Hindawi
Mathematical Problems in Engineering
Volume 2022, Article ID 2946091, 9 pages
https://doi.org/10.1155/2022/2946091

Research Article

An Optimization Algorithm of Time-Delayed Feedback Control Parameters for Quarter Vehicle Semiactive Suspension System

Yong Guo,[1] **Yuh-Chung Hu**,[2] **and Chuan-Bo Ren**[3]

[1]*School of Mechanical Engineering, Shandong University of Technology, Zibo 255000, China*
[2]*Department of Mechanical and Electromechanical Engineering, National Ilan University, Shenlung Road, Yilan 26047, Taiwan*
[3]*School of Transportation and Vehicle Engineering, Shandong University of Technology, Zibo 255000, China*

Correspondence should be addressed to Chuan-Bo Ren; chuanbor@sdut.edu.cn

Received 6 August 2021; Revised 29 August 2021; Accepted 1 September 2021; Published 30 April 2022

Academic Editor: Kuei-Hu Chang

Time-delayed feedback control is commonly used on the vehicle semiactive suspension system to improve ride comfort and safety. However, its performance on the suppression of road random excitation is less significant than that on the suppression of simple harmonic excitation. Therefore, this paper proposes a strategy of time-delayed feedback control with the vertical displacement of wheel and the method of optimizing its parameters based on equivalent harmonic excitation. The optimal parameters of the time-delayed feedback control are obtained in this way for the vehicle semiactive suspension system in its effective frequency band. The results of numerical simulation verify that the time-delayed feedback control with vertical wheel displacement and its parameter optimization based on equivalent harmonic excitation can significantly improve the ride comfort and stability. Its performance is much better than that of the passive suspension system.

1. Introduction

Modern automobile is developing in the direction of safety, comfort, energy saving, environmental protection, and intelligence. People's pursuit of automobile ride comfort and overall quality is also increasing. As one of the important components of modern automobile, the main function of vehicle suspension system is to alleviate the impact and vibration caused by road roughness and improve the ride comfort of vehicles, as well as maintaining the adhesion between the tire and the ground, so as to improve the handling stability of the vehicle. In general, the conventional passive suspension system consists of elastic components and damper with fixed parameters. Although it has the advantages of simple structure and low cost, its structural parameters cannot be adjusted automatically in complex and changeable external conditions and different driving states of vehicles, and the passive suspension system can achieve the optimal performance only under a specific combination of speed and road conditions. In order to improve the ride performance of vehicles, the present research on active and semiactive suspension systems, which can improve the driving characteristics according to the driving states of the vehicle and road conditions, focuses mainly on the control algorithm, stability characteristics, and test methods [1–4]. However, due to a series of factors, such as signal acquisition, transmission, controller calculation, and actuation delay, several time delay factors inevitably arise. The existence of time delay reduces the performance of the suspension system. It may even lead to the instability of the suspension system, thus affecting the safety of vehicle driving [5, 6]. In the past, time delay was usually regarded as an unfavorable factor leading to the instability of the active and semiactive suspension systems of vehicles. Therefore, in order to improve the control accuracy and stability of the suspension system of vehicles, most researchers reduce or eliminate the adverse effects of time delay by combining the time delay compensation technology with the modern control technology [7–9]. However, from the in-depth study of time delay, it is found that time delay can also change the effective frequency range of the saturation control [10–12]. The vibration control effect of the main system can be improved effectively by adjusting the time delay as a feedback control parameter. Olgac and Holm-Hansen [13] introduced the time-delayed feedback control in the dynamic vibration absorber for the first time and proposed the concept of delayed resonator. Their results showed that if appropriate delay feedback control parameters (feedback gain and feedback delay) are chosen, the delayed resonator can completely absorb the vibration of the main system under harmonic excitation; that is, the vibration of the main system will disappear. Liu and Sun et al. [14] constructed a vehicle suspension model of two-degree-of-freedom based on the time-delayed feedback control of the vertical acceleration of the body and optimized the time-delayed feedback control parameters of the vehicle suspension system under different harmonic excitation frequencies of road. The optimization results showed that, in comparison to the passive suspension system, the vibration isolation effect of the vehicle suspension system under the optimal time-delayed feedback control was effectively improved. The acceleration amplitude of the vehicle body could also be reduced by at least 19.60%. Yan and Fang [15] studied a vehicle suspension system of two-degree-of-freedom based on time-delayed feedback control of the vertical velocity of the body. They used a genetic algorithm to optimize the structural parameters and control gain of suspension. The optimization results showed that, under a certain inherent time delay, the acceleration amplitude in the optimized sprung mass was reduced by 22.7%. Qu and Ren et al. [16] constructed a four-degree-of-freedom vehicle suspension model based on dual-delayed feedback control, and the particle swarm optimization algorithm was adopted to study the optimization of time-delayed feedback control parameters of vehicle suspension system under road random excitation. The results show that, compared with the passive suspension system, the acceleration amplitude of the vehicle body is 15.1% lower than that before optimization. Huang and Zhao [17] introduced a time-delayed feedback control into the mode of a two-degree-of-freedom semiactive suspension system to suppress the lateral vibration of the high-speed train, and the results show that, compared with the passive suspension system, the amplitude of the body's lateral vibration could be suppressed about 50% when the suitable values of damping coefficient and time delay are chosen.

In view of this, from the perspective of the utilization of time delay, it is rare to introduce time delay into the field of vehicle engineering applications, and the key to improving the performance of the vehicle suspension control system is to obtain the optimal time-delayed feedback control parameters (feedback gain and feedback delay). However, in the current research of quarter vehicle semiactive suspension system based on time-delayed feedback control, there are still the following main problems:

(1) Generally, only the damping effect of the vertical acceleration of the vehicle body near the resonance frequency point under the optimal time-delayed feedback control is considered. But the output response of other suspension performance evaluation indexes such as suspension dynamic stroke and tire dynamic displacement is not accurately understood. However, these three kinds of performance evaluation indicators are usually contradictory; that is to say, if we want to get better riding comfort, it will usually lead to the reduction of driving stability. Similarly, if we improve the driving stability, it will usually lead to the reduction of riding comfort. Therefore, only taking the output response of the vertical acceleration of the vehicle body as the evaluation index of the vibration reduction effect of the vehicle suspension system, there are some limitations in the evaluation of the comprehensive performance of the vehicle semiactive suspension system based on the time-delayed feedback control.

(2) Generally, the road conditions of vehicles are dominated by simple road harmonic excitation. The vibration reduction effect of vehicle semiactive suspension system based on time-delayed feedback control under road harmonic excitation is relatively good, but the vibration reduction effect under road random excitation is usually not as significant as that under road harmonic excitation. Therefore, in order to solve the problem of a complex time-varying excitation, such as external multifrequency, the general method for solving the appropriate time-delayed feedback control parameters still needs to be explored further.

In view of the above problems, this paper takes the two-degree-of-freedom quarter car model based on time-delayed feedback control as the basic research object; a strategy of time-delayed feedback control with vertical displacement of the wheel and an optimization method of time-delayed feedback control parameters based on "equivalent harmonic excitation" are proposed for vehicle semiactive suspension system with time-delayed feedback control. In the final case analysis, the optimal values of time-delayed feedback control parameters of the vehicle suspension system in the effective frequency band are obtained through particle swarm optimization algorithm. The numerical simulation results of time-domain response show that, compared with the passive suspension system, the semiactive suspension system based on time-delayed feedback control with wheel displacement can significantly improve the ride comfort and stability of the vehicle under the optimal time-delayed feedback control, where the comprehensive performance of the suspension is improved.

2. Mechanical Model of the Time-Delayed Feedback Control for Vehicle Suspension System

In this paper, taking magnetorheological damper as active control actuator, a time-delayed feedback control based on wheel displacement is introduced into vehicle passive suspension system. The mechanical model of vehicle semiactive suspension system based on time-delayed feedback control of wheel displacement is established, and the model is shown in Figure 1.

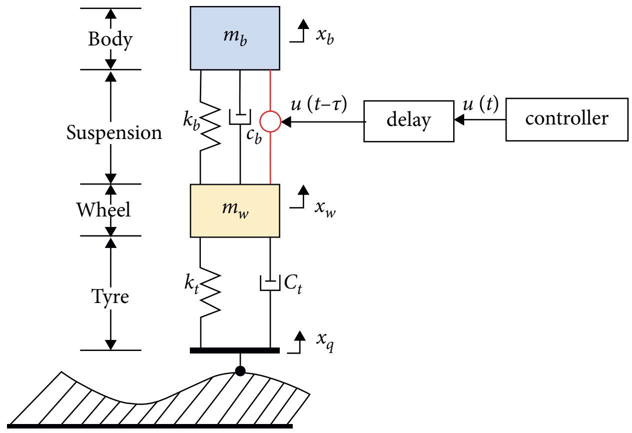

Figure 1: Mechanical model for controlling time-delayed vibration reduction in vehicle suspension system.

Table 1: Model parameters of vehicle suspension system.

Parameter	Value	Parameter	Value
m_b (kg)	136.05	mw (kg)	24.29
cb (N•s•m^{-1})	153	ct (N•s•m^{-1})	15
kb (N•m^{-1})	10200	kt (N•m^{-1})	9800

The design parameters of vehicle semiactive suspension system are shown in Table 1 where mb is the quarter vehicle body mass (sprung mass), m_w is the unsprung mass, c_b is the damper coefficient of semiactive suspension, k_b is the spring stiffness of semiactive suspension, c_t is the damper coefficient of tire, k_t is the equivalent stiffness of tire, and $u(t\text{-}\tau)$ is the time-delayed feedback control force between sprung mass and unsprung mass. In addition, x_b is the vertical displacement of sprung mass, x_w is the vertical displacement of unsprung mass, and x_q is the vertical displacement of road roughness excitation.

According to the mechanical model for controlling the time-delayed vibration reduction as shown in Figure 1, the origin of the coordinate system is selected at the respective equilibrium positions of the wheel and the body. The equation of dynamic motion of the vehicle suspension system is formulated as follows:

$$\begin{cases} m_b\ddot{x}_b + c_b\dot{x}_b - c_b\dot{x}_w + k_bx_b - k_bx_w - u(t-\tau) = 0, \\ m_w\ddot{x}_w - c_b\dot{x}_b + (c_b + c_t)\dot{x}_w - k_bx_b + (k_b + k_t)x_w \\ \quad + u(t-\tau) = c_t\dot{x}_q + k_tx_q. \end{cases} \tag{1}$$

In practical applications, in view of the flexibility of sensor selection, the type of time-delayed feedback (time-delayed feedback control signal) mainly includes the single time-delayed feedback type of displacement [18], velocity [19], acceleration [20], and the combination of them [21]. The time-domain function based on the time-delayed feedback with vertical displacement of the vehicle body can be expressed as follows:

$$u(t-\tau) = g_w \cdot x_w(t-\tau_w), \tag{2}$$

where gw represents the coefficient of the time-delayed feedback gain and τw is the time delay (inherent and active time delay). The time delay from signal acquisition to input and the time delay of the MR damper response are the inherent feedback delay of the system, while the active time delay is an artificial time delay in the control loop of the suspension, which can be regarded as an adjustable parameter for improving the suspension performance of the control system. When $u(t\text{-}\tau)$ disappears, the semiactive suspension system based on the time-delayed vibration reduction control will degenerate into a vibration reduction system of the passive suspension.

Equations (1) to (2) are simplified and identified by matrix equation, and the result can be written as

$$\mathrm{M}\ddot{\mathrm{X}} + \mathrm{C}\dot{\mathrm{X}} + (\mathrm{K} + \mathrm{A})\mathrm{X} = \mathrm{K_t}\mathrm{X_q} + \mathrm{C_t}\dot{\mathrm{X}}_\mathrm{q}. \tag{3}$$

Let the root of the characteristic equation of (3) be s; after Laplace transformation, (4) can be obtained as

$$\left(\mathrm{M}s^2 + \mathrm{C}s + \mathrm{K} + A(s)\right)\mathrm{X}(s) = [\mathrm{K_t} + \mathrm{C_t}s]\mathrm{X_q}(s), \tag{4}$$

where

$$\begin{gathered} \mathrm{M} = \begin{bmatrix} m_b & 0 \\ 0 & m_w \end{bmatrix}, \\ \mathrm{C} = \begin{bmatrix} c_b & -c_b \\ -c_b & c_b + c_t \end{bmatrix}, \\ \mathrm{K} = \begin{bmatrix} k_b & -k_b \\ -k_b & k_b + k_t \end{bmatrix}, \\ \mathrm{A}(s) = \begin{bmatrix} 0 & -g_w \cdot e^{-s\tau_w} \\ 0 & g_w \cdot e^{-s\tau_w} \end{bmatrix}. \end{gathered} \tag{5}$$

And

$$\begin{gathered} \mathrm{K_t} = \begin{bmatrix} 0 & 0 \\ 0 & k_t \end{bmatrix}, \\ \mathrm{C_t} = \begin{bmatrix} 0 & 0 \\ 0 & c_t \end{bmatrix}, \\ \mathrm{X}(s) = \begin{Bmatrix} X_b(s) \\ X_w(s) \end{Bmatrix}, \\ \mathrm{X_q}(s) = \begin{Bmatrix} 0 \\ X_q(s) \end{Bmatrix}. \end{gathered} \tag{6}$$

3. Stability Analysis of Vehicle Suspension System with Time-Delayed Feedback Control

The feedback gain and feedback delay will affect the stability of the system after the introduction of time-delayed feedback control. Therefore, in order to ensure that the semiactive suspension system can work in a stable state, it is necessary to analyze its stability. According to (4), let $\mathrm{M}s^2 + \mathrm{C}s + K + \mathrm{A}(s) = G(s)$, and the expression of $G(s)$ can be derived by the following:

$$\mathrm{G}(s)=\begin{bmatrix} m_b s^2+c_b s+k_b & -c_b s-k_b-g_w\cdot e^{-s\tau_w} \\ -c_b s-k_b & m_w s^2+(c_b+c_t)s+k_t+k_b+g_w\cdot e^{-s\tau_w}\end{bmatrix}. \tag{7}$$

According to the nonzero solution condition of the equation, the characteristic equation of vehicle suspension system based on time-delayed feedback control with wheel displacement can be obtained as follows:

$$P(s)+Q(s)e^{-\tau_w s}=0, \tag{8}$$

where

$$\begin{aligned} P(s) &= m_b m_w s^4+(c_b m_b+c_t m_b+c_b m_w)s^3 \\ &\quad +(k_b m_b+k_t m_b+k_b m_w+c_b c_t)s^2 \\ &\quad +(c_b k_t+c_t k_b)s+k_b k_t, \\ Q(s) &= -g m_b s^2. \end{aligned} \tag{9}$$

According to the Routh–Hurwitz criterion, the necessary and sufficient condition for system stability is that all the characteristic roots of (8) have negative real parts, and the critical condition for system instability is that the characteristic roots of (8) are only pure virtual roots $s=\mathrm{i}\omega c$. Therefore, substituting $s=\mathrm{i}\omega c$ into (8) and using Euler's formula to separate the real and imaginary parts of the equation, the expressions of $\sin(\omega_c\tau_w)$ and $\cos(\omega_c\tau_w)$ can be obtained. According to $\sin^2(\omega_c\tau_w)+\cos^2(\omega_c\tau_w)=1$, the available high order polynomial (10) is as follows:

$$a_8\omega_c^8++a_6\omega_c^6+a_4\omega_c^4+a_2\omega_c^2+a_0=0. \tag{10}$$

In equation (10), a_8, a_6, a_4, a_2, and a_0 are the polynomial coefficients related to the suspension.

System parameters and their expression can be derived by the following:

$$\begin{aligned} a_8 &= m_b^2+m_w^2, \\ a_6 &= \left(c_b^2+c_t^2+2c_b c_t\right)m_b^2+\left(c_b^2-2k_b m_b\right)m_w^2 \\ &\quad -2(k_b+k_t)m_b^2 m_w+2c_b^2 m_b m_w, \\ a_4 &= \left(c_b^2c_t^2+2c_b^2k_t-2c_t^2k_b-2c_t^2k_b\right)m_b \\ &\quad +\left(k_b^2+k_t^2+2k_bk_t\right)m_b^2+\left(2k_b^2+4k_bk_t\right)m_bm_w \\ &\quad +k_b^2m_w^2-2c_b^2k_tm_w+\left(-g^2+k_t^2\right)m_b^2, \\ a_2 &= c_b^2k_t^2+c_t^2k_b^2-\left(2k_t^2k_b+2k_b^2k_t\right)m_b-2k_b^2k_tm_w, \\ a_0 &= k_b^2k_t^2. \end{aligned} \tag{11}$$

It can be seen from (10) that when the hysteresis feedback gain gw is a certain value, the coefficient a_λ ($\lambda=0, 2, 4, 6, 8$) determines the value of the root of the equation. The number of positive real roots of (10) is defined as N. If $N=0$, the stability of the system does not switch. That is to say, the stability of the system does not change when τ is taken as any positive real number. If $\mathrm{N}\neq 0$, the root number of the system is N, and for each ω^{cm} ($m=1, 2, \ldots, \mathrm{N}$), there are infinitely critical delay values τ^{cn} ($n=1, 2, \ldots, \infty$) corresponding with it. When τ_w crosses from $\tau_{wcn}-\varepsilon$ to $\tau_{wcn}+\varepsilon$ ($0<\varepsilon<1$), the changing trend of the real part of the characteristic root of the equation can be determined by the following equation:

$$RT=\operatorname{sgn}\left[\operatorname{Re}\left(\frac{\partial s}{\partial\tau}\right)|s=\omega_{cm}i, i=\sqrt{-1}\right]. \tag{12}$$

If $\mathrm{RT}=1$, it means that the number of unstable characteristic roots of the characteristic (12) increases by 2, when τw crosses the critical value from left to right. If $\mathrm{RT}=-1$, it means that the number of unstable characteristic roots of the characteristic (12) decreases by 2 [22, 23]. Different feedback gains g are selected in turn, and the above analysis process is repeated; the stability partition diagram on the (g, τ) plane can be obtained as shown in Figure 2. It can be seen from the stability partition in Figure 2 that the system is stable when (g, τ) is located in the shaded region, and the system is unstable when (g, τ) is located in the blank region.

4. Optimization of Time-Delayed Feedback Control Parameters in Vehicle Semiactive Suspension System

4.1. Construction of Optimization Objective Function. According to (4), the transfer function matrix H(s) of vehicle suspension system with time-delayed feedback control can be obtained as

$$\mathrm{H}(s)=\frac{\mathrm{X}(s)}{\mathrm{X_q}(s)}=\frac{\mathrm{K_t}+\mathrm{C_t}s}{\mathrm{M}s^2+\mathrm{C}s+\mathrm{K}+A(s)}. \tag{13}$$

With the frequency-domain analysis method, $\mathrm{i}\omega$ is used to replace the operator s in (13), and the frequency response transfer function matrix can be obtained as

$$\mathrm{H}(i\omega)=\frac{\mathrm{X}(i\omega)}{\mathrm{X_q}(i\omega)}=\frac{\mathrm{K_t}+\mathrm{C_t}(i\omega)}{-\omega^2\mathrm{M}+i\omega\mathrm{C}+\mathrm{K}+A(i\omega)}. \tag{14}$$

For the quarter car model, the vertical acceleration of the vehicle body is an important characteristic to measure the vibration effect of the vehicle suspension system. The minimum dimensionless amplitude frequency characteristic function of vehicle body vertical acceleration in the effective frequency band is taken as the objective function [24, 25], and the objective function $J(g_w, \tau_w)$ is established, as shown in (15)

$$\min J(g_w,\tau_w)=\frac{|\mathrm{H}(i\omega)|_{\ddot{x}_b\sim x_q}-\min|\mathrm{H}(i\omega)|_{\ddot{x}_b\sim x_q}}{\max|\mathrm{H}(i\omega)|_{\ddot{x}_b\sim x_q}-\min|\mathrm{H}(i\omega)|_{\ddot{x}_b\sim x_q}},$$
$$s.t.\begin{cases} \underline{g}_w\leq g_w\leq\overline{g}_w, \\ \underline{\tau}_w\leq\tau_w\leq\overline{\tau}_w. \end{cases} \tag{15}$$

Obviously, the smaller the value of the objective function, the better the ride comfort of the vehicle. However, the selection of time-delayed feedback control parameters should not only meet the requirements of vehicle ride

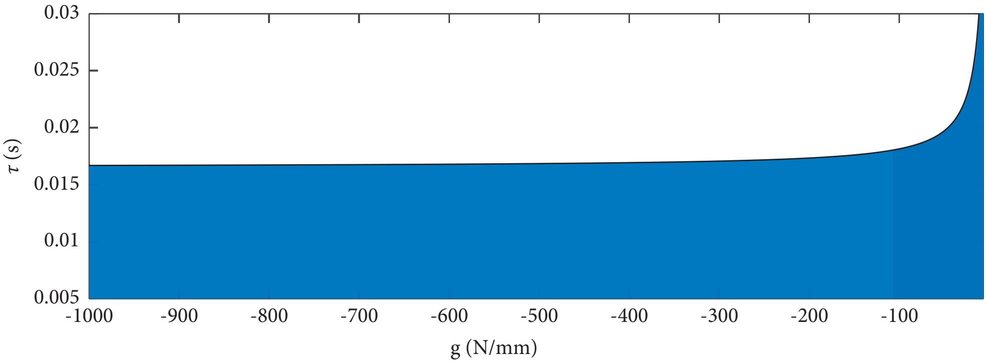

Figure 2: Division of suspension full time delay stability region under wheel displacement feedback control.

comfort to the greatest extent but also consider other performance evaluation indexes of suspension system, such as suspension dynamic travel and tire dynamic displacement output response, so as to comprehensively improve the comprehensive performance of suspension system. Therefore, according to the constraint range of the feedback control parameters, which can satisfy the full delayed stability of the suspension system shown in Figure 2, the numerical search range of the proposed feedback gain is $-k_b \le g_w \le k_b$; if the numerical search range of feedback delay τ_w is set as $0.0001 \le \tau_w \le 0.01$, the constraint condition of the time-delayed feedback control parameters is expressed as (16)

$$\begin{aligned} -k_b \le g_w \le k_b, \\ 0.0001 \le \tau_w \le 0.01. \end{aligned} \tag{16}$$

4.2. Time-Domain Model of Road Roughness. Establishing a reasonable time-domain model of road roughness is one of the most important issues in the follow-up study of dynamic characteristics of vehicle suspension system under random road excitation. At present, the main methods to simulate the road random excitation model at home and abroad are the harmonic superposition method, Fourier inverse change method, filtered white noise method, and time series model method. In view of the time-delayed vibration reduction effect of the vehicle body under the condition of road harmonic excitation which is remarkable, the time-domain model of random road excitation that is based on the harmonic superposition method is selected [26, 27]. The time-domain model can be expressed as

$$x_q(t) = \sum_{1}^{k} \sqrt{2G_q\left(f_{mid-j}\right) \cdot \Delta f_j} \cdot \ \sin\left(2\pi f_{mid-j} t + \theta_j\right), \quad (j = 1, 2, 3, \ldots k). \tag{17}$$

Due to the randomness of road excitation in the process of vehicle driving, the structural parameters that usually need to be selected in the establishment of a concrete pavement excitation model mainly include pavement grade roughness coefficient Gq ($f_{\text{mid-j}}$), effective time and frequency interval (f_1; f_2), and the number of parts k to separate them. The selection range of time frequency should include the main natural frequency of vehicle vibration. The division of time and frequency interval should take into account the operation efficiency and the fitting accuracy of the given road spectrum.

4.3. Optimization of Time-Delayed Feedback Control Parameters in Effective Frequency Band. In this paper, it is assumed that the vehicle travels at a speed of v = 60 km/h on the C-class road surface, so the road roughness coefficient is $Gq(f) = 256 \times 10^{-6} \text{m}^3$. Generally, the frequency range of vehicle vibration caused by road roughness is about 0.25 Hz–15 Hz. When it is lower than 15 Hz, the body motion can be assumed to be a simple rigid body motion. When it is higher than 15 Hz, the dynamic modeling requires an accurate description of the vehicle structure. In this paper, the lower limit and the upper limit of the time frequency of the road acting on the tire excitation are taken as $f_1 = 0.25$ Hz and $f_2 = 15$ Hz, respectively.

According to the product of time frequency and space frequency $f = \text{v}\cdot\text{n}$, the lower limit of effective spatial frequency $n_1 = 0.015\ \text{m}^{-1}$ and the upper limit of effective spatial frequency $n_2 = 0.9\ \text{m}^{-1}$ of road roughness power spectral density are obtained. In order to avoid frequency aliasing, the sampling interval is set as Δl, Δl ≤ 0.5·v·f2−1, which can be obtained from the sampling theorem. If the number of sampling points is N, the length of simulated road roughness shall be $L = N\cdot\Delta\text{l}$, and the resolution of sampling spatial frequency shall be Δn = L−1. In order to ensure the validity of the lower limit frequency n1, then $\Delta\text{n} \le n1$, so $L \ge n_{1-1} = \text{v}\cdot \text{f}_{1-1}$. According to the constraint of sampling interval Δl, Δl can be selected as $0.015\ \text{m}^{-1}$. The effective spatial frequency interval (n1, n2) of road roughness power spectral density is divided into $k = 59$ small areas in turn. The center frequency of each space cell is $\text{n}_{\text{mid}-\text{j}} = 0.015\cdot j + 0.0075$ for j = (1,2,3, . . ., 59). Then the interval length of time frequency is Δfj = 0.25, which leads to $\text{f}_{\text{mid}-\text{j}} = 0.25\cdot j + 0.125$ for j = (1,2, . . ., 59).

As the time-domain model of random road roughness shown in equation (17) and the constraint conditions of time-delayed feedback control parameters shown in (16) are referenced, the particle swarm optimization algorithm with

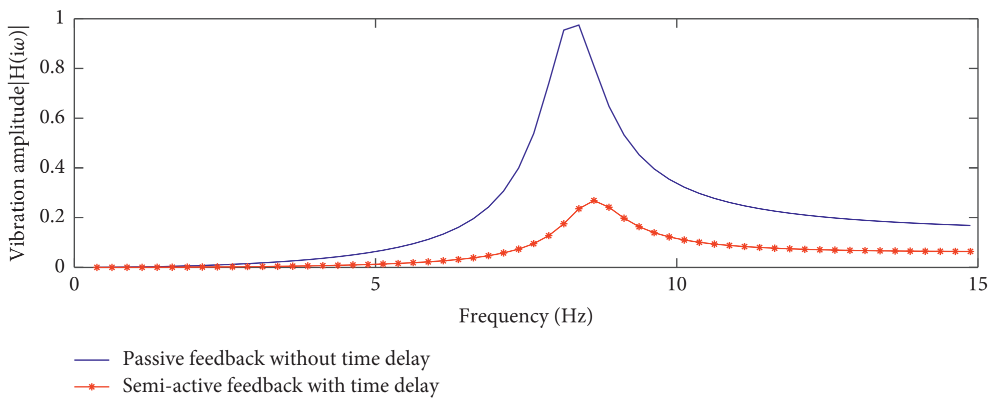

Figure 3: Amplitude frequency characteristic curve of the vertical acceleration of the vehicle body.

$J(g_w, \tau_w)$ as fitness function is constructed by using equivalent harmonic excitation method. That is, in the effective frequency band (f_1; f_2), a series of discrete frequency points are selected at equal intervals and divided into multiple cells. By taking the harmonic excitation at the center frequency of each cell as the optimization strategy of the equivalent harmonic excitation of the whole cell, the particle swarm optimization algorithm with the vertical acceleration of the vehicle body as the optimization objective function is constructed so as to obtain the optimal value of the delay feedback control parameters in the effective frequency band.

In view of the characteristics that the vibration reduction effect of vehicle semiactive suspension system under road random excitation is usually not as significant as that under road harmonic excitation, the reason is that the corresponding optimization objective function is mostly constructed by Fourier transform and frequency-domain transfer function, but the time-varying state of external excitation is not considered in the optimization process of time-delayed feedback control parameters, and the time-delayed feedback control parameters obtained by optimization solution are only local optimal values. In this paper, in order to avoid the optimization result of time-delayed feedback control parameters as local optimal value as far as possible, according to the expression of amplitude frequency characteristic function based on vertical acceleration of vehicle body shown in (18),

$$\left|H\left(i\omega_{mid-j}\right)\right|_{\ddot{x}_b\sim x_q} = \omega^2\left|\frac{X_b\left(i\omega_{mid-j}\right)}{X_q\left(i\omega_{mid-j}\right)}\right|, \quad (18)$$

where the amplitude of the road excitation at the center frequency of each interval (i.e., $|x_q\,(f_{mid-j})|$) is assumed to be the peak value of the road excitation in the effective frequency band (f_1; f_2), which can effectively reduce the amplitude of vertical acceleration based on the center frequency of each interval in the whole frequency band.

Finally, the particle swarm optimization algorithm with the vertical acceleration of the vehicle body as the optimization objective function is constructed to obtain the optimal value of the time-delayed feedback control parameters in the effective frequency band. On the basis of this optimization strategy, 200 particles are randomly selected for iterative optimization, and the optimal value of the time-delayed feedback control parameters of the vehicle semiactive suspension system in the effective frequency band is obtained, that is, (gwop, τwop) = (−8356.7 N/m, 0.0056 s), and the amplitude frequency characteristic curve of the vertical acceleration of the vehicle body in the effective frequency band is obtained as shown in Figure 3.

4.4. Results and Discussion. According to the frequency-domain simulation results of the vertical acceleration of the vehicle body shown in Figure 3, it can be seen that, compared with the vertical acceleration amplitude of the passive control suspension system, at the frequency $f = 8.3750$ Hz, the corresponding amplitude of the vehicle body vertical acceleration decreases from 0.9746 of the passive control to 0.2357 of the time-delayed feedback control, and at the frequency $f = 8.6250$ Hz, the corresponding amplitude of the vehicle body vertical acceleration decreases from 0.8100 of the passive control to 0.2687 of the time-delayed feedback control, while the peak values of passive suspension system and delayed feedback control suspension system in the whole frequency band are 0.9746 and 0.2687, respectively. Therefore, the amplitude frequency characteristics of the vertical acceleration of the vehicle body are effectively improved under the optimal time-delayed feedback control.

5. Time-Domain Simulation Analysis of Vehicle Semiactive Suspension System

In order to verify the correctness of the optimization results of time-delayed feedback control parameters, the time-domain response of various performance evaluation indexes of vehicle semiactive suspension system based on time-delayed feedback control under road excitation needs to be numerically simulated, analyzed, and verified.

5.1. Time-Domain Simulation Analysis under Harmonic Road Excitation. Based on the center frequency of each section in the effective frequency band, $f_{mid-1} = 0.375$ Hz is selected. Then the road excitation is simple harmonic excitation; that is, $x_q(t) = 0.0123\ \sin\ (2\pi f_{mid-1} t + \theta_{-1})$. According to the

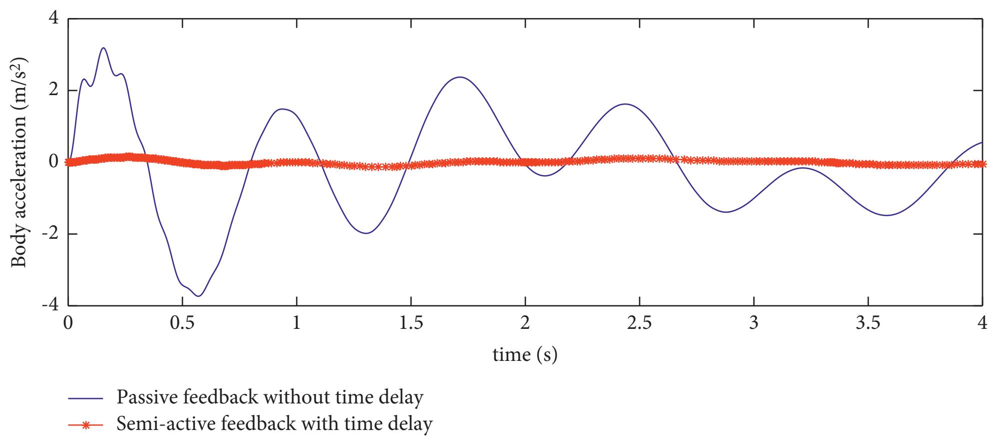

FIGURE 4: Time-domain response characteristic curve of vehicle body vertical acceleration.

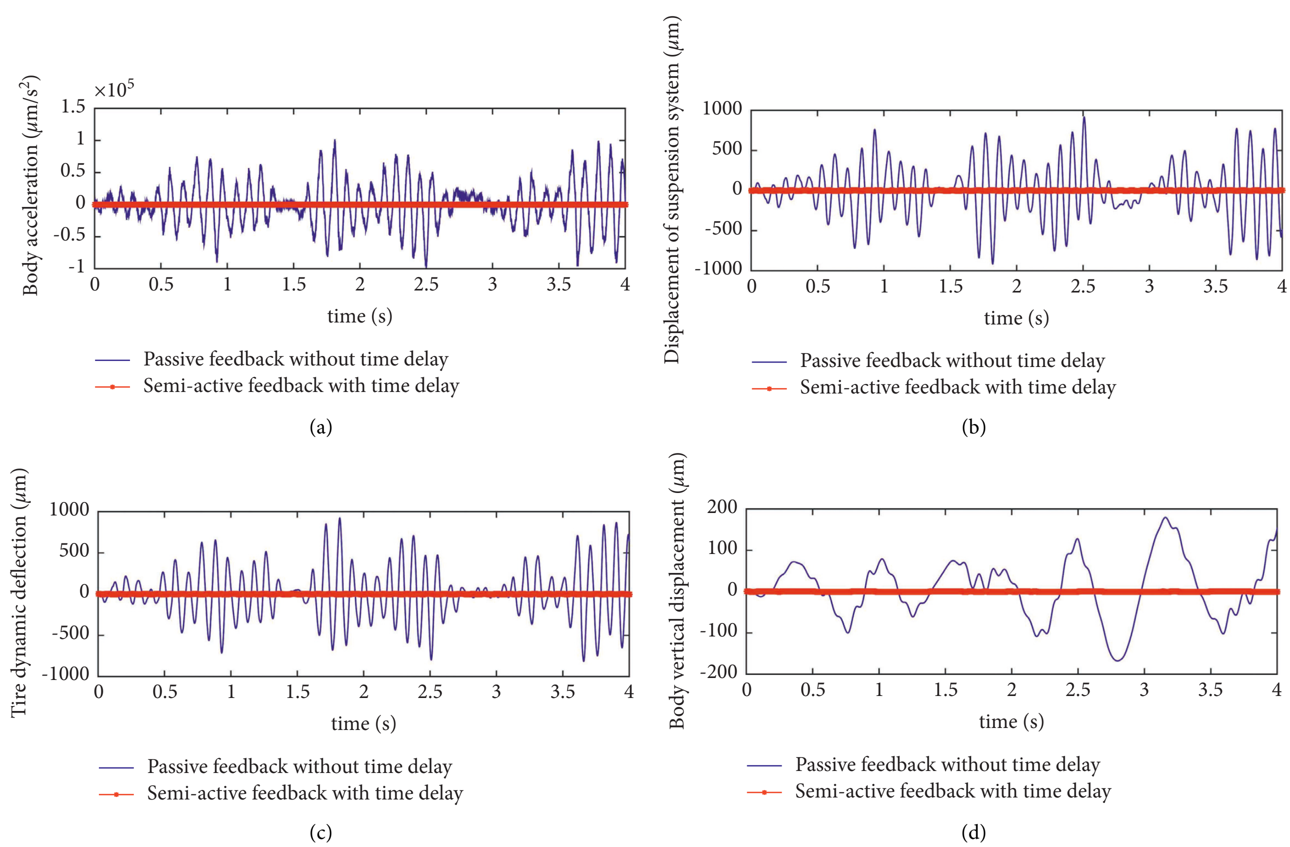

FIGURE 5: Time-domain simulation results of various performance evaluation indexes of vehicle suspension system. (a) Body vertical acceleration. (b) Suspension dynamic stroke. (c)Tire dynamic displacement. (d) Vertical displacement of vehicle body.

optimal value of time-delayed feedback control parameters (gwop; τwop), through numerical simulation analysis, the time-domain response of vehicle body vertical acceleration based on time-delayed feedback control of wheel displacement can be obtained, as shown in Figure 4.

5.2. Results and Discussion under Harmonic Road Excitation. Time-domain simulation results show that, compared with the passive control, the time-delayed feedback control significantly reduces the amplitude of vertical acceleration of vehicle body under harmonic excitation, and the corresponding root mean square value is reduced from $1.5730\,\mathrm{m{\bullet}s^{-2}}$ in passive control to $0.0699\,\mathrm{m{\bullet}s^{-2}}$ in time-delayed feedback control. So the optimization efficiency is 95.5587%.

5.3. Time-Domain Simulation Analysis under Random Road Excitation. With the C-level road roughness as the

Table 2: Comparison of RMS of various performance evaluation indexes of passive suspension and time-delayed feedback control suspension system.

Performance evaluation index	Passive suspension	Time-delayed feedback suspension	Difference (%)
Vertical acceleration (μm/s2)	14238	8.6600	99.9392
Suspension dynamic travel (μm)	173.46	2.3013	98.6733
Wheel displacement (μm)	154.34	2.1973	98.5764
Vertical displacement (μm)	87.576	0.6952	99.2062

excitation and the help of the Matlab platform, according to the optimal value of the time-delayed feedback control parameters, the time-domain response simulation results of the vehicle semiactive suspension system and vehicle passive control suspension system are compared and analyzed. The simulation results of vehicle vertical acceleration, suspension dynamic travel, and tire dynamic displacement are items that are compared and analyzed. The time-domain simulation results are shown in Figure 5.

5.4. Results and Discussion under Random Road Excitation. According to the comparison of the root mean square values of various performance evaluation indexes of vehicle suspension system under random road excitation shown in Table 2, the following results can be drawn:

(1) Compared with the passive control, the time-delayed feedback control significantly reduces the amplitude of vertical acceleration, and the RMS value of vertical acceleration decreases from 14238 μm/s2 of passive control to 8.6600 μm/s2 of time-delayed feedback control. Therefore, the optimization efficiency is 99.9392%.

(2) Compared with the passive control, the time-delayed feedback control significantly reduces the amplitude of suspension dynamic travel, and the RMS value of vertical acceleration decreases from 173.46 μm of passive control to 2.3013 μm of time-delayed feedback control. Therefore, the optimization efficiency is 98.6733%.

(3) Compared with the passive control, the time-delayed feedback control significantly reduces the amplitude of tire dynamic displacement, and the RMS value of vertical acceleration decreases from 154.34 μm of passive control to 2.1973 μm of time-delayed feedback control. Therefore, the optimization efficiency is 98.5764%.

(4) Compared with the passive control, the time-delayed feedback control significantly reduces the amplitude of vertical displacement, and the RMS value of vertical acceleration decreases from 87.576 μm of passive control to 0.6952 μm of time-delayed feedback control. Therefore, the optimization efficiency is 99.2062%.

6. Conclusions

In this paper, the mechanical model of vehicle semiactive suspension system based on time-delayed feedback control of wheel displacement is studied. In the example analysis, the optimization strategy based on "equivalent harmonic excitation" and particle swarm optimization algorithm are adopted to optimize and solve the time-delayed feedback control parameters involved in the model in this paper. Then, through numerical simulation analysis and verification, the following conclusions are drawn:

(1) Compared with the passive suspension system, the vehicle semiactive suspension system based on time-delayed feedback control with wheel displacement under the optimal time-delayed feedback control can effectively broaden the vibration absorption bandwidth of the vehicle suspension system.

(2) Compared with the passive suspension system, the vehicle semiactive suspension system based on time-delayed feedback control with wheel displacement can significantly improve the ride comfort and stability of the vehicle under the optimal time-delayed feedback control, which means that the comprehensive performance of the suspension is improved.

(3) The effectiveness of the time-delayed feedback control strategy based on wheel displacement and the parameter optimization strategy based on "equivalent harmonic excitation" is verified, which provides a theoretical reference for the optimal design of time-delayed feedback control parameters of the vehicle suspension system.

Data Availability

The data used to support the findings of this study have not been made available so as to ensure the privacy and anonymity of the people involved.

Conflicts of Interest

The authors declare no potential conflicts of interest with respect to the research, authorship, and/or publication of this paper.

Acknowledgments

The authors disclosed receipt of the following financial support for the research, authorship, and/or publication of this paper. This study was supported by the National Natural Science Foundation of China (Grant no. 51275280).

References

[1] R. C. Wang, L. Chen, X. L. Zhang, and X. H. Zhu, "Design and test of semi-active air suspension system of vehicle," *Transactions of the Chinese Society for Agricultural Machinery*, vol. 43, no. 4, pp. 6–9, 2012.

[2] K. H. Guo, W. H. Yu, X. J. Zhang, F. W. Ma, and F. Q. Zhao, "Semi-active suspension adaptive control strategy," *Journal of Hunan University*, vol. 40, no. 2, pp. 39–44, 2013.

[3] I. Mihai and F. Andronic, "Behavior of a semi-active suspension system versus a passive suspension system on an uneven road surface," *Mechanics*, vol. 20, no. 1, pp. 64–69, 2014.

[4] C. H. Zhao, S. A. Chen, and J. H. Wang, "Influences of stiffness and damping parameters on control of active suspension based on LQG," *Transactions of the Chinese Society for Agricultural Machinery*, vol. 46, no. 12, pp. 30–308, 2015.

[5] L. Chen, R. C. Wang, H. B. Jiang, L. K. Zhou, and S. H. Wang, "Time delay on semi-active suspension and control system," *Chinese Journal of Mechanical Engineering*, vol. 42, no. 1, pp. 130–133, 2006.

[6] G. Wang, C. Z. Chen, and S. B. Yu, "Finite frequency multi-objective control of vehicle active suspension with road preview information," *Transactions of the Chinese Society of Agricultural Engineering*, vol. 46, no. 12, pp. 294–300, 2015.

[7] G. Song and C. C. Xu, "Stochastic optimal preview control of active vehicle suspension with time-delay consideration," *Transactions of the Chinese Society of Agricultural Engineering*, vol. 44, no. 6, pp. 1–7, 2013.

[8] G. Yan, M. X. Fang, Q. F. Dong, and R. J. Ji, "Time-delayed feedback control of vehicle suspension system based on state Transformation," *Transactions of the Chinese Society of Agricultural Engineering*, vol. 34, no. 10, pp. 54–61, 2018.

[9] W. Q. Fu, H. Pang, and K. Liu, "Modeling and Stability analysis of semi-active suspension with sky-hook damper coefficients considering time delay," *Mech Sci Technol Aerosp Eng*, vol. 36, no. 2, pp. 213–218, 2017.

[10] J. Xu, K. W. Chung, and Y. Y. Zhao, "Delayed saturation controller for vibration suppression in a stainless-steel beam," *Nonlinear Dynamics*, vol. 62, no. 1-2, pp. 177–193, 2010.

[11] Y.-Y. Zhao and J. Xu, "Performance analysis of passive dynamic vibration absorber and semi-active dynamic vibration absorber with delayed feedback," *International Journal of Nonlinear Sciences and Numerical Stimulation*, vol. 8, no. 4, pp. 607–614, 2007.

[12] Y. Y. Zhao and J. Xu, "Using the delayed feedback control and saturation control to suppress the vibration of the dynamical system," *Nonlinear Dynamics*, vol. 67, no. 1, pp. 735–753, 2012.

[13] N. Olgac and B. T. Holm-Hansen, "A novel active vibration absorption technique: delayed resonator," *Journal of Sound and Vibration*, vol. 176, no. 1, pp. 93–104, 1994.

[14] J. J. Liu, Y. X. Sun, and S. Li, "Parameter optimization of time-delayed feedback control for vehicle suspension system in finite frequency band," *J Mech Elec Eng*, vol. 37, no. 1, pp. 54–58, 2020.

[15] G. Yan and M. X. Fang, "Time-delay speed feedback control of vehicle suspension system," *Science Technology and Engineering*, vol. 20, no. 20, pp. 8367–8372, 2020.

[16] Y. W. Qu, C. B. Ren, and J. L. Zhou, "Semi-active suspension control by using dual-delayed feedback based on particle swarm optimization," *Journal of Guangxi University*, vol. 41, no. 5, pp. 1368–1376, 2016.

[17] X. W. Huang and Y. Y. Zhao, "The effect of delayed feedback control on lateral semi-active suspension system for high-speed train," *Advanced Materials Research*, vol. 753-755, pp. 1795–1799, 2013.

[18] W. A. El-Ganaini, A. Kandil, M. Eissa, and M. Kamel, "Effects of delayed time active controller on the vibration of a nonlinear magnetic levitation system to multi excitations," *Journal of Vibration and Control*, vol. 22, no. 5, pp. 1257–1275, 2016.

[19] J. Xu, Y. Chen, and K. W. Chung, "An improved time-delay saturation controller for suppression of nonlinear beam vibration," *Nonlinear Dynamics*, vol. 82, no. 4, pp. 1691–1707, 2015.

[20] F. An, J. Li, and W. Zhang, "Advances and review on the research of the time-delayed acceleration feedback control," *Canadian Journal of Mathematics*, vol. 32, no. 6, pp. 985–991, 2015.

[21] M. Eissa, A. Kandil, W. A. El-Ganaini, and M. Kamel, "Analysis of a nonlinear magnetic levitation system vibrations controlled by a time-delayed proportional-derivative controller," *Nonlinear Dynamics*, vol. 79, no. 2, pp. 1217–1233, 2015.

[22] Z. H. Wang and H. Y. Hu, "Delay-independent stability of retarded dynamic systems of multiple degrees of freedom," *Journal of Sound and Vibration*, vol. 226, no. 1, pp. 57–81, 1999.

[23] Z. H. Wang and H. Y. Hu, "Stability switches of time-delayed dynamic systems with unknown parameters," *Journal of Sound and Vibration*, vol. 233, no. 2, pp. 215–233, 2000.

[24] Q. Wang, C. Ren, J. Zhou, and L. Zhang, "The double-delay reducing vibration control for five-degree-of-freedom half-vehicle model in idle condition," *Journal of Low Frequency Noise, Vibration and Active Control*, vol. 39, no. 1, pp. 203–215, 2019.

[25] K. Zhu, C. B. Ren, and F. Wang, "Research on vibration reduction of 1/4 car model with time delay feedback control," *Journal of Shandong University of Technology*, vol. 30, no. 2, pp. 31–35, 2016.

[26] F. T. K. Au, Y. S. Cheng, and Y. K. Cheung, "Effects of random road surface roughness and long-term deflection of prestressed concrete girder and cable-stayed bridges on impact due to moving vehicles," *Computers & Structures*, vol. 79, no. 8, pp. 853–872, 2001.

[27] W. Schiehlen and B. Hu, "Spectral simulation and shock absorber identification," *International Journal of Non-linear Mechanics*, vol. 38, no. 2, pp. 161–171, 2003.