Hindawi
Mathematical Problems in Engineering
Volume 2022, Article ID 2553811, 13 pages
https://doi.org/10.1155/2022/2553811

Research Article

Analysis of T-Spherical Fuzzy Matrix and Their Application in Multiattribute Decision-Making Problems

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Received 31 May 2022; Accepted 19 July 2022; Published 27 September 2022

Academic Editor: A. M. Bastos Pereira

The aim of this study is to introduce an innovative concept of T-spherical fuzzy matrix, which is a hybrid structure of fuzzy matrix and T-spherical fuzzy set. This article introduces the square T-spherical fuzzy matrix and constant T-spherical fuzzy matrix and discusses related properties. Determinant and the adjoint of a square T-spherical fuzzy matrix are also established, and some related properties are investigated. An application of the T-spherical fuzzy matrix in decision-making problem with an illustrative example is discussed here. Then, in the end, to check capability and viability, a practical demonstration of the planned approach has also been explained.

1. Introduction

In real life, sometimes it is necessary to compare two different things from different perspectives while dealing with different problems related to machine learning, namely, decision-making (DM), and image processing. An abundance of data is available in fuzzy and nonfuzzy situations concerned with the application. Different comparative measures may be applicable for various problems. The under-consideration article, for the most part, is related to the multi-attribute decision-making (MADM) problems. MADM is the important problem of deciding science, whose objective is to get the best choice from the group of similar choices. Originally in DM, one needed to evaluate the alternate options by many other categories. The non-cooperative behavior management for personalized individual semantic-based social network group decision-making is developed in [1], the group consensus-based travel destination evaluation method with online reviews is discussed in [2], and the comprehensive star rating approach for cruise ships based on interactive group DM with personalized individual semantics is performed in [3]. In order to regulate it, the concept of a fuzzy set (FS) was initiated by Zadeh [4]. It was a helpful tool to deal with uncertainties in real-life problems. Some prominent developments in these directions are mentioned. The fundamental theory of fuzzy sets with illustrative examples has been discussed in [5], some aggregation procedures, choice problems, and treatment of attributes are examined in [6], DM approaches to vowel and speaker recognition are studied in [7], multiple objective DM is discussed in [8], and fuzzy sets and fuzzy decision-making are discussed by Li and Yen [9]. Following this new direction in fuzzy theory, the idea of a fuzzy matrix (FM) was initiated in [10]. Later on, some operations and generalizations on FM such that FM with row and column have been developed in [11], and interval-valued FM with rows and columns is discussed by Pal [12]. The study of bipolar FM has been developed in [13]. The generalized FMs are discussed in [14]. Pradhan and Pal [15] developed the concept of the triangular FM norm and its properties. Ragab discussed the adjoint and determinant of square FM in [16], and he further developed the concept of min-max composition of FMs [17].

The FS theory has not been able to deliver in some conditions. In particular, in clear information, the complement of the participation degree (PD) is equal to the nonparticipation degree (NPD). In such cases, the NPD is not the complement of the PD. In this situation, the PD and

NPD are needed. To handle the situation, Atanassov introduced the concept of intuitionistic fuzzy set (IFS) [18], which describes the PD and the NPD of an element or object. Following this new direction in fuzzy theory, the idea of an intuitionistic fuzzy determinant was initiated in [19]. Later on, some operations and generalizations on intuitionistic FM (IFM) such that interval-valued IFM have been examined by Khan and Pal [20], the concept of generalized inverse of block IFM is discussed in [21], and intuitionistic fuzzy incline matrix and determinant have been developed in [22]. Furthermore, Sriram and Murugadas [23] developed the concept of semiring of IFM, and he also studies the α-cut of IFM [24].

An IFS is a better tool than Zadeh's FS as it describes the NPD as well. But IFS has not been able to deliver in some conditions. For example, if a person is given 0.7 PD and 0.5 NPD, in that condition IFS will be unable to manage it, i.e., $0.7 + 0.5 = 1.2 > 1$. In that condition, IFS has not been kept in mind. In the same way, some problems were faced in real-life matters, where the IFS was also deviated. To handle the situation, Yager [25, 26] initiated the system of Pythagorean FSs (PyFSs), having the condition $(\text{PD})^2 + (\text{NPD})^2 \in [0, 1]$. Following this new direction in fuzzy theory, the idea of Pythagorean FM (PyFM) was initiated in [27]. Later on, some operations and generalizations on PyFM were developed in [28, 29].

In various fields of real life, it turns out that to represent a physical phenomenon two components are not enough. For example, a disease may have three aspects: positive, neutral, and negative. To handle such type of data, the IFS model is not sufficient. To overcome these limitations, Cuong initiated the concept of picture fuzzy set (PFS) in [30, 31], which described the PD, abstained degree (AD), and NPD of an element or object. Some picture fuzzy operators are discussed in [32, 33]. In the generalization of PFSs, the new concept of picture fuzzy matrix (PFM) was introduced in [34].

The PFSs extend the model of FSs and IFSs, but there is still a limitation in the structure. For example, if a person is given 0.6 PD, 0.4 AD, and 0.3 NPD, in that condition PFS will be unable to manage it, i.e., $0.6 + 0.4 + 0.3 = 1.3 > 1$. In that condition, PFS has not been kept in mind. In the same way, some problems were faced in real-life matters, where the PFS was also deviated. To handle the situation, the concept of T-spherical fuzzy set (TSFS,) which rectifies these limitations, was proposed in [35] having the condition $(\text{PD})^n + (\text{AD})^n + (\text{NPD})^n \in [0, 1]$. Some new similarity measures for TSFSs have been developed by Ullah et al. [36] and Saad and Rafiq [37]. The divergence measure of TSFSs with their applications in pattern recognition has been discussed in [38]. A study of correlation coefficients based on TSFSs has been examined in [39]. Algorithms based on improved interactive aggregation operators are discussed in [40], and immediate probabilistic interactive averaging aggregation operators are discussed in [41]. With application in MADM problems, the Einstein hybrid aggregation operators based on TSFS are discussed in [42]. Wu et al. [38] discuss the divergence measure of TSFSs and their applications. Quek et al. [43] discussed the generalized T-spherical fuzzy weighted aggregation operators on neutrosophic sets. Based on TSFSs, the shortest path problem and the DM approach are discussed by Zedam et al. [44].

Several studies then explore the concepts of DM in FM and the PFM model. But there are some limitations, as we are not independent to assign the values to all participation grades while investigating the data are in picture fuzzy form [34]. In this situation, we needed a structure in the FM theory that is independent to assign the values of different grades that are involved in it. Also, we are forbidden to treat the data in T-spherical fuzzy (TSF) context. Keeping in view the importance of the matrix theory, the fuzzy matrix, and the broad domain of TSFSs, our aim is to develop the hybrid structure of FM and TSFSs named as T-spherical fuzzy matrix (TSFM). The following point shows the importance of the proposed work. A lot of objectives are under consideration to emphasize the need to build this model. Some of these objectives are mentioned as follows:

(1) The foremost aim to build this model is to overcome the research loopholes that are found in the existing methodologies. The FM and TSFS may also be involved together in decision analysis.

(2) To discuss the concepts of a square T-spherical fuzzy matrix, constant T-spherical fuzzy matrix and constant square T-spherical fuzzy matrix and study their related properties.

(3) To present multiattribute decision-making (MADM) algorithm to solve the decision-making problems, the approach has been illustrated with a numerical example.

This article is further separated into various sections. Section 2 reviews some of the essentials of the developed work. Section 3 introduces a new concept as TSFM and its features. In section 4, we initiated the decision-making algorithm for solving the problems and provided the numerical examples for justification. Section 5 provides a comparative study of the work with the existing studies. Finally, Section 6 concludes the paper.

2. Preliminaries

Here, the notions discussed provided a foundation for our work. From now onward, we use t, i, and f that act as PD, AD, and NPD, respectively. Furthermore, m_{xyt}, m_{xyi}, and m_{xyf} mentioned the PD of the xy^{th} element of M, AD of the xy^{th} element of M, and NPD of the xy^{th} element of M, respectively. Furthermore, P_j denotes the set of permutations on $\{1, 2, \ldots, j\}$, $P_{j_y j_x}$ $(P_{j_x j_y})$ is a set of all permutations of a set j_x over j_y (j_x over j_y), and X acts as a universal set.

Definition 1 (see [18]). An IFS is of the form $A = \{x, t_A(x), f_A(x) | x \in X\}$, where t and f are functions from X to an element in the unit interval $[0, 1]$ with a restriction $0 \leq t + f \leq 1$, and $r(x) = 1 - (t + f)$ is the refusal degree (RD) of x in A. Here, (t, f) is an intuitionistic fuzzy number (IFN).

Definition 2 (see [19]). An IFM M of order $j \times k$ is of the form $M = (m_{xyt}, m_{xyf})$, where m_{xyt} and $m_{xyf} \in [0,1]$ with the condition $0 \leq m_{xyt} + m_{xyf} \leq 1$ for $x = 1, 2, \ldots, j$ and $y = 1, 2, \ldots, k$. An IFM is said to be square IFM (SIFM) if $j = k$.

Definition 3 (see [34]). A PFS is of the form $A = \{x, t_A(x), i_A(x), f_A(x) | x \in X\}$, where t, i, and f are functions from X to an element in the unit interval $[0,1]$ with a restriction $0 \leq t + i + f \leq 1$, and $r(x) = 1 - (t + i + f)$ is the RD of x in A, where (t, i, f) is a picture fuzzy number (PFN).

Definition 4 (see [34]). A PFM M of order $j \times k$ is of the form $M = (m_{xyt}, m_{xyi}, m_{xyf})$, where m_{xyt}, m_{xyi}, and $m_{xyf} \in [0,1]$ with the condition $0 \leq m_{xyt} + m_{xyi} + m_{xyf} \leq 1$ for $x = 1, 2, \ldots, j$ and $y = 1, 2, \ldots, k$.

Remarks 1 (see [34])

(1) A PFM is said to be square PFM (SIFM) if $j = k$
(2) An identity PFM I of order j is SPFM with all diagonal entries $(1, 0, 0)$ and others $(0, 1, 1)$
(3) A null PFM of order j is the SPFM with all entries $(0, 1, 1)$

Definition 5 (see [34]). For two SPFM $M_1 = (m_{1xyt}, m_{1xyi}, m_{1xyf})$ and $M_2 = (m_{2xyt}, m_{2xyi}, m_{2xyf})$, the product is defined as $M = M_1 \times M_2 = (m_{xyt}, m_{xyi}, m_{xyf})$, where $m_{xyt} = \vee_u (m_{1xut} \wedge m_{2uyt})$, $m_{xyi} = \vee_u (m_{1xui} \wedge m_{2uyi})$, and $m_{xyi} = \wedge_u (m_{1xui} \vee m_{2uyi})$ for $x, y = 1, 2, \ldots, j$. Here, u runs from 1 to j.

Definition 6 (see [34]). For a SPFM $M = (m_{xyt}, m_{xyi}, m_{xyf})$ of order j, the $|M|$ is defined as follows:

$$
\begin{aligned}
|M| = & \bigvee_{\delta \in P_j} \left(m_{1\delta(1)t} \wedge m_{2\delta(2)t} \wedge \ldots \wedge m_{j\delta(j)t} \right) \\
& \bigwedge_{\delta \in P_j} \left(m_{1\delta(1)i} \wedge m_{2\delta(2)i} \wedge \ldots \wedge m_{1\delta(j)i} \right) \\
& \bigwedge_{\delta \in P_j} \left(m_{1\delta(1)f} \vee m_{2\delta(2)f} \vee \ldots \vee m_{j\delta(j)f} \right).
\end{aligned}
\tag{1}
$$

Definition 7 (see [34]). For a SPFM $M = (m_{xyt}, m_{xyi}, m_{xyf})$ of order j, the $\operatorname{adj}(M)$ is defined as $\operatorname{adj}(M) = R = (r_{xyt}, r_{xyi}, r_{xyf})$, where

$$
\begin{aligned}
r_{xyt} &= \bigvee_{\delta \in P_{j_y j_x}} \bigwedge_{u \in j_x} m_{u\delta(u)t}, \\
r_{xyi} &= \bigwedge_{\delta \in P_{j_y j_x}} \bigwedge_{u \in j_x} m_{u\delta(u)i}, \\
r_{xyf} &= \bigwedge_{\delta \in P_{j_y j_x}} \bigvee_{u \in j_x} m_{u\delta(u)f}.
\end{aligned}
\tag{2}
$$

Here, $j_x = \{1, 2, \ldots, j\} - \{x\}$.

The PFSs extend the model of FSs and IFSs, but there is still a limitation in the structure. For example, if a person is given 0.6 PD, 0.4 AD, and 0.3 NPD, in that condition PFS will be unable to manage it, i.e., $0.6 + 0.4 + 0.3 = 1.3 > 1$. In that condition, PFS has not been kept in mind. In the same way, some problems were faced in real-life matters, where the PFS was also deviated. To handle the situation, the concept of spherical fuzzy set (SFS) and TSFS, which rectifies these limitations, was proposed in [30] having the conditions $(\mathrm{PD})^2 + \mathrm{HD}^2 + (\mathrm{NPD})^2 \in [0,1]$ and $(\mathrm{PD})^n + \mathrm{HD}^n + (\mathrm{NPD})^n \in [0,1]$, respectively. This shows the importance and advantages of TSFSs over existing fuzzy structures.

Definition 8 (see [35]). A SFS is of the form $A = \{x, t_A(x), i_A(x), f_A(x) | x \in X\}$, where t, i, and f are functions from X to an element in the unit interval $[0,1]$ with a restriction of $0 \leq t^2 + i^2 + f^2 \leq 1$, and $r(x) = \sqrt{1 - (t^2 + i^2 + f^2)}$ is the RD of x in A, where (t, i, f) is a spherical fuzzy number (SFN).

Definition 9 (see [35]). A TSFS is of the form $A = \{x, t_A(x), i_A(x), f_A(x) | x \in X\}$, where t, i, and f are function from X to an element in the unit interval $[0,1]$ with a restriction of $0 \leq t^n + i^n + f^n \leq 1$ for $n \in \mathbb{Z}$, and $r(x) = \sqrt[n]{1 - (t^n + i^n + f^n)}$ is the RD of x in A, where (t, i, f) is a T-spherical fuzzy number (TSFN).

3. T-Spherical Fuzzy Matrix

Here, we will define a novel concept TSFM, in the generalization of PFM.

Definition 10. A TFM M of order $j \times k$ is of the form $M = (m_{xyt}, m_{xyi}, m_{xyf})$, where m_{xyt}, m_{xyi}, and $m_{xyf} \in [0,1]$ with the condition $0 \leq m_{xyt}^n + m_{xyi}^n + m_{xyf}^n \leq 1$, $n \in \mathbb{Z}$ for $x = 1, 2, \ldots, j$ and $y = 1, 2, \ldots, k$.

Remarks 2

(1) For $n = 2$, a TSFM becomes spherical fuzzy matrix
(2) A TSFM is said to be square TSFM (SIFM) if $j = k$
(3) An identity TSFM I of order j is STSFM with all diagonal entries $(1, 0, 0)$ and others $(0, 1, 1)$
(4) A null TSFM of order j is the STSFM with all entries $(0, 1, 1)$

Definition 11. For two STSFM $M_1 = (m_{1xyt}, m_{1xyi}, m_{1xyf})$and $M_2 = (m_{2xyt}, m_{2xyi}, m_{2xyf})$, the product is defined as follows: $M = M_1 \times M_2 = (m_{xyt}, m_{xyi}, m_{xyf})$, where $m_{xyt}^n = \vee_k (m_{1xkt}^n \wedge m_{2kyt}^n)$, $m_{xyi}^n = \wedge_k (m_{1xki}^n \wedge m_{2kyi}^n)$, and $m_{xyi}^n = \wedge_k (m_{1xkt}^n \vee m_{2kyt}^n)$ for $x, y = 1, 2, \ldots, j$. Here, k runs from 1 to j.

Definition 12. For two STSFM $M_1 = (m_{1xyt}, m_{1xyi}, m_{1xyf})$ and $M_2 = (m_{2xyt}, m_{2xyi}, m_{2xyf})$, $M_1 \leq M_2$ if $m_{1xyt}^n \leq m_{2xyt}^n$, $m_{1xyi}^n \leq m_{2xyi}^n$ and $m_{1xyf}^n \geq m_{2xyf}^n$ for $x, y = 1, 2, \ldots, j$.

Definition 13. For a STSFM $M = (m_{xyt}, m_{xyi}, m_{xyf})$, the multiplication by a TSFN $z = (z_1, z_2, z_3)$ is defined as $z.M = (z_1^n.m_{xyt}^n, z_2^n.m_{xyi}^n, z_3^n.m_{xyf}^n)$.

Definition 14. Let a STSFM $M = (m_{xyt}, m_{xyi}, m_{xyf})$ and TSFN $z = (z_1, z_2, z_3)$, for $(m_{xyt}, m_{xyi}, m_{xyf}) \geq (z_1, z_2, z_3)$ means that, $m^n_{xyt} \geq z^n_1$, $m^n_{xyi} \geq z^n_2$ and $m^n_{xyf} \leq z^n_3$.

Definition 15. For two STSFM $M_1 = (m_{1xyt}, m_{1xyi}, m_{1xyf})$ and $M_2 = (m_{2xyt}, m_{2xyi}, m_{2xyf})$, the operations of product, supremum, infimum, transpose, and complement are defined as follows:

(1) $M_1 \odot M_2 = (m^n_{1xyt}.m^n_{2xyt}, m^n_{1xyi}.m^n_{2xyi}, m^n_{1xyf}.m^n_{2xyf})$
(2) $M_1 \vee M_2 = (m^n_{1xyt} \vee m^n_{2xyt}, m^n_{1xyi} \wedge m^n_{2xyi}, m^n_{1xyf} \wedge m^n_{2xyf})$
(3) $M_1 \wedge M_2 = (m^n_{1xyt} \wedge m^n_{2xyt}, m^n_{1xyi} \wedge m^n_{2xyi}, m^n_{1xyf} \vee m^n_{2xyf})$
(4) $M^t_1 = (m^n_{1yxt}, m^n_{1yxi}, m^n_{1yxf})$
(5) $M^c_1 = (m^n_{1xyf}, m^n_{1xyi}, m^n_{1xyt})$

3.1. Some Properties on Square T-Spherical Fuzzy Matrix. Here, we will discuss some ground properties of STSFM.

Proposition 1. *For three STSFM $M_1 = (m_{1xyt}, m_{1xyi}, m_{1xyf})$, $M_2 = (m_{2xyt}, m_{2xyi}, m_{2xyf})$, and $M_3 = (m_{3xyt}, m_{3xyi}, m_{3xyf})$, 1 to 7 holds.*

(1) $M_1 \wedge M_2 = M_2 \wedge M_1$
(2) $M_1 \vee M_2 = M_2 \vee M_1$
(3) $(M^t_1)^{\mathbf{t}} = M_1$
(4) $(M^t_1)^t = (M^t_1)^{\mathbf{c}}$
(5) $M_1 \wedge (M_2 \vee M_3) = (M_1 \wedge M_2) \vee (M_1 \wedge M_3)$
(6) $M_1 \vee (M_2 \wedge M_3) = (M_1 \vee M_2) \wedge (M_1 \vee M_3)$
(7) $(z.M_1)^t = (z.M^t_1)$

Proof

(1) We have

$$\begin{aligned} M_1 \wedge M_2 &= \left(m^n_{1xyt} \wedge m^n_{2xyt}, m^n_{1xyi} \wedge m^n_{2xyi}, m^n_{1xyf} \vee m^n_{2xyf}\right) \\ &= \left(m^n_{2xyt} \wedge m^n_{1xyt}, m^n_{2xyi} \wedge m^n_{1xyi}, m^n_{2xyf} \vee m^n_{1xyf}\right) \\ &= M_2 \wedge M_1. \end{aligned} \tag{3}$$

(2) We have

$$\begin{aligned} M_1 \vee M_2 &= \left(m^n_{1xyt} \vee m^n_{2xyt}, m^n_{1xyi} \wedge m^n_{2xyi}, m^n_{1xyf} \wedge m^n_{2xyf}\right) \\ &= \left(m^n_{2xyt} \vee m^n_{1xyt}, m^n_{2xyi} \wedge m^n_{1xyi}, m^n_{2xyf} \wedge m^n_{1xyf}\right) \\ &= M_2 \vee M_1. \end{aligned} \tag{4}$$

(3) We have

$$\begin{aligned} M^t_1 &= \left(m^n_{1xyt}, m^n_{1xyi}, m^n_{1xyf}\right)^t = \left(m^n_{1yxt}, m^n_{1yxi}, m^n_{1yxf}\right), \\ \left(M^t_1\right)^{\mathbf{t}} &= \left(m^n_{1yxt}, m^n_{1yxi}, m^n_{1yxf}\right)^t = \left(m^n_{1xyt}, m^n_{1xyi}, m^n_{1xyf}\right) = M_1, \\ M^c_1 &= \left(m^n_{1xyf}, m^n_{1xyi}, m^n_{1xyt}\right), \\ \left(M^c_1\right)^t &= \left(m^n_{1xyf}, m^n_{1xyi}, m^n_{1xyt}\right)^t = \left(m^n_{1yxf}, m^n_{1yxi}, m^n_{1yxt}\right). \end{aligned} \tag{5}$$

Now,

$$\begin{aligned} M^t_1 &= \left(m^n_{1xyt}, m^n_{1xyi}, m^n_{1xyf}\right)^t = \left(m^n_{1yxt}, m^n_{1yxi}, m^n_{1yxf}\right), \\ \left(M^t_1\right)^c &= \left(m^n_{1yxt}, m^n_{1yxi}, m^n_{1yxf}\right)^c = \left(m^n_{1yxf}, m^n_{1yxi}, m^n_{1yxt}\right). \end{aligned} \tag{6}$$

Thus, $(M^t_1)^t = (M^t_1)^{\mathbf{c}}$.

(4) We have

$$\begin{aligned} &M_1 \wedge (M_2 \vee M_3) \\ &= \left(m^n_{1xyt}, m^n_{1xyi}, m^n_{1xyf}\right) \wedge \left(\left(m^n_{2xyt} \vee m^n_{3xyt}, m^n_{2xyi} \wedge m^n_{3xyi}, m^n_{2xyf} \wedge m^n_{3xyf}\right)\right) \\ &= \left(m^n_{1xyt} \wedge \left(m^n_{2xyt} \vee m^n_{3xyt}\right), m^n_{1xyi} \wedge \left(m^n_{2xyi} \wedge m^n_{3xyi}\right), m^n_{1xyf} \vee \left(m^n_{2xyf} \wedge m^n_{3xyf}\right)\right) \\ &= \left(\left(m^n_{1xyt} \wedge m^n_{2xyt}\right) \vee \left(m^n_{2xyt} \wedge m^n_{3xyt}\right), \left(m^n_{1xyi} \wedge m^n_{2xyi}\right) \wedge \left(m^n_{2xyi} \wedge m^n_{3xyi}\right), \left(m^n_{1xyf} \vee m^n_{2xyf}\right) \wedge \left(m^n_{2xyf} \vee m^n_{3xyf}\right)\right) \\ &= \left(\left(m^n_{1xyt} \wedge m^n_{2xyt}, m^n_{1xyi} \wedge m^n_{2xyi}, m^n_{1xyf} \vee m^n_{2xyf}\right) \vee \left(m^n_{2xyt} \wedge m^n_{3xyt}, m^n_{2xyi} \wedge m^n_{3xyi}, m^n_{2xyf} \vee m^n_{3xyf}\right)\right) \\ &= (M_1 \wedge M_2) \vee (M_1 \wedge M_3). \end{aligned} \tag{7}$$

(5) We have

$$\begin{aligned}
M_1\vee(M_2\wedge M_3) &= \left(m^n_{1xyt}, m^n_{1xyi}, m^n_{1xyf}\right)\vee\left(\left(m^n_{2xyt}\wedge m^n_{3xyt}, m^n_{2xyi}\wedge m^n_{3xyi}, m^n_{2xyf}\vee m^n_{3xyf}\right)\right)\\
&= \left(m^n_{1xyt}\vee\left(m^n_{2xyt}\wedge m^n_{3xyt}\right), m^n_{1xyi}\wedge\left(m^n_{2xyi}\wedge m^n_{3xyi}\right), m^n_{1xyf}\wedge\left(m^n_{2xyf}\vee m^n_{3xyf}\right)\right)\\
&= \left(\left(m^n_{1xyt}\vee m^n_{2xyt}\right)\wedge\left(m^n_{2xyt}\vee m^n_{3xyt}\right), \left(m^n_{1xyi}\wedge m^n_{2xyi}\right)\wedge\left(m^n_{2xyi}\wedge m^n_{3xyi}\right), \left(m^n_{1xyf}\wedge m^n_{2xyf}\right)\vee\left(m^n_{2xyf}\wedge m^n_{3xyf}\right)\right)\\
&= \left(\left(m^n_{1xyt}\vee m^n_{2xyt}, m^n_{1xyi}\wedge m^n_{2xyi}, m^n_{1xyf}\wedge m^n_{2xyf}\right)\wedge\left(m^n_{2xyt}\vee m^n_{3xyt}, m^n_{2xyi}\wedge m^n_{3xyi}, m^n_{2xyf}\wedge m^n_{3xyf}\right)\right)\\
&= (M_1\vee M_2)\wedge(M_1\vee M_3),\\
(z.M_1)^t &= \left(\left(z^n_1, z^n_2, z^n_3\right).\left(m^n_{1xyt}, m^n_{1xyi}, m^n_{1xyf}\right)\right)^t\\
&= \left(z^n_1\wedge m^n_{1xyt}, z^n_2\wedge m^n_{1xyi}, z^n_3\vee m^n_{1xyf}\right)^t\\
&= \left(z^n_1\wedge m^n_{1yxt}, z^n_2\wedge m^n_{1yxi}, z^n_3\vee m^n_{1yxf}\right)\\
&= (z^n_1, z^n_2, z^n_3).\left(m^n_{1yxt}, m^n_{1yxi}, m^n_{1yxf}\right) = \left(z.M^t_1\right).
\end{aligned}\tag{8}$$

□

Proposition 2. *For three STSFM $M_1 = (m_{1xyt}, m_{1xyi}, m_{1xyf})$, $M_2 = (m_{2xyt}, m_{2xyi}, m_{2xyf})$, and $M_3 = (m_{3xyt}, \backslash m_{3xyi}, m_{3xyf})$,*

(1) $M_1\vee M_2 \geq M_1\wedge M_2$

(2) $M_1\vee M_3 \geq M_2\vee M_3$, when $M_1 \geq M_2$

(3) $M_1\wedge M_3 \leq M_2\wedge M_3$, when $M_1 \leq M_2$

Proof

(1) For two STSFM $M_1 = (m_{1xyt}, m_{1xyi}, m_{1xyf})$ and $M_2 = (m_{2xyt}, m_{2xyi}, m_{2xyf})$ of order j,

$$\begin{aligned}
M_1\vee M_2 &= \left(m^n_{1xyt}\vee m^n_{2xyt}, m^n_{1xyi}\wedge m^n_{2xyi}, m^n_{1xyf}\wedge m^n_{2xyf}\right),\\
M_1\wedge M_2 &= \left(m^n_{1xyt}\wedge m^n_{2xyt}, m^n_{1xyi}\wedge m^n_{2xyi}, m^n_{1xyf}\vee m^n_{2xyf}\right).
\end{aligned}\tag{9}$$

Taking

$$\begin{aligned}
M_1 &= (0.50, 0.46, 0.64),\\
M_2 &= (0.52, 0.49, 0.36) \quad \text{for } n = 3,
\end{aligned}\tag{10}$$

$$\begin{aligned}
M_1\vee M_2 &= (0.13\vee 0.14, 0.10\wedge 0.12, 0.26\wedge 0.05)\\
&= (0.14, 0.10, 0.05),\\
M_1\wedge M_2 &= (0.13\wedge 0.14, 0.10\wedge 0.12, 0.26\vee 0.05)\\
&= (0.13, 0.10, 0.26).
\end{aligned}\tag{11}$$

It is clear that $m^n_{1xyt}\vee m^n_{2xyt} \geq m^n_{1xyt}\wedge m^n_{2xyt}$, $m^n_{1xyi}\wedge m^n_{2xyi} \geq m^n_{1xyi}\wedge m^n_{2xyi}$, and $m^n_{1xyf}\wedge m^n_{2xyf} \leq m^n_{1xyf}\vee m^n_{2xyf}$. So,

$$M_1\vee M_2 \geq M_1\wedge M_2. \tag{12}$$

(2) For three STSFM $M_1 = (m_{1xyt}, m_{1xyi}, m_{1xyf})$, $M_2 = (m_{2xyt}, m_{2xyi}, m_{2xyf})$, and $M_3 = (m_{3xyt}, m_{3xyi}, m_{3xyf})$ of order j,

$M_1 \geq M_2 \Rightarrow m^n_{1xyt} \geq m^n_{2xyt}$, $m^n_{1xyi} \geq m^n_{2xyi}$ and $m^n_{1xyf} \leq m^n_{2xyf}$

When $m^n_{1xyt} \geq m^n_{2xyt} \geq m^n_{3xyt}$, $m^n_{1xyt}\vee m^n_{3xyt} = m^n_{1xyt}$ and $m^n_{2xyt}\vee m^n_{3xyt} = m^n_{2xyt}$.

When $m^n_{1xyt} \geq m^n_{3xyt} \geq m^n_{2xyt}$, $m^n_{1xyt}\vee m^n_{3xyt} = m^n_{1xyt}$ and $m^n_{2xyt}\vee m^n_{3xyt} = m^n_{2xyt}$.

When $m^n_{3xyt} \geq m^n_{1xyt} \geq m^n_{2xyt}$, $m^n_{1xyt}\vee m^n_{3xyt} = m^n_{3xyt}$ and $m^n_{2xyt}\vee m^n_{3xyt} = m^n_{3xyt}$.

Now, when $m^n_{1xyi} \geq m^n_{2xyi} \geq m^n_{3xyi}$, $m^n_{1xyi}\wedge m^n_{3xyi} = m^n_{3xyi}$ and $m^n_{2xyi}\wedge m^n_{3xyi} = m^n_{3xyi}$.

When $m^n_{1xyi} \geq m^n_{3xyi} \geq m^n_{2xyi}$, $m^n_{1xyi}\wedge m^n_{3xyi} = m^n_{3xyi}$ and $m^n_{2xyi}\wedge m^n_{3xyi} = m^n_{2xyi}$.

When $m^n_{3xyi} \geq m^n_{1xyi} \geq m^n_{2xyi}$, $m^n_{1xyi}\wedge m^n_{3xyi} = m^n_{1xyi}$ and $m^n_{2xyi}\wedge m^n_{3xyi} = m^n_{2xyi}$.

Also, when $m^n_{1xyf} \leq m^n_{2xyf} \leq m^n_{3xyf}$, $m^n_{1xyf}\wedge m^n_{3xyf} = m^n_{1xyf}$ and $m^n_{2xyf}\wedge m^n_{3xyf} = m^n_{2xyf}$.

When $m^n_{1xyf} \leq m^n_{3xyf} \leq m^n_{2xyf}$, $m^n_{1xyf}\wedge m^n_{3xyf} = m^n_{1xyf}$ and $m^n_{2xyf}\wedge m^n_{3xyf} = m^n_{3xyf}$.

When $m^n_{3xyf} \leq m^n_{1xyf} \leq m^n_{2xyf}$, $m^n_{1xyf}\wedge m^n_{3xyf} = m^n_{3xyf}$ and $m^n_{2xyf}\wedge m^n_{3xyf} = m^n_{3xyf}$.

Therefore, $\Rightarrow m^n_{1xyt}\vee m^n_{3xyt} \geq m^n_{2xyt}\vee m^n_{3xyt}$, $m^n_{1xyt}\wedge m^n_{3xyt} \geq m^n_{2xyt}\wedge m^n_{3xyt}$ and $m^n_{1xyt}\wedge m^n_{3xyt} \leq m^n_{2xyt}\wedge m^n_{3xyt}$ for $x, y = 1, 2, \ldots, j$. Consequently, $M_1\vee M_3 \geq M_2\vee M_3$.

(3) The proof is similar to 2.

While investigating the TSFSs, the convex combination (CC) defined in [34] has found some limitations, as we are not independent to assign the values to the PD, RD, and NPD. To overcome the problem, we will define the CC of two matrixes in a T-spherical fuzzy context.

Definition 16. For two STSFM M_1 and M_2 of order j, the CC is denoted and defined as follows:

$$M_1 * M_2 = \left(m_{xyt}, m_{xyi}, m_{xyf}\right), \tag{13}$$

where $m^n_{xyt} = \delta m^n_{1xyt} + (1-\delta)m^n_{2xyt}, m^n_{xyi} = \delta m^n_{1xyi} + (1-\delta)m^n_{2xyi}$, and $m^n_{xyf} = \delta m^n_{1xyf} + (1-\delta)m^n_{2xyf}$ for $x, y = 1, 2, \ldots, j$, $0 \le \delta \le 1$.

So, it is observed that the CC of two STSFM is the CC of their entries.

Definition 17. A STSFM $M = (m_{xyt}, m_{xyi}, m_{xyf})$ is called idempotent w.r.t. some operation ". ∘" if $M \circ M = M$.

Proposition 3. *Every STSFM is idempotent concerning CC "$*$."*

Proof. For an STSFM $M = (m_{xyt}, m_{xyi}, m_{xyf})$ of order j, let $M * M = (m_{1xyt}, m_{1xyi}, m_{1xyf})$, where $m^n_{1xyt} = \delta m^n_{xyt} + (1-\delta)m^n_{xyt} = m^n_{xyt}$

$$m^n_{x1yi} = \delta m^n_{xyi} + (1-\delta)m^n_{xyi} = m^n_{xyi}. \tag{14}$$

$m^n_{1xyf} = \delta m^n_{xyf} + (1-\delta)m^n_{xyf} = m^n_{xyf}$ for $x, y = 1, 2, \ldots, j$, $0 \le \delta \le 1$.

Therefore, $M * M = M$.

3.2. Determinant of STSFM. Here, we will define determinants and some related results along with a numerical example.

Definition 18. For a TSFM M of order j, the $|M|$ is defined as follows:

$$\begin{aligned} |M| = &\bigvee_{\delta \in P_j} \left(m^n_{1\delta(1)t} \wedge m^n_{2\delta(2)t} \wedge \ldots \wedge m^n_{j\delta(j)t}\right) \\ &\bigwedge_{\delta \in P_j} \left(m^n_{1\delta(1)i} \wedge m^n_{2\delta(2)i} \wedge \ldots \wedge m^n_{j\delta(j)i}\right) \\ &\bigwedge_{\delta \in P_j} \left(m^n_{1\delta(1)f} \vee m^n_{2\delta(2)f} \vee \ldots \vee m^n_{j\delta(j)i}\right). \end{aligned} \tag{15}$$

Example 1. Let M be a STSFM of order 3, for $n = 3$

$$M = \begin{pmatrix} (0.63, 0.53, 0.31) & (0.63, 0.43, 0.53) & (0.53, 0.33, 0.51) \\ (0.41, 0.26, 0.47) & (0.42, 0.18, 0.44) & (0.25, 0.46, 0.43) \\ (0.51, 0.38, 0.45) & (0.71, 0.34, 0.34) & (0.50, 0.50, 0.50) \end{pmatrix}. \tag{16}$$

To find determinant, it is necessary to find all permutations on $\{1, 2, 3\}$.

$$\begin{aligned} &\delta_1 = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}, \delta_2 = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}, \delta_3 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}, \\ &\delta_4 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}, \delta_5 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}, \delta_6 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}. \end{aligned} \tag{17}$$

The participation degree of $|M|$ is as follows:

$$\begin{aligned} &\left(m^3_{1\delta_1(1)t} \wedge m^3_{2\delta_1(2)t} \wedge m^3_{3\delta_1(3)t}\right) \vee \left(m^3_{1\delta_2(1)t} \wedge m^3_{2\delta_2(2)t} \wedge m^3_{3\delta_2(3)t}\right) \vee \\ &\cdot \left(m^3_{1\delta_3(1)t} \wedge m^3_{2\delta_3(2)t} \wedge m^3_{3\delta_3(3)t}\right) \vee \left(m^3_{1\delta_4(1)t} \wedge m^3_{2\delta_4(2)t} \wedge m^3_{3\delta_4(3)t}\right) \vee \\ &\cdot \left(m^3_{1\delta_5(1)t} \wedge m^3_{2\delta_5(2)t} \wedge m^3_{3\delta_5(3)t}\right) \vee \left(m^3_{1\delta_6(1)t} \wedge m^3_{2\delta_6(2)t} \wedge m^3_{3\delta_6(3)t}\right) \\ &= \left(m^3_{11t} \wedge m^3_{22t} \wedge m^3_{33t}\right) \vee \left(m^3_{11t} \wedge m^3_{23t} \wedge m^3_{32t}\right) \vee \left(m^3_{12t} \wedge m^3_{21t} \wedge m^3_{33t}\right) \vee \\ &\left(m^3_{12t} \wedge m^3_{23t} \wedge m^3_{31t}\right) \vee \left(m^3_{13t} \wedge m^3_{21t} \wedge m^3_{32t}\right) \vee \left(m^3_{13t} \wedge m^3_{22t} \wedge m^3_{31t}\right) \\ &= (0.2500 \wedge 0.0741 \wedge 0.1250) \vee (0.2500 \wedge 0.0156 \wedge 0.3589) \\ &\vee (0.2500 \wedge 0.0689 \wedge 0.1250) \vee (0.2500 \wedge 0.0156 \wedge 0.1327) \\ &\vee (0.1489 \wedge 0.0689 \wedge 0.3579) \vee (0.1489 \wedge 0.0741 \wedge 0.1327) \\ &= 0.0741 \vee 0.0156 \vee 0.0689 \vee 0.0156 \vee 0.0689 \vee 0.0741 \\ &= 0.0741. \end{aligned} \tag{18}$$

The refusal degree of $|M|$ is as follows:

$$\begin{aligned}
&\left(m^3_{1\delta_1(1)i}\wedge m^3_{2\delta_1(2)i}\wedge m^3_{3\delta_1(3)i}\right)\wedge\left(m^3_{1\delta_2(1)i}\wedge m^3_{2\delta_2(2)i}\wedge m^3_{3\delta_2(3)i}\right)\wedge,\\
&\cdot\left(m^3_{1\delta_3(1)i}\wedge m^3_{2\delta_3(2)i}\wedge m^3_{3\delta_3(3)i}\right)\wedge\left(m^3_{1\delta_4(1)i}\wedge m^3_{2\delta_4(2)i}\wedge m^3_{3\delta_4(3)i}\right)\wedge,\\
&\cdot\left(m^3_{1\delta_5(1)i}\wedge m^3_{2\delta_5(2)i}\wedge m^3_{3\delta_5(3)i}\right)\wedge\left(m^3_{1\delta_6(1)i}\wedge m^3_{2\delta_6(2)i}\wedge m^3_{3\delta_6(3)i}\right),\\
&=\left(m^3_{11i}\wedge m^3_{22i}\wedge m^3_{33i}\right)\wedge\left(m^3_{11i}\wedge m^3_{23i}\wedge m^3_{32i}\right)\wedge\left(m^3_{12i}\wedge m^3_{21i}\wedge m^3_{33i}\right)\wedge,\\
&\left(m^3_{12i}\wedge m^3_{23i}\wedge m^3_{31i}\right)\wedge\left(m^3_{13i}\wedge m^3_{21i}\wedge m^3_{32i}\right)\wedge\left(m^3_{13i}\wedge m^3_{22i}\wedge m^3_{31i}\right),\\
&=(0.1489\wedge 0.0058\wedge 0.1250)\wedge(0.1489\wedge 0.0973\wedge 0.0393),\\
&\wedge(0.0795\wedge 0.0176\wedge 0.1250)\wedge(0.0795\wedge 0.0973\wedge 0.0549),\\
&\wedge(0.0359\wedge 0.0176\wedge 0.0393)\wedge(0.0359\wedge 0.0058\wedge 0.0549),\\
&=0.0058\wedge 0.0393\wedge 0.0176\wedge 0.0549\wedge 0.0176\wedge 0.0058,\\
&=0.0058.
\end{aligned}\tag{19}$$

The nonparticipation degree of $|M|$ is as follows:

$$\begin{aligned}
&\left(m^3_{1\delta_1(1)f}\vee m^3_{2\delta_1(2)f}\vee m^3_{3\delta_1(3)f}\right)\wedge\left(m^3_{1\delta_2(1)f}\vee m^3_{2\delta_2(2)f}\vee m^3_{3\delta_2(3)f}\right)\wedge,\\
&\cdot\left(m^3_{1\delta_3(1)f}\vee m^3_{2\delta_3(2)f}\vee m^3_{3\delta_3(3)f}\right)\wedge\left(m^3_{1\delta_4(1)f}\vee m^3_{2\delta_4(2)f}\vee m^3_{3\delta_4(3)f}\right)\wedge,\\
&\cdot\left(m^3_{1\delta_5(1)f}\vee m^3_{2\delta_5(2)f}\vee m^3_{3\delta_5(3)f}\right)\wedge\left(m^3_{1\delta_6(1)f}\vee m^3_{2\delta_6(2)f}\vee m^3_{3\delta_6(3)f}\right),\\
&=\left(m^3_{11f}\vee m^3_{22f}\vee m^3_{33f}\right)\wedge\left(m^3_{11f}\vee m^3_{23f}\vee m^3_{32f}\right)\wedge,\\
&\cdot\left(m^3_{12f}\vee m^3_{21f}\vee m^3_{33f}\right)\wedge\left(m^3_{12f}\vee m^3_{23f}\vee m^3_{31f}\right)\wedge,\\
&\cdot\left(m^3_{13f}\vee m^3_{21f}\vee m^3_{32f}\right)\wedge\left(m^3_{13f}\vee m^3_{22f}\vee m^3_{31f}\right),\\
&=(0.0298\vee 0.0852\vee 0.1250)\wedge(0.0298\vee 0.0795\vee 0.0393),\\
&\wedge(0.1489\vee 0.1038\vee 0.1250)\wedge(0.1489\vee 0.0795\vee 0.0911),\\
&\wedge(0.1327\vee 0.1038\vee 0.0393)\wedge(0.1327\vee 0.0852\vee 0.0911),\\
&\wedge(0.1327\vee 0.1038\vee 0.0393)\wedge(0.1327\vee 0.0852\vee 0.0911),\\
&=0.1250\wedge 0.0795\wedge 0.1489\wedge 0.1489\wedge 0.1327\wedge 0.1327,\\
&=0.0795.
\end{aligned}\tag{20}$$

From equations (18)–(20),

$$|M|=(0.0741,0.0058,0.0795).\tag{21}$$

Proposition 4. *Let M be a STSFM, and M^t is a transpose of M. Then, $|M^t|=|M|$.*

Proof. It is trivial, so we omit here.

Proposition 5. *Let M be a STSFM. If a row is multiplied by a TSFN $z=(z_1,z_2,z_3)$, then*

$$|z.M|=z.|M|.\tag{22}$$

Proof. It is trivial, so we omit here.

3.3. Adjoint of STSFM. Here, we will define adjoint and some related results on it.

Definition 19. For a TSFM M of order j, the $\mathrm{adj}(M)$ is defined as follows: $\mathrm{adj}(M)=R=(r_{xyt},r_{xyi},r_{xyf})$, where

$$\begin{aligned}
r^n_{xyt}&=\bigvee_{\delta\in P_{j_yj_x}}\bigwedge_{u\in j_x}m^n_{u\delta(u)t},\\
r^n_{xyi}&=\bigwedge_{\delta\in P_{j_yj_x}}\bigwedge_{u\in j_x}m^n_{u\delta(u)i},\\
r^n_{xyf}&=\bigwedge_{\delta\in P_{j_yj_x}}\bigvee_{u\in j_x}m^n_{u\delta(u)f}.
\end{aligned}\tag{23}$$

Here, $j_x = \{1, 2, \ldots, j\} - \{x\}$.

Example 2. Let M be a STSFM of order 3, for $n = 3$,

$$M = \begin{pmatrix} (0.40, 0.60, 0.30) & (0.30, 0.30, 0.30) & (0.30, 0.40, 0.50) \\ (0.60, 0.40, 0.30) & (0.40, 0.40, 0.40) & (0.40, 0.50, 0.40) \\ (0.30, 0.30, 0.30) & (0.40, 0.50, 0.40) & (0.40, 0.60, 0.30) \end{pmatrix}. \tag{24}$$

For $x = 1$ and $y = 1$, $j_y = \{1, 2, 3\} - \{1\} = \{2, 3\}$ and $j_x = \{1, 2, 3\} - \{1\} = \{2, 3\}$. Then, $P_{j_y j_x}$ is as follows: $\begin{pmatrix} 2 & 3 \\ 2 & 3 \end{pmatrix}$ and $\begin{pmatrix} 2 & 3 \\ 3 & 2 \end{pmatrix}$.

So,

$$\begin{aligned} \left(m^n_{22t} \wedge m^n_{33t}\right) \vee \left(m^n_{23t} \wedge m^n_{32t}\right) &= (0.0640 \wedge 0.0640) \vee (0.0640 \wedge 0.0640) = 0.0640 \vee 0.0640 = 0.0640, \\ \left(m^n_{22i} \wedge m^n_{33i}\right) \vee \left(m^n_{23i} \wedge m^n_{32i}\right) &= (0.0640 \wedge 0.2160) \vee (0.1250 \wedge 0.1250) = 0.0640 \vee 0.1250 = 0.1250, \\ \left(m^n_{22f} \vee m^n_{33f}\right) \wedge \left(m^n_{23f} \vee m^n_{32f}\right) &= (0.0640 \vee 0.0270) \wedge (0.0640 \vee 0.0640) = 0.0640 \wedge 0.0640 = 0.0640. \end{aligned} \tag{25}$$

For $x = 1$ and $y = 2$, $j_y = \{1, 2, 3\} - \{1\} = \{2, 3\}$ and $j_x = \{1, 2, 3\} - \{2\} = \{1, 3\}$. Then, $P_{j_y j_x}$ is as follows: $\begin{pmatrix} 1 & 3 \\ 2 & 3 \end{pmatrix}$ and $\begin{pmatrix} 1 & 3 \\ 3 & 2 \end{pmatrix}$

So,

$$\begin{aligned} \left(m^n_{12t} \wedge m^n_{33t}\right) \vee \left(m^n_{13t} \wedge m^n_{32t}\right) &= (0.0270 \wedge 0.2160) \vee (0.0270 \wedge 0.0640) = 0.0270 \vee 0.0270 = 0.0270, \\ \left(m^n_{12i} \wedge m^n_{33i}\right) \vee \left(m^n_{13i} \wedge m^n_{32i}\right) &= (0.0270 \wedge 0.2160) \vee (0.0640 \wedge 0.1250) = 0.0270 \vee 0.0640 = 0.0640, \\ \left(m^n_{12f} \vee m^n_{33f}\right) \wedge \left(m^n_{13f} \vee m^n_{32f}\right) &= (0.0270 \vee 0.0270) \wedge (0.1250 \vee 0.0640) = 0.0270 \wedge 0.1250 = 0.0270. \end{aligned} \tag{26}$$

Calculating similarly,

$$\operatorname{adj}(M) = \begin{pmatrix} (0.0640, 0.1250, 0.0640) & (0.0270, 0.0640, 0.0270) & (0.0270, 0.0270, 0.0640) \\ (0.0640, 0.0640, 0.0270) & (0.0640, 0.2160, 0.0270) & (0.0640, 0.1250, 0.0640) \\ (0.0640, 0.0640, 0.0640) & (0.0640, 0.1250, 0.0270) & (0.0640, 0.0640, 0.0270) \end{pmatrix}. \tag{27}$$

Proposition 6. *Let M be a STSFM. Then, $adj(M^t) = (adj(M))^t$.*

Proof. Let M be a STSFM of order j and $\operatorname{adj}(M) = (r_{xyt}, r_{xyi}, r_{xyf})$, then

$$\begin{aligned} M^t &= \left(m^n_{1xyt}, m^n_{1xyi}, m^n_{1xyf}\right)^t \\ &= \left(m^n_{1yxt}, m^n_{1yxi}, m^n_{1yxf}\right). \end{aligned} \tag{28}$$

Then, by definition of adjoint, we have

$$\operatorname{adj}\left(M^t\right) = \left(r_{yxt}, r_{yxi}, r_{yxf}\right), \tag{29}$$

where

$$\begin{aligned} r^n_{yxt} &= \bigvee_{\delta \in P_{j_x j_y}} \bigwedge_{u \in j_y} m^n_{u\delta(u)t}, r^n_{yxi} = \bigwedge_{\delta \in P_{j_x j_y}} \bigwedge_{u \in j_y} m^n_{u\delta(u)i}, r^n_{yxf} \\ &= \bigwedge_{\delta \in P_{j_x j_y}} \bigvee_{u \in j_y} m^n_{u\delta(u)f}. \end{aligned} \tag{30}$$

Here, $j_y = \{1, 2, \ldots, j\} - \{y\}$.
Hence,

$$\operatorname{adj}(M) = \left(r_{xyt}, r_{xyi}, r_{xyf}\right), \tag{31}$$

where

$$\begin{aligned} r^n_{xyt} &= \bigvee_{\delta \in P_{j_y j_x}} \bigwedge_{u \in j_x} m^n_{u\delta(u)t}, r^n_{xyi} = \bigwedge_{\delta \in P_{j_y j_x}} \bigwedge_{u \in j_x} m^n_{u\delta(u)}, r^n_{xyf} \\ &= \bigwedge_{\delta \in P_{j_y j_x}} \bigvee_{u \in j_x} m^n_{u\delta(u)f}. \end{aligned} \tag{32}$$

Here, $j_x = \{1, 2, \ldots, j\} - \{x\}$.

$$\operatorname{adj}(M)^t = \left(r_{yxt}, r_{yxi}, r_{yxf}\right), \tag{33}$$

where

$$\begin{aligned} r^n_{yxt} &= \bigvee_{\delta \in P_{j_x j_y}} \bigwedge_{u \in j_y} m^n_{u\delta(u)t}, r^n_{yxi} = \bigwedge_{\delta \in P_{j_x j_y}} \bigwedge_{u \in j_y} m^n_{u\delta(u)i}, r^n_{yxf} \\ &= \bigwedge_{\delta \in P_{j_x j_y}} \bigvee_{u \in j_y} m^n_{u\delta(u)f}. \end{aligned} \tag{34}$$

Here, $j_y = \{1, 2, \ldots, j\} - \{y\}$.
From equations (29) and (33),

$$\operatorname{adj}\left(M^t\right) = \operatorname{adj}(M)^t. \tag{35}$$

Definition 20. A STSFM M of order j is called a CSTSFM if $(m^n_{xkt}, m^n_{xki}, m^n_{xkf}) = (m^n_{ykt}, m^n_{yki}, m^n_{ykf})$ for $x, y, k = 1, 2, \ldots j$.

Proposition 7. *For a CSTSFM M, the $adj(M)^t$ is constant.*

Proof. It is trivial, so we omit here.

Proposition 8. *For a CSTSFM M, the $M.adj(M)$ is constant.*

Proof. It is trivial, so we omit here.

4. Proposed Decision-Making Algorithm and Illustration

The TSFM is the most generalized idea in fuzziness; it is applicable for various DM problems. Let promotion test is passed by the n administrative officers (AOs). Over goal is to pick m out of n based on the AO's approach to government (Govt) performance, because all members of Govt are from different groups. The solution of this problem is to find out how close the AO's ideology is to the Govt performance. The performance is a linguistic term and has no special meaning. We use fuzzy logic to handle such conditions, more specifically T-spherical fuzzy logic. This is the most generalized fuzzy structure in the existing fuzzy theory. The choice of AOs must meet certain conditions, and many counts are required. The algorithm defined below finds the appropriate AOs among many candidates.

The proposed algorithm is depicted in Figure 1, as a flowchart. The step-by-step explanation of the proposed algorithm is given as Algorithm 1.

Example 3. Let AA_1, AA_2, and AA_3 are three political parties coming from different Govts. G_1, G_2, and G_3. The A_1, A_2, A_3, A_4, and A_5 are AOs qualified for promotion. Now, a TSFM $M_1 = (m_{1xyt}, m_{1xyi}, m_{1xyf})$ of order 5×3, which shows the view of AOs to the party-backed Govt.

$$M_1 = \begin{matrix} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{matrix} \begin{pmatrix} (0.70, 0.60, 0.65) & (0.70, 0.80, 0.30) & (0.60, 0.60, 0.60) \\ (0.64, 0.64, 0.40) & (0.46, 0.30, 0.80) & (0.70, 0.60, 0.70) \\ (0.50, 0.70, 0.50) & (0.70, 0.50, 0.60) & (0.64, 0.50, 0.40) \\ (0.46, 0.30, 0.60) & (0.60, 0.50, 0.40) & (0.46, 0.40, 0.80) \\ (0.46, 0.30, 0.80) & (0.40, 0.70, 0.80) & (0.50, 0.70, 0.50) \end{pmatrix}. \tag{36}$$

The works performed by the Govt and their commitments are in M_2 during the election period, followed by the party.

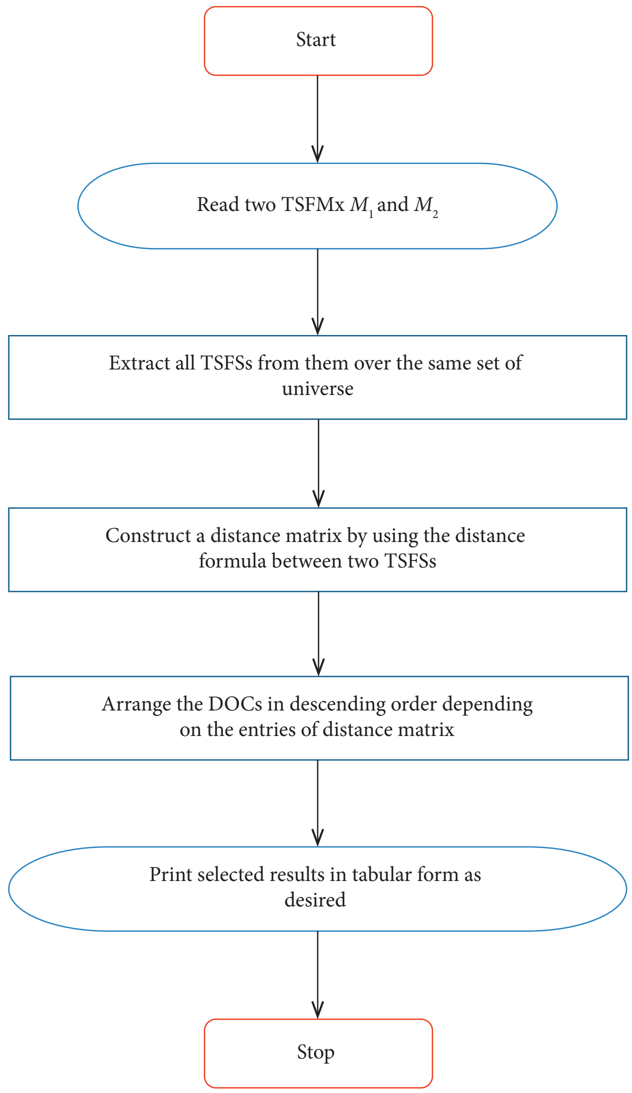

FIGURE 1: Flowchart of the proposed algorithm.

Aim: Obtain a picked list of AOs based on the AO's approach to Govt performance.
Input: From two given TSFM, first indicates the AO's view of the Govt by the political party, and the second one "the work done by the Govt. during election period."
Output: For different Govts, the selected list of AOs.
Step 1: Extract all TSFSs from the given TSFM over the set of parties.
Step 2: Using the distance formula between two TSFSs, compute a distance matrix as follows:

$$\gamma(A_1, A_2) = \sqrt{1/3p \sum_{u=1}^{p} [(x_u - x_u')^2 + (y_u - y_u')^2 + (z_u - z_u')^2 + (s_u - s_u')^2]},$$

where $x_u = t_{A_1}^n(a_u)$, $y_u = i_{A_1}^n(a_u)$, $z_u = f_{A_1}^n(a_u)$, and $s_u = 1 - x_u - y_u - z_u$,
$x_u' = t_{A_2}^n(a_u)$, $y_u' = i_{A_2}^n(a_u)$, $z_u' = f_{A_2}^n(a_u)$, and $s_u' = 1 - x_u' - y_u' - z_u'$ are PD, RD, NPD, and HD of a_u: $u = \{1, 2, \ldots, p\}$ in A_1 and A_2, respectively, where is a universal set understudy?
Step 3: Arrange the degree of closeness (DOCs) in descending order based on their distance to find selected AOs.

ALGORITHM 1

$$M_2 = \begin{matrix} G_1 \\ G_2 \\ G_3 \end{matrix} \begin{pmatrix} (0.30, 0.60, 0.80) & (0.80, 0.70, 0.40) & (0.70, 0.40, 0.70) \\ (0.70, 0.70, 0.40) & (0.40, 0.34, 0.50) & (0.40, 0.80, 0.40) \\ (0.50, 0.60, 0.40) & (0.50, 0.40, 0.80) & (0.40, 0.34, 0.50) \end{pmatrix}. \quad (37)$$

In M_1 and M_2, AA_1, AA_2, and AA_3 represent the three columns, respectively.

Step 1. The TSFM is extracted as follows:

$$\begin{aligned}
A_1 &= \{(AA_1, (0.70, 0.60, 0.65)), (AA_2, (0.70, 0.80, 0.30)), (AA_3, (0.60, 0.60, 0.60))\},\\
A_2 &= \{(AA_1, (0.64, 0.64, 0.40)), (AA_2, (0.46, 0.30, 0.80)), (AA_3, (0.70, 0.60, 0.70))\},\\
A_3 &= \{(AA_1, (0.50, 0.70, 0.50)), (AA_2, (0.70, 0.50, 0.60)), (AA_3, (0.64, 0.50, 0.40))\},\\
A_4 &= \{(AA_1, (0.46, 0.30, 0.60)), (AA_2, (0.60, 0.50, 0.40)), (AA_3, (0.46, 0.40, 0.80))\},\\
A_5 &= \{(AA_1, (0.46, 0.30, 0.80)), (AA_2, (0.40, 0.70, 0.80)), (AA_3, (0.50, 0.70, 0.50))\},\\
G_1 &= \{(AA_1, (0.30, 0.60, 0.80)), (AA_2, (0.80, 0.70, 0.40)), (AA_3, (0.70, 0.40, 0.70))\},\\
G_2 &= \{(AA_1, (0.70, 0.70, 0.40)), (AA_2, (0.40, 0.34, 0.50)), (AA_3, (0.40, 0.80, 0.40))\},\\
G_3 &= \{(AA_1, (0.50, 0.60, 0.40)), (AA_2, (0.50, 0.40, 0.80)), (AA_3, (0.40, 0.34, 0.50))\}.
\end{aligned} \quad (38)$$

Step 2. The distance matrix D is computed, by applying the distance formula for $n = 3$.

$$D = \begin{matrix} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{matrix} \begin{pmatrix} 0.1789 & 0.3339 & 0.3384 \\ 0.3135 & 0.2726 & 0.2731 \\ 0.2435 & 0.2586 & 0.1808 \\ 0.2971 & 0.3019 & 0.2816 \\ 0.2695 & 0.3545 & 0.2685 \end{pmatrix}, \quad (39)$$

where columns show the G_1, G_2, and G_3, respectively.

Step 3. The results made from the matrix D are as follows:

$$\begin{aligned}
&DOC(A_2, G_1) > DOC(A_4, G_1) > DOC(A_5, G_1) > DOC(A_3, G_1) > DOC(A_1, G_1),\\
&DOC(A_5, G_2) > DOC(A_1, G_2) > DOC(A_4, G_2) > DOC(A_2, G_2) > DOC(A_3, G_2),\\
&DOC(A_1, G_3) > DOC(A_4, G_3) > DOC(A_2, G_3) > DOC(A_5, G_3) > DOC(A_3, G_3).
\end{aligned} \quad (40)$$

As a result, the selected list of AOs is presented in Table 1.

For first government, A_2 and A_4, for second A_5 and A_1, and for third A_1 and A_4 are selected. Also, note that A_1 and A_4 are selected for more than one Govt.

5. Comparative Study

Here, we will analyze the proposed work with existing work and compare it in the light of suitable examples.

Remarks 3. We consider Definition 10:

(1) For $n = 2$, TSFM becomes SFM

(2) For $n = 1$, TSFM becomes PFM, developed in [34]

(3) For $n = 2$ and $m_{xyi} = 0$, TSFM becomes PyFM, developed in [27]

(4) For $n = 1$ and $m_{xyi} = 0$, TSFM becomes IFM, developed in [19]

(5) For $n = 1$, $m_{xyi} = 0$, and $m_{xyf} = 0$, TSFM becomes FM, developed in [10]

From the above remarks, it is clear that TSFM is most generalized among all existing fuzzy matrix structures.

Another advantage of our proposed work is that it can be used where all existing structures failed to find the results. Considering Example 3, the sum of all grades of the data given in the matrix M_1 exceeds from the unit interval $[0, 1]$ for $n = 1$, in Table 2, so the information is not in picture fuzzy form and the method proposed in [34] is unable to handle the information. By observing Table 3, it is seen that the sum of all grades of the data given in M_1 is also rose above from the unit interval $[0, 1]$ for $n = 2$, so the information is not spherical fuzzy form and the so far proposed

Table 1: Selected AOs.

Govts.	Selected as
$\mathbf{G}_1$	A_2, A_4
$\mathbf{G}_2$	A_5, A_1
$\mathbf{G}_3$	A_1, A_4

Table 2: Sum of all grades of $\mathbf{M}_1$ for $\mathbf{n} = 1$.

	$\mathbf{AA}_1$	$\mathbf{AA}_2$	$\mathbf{AA}_3$
$\mathbf{A}_1$	1.95	1.80	1.80
$\mathbf{A}_2$	1.68	1.56	2.00
$\mathbf{A}_3$	1.70	1.80	1.54
$\mathbf{A}_4$	1.36	1.50	1.66
$\mathbf{A}_5$	1.56	1.90	1.70

Table 3: Sum of all grades of $\mathbf{M}_1$ for $\mathbf{n} = 2$.

	$\mathbf{AA}_1$	$\mathbf{AA}_2$	$\mathbf{AA}_3$
$\mathbf{A}_1$	1.27	1.22	1.08
$\mathbf{A}_2$	0.98	0.94	1.34
$\mathbf{A}_3$	0.99	1.10	0.82
$\mathbf{A}_4$	0.66	0.77	1.01
$\mathbf{A}_5$	0.94	1.29	0.99

Table 4: Sum of all grades of $\mathbf{M}_1$ for $\mathbf{n} = 3$.

	$\mathbf{AA}_1$	$\mathbf{AA}_2$	$\mathbf{AA}_3$
$\mathbf{A}_1$	0.83	0.88	0.65
$\mathbf{A}_2$	0.59	0.64	0.90
$\mathbf{A}_3$	0.59	0.68	0.45
$\mathbf{A}_4$	0.34	0.41	0.67
$\mathbf{A}_5$	0.64	0.92	0.59

methods are unable to handle the information. From Table 4, it is observed that the data are in T-spherical fuzzy form for $n = 3$. The proposed method is only to handle such type of data, which shows the importance of the proposed article.

Sum of all grades of the data given in M_1 for $n = 2$ is given in Table 3.

Sum of all grades of the data given in M_1 for $n = 3$ is given in Table 4.

From all the above discussion, it is clear that TSFM is the most generalized in all the existing fuzzy structures.

6. Conclusions

In this paper, a concept of T-spherical fuzzy matrix is presented by taking the importance of the matrix theory, fuzzy matrix, and the T-spherical fuzzy sets. The key findings of the present study are listed as below as follows:

(1) The concept of T-spherical fuzzy matrix is introduced, which is an extension of matrix and fuzzy matrix.

(2) The concepts of a square T-spherical fuzzy matrix, constant T-spherical fuzzy matrix, and constant square T-spherical fuzzy matrix with their related properties are defined, and their related properties are investigated with examples.

(3) Determinant and adjoint of square T-spherical fuzzy matrix with their related results are discussed.

(4) An algorithm for multiattribute decision-making problems is presented to solve the decision-making problems.

(5) A numerical example is solved using the developed algorithm, where the appropriate AOs among many candidates are selected. A comparative study has been made to show the importance and novelty of the proposed work.

In our next study, our aim is to explore the concept. In further, our aim is to extend the proposed work to develop some applicable results in the matrix theory in the context of T-spherical fuzzy matrix and to utilize them in decision-making problems.

Data Availability

No data were used to support this study.

Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

References

[1] Y. Gao and Z. Zhang, "Consensus reaching with non-cooperative behaviour management for personalized individual semantics-based social network group decision making," *Journal of the Operational Research Society*, vol. 1, 2021.

[2] J. Wu, Q. Hong, M. Cao, Y. Liu, and H. Fujita, "A group consensus-based travel destination evaluation method with online reviews," *Applied Intelligence*, vol. 52, no. 2, pp. 1306–1324, 2022.

[3] M. Cao, Y. Liu, T. Gai, M. Zhou, H. Fujita, and J. Wu, "A comprehensive star rating approach for cruise ships based on interactive group decision making with personalized individual semantics," *Journal of Marine Science and Engineering*, vol. 10, no. 5, p. 638, 2022.

[4] L. A. Zadeh, "Fuzzy sets," *Information and Control*, vol. 8, no. 3, pp. 338–353, 1965.

[5] W. Pedrycz and F. Gomide, *An Introduction to Fuzzy Sets: Analysis and Design*, MIT Press, Cambridge, MA, USA, 1998.

[6] M. Roubens, "Fuzzy sets and decision analysis," *Fuzzy Sets and Systems*, vol. 90, no. 2, pp. 199–206, 1997.

[7] S. K. Pal and D. D. Majumder, "Fuzzy sets and decision making approaches in vowel and speaker recognition," *IEEE Transactions on Systems, Man, and Cybernetics*, vol. 7, no. 8, pp. 625–629, 1977.

[8] R. R. Yager, "Multiple objective decision-making using fuzzy sets," *International Journal of Man-Machine Studies*, vol. 9, no. 4, pp. 375–382, 1977.

[9] H. Li and V. C. Yen, *Fuzzy Sets and Fuzzy Decision-Making*, CRC Press, Boca Raton, FL, USA, 1995.

[10] H. Hashimoto, "Canonical form of a transitive matrix," *Fuzzy Sets and Systems*, vol. 11, no. 1–3, pp. 157–162, 1983.

[11] M. Pal, "Fuzzy matrices with fuzzy rows and columns," *Journal of Intelligent and Fuzzy Systems*, vol. 30, no. 1, pp. 561–573, 2015.

[12] M. Pal, "Interval valued fuzzy matrices with interval valued rows and columns," *Fuzzy Inf Eng*, vol. 7, pp. 335–368, 2015.

[13] M. Pal and S. Mondal, "Bipolar fuzzy matrices," *Soft Computing*, vol. 23, pp. 9885–9897, 2019.

[14] K. H. Kim and F. W. Roush, "Generalized fuzzy matrices," *Fuzzy Sets and Systems*, vol. 4, no. 3, pp. 293–315, 1980.

[15] R. Pradhan and M. Pal, "Triangular fuzzy matrix norm and its properties," *Journal of Fuzzy Mathematics*, vol. 25, no. 4, pp. 823–834, 2017.

[16] M. G. Ragab and E. G. Emam, "The determinant and adjoint of a square fuzzy matrix," *Fuzzy Sets and Systems*, vol. 61, no. 3, pp. 297–307, 1994.

[17] M. G. Ragab, "Emam EG on the min–max composition of fuzzy matrices," *Fuzzy Sets and Systems*, vol. 75, no. 1, pp. 83–92, 1995.

[18] K. T. Atanassov, "Intuitionistic fuzzy sets," *Fuzzy Sets and Systems*, vol. 20, no. 1, pp. 87–96, 1986.

[19] M. Pal, "Intuitionistic fuzzy determinant," *U. J. Phys Sci*, vol. 7, pp. 87–93, 2001.

[20] S. K. Khan and M. Pal, "Interval-valued Intuitionistic Fuzzy Matrices," 2014, https://arxiv.org/abs/1404.6949.

[21] R. Pradhan and M. Pal, "Generalized inverse of block intuitionistic fuzzy, matrices," *International Journal of Applications of Fuzzy Sets and Artificial Intelligence*, vol. 3, pp. 23–38, 2013.

[22] S. Mondal and M. Pal, "Intuitionistic fuzzy incline matrix and determinant," *Annals of Fuzzy Mathematics and Informatics*, vol. 8, no. 1, pp. 19–32, 2014.

[23] S. Sriram and P. Murugadas, "On semiring of intuitionistic fuzzy matrices," *Applied Mathematical Sciences*, vol. 4, no. 23, pp. 1099–1105, 2010.

[24] S. Sriram and P. Murugadas, "α -cut of intuitionistic fuzzy matrices," *Journal of Fuzzy Mathematics*, vol. 20, no. 2, pp. 307–318, 2012.

[25] R. R. Yager, "Pythagorean fuzzy subsets," in *Proceedings of the IFSA World Congress and NAFIPS Annual Meeting (IFSA/NAFIPS), Joint*, IEEE, Edmonton, AB, Canada, June 2013.

[26] R. R. Yager and A. M. Abbasov, "Pythagorean membership grades, complex numbers, and decision making," *International Journal of Intelligent Systems*, vol. 28, no. 5, pp. 436–452, 2013.

[27] I. Silambarasan and S. Sriram, "Algebraic operations on Pythagorean fuzzy matrices," *Mathematical Sciences International Research Journal*, vol. 7, no. 2, pp. 406–414, 2018.

[28] I. Silambarasan and S. Sriram, "New operations for pythagorean fuzzy matrices," *Indian Journal of Science and Technology*, vol. 12, no. 20, pp. 1–7, 2019.

[29] I. Silambarasan and S. Sriram, "Hamacher operations on Pythagorean fuzzy matrices," *Journal of Applied Mathematics and Computational Mechanics*, vol. 18, no. 3, 2019.

[30] B. Cuong, "Picture fuzzy sets-First results. Part 1," in *Seminar Neuro-Fuzzy Systems with Applications*China, 2013.

[31] B. C. Cường, "Picture fuzzy sets," *Journal of Computer Science and Cybernetics*, vol. 30, no. 4, p. 409, 2014.

[32] K. Ullah, "Picture fuzzy maclaurin symmetric mean operators and their applications in solving multiattribute decision-making problems," *Mathematical Problems in Engineering*, 2021.

[33] M. Saad, A. Rafiq, and L. Perez-Dominguez, "Methods for multiple attribute group decision making based on picture fuzzy dombi hamy mean operator," *Journal of Computational and Cognitive Engineering*, vol. 12, 2022.

[34] S. Dogra and M. Pal, "Picture fuzzy matrix and its application," *Soft Computing*, vol. 24, pp. 9413–9428, 2020.

[35] T. Mahmood, K. Ullah, Q. Khan, and N. Jan, *An Approach toward Decision-Making and Medical Diagnosis Problems Using the Concept of Spherical Fuzzy Sets Neural Comput & Applic*, Bejing, China, 2018.

[36] K. Ullah, T. Mahmood, and N. Jan, "Similarity measures for T-spherical fuzzy sets with applications in pattern recognition," *Symmetry*, vol. 10, no. 6, p. 193, 2018.

[37] M. Saad and A. Rafiq, "Novel similarity measures for t-spherical fuzzy sets and their applications in pattern recognition and clustering," *Journal of Intelligent and Fuzzy Systems*, vol. 12, pp. 1–11, 2020.

[38] M. Q. Wu, T. Y. Chen, and J. P. Fan, "Divergence measure of T-spherical fuzzy sets and its applications in pattern recognition," *IEEE Access*, vol. 8, pp. 10208–10221, 2019.

[39] K. Ullah, H. Garg, T. Mahmood, N. Jan, and Z. Ali, "Correlation coefficients for T-spherical fuzzy sets and their applications in clustering and multi-attribute decision making," *Soft Computing*, vol. 24, no. 3, pp. 1647–1659, 2020.

[40] H. Garg, M. Munir, K. Ullah, T. Mahmood, and N. Jan, "Algorithm for T-spherical fuzzy multi-attribute decision making based on improved interactive aggregation operators," *Symmetry*, vol. 10, no. 12, p. 670, 2018.

[41] S. Zeng, H. Garg, M. Munir, T. Mahmood, and A. Hussain, "A multi-attribute decision making process with immediate probabilistic interactive averaging aggregation operators of T-spherical fuzzy sets and its application in the selection of solar cells," *Energies*, vol. 12, no. 23, p. 4436, 2019.

[42] M. Munir, H. Kalsoom, K. Ullah, T. Mahmood, and Y. M. Chu, "T-spherical fuzzy Einstein hybrid aggregation operators and their applications in multi-attribute decision making problems," *Symmetry*, vol. 12, no. 3, p. 365, 2020.

[43] S. G. Quek, G. Selvachandran, M. Munir et al., "Multi-attribute multi-perception decision-making based on generalized T-spherical fuzzy weighted aggregation operators on neutrosophic sets," *Mathematics*, vol. 7, no. 9, p. 780, 2019.

[44] L. Zedam, N. Jan, E. Rak, T. Mahmood, and K. Ullah, "An approach towards decision-making and shortest path problems based on T-spherical fuzzy information," *International Journal of Fuzzy Systems*, vol. 22, no. 5, pp. 1521–1534, 2020.