New Analytical Solution of the Large Deflection Problem of a Circular Thin Plate with Edge Clamped but Sliding Freely Under a Combined Loading

1. Introduction

The large deflection problems of circular thin plates are typical nonlinear problems in elastic mechanics. It is difficult to obtain exact solutions to these large deflection problems, so people mainly study their numerical solutions and approximate analytical solutions by various methods and theories. Vincent [1] applied the perturbation method to solve the bending problem of a circular thin plate under a uniformly distributed load; Qian [2] used the central deflection as the perturbation parameter to solve the large deflection problems by the perturbation method; Nguyen-Van et al. [3] studied the large deflection problems of plates and cylindrical shells with an efficient four-node flat element with mesh distortions; Madyan et al. [4] proposed an automated Ritz method for the large deflection problem of plates with mixed boundary conditions; Yu et al. [5] studied the nonlinear analysis for the extreme large bending deflection of a rectangular plate on nonuniform elastic foundations; Wang et al. [6] proposed a wavelet method for the bending of circular plate with large deflection; Sladek et al. [7] gave an iterative solution of large deflection problem of thin plates by the local boundary integral equation method; Yun et al. [8] used the homotopy perturbation method to solve the large deflection problem of a circular plate; Yun et al. [9] obtained new approximate analytical solution of the large deflection problem of a uniformly loaded thin circular plate with edge simply hinged by the Adomian decomposition method; Gbeminiyi et al. [10] considered free vibration analysis of thin isotropic rectangular plates submerged in fluid by the Adomian decomposition method and so on.

For the large deflection problem of circular plates under the combined action of the uniform load and centrally concentrated load, the central deflection at this time is zero. Therefore, the central deflection of the plate cannot be selected as the perturbation parameter. As for this, Hu [11] used the double parameter perturbation method to solve the problem, where the uniform load and centrally concentrated load are chosen as perturbation parameters. To improve the perturbation solution, we reconsider the large deflection problem of a circular plate under the combined action of the uniform load and...
centrally concentrated load in this paper by the Adomian decomposition method.

The Adomian decomposition method [12–16] is a practical technique for solving initial boundary value problems for differential equations. The advantage of the Adomian decomposition method is that linearization and perturbation parameters are no longer required. Therefore, the Adomian decomposition method is widely used in mathematical physics problems, such as singular initial value problems [17], Lane–Emden equations [18], Klein–Gordon equation [19], fractional differential equations [20–24], others [25–29], and so on. Many scholars have improved the Adomian decomposition method [30–33] to deal with the linear and nonlinear deflections of plates. Rach’s [34, 35] study confirms the new application of the Adomian decomposition method.

Based on the above research, in order to optimize the perturbation solution in the citation [11], we will discuss the influence of a finite series expansion with the uniform load on the Adomian approximation solution. By appropriately selecting a certain form of finite series expansion of uniform load, the purpose of optimizing the perturbation approximate solution is achieved.

The specific content of this paper is arranged as follows: in Section 2, the Adomian decomposition method is to be reviewed; in Section 3, the governing equations for the deflection problem are to be reviewed; in Section 4, the large deflection problem of a circular plate under the combined action of uniform load and centrally concentrated load with edge clamped but sliding freely is to be solved by the Adomian decomposition method; and in Section 5, the mechanical properties of the circular plates are discussed.

2. Review of the Adomian Decomposition Method

According to the citation [9], the ordinary differential equations are considered as follows:

\[ L_1 (w) + N_1 (w, s) + f_1 (x) = 0, \quad (1) \]
\[ L_2 (s) + N_2 (w, s) + f_2 (x) = 0, \quad (2) \]

and the boundary conditions

\[ \text{Condition1: } \bar{L}_1 (s) = 0, \quad \bar{L}_2 (w) = 0, \quad (3) \]
\[ \text{Condition2: } s < + \infty, \quad w < + \infty, \quad (4) \]

where, \( L_1 \) and \( L_2 \) are denoted the highest-order derivatives in equations (1) and (2); \( N_1, N_2 \) are nonlinear operators; \( f_1 \) and \( f_2 \) are known functions about the independent variable \( x \); \( s, w \) are dependent variables about the independent variable \( x \); \( \bar{L}_1 \) and \( \bar{L}_2 \) are linear operators.

Through equations (1) and (2) and boundary conditions (3) and (4), the inverse operators \( L_1^{-1}, L_2^{-1} \) are determined. Acting on the inverse operators \( L_1^{-1} \) and \( L_2^{-1} \) on both sides of equations (1) and (2), one obtains

\[
\begin{align*}
  w &= \phi - L_1^{-1} f_1 (x) - L_1^{-1} (N_1 (w, s)), \\
  s &= \phi - L_2^{-1} f_2 (x) - L_2^{-1} (N_2 (w, s)),
\end{align*}
\]

(5)

where, \( L_1^{-1} L_1 (w) = w - \phi \) and \( L_2^{-1} L_2 (s) = s - \phi \).

Assuming \( w, s \) are as the following infinite series, and the nonlinear terms are expanded into the following series:

\[
\begin{align*}
  w &= \sum_{n=0}^{\infty} w_n, \\
  s &= \sum_{n=0}^{\infty} s_n,
\end{align*}
\]

(6)

\[
\begin{align*}
  N_1 (w, s) &= \sum_{n=0}^{\infty} A_n, \\
  N_2 (w, s) &= \sum_{n=0}^{\infty} B_n,
\end{align*}
\]

(7)

where

\[
A_n = \frac{1}{n!} \frac{d^n}{dx^n} \left[ N_1 \left( \sum_{i=0}^{\infty} \lambda_i^n w_n, \sum_{i=0}^{\infty} \lambda_i^n s_n \right) \right]_{x=0}, \quad n=0,1,2,\ldots,
\]

(8)

\[
B_n = \frac{1}{n!} \frac{d^n}{dx^n} \left[ N_2 \left( \sum_{i=0}^{\infty} \lambda_i^n w_n, \sum_{i=0}^{\infty} \lambda_i^n s_n \right) \right]_{x=0}, \quad n=0,1,2,\ldots.
\]

(9)

Substituting equations (6) and (7) into equations (1) and (2) and (5), one gets

\[
\sum_{n=0}^{\infty} w_n = \phi - L_1^{-1} f_1 (x) - L_1^{-1} \left( \sum_{n=0}^{\infty} A_n \right), \quad (10)
\]

\[
\sum_{n=0}^{\infty} s_n = \phi - L_2^{-1} f_2 (x) - L_2^{-1} \left( \sum_{n=0}^{\infty} B_n \right). \quad (11)
\]

From (10) and (11), the following recursive formulas are constructed:

\[
\begin{align*}
  w_0 &= \phi - L_1^{-1} f_1 (x), \\
  w_n &= -L_1^{-1} \left( \sum_{i=0}^{n} A_i \right), \quad n \geq 1,
\end{align*}
\]

(12)

\[
\begin{align*}
  s_0 &= \phi - L_2^{-1} f_2 (x), \\
  s_n &= -L_2^{-1} \left( \sum_{i=0}^{n} B_i \right), \quad n \geq 1.
\end{align*}
\]

(13)

From (12) and (13), one can determine \( w_0, s_0, w_1, s_1, \ldots \).

Thus, one obtains the \( n \)-term Adomian approximate solutions.

\[
\begin{align*}
  w &\approx \sum_{i=0}^{n} w_i, \\
  s &\approx \sum_{i=0}^{n} s_i.
\end{align*}
\]

(14)
3. Review of the Governing Equations for the Deflection Problem

The governing equations of the large deflection problem [11] are the famous Von Kármán equations as follows:

$$
\begin{align*}
&\left. D \frac{d}{dr} \left( \frac{1}{r} \frac{d}{dr} \right) \left( r \frac{d}{dr} \right) \right| w + N^r \frac{d}{dr} + Q + P \frac{1}{2 \pi r} = 0, \\
&\left. r \frac{d}{dr} \left( \frac{1}{r} \frac{d}{dr} \right) \left( r^2 N^r \right) + \frac{E h}{2} \left( \frac{d^2}{dr^2} w \right)^2 = 0,
\end{align*}
$$

(15)

where, the radius of the circular thin plate is \( r \), the thickness is \( h \), \( Q \) and \( P \) are uniform load acting on circular plates and concentrated load acting on central points, \( N^r \) and \( w \) are radial thin film internal force and thin plate deflection, \( E \) and \( \sigma \) are Young’s elastic modulus and Poisson’s ratio, and \( D \) is flexural rigidity of the plate which stands for \( D = (E h^3)/4(1 - \mu^2) \).

The boundary conditions of a circular thin plate with an edge clamped but sliding freely under the combined action of uniformly distributed load and centrally concentrated load are as follows:

$$
\begin{align*}
& r = a: \ w = 0, \\
& \frac{d w}{d r} = 0, \quad N^r = 0, \\
& r = 0: \ N^r < + \infty, \quad w < + \infty.
\end{align*}
$$

(16)

Through dimensionless transformations [11]

$$
\begin{align*}
\xi &= \frac{r}{a}, \\
x &= \xi^2, \\
y &= \sqrt{3(1 - \sigma^2)} \frac{w}{h}, \\
s &= 3(1 - \sigma^2) \frac{a^2 N^r}{E h}, \\
q &= \frac{3}{4}(1 - \sigma^2) \sqrt{3(1 - \sigma^2)} \frac{a^4 Q}{E h}, \\
p &= \frac{3}{4}(1 - \sigma^2) \sqrt{3(1 - \sigma^2)} \frac{a^2 P}{\pi E h}.
\end{align*}
$$

(17)

The governing equations (15) and the boundary conditions (16) become as follows:

$$
\begin{align*}
\frac{d^2}{dx^2} \left( x \frac{dy}{dx} \right) &= q + P \frac{1}{x}, \\
\frac{d^2}{dx^2} (x s) + \frac{1}{2} \left( \frac{dy}{dx} \right)^2 &= 0,
\end{align*}
$$

(18)

with

$$
\begin{align*}
\left\{ \begin{array}{l}
\chi = 1: \ y = 0, \quad \frac{dy}{dx} = 0, \quad s = 0, \\
\chi = 0: \ y < + \infty, \quad s < + \infty.
\end{array} \right.
$$

(19)

4. Solution of the Problem

4.1. Determination of the Inverse Operators. Firstly, integrating from 1 to \( x \) on both sides of the first equation of (18), then integrating from 0 to \( x \) again as follows:

$$
\int_0^x \int_0^x \frac{d^2}{dx^2} \left( x \frac{dy}{dx} \right) dx dx = \int_0^x \int_0^x \left( \frac{dy}{dx} + q + \frac{P}{x} \right) dx dx.
$$

(20)

From above equations, consider the boundary conditions (19), one obtains

$$
y(x) = y''(1) + \frac{1}{x} \int_0^1 \left( s y' (x) + q + \frac{P}{x} \right) dx dx
$$

(21)

Secondly, integrating from 1 to \( x \) on both sides of (21), consider (19), one gets

$$
y(x) = -(x - 1) \int_0^1 \left( s y' (x) + q + \frac{P}{x} \right) dx dx
$$

(22)

Thus, the first inverse operator \( L_1^{-1} \) is determined as follows:

$$
L_1^{-1} = -(x - 1) \int_0^1 dx dx + \int_0^1 \frac{1}{x} \int_0^1 \int_0^1 dx dx dx.
$$

(23)

Similarly, consider the second equation of equation (18) and the boundary conditions (19), the second inverse operator \( L_2^{-1} \) is determined as follows:

$$
L_2^{-1} = - \int_0^1 dx dx + \int_0^1 \frac{1}{x} \int_0^1 dx dx.
$$

(24)

4.2. Undecomposition of Uniform Load \( q \). Applying the inverse operators \( L_1^{-1} \) and \( L_2^{-1} \) to the both sides of (18), one obtains

$$
\begin{align*}
& y(x) = L_1^{-1} \left( s(x) \frac{dy(x)}{dx} + q + \frac{P}{x} \right), \\
& s(x) = L_2^{-1} \left( -\frac{1}{2} \left( \frac{dy(x)}{dx} \right)^2 \right).
\end{align*}
$$

(25)

According to the Adomian decomposition method, we assume that \( y(x) \), \( s(x) \) are infinite series as follows:

$$
\begin{align*}
y(x) &= \sum_{n=0}^{\infty} y_n, \\
s(x) &= \sum_{n=0}^{\infty} s_n.
\end{align*}
$$

(26, 27)
And the nonlinear terms \( s(x) \cdot \frac{dy(x)}{dx} \) and \( (dy(x)/dx)^2 \) are divided into infinite series as follows:

\[
\left\{ \begin{array}{l}
\frac{d}{dx} \sum_{n=0}^{\infty} A_n - \frac{1}{2} \left( \frac{dy(x)}{dx} \right)^2 = \sum_{n=0}^{\infty} B_n, \\
\end{array} \right. 
\] 

(28)

where

\[
A_n = \frac{1}{n!} \frac{d^n}{dx^n} \left[ \left( \sum_{n=0}^{\infty} \lambda^n s_n \right) \left( \sum_{n=0}^{\infty} \frac{dy_n}{dx} \right) \right]_{x=0}, \quad n = 0, 1, 2, \ldots,
\]

(29)

\[
B_n = \frac{1}{n!} \frac{d^n}{dx^n} \left[ -\frac{1}{2} \left( \sum_{n=0}^{\infty} \frac{dy_n}{dx} \right)^2 \right]_{x=0}, \quad n = 0, 1, 2, \ldots
\]

(30)

From (29) and (30), one gets

\[
\left\{ \begin{array}{l}
A_0 = s_0 y'_0, \\
A_1 = s_1 y'_0 + s_0 y'_1, \\
A_2 = s_2 y'_0 + s_1 y'_1 + s_0 y'_2, \\
B_0 = -\frac{1}{2} (y_0)'^2, \\
B_1 = -y_0 y'_1, \\
B_2 = -\frac{1}{2} (y'_1)^2 - y_0 y'_2, \\
\end{array} \right.
\]

(31)

Substituting (26) and (27) and (28) into (25), one obtains

\[
\sum_{n=0}^{\infty} y_n = L_1^{-1} \left( \sum_{n=0}^{\infty} A_n + q + \frac{p}{x} \right) \sum_{n=0}^{\infty} s_n = L_2^{-1} \left( \sum_{n=0}^{\infty} B_n \right). 
\]

(32)

According the above equations, considering the boundary conditions (19), we construct the following recursive formulas:

\[
\left\{ \begin{array}{l}
y_0 = L_1^{-1} \left( q + \frac{p}{x} \right), \\
s_0 = L_2^{-1} (B_0). \\
y_n = L_1^{-1} (A_{n-1}), \quad n \geq 1, \\
s_n = L_2^{-1} (B_n), \quad n \geq 1.
\end{array} \right.
\]

(33)

(34)

From (33), one gets

\[
y_0 = p + \frac{q}{4} - px - \frac{q x^2}{2} + \frac{p q x^2}{4} + px \log [x], \\
s_0 = \frac{7 p^2}{8} + \frac{11 p q}{36} + \frac{q^2}{8} - \frac{7 p^2 x}{8} - \frac{3 p q x}{8} - \frac{q^2 x}{16} \\
+ \frac{5}{72} p q x^2 + \frac{q^2 x^2}{24} - \frac{q^2 x^2}{96} + \frac{3}{4} p^2 x \log [x] \\
+ \frac{1}{4} p q x \log [x] - \frac{1}{12} p q x^2 \log [x] - \frac{1}{4} p^2 x \log [x]^2.
\]

(35)

Then, substituting \( y_0 \) and \( s_0 \) into (34), one obtains \( y_1, s_1 \) as follows:

\[
y_1 = \frac{1}{2073600} \left( \begin{array}{c}
800 p^2 (191 - 658 x + 476 x^2) - 75 p^2 q (-1419 + 4081 x - 3167 x^2 + 577 x^3)
+ 2 p q (12946 - 31439 x + 24361 x^2 - 7839 x^3 + 711 x^4)
- 30 q^3 (-73 + 149 x - 121 x^2 + 59 x^3 - 16 x^4 + 2 x^5)
- 360 p x (20 p^2 (-63 + 64 x) - 5 p q x (88 - 104 x + 19 x^2))
+ q^2 (-45 + 60 x - 25 x^2 + 3 x^3)
\end{array} \right), \\
\log [x] + 1800 p^2 (104 p + q (24 - 5 x)) x \log [x] - 28800 p^2 x \log [x]^2, \\
\quad (1 + x)
\]

(36)

\[
s_1 = \frac{1}{15240960000} \left( \begin{array}{c}
30625 p^4 (64241 - 58015 x + 25649 x^2)
- 98 p q (16963529 + 1897456 x - 12355404 x^2 + 2852721 x^3)
+ 245 p^2 q (2226252 - 2864823 x + 2181927 x^2 - 792323 x^3 + 98173 x^4)
- 75 p^3 q (-1091295 + 1573080 x - 1283375 x^2 + 571275 x^3)
- 125652 x^4 + 9245 x^5
+ 7875 q (599 - 955 x + 823 x^2 - 437 x^3 + 151 x^4 - 31 x^5 + 3 x^6)
\end{array} \right), \\
\log [x] + 41400 p^2 x^2 (450 p^2 (-56 + 65 x) + p q (8800 + 14100 x - 3861 x^2)
+ 10 q^2 (-90 + 180 x - 99 x^2 + 13 x^3)
- 882000 p^3 (470 p + 3 q (40 - 11 x)) x \log [x]^2 + 5292000 p^3 x \log [x]^2
\end{array} \right).
\]
Similarly, one can determine \( y_2, s_2, y_3, s_3, \ldots \) one by one from (34). Thus, one gets the \( n \)-term Adomian approximate solutions.

\[
\begin{align*}
    y &\approx \sum_{i=0}^{n-1} y_i = y_0 + y_1 + \cdots + y_{n-1}, \\
    s &\approx \sum_{i=0}^{n-1} s_i = s_0 + s_1 + \cdots + s_{n-1}.
\end{align*}
\]

(37)

4.3. Decomposition of Uniform Load \( q \). Assuming \( q \) is the finite series as follows:

\[
q = \sum_{n=0}^{m} q_n,
\]

(38)

where \( m \) is the order of the approximate solution. Now substituting (38) into (25), one gets

\[
\begin{align*}
    \begin{cases}
        y_n = L_1^{-1} \left( \sum_{i=0}^{m} A_i + \sum_{i=0}^{m} q_i P \right) \\
        s_n = L_2^{-1} \left( \sum_{i=0}^{m} B_i \right)
    \end{cases}
\end{align*}
\]

(39)

According to the above equations, we construct the following recursive formulas:

\[
\begin{align*}
    y_1 &= \frac{1}{2073600} \left( -28800 p^3 x^3 \log |x|^3 + 1800 p^3 x^3 \log |x|^2 (104 p + (24 - 5 x) q_0) - 360 p x^2 \log |x| \right) \\
        &+ \left( 20 p^2 (-63 + 64 x) - 5 p q_0 (88 - 104 x + 19 x^2) + q_0 (-45 + 60 x - 25 x^2 + 3 x^3) \right) \frac{1}{2} \\
        &+ (-1 + x) \left( 800 p^3 (191 - 658 x + 476 x^2) - 75 p^2 q_0 (-1419 + 4081 x - 3167 x^2 + 577 x^3) \right) \\
        &+ 2 p q_0^2 (12946 - 31439 x + 24361 x^2 - 7839 x^3 + 711 x^4) - 30 (-1 + x) (73 - 76 x + 45 x^2 - 14 x^3 + 2 x^4) q_0^3 - 17280 q_1 \\
    s_1 &= \frac{1}{1524906000} \left( 529200000 p^4 x^3 \log |x|^3 - 88200000 p^3 x^3 \log |x|^2 \right) \\
        &+ \left( 450 p^2 (-56 + 65 x) + p q_0 (-8800 + 14100 x - 3861 x^2) + 10 q_0^2 (-90 + 180 x - 99 x^2 + 13 x^3) \right).
\end{align*}
\]

(40)

From (40), one gets

\[
\begin{align*}
    y_0 &= p - px + px \log |x| + \frac{q_0}{4} - \frac{1}{2} x q_0 + \frac{1}{4} x^2 q_0, \\
    s_0 &= \frac{7 p^2}{8} - \frac{7 p^3 x}{8} + \frac{3}{4} p^2 x \log |x| - \frac{1}{4} p x \log |x|^2 \\
        &+ \frac{11}{36} p q_0 - \frac{3}{8} p x q_0 + \frac{5}{72} p x^2 q_0 \\
        &+ \frac{1}{4} p x \log |x| q_0 - \frac{1}{12} p x^2 \log |x| q_0 + \frac{q_0^2}{32} - \frac{1}{16} x q_0^2 \\
        &+ \frac{1}{24} x^2 q_0 - \frac{1}{96} x^3 q_0.
\end{align*}
\]

(41)

Then, substituting \( y_0 \) and \( s_0 \) into (41), one obtains \( y_1, s_1, s_2 \).

\[
\begin{align*}
    y_0 &= L_1^{-1} \left( q_0 \right), \\
    s_0 &= L_2^{-1} \left( B_0 \right), \\
    y_n &= L_1^{-1} \left( A_{n-1} \right) + L_1^{-1} \left( q_n \right), \quad 1 \leq n \leq m, \\
    s_n &= L_2^{-1} \left( B_n \right), \quad 1 \leq n \leq m.
\end{align*}
\]

(42)

\[
\begin{align*}
    y_0 &= -210 p x \log |x| \\
        &+ \left( -98 p^3 (-16963529 + 18974596 x - 12355404 x^2 + 2852721 x^3) q_0 \\
        &+ 245 p^2 (-222625 - 2864823 x + 2181927 x^2 - 792323 x^3 + 98173 x^4) q_0^3 \\
        &- 75 p (-1091295 + 1573080 x - 1283375 x^2 + 571275 x^3 - 125652 x^4 + 9245 x^5) q_0^3 + 245 p^2 (-222625 - 2864823 x + 2181927 x^2 - 792323 x^3 + 98173 x^4) q_0^3 \\
        &+ 7875 q_0 (599 - 955 x + 823 x^2 - 437 x^3 + 151 x^4 - 31 x^5 + 3 x^6) q_0^3 - 40320 (3 - 3 x + x^2) q_0^3 \right)
\end{align*}
\]

(43)
Figure 1: The \( y \) with the \( x \) of 4th-order approximation (\( y_1: q \) is not decomposed; \( y_2: q \) is decomposed).

Figure 2: The \( s \) with the \( x \) of 4th-order approximation (\( y_1: q \) is not decomposed; \( y_2: q \) is decomposed).

Figure 3: The Error1 curves with the \( z = 0.5 \) of 4th-order approximation.

Figure 4: The Error2 curves with the \( z = 0.5 \) of 4th-order approximation.

Figure 5: The \( y-q \) curve with the \( x = 0.5 \) (\( y_1: q \) is not decomposed; \( y_2: q \) is decomposed).

Table 1: The comparison of \( \|\text{Error1}\|^2 \).

<table>
<thead>
<tr>
<th>Order number</th>
<th>PM [11]</th>
<th>Our results</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( 4.05207 \times 10^{-4} )</td>
<td>( 4.05207 \times 10^{-4} )</td>
</tr>
<tr>
<td>2</td>
<td>( 2.28091 \times 10^{-7} )</td>
<td>( 7.81048 \times 10^{-8} )</td>
</tr>
<tr>
<td>3</td>
<td>( 2.27987 \times 10^{-10} )</td>
<td>( 8.44018 \times 10^{-11} )</td>
</tr>
<tr>
<td>4</td>
<td>( 2.97820 \times 10^{-13} )</td>
<td>( 7.28932 \times 10^{-14} )</td>
</tr>
</tbody>
</table>

Table 2: The comparison of \( \|\text{Error2}\|^2 \).

<table>
<thead>
<tr>
<th>Order number</th>
<th>PM [11]</th>
<th>Our results</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>( 1.02327 \times 10^{-10} )</td>
<td>( 1.87307 \times 10^{-11} )</td>
</tr>
<tr>
<td>3</td>
<td>( 2.27542 \times 10^{-13} )</td>
<td>( 4.45079 \times 10^{-14} )</td>
</tr>
<tr>
<td>4</td>
<td>( 4.26766 \times 10^{-16} )</td>
<td>( 6.48961 \times 10^{-17} )</td>
</tr>
</tbody>
</table>
Consequently, one can obtain the \( m + 1 \) term approximate solutions as follows:

\[
y' = \sum_{i=0}^{m+1} y_i,
\]

\[
s = \sum_{i=0}^{m+1} s_i.
\]  

(44)

5. Discussions and Conclusions

Based on the new estimation of the convergence and approximation error of the Adomian decomposition method derived by Cherruault and Adomian [36], to characterize the accuracy of the approximate solution, we define the following two error functions as follows [8, 9]:

\[
\text{Error1}(y, s) = \frac{d^2}{dx^2} \left( x \frac{dy}{dx} \right) - s \frac{dy}{dx} - q - g(x)
\]

\[
\text{Error2}(y, s) = \frac{d^2}{dx^2} (xs) + \frac{1}{2} \left( \frac{dy}{dx} \right)^2.
\]

(45)

Then, we use \( \|\text{Error1}\|_2^2 \) and \( \|\text{Error2}\|_2^2 \) to characterize the accuracy of the approximate solutions to the problems (18) and (19), where \( \|\cdot\|_2 \) denote the \( L^2 \)-norm. It is clear that \( \|\text{Error1}\|_2^2 = \|\text{Error2}\|_2^2 = 0 \), when \( y(x), s(x) \) are the exact solutions of (18) and (19). For approximate solutions, \( \|\text{Error1}\|_2^2 \) and \( \|\text{Error2}\|_2^2 \) represent the errors of the solutions.

When the uniform load \( q \) is not decomposed, the results obtained in this paper are exactly the same as in [11]. When the uniform load \( q \) is decomposed, obviously, the \( m \) th-order approximate solutions (44) depend on the independent variable \( x \) and parameters \( q_i (i = 0, 1, 2, \ldots, m) \). Therefore, by selecting the suitable values of \( q_i (i = 0, 1, 2, \ldots, m) \), we can improve the accuracy of the approximate solutions. The following selections are an example of values of these parameters \( q_i (i = 0, 1, 2, \ldots, m) \):

\[
\begin{align*}
\text{m} = 1: & \quad q_0 = q, \\
\text{m} = 2: & \quad q_0 = q - 10^{-2} q, \quad q_1 = 10^{-2} q, \\
\text{m} \geq 3: & \quad q_i = 10^{-2i} q - 10^{-2(i+1)} q, \quad (2 \leq i < m - 1), \\
& \quad q_m = 10^{-2m} q.
\end{align*}
\]

(46)

When \( q = 0.5 \) and \( p = 0.2 \), the dimensionless internal stress and deflection are given in Figures 1 and 2. At this time, the error function curves are given in Figures 3 and 4. For the large deflection problem of circular plates under the combined action of uniform load and centrally concentrated load, the central deflection at this time is 0. The graph of dimensionless deflection \( y \) at \( x = 0.5 \) with the uniform load \( q \) is given in Figure 5. The error comparisons of the later results in this paper with [11] are shown in Table 1 and 2. From error comparisons, one can see the result in this paper is better than in [11]. Other parameters are basically the same.

Data Availability

All data used in this study are available upon request from the corresponding author.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

This work was supported by National Natural Science Foundation of China (No. 12161064), Natural Science Foundation of Inner Mongolia (No. 2020LH01003), and Key Subject Team of Inner Mongolia University of Technology (ZD202018).

References


