Research Article

Effects of Supply Reliability, Risk Aversion, and Wealth on Retailer’s Optimal Order Strategy

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This study examines the effects of supply reliability, risk aversion, and wealth on the optimal order strategy of retailers in the case of uncertain demand by measuring the degree of risk aversion. A more practical model of optimal ordering strategy is proposed, considering supply reliability, demand uncertainty, risk aversion, and retailer wealth, in which two random variables, supply reliability factors and demand, are introduced into the retailer’s function of expected utility. To avoid nonconvergence at both ends, the demand follows a triangular rather than a normal distribution. It is found that the optimal order quantity will increase with the improvement of supply reliability when the risk-averse degree is fixed. The results also show that the optimal order quantity of risk-averse retailers is smaller than that of risk-neutral retailers. Additionally, the optimal order quantity for the risk-averse retailer decreases as the degree of risk aversion increases, when supply reliability is fixed. Further research shows that the retailer is a constant absolute risk aversion (CARA). That means retailer’s wealth has nothing to do with the risk aversion and changes in the retailer’s wealth will not affect the retailer’s optimal order quantity. This study provides valuable insights for sustainable supply chain management and marketing.

1. Introduction

Adam Smith pointed out in his book, “The Theory of National Wealth,” that division of labour is an important feature of the continuous development of productive forces. However, to provide high-quality products or services to the market, it is insufficient for enterprises to rely solely on the division of labour. They also need to coordinate or organize increasingly segmented work. The supply chain is the main body of division of labour and coordination, playing an essential role in solving various supply and demand contradictions.

The sustainability and effectiveness of supply chain management determine the competitive advantage of enterprises that must simultaneously consider the reliability of supply when facing uncertain demand [1, 2]. Because the success of an enterprise depends on balanced supply and demand [3], both demand and supply uncertainties can be a major obstacle to realizing this goal. Against the backdrop of economic globalization, supply chain networks have expanded rapidly, resulting in the strengthening of relationships among supply chain members. This development trend not only improves the efficiency and resiliency of existing supply chain networks but also increases the possibility of risk diffusion among supply chain companies, with individual enterprises putting the entire supply chain at risk, especially in an uncertain COVID-19 environment [4]. Consequently, the issue of uncertainties throughout supply chain in the process of integrated optimization has become a major focus of scholars and practitioners [5, 6].

Demand uncertainty is an important part of supply chain management and determines the difficulty of supply chain management [7], which is inherent in almost all actual business environments and has been extensively researched in the inventory management literature [8]. Although supply uncertainty has received less scholarly attention, it can also
have a significant impact on an enterprise’s bottom-line performance. Uncertainty in supply prevents companies from meeting consumer demand within a valid sales period, resulting in supply chain losses and risks [9]. The causes of supply uncertainty are multifaceted and can be divided into operational and disruptive risks [10–12].

Operational risks mainly involve daily disturbances in supply chain operation, such as demand fluctuations and transportation delays, while disruption risks are low-frequency and high-impact events [13]. The recent coronavirus disease (COVID-19) pandemic, which represents one of the major disruptions encountered after the SARS outbreak in 2003, drastically reduced supply availability in global supply chains [14]. Clearly, the uncertainty of both demand and supply is a greater challenge that enterprises must simultaneously face in supply chain management. Retail operations, such as news vendors, also face uncertainty regarding demand and supply. Therefore, it is of great significance to study the effect of supply reliability and risk aversion on the order quantity of retailers under supply and demand uncertainty; however, there are limited relevant studies.

Research that properly analyzes the impact of supply reliability and risk aversion on the optimal order strategy of retailers is lacking. This gap includes a lack of supply reliability modelling, which simultaneously considers supply reliability factors, risk aversion, and uncertain demand. Given the prevalence of supply disruptions, such as those caused by the COVID-19 pandemic, it is imperative and timely to study the effect of supply reliability and risk aversion on the optimal order strategy of retailers. Unlike previous studies, this study integrates supply reliability factors, risk aversion, and uncertain demand into a retailer’s optimal order model based on the utility maximisation of expected wealth rather than maximisation of expected wealth. The effect of supply reliability and risk aversion on retailers’ optimal order quantity is determined and verified through numerical analysis. In this paper, the definition and measurement of risk aversion is based on Arrow-Pratt [15] and Thomas [16] decreasing absolute risk aversion (DARA) hypothesis and measurement. The market demand in this study follows a triangular rather than a normal distribution, which is also a novelty of this study. Therefore, this study is consistent with the practice of retailers in supply chain management and has theoretical reference significance.

The main contributions of this paper include the following three aspects. (1) We propose a more practical model of optimal ordering strategy, considering supply reliability, demand uncertainty, risk aversions, and retailer wealth, in which two random variables—supply reliability factors and demand—are introduced into the retailer’s function of expected utility. (2) The demand follows a triangular distribution rather than a normal distribution to avoid nonconvergence at both ends of the normal distribution. (3) We examine the effect of supply reliability on the optimal order quantity when the degree of risk aversion is the same, which can only be achieved by introducing a supply reliability factor.

The remainder of this paper is organized as follows. Section 2 reviews the relevant literature. Section 3 develops the models after the description and assumptions. Section 4 examines the effect of supply reliability on the optimal order strategy. Section 5 examines the effects of risk aversion and wealth on the optimal order strategy. The numerical analysis is presented in Section 6, and Section 7 provides the summary and conclusions.

2. Literature Review

Much of the literature on newsvendor problems has considered the issue of order quantity and the effect of uncertainty, which are based on one of three conditions: demand uncertainty, supply uncertainty, and risk aversion. Therefore, the literature related to this study includes three aspects: the newsvendor problem with uncertainty in demand, uncertainty in supply, and risk aversion.

2.1. The Newsvendor Problem of Demand Uncertainty

The newsvendor scenario determines the optimal order quantity when demand is uncertain. Whitin [17] was the first to propose the newsvendor model. Since then, several extensions of the classical model have been proposed [18] to solve the problem of optimal order quantity with uncertain demand. Li et al. [19] considered a single-period inventory problem in the presence of demand uncertainty. They developed two models in which the objectives are the maximisation of profit versus the optimal solution for order quantity. Li and Zheng [20] studied the joint inventory replenishment and pricing problem for production systems with stochastic demand and yield to maximise total discounted profit. They found that the optimal replenishment policy is of the threshold type. Fei et al. [21] investigated joint inventory and pricing strategies for perishable and alternative products under uncertain demand. They developed a model that addresses inventory and dynamic pricing decisions for multiple perishable and alternative products over a multicyle life cycle. Rajesh [22] measured the barriers to resilience in manufacturing supply chains using the grey clustering and VIKOR approaches, as supply chain uncertainty becomes more prevalent.

An important extension incorporates advanced selling strategies to reduce demand uncertainty. Few studies have focused on advance selling from retailers to consumers. Prasad et al. [23] examined inventory decisions and the advance selling price in a two-period setting, finding that an advance selling strategy is not always optimal for retailers but is contingent on the parameters of the consumers and the market. Feng et al. [24] investigated the impact of introducing a presale channel on inventory risk and out-of-stock risk. Wang et al. [25] examined the profitability of omni-channel preordering (i.e., a new advance selling strategy for retailers) and found that the retailers were more likely to order a smaller quantity when the traditional online preordering option was used. Zhang et al. [26] examined optimal pricing and inventory decisions under two strategies in three situations of total market size based on the newsvendor model. They showed that probabilistic selling always brings more benefit to the retailer and performs better than
inventory substitution. Zhang et al. [27] studied partial refunds in advance selling as a means of strategic price commitment for the service industry. They found that a partial refund strategy, in which a fee is cancelled as a price commitment mechanism, is capable of becoming the optimal strategy.

2.2. The Newsvendor Problem of Supply Uncertainty. Compared with demand uncertainty, there are fewer studies on the supply uncertainty of the newsvendor problem. While alone research combined with risk aversion. Liu et al. [28] examined the effect of supply uncertainty in a retail setting with joint marketing and inventory decisions. Their research showed that retailers are willing to pay more to improve supply reliability for products at a higher price and found that the prioritisation of adopting new technologies should be for situations where the company can effectively induce greater demand through promotional efforts. Xanthopoulos et al. [29] developed single-period (newsvendor-type) inventory models to capture the trade-off between disruption risks and inventory policies in a dual-source supply chain network where both supply channels are vulnerable to disruption risks. Some studies have extended supply uncertainty to supply chain uncertainty. Nagarajan et al. [30] extended their research to the impact of environmental uncertainty on supply chain flexibility. They found that companies must strive to improve the quality of information to ensure improved supply chain flexibility. Michael [31] identified the key factors of logistics capability and found a negative relationship between supply chain uncertainty and logistics capability among Australian express companies. Michael’s empirical results (2017) also showed that supply uncertainty has a negative effect on logistics performance, the greatest effect of supply uncertainty being from outside firms in the Australian express industry. Esmaeili-Najafabadi et al. [32] investigated outsourcing strategies with supply uncertainties and two types of demand for risk-averse decision makers. Zhao et al. [33] investigated the optimal decisions of a supply chain with a risk-averse retailer and one risk-neutral supplier to derive the condition for the supply chain to be coordinated. Recently, the COVID-19 pandemic has severely disrupted different supply chain sectors worldwide. Rahman et al. [34] developed an agent-based model for supply chain recovery to alleviate the problems caused by exceptional disruptive events. Mohammadivjdan et al. [35] examined the problem of allocating procurement quantities to multiple suppliers ahead of a selling season with uncertain demand. They showed that supply uncertainty has an impact on the conditional service level of the newsvendor.

2.3. The Newsvendor Problem of Risk Aversion. People’s attitudes toward risk are classified into risk neutrality, risk aversion, and risk-taking. The newsvendor model is widely employed in the literature and is typically based on the assumption of risk neutrality [36–38]. This study restricts our attention to risk aversion to focus on the effects of risk aversion on the optimal order quantity. An early paper by Baron [39] studied the comparative static impact of changes in newsvendor risk aversion on the optimal order quantity. Based on Baron’s results, Eckhoudt et al. [40] further examined the comparative static impact of changes in various other costs and changes in demand risk on risk-averse newsvendors. Keren and Pliskin [41] studied a benchmark solution to the risk-averse newsvendor problem when the random demand faced by the expected-utility newsvendor is uniformly distributed, proving that the greater the degree of risk aversion, the smaller the optimal order quantity. Liu et al. [28] studied the effect of supply reliability on retailer performance under joint marketing and inventory decisions. The results show that firms that adopt new technologies can effectively induce greater demand through promotional efforts. Giri [42] developed a model from the perspective of a low-risk-averse retailer and quantified the risk using an exponential utility function. Through numerical experiments, they showed how the resulting dual-sourcing strategies differ from those obtained in the risk-neutral analysis. They found that the optimal order quantity of a risk-averse retailer is less than that of a risk-neutral retailer. Thomas [16] presented a theoretical framework of risk-aversion measurement, paving the way for direct utility calculations. Wang et al. [43] examined the optimal inventory decisions for a risk-averse retailer when offering layaway and found that the optimal order quantity depends on different loss functions and demand distribution functions. Li and Jiang [44] examined the impact of return policy and retailers’ risk aversion on the behaviour of supply chain members, finding that increasing the level of retailer risk aversion may result in a smaller expected utility for the retailer and a larger profit for the supplier. Some literature extends risk aversion to other issues beyond the newsvendor problem [25, 45]. Bonzelet [46] analyzed how increasing relative risk aversion impacts order decisions of retailer under two coordinating contracts. They found that under real option contracts, risk-averse retailers order more than under buyback contracts.

In summary, the literature reviewed above has studied strategies such as optimal price, inventory, coordination, outsourcing, and supply chain recovery from the aspects of uncertain demand, uncertain supply, and risk aversion, respectively. This study examines the effects of supply reliability and risk aversion on the optimal ordering strategy when retailers face uncertain demand, which differs from the majority of previous studies.

3. Model Formulation

3.1. Description and Assumptions. A retailer who sells products and faces the newsvendor problem in a single selling season needs to decide on the order quantity prior to the start of the selling season due to the long supply lead-time factor. However, the available stock to meet the demand would be less than the order quantity owing to various uncertainty factors of the supply, which is more realistic. The supply reliability factor, a random variable, is introduced as the supply instability factor faced by retailers in this study. When retailers are subject to uncertain demand and supply, their profits can be risky, which means that retailers with
initial wealth face a gamble. This study attempts to resolve how supply reliability and risk aversion affect the ordering decisions of the retailer. This section presents the assumptions regarding retailers and the models based on these assumptions. Table 1 lists the definitions of the variables associated with the model.

It is assumed that a retailer who faces uncertain demand, $D$, in the market with intense competition has to determine $Q$, the number of products to order ex ante. The random variable $D$ is fully characterized by its distribution function, $F(D)$, and its density function, $f(D)$, which is assumed to have its support contained in $(a, b)$. Consider a retailer with initial wealth $w_0$ who orders products at a unit price $c$ and resells them at a price $p > c$. All unsold newspapers are returned to the publisher at the salvage price $v < c$. Of course, the retailer is allowed to procure additional products if demand exceeds his original order, but at a higher cost $\tilde{c}$. A reasonable assumption is that $0 < v < c < \tilde{c} < p$.

However, the actual inventory available to meet the demand during the selling season is uncertain because the supply is not completely reliable. Specifically, the available inventory to meet the demand is given by $\varepsilon Q$, where $\varepsilon$ is introduced as a positive random variable. We define $\varepsilon$ as the supply reliability factor. Assume that $G(\varepsilon)$ and $g(\varepsilon)$ denote the distribution and density functions of $\varepsilon$, respectively, where $G(\varepsilon)$ is twice differentiable and the support of $\varepsilon$ is contained in $(0, 1)$; that is, the actual available inventory to meet demand is always less than $Q$ because of supply uncertainty. However, this analysis also applies to general cases in which $\varepsilon$ can be greater than one.

Based on Liu’s [28] assumptions, the total procurement cost of retailer depends on the assumption that the retailer is responsible for the unavailable inventory $(1 - \varepsilon)Q$. In this study, it is assumed that the retailer’s total procurement cost is equal to $cQ$ because supply reliability is primarily the responsibility of the retailer who has to pay for the entire order. Analogously, the total salvage value for unsold products at the end of the selling season depends on the assumption of whether the unavailable inventory $(1 - \varepsilon)Q$ would appear after the selling season. In this study, it is assumed that the unavailable inventory can be recovered at the end of the selling season because the uncertainty in supply is due to misplaced inventory or late delivery.

Assume that the retailer’s aversion function over ultimate wealth distributions is represented by $u(\cdot)$, which is the expected utility type denoting the utility of wealth. In this study, the analysis of the retailer’s risk attitude focuses on two types: risk neutrality and risk aversion. When the retailer is risk-averse, $u(\cdot)$ is defined as strictly increasing and is a concave function. When risk-neutral, $u(\cdot)$ is a linear utility function. For analytical ease, $u(\cdot)$ is also assumed to be three times differentiable. It is also assumed that $u(w(D, Q))$ is valid for any feasible value of wealth.

### 3.2. Modelling

Consistent with these assumptions, let $W(D, Q)$ be the expected wealth of the retailer, who is endowed with the following wealth function:

$$W(D, Q) = W_0 + pD - cQ + v \max(\varepsilon Q - D, 0) - \tilde{c} \cdot \max(D - \varepsilon Q, 0) + v(1 - \varepsilon)Q. \quad (1)$$

Or equivalently, it is expressed as a piecewise linear wealth function, as follows:

$$W(D, Q) = \begin{cases} W_-(D, Q) = W_0 + (p - v)D - (c - v)Q (0 < D < \varepsilon Q), \\ W_+(D, Q) = W_0 + (p - \tilde{c})D - (c - \tilde{v})Q + (\tilde{c} - v)\varepsilon Q (\varepsilon Q < D). \end{cases} \quad (2)$$

The retailer’s optimization problem can be formulated as follows:

$$\max W(D, Q) = \max \left[ W_0 + pD - cQ + v \max(\varepsilon Q - D, 0) - \tilde{c} \cdot \max(D - \varepsilon Q, 0) + v(1 - \varepsilon)Q \right]$$

$$= \max \left[ W_0 + pD + (v - \tilde{c}) \cdot \max(\varepsilon Q, D) + (\tilde{c} - v)\varepsilon Q + (v - c)Q - v D \right]. \quad (3)$$
We assume that the retailer is an expected utility maximiser. Since demand is a random variable, the objective function is as follows:

\[
\max_{Q} E \left[ u(W(D, Q)) \right] = \max_{Q} \int_{a}^{b} \left( W_{0} + pD + (v - \bar{c})Q + \int_{a}^{Q} f(D) dD - (c - v)Q \right) g(\varepsilon) d\varepsilon
\]

The first-order derivative condition for an optimal solution is as follows:

\[
\frac{\partial E[u(W(D, Q))]}{\partial Q} = 0
\]

\[
= -(c - v) \int_{a}^{Q} u' \left[ W_{0} + (p - v)D + (c - v)Q \right] f(D) dD g(\varepsilon) d\varepsilon + \int_{a}^{b} u' \left[ W_{0} + (p - \bar{c})D - (c - v)Q + (\bar{c} - v)Q \right] f(D) dD g(\varepsilon) d\varepsilon.
\]

The condition that there is a solution to the above equation is as follows:

\[
(\bar{c} - v)\varepsilon - (c - v) > 0.
\]
\[ \begin{align*}
\bar{c} - v = & \frac{\int_a^n \int_0^a f(D) g(e) dD \, d\epsilon \, d\epsilon}{\int_a^n \int_0^a (c - v) f(D) dD \, d\epsilon} \\
& + \int_m^n \int_m^b u'(W_0 + (p - \bar{c})D - (c - v)Q + (\bar{c} - v)e\epsilon) f(D) dD \, d\epsilon \\
& + \int_m^n \int_m^b u'(W_0 + (p - \bar{c})D - (c - v)Q + (\bar{c} - v)e\epsilon) f(D) dD \, d\epsilon
\end{align*} \]

(7)

4. Effect of Supply Reliability on the Optimal Order Strategy

4.1. Order Strategy with Risk Neutrality. A risk-neutral retailer is endowed with the linear utility function:

\[ u(x) = ax + bx, \tag{8} \]

where \( u' > 0, u'' = 0 \). It is assumed that the supply reliability factor of the retailer is uniformly distributed, where the density and distribution functions are as follows:

\[ g(e) = \frac{1}{n-m}, \quad 0 \leq m \leq n \leq 1, \tag{9} \]

\[ G(e) = \frac{\epsilon - m}{n-m} \tag{10} \]

By substituting (8) and (9) into (7), we obtain the following critical ratio:

\[ \bar{c} - v = \frac{\int_m \int_0^a f(D) dD \, d\epsilon \, d\epsilon + \int_m^b \int_m^b f(D) dD \, d\epsilon}{\int_m \int_0^a (c - v) f(D) dD \, d\epsilon}, \]

\[ \frac{\bar{c} - v}{c - v} = \frac{n-m}{\int_m^b \int_m^b f(D) dD \, d\epsilon}, \]

\[ \int_m^b \int_m^b f(D) dD \, d\epsilon = (n-m) \left( \frac{m + n}{2} - \frac{c - v}{\bar{c} - v} \right). \tag{11} \]

Assume that the market demand faced by retailers follows a triangular distribution, which avoids the drawbacks of the normal distribution not converging at both ends; thus, it is closer to the actual case. Density and distribution functions are expressed as follows:

\[ f(D) = \begin{cases} 
\left( \frac{2}{b-a} \right)^2 (D-a), & 0 \leq a \leq D \leq a + \frac{b}{2} \\
\left( \frac{2}{b-a} \right)^2 (b-D), & a + \frac{b}{2} \leq D \leq b,
\end{cases} \tag{12} \]

\[ F(D) = \begin{cases} 
\frac{2(D-a)^2}{(b-a)^2}, & 0 \leq a \leq D \leq a + \frac{b}{2} \\
1 - \frac{2(b-D)^2}{(b-a)^2}, & a + \frac{b}{2} \leq D \leq b, \end{cases} \tag{13} \]

where \( a \geq 0, b \geq 0 \), and the probability density function of random demand is illustrated in Figure 1.

By substituting (13) into (10), we get the following.

\[ \int_m^b \int_m^b f(D) dD \, d\epsilon = \frac{2}{b-a} [\phi(n) - \phi(m)] \tag{14} \]

\[ \phi(\epsilon) = \frac{Q^2}{4} \epsilon^4 - \frac{2aQ}{3} \epsilon^3 + \frac{1}{2} \frac{b^2}{\epsilon^2}. \]

When \( \frac{a+b}{2} \leq Q \leq \frac{a+b}{2}, \) by substituting (17), (9), and (11) into (7), we obtain the following:

4.2. Order Strategy with Risk Aversion. A risk-averse retailer is endowed with a concave utility function:

\[ u(x) = 1 - e^{-rx}, \tag{16} \]

where \( u' > 0, u'' < 0 \). Assume that the supply reliability factor and the random demand for the risk-averse retailer are the same as those of the risk-neutral retailer.

\[ \int_m^b \int_m^b f(D) dD \, d\epsilon = \frac{2}{b-a} [\phi(n) - \phi(m)], \]

\[ \phi(\epsilon) = \frac{Q^2}{4} \epsilon^4 - \frac{2bQ}{3} \epsilon^3 + \frac{1}{2} \frac{b^2}{\epsilon^2}. \tag{15} \]
\[
\frac{\bar{c} - \nu}{c - \nu} = \int_{m}^{n} \left[ \int_{Q}^{b} \left( 1 - e^{-r \left[ W_{x}(p - \nu)D + (c - \nu)Q \right] } \right) - (2/b - a)^{2} (D - a) dD \right] \frac{1/n - m \epsilon}{1/n - m \epsilon} \\
+ \int_{m}^{n} \left[ \int_{Q}^{b} \left( 1 - e^{-r \left[ W_{x}(p - \check{c})D + (c - \nu)Q + (\check{c} - \nu)Q \right] } \right) - (2/b - a)^{2} (D - a) dD \right] \frac{1/n - m \epsilon}{1/n - m \epsilon} \\
\int_{m}^{n} \left[ \int_{Q}^{b} \left( 1 - e^{-r \left[ W_{x}(p - \check{c})D + (c - \nu)Q + (\check{c} - \nu)Q \right] } \right) - (2/b - a)^{2} (D - a) dD \right] \frac{1/n - m \epsilon}{1/n - m \epsilon} \\
\]

Let
\[
\frac{\bar{c} - \nu}{c - \nu} = \frac{(Q) + h(Q)}{j(Q)} \tag{18}
\]

where \( Q = (2/b - a)^{2} \cdot e^{-r \left( m_{0} + (c - \nu)Q \right) } \), \( 1/n - m \cdot [-1/r \left( p - v \right) Q \cdot e^{-r \left( p - v \right) \nu Q} \cdot (a - 2/r \left( p - v \right) - mQ) - 1/r \left( p - v \right) Q \cdot e^{-r \left( p - v \right) mQ} \cdot (a - 2/r \left( p - v \right) - mQ) + (1/r \left( p - v \right) - a) (n - m)]. \)

\[
h(Q) = \left( \frac{2}{b - a} \right)^{2} \cdot e^{-r \left( m_{0} + (c - \nu)Q \right)} \cdot \frac{1}{n - m} \cdot \left[ \frac{-1}{r \left( p - v \right) Q} \cdot e^{-r \left( p - v \right) Q} \cdot \left( aQ - a + \frac{1}{r \left( p - \check{c} \right)} + \frac{1}{r \left( p - v \right)} \right) \right] \\
+ \frac{1}{r \left( p - v \right) Q} \cdot e^{-r \left( p - v \right) Q} \cdot \left( mQ + \frac{m}{r \left( p - \check{c} \right)} - am + \frac{2m}{r \left( p - v \right) Q} + \frac{1}{Q} \left( p - \check{c} \right) - \frac{a}{r \left( p - v \right) Q} \right) \\
\frac{2}{r \left( p - v \right) Q} \left( e^{-r \left( \check{c} - \nu \right) Q} - e^{-r \left( p - v \right) Q} \right) + \left( b - a + \frac{1}{r \left( p - \check{c} \right)} \right) \cdot e^{-r \left( p - v \right) Q} \left[ ne^{-r \left( \check{c} - \nu \right) Q} - me^{-r \left( \check{c} - \nu \right) mQ} + \frac{1}{r \left( p - v \right) Q} \left( e^{-r \left( \check{c} - \nu \right) Q} - e^{-r \left( p - v \right) \check{c} Q} \right) \right] \tag{19}
\]

When \( a + b/2e \leq Q \leq b/e \), by substituting (17), (9), and (11) into (7), we obtain the following:

\[
\frac{\bar{c} - \nu}{c - \nu} = \int_{m}^{n} \left[ \int_{Q}^{b} \left( 1 - e^{-r \left[ W_{x}(p - \nu)D + (c - \nu)Q \right] } \right) - (2/b - a)^{2} (b - D) dD \right] \frac{1/n - m \epsilon}{1/n - m \epsilon} \\
\int_{m}^{n} \left[ \int_{Q}^{b} \left( 1 - e^{-r \left[ W_{x}(p - \check{c})D + (c - \nu)Q + (\check{c} - \nu)Q \right] } \right) - (2/b - a)^{2} (b - D) dD \right] \frac{1/n - m \epsilon}{1/n - m \epsilon} \\
\int_{m}^{n} \left[ \int_{Q}^{b} \left( 1 - e^{-r \left[ W_{x}(p - \check{c})D + (c - \nu)Q + (\check{c} - \nu)Q \right] } \right) - (2/b - a)^{2} (b - D) dD \right] \frac{1/n - m \epsilon}{1/n - m \epsilon} \\
\frac{\bar{c} - \nu}{c - \nu} = \Theta \left( Q \right) + \Phi \left( Q \right) \tag{20}
\]

where
\[ \varnothing (Q) = \begin{bmatrix} -\frac{1}{r(p-v)Q} \cdot e^{-r(p-v)Q} \cdot \left( mQ + \frac{2}{r(p-v)} - b \right) \\ + \frac{1}{r(p-v)Q} \cdot e^{-r(p-v)Q} \cdot \left( mQ + \frac{2}{r(p-v)} - b \right) + \left( b - \frac{1}{r(p-v)} \right) \cdot (n-m) \\ - \frac{2}{\beta - \alpha} \cdot e^{-\beta (\omega - v)Q} \cdot \frac{1}{n-m} \end{bmatrix} \]

\[ \varphi (Q) = \left( \frac{2}{\beta - \alpha} \right)^2 \cdot e^{-\beta (\omega - v)Q} \cdot \frac{1}{n-m} \cdot \begin{bmatrix} -\frac{1}{r(p-v)Q} \cdot e^{-r(p-v)Q} \cdot \left( b - nQ - \frac{1}{r(p-v)} - \frac{1}{r(p-v)} \right) \\ + \frac{1}{r(p-v)Q} \cdot e^{-r(p-v)Q} \cdot \left( b - nQ - \frac{1}{r(p-v)} - \frac{1}{r(p-v)} \right) \\ - \frac{1}{r(p-v)Q} \cdot e^{-\beta (\omega - v)Q} \cdot \left( e^{-r(p-v)Q} - e^{-r(p-v)Q} \right) \end{bmatrix} \]

Equations (17) and (20) can be expressed as a function of \( Q \) with respect to \( m \). It can be proved that the first derivative of the function \( Q \) with respect to \( m \) is greater than zero, namely, \( \partial Q / \partial m > 0 \). In other words, when \( r \) is fixed, \( Q \) increases (decreases) as \( m \) increases (decreases). Equation (16) is a function expression of order quantity with respect to supply reliability when the retailer is risk neutral and equation (20) is a function expression of order quantity with respect to supply reliability when the retailer is risk averse.

These two function expressions show that the optimal order quantity is directly related to the supply reliability regardless of whether the retailer’s risk attitude is neutral or averse. When the first derivative of the two functions with respect to supply reliability is greater than zero, it means that supply reliability positively affects the optimal order quantity because a higher supply reliability of the supplier can ensure the retailer of the customer’s demand satisfaction. Therefore, the following propositions can be obtained.
Proposition 1. The optimal order quantity of the retailer increases with an improvement in supply reliability when the risk-averse degree is fixed.

5. Effect of Risk Aversion and Wealth on Optimal Order Strategy

5.1. Effect of Risk Neutral and Risk Aversion on Order Strategy. Assume that the optimal order for the risk-neutral retailer is \( Q^*_1 \) and that for the risk-averse retailer is \( Q^*_2 \). According to (11), \( Q^*_1 \) is given by the following:

\[
\int_{m}^{n} \epsilon F(\epsilon Q_1^*) \, d\epsilon = (n - m) \left( \frac{m + n}{2} - \frac{c - v}{p - v} \right). \tag{22}
\]

The utility function for the risk-averse retailer is defined as concave, where \( (x) \) is increasing and \( u'(x) \) is decreasing. From (2), it is easy to get \( W(Q^*, D_1) > W(Q^*, eQ^*) > W(Q^*, D_2) \) for \( eQ^* < D_2 \). Whenever \( u \) is strictly concave, the following inequality holds:

\[
u'(W(Q^*, D_1)) > u'(W(Q^*, eQ^*)) > u'(W(Q^*, D_2)). \tag{23}
\]

Replacing (21) in (5) yields the following inequality:

\[
\frac{\partial E(u)}{\partial Q} = -(c - v) \left[ \int_{m}^{n} u'(W(Q^*, D_1)) f(D) \, dD \right] g(\epsilon) \, d\epsilon,
\]

\[
+ \int_{0}^{1} [(p - v)\epsilon - (c - v)] \left[ \int_{0}^{\epsilon} u'(W(Q^*, D_2)) f(D) \, dD \right] g(\epsilon) \, d\epsilon < 0. \tag{24}
\]

Because

\[
-(c - v) \int_{m}^{n} \left[ \int_{0}^{\epsilon} u'(W(Q^*, eQ^*)) f(D) \, dD \right] g(\epsilon) \, d\epsilon
\]

\[
+ \int_{m}^{n} [(p - v)\epsilon - (c - v)] \left[ \int_{0}^{\epsilon} u'(W(Q^*, eQ^*)) f(D) \, dD \right] g(\epsilon) \, d\epsilon > 0. \tag{25}
\]

\( Q^*_2 \) is given by

\[
\int_{m}^{n} \epsilon F(\epsilon Q_2^*) \, d\epsilon < \int_{m}^{n} \epsilon F(\epsilon Q_1^*) \, d\epsilon = (n - m) \left( \frac{m + n}{2} - \frac{c - v}{p - v} \right). \tag{26}
\]

(20) considers supply stability and is an increasing function, that is, the function increases as \( Q \) increases. (24) shows that when the supply stability is fixed and faced with the same demand, risk-averse decision makers and risk-neutral decision makers have different optimal order
Proposition 2. The optimal order quantity of risk-averse retailers considering supply reliability is smaller than that of risk-neutral retailers.

5.2. Effect of Retailers with Different Risk Aversions on Order Strategy. Arrow-Pratt [47] postulated the well-known property of DARA, which is expressed as \( \text{ARA} = -u''(x)/u'(x) \). For fixed risk, this property suggests that individuals are willing to pay more to avoid risk when they are poor. Arrow-Pratt [12, 13] postulated the well-known property of absolute risk aversion measurement, which is expressed as \( \pi = 1/2\sigma^2 \left[-u''(W_0)/u'(u_0)\right] \), and the measurement of risk depends entirely on the value in square brackets, which is called absolute risk aversion, \( \text{ARA} = -u''(x)/u'(x) \). According to literature [14, 47], the change of risk aversion degree on the order quantity can be obtained:}

\[
\frac{dh(u(W(Q)))}{dQ} = - (c - s) \int_m^{\beta Q} h'[u(W(\cdot))] u'(W) f(D)dD \times \frac{dQ}{dQ} + \frac{dh}{dQ} < (c - s) \int_m^{\beta Q} u'(W) f(D)dD \times \frac{dQ}{dQ} \]

5.3. Effect of Retailer Wealth on Ordering Strategies. The utility function of the retailer in this article takes the exponential utility function, \( u(W) = 1 - e^{-\rho W} \). According to the definition of risk measures in Arrow-Pratt [47], the absolute risk aversion of retailers is \( \text{ARA} = -u''(W)/u'(W) = r > 0 \). Through the first derivative of the retailer’s absolute risk aversion to wealth, the relationship between the retailer’s risk aversion degree and wealth can be obtained. From the first-order derivative \( d(\text{ARA})/dW = 0 \) of the retailer’s absolute risk aversion degree with respect to wealth, it can be known that the retailer is a constant absolute risk aversion (CARA), that is, the retailer’s risk aversion degree has nothing to do with wealth. Since the retailer’s wealth does not affect the risk aversion, nor does the optimal order quantity, Proposition 4 can be obtained.

Proposition 4. When the retailer’s utility function is constant absolute risk aversion (CARA) and supply reliability remains unchanged, the retailer’s wealth has nothing to do
with the risk aversion. Therefore, changes in the retailer's wealth will not affect the optimal order quantity.

6. Numerical Analysis

To see the effects that risk aversion and supply reliability can have on optimal orders, consider the following simple example of a risk-averse retailer whose properties satisfy constant absolute risk aversion. This property can be represented by the utility function \( u(W) = 1 - e^{-rW} \), where \( r \) represents the retailer's degree of risk aversion. Let \( W_0 = 5000, p = 45, c = 25, v = 5, \bar{c} = 35, a = 0, b = 300, n = 1 \) and let \( D \in (10, 1000) \) and \( \varepsilon \in (m, 1) \). Straightforward calculations using MATLAB yield the optimal order quantities for different degrees of risk aversion as \( m \) changes, as shown in Tables 2 and 3.

A comparison of Tables 2 and 3 shows that the retailer's order increases with increasing supply reliability when the degree of risk aversion is fixed. In the case where \( r = 0.0001 \),

<table>
<thead>
<tr>
<th>Risk averse</th>
<th>Optimal order</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r = 0.000001 )</td>
<td>14</td>
</tr>
<tr>
<td>( r = 0.00001 )</td>
<td>13</td>
</tr>
<tr>
<td>( r = 0.0001 )</td>
<td>12</td>
</tr>
<tr>
<td>( r = 0.001 )</td>
<td>8</td>
</tr>
<tr>
<td>( r = 0.01 )</td>
<td>2</td>
</tr>
<tr>
<td>( r = 0.1 )</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Risk averse</th>
<th>Optimal order</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r = 0.000001 )</td>
<td>5</td>
</tr>
<tr>
<td>( r = 0.00001 )</td>
<td>4</td>
</tr>
<tr>
<td>( r = 0.0001 )</td>
<td>3</td>
</tr>
<tr>
<td>( r = 0.001 )</td>
<td>2</td>
</tr>
<tr>
<td>( r = 0.01 )</td>
<td>0</td>
</tr>
</tbody>
</table>
when \( m = 0.45 \), the retailer's order is 3, and when the supply reliability increases to 0.8 \((m = 0.8)\), the retailer's order increases to 12, which numerically verifies Proposition 1.

From Tables 2 and 3, it can be found that the retailer's optimal order quantity decreases as the degree of risk aversion increases when supply reliability is fixed, which numerically verifies Proposition 3. In the case where \( r = 0.01 \) the retailer is so risk-averse that he is reluctant to order even a single product for fear of losing the cost of 25.

7. Conclusions and Managerial Implications

7.1. Conclusions. Based on the risk-averse retailer facing an environment of supply interruption and demand uncertainty, this study makes the following three contributions to the modeling of the retailer's objective function. First, there are two different random variables in the model, the supply reliability factor and uncertain demand, which are more in line with the actual case. Second, market demand follows a triangular distribution instead of a normal distribution to avoid the shortcomings of nonconvergence at both ends, making the model more in line with the actual case. Finally, the wealth utility for retailers adopts an exponential function, and the measure of the retailer's risk aversion adopts a decreasing absolute risk aversion.

This study assumes that retailers exhibit ARA. From the perspective of the impact of supply reliability, it is found that the optimal order quantity of retailer increases with an improvement in supply reliability when the risk-averse degree is fixed. From the perspective of the impact of risk attitude, the results show that the optimal order quantity of risk-averse retailers is smaller than that of risk-neutral retailers when compared. Furthermore, the optimal order quantity of retailer decreases as the degree of risk aversion increases, when supply reliability is fixed. From the perspective of the impact of wealth, retailer is a constant absolute risk aversion (CARA). That means the retailer's wealth does not affect the risk aversion and changes in the retailer's wealth will not affect the retailer's optimal order quantity [21, 48–50].

7.2. Managerial Implications. The findings show that supply reliability has a positive effect on order quantity, while the degree of risk aversion has a negative effect on order quantity and changes in the retailer's wealth do not affect the optimal order quantity. These findings will provide some managerial and practical implications. (1) Suppliers should strive to improve supply reliability and avoid supply interruption, which is not only an inevitable requirement of sustainable supply chain but also a prerequisite to achieve a win-win situation between suppliers and retailers. (2) Risk-averse retailers order less than risk-neutral retailers for prudence reasons. In general marketing practice, market researchers should grasp this objective law and adjust their marketing strategies according to the actual situation. For example, suppliers may increase the risk type management of retailers in customer relationship management to form a reasonable portfolio of customer risk propensity. (3) When the retailer exhibits constant absolute risk aversion, the retailer's wealth change will not affect the optimal order quantity. This means that in the case of a shortage of funds, the supplier can take some delayed payment or promotional measures to supply the retailer because the retailer's shortage of funds does not affect the optimal order quantity. (4) Suppliers should not ignore that, facing the same risk, retailers with different levels of risk aversion perceive risk differently. This difference of risk attitude may be related to gender, age, personality, etc., which provides space for marketers to develop the market. We believe that the implications of these results are particularly relevant for practitioners and policymakers.

In the future, it would be interesting to explore how to reduce the risk of supply interruption and improve supply reliability, thus improving the sustainability of the supply chain. Furthermore, risk-taking retailers are not considered in this paper, which will also be included in our future research work.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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References


