Hindawi
Mathematical Problems in Engineering
Volume 2022, Article ID 2793538, 10 pages
https://doi.org/10.1155/2022/2793538

Research Article

Decision-Making with Risk under Interval Uncertainty Based on Area Metrics

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Received 9 February 2022; Revised 11 March 2022; Accepted 17 March 2022; Published 14 April 2022

Academic Editor: Darko Božanić

From the perspective of D-S evidence theory and area measurement, a risk-based comprehensive decision-making method that considers both the expected utility and the uncertainty of the scheme is proposed under the interval uncertainty environment of attribute values. The upper and lower bounds of the synthetic probability distribution of attributes values in different natural states are constructed based on the belief measure and plausibility measure. Based on the area measurement, a method for calculating the expected utility of each scheme is proposed. To reflect the influence of the uncertainty in the evaluation value of each scheme attribute on the final decision result, two indexes are defined: the evaluation uncertainty of attributes (EUA) and the uncertainty of the expected utility of scheme (UEU). Finally, considering the expected value of the expected utility and its uncertainty, three decision methods, namely, risk-neutral, risk-averse, and risk-preference, are constructed. An example is considered to show that the proposed method is effective and practical, and the uncertainty of the expected utility has a significant impact on the result of risky decisions. The new method can solve the problems of existing methods that overlook the impact of epistemic uncertainty on the decision-making process.

1. Introduction

As a special form of multiattribute decision-making, risky decision-making is characterized by the presence of different natural states in the decision-making process, each of which has a certain occurrence probability, and the attribution of values as the natural state changes. Risky decision-making is common in investment decision-making [1, 2], emergency decision-making [3, 4], ecological risk assessment [5, 6], and other fields and has attracted extensive attention in recent years.

Due to the complexity, uncertainty, and unpredictability of risky decision-making problems, it is often difficult to accurately predict information such as attribute values and natural state occurrence probabilities during the decision-making process, leading to epistemic uncertainty, which has been described in various ways, such as fuzzy numbers/intuitive fuzzy sets [7, 8], interval numbers [9, 10], and linguistic variables [11, 12]. To obtain the final decision-making conclusion in different uncertain environments, various methods of converting risky decision-making into deterministic decision-making have been proposed.

Generally, two approaches are used to solve the risk decision-making problem. In the first approach, the interval probability is transformed into a point probability. Reference [13] used the continuous ordered weighted average (C-OWA) operator to convert the interval probability into a point probability. Reference [14] proposed an interval probability conversion method based on the Monte Carlo simulation method. Reference [10] proposed another interval probability conversion method based on belief and plausibility measures to transform interval risky decision-making into deterministic decision-making. These methods rank the decisions based on the expected utility theory without considering the psychological factors of decision-makers. In the second approach, the psychological and behavioral factors of decision-makers are accounted for. Representative methods mainly include prospect theory-based methods and regret theory-based methods. Reference [15] calculated the weighted prospect value (interval number) of each scheme and used the expected value of the interval number as the basis for deterministic decision-making. Reference [16] calculated the value of the potential response result related to each criterion based on cumulative prospect theory and determined the prospect value of each alternative by aggregating the values and weights of the response results, based on which the alternatives were sorted. Considering that it is difficult for prospect theory-based methods to determine reference point information, some researchers have investigated risky decision-making methods based on regret theory. Reference [17] proposed a decision analysis method that considers the regret-aversion psychological behaviors of decision-makers. In this method, the alternatives are sorted based on the calculated overall regret value and overall gratification value of each alternative relative to other alternatives. Reference [18] proposed the VIKOR method based on regret theory. A decision-making mechanism coefficient was introduced to measure the impact of the maximum group utility value and the minimum individual regret value on the decision-making result, and an optimization model was constructed and then solved to obtain the final decision-making result.

The aforementioned methods can be used to address risky decision-making problems from different perspectives. However, previous studies have focused on transforming risky decision-making problems into deterministic decision-making problems while overlooking the influence of uncertainty information in the decision-making process on the decision-making result. Because the attribute values and natural state occurrence probabilities of different schemes often contain massive amounts of uncertainty information, uncertainty is always present regardless of the description method used (e.g., intuitionistic fuzzy sets, interval numbers, and linguistic variables). As the uncertainty of a scheme increases, the uncertainties contained in the expected utility value or prospect value increase, so ignoring the influence of these uncertainties and only sorting the schemes based on the mathematical expectation of the expected utility or prospect value may lead to irrational decisions. For example, the expected values of the expected utility of schemes A and B are 1 million yuan and 0.9 million yuan, respectively, and scheme A is superior to scheme B if the schemes are sorted according to the expected value; however, if the uncertainties of the expected utility of schemes A and B are 300,000 yuan and 30,000 yuan, respectively, then, for risk-averse decision-makers, scheme B is superior to scheme A.

Current methods to deal with uncertainty include probability theory [19, 20], fuzzy theory [21–23], and Dempster–Shafer (D-S) evidence theory [24, 25]. D-S evidence theory has a strong ability to deal with epistemic uncertainty. Compared with probability theory, fuzzy theory, and other approaches, it can be used to evaluate and quantify the existing uncertainty only by using the obtained information without any additional assumptions, for example, by assuming a random distribution and a membership function. Based on the above analysis, in this paper, from the perspective of D-S evidence theory, we consider the case in which the attribute value is an interval number and construct the upper and lower bounds of the comprehensive probability distribution of the attribute evaluation values in various natural states based on the plausibility measure and belief measure. We propose an expected utility value calculation method based on area metrics. In addition, we consider the influence of the uncertainty in the final decision evaluation information by defining two indicators of the scheme: the evaluation uncertainty of attributes (EUA) and the uncertainty of the expected utility of schemes (UEU). Finally, we make a comprehensive decision by simultaneously considering the expected utility and the UEU based on the different risk preferences of decision-makers (risk-preferred, risk-averse, and risk-neutral). The new evaluation framework considers the preferences of decision-makers and their aversion to risk and can thus provide a more comprehensive basis for decision-makers with different risk preference types when making decisions in the real world.

2. D-S Evidence Theory

D-S evidence theory is an uncertainty reasoning method proposed by A. P. Dempster and further expanded by his student G. Shafer. It is based on the frame of discernment, which represents a nonempty set containing all possible results that are generally expressed as a nonempty set Θ.

Definition 1. [26]: Basic probability assignment (BPA) is a mapping from a power set to interval numbers [0, 1], i.e., m: $2\Theta \longrightarrow [0, 1]$. The reliability of a set A is denoted as $m(A)$, which represents the degree of confidence in A but not any subset of A. Reliability has the following basic attributes:

$$\begin{cases} m(\varnothing) = 0, \\ 0 \le m(A) \le 1, \ \forall A \subseteq \Theta, \\ \sum\limits_{A \subseteq \Theta} m(A) = 1 \end{cases}. \tag{1}$$

If $m(A) > 0$, then A is called a focal element.

Definition 2. [27]: For a proposition A, the degree of confidence in this proposition can be represented by interval numbers $[Bel(A), Pl(A)]$, and $Bel(A)$ and $Pl(A)$ are both numbers between 0 and 1, as shown in Figure 1. $Bel(\cdot)$ and $Pl(\cdot)$ are called the belief function and the plausibility function, respectively, and are defined as follows:

$$\begin{aligned} Bel(A) &= \sum_{B \subseteq A} m(B), \\ Pl(A) &= \sum_{B \cap A \neq \varnothing} m(B). \end{aligned} \tag{2}$$

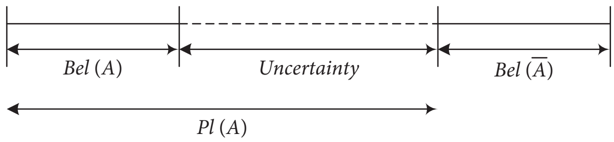

Figure 1: Belief function and plausibility function.

3. Risky Decision-Making Method Based on an Area Measure

3.1. Problem Description. In a risky multiattribute decision-making problem, there are n schemes, denoted as $\mathbf{a} = \{a_1, a_2, \cdots, a_n\}$, $a_i\,(i = 1, 2, \cdots, n) \in \ell$; 1 is the decision space with N natural states, denoted as $W = \{W_1, W_2, \cdots, W_N\}$; the probability of the occurrence of the j^{th} natural state $W_j\,(j = 1, 2, \cdots, N)$ is $p_j\,(j = 1, 2, \cdots, N)$; and there are m decision attributes, denoted as $\mathbf{C} = \{C_1, C_2, \cdots, C_m\}$, with attribute weights of $\omega = \{\omega_1, \omega_2, \cdots, \omega_m\}$ that satisfy $\omega_k > 0\,(k = 1, 2, \cdots, m)$ and $\sum \omega_k = 1$.

In general, attributes $C_k\,(k = 1, 2 \cdots, m)$ are evaluated with two types of indicators: benefit and cost. For benefit-type indicators, a greater value is better, while for cost-type indicators, a smaller value is better.

For the j^{th} natural state, the decision-maker's evaluation value of attribute $C_k\,(k = 1, 2 \cdots, m)$ is an interval number $[x_{jk}^L, x_{jk}^U]$, and the expected utility of each scheme according to the expected monetary value criterion is as follows:

$$E_i = \sum_{j=1}^{N} p_j u_{ij}, \tag{3}$$

where u_{ij} is the utility value of scheme a_i in natural state W_j.

3.2. Area Metrics Definition of Attribute Evaluation Value. For attribute $C_k\,(k = 1, 2 \cdots, m)$ under scheme $a_i\,(i = 1, 2, \cdots, n)$, the decision information for different natural states is a set of data, as shown in Table 1.

For N natural states, the evaluation values can be expressed as a set of D-S evidence theory focal elements:

$$\begin{cases} h_{i,1k} = \left[x_{i,1k}^L, x_{i,1k}^U\right] \\ h_{i,2k} = \left[x_{i,2k}^L, x_{i,2k}^U\right] \\ \cdots \\ h_{i,Nk} = \left[x_{i,1k}^L, x_{i,1k}^U\right] \end{cases}. \tag{4}$$

The BPA corresponding to each focal element is as follows:

$$\begin{cases} m\left(h_{i,1k}\right) = p_1 \\ m\left(h_{i,2k}\right) = p_2 \\ \cdots \\ m\left(h_{i,Nk}\right) = p_N \end{cases}. \tag{5}$$

Based on (4), the upper and lower bounds of attribute C_k can be obtained as follows:

Table 1: Decision information of C_k under scheme a_i.

W_1	W_2	$\cdots$	W_N
$[x_{i,1k}^L, x_{i,1k}^U]$	$[x_{i,2k}^L, x_{i,2k}^U]$	$\cdots$	$[x_{i,Nk}^L, x_{i,Nk}^U]$

$$\begin{cases} X_{i,k}^L = \min\left(x_{i,1k}^L, x_{i,2k}^L, \cdots, x_{i,Nk}^L\right) \\ X_{i,k}^U = \max\left(x_{i,1k}^U, x_{i,2k}^U, \cdots, x_{i,Nk}^U\right) \end{cases}. \tag{6}$$

Based on the above information, the belief function and plausibility function of the attribute evaluation value of attribute C_k can be calculated as follows:

$$Bel_{i,k}\left(x<x^*\right) = \begin{cases} \sum\limits_{\sup\left(h_{i,jk}\right)<x^*} m\left(h_{i,jk}\right) x^* \in \left[X_{i,k}^L, X_{i,k}^U\right] \\ 1 x^* > X_{i,k}^U \\ 0 x^* < X_{i,k}^L \end{cases}, \tag{7}$$

$$Pl_{i,k}\left(x<x^*\right) = \begin{cases} \sum\limits_{\inf\left(h_{i,jk}\right)<x^*} m\left(h_{i,jk}\right) \left[X_{i,k}^L, X_{i,k}^U\right] \\ 1 x^* > X_{i,k}^U \\ 0 x^* < X_{i,k}^L \end{cases}. \tag{8}$$

In this manner, the upper and lower bounds of the comprehensive probability distribution of attribute C_k are constructed; $Bel_{i,k}\,(x < x^*)$ is the lower bound, and $Pl_{i,k}\,(x < x^*)$ is the upper bound, as shown in Figure 2.

In Figure 2, $Bel_{i,k}\,(x < x^*)$ represents the lower bound of the comprehensive probability distribution of evaluation values in various natural states, and $Pl_{i,k}\,(x < x^*)$ represents the upper bound of the comprehensive probability distribution, while the actual probability distribution $P_{i,k}\,(x < x^*) \in [Bel_{i,k}\,(x < x^*), Pl_{i,k}\,(x < x^*)]$ is shown as the double-dotted line in Figure 2.

Definition 3. Area metric of the attribute evaluation value (AMA). For $Pl_{i,k}\,(x < x^*)$, the area metric is defined as follows:

$$A_{i,k}^L = \int_0^1 Pl_{i,k}^{\ -1}\,(x < x^*)\mathrm{d}x. \tag{9}$$

Clearly, a greater evaluation value of attribute C_k indicates that $Pl_{i,k}\,(x < x^*)$ is closer to the right side of the coordinate axis and greater values of $A_{i,k}^L$; this function can reflect the size of the evaluation value of attribute C_k. If C_k is a benefit-type index, then a value of $A_{i,k}^L$ is better; if C_k is a cost-type index, a smaller value of $A_{i,k}^L$ is better. As indicated by (9), the area metric index $A_{i,k}^L$ is a point value that realizes the transformation from a random probability distribution to a deterministic index and is thus beneficial to subsequent decision-making.

Similarly, the area measure for the lower bound of the probability of attribute C_k can be obtained as follows:

$$A_{i,k}^U = \int_0^1 Bel_{i,k}^{\ -1}\,(x < x^*)\mathrm{d}x. \tag{10}$$

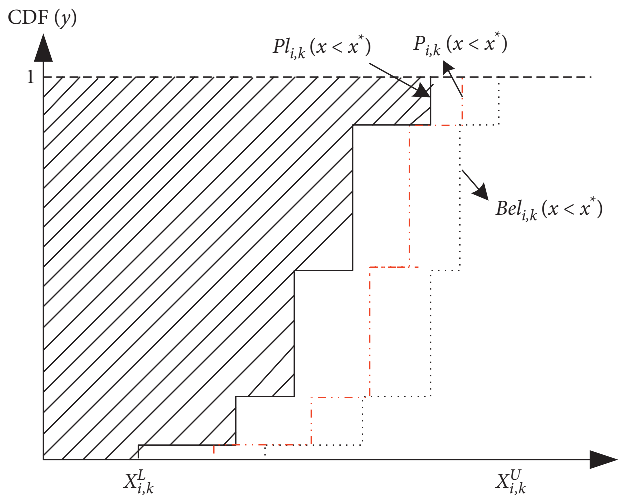

Figure 2: Comprehensive probability distribution of attribute C_k.

Then, the AMA indicator of attribute C_k is $A_{i,k} = [A^L_{i,k}, A^U_{i,k}]$, and its expected value is the median of the interval:

$$\overset{\%}{A}_{i,k} = \frac{\left(A^L_{i,k} + A^U_{i,k}\right)}{2}. \qquad (11)$$

For all attributes, the AMA expectation vector can be calculated as follows:

$$\overset{\%}{A}_i = \left(\overset{\%}{A}_{i,1}, \overset{\%}{A}_{i,2}, \cdots, \overset{\%}{A}_{i,m}\right). \qquad (12)$$

Definition 4. Area metric of the expected utility (AME) of the scheme. Based on the AMA indicator of each attribute, the AME value of scheme a_i is as follows:

$$\widehat{E}_i = \sum_{k=1}^{m} \omega_k \widetilde{A}_{i,k}. \qquad (13)$$

Because the evaluation value of attribute C_k is an interval number, it describes the epistemic uncertainty of the decision-maker on the value of the attribute; a greater epistemic uncertainty indicates a greater uncertainty of the expected utility value $\widehat{E}_i$ reflected in the final scheme. The greater the uncertainty is, the greater the expected volatility of the scheme is, and the worse the worst-case scenario of its expected utility is. This information is also an important indicator in decision-making. Therefore, in this study, we define the EUA and UEU of an attribute to reflect the information.

Definition 5. EUA of an attribute. The evaluation uncertainty of C_k is the area enclosed between $Bel_{i,k}(x<x^*)$ and $Pl_{i,k}(x<x^*)$:

$$EUA_{i,k} = \int_{X^L_{i,k}}^{X^U_{i,k}} \left(Pl_{i,k}{}^{-1}(x<x^*) - Bel_{i,k}{}^{-1}(x<x^*)\right)\mathrm{d}x. \qquad (14)$$

Based on (14), the greater the $EUA_{i,k}$ value is, the greater the EUA of attribute C_k is, and vice versa; if $Bel_{i,k}(x<x^*) = Pl_{i,k}(x<x^*)$, i.e., if the epistemic uncertainty disappears and only random uncertainty remains, then the probability envelope is transformed into a deterministic probability distribution $P_{i,k}(x<x^*)$, where the EUA of attribute C_k is zero.

Definition 6. UEU of a scheme. For all attributes, the EUA indicator vector is given as follows:

$$EUA_i = \left(EUA_{i,1}, EUA_{i,2}, \cdots, EUA_{i,m}\right). \qquad (15)$$

The UEU indicator of scheme a_i is defined as follows:

$$UEU_i = \sum_{k=1}^{m} \omega_k EUA_{i,k}. \qquad (16)$$

In summary, $\widehat{E}_i$ reflects the expected value of the expected utility of scheme a_i, and UEU_i reflects the uncertainty of the expected utility of scheme a_i; a greater $\widehat{E}_i$ value is better, while a smaller UEU_i value is better. These two indicators need to be considered when making decisions.

3.3. Decision-Making Algorithm. The diagram of the proposed decision-making algorithm is shown in Figure 3.

Step 1. If the dimensions and scales of the attribute evaluation values of $C_1, C_2, \cdots, C_m$ are identical, then go to Step 2 directly; otherwise, first perform nondimensionalization as follows:

If the evaluation value of attribute a_i of scheme C_k in the j^{th} natural state is the interval number $h_{i,jk} = [x^L_{i,jk}, x^U_{i,jk}]$, then for benefit-type attributes, the upper and lower bounds of the interval after nondimensionalization are as follows:

$$\begin{cases} h^U_{i,jk} = \dfrac{x^U_{i,jk}}{\sum_i \left(x^L_{i,jk} + x^U_{i,jk}\right)/2n} \\ h^L_{i,jk} = \dfrac{x^L_{i,jk}}{\sum_i \left(x^L_{i,jk} + x^U_{i,jk}\right)/2n} \end{cases}. \qquad (17)$$

For cost-type attributes, the upper and lower bounds of the interval after nondimensionalization are as follows:

$$\begin{cases} h^U_{i,jk} = \dfrac{1/x^L_{i,jk}}{\sum_i \left(1/x^L_{i,jk} + 1/x^U_{i,jk}\right)/2n} \\ h^L_{i,jk} = \dfrac{1/x^U_{i,jk}}{\sum_i \left(1/x^L_{i,jk} + 1/x^U_{i,jk}\right)/2n} \end{cases}. \qquad (18)$$

Step 2. Construct the upper and lower bounds ($Pl_{i,k}(x<x^*)$ and $Bel_{i,k}(x<x^*)$) of the probability distribution of the evaluation values of attribute C_k using equations (8) and (9).

Step 3. Calculate the area metric index $\widetilde{A}_{i,k}$ and the $EUA_{i,k}$ of attribute C_k using equations (10) and (15).

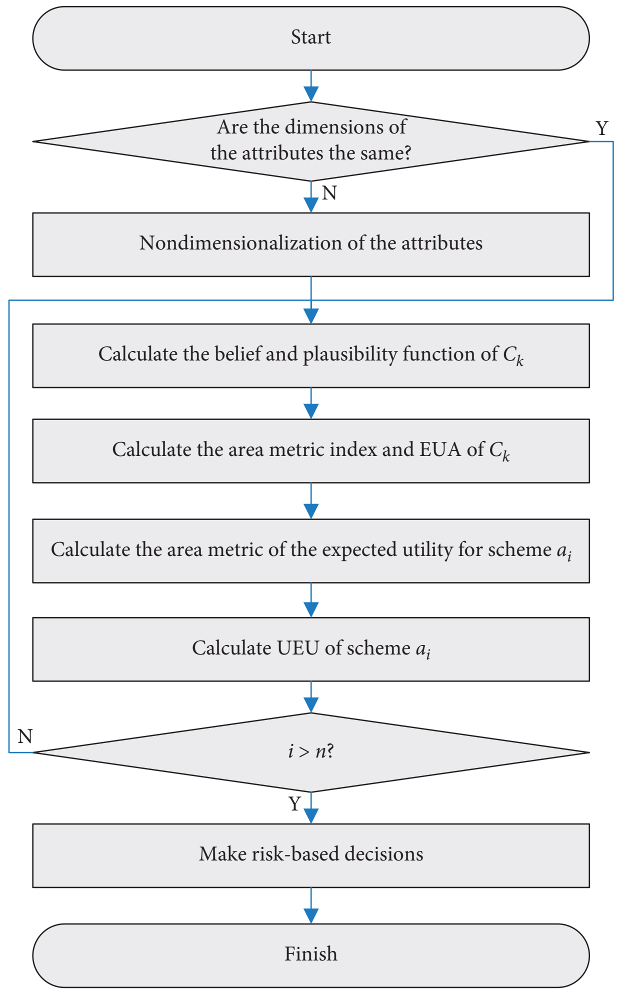

Figure 3: Diagram of the proposed decision-making algorithm.

Step 4. Calculate the $\widehat{E}_i$ of scheme a_i using equation (14).

Step 5. Calculate the UEU_i of scheme a_i using equations (16) and (17).

Step 6. Repeat Steps 2 to 5 to calculate the $\widehat{E}_i$ and UEU_i values of all n schemes.

Step 7. The decision-maker makes risk-based decisions on n schemes according to the following principles:

(1) Risk-neutral decision-makers: Decisions are made directly according to the order of $\widehat{E}_i$. If the $\widehat{E}_i$ values of the two schemes are identical, the scheme with a smaller UEU_i value is preferred.

(2) Risk-averse decision-makers: Set the risk aversion coefficient to $\alpha\,(0 \leq \alpha \leq 1)$ and sort the schemes using the following equation:

$$\widehat{E}_i^L = \widehat{E}_i - \alpha \cdot UEU_i. \tag{19}$$

(3) Risk-preferred decision-makers: Set the risk preference coefficient to $\beta\,(0 \leq \beta \leq 1)$ and sort the schemes using the following equation:

$$\widehat{E}_i^L = \widehat{E}_i + \beta \cdot UEU_i. \tag{20}$$

4. Case Study

A new energy vehicle is to be selected to support the company plans to invest in a power battery project. There are four investment schemes for selection: ternary lithium batteries, lithium iron phosphate batteries, nickel-metal hydride batteries, and hydrogen fuel cells, denoted as $\mathbf{a} = \{a_1, a_2, a_3, a_4\}$. The attributes of the schemes include sales volume C1 (unit: 10,000 units/year), rate of return C2 (unit: %/year), R&D cost C3 (unit: 10,000 yuan/unit), and payback period C4 (unit: year). Of these attributes, C1 and C2 are benefit-type indicators, and C3 and C4 are cost-type indicators. The decision-maker assigns weights to the four attributes as $\omega = (0.35, 0.2, 0.2, 0.25)$. In addition, after the product is put on the market, there are three natural states, $\mathbf{W} = \{W_1, W_2, W_3\}$, corresponding to fast-selling, fair, and slow-selling, respectively. The probabilities of occurrence of the three natural states are determined by experts to be $\mathbf{p} = (0.5, 0.3, 0.2)$. The risk decision information of each scheme is shown in Tables 2-4.

First, the data in Tables 2-4 are nondimensionalized, and the results are shown in Tables 5-7.

Next, the upper and lower bounds ($Pl_k(x < x^*)$ and $Bel_k(x < x^*)$) of the comprehensive evaluation probability distribution of attribute C_k are constructed. Taking the attribute C_1 of scheme a_1 as an example, the probability distribution of the evaluation values of C_1 can be obtained through (7) and (8), as shown in Figure 4.

Using (9) and (10), $A_{1,1}^L = 0.8910$ and $A_{1,1}^U = 1.1160$ can be obtained. Thus, (11) yields the expected value of the evaluation value of $C_1 \widetilde{A}_{i,k} = 1.0035$, and the evaluation uncertainty is $EUA_{1,1} = 0.2250$. Similarly, the expected values and EUA values of attributes C2–C4 can be calculated, as listed in Table 8.

Similarly, the comprehensive evaluation results of each attribute of scheme a_2-a_4 can be obtained, as shown in Tables 9-11.

Assuming the coefficient of risk aversion and the coefficient of risk preference are $\alpha = 1$ and $\beta = 1$, respectively, and using Tables 8-11 and (13) and (16), the expected values ($\widehat{E}_i$) and uncertainty values (UEU_i) of the expected utility of the four alternatives can be calculated. The results are listed in Table 12.

Based on the calculation results in Table 12, the comprehensive evaluation results for the risk-preferred, risk-averse, and risk-neutral cases are obtained using the decision-making method described in Step 7 of Section 2.3, as shown in Table 13.

Table 2: Risk decision information table of each scheme (natural state W_1).

Attribute		C_1	C_2	C_3	C_4
Scheme	a_1	[45, 60]	[15, 20]	[3.2, 3.6]	[4.5, 6.0]
	a_2	[42, 54]	[18, 22]	[3.1, 3.4]	[5.5, 6.5]
	a_3	[38, 46]	[12, 18]	[2.5, 2.8]	[4.0, 5.0]
	a_4	[40, 70]	[13, 17]	[3.8, 4.3]	[5.0, 7.0]

Table 3: Risk decision information table of each scheme (natural state W_2).

Attribute		C_1	C_2	C_3	C_4
Scheme	a_1	[31, 35]	[12, 16]	[3.5, 4.1]	[5.5, 7.0]
	a_2	[22, 34]	[13, 17]	[3.4, 3.9]	[6.5, 7.5]
	a_3	[27, 30]	[10, 11]	[3.0, 3.2]	[4.8, 6.4]
	a_4	[24, 39]	[11, 13]	[4.2, 4.5]	[6.5, 8.5]

Table 4: Risk decision information table of each scheme (natural state W_3).

Attribute		C_1	C_2	C_3	C_4
Scheme	a_1	[12, 15]	[8, 12]	[3.7, 4.4]	[8.5, 10.0]
	a_2	[10, 13]	[7, 10]	[3.8, 4.2]	[10.0, 12.0]
	a_3	[11, 14]	[6, 9]	[3.3, 3.5]	[9.0, 10.5]
	a_4	[10, 18]	[5, 10]	[4.4, 4.7]	[12.0, 13.0]

Table 5: Risk decision information table of each scheme (natural state W_1).

Attribute		C_1	C_2	C_3	C_4
Scheme	a_1	[0.91, 1.21]	[0.88, 1.18]	[0.90, 1.01]	[0.87, 1.17]
	a_2	[0.85, 1.09]	[1.06, 1.30]	[0.95, 1.04]	[0.81, 0.95]
	a_3	[0.76, 0.93]	[0.71, 1.06]	[1.16, 1.30]	[1.05, 1.31]
	a_4	[0.81, 1.41]	[0.77, 1.00]	[0.75, 0.85]	[0.75, 1.05]

Table 6: Risk decision information table of each scheme (natural state W_2).

Attribute		C_1	C_2	C_3	C_4
Scheme	a_1	[31, 35]	[12, 16]	[3.5, 4.1]	[5.5, 7.0]
	a_2	[22, 34]	[13, 17]	[3.4, 3.9]	[6.5, 7.5]
	a_3	[27, 30]	[10, 11]	[3.0, 3.2]	[4.8, 6.4]
	a_4	[24, 39]	[11, 13]	[4.2, 4.5]	[6.5, 8.5]

Table 7: Risk decision information table of each scheme (natural state W_3).

Attribute		C_1	C_2	C_3	C_4
Scheme	a_1	[31, 35]	[12, 16]	[3.5, 4.1]	[5.5, 7.0]
	a_2	[22, 34]	[13, 17]	[3.4, 3.9]	[6.5, 7.5]
	a_3	[27, 30]	[10, 11]	[3.0, 3.2]	[4.8, 6.4]
	a_4	[24, 39]	[11, 13]	[4.2, 4.5]	[6.5, 8.5]

As shown in Table 13, when deciding about the four alternatives, the risk-neutral, risk-averse, and risk-preferred decision-makers show completely different decision-making results.

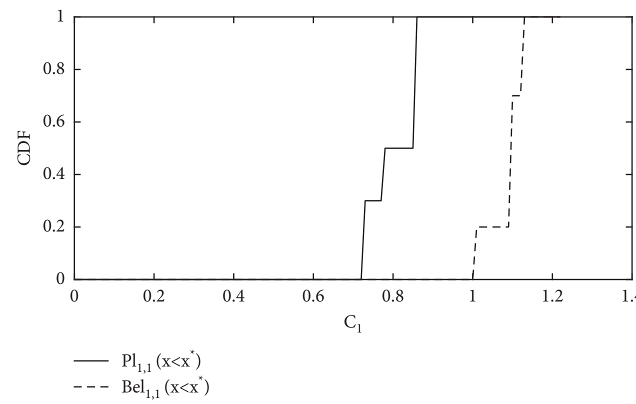

Figure 4: Comprehensive probability distribution of attribute a_1.

Table 8: Comprehensive evaluation results of each attribute under scheme a_1.

Attribute	C_1	C_2	C_3	C_4
$A^L_{1,k}$	0.8910	0.8970	0.8950	0.8440
$A^U_{1,k}$	1.1160	1.1700	1.0290	1.0950
$\widetilde{A}_{1,k}$	1.0035	1.0335	0.9620	0.9695
$EUA_{1,k}$	0.2250	0.2730	0.1340	0.2510

Table 9: Comprehensive evaluation results of each attribute under scheme a_2.

Attribute	C_1	C_2	C_3	C_4
$A^L_{1,k}$	0.7950	0.9940	0.9350	0.7720
$A^U_{1,k}$	1.0810	1.2840	1.0470	0.9000
$\widetilde{A}_{1,k}$	0.9380	1.1390	0.9910	0.8360
$EUA_{1,k}$	0.2860	0.2900	0.1120	0.1280

Table 10: Comprehensive evaluation results of each attribute under scheme a_3.

Attribute	C_1	C_2	C_3	C_4
$A^L_{1,k}$	0.7650	0.7280	1.1430	0.9570
$A^U_{1,k}$	0.9220	0.9990	1.2510	1.2080
$\widetilde{A}_{1,k}$	0.8435	0.8635	1.1970	1.0825
$EUA_{1,k}$	0.1570	0.2710	0.1080	0.2510

Table 11: Comprehensive evaluation results of each attribute under scheme a_4.

Attribute	C_1	C_2	C_3	C_4
$A^L_{1,k}$	0.7450	0.7580	0.7820	0.7060
$A^U_{1,k}$	1.2890	1.0380	0.8640	0.9310
$\widetilde{A}_{1,k}$	1.0170	0.8980	0.8230	0.8185
$EUA_{1,k}$	0.5440	0.2800	0.0820	0.2250

Table 12: Expected utility evaluation results of four alternatives.

Attribute		$\widehat{E}_i$	UEU_i
Scheme	a_1	0.9927	0.2229
	a_2	0.9633	0.2125
	a_3	0.9780	0.1935
	a_4	0.9048	0.3191

Table 13: Comprehensive evaluation results of alternative schemes under different risk preferences.

Attribute		Risk-neutral		Risk-averse		Risk-preferred	
		Value	Rating	Value	Rating	Value	Rating
Alternatives	a_1	0.9927	1	0.7698	2	1.2156	2
	a_2	0.9633	3	0.7508	3	1.1758	3
	a_3	0.9780	2	0.7845	1	1.1715	4
	a_4	0.9048	4	0.5857	4	1.2239	1

Table 14: Comparison of the proposed method and other methods for different α and β.

Method	Condition	a_1	a_2	a_3	a_4
Proposed method	$\alpha = 0.1$	1	3	2	4
	$\alpha = 0.3$	1	3	2	4
	$\alpha = 0.5$	1	3	2	4
	$\alpha = 0.7$	1	4	3	2
	$\alpha = 0.9$	1	3	4	2
	$\beta = 0.1$	1	3	2	4
	$\beta = 0.3$	1	3	2	4
	$\beta = 0.5$	1 (tied)	3	1 (tied)	4
	$\beta = 0.7$	2	3	1	4
	$\beta = 0.9$	2	3	1	4
[9]	—	1	3	2	4

Table 15: Risk decision information table of each scheme (natural state W_1, uncertainty increased by 20%).

Attribute		C_1	C_2	C_3	C_4
Scheme	a_1	[0.94, 1.18]	[0.91, 1.15]	[0.911, 0.999]	[0.9, 1.14]
	a_2	[0.874, 1.066]	[1.084, 1.276]	[0.959, 1.031]	[0.824, 0.936]
	a_3	[0.777, 0.913]	[0.745, 1.025]	[1.174, 1.286]	[1.076, 1.284]
	a_4	[0.87, 1.35]	[0.793, 0.977]	[0.76, 0.84]	[0.78, 1.02]

Table 16: Risk decision information table of each scheme (natural state W_2, uncertainty increased by 20%).

Attribute		C_1	C_2	C_3	C_4
Scheme	a_1	[1.033, 1.137]	[0.961, 1.209]	[0.905, 1.025]	[0.935, 1.135]
	a_2	[0.76, 1.08]	[1.032, 1.288]	[0.944, 1.056]	[0.863, 0.967]
	a_3	[0.9, 0.98]	[0.778, 0.842]	[1.147, 1.203]	[1.033, 1.297]
	a_4	[0.839, 1.231]	[0.865, 0.985]	[0.816, 0.864]	[0.773, 0.957]

Table 17: Risk decision information table of each scheme (natural state W_3, uncertainty increased by 20%).

Attribute		C_1	C_2	C_3	C_4
Scheme	a_1	[0.953, 1.137]	[0.998, 1.382]	[0.907, 1.043]	[0.702, 0.798]
	a_2	[0.793, 0.977]	[0.866, 1.154]	[0.94, 1.02]	[0.582, 0.678]
	a_3	[0.873, 1.057]	[0.746, 1.034]	[1.127, 1.183]	[0.671, 0.759]
	a_4	[0.832, 1.328]	[0.65, 1.13]	[0.836, 0.884]	[0.534, 0.566]

Table 18: Risk decision information table of each scheme (natural state W_1, uncertainty decreased by 20%).

Attribute		C_1	C_2	C_3	C_4
Scheme	a_1	[0.88, 1.24]	[0.85, 1.21]	[0.889, 1.021]	[0.84, 1.2]
	a_2	[0.826, 1.114]	[1.036, 1.324]	[0.941, 1.049]	[0.796, 0.964]
	a_3	[0.743, 0.947]	[0.675, 1.095]	[1.146, 1.314]	[1.024, 1.336]
	a_4	[0.75, 1.47]	[0.747, 1.023]	[0.74, 0.86]	[0.72, 1.08]

Risk-neutral decision-makers conclude that scheme a_1 is the best, and they sort the schemes as follows: $a_1 > a_3 > a_2 > a_4$.

Risk-averse decision-makers conclude that scheme a_3 is the best, and they sort the schemes as follows: $a_3 > a_1 > a_2 > a_4$.

Table 19: Risk decision information table of each scheme (natural state W_2, uncertainty decreased by 20%).

Attribute		C_1	C_2	C_3	C_4
Scheme	a_1	[1.007, 1.163]	[0.899, 1.271]	[0.875, 1.055]	[0.885, 1.185]
	a_2	[0.68, 1.16]	[0.968, 1.352]	[0.916, 1.084]	[0.837, 0.993]
	a_3	[0.88, 1]	[0.762, 0.858]	[1.133, 1.217]	[0.967, 1.363]
	a_4	[0.741, 1.329]	[0.835, 1.015]	[0.804, 0.876]	[0.727, 1.003]

Table 20: Risk decision information table of each scheme (natural state W_3, uncertainty decreased by 20%).

Attribute		C_1	C_2	C_3	C_4
Scheme	a_1	[0.907, 1.183]	[0.902, 1.478]	[0.873, 1.077]	[0.678, 0.822]
	a_2	[0.747, 1.023]	[0.794, 1.226]	[0.92, 1.04]	[0.558, 0.702]
	a_3	[0.827, 1.103]	[0.674, 1.106]	[1.113, 1.197]	[0.649, 0.781]
	a_4	[0.708, 1.452]	[0.53, 1.25]	[0.824, 0.896]	[0.526, 0.574]

Table 21: Comparison of the proposed method and other methods under different uncertainties.

Condition	Method	a_1	a_2	a_3	a_4
Uncertainty decreased by 20%	Proposed method $\alpha = 0.5$	1	3	2	4
	Proposed method $\beta = 0.5$	1	3	2	4
	[9]	1	3	2	4
Uncertainty increased by 20%	Proposed method $\alpha = 0.5$	1	4	3	2
	Proposed method $\beta = 0.5$	2	3	1	4
	[9]	1	3	2	4

Risk-preferred decision-makers conclude that scheme a_4 is the best, and they sort the schemes as follows: $a_4 > a_1 > a_2 > a_3$.

By carefully analyzing the results in Table 13, although the expected value of the expected utility of scheme a_1 is the greatest, its uncertainty is also higher (ranks second), so it ranks first when its uncertainty is ignored; however, when considering the risk of uncertainty during decision-making, scheme a_1 is no longer the best choice. Scheme a_4 has the greatest uncertainty and the greatest risk, but from the perspective of risk-preferred decision-makers, it also has the greatest opportunity and enables the highest return in the best case, so it is the best choice for risk-preferred decision-makers.

5. Validation of Results

To further verify the proposed method, the risk preference coefficient α and the risk aversion coefficient β are set to different values, and the schemes are sorted using the proposed method. The results are then compared with the ranking results in [9], as shown in Table 14.

As shown in Table 14, when the risk preference coefficient α and the risk aversion coefficient β are set to low values, the ranking results of the four schemes are identical and consistent with the ranking results of [9]: $a_1 > a_3 > a_2 > a_4$. When α and β are set to high values, the ranking results begin to change; for example, when $\alpha = 0.7$, $a_1 > a_4 > a_3 > a_2$, and when $\beta = 0.7$, $a_2 > a_3 > a_1 > a_4$. The ranking result is associated with the values of α and β and the values of $\widehat{E}_i$ and UEU_i.

To assess the influence of attribute uncertainty on the decision-making results, the uncertainty of the estimated values of the various attributes in Tables 5–7 under different natural states is reduced by 20% and expanded by 20%, respectively. The results are shown in Tables 15–20.

For $\alpha = 0.5$ and $\beta = 0.5$, the schemes are sorted, and the results are compared with the results from [9], as shown in Table 21.

As shown in Table 21, when the uncertainties in the attributes are reduced by 20%, the ranking results of the four schemes are identical and consistent with the ranking results of [9], i.e., $a_1 > a_3 > a_2 > a_4$. However, when the uncertainties are increased by 20%, the ranking results begin to change. For example, for $\alpha = 0.5$, $a_1 > a_4 > a_3 > a_2$, while for $\beta = 0.5$ and the method in [9], the results are $a_2 > a_3 > a_1 > a_4$ and $a_1 > a_3 > a_2 > a_4$, respectively.

This case study demonstrates that uncertainty in decision-making information can have a great impact on the final decision-making result and is thus an important factor that must be considered in risky decision-making. In view of previous studies, regardless of the method used, uncertain decision-making information is converted into accurate information to make final decisions. Clearly, these decision-making methods overlook uncertainty, which may lead decision-makers to overlook risks and make incorrect choices.

6. Conclusion

In multiattribute risky decision-making processes, the attribute evaluation information of a scheme often contains interval epistemic uncertainty, which has a significant impact on the decision outcome. From the perspective of D-S evidence theory, in this paper, we construct the area metric indicator AME for the expected utility of the scheme to measure the expected value of the expected utility of the scheme; we also construct the uncertainty index UEU of the expected utility of the scheme to measure the risks and opportunities of the expected utility of alternative schemes so that quantitative risk and opportunity measures for decision-makers with different risk preferences can be provided. When comparing and selecting schemes, decision-makers must comprehensively consider the area metric index AME and the uncertainty index UEU of the expected value of the expected utility to make decisions that are more aligned with reality.

The main contributions of the risk-based decision-making method proposed in this paper are as follows:

(1) The area metric of the attribute evaluation value is proposed. The calculation process of the index does not require any artificial assumptions, and the results are more objective.

(2) Different from the existing methods that only consider the expected utility index, the method proposed in this paper establishes the expected utility uncertainty index at the same time. Decision-makers can comprehensively evaluate alternatives according to the two indexes and draw more objective and consistent conclusions.

(3) The proposed evaluation framework considers the preferences of decision-makers and their aversion to risk, so it provides a more comprehensive basis for decision-makers with different risk preference types when making decisions in the real world.

In future work, more complex application scenarios will be explored. For example, the uncertainty of attribute weights and the uncertainty of natural state probability will be considered [28].

Data Availability

The data were curated by the authors and are available upon request.

Conflicts of Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgments

This work was supported by the National Natural Science Foundation of China (Grant nos. 71702156 and U183010080) and the Research Foundation of Southwest University of Science and Technology (Grant no. 21zx7125).

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