Research Article

Scaling Laws of Spatial Visitation Frequency on Motorized Travel

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The scaling law is a functional relationship between two parameters. In the field of urban studies, the distribution of various phenomena follows the scaling law, e.g., travel time, travel distance, and visitation frequency. To capture the scaling law of spatial visitation frequency on a motorized vehicle, this study carried out a numerical study based on the license plate recognition (LPR) data of three cities, Hangzhou, Guiyang, and Xiaoshan in China. Firstly, according to the LPR data, vehicles are classified into different types, including foreign vehicles, on-demand ride-sourcing cars, and taxis. Then, the spatial visitation frequency of different vehicle types, different cities, and different time periods was calculated. The results show that the scaling law of the spatial visitation frequency always conforms to the exponential function, no matter the city types, time periods, or type of vehicles.

1. Introduction

Exploring the spatial distribution of human mobility patterns is of great importance for traffic forecasting and transportation planning. With the development of Internet and traffic information technology, the means of traffic data collection have been transformed into mass, accurate, low-cost, and widely distributed data sources. The license plate recognition (LPR) data is representative of these merging data sources. The emergence of these data sources offers us an opportunity to study human spatial mobility patterns on a large scale.

Scaling laws may shed light on exploring human spatial mobility patterns. The scaling law is a rule that describes the scale invariance found in many natural phenomena. A scaling law can be described as an expression in which when one parameter changes the other parameter changes in a corresponding proportion.

The scaling laws reveal statistical patterns of human mobility by finding probabilistic distributions of mobility variables. The main research referring to the scaling laws of mobility is concentrated on spatial visitation frequency. Studies on spatial visitation frequency aim at estimating the probability of an individual’s arrival at a specific location, by assuming that the higher the historical visitation frequency to a location, the greater the possibility of attracting visitors. The scaling laws of spatial visitation frequency can be measured by two approaches: frequency-ranking laws and frequency probability distributions. The frequency-ranking laws can be expressed by the relationship between the visitation frequency of a location and the rank of this location ordered by its visitation frequency. Previous studies have found that the frequency-ranking law of spatial visitation frequency conforms to various functional forms such as exponential distribution and power-law distribution. For example, some studies believe that travel distance conforms to power-law distribution with adjustable parameters [1], and travel time is fitted to exponential function [2]. GPS data of taxis were used to explore the frequency-ranking law of taxis, and the relationship came out to conform to the exponential distribution [3]. Meanwhile, the frequency probability distribution is generally accepted to follow a power-law distribution.

Based on the characteristics of LPR data, this is a good data source to verify the universality of the scaling law for different vehicle types and different cities. Therefore, this
article wants to obtain the visitation frequency of different vehicle types by using LPR data and explore the scaling laws of the spatial visitation frequency of different vehicle types in different cities. This work will further verify whether the scaling laws of the spatial visitation frequency under different situations are universality.

The remainder of this article is structured as follows. Section 2 summarizes findings from the previous literature. Section 3 describes the LPR dataset, the process of car types classification, and the calculation of spatial visitation frequency. Section 4 describes the process of approaching the scaling laws. Section 5 analyzes the frequency-ranking laws for different cities, different vehicle types, and different time periods. Section 6 summarizes the contribution of this work and gives future directions for research.

2. Literature Review

Travel is generally defined as the movement of personal location and can be measured by a series of characteristics, including travel time, travel frequency, travel mode, and travel purpose. Generally speaking, the arrival of residents at a certain location conforms to Poisson distribution. However, in recent years, the laws conforming to non-Poisson distribution have attracted much attention, which is represented by some research exploring the scaling laws of human mobility patterns. At present, in the field of human mobility, the main research on the scaling law is focused on spatial frequency and individual position movement [4]. The visitation frequency of a location represents the possibility for residents to visit a certain location. The scaling law of spatial visitation frequency can be expressed in the form of the frequency-ranking law, which is the relationship between the visitation frequency of a location and the rank of this location ordered by its visitation frequency. It is assumed that the higher the historical visitation frequency of a spatial location, the more attractive it is to residents, and the greater the probability that the next arrival will occur at a certain location.

Previous studies have discussed the application of scaling laws in transportation research. For example, Tachet et al. [5] found that ride-sharing potentials of several cities follow the same scaling law, and based on this scaling law, they put forward a method to estimate ride-sharing potentials with basic urban parameters. Kwon [6] explored the relationship between population and urban bus system and found that public transportation accessibility and population follow an exponential scaling law with the exponent lower than 2/3. Yan et al. [7] studied the travel patterns of some volunteers. The results turned out that there was no obvious regularity in the distribution of destinations at the individual level. When it comes to the aggregated level of all volunteers, the scaling law followed a power law with an exponential cutoff.

Many researchers utilized different data sources to explore the scaling laws of spatial visitation frequency. It is found that the frequency-ranking laws may conform to several different distributions, including exponential distribution, exponential distribution with cutoff, power-law distribution, and power-law distribution with cutoff. Giannotti et al. [8] and Liang et al. [9] used GPS data of private cars and taxis and found that the frequency-ranking law accorded with an exponential distribution. Chen et al. [10] used GPS data of on-demand ride-sourcing cars and found that the frequency-ranking law followed an exponential distribution with a cutoff. Song et al. [12] used mobile phone data and found that the frequency-ranking law conformed to a power-law distribution. Hasan et al. [13] used social media data from Twitter and found that the frequency-ranking law of check-in locations accorded to a power-law distribution with a cutoff.

LPR data is an emerging data source with a large amount, wide distribution, and high accuracy. Many studies have applied LPR data to analyze travel behavior and traffic conditions. In terms of the travel behavior, Yao et al. [14] proposed nine features reflecting the commuting travel behavior using the one-month LPR data, and then analyzed the commuting pattern of vehicles. Shen et al. [15] used a geographically and temporally weighted regression (GTWR) model to analyze the influence of land use and household properties on automobile travel demand using the LPR data. In terms of the traffic conditions, Wang et al. [16] proposed a travel time estimation model based on LPR data. Luo et al. [17] proposed a queue length estimation method for signalized intersections based on LPR data. Zhan et al. [18] used LPR data to reconstruct the equivalent cumulative arrival-departure curve of each lane to capture queue length and used queue length as a performance indicator for real-time traffic control applications. Luo et al. [19] proposed a grouped travel time estimation method in signalized arterials using LPR data. LPR data has become a common database used in current traffic flow research [20–22]. However, LPR data has seldom been utilized in research on scaling laws.

In view of the advantage of LPR data in scaling law research, this article will use LPR data to explore scaling laws of spatial visitation frequency in the form of frequency-ranking laws and carry out a comparison of scaling laws between cities, vehicle types, and time periods.

3. Data Collection and Analysis

3.1. License Plate Recognition Data. The LPR data is obtained by license plate recognition devices on the road. Information on license plate numbers and passing time is recorded based on video recognition technology. With LPR data, we can uniquely identify a car by its license plate number. Hence, multidimensional information such as time, location, vehicle features, and vehicle usage features of individual vehicles can be obtained.

This article used LPR datasets from three cities, Hangzhou, Guiyang, and Xiaoshan to explore scaling laws of spatial visitation frequency in the form of frequency-ranking laws. Due to the even distribution of detection devices and a large number of devices, accurate travel information of individual vehicles can be obtained. The LPR dataset
3.2. Vehicle Types Classification. With the overwhelming number of online car-hailing services in China, such as Didi and Uber, it is essential to estimate the travel characteristics of on-demand ride-sourcing cars. This article classifies cars into three types such as taxis, on-demand ride-sourcing cars, and foreign cars. Therefore, we could study the frequency-ranking laws with different types of vehicles.

To classify vehicle types, we comprehensively take into account the vehicle license plate number and vehicle type from LPR data, and the average daily use frequency of cars during the whole month (June 2016). Large vehicles, including shuttle buses and trucks, are partitioned out by the vehicle type field of LPR data. Foreign cars and local taxis can be distinguished by their license plates. However, there are still private cars, whose license plates have the same characteristics as local taxis. To distinguish local taxis from these private cars, researchers examined daily use frequency and set a threshold.

\[ N_t(i) = \begin{cases} 1, & F(i) \geq F_{T, t}^r, \\ 0, & F(i) < F_{T, t}^r, \end{cases} \]

where \( F(i) \) is the daily use frequency of car \( i \) that can be acquired from LPR data. \( F_{T, t}^r \) is a threshold to differentiate taxis from private cars. For every controversial car \( i \), researchers check its \( N_t(i) \) value to determine whether it is a taxi. Similarly, we define heavily used local private cars as on-demand ride-sourcing cars. For every local private car \( i \), researchers check its \( N_t(i) \) value to determine whether it is an on-demand ride-sourcing car.

\[ N_s(i) = \begin{cases} 1, & F(i) \geq F_{T, s}^r, \\ 0, & F(i) < F_{T, s}^r, \end{cases} \]

3.3. Spatial Visitation Frequency. The spatial visitation frequency is measured with the traffic volume arriving at each grid area. Traffic volume is obtained from LPR data, the process of which includes two procedures: (1) obtaining each trip at the level of individual cars. Through sorting all records of the LPR data by car plate and detection timestamp, we can obtain a set of equipment points every car has traveled. Match the device ID with the corresponding intersection ID, so that a trip chain of each car with a timestamp is obtained. The raw trip chains should be broken at the lingering point. By managing the spatiotemporal relationship of every adjacent point in the trip chain, we recognize the lingering points and carry out the trip chain break. (2) Obtain traffic volume by aggregating individual trip records at grid area and time period level. The traffic volume of different grid areas can be obtained by matching detection devices with their corresponding grid area number.

4. The Scaling Law Model

Spatial arrival of vehicles is a highly random process. To describe the spatial arrival of cars, the spatial locations are sorted in descending order according to the visitation frequency of cars. \( L_1, L_2, \ldots, L_s \) represent the ranked location number, which are also the ranking of visitation possibilities. For example, location \( L_s \) is the \( s \)th most likely to be visited. Visitation frequency \( f(r) \) is defined as the number of trips arriving at a location \( L_r \). The frequency-ranking law between spatial visitation frequency \( f(r) \) and ranking \( r \) is the research topic of this article. The frequency-ranking law embodies the following two mechanisms.

(1) Preferential attachment mechanism: the probability of new visits arriving at a certain location is related to

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<th>Information</th>
<th>Explanation</th>
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<td>2150091</td>
<td>Serial number of the device</td>
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<tr>
<td>XY</td>
<td>(120.128, 30.265)</td>
<td>Coordinates of the device</td>
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<td>Serial number of the record</td>
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<tr>
<td>DEV_ID</td>
<td>2147726</td>
<td>Serial number of the device</td>
</tr>
<tr>
<td>CAR_NUM</td>
<td>ZHE-AXXX</td>
<td>License plate number</td>
</tr>
<tr>
<td>CAR_TYPE</td>
<td>1</td>
<td>Type of vehicle</td>
</tr>
<tr>
<td>CAP_DATE</td>
<td>2016/6/13 09:09:15</td>
<td>Acquisition time of the record</td>
</tr>
</tbody>
</table>

Table 1: Log sheet of detector devices.

Table 2: Log sheet of car records.

Figure 1 illustrates the process of determining two aforementioned thresholds \( F_{T, t}^r \) and \( F_{T, s}^r \). The inflection points of curves are considered as the demarcation point between ordinarily used cars (local private cars) and heavily used cars (taxis and on-demand ride-sourcing cars).
the visitation frequency of visits to this location during historical periods.

\[ p(L_r) \propto f(r), \]  

(3)

where \( p(L_r) \) is the probability of new visits arriving at a certain location \( r \).

(2) Distinct visited locations growth mechanism: when new locations are visited, the number of distinct visited locations grows. This mechanism means that the number of distinct visited locations \( S_n \) always increases with the increase of the total trip \( n \).

Combining the distinct visited locations growth mechanism with the preferential attachment mechanism, there is the following relationship between the time of the first trip arrived at location \( L_r \), the ranking \( r \), and the number \( S(n) \) of distinct visited locations.

\[ S(n) = k(L_r) = r, \]  

(4)

where \( k(L_r) \) is the time of the first trip arrived at location \( L_r \).

Explain the above formula. Suppose that in a period of time, a total of \( n \) trips have been completed and a total of \( S(n) \) distinct locations have been visited (that is, location \( L_1, L_2, \ldots, L_r \)). Then according to the preferential attachment mechanism, the earlier a location was visited, the more likely it will be visited again. Therefore, the ranking \( r \) of the accessibility of a location is equal to the time of its first visitation happened \( k(L_r) \). It can be known from the distinct visited locations growth mechanism that when location \( L_r \) is first visited by the \( n \),th trip, it is the \( k \)th location to be visited for the first time, and therefore, the number of distinct visited locations \( S(n) \) is equal to \( k(L_r) \).

According to the actual data, the frequency-ranking law between \( f(r) \) and \( r \) can be obtained. We found that the relationship between \( f(r) \) and \( r \) follows the following exponential relationship and calibrating parameters in the exponential expression are as follows.

\[ f(r, n) = \alpha g(n)^{-r}, \]  

(5)

After calibrating parameters \( \alpha \) and \( g \), a logarithmic relationship between \( g \) and the total trip \( n \) exists as follows:

\[ f(r, n) = \alpha g(n)^{-r}, \]  

\[ g(n) = c \log(n) + d, \]  

(6)

where \( c \) and \( d \) are constants in the logarithmic expression of \( g(n) \). \( c \) and \( d \) can be calibrated by actual data. Thus, parameter \( \alpha \) can be calibrated in the following formula.

\[ \frac{1 - \alpha g(n)}{1 - \alpha} = n. \]  

(7)
Lü et al. [23] gave the mathematical formula of Distinct Visited Locations Growth Mechanism in his research, that is, the functional relationship between $S(n)$ and $n$. As the number of trips $n$ increases, he divides the mechanism of the growth pattern of distinct visited locations into three phases. In the first stage, $S(n) \approx n$; in the second stage, $S(n) \approx \log \alpha n[1 + n(V\alpha)^{-1}]$; and in the third stage, almost all locations have been visited, $S(n) \approx V$.

5. Results

LPR datasets of three cities, Hangzhou, Xiaoshan, and Guiyang, were used to study the scaling laws of the spatial visitation frequency of cars. According to the urban layouts, the three cities are all divided into spatial area grids of the same size ($2\text{km} \times 2\text{km}$). Therefore, a comparative study on the performance of the scaling laws in different cities can be carried out, and the universality of the scaling laws can be discussed. Spatial visitation frequency is expressed by the number of arrivals of a spatial area grid, obtained from the LPR data. Next, we construct and compare the scaling laws of different car types and different time periods to further test the universality of the findings.

5.1. Scaling Laws of Spatial Visitation Frequency. The method described in Section 3 is used to process the LPR data and obtain the frequency-ranking laws of spatial visitation frequency. Figure 2 shows the frequency-ranking relationship of the three cities. Spatial area grids were sorted and numbered from the highest visitation frequency to the lowest. Obviously, the relationship between the logarithm form of visitation frequency $\log (r)$ and ranking $r$ is almost a straight line. Therefore, the frequency-ranking law of spatial visitation frequency is in accordance with the exponential function. This feature is shared in the three cities. Based on the expression of $r = f(r)$ in Section 4, we can assume that the slope of the straight line between $r$ and $\log (r)$ is $-\log \alpha$, and the intercept is $g \log \alpha$. Thus, the parameters $\alpha$ and $g$ can be calibrated from the real data of different cities, which are shown in Table 3.

Then, we will explore the relationship between parameters and the city scale. From Section 3, we assume that the parameter $g$ is positively correlated to the number of total trips $n$, but this correlation only refers to the growth process of $g(n)$ of a single city. Our real data conforms to the third phase, where almost every area grid has been visited and $S(n) \cup V$. This article focuses on the frequency-ranking laws of different cities with various city scales. Comparing the calibration results of three cities, it can be concluded that the smaller the scale of a city ($V$) and the greater the number of total trips ($n$), the frequency-ranking laws have a relatively larger $\alpha$ and smaller $g$.

The visitation frequency used in this section is the number of total trips of all cars during the whole day. In the following two sections, the visitation frequency is divided into different vehicle types and different time periods, to further check the validity of the scaling laws, and also to analyze the compositions and their composition ratio of the visitation frequency used in this section.

5.2. Scaling Laws of Different Vehicle Types. This article also discussed the validity of the scaling laws when vehicles are divided into different types. The method described in Section
Table 4: Parameters of scaling laws of spatial visitation frequency of different car types in Hangzhou.

<table>
<thead>
<tr>
<th>Car type</th>
<th>$a$</th>
<th>$g$</th>
<th>$n$</th>
<th>$S(n)/V$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Foreign cars</td>
<td>1.0272</td>
<td>342.41</td>
<td>21394</td>
<td>123</td>
</tr>
<tr>
<td>On-demand ride-sourcing cars</td>
<td>1.0315</td>
<td>300.92</td>
<td>15564</td>
<td>123</td>
</tr>
<tr>
<td>Taxi</td>
<td>1.0388</td>
<td>219.87</td>
<td>6352</td>
<td>123</td>
</tr>
</tbody>
</table>

![Figure 4](image)

Figure 4: Scaling laws of spatial visitation frequency in different time periods. (a) Guiyang, (b) Hangzhou, and (c) Xiaoshan.

3 is utilized to divide vehicle types, including foreign cars, on-demand ride-sourcing cars, and taxis. Taking Hangzhou city as an example, Figure 3 shows the frequency-ranking laws of different car types, and Table 4 shows the corresponding parameters in frequency-ranking laws calibrated from the real data. It can be concluded that the frequency-
ranking laws apply to both whole cars and every individual category of cars.

Visitation frequencies of different car types are all from the same city, Hangzhou, so they share the same parameter of city scale $V$. Comparing the calibration results of three car types, it can be concluded that the greater the number of total trips ($n$), the frequency-ranking laws have a relatively smaller $\alpha$ and larger $g$.

5.3. Scaling Laws of Different Time Periods. The scaling law of spatial visitation frequency is explored in four time periods,
such as morning peak (7:00–10:00), evening peak (17:00–20:00), flat hump (11:00–14:00), and late night (23:00–2:00).

Figure 4 shows the frequency-ranking laws in different time periods. By constructing frequency-ranking laws of historical spatial visitation frequency in different cities, we address that the frequency-ranking laws conform to the exponential relationship, and the parameters of the exponential expression are associated with both the city scale and the number of total trips. Then, the frequency-ranking laws of different car types, different time periods, and both the different vehicle types and time periods are constructed.

The results show that the scaling laws are widespread for different cities, different vehicle types, and different time periods. By constructing frequency-ranking laws of historical spatial visitation frequency in different cities, we address that the frequency-ranking laws conform to the exponential relationship, and the parameters of the exponential expression are associated with both the city scale and the number of total trips.

The frequency-ranking laws of different car types and different time periods are shown in Figure 5 and Table 6. Visitation frequencies of both different car types and different time periods from the same city share the same parameter of city scale $V$. Comparing the calibration results, an interesting phenomenon can be addressed that for the same car type and varying time periods, the greater the number of total trips ($n$), the frequency-ranking laws have a relatively smaller $\alpha$ and larger $g$.

Visitation frequencies of different time periods from the same city share the same parameter of city scale $V$. Comparing the calibration results of late night with the other three time periods, it can be concluded that the greater the number of total trips ($n$), the frequency-ranking laws have a relatively smaller $\alpha$ and larger $g$.

The frequency-ranking laws of both different car types and different time periods are shown in Figure 5 and Table 6. Visitation frequencies of both different car types and different time periods from the same city share the same parameter of city scale $V$. Comparing the calibration results, an interesting phenomenon can be addressed that for the same car type and varying time periods, the greater the number of total trips ($n$), the frequency-ranking laws have a relatively smaller $\alpha$ and larger $g$, while for the same time period and varying vehicle types, the greater $n$, the frequency-ranking laws have a relatively larger $\alpha$ and smaller $g$.

5.4. Discussion. The findings from the results of the case study using LPR data from three cities in China are as follows. The relationship between spatial visitation and ranking is in accordance with the exponential function expressed by $f(r) = 10^{\alpha r^g}$. The parameters $\alpha$ and $g$ tend to be valued as $V$ stands for city scale and $n$ stands for the number of total trips.

In the first case, for different city scales, as the city scale decreases and the number of total trips increases, the parameter $\alpha$ increases and the parameter $g$ decreases. In the other case, for the same city scale, the parameters $\alpha$ and $g$ change as $n$ changes. With the increase of $n$, $\alpha$ decreases and $g$ increases on the condition of different vehicle types and different time periods.

6. Conclusions

In this article, we verify that the scaling laws between dynamic spatial visitation frequency and the ranking are in accordance with the exponential distribution using the LPR data in three cities, Hangzhou, Guiyang, and Xiaoshan. The results show that the scaling laws are widespread for different cities, different vehicle types, and different time periods. By constructing frequency-ranking laws of historical spatial visitation frequency in different cities, we address that the frequency-ranking laws conform to the exponential relationship, and the parameters of the exponential expression are associated with both the city scale and the number of total trips. Then, the frequency-ranking laws of different car types, different time periods, and both the different vehicle types and time periods are constructed. Also, the relationship conforms to exponential functions influenced by the number of total trips.

The results guarantee the wide applicability of the scaling laws, which can be applied to traffic demand forecasting. The main contributions of this article are as follows:

(1) This article found that the scaling laws of spatial visitation frequency do exist and follow the exponential function. Also, this article provides a method to calibrate the scaling laws using LPR data.

(2) With varying cities, vehicle types, and time periods, bunch of scaling laws were constructed in this article, and the common and different points were thoroughly analyzed.

This article only uses LPR data to model the scaling laws. To make the results more convincing, multiple data sources should be used for calibration and validation of scaling laws for further study.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.
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