, 0.12
ALT ₄ 0.36, 0.24, 0.08	0.4, 0.23, 0.34	0.08, 0.72, 0.06	0.23, 0.14, 0.15	0.28, 0.28, 0.28	0.59, 0.10, 0.20
ALT ₅ 0.40, 0.30, 0.14	0.56, 0.33, 0.07	0.64, 0.12, 0.21	0.56, 0.13, 0.11	0.51, 0.10, 0.36	0.35, 0.39, 0.12

TABLE 3: Calculated values of different criteria with information measure.

	CR_1	CR_2	CR_3	CR_4	CR_5	CR_6
P_{FDMV}	0.82895	0.72463	0.48285	0.65164	0.69809	0.66986

TABLE 4: The weights for the defined criteria throughout the alternatives with use of values P_{FDMW} .

$\hat{W}_1$	$\hat{W}_2$	$\hat{W}_3$	$\hat{W}_4$	$\hat{W}_5$	$\hat{W}_6$
0.08798	0.14165	0.26602	0.17920	0.15530	0.16982

TABLE 5: The values for ALT^+ and ALT^- .

ALT^+	0.46, 0.05, 0.08	0.57, 0.10, 0.07	0.81, 0.03, 0.04	0.78, 0.02, 0.11	0.84, 0.04, 0.08	0.77, 0.04, 0.05
ALT^-	0.24, 0.32, 0.14	0.18, 0.33, 0.34	0.08, 0.72, 0.21	0.23, 0.24, 0.23	0.17, 0.58, 0.36	0.05, 0.66, 0.20

TABLE 6: The computed values for $\hat{G}_{ui}$, $\hat{I}_{Ri}$, and $\hat{I}_c$.

	ALT_1	ALT_2	ALT_3	ALT_4	ALT_5
$\hat{G}_{ui}$	0.3038	0.4965	0.3154	0.6652	0.4024
$\hat{I}_{Ri}$	0.134	0.1831	0.099	0.2425	0.1103
$\hat{I}_c$	0.1219	0.5596	0.016	1	0.1757

TABLE 7: The computed values for $\hat{G}_{ui}$, $\hat{I}_{Ri}$, and $\hat{I}_c$.

	ALT_1	ALT_2	ALT_3	ALT_4	ALT_5
$\hat{G}_{ui}$	I	IV	II	V	III
$\hat{I}_{Ri}$	III	IV	I	V	II
$\hat{I}_c$	II	IV	I	V	III

- (5) Table 6 briefs the computed values for $\hat{G}_{ui}$, $\hat{I}_{Ri}$, and $\hat{I}_c$ by using equations (39), (40), and (41).
- (6) In accordance with the computed values for $\hat{G}_{ui}$, $\hat{I}_{Ri}$, and $\hat{I}_c$, the ranking of the alternatives is shown in Table 7.
- (7) The indexing shows ALT_1 and ALT_3 located at 2nd and 1st position in Table 7.

The CONI condition $[\hat{I}(ALT)^1 - \hat{I}(ALT)^3] = 0.1219 - 0.016 = 0.1059 \geq \frac{1}{6-1} = 0.2$ does not hold.

Therefore, we proceed for the compromise solution, i.e.,

$$\begin{aligned}
 [\hat{I}(ALT^1) - \hat{I}(ALT^3)] &= 0.1219 - 0.016 \\
 &= 0.1059 < \frac{1}{6-1} = 0.2, \\
 [\hat{I}(ALT^2) - \hat{I}(ALT^3)] &= 0.5596 - 0.016 \\
 &= 0.5436 > \frac{1}{6-1} = 0.2, \\
 [\hat{I}(ALT^4) - \hat{I}(ALT^3)] &= 1 - 0.016 \\
 &= 0.984 > \frac{1}{6-1} = 0.2, \\
 [\hat{I}(ALT^5) - \hat{I}(ALT^3)] &= 0.1757 - 0.016 \\
 &= 0.1597 < \frac{1}{6-1} = 0.2.
 \end{aligned} \tag{47}$$

Thus, ALT^1 , ALT^3 , and ALT^5 hold the CONII condition, and the alternatives ALT^1 , ALT^3 , and ALT^5 are the compromise solution.

TABLE 8: The picture fuzzy discriminant measures E_D^+ and E_D^- are computed for distinct attributes from ALT^+ and ALT^- .

	ALT_1	ALT_2	ALT_3	ALT_4	ALT_5
E_D^+	0.42005	0.48385	0.40190	0.63509	0.33341
E_D^-	0.59931	0.57940	0.72670	0.41747	0.61122

TABLE 9: The value of relative closeness coefficient (R_{cc}).

	ALT_1	ALT_2	ALT_3	ALT_4	ALT_5
	0.5879	0.5449	0.6438	0.3966	0.6470

TABLE 10: The indexing of the alternatives in descending order.

ALT_1	ALT_2	ALT_3	ALT_4	ALT_5
III	IV	II	V	I

TOPSIS approach

Proceed with the same manner up to Step 4.

- (8) The picture fuzzy discriminant measures E_D^+ and E_D^- are computed for distinct attributes from ALT^+ and ALT^- , respectively, with use of equation (46) and listed in Table 8.
- (9) The value of relative closeness coefficient (R_{cc}) degree with use of (47) is mentioned in Table 9.

Finally, the indexing of the alternatives in descending order list is shown in Table 10.

From the indexing of the alternatives, it is concluded that ALT_5 is the most desirable alternative.

7. Comparative Evaluation

To verify the effectiveness and practicability of the proposed information measure with TOSIS and VIKOR approaches, the result prevails in such a way that the TOPSIS method provides the best alternatives ALT_5 whereas VIKOR has a compromise solution, i.e., ALT^1 , ALT^3 , and ALT^5 . Thus, it is quite natural to think about the most benefited alternative. The authors [40] have briefly explained the relative difference between these two approaches for selection of best alternative. The feasible compromise solution in the VIKOR approach formed a consent of mutual compromises focussed on optimising $\hat{G}_{ui}$ and minimising $\hat{I}_{Ri}$; however, the TOPSIS approach adopts the feasible solution without considering the relative importance of the distances which are nearest and farthest from an ideal solution. In the perspective of the proposed example, if the results are interpreted in the broader sense, then it shows that national party ALT_5 is the choice of the peoples in majority and would be able to form the government. In compromise solution, it reveals that the parties ALT^1 , ALT^3 , and ALT^5 are also the choice of the peoples which leads the alliance situation to establish the

government. There is also the possibility of an alliance between the parties either two or three to have a majority criterion to form the government.

8. Concluded Remarks

Till now, several mindsets have developed different approaches or methods of information measure to explore the information in picture fuzzy environment, and still the researchers are working hard to develop such information measures which are more reliable, precise, and informative reasonably with existing measure techniques. The proposed picture novel entropy was validated with the two different approaches. The results reveal that the best alternative is analogous in both approaches and proved that information measure is quite appropriate to solve the complex issues on M_{CDM} system. The trustworthiness and practical applicability of the proposed work proved to improve the voting system and the opinion of the peoples about the parties to the national importance. The proposed model could be used to predict the future outcomes of the polling. The novel research work is significantly suitable to the M_{CDM} complex multivariable problems, relating to the neutral factors or variables. The role of neutral variables in complex issues affects the result of decision-makers. This advantage of the proposed work facilitates the decision-makers to handle the ambiguity that arises due to neutral variables in complex issues. Therefore, it concluded that novel picture information measure is more reliable, sophisticated, and informative as well as simpler, to tackle the uncertainty or vagueness more precisely and effectively in a picture fuzzy environment.

8.1. The Advantage and Future Scope of the Current Research.

The present research work could be extensively applied as a model to predict the forthcoming results in the field of data analysis, i.e., healthcare, agriculture, smart cities, and many more approaches in relation to sustainable development. Further, it can be explored to P_{FSS} in terms of divergence measure, intervalued, and many more. The proposed research work could be bridged between the complex evidence theory as well as quantum mechanics theory to envisage the human decision behaviours in decision theory. Moreover, the presented research work model is also utilized in intelligent generalized quality-based approach which is capable enough to fuse the complex multivariable informations. In fact, decision-makers are comfortable to express the human judgments or suggestions in the linguistic form in picture fuzzy environment. Therefore, present research works can be utilized in fuzzy linguistic models.

Abbreviations

F_{ss} :	Fuzzy set
F_{sss} :	Fuzzy set
I_{Fs} :	Intuitionistic fuzzy set
I_{Fss} :	Intuitionistic fuzzy set
P_{Fs} :	Picture fuzzy set
P_{Fss} :	Picture fuzzy set

M_{CDM} :	Multicriteria decision-making
P_{PFS} :	Pythagorean fuzzy sets
TOPSIS:	Technique for order preference by similarity to ideal solution
VIKOR:	VIse Kriterijumska Optimizacija I Kompromisno Resenje
μ :	Degree of membership function
ν :	Degree of nonmembership function
η :	Degree of neutral-membership function
π :	Degree of hesitency or refusal
G_{ui} :	Group efficacy or group utility
I_{Ri} :	Entity regret or individual regret.

Data Availability

The data used to support the findings of the study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this article.

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