Research Article

Efficient Control Charting Scheme for the Process Location with Application in Automobile Industry

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The recently introduced triple exponentially weighted moving average (TEWMA) chart is the extended form of the classical EWMA and double EWMA (DEWMA) charts. On the other hand, the auxiliary information-based (AIB) homogeneously weighted moving average (HWMA AIB) chart is used for the monitoring of process location shifts efficiently as compared to the HWMA chart. Combining TEWMA with HWMA features is a new idea and is not seen in the literature until now. So, the objective of this study is to combine the idea of HWMA AIB and TEWMA charts and propose an AIB triple HWMA, symbolized as THWMA AIB chart to further improve the process location shift monitoring. The proposed THWMA AIB chart is developed by combining the AIB double HWMA plotting statistic into the other HWMA chart. The numerical results are computed using the Monte Carlo simulation method. Average run length (ARL), relative ARL, performance comparison index, and extra quadratic loss are used as comparison tools for the proposed THWMA AIB chart with the classical EWMA, TEWMA, mixed HWMA-CUSUM (MHC), AIB EWMA (EWMA AIB), HWMA, double HWMA (DHWMA), HWMA AIB, AIB mixed EWMA-CUSUM (MEC AIB), and AIB mixed CUSUM-EWMA (MCE AIB) charts. Finally, a practical application is also provided for users to demonstrate the proposed study’s vitality.

1. Introduction

Variations are an integral part of all kinds of production and non-production processes. These variations are categorized as common and special variations. Generally, these special variations can result in process parameters (location and/or dispersion) shifts. These shifts are classified into three sizes as small, moderate, and large. Control charts are widely used in the statistical process control (SPC) toolkit to identify these special variations. For tracking large shifts, Shewhart charts [1] are widely used; however, for small to moderate shifts, cumulative sum (CUSUM) [2] and exponentially weighted moving average (EWMA) [3] are used.

In the SPC literature, many enhancements and modifications on control charts continuously fulfill the practitioner’s requirement of quickly detecting shifts. In this regard, Shamma and Shamma [4] extended the classical EWMA chart and suggested an improved double EWMA (DEWMA) chart. The DEWMA chart is more responsive than the classical EWMA chart. Alevizakos, et al. [5] proposed a triple EWMA (TEWMA) chart, the extended version of the EWMA and DEWMA charts for process location. The TEWMA chart is handy for detecting smaller process location shifts. After that, Alevizakos, et al. [6] recommended one-sided and two-sided TEWMA charts for the time between events, regarded as TEWMA TBE charts, and


Hunter [29] points out the drawback of classical EWMA statistic is that the freshest observations are given more weight than previous observations. To resolve this drawback, Abbas [30] proposed a homogeneous weighted moving average (HWMA) chart that assigns particular weights to recent process values and homogeneously allocates the remaining weights to the old process values. This approach enhances the effectiveness of the HWMA chart as compared to its competitor’s charts. Later Adegoke, et al. [28] extended the Abbas [30] work to propose the AIB HWMA (HWMA\_AIB) chart is more effective for smaller shift monitoring. Subsequently, Adeoti and Koleoso [31] and Abid, et al. [32] enhanced the existing work by introducing a hybrid-HWMA (HHWMA) and double HWMA (DHWMA) charts, respectively. Also, Thanwane, et al. [33] presented an HWMA chart for the autocorrelated process under the assumption of estimated parameters. Unlike process location monitoring, Riaz, et al. [34] utilized the HWMA chart concept for efficient process dispersion monitoring. Recently, Abid, et al. [35] introduced a mixed HWMA-CUCUM (MHC) chart for process location for efficient process monitoring. Also, Rasheed, et al. [36] introduced homogeneously mixed memory control chart for process location.

Motivated by the extraordinary performances of TEWMA and HWMA\_AIB charts, we are aimed to combine the features of TEWMA and HWMA\_AIB charts and proposed a more efficient AIB triple HWMA (symbolized as (THWMA\_AIB)) chart for process location. The merging of TEWMA with HWMA features in the presence of auxiliary information is a new idea and is not seen in the literature till now and it is expected that the combining of TEWMA and HWMA\_AIB charts features will further boost the performance of proposed THWMA\_AIB chart. Average run length (ARL), relative ARL (RARL), extra quadratic loss (EQL), and performance comparison index (PCI) are used for the performance comparison with existing counterparts. Besides, the Monte Carlo simulation method is used for obtaining numerical measures. The classical EWMA, TEWMA, MHC, EWMA\_AIB, HWMA, DHWMA, HWMA\_AIB, MEC\_AIB and MCE\_AIB charts are considered for the comparison. Additionally, the proposed THWMA\_AIB chart is implemented in a real-world scenario to demonstrate its utility in practice.

The remainder of the research article is structured as follows: existing charts are described in Section 2. Additionally, Section 3 enlisted the design structure and special cases of proposed (THWMA\_AIB) chart. Furthermore, the next section provides the performance evaluation and comparison against classical EWMA, TEWMA, MHC, EWMA\_AIB, HWMA, HWMA\_AIB DHWMA, MEC\_AIB, and MCE\_AIB charts. Also, the application of (THWMA\_AIB) chart is included in Section 5. The last section describes the overall summary, conclusions, and recommendations.

2. Existing Methods

This section describes the detailed methodology of the existing HWMA\_AIB and TEWMA charts for process location monitoring.

2.1. Variable of Interest and Auxiliary Information.

Assume that the process variable $Y$ is normally distributed with a mean $\mu_Y + \delta \sigma_Y$ and variance $\sigma_Y^2$. Let $Y_i = \sum_{i=1}^{n} Y_{it}/n$ and $S_{Y_i}^2 = \sum_{i=1}^{n} (Y_{it} - Y_i)^2/(n - 1)$ represents the sample mean and variance of $Y$ of size $n$. If $\delta = 0$, the process is in-control (IC); otherwise, it is out-of-control (OOC). So, for the IC situation, $Y_i$ and $S_{Y_i}^2$ are mutually independent identically distributed. Let $X$ be an auxiliary variable of $Y$. The $X$ and $Y$ follow a bivariate normal distribution (BND) (i.e., $(Y, X) \sim N(\mu_Y, \mu_X, \sigma_Y, \sigma_X, \rho)$, where $\mu_X$ represents the mean and $\sigma_X$ represents the standard deviation of $X$. Also,
the \( \rho \) is the correlation coefficient corresponding to \( X \) and \( Y \). The AIB regression estimator for process location is given as follows:

\[
R_t = \bar{Y}_t + b_{YX} (\mu_X - \bar{X}_t),
\]

where \( b_{YX} = \rho (\sigma_Y / \sigma_X) \), \( E(R_t) = \mu_Y \) and \( \text{Var}(R_t) = \sigma_Y^2 - b_{YX}^2 \sigma_X^2 / n \).

2.2. HWMA\_AIB Chart. Adegoke, et al. [28] introduced HWMA\_AIB chart to track changes in process location. The plotting statistic of the HWMA\_AIB chart is given as:

\[
\mu_0 - L_{\text{HWMA\_AIB}} \sigma_Y \sqrt{\left( \lambda^2 / n \right)} \left( 1 - \rho^2 \right), \quad \text{if } t = 1
\]

\[
LCL_{\text{(HWMA\_AIB)\_t}} = \mu_0 - L_{\text{HWMA\_AIB}} \sigma_Y \sqrt{\left( \lambda^2 / n \right)} \left( 1 - \rho^2 \right) \left( 1 - \rho \right), \quad \text{if } t > 1
\]

\[
UCL_{\text{(HWMA\_AIB)\_t}} = \mu_0 + L_{\text{HWMA\_AIB}} \sigma_Y \sqrt{\left( \lambda^2 / n \right)} \left( 1 - \rho^2 \right), \quad \text{if } t = 1
\]

\[
SCL_{\text{(HWMA\_AIB)\_t}} = \mu_0 + L_{\text{HWMA\_AIB}} \sigma_Y \sqrt{\left( \lambda^2 / n \right)} \left( 1 - \rho^2 \right) 
\]

The \( L_{\text{HWMA\_AIB}} \) represents the coefficient of control limits. If \( H_t > UCL_{\text{(HWMA\_AIB)\_t}} \) or \( H_t < LCL_{\text{(HWMA\_AIB)\_t}} \), the process is OOC; otherwise, IC.

2.3. TEWMA Chart. Alevizakos et al. [5] designed a TEWMA chart for efficient process location monitoring. The plotting statistics of the TEWMA chart are defined as

\[
\text{var}(TE_t) = \theta \lambda^2 / 4 \left[ \left( t + 1 \right) \theta \left( \lambda \left( 1 / \theta \right) - 1 \right) - \left[ \left( t + 1 \right) \theta \left( \lambda \left( 1 / \theta \right) - 1 \right) \right]^2 - 12 \left[ \left( t + 1 \right) \theta \left( \lambda \left( 1 / \theta \right) - 1 \right) \right] - 24 \left[ \left( t + 1 \right) \theta \left( \lambda \left( 1 / \theta \right) - 1 \right) \right] + 24 \left[ \left( \lambda + 1 \right) \theta \left( \lambda \left( 1 / \theta \right) - 1 \right) \right] \right]
\]

respectively, where \( \theta = (1 - \lambda)^2 \). The time-varying control limits of the TEWMA chart are defined as:

\[
\begin{align*}
LCL_{(\text{TEWMA})t} &= \mu_0 - L_{\text{TEWMA}} \sqrt{\text{var}(TE_t)}, \\
UCL_{(\text{TEWMA})t} &= \mu_0 + L_{\text{TEWMA}} \sqrt{\text{var}(TE_t)},
\end{align*}
\]

where \( L_{\text{TEWMA}} = L_{\text{TEWMA}}(\mu > 0) \) is the control limits coefficient of the TEWMA chart. The process will be OOC if any \( T_{E_t} > UCL_{(\text{TEWMA})t} \) or \( T_{E_t} < LCL_{(\text{TEWMA})t} \).

HWMA\_AIB chart is the extended form of HWMA chart, used when interest variable is observed along with auxiliary information variable. The plotting statistic of the HWMA chart given as:

\[
H_t = \lambda_1 R_t + (1 - \lambda_1) R_{t-1},
\]

where \( \bar{Y}_t \) is the sample average of \( t \)th sample and \( \lambda_1 \in (0, 1] \) is the smoothing constant, and \( R_{t-1} = \sum_{i=1}^{t-1} R_i / t - 1 \) is the mean of preceding \( t - 1 \) observations. The control limits of the HWMA\_AIB chart are:

\[
\begin{align*}
LCL_{\text{(HWMA\_AIB)\_t}} &= \mu_0 - L_{\text{HWMA\_AIB}} \sigma_Y \sqrt{\left( \lambda^2 / n \right)} \left( 1 - \rho^2 \right), \quad \text{if } t = 1 \\
LCL_{\text{(HWMA\_AIB)\_t}} &= \mu_0 - L_{\text{HWMA\_AIB}} \sigma_Y \sqrt{\left( \lambda^2 / n \right)} \left( 1 - \rho^2 \right) \left( 1 - \rho \right), \quad \text{if } t > 1 \\
UCL_{\text{(HWMA\_AIB)\_t}} &= \mu_0 + L_{\text{HWMA\_AIB}} \sigma_Y \sqrt{\left( \lambda^2 / n \right)} \left( 1 - \rho^2 \right), \quad \text{if } t = 1 \\
UCL_{\text{(HWMA\_AIB)\_t}} &= \mu_0 + L_{\text{HWMA\_AIB}} \sigma_Y \sqrt{\left( \lambda^2 / n \right)} \left( 1 - \rho^2 \right) \left( 1 - \rho \right), \quad \text{if } t > 1
\end{align*}
\]

\[
\begin{align*}
E_t &= \lambda Y_t + \lambda E_{t-1}, \\
DE_t &= \lambda E_t + \lambda DE_{t-1}, \\
TE_t &= \lambda E_t + \lambda TE_{t-1},
\end{align*}
\]

where \( \lambda \in (0, 1] \) is a TEWMA constant. The initial values of \( E_0, DE_0, TE_0 \) are all equal to \( \mu_0 \). The mean and variance of the \( TE_t \) are \( E(TE_t) = \mu_0 \) and

3. Proposed Method

The methodology for the proposed chart is described in this section. Subsection 3.1 covers the design structure of the proposed THWMA\_AIB chart. Besides, the special cases of the THWMA\_AIB chart are given in Subsection 3.2.

3.1. Proposed THWMA\_AIB Chart. To construct the proposed THWMA\_AIB chart, the plotting statistic of the THWMA\_AIB chart is given as:
\[ H_t = \lambda_1 R_t + (1 - \lambda_1)R_{t-1}, \]
\[ DH_t = \lambda_2 H_t + (1 - \lambda_2)R_{t-1}, \]
\[ TH_t = \lambda_3 DH_t + (1 - \lambda_3)R_{t-1}. \]

The \( TH_t \) statistic in Equation (9) can be expressed as:
\[ TH_t = \lambda^3 R_t + \left[ \frac{1 - \lambda^3}{t - 1} \right] R_{t-1} + \cdots + \left[ \frac{1 - \lambda^3}{1 - 1} \right] R_1. \]

Also, for \( TH_t \), \( \text{var}(TH_t) = \frac{\lambda^6}{n}(1 - \rho^2)^2 \sigma_i^2 \) for \( t = 1 \), \( \text{var}(TH_t) = \frac{\lambda^6}{n}(1 - \rho^2)^2 \sigma_i^2 + \frac{(1 - \lambda^3)^2}{n(t - 1)}(1 - \rho^2)^2 \sigma_i^2 \) for \( t > 1 \). The proposed \( \text{THWMA}_{\text{AIB}} \) chart’s control limits are written as:
\[ \mu_0 - L_{\text{THWMA}_{\text{AIB}}} = \mu_0 - L_{\text{THWMA}_{\text{AIB}}} \sqrt{\frac{\lambda^6}{n} \left( 1 - \rho^2 \right)}, \quad \text{if } t = 1, \]
\[ \mu_0 - L_{\text{THWMA}_{\text{AIB}}} = \mu_0 - \frac{\lambda^6}{n} \left( 1 - \rho^2 \right) \left( 1 - \rho^2 \right), \quad \text{if } t > 1, \]
\[ \mu_0 + U_{\text{THWMA}_{\text{AIB}}} = \mu_0 + \frac{\lambda^6}{n} \left( 1 - \rho^2 \right), \quad \text{if } t = 1, \]
\[ \mu_0 + U_{\text{THWMA}_{\text{AIB}}} = \mu_0 + \frac{\lambda^6}{n} \left( 1 - \rho^2 \right) \left( 1 - \rho^2 \right), \quad \text{if } t > 1. \]

Case 1. When \( \rho = 0 \) and \( \lambda_3 = 1 \), the proposed \( \text{THWMA}_{\text{AIB}} \) chart tends to the DHWMA chart.

Proof. When \( \rho = 0 \), then the difference \( R_t \) estimator reduces to
\[ R_t^{(1)} = \overline{Y}_t. \]

Now substitute the resulted \( R_t^{(1)} \) in Equation (6) to obtain
\[ H_t^{(1)} = \lambda_1 R_t^{(1)} + (1 - \lambda_1)R_{t-1}^{(1)} \]
\[ \text{(14)} \]

Then the Equation (7) can be written as
\[ \text{(13)} \]

where \( L_{\text{THWMA}_{\text{AIB}}} \) is the control limits co-efficient. The \( TH_t \) statistic is plotted along with \( L_{\text{THWMA}_{\text{AIB}}} \) and \( U_{\text{THWMA}_{\text{AIB}}} \). The process is declared IC if \( TH_t > L_{\text{THWMA}_{\text{AIB}}} \) or \( TH_t < U_{\text{THWMA}_{\text{AIB}}} \); otherwise, OOC.

3.2. Special Cases of \( \text{THWMA}_{\text{AIB}} \) Chart. The proposed \( \text{THWMA}_{\text{AIB}} \) chart reduced to some existing charts including HWMA, DHWMA, and HWMA\text{}\text{AIB} charts by considering special values of parameters. The special cases, along with their proves, are provided here.

Case 1. When \( \rho = 0 \) and \( \lambda_3 = 1 \), the proposed \( \text{THWMA}_{\text{AIB}} \) chart tends to the DHWMA chart.

Proof. When \( \rho = 0 \), then the difference \( R_t \) estimator reduces to
\[ R_t^{(1)} = \overline{Y}_t. \]

Now substitute the resulted \( R_t^{(1)} \) in Equation (6) to obtain
\[ H_t^{(1)} = \lambda_1 R_t^{(1)} + (1 - \lambda_1)R_{t-1}^{(1)}. \]

Now substitute Equation (14) in Equation (8) which provides the following:
\[ TH_t^{(1)} = \lambda_3 DH_t^{(1)} + (1 - \lambda_3)R_{t-1}^{(1)}. \]

Case 2. The proposed \( \text{THWMA}_{\text{AIB}} \) chart converts to THWMA chart at \( \rho = 0 \).

Proof. Consider estimator \( R_t^{(1)} \) which is given in Equation (12) when \( \rho = 0 \) (i.e., \( R_t = \overline{Y}_t \)). Based on \( R_t^{(1)} \), the \( H_t^{(1)} \) of Equation (13) will be as follows:
\[ H_t^{(1)} = \lambda_1 R_t^{(1)} + (1 - \lambda_1)R_{t-1}^{(1)}. \]

Substituting the \( H_t^{(1)} \) in the plotting statistic of the \( DH_t \), (presented as \( DH_t^{(2)} \) in this case) we get:
\[ DH_t^{(2)} = \lambda_2 H_t^{(1)} + (1 - \lambda_2) R_{t-1}^{(1)}, \]  
\[ TH_t^{(2)} = \lambda_2 DH_t^{(1)} + (1 - \lambda_2) R_{t-1}^{(1)}. \]  

The \( TH_t^{(2)} \) in Equation (20) is the THWMA chart with no correlation. Hence, the proposed THWMA\_AIB chart reduced to the statistic of the THWMA chart for \( \rho = 0 \).

Case 3. When \( \lambda_2 = 1 \) and \( \lambda_3 = 1 \), the proposed THWMA\_AIB chart tends to the HWMA\_AIB chart.

Proof. When \( \lambda_3 = 1 \) and \( \lambda_3 = 1 \), then the plotting statistic of the proposed THWMA\_AIB chart, symbolized as \( TH_t^{(3)} \) can be written as

\[ TH_t^{(3)} = \lambda_1 R_t + (1 - \lambda_1) R_{t-1}. \]  

The \( TH_t^{(3)} \) in Equation (19) is similar to the HWMA\_AIB statistic proposed by Adegoke, et al. [28] except for their notations. Hence, the statistic of proposed THWMA\_AIB chart becomes the statistic of HWMA\_AIB when \( \lambda_2 = 1 \) and \( \lambda_3 = 1 \).

Case 4. The proposed THWMA\_AIB chart converts to the HWMA chart at \( \rho = 0 \), \( \lambda_2 = 1 \), and \( \lambda_3 = 1 \).

Proof. Consider estimator \( R_t^{(1)} \) which is given in Equation (12) when \( \rho = 0 \) (i.e., \( R_t^{(1)} = \overline{Y}_t \)). Based on \( R_t^{(1)} \), the \( H_t^{(1)} \) of Equation (13) will be as follows:

\[ H_t^{(4)} = \lambda_1 R_t^{(1)} + (1 - \lambda_1) R_{t-1}^{(1)}. \]  

Substituting the \( H_t^{(1)} \) in the plotting statistic of Equation (7) and (8), we get

\[ DH_t^{(4)} = \lambda_2 H_t^{(4)} + (1 - \lambda_2) R_{t-1}^{(1)}, \]  
\[ TH_t^{(4)} = \lambda_2 DH_t^{(4)} + (1 - \lambda_2) R_{t-1}^{(1)}. \]  

Now put \( \lambda_2 = 1 \) and \( \lambda_3 = 1 \), then the plotting statistic of the proposed THWMA\_AIB chart, symbolized as \( TH_t^{(4)} \) can be written as

\[ TH_t^{(4)} = \lambda_1 R_t^{(1)} + (1 - \lambda_1) R_{t-1}. \]  

The \( TH_t^{(4)} \) in Equation (22) is identical to the HWMA statistic. This shows that the THWMA\_AIB chart is identical to the HWMA chart by Abbas (2018) for \( \rho = 0 \), \( \lambda_2 = 1 \), and \( \lambda_3 = 1 \).

Case 5. When \( \lambda_3 = 1 \), the proposed THWMA\_AIB chart tends to the double HWMA\_AIB (DHWMA\_AIB) chart.

Proof. When \( \lambda_3 = 1 \), then the plotting statistic of the proposed THWMA\_AIB chart, symbolized as \( TH_t^{(5)} \) can be written as

\[ TH_t^{(5)} = \lambda_2 DH_t + (1 - \lambda_2) R_{t-1}. \]  

The \( TH_t^{(5)} \) in Equation (23) is a DHWMA\_AIB statistic. Hence, the statistic of proposed THWMA\_AIB chart becomes the statistic of DHWMA\_AIB for \( \lambda_3 = 1 \).

4. Performance Evaluation Measures

This section introduces the performance evaluation measures to analyze the charts’ performance. The Monte Carlo simulation detail is given in Subsection 4.1. Likewise, the description of the ARL is enlisted in Subsection 4.2. Similarly, the overall performance evaluation measures are defined in Subsection 4.3. The choices of parameters of the proposed THWMA\_AIB chart is given in Subsection 4.4.

4.1. Monte Carlo Simulation. The Monte Carlo simulation procedure is regarded as a computational technique for obtaining numerical results for evaluating the performance of the proposed THWMA\_AIB chart. Monte Carlo simulation with \( 10^5 \) iterations is conducted for each displacement of \( \delta \) using R software to obtain the ARL of the proposed THWMA\_AIB chart. The shift is reflected in the process mean as: \( \mu_Y \) to \( \mu_Y + \delta \sigma_Y \), where \( \delta = 0.00, 0.05, 0.10, 0.20, 0.25, 0.50, 1.00, 1.50, 2.00, 2.50, 3.00, \) and \( 5.00 \). The proposed THWMA\_AIB chart is constructed by following given below guidelines:

(i) Generate random observations from \((Y_1, X_1) \sim N(\mu_Y, \mu_X, \sigma_Y, \sigma_X, \rho)(i = 1, 2, 3, \ldots)\).

(ii) Calculate the \( R_t \) estimator from Equation (1).

(iii) Calculate the \( H_t \) statistics of the THWMA\_AIB chart from Equation (6) using \( R_t \) estimator.

(iv) Use the \( DH_t \) statistic as an input in Equation (8) to obtain the \( TH_t \) statistic.

(v) Chose \( L_{\text{THWMA\_AIB}} \) along with other desired parameters \((\lambda, \rho)\) for desired IC ARL denoted as \( \text{ARL}_0 \).

(vi) Compute \( LCL_{\text{THWMA\_AIB}} \) and \( UCL_{\text{THWMA\_AIB}} \) based on \( L_{\text{THWMA\_AIB}} \) and \( \lambda \).

(vii) Plot the \( TH_t \) statistic against the \( LCL_{\text{THWMA\_AIB}} \) and \( UCL_{\text{THWMA\_AIB}} \).

(viii) If \( TH_t > UCL_{\text{THWMA\_AIB}} \) or \( TH_t < LCL_{\text{THWMA\_AIB}} \) record sequence order which is called run length (RL).

(ix) Repeat steps (i)-(viii) \( 10^5 \) times.

(x) Calculate the average of \( 10^5 \) RL, which is \( \text{ARL}_0 \). If it is desired \( \text{ARL}_0 \), otherwise, adjust the \( L_{\text{Proposed}} \) accordingly and repeat from steps (i)–(ix) until will not get desired \( \text{ARL}_0 \).

(xi) For OOC ARL values, considered \((Y_1, X_1) \sim N(\mu_Y, \mu_X, \sigma_Y, \sigma_X, \rho)\) and repeat from steps (ii)-(x).

4.2. Individual Performance Measure. The average run length (ARL) is commonly used to evaluate a chart’s performance at a single shift. The ARL is listed as IC ARL (ARL\(_0\)) and OOC ARL (ARL\(_1\)). For IC state, the ARL\(_0\)
2.4. Effect of Parameters Choices. The design parameters ($\lambda$, $L_{THWMAAIB}$) of the proposed THWMAAIB chart have its effect on the detection ability. Various combinations of these parameters are chosen, and hence corresponding ARL, the standard deviation of RL (SDRL), and median of RL (MDRL) are computed. The parameter $\lambda$ is set as 0.10, 0.25, 0.50, and 0.75 to find the values of $L_{THWMAAIB}$ to obtain ARL$_0 = 500$. Various $\rho$ like 0.00, 0.25, 0.50, 0.75, and 0.95 are assumed for this study. Numerical results of the proposed THWMAAIB chart are presented in Tables 1–4.

5. Evaluation and Performance Comparison

This section presents comprehensive comparisons of the proposed THWMAAIB chart with classical EWMA [3], EWMA$_{AIB}$ [16], HWMA [30], HWMA$_{AIB}$ [28], DHWMA [32], TEWMA [5], MCEAIB and MEC$_{AIB}$ [22], and MHC [35] charts. More detail is provided here.

5.1. Proposed versus Classical EWMA Chart. The proposed THWMAAIB chart provides superior detection ability for various values of $\lambda$ against the classical EWMA chart. For instance, at $\rho = 0.75, \lambda = 0.25, \delta = 0.50$, the proposed THWMAAIB chart has an ARL$_1$ value of 7.74, while the classical EWMA chart has an ARL$_1$ value of 47.36 (see Tables 2 and 5). The supremacy of the proposed THWMAAIB chart compared to the classical EWMA chart can be seen in Figure 1. Evaluating the overall efficiency with $\lambda = 0.10$, the THWMAAIB chart has smaller EQL, RARL, and PCI (i.e., 9.38, 1.000, 1.000) values against the classical EWMA chart EQL, RARL, and PCI (i.e., 13.38, 1.43, 2.23) values (see Table 6).

5.2. Proposed versus TEWMA Chart. The proposed THWMAAIB chart is superior to the TEWMA chart. For example, at $\delta = 0.25$, the proposed THWMAAIB chart ($\lambda = 0.10, \rho = 0.50$) yields ARL$_1 = 13.34$, whereas the TEWMA chart gives ARL$_1$ equal to 74.21 (see Table 1 and 5). The superiority of the THWMAAIB chart than the TEWMA chart can also be found in Figure 1. Additionally, for a specific range of shifts, the EQL, RARL, and PCI values for proposed THWMAAIB chart are smaller than the TEWMA chart. For example, at $\lambda = 0.50$, for THWMAAIB and TEWMA charts, the EQL, RARL, and PCI values are (13.03, 1.000, 1.000) and (15.29, 1.26, 1.17), respectively (see Table 6).

5.3. Proposed versus MHC Chart. The proposed THWMAAIB chart is better for tracking changes in process location than the MHC chart. For example, with $\lambda = 0.10$ and $\delta = 0.25, 0.50, 0.75$, the ARL$_1$ values of the MHC chart are (25.00, 13.00, 9.00), while the ARL$_1$ values of proposed THWMAAIB chart ($\rho = 0.50$) are (13.34, 6.01, 3.83) (see Table 1 and 5). Figure 1 also highlighted the dominant position of the THWMAAIB chart on the MHC chart. Also, the EQL, PCI, and RARL values are significantly higher for the MHC chart as compared to THWMAAIB chart. For illustration, at $\lambda = 0.25$, the THWMAAIB chart has EQL = 10.35, PCI = 1.00, and RARL = 1.00, whereas the MHC chart has EQL = 22.05, PCI = 2.13, and RARL = 1.98 measures (see Table 6).

5.4. Proposed versus EWMA$_{AIB}$ Chart. The proposed THWMAAIB chart offers good performance against the EWMA$_{AIB}$ chart. As an illustration, at $\lambda = 0.10, \rho = 0.50$ and $\delta \in (0.25, 0.50)$, the ARL$_1$ (80.66, 22.04) values of the EWMA$_{AIB}$ chart are larger than the ARL$_1$ (13.34, 6.01) values of the THWMAAIB chart (see Tables 1 and 5). Visual
representation confirms the supremacy of the proposed THWMAIB chart over EWMAIB chart (see Figure 2). The dominance of the THWMAIB chart against the EWMAIB chart is seen in the EQL, RARL, and PCI values for a certain range of shifts. As an illustration, at $\lambda = 0.10$, the proposed THWMAIB chart has EQL = 9.38, PCI = 1.00, and RARL = 1.00, while the EWMAIB chart has EQL = 11.78, PCI = 1.26, RARL = 1.82 (see Table 6). This reveals the superiority of the proposed THWMAIB chart.

5.5. Proposed versus HWMA Chart. The proposed THWMAIB chart has lower ARL1 values than the HWMA chart. Suppose that, for $\lambda = 0.50$, $\delta = 0.25$, the ARL1 value of the proposed THWMAIB chart with $\rho = 0.50$ is 68.01, while the ARL1 value of the HWMA chart is 218.06 (see Tables 3 and 5). In other words, the proposed chart can track a quick shift in the process parameter than the HWMA chart (see Figure 3). The overall performance metrics suggest that the effectiveness of proposed THWMAIB than the HWMA chart. For instance, at $\lambda = 0.50$, the EQL values for HWMA and DHWMAIB are 16.25 and 13.03, respectively (see Table 6).

5.6. Proposed versus DHWMA Chart. The proposed THWMAIB chart is better as compared to the DHWMA chart. As an illustration, if $\lambda = 0.10, \delta = 0.25$, the values of...
ARL1 for DHWMA and THWMA AIB (\(\rho = 0.25\)) are 34.76 and 17.40, respectively (see Table 1 versus Table 5 and Figure 3). The overall performance measures highlight the superiority of the THWMA AIB chart. For instance, at \(\lambda = 0.10\), the EQL, PCI, and RARL values of the DHWMA and THWMA AIB charts are presented as (10.76, 1.15, and 1.38), and (9.38, 1.00, and 1.00), respectively (see Table 6).

5.7. Proposed versus HWMA AIB Chart. The proposed THWMA AIB chart detects earlier shift than the HWMA AIB chart. If \(\lambda = 0.10\), \(\rho = 0.50\), and \(\delta = (0.25, 0.50, 0.75)\) the ARL1 values of the charts are (65.89, 22.63, 11.79) and (13.34, 6.01, 3.83) respectively, for HWMA AIB and THWMA AIB charts (see Tables 1 and 5). Figure 3 also reveals the better performance of the proposed THWMA AIB against the HWMA AIB chart. Likewise, the HWMA AIB chart EQL, RARL, and PCI (i.e., 14.55, 1.12, and 1.23) values are higher than the EQL, RARL, and PCI (13.03, 1.00, and 1.00) values of the THWMA AIB chart at \(\lambda = 0.50\), \(\rho = 0.50\) (see Table 6).

5.8. Proposed versus MEC AIB Chart. By comparing the ARL measures, the proposed THWMA AIB chart with the MEC AIB chart shows that the THWMA AIB chart is more sensitive for all \(\delta\) and \(\lambda\) values. For instance, with \(\lambda = 0.25\), \(\rho = 0.50\), \(\delta = (0.25, 0.50, 0.75)\) the ARL1 values of the charts are (35.07, 11.65, 6.31) for THWMA AIB chart, while the (ARL1 = 65.26, 22.14, 12.61) for MEC AIB (\(k = 0.50\)) chart (see Tables 2 and 5). The proposed THWMA AIB chart has smaller EQL, RARL, and PCI measures as compared to the MEC AIB chart (see Table 6). Figure 2 illustrates that the proposed THWMA AIB is functioning well and detects shifts earlier in the process. These findings demonstrate that the proposed THWMA AIB chart performs better than the MEC AIB chart.

5.9. Proposed versus MCE AIB Chart. The ARL study indicates that the proposed THWMA AIB chart works much better than the MCE AIB chart for each of the choices formed by \(\lambda, \delta\) (see Tables 1–4 versus 5). As an illustration, Table 3 and 5 shows that at \((\lambda = 0.50, \delta = (0.25, 0.50, 0.75)), \rho = 0.50\), the ARL1 values of the proposed THWMA AIB chart are (68.01, 23.02, 11.93), and the corresponding values of ARL1 for MCE AIB chart (\(k = 0.50\)) are (111.47, 28.62, 13.55). Figure 2 indicates that the proposed THWMA AIB chart is better than the MCE AIB chart. In the overall performance scenario, the proposed THWMA AIB chart also exhibits overperformance than MCE AIB chart (see Table 6).
5.1. Main Findings of the Study. Important findings of the proposed THWMA\textsubscript{AIB} chart are listed below:

(i) The THWMA\textsubscript{AIB} statistic improves the detection ability of the proposed chart.

(ii) The efficiency of the proposed THWMA\textsubscript{AIB} chart is strengthened with the appropriate inclusion of auxiliary information in the structure (see, Tables 1–5).

(iii) Unlike the classic CUSUM, MHC, EWMA, HWMA, DHWMA, HWMA\textsubscript{AIB}, EWMA\textsubscript{AIB}, MEC\textsubscript{AIB} and MCE\textsubscript{AIB} charts, the ARL\textsubscript{1} values of the proposed THWMA\textsubscript{AIB} chart are smaller at various parameter values.

(iv) Overall performance evaluations demonstrate that the THWMA\textsubscript{AIB} chart is dominant over the other competing charts included in this research (see Subsections 5.1–5.9).

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|}
\hline
$\rho$ & $\delta$ & 0.00 & 0.05 & 0.10 & 0.20 & 0.25 & 0.50 & 0.75 & 1.00 & 1.50 & 2.00 & 2.50 & 3.00 & 5.00 \\
\hline
ARL & 499.25 & 401.97 & 259.88 & 115.66 & 84.14 & 29.02 & 15.03 & 9.45 & 5.00 & 3.33 & 2.45 & 1.88 & 1.03 & \\
SDRL & 428.13 & 344.00 & 216.62 & 87.43 & 61.14 & 18.37 & 8.78 & 5.20 & 2.42 & 1.52 & 1.17 & 0.98 & 0.18 & \\
MDRL & 387.00 & 311.00 & 202.00 & 95.00 & 70.00 & 25.00 & 13.00 & 9.00 & 5.00 & 3.00 & 3.00 & 1.00 & 1.00 & \\
\hline
ARL & 496.96 & 396.92 & 251.97 & 110.33 & 80.30 & 27.49 & 14.36 & 8.97 & 4.76 & 3.18 & 2.35 & 1.79 & 1.02 & \\
SDRL & 427.47 & 339.79 & 210.10 & 83.17 & 57.95 & 17.21 & 8.31 & 4.89 & 2.29 & 1.45 & 1.14 & 0.94 & 0.15 & \\
MDRL & 384.00 & 306.00 & 197.00 & 90.50 & 67.00 & 24.00 & 13.00 & 8.00 & 4.00 & 3.00 & 3.00 & 1.00 & 1.00 & \\
\hline
ARL & 498.49 & 378.78 & 226.01 & 94.11 & 68.01 & 23.02 & 11.93 & 7.50 & 4.06 & 2.75 & 1.99 & 1.50 & 1.00 & \\
SDRL & 428.74 & 325.16 & 186.07 & 69.40 & 47.70 & 14.12 & 6.76 & 3.97 & 1.89 & 1.27 & 1.02 & 0.79 & 0.06 & \\
MDRL & 384.00 & 291.00 & 177.00 & 78.00 & 58.00 & 20.00 & 11.00 & 7.00 & 4.00 & 3.00 & 3.00 & 1.00 & 1.00 & \\
\hline
\end{tabular}
\caption{Table 3: ARL values of the proposed THWMA\textsubscript{AIB} chart for $\lambda = 0.50$ and ARL$_0 = 500$.}
\end{table}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|}
\hline
$\rho$ & $\delta$ & 0.00 & 0.05 & 0.10 & 0.20 & 0.25 & 0.50 & 0.75 & 1.00 & 1.50 & 2.00 & 2.50 & 3.00 & 5.00 \\
\hline
ARL & 498.97 & 342.43 & 166.97 & 63.23 & 45.01 & 14.81 & 7.71 & 4.96 & 2.81 & 1.86 & 1.32 & 1.08 & 1.00 & \\
SDRL & 427.80 & 276.15 & 133.22 & 44.19 & 29.98 & 8.64 & 4.13 & 2.41 & 1.30 & 0.97 & 0.65 & 0.32 & 0.00 & \\
MDRL & 385.00 & 251.00 & 133.00 & 53.00 & 39.00 & 13.00 & 7.00 & 5.00 & 3.00 & 1.00 & 1.00 & 1.00 & 1.00 & \\
\hline
ARL & 498.09 & 227.88 & 95.21 & 33.33 & 23.29 & 7.57 & 4.11 & 2.76 & 1.52 & 1.07 & 1.00 & 1.00 & 1.00 & \\
SDRL & 428.96 & 188.01 & 70.38 & 21.35 & 14.30 & 4.03 & 1.91 & 1.28 & 0.80 & 0.30 & 0.06 & 0.01 & 0.00 & \\
MDRL & 387.00 & 179.00 & 79.00 & 29.00 & 21.00 & 7.00 & 4.00 & 3.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & \\
\hline
ARL & 498.09 & 155.14 & 58.13 & 19.54 & 13.54 & 4.55 & 2.60 & 1.70 & 1.04 & 1.00 & 1.00 & 1.00 & 1.00 & \\
SDRL & 428.96 & 155.14 & 58.13 & 19.54 & 13.54 & 4.55 & 2.60 & 1.70 & 1.04 & 1.00 & 1.00 & 1.00 & 1.00 & \\
MDRL & 387.00 & 123.01 & 40.07 & 11.73 & 7.83 & 4.21 & 2.17 & 0.90 & 0.23 & 0.02 & 0.00 & 0.00 & 0.00 & \\
\hline
\end{tabular}
\caption{Table 4: ARL values of the proposed THWMA\textsubscript{AIB} chart for $\lambda = 0.75$ and ARL$_0 = 500$.}
\end{table}
### Table 5: ARL values of some existing charts when $\text{ARL}_0 = 500$.  

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<tr>
<th>$\delta$</th>
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<th>0.50</th>
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<th>2.00</th>
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<tr>
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<td>7.85</td>
<td>3.85</td>
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<tr>
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<td>9.00</td>
<td>7.00</td>
<td>5.00</td>
<td>4.00</td>
<td>4.00</td>
<td>4.00</td>
<td>3.00</td>
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<td>MHC ($k = 0.50$)</td>
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<td>4.47</td>
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<td>16.19</td>
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<td>19.31</td>
<td>10.19</td>
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<td>2.19</td>
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<tr>
<td>MHC ($k = 0.50$)</td>
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<td>24.04</td>
<td>14.75</td>
<td>9.00</td>
<td>6.00</td>
<td>4.00</td>
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<td>2.32</td>
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### Table 6: EQL, PCI, and RARL measures of the proposed versus existing charts.  

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<td>1.43</td>
<td>1.45</td>
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<td>EWMA</td>
<td>HWMA</td>
<td>DHWMA</td>
<td>HWMA_AIB</td>
<td>EWMA_AIB</td>
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<td>1.15</td>
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<td>1.12</td>
<td>1.27</td>
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<td>RARL</td>
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<td>1.61</td>
<td>1.45</td>
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<td>1.23</td>
<td>1.28</td>
<td>1.29</td>
<td>1.65</td>
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</tr>
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</table>
(v) The proposed THWMA_{AIB} chart provides optimal results for larger values \( \rho \).

(vi) The efficiency of the ARL\(_{1} \) measures is increased by using the proposed THWMA_{AIB} chart.

(vii) As \( \lambda \) increases, the control limit coefficient \( L_{THWMA_{AIB}} \) of the proposed THWMA_{AIB} chart also increases.

(viii) The proposed THWMA_{AIB} chart is more effective for larger \( \rho \) and smaller \( \lambda \) in term of ARL\(_{1} \) performance (see Figures 4 and 5). For example, the minimal value of ARL\(_{1} \) is 4.11 at \( \delta = 0.25, \lambda = 0.10, \) and \( \rho = 0.95 \).

### 6. Case Study

This section contains the case study of the THWMA_{AIB} versus DHWMA, HWMA_{AIB} and HWMA charts. In this regard, a data set is considered from the study of Constable, et al. [39]. The data represents the measurements for adjacent parts of the braking system of vehicles, which contains the study variable \( Y \) and auxiliary variable \( X \). Forty-five data values are taken from the IC process, and this data set is used to estimate unknown parameters. These estimates are given as; \( \bar{X} = 210.24, \bar{Y} = 210.18, s_{X} = 1.23, s_{Y} = 1.17, \) and \( r = 0.54 \). In view of these estimates as the known parameters, two datasets were generated from a BND. Data set-I consists of fifteen samples with \( \mu_{X} = 210.24, \mu_{Y} = 201.18, \sigma_{X} = 1.23, \sigma_{Y} = 1.17, \) and \( \rho = 0.54 \) and data set-II consists of fifteen samples with \( \mu_{X} = 210.24, \mu_{Y} = 201.88, \sigma_{X} = 1.23, \sigma_{Y} = 1.17, \) and \( \rho = 0.54 \) (see Table 7). This fashion of perturbing the parameters in such a way can be seen from the study of Anwar, et al. [40].

For the comparison, the proposed THWMA_{AIB} chart is considered along with the existing DHWMA, HWMA_{AIB}, and HWMA charts. Furthermore, the parameters of the proposed THWMA_{AIB} chart are \( L_{THWMA_{AIB}} = 1.90, \lambda = 0.25, \) \( \rho = 0.54 \) with ARL\(_{0} = 500 \), and the parameters of the DHWMA chart are \( L_{DHWMA} = 2.7424, \lambda = 0.25, \) with ARL\(_{0} = 500 \). Likewise, the parameters of HWMA_{AIB} chart...
Table 7: Application of proposed THWMA\textsubscript{AIR} versus HWMA, DHWMA, and HWMA\textsubscript{AIR} control charts.

<table>
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Plotting statistic

| LCL\textsubscript{HWMA\textsubscript{AIR}} |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |
| UCL\textsubscript{HWMA\textsubscript{AIR}} |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |

DHWMA

| LCL\textsubscript{DHWMA\textsubscript{AIR}} |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |
| UCL\textsubscript{DHWMA\textsubscript{AIR}} |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |

Plotting statistic

| LCL\textsubscript{THWMA\textsubscript{AIR}} |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |
| UCL\textsubscript{THWMA\textsubscript{AIR}} |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |

Plotting statistic

| LCL\textsubscript{THWMA\textsubscript{AIR}} |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |
| UCL\textsubscript{THWMA\textsubscript{AIR}} |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |

Plotting statistic

| LCL\textsubscript{THWMA\textsubscript{AIR}} |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |
| UCL\textsubscript{THWMA\textsubscript{AIR}} |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |

Plotting statistic

| LCL\textsubscript{THWMA\textsubscript{AIR}} |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |
| UCL\textsubscript{THWMA\textsubscript{AIR}} |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |

Plotting statistic

| LCL\textsubscript{THWMA\textsubscript{AIR}} |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |
| UCL\textsubscript{THWMA\textsubscript{AIR}} |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |

Plotting statistic

| LCL\textsubscript{THWMA\textsubscript{AIR}} |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |
| UCL\textsubscript{THWMA\textsubscript{AIR}} |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |
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are $L_{\text{HWMA}} = 3.075$, $\lambda = 0.25$, and $\text{ARL}_0 = 500$. From Table 7 and Figures 6–9 the proposed THWMA $\text{AIB}$ chart tracks the first OOC signal at the 5th sample, whereas the DHWMA, HWMA $\text{AIB}$, and HWMA charts track the first OOC signal at the 18th, 8th, and 9th samples, respectively. Overall, the proposed THWMA $\text{AIB}$ chart highlights 21 OOC points, while the DHWMA, HWMA $\text{AIB}$, and HWMA charts highlight 13, 9, and 9 OOC points, respectively. This shows that the proposed THWMA $\text{AIB}$ chart is more sensitive than the existing DHWMA, HWMA $\text{AIB}$, and HWMA charts.

7. Summary, Conclusions, And Recommendations

The objective of this study is to enhance homogeneously weighted moving average (HWMA) and double HWMA (DHWMA) charts and propose an auxiliary-information-based triple HWMA, symbolized as THWMA $\text{AIB}$ chart to further improve the process location shift monitoring. To evaluate the performance of the proposed THWMA $\text{AIB}$ chart against other charts, an algorithm is developed in R software using the Monte Carlo simulation technique to obtain numerical results. The analysis based on average run length, extra quadratic loss, performance comparison index, and relative average run length reveals the proposed THWMA $\text{AIB}$ chart over-performed against EWMA, EWMA $\text{AIB}$, HWMA, HWMA $\text{AIB}$, DHWMA, TEWMA, MCE $\text{AIB}$, MEC $\text{AIB}$, and MHC charts. Finally, to demonstrate the proposed THWMA $\text{AIB}$ chart’s utility from a practical perspective, a real-world application is included. This study can be extended for multivariate and more than one auxiliary characteristic in the model.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.
Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors’ Contributions


References


