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Research Article

The Effects of Magneto-Radiative Parameters on the Heat Transfer Mechanism in H_2O Composed by Cu-Al_2O_3 Hybrid Nanomaterial: Numerical Investigation

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The analysis of thermal performance in second generation of nanofluids (hybrid nanofluids) attained much attention of the researchers, scientists, engineers, and industrialists. These fluids have ultra-high thermal characteristics due to which their broad applications could be found in many areas of technological world. Therefore, a novel analysis regarding the heat transfer is conducted over a stretched surface by considering combined convection, thermal radiations, and magnetic field. The hybrid nanofluid is synthesized by Cu-Al_2O_3 guest hybrid-nanomaterial and host liquid H_2O. The hybrid flow model is solved numerically and decorated the results over the region of interest. It is drawn that the velocity drops by increasing the strength of Cu-Al_2O_3 fraction and applied Lorentz forces. Furthermore, the thermal performance of Cu-Al_2O_3/H_2O augmented against stronger thermal radiations, volumetric fraction, and magnetic field effects.

1. Introduction

Hybrid nanofluids are a new generation of nanofluids with ultra-high thermal performance than conventional nanofluids. Hybrid nanofluids are composed by binary mixture of nanoparticles of various metallic, nonmetallic, carbide, and CNT nanomaterials suspended in the host liquid. Thermal conductivity of the nanoparticles is an important part in the composition of nanofluids that improves the heat transport rate of the nanofluids significantly. The advancement in the nanofluids became very famous among the researchers and engineers due their superior heat transport mechanism. Thus, investigators and engineers paved their attention on such significant nanofluids and studies from different aspects. The nanofluids almost determined the problems of manufacturers and engineers about to huge amount of heat transfer for different manufacturing processes. Hence, the researchers investigated the influences of nanofluids on the flow characteristics under different conditions. The applications of these fluids are broadly found in medical, microelectronics, momentum, sailing buildings, microfluidics, civil engineering, for the detection of cancer cells in human bodies, paint industries, aerodynamics, chemical engineering and cooling of building, etc.

Keeping in view the broad applications of this new generation of the nanofluids, researchers paid much attention to investigate the flow characteristics more specifically thermal performance. Therefore, Takabi et al. [1]

worked on laminar convection flow of the nanofluid (composed by Cu-Al_2O_3) and discussed significant results regarding heat transport by altering various flow quantities. Takabi and Shoshouhmand [2] explained the heat transfer in the hybrid nanofluid (Cu-Al_2O_3/water). Suresh et al. [3] described the influence of hybrid nanofluid (Cu-Al_2O_3/water) in heat transference. Morain [4] organized an experimental study for thermal performance in the hybrid nanoliquid. They synthesized the fluid mixture by adding nanoadditives of Cu and aluminum oxides in the host liquid. They chose water as a host fluid and then performed the analysis over the synthesized hybrid nanoliquid. Suresh et al. [5] explored the influence of $\mathrm{Cu}-\mathrm{Al_2O_3}$/water hybrid nanofluids regarding the heat transfer. Another study for turbulent flow is conducted by Suresh et al. [6] by using the same hybrid nanoliquid. Some other imperative analysis on the heat transport mechanism in the hybrid nanoliquid under certain flow assumptions and conditions are reported in [7, 8].

Ahmad and Khan [9] examined the behaviour of heat and mass transfer in the fluid over a surface with elasticity characteristics and provided a comprehensive detail about the flow regimes. They analyzed the model numerically. Kumar and Bandari [10] worked in the melting temperature transference of nanofluid and stretching surface. The model comprised the effects of Brownian motion and thermophoresis and then investigated the alterations in the heat and mass transfer due to these physical parameters. Khan and Pop [11] organized and discussed a laminar flow over a stretchable surface and explored the flow characteristic over a semi-infinite domain. Mohimanianpour and Rashidi [12] performed the analysis of steady-state flow behaviour under boundary-layer approximation theory (BLAT) and discussed the results in brevity. They tackled the nonlinear problem via HAM and decorated the results. Bhargava et al. [13] studied combined convection effects in micropolar fluid. They modeled the problem over a porous surface which is stretchable.

The heat transfer can be described magnetic effect of flow over stretching sheet which is explained by Chakrabarti et al. [14]. Ahmad [15] designed a flow model under the impacts of magnetic field and unsteady effects. For mathematical investigation, they utilized quasi-linearization technique and performed the results. Rashidi et al. [16] reported the parametric study and optimization of entropy generation in unsteady MHD flow past a stretching rotational disk with particle swarm optimization (PSO) algorithm, HAM, and artificial neural network (ANN). Sheikholeslami et al. [17] worked on the Lattice Boltzmann method to study the magneto-hydrodynamic flow using $\mathrm{Cu}-\mathrm{water}$ nanofluid. Fang [19] presented MHD flow above a nonlinearly affecting surface. Devi and Suriyakumar [20] combined the properties of magnetic field on the Blasius and Sakiadis flow composed by Cu and Al_2O_3 nanoparticles. The effects of flow parameters in the flow model on the velocity, high temperature, skin friction coefficient, and local heat transfer were explained comprehensively.

The second generation of nanofluids titled as hybrid nanofluid took much attention of the researchers and scientists due to their ultra-high thermal performance rate. These are extensively used in cooling systems and for other industrial and technological purposes. In the view of extensive uses of such fluids, Rashidi et al. [21] reported the energy transport mechanism in the hybrid nanofluid. The study is organized in lid driven cavity of square shaped. In ordered to enhance the energy efficiency, they plugged the influences of mixed convection on the square boundaries and reported the significant results regarding the heat transport mechanism. The analysis of non-Newtonian fluids has their own importance in various industries. In this regard, a study is conducted by Nazari et al. [22]. They reported that, by enhancing the volumetric fraction and Darcy number, the heat transport rate rises.

The investigation of two-phase nanofluid flow synthesized by hafnium nanoparticles in the presence of slip effects is discussed by Ellahi et al. [23]. The model is tackled analytically and plotted the results for particles phase and fluid phase with a comprehensive detail. A study regarding the heat transfer by considering multiple nanomaterials (MWCNTs, Cu, and Al_2O_3) in cavity is explored by Goodarzi et al. [24]. The results are obtained against various aspect ratios under the influence of natural convection and conductive heat transport. Mixed convection which is a combination of two physical phenomena known as natural and forced convection significantly alters the fluid behaviour and its temperature. In 2020, Yousefzadeh et al. [25] analyzed the fluid dynamics under the impacts of mixed convection by taking various heat transfer areas.

2. Description of the Problem

The study of heat transfer over a stretchable surface is organized for two different generation of the fluids known as nanofluid (Cu–H_2O) and hybrid nanofluid (Cu–Al_2O_3–H_2O). The sheet is situated in the Cartesian frame. The following assumptions are made during the analysis of aforementioned fluids:

(i) The flow is steady, laminar, and incompressible
(ii) The surface is stretchable in unidirectional
(iii) The flow is two dimensional
(iv) The guest nanomaterial (Cu-Al_2O_3) and the host liquid (H_2O) are thermally compatible
(v) The surface is maintained at temperature T_{w} and velocity U_{w}
(vi) The effects of thermal radiations, combined convection, and magnetic field are taken during the study
(vii) The surface is stretched with the velocity $Uw = ax$ and the temperature and velocity asymptotically vanish at extreme position of the surface

Figure 1 elaborates the flow configuration of the fluids.

The basic constitutive relations for the particular flow are described by the following equations [26]:

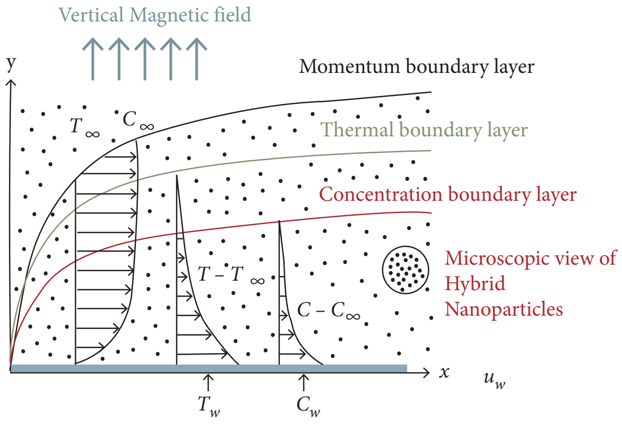


FIGURE 1: Flow configuration and geometrical coordinates.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$\rho_{hnf}\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\mu_{hnf}\frac{\partial^2 u}{\partial y^2}+g(\rho\beta)_{hnf}(T-T_\infty) - \sigma_{hnf}B_0^2u, \tag{2}$$

$$\left(\rho C_p\right)_{hnf}\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=K_{hnf}\frac{\partial^2 T}{\partial y^2}-\frac{\partial q_r}{\partial y}. \tag{3}$$

Appropriate boundary conditions are presented as

$$\begin{aligned} &\text{at } y=0,\\ &u=U_w=ax,\\ &v=v_0,\\ &T=T_w,\\ &\text{at } y\longrightarrow\infty.u\longrightarrow 0,\ T\longrightarrow T_\infty. \end{aligned} \tag{4}$$

For the particular flow configuration, the velocity components u, v and the stream function are defined as $u=\partial\psi/\partial y$ and $v=-\partial\psi/\partial x$ wherever ψ represents stream function. Hence, we get the values of u, v as below:

$$\begin{aligned} u&=axF'(\eta),\\ v&=-\sqrt{av_f}F(\eta). \end{aligned} \tag{5}$$

In equations (2) and (3), ρ_{hnf}, $(\rho C_p)_{hnf}$ symbolizes the density and heat capacity, μ_{hnf} denotes the dynamic viscosity, g represents the gravity acceleration, and β_{hnf} is thermal expansion. Now, for the hybrid nanofluids, the expression for ρ_{hnf}, $(\rho C_p)_{hnf}$, and $(\rho\beta)_{hnf}$ are reported as

$$\begin{aligned} \rho_{hnf}&=\left\{(1-\phi_2)(1-\phi_1)\rho_f+(1-\phi_2)\phi_1\rho_{s1}\right\}+\phi_2\rho_{s2},\\ \left(\rho C_p\right)_{hnf}&=(1-\phi_2)\left\{(1-\phi_1)\left(\rho C_p\right)_f+\phi_1\left(\rho C_p\right)_{s1}\right\}\\ &\quad+\phi_2\left(\rho C_p\right)_{s2},\\ (\rho\beta)_{hnf}&=\left\{(1-\phi_1)(1-\phi_2)(\rho\beta)_f+\phi_1(1-\phi_2)(\rho\beta)_{s1}\right\}\\ &\quad+\phi_2(\rho\beta)_{s2},\\ \frac{\sigma_{hnf}}{\sigma_{bf}}&=\frac{\sigma_{s2}+2\sigma_{bf}-2\phi_2\left(\sigma_{bf}-\sigma_{s2}\right)}{\sigma_{s2}+2\sigma_{bf}+\phi_1\left(\sigma_{bf}-\sigma_{s2}\right)}. \end{aligned} \tag{6}$$

The dynamic viscosity and thermal conductivity of hybrid nanofluid are described as

$$\begin{aligned} \frac{\mu_{hnf}}{\mu_f}&=\frac{1}{(1-\phi_1)^{2.5}(1-\phi_2)^{2.5}},\\ K_{hnf}&=K_f\left[\frac{\left(\left(K_{s2}+2A_2\right)-\phi_2\left(A_2-K_{s2}\right)\right)}{\left(\left(K_{s2}+2A_2\right)+\phi_2\left(A_2-K_{s2}\right)\right)}\right],\\ A_2&=\frac{\left(\left(K_{s1}+k_f\right)-2\phi_1\left(K_f-K_{s1}\right)\right)}{\left(\left(K_{s1}+2f\right)+\phi_1\left(K_f-K_{s1}\right)\right)}. \end{aligned} \tag{7}$$

Here, ϕ_1 and ϕ_2 represent the fraction factor of the used nanomaterials, respectively. Furthermore, K_f, K_{hnf} are the thermal conductivities of the host and hybrid nanoliquid, respectively. Suppose that the velocity and temperature of the stretching sheet are described as

$$\begin{aligned} U_w(x,0)&=ax,\\ T_w(x,0)&=T_\infty+bx, \end{aligned} \tag{8}$$

where a, b, and c are constants. The dimensionless stream function F and dimensionless temperature θ are described as

$$\begin{aligned} \eta&=\sqrt{\frac{a}{v_f}}y,\\ \psi&=\sqrt{av_f}\,xF(\eta),\\ \theta(\eta)&=\frac{T-T_\infty}{T_w-T_\infty},\\ q_r&=-\left(\frac{4\sigma^*}{3k^*}\right)\frac{\partial T^4}{\partial y}, \end{aligned} \tag{9}$$

where η represents the similarity variable. Furthermore, $\psi(x,y)$ is the stream function and q_r denotes the radiative heat flux. Finally, the following hybrid-nanoliquid flow model is obtained after incorporating the effective empirical correlations and similarity equations:

$$F'''(\eta) - (1-\phi_1)^{2.5}(1-\phi_2)^{2.5}\left[\left\{(1-\phi_2)\left((1-\phi_1)+\phi_1\frac{\rho_{s1}}{\rho_f}\right)+\phi_2\frac{\rho_{s2}}{\rho_f}\right\}\left\{(F'(\eta))^2 - F(\eta)F''(\eta)\right\}\right]$$

$$-(1-\phi_1)^{2.5}(1-\phi_2)^{2.5}\left\{\frac{\sigma_{s2}+2\sigma_{bf}-2\phi_2(\sigma_{bf}-\sigma_{s2})}{\sigma_{s2}+2\sigma_{bf}+\phi_1(\sigma_{bf}-\sigma_{s2})}\right\}, \tag{10}$$

$$MF'(\eta) + (1-\phi_1)^{2.5}(1-\phi_2)^{2.5}\left\{(1-\phi_2)\left((1-\phi_1)+\phi_1\frac{(\rho\beta)_{s1}}{(\rho\beta)_f}\right)+\phi_2\frac{(\rho\beta)_{s2}}{(\rho\beta)_f}\right\}\lambda\theta(\eta) = 0,$$

$$\frac{1}{\mathrm{Pr}}\frac{\left(K_{hnf}/K_{bf} + Rd\left(1+(\theta_w-1)\right)^3\right)}{(\rho C_p)_{hnf}}\theta''(\eta) + F(\eta)\theta'(\eta) = 0. \tag{11}$$

The transformed boundary conditions become

$$\begin{aligned} F(0) &= 0,\\ \theta(0) &= 1,\\ F'(0) &= 1,\\ F'(\eta) &\longrightarrow 0,\\ \theta(\eta) &\longrightarrow 0, \text{ at } \eta \longrightarrow \infty, \end{aligned} \tag{12}$$

$$\begin{aligned} C_f &= \frac{2U_w^{-2}\tau_w}{\rho_f},\\ Nu_x &= \frac{xq_w(T_w-T_\infty)^{-1}}{K_f}. \end{aligned} \tag{13}$$

Here, Re is denoted the Reynolds number, Gr is denoted the Grashof number, and λ represents the buoyancy parameter given by the following expressions:

$$\begin{aligned} Re_x &= U_w x v_f^{-1},\\ Gr_x &= g\beta_f(T_w-T_\infty)v_f^{-2}x^3,\\ \lambda &= Gr_x Re_x^{-2}. \end{aligned} \tag{14}$$

Moreover, $Pr = v_f/\propto_f$ denotes Prandtl number and $M = \sigma_f B_0^2/a\rho_f$ is the magnetic interaction parameter for the hybrid nanofluids. The skin friction coefficient is symbolized C_f and heat transfer capacity that, said in the Nusselt number Nu_x, explains the following.Wherever τ_w is represented the shear stresses and heat flux is denoted q_w,

$$\begin{aligned} \tau_w &= \mu_{hnf}\left(\frac{\partial u}{\partial y}\right)_{y=0},\\ q_w &= -K_{hnf}\left(\frac{\partial T}{\partial y}\right)_{y=0}. \end{aligned} \tag{15}$$

Applying transformation in the nondimensional expressions, we obtained the following expressions:

$$\begin{aligned} C_f Re_x^{1/2} &= 2\left((1-\phi_1)^{-2.5}(1-\phi_2)^{-2.5}\right)F''(0),\\ \frac{Nu_x}{Re_x^{1/2}} &= -\left(\frac{K_{hnf}}{K_{bf}}\right)\theta'(0). \end{aligned} \tag{16}$$

3. Mathematical Analysis

Many physical phenomena can be modeled as a system of highly coupled nonlinear differential equations over a bounded or semi-infinite regions. Actually, such models are very tedious due to high nonlinearity and impossible to tackle in the form of closed solution. However, numerical techniques are reliable under such circumstances. Therefore, RK technique and its coupling with the shooting method is applied on the model under consideration. The model is described in (10) and (11) along with conditions defined over the surface and away from it given in equation (12). Actually, aforementioned technique works for the system of first-order ODEs. In this regard, firstly, we fixed the following transformations to get the desired system:

$$(y_1, y_2, y_3, y_4, y_5, y_6) = \left(F, F'', F''', \theta, \theta', \theta''\right). \tag{17}$$

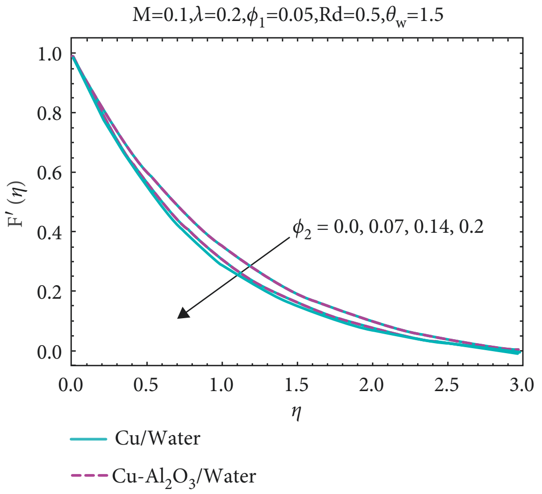


Figure 2: Effect of ϕ_2.

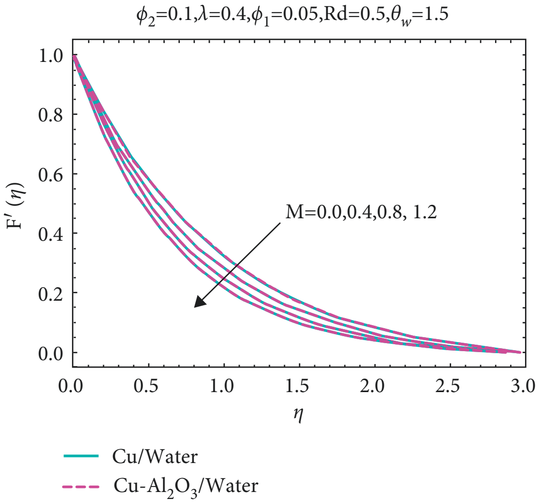


Figure 3: Effect of M.

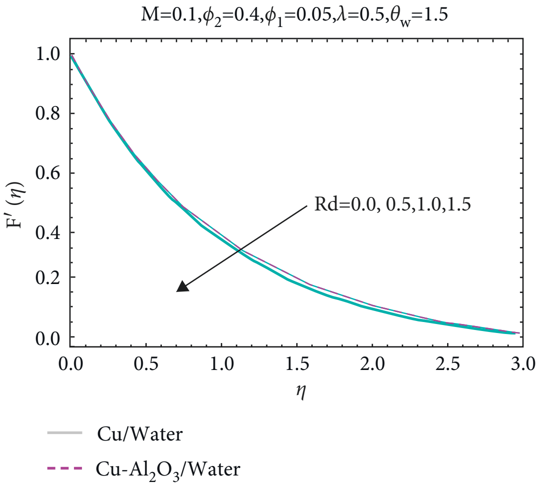
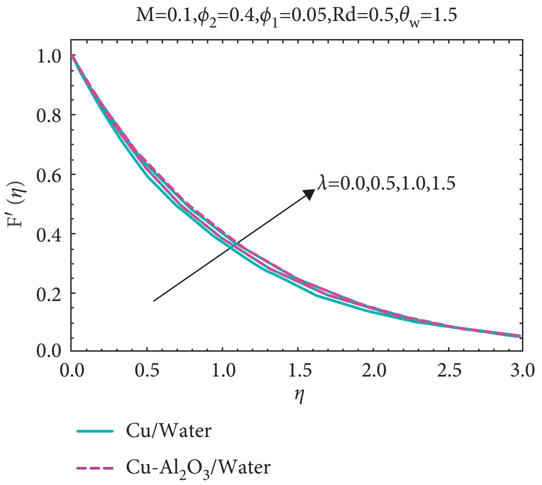
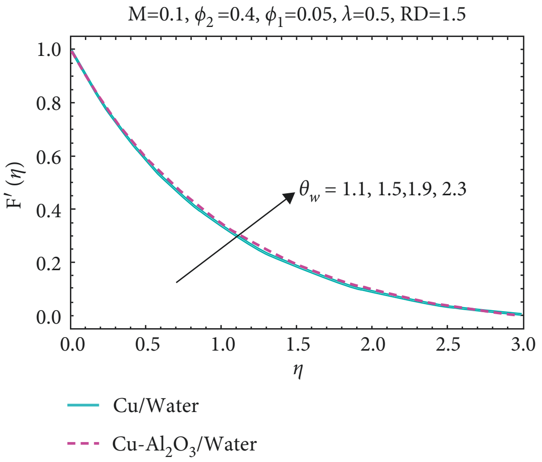


Figure 4: Effect of Rd.



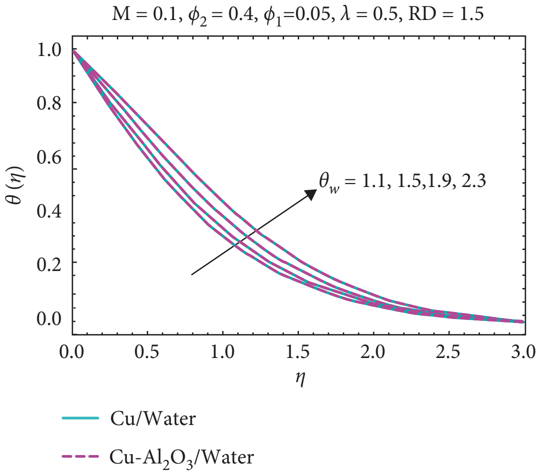
$M=0.1, \phi_2=0.4, \phi_1=0.05, Rd=0.5, \theta_w=1.5$

FIGURE 5: Effect of λ.



$M=0.1, \phi_2=0.4, \phi_1=0.05, \lambda=0.5, RD=1.5$

FIGURE 6: Effect of. θ_w



$M = 0.1, \phi_2 = 0.4, \phi_1=0.05, \lambda = 0.5, RD = 1.5$

FIGURE 7: Effect of θ_w.

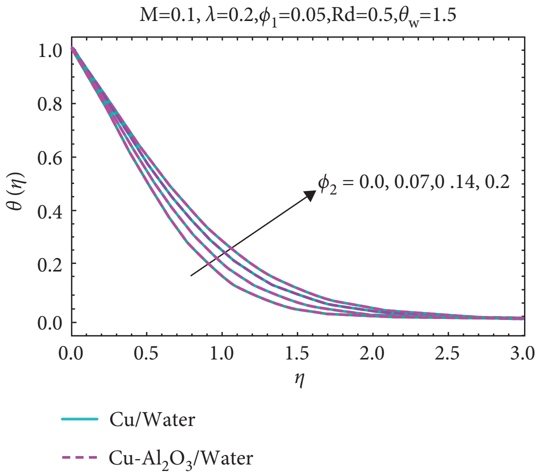


Figure 8: Effect of ϕ_2.

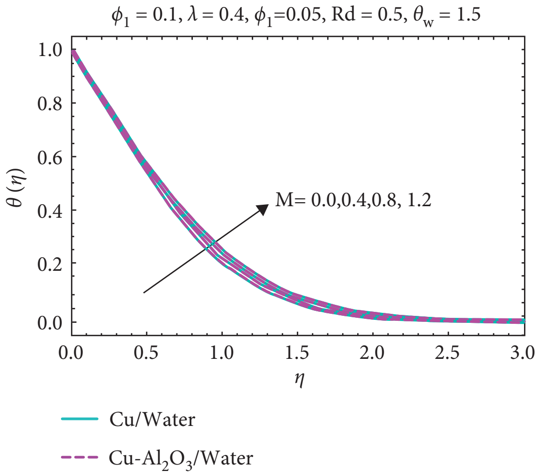


Figure 9: Effect of M.

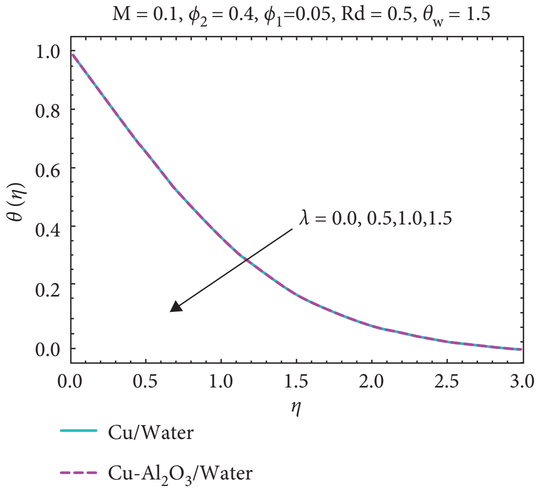


Figure 10: Effect of. λ.

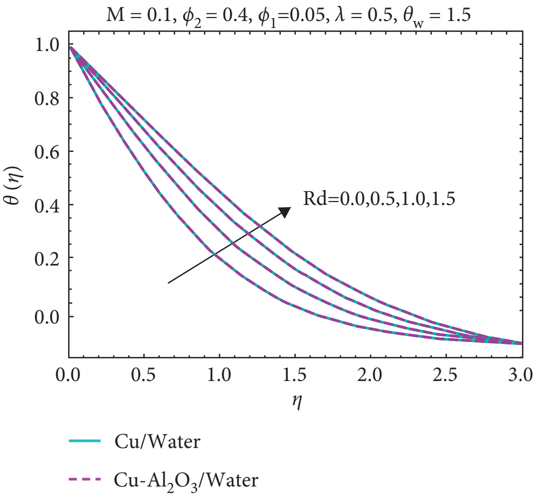


Figure 11: Effect of Rd.

By inducing these transformations in the hybrid model defined in (10) and (11), the following version is obtained:

$$y_2' = (1-\phi_1)^{2.5}(1-\phi_2)^{2.5}\Bigg[\bigg\{(1-\phi_2)\bigg((1-\phi_1)+\phi_1\frac{\rho_{s1}}{\rho_f}\bigg)+\phi_2\frac{\rho_{s2}}{\rho_f}\bigg\}.\{(y_1')^2 - y_1y_2'\}\Bigg]$$
$$+(1-\phi_1)^{2.5}(1-\phi_2)^{2.5}\Bigg(\frac{\sigma_{s2}+2\sigma_{bf}-2\phi_2(\sigma_{bf}-\sigma_{s2})}{\sigma_{s2}+2\sigma_{bf}+\phi_1(\sigma_{bf}-\sigma_{s2})}My_1' - \bigg\{(1-\phi_2)\bigg((1-\phi_1)+\phi_1\frac{(\rho\beta)_{s1}}{(\rho\beta)_f}\bigg)+\phi_2\frac{(\rho\beta)_{s2}}{(\rho\beta)_f}\bigg\}\lambda y_4\Bigg), \tag{18}$$

$$y_5' = -\frac{\{(1-\phi_1)(1-\phi_2)+\phi_1(1-\phi_2)(\rho C_p)_{s1}/(\rho C_p)_f + \phi_2(\rho C_p)_{s2}/(\rho C_p)_f\}}{\big(K_{hnf}/K_{bf}+Rd(1+(\theta_w-1))^3\big)}Pr\{y_1y_1'\}. \tag{19}$$

The model described in (18) and (19) is a desired system on which the proposed numerical technique is applicable. For said purpose, MATHEMATICA 10.0 code is generated for the model and plotted the results for the preeminent flow quantities.

4. Results and Discussion

This section is fixed to explore the results for the velocity and temperature behaviour of the hybrid as well conventional nanofluid. The results are decorated against the various preeminent flow parameters over the desired region.

M is Magnetic parameter, λ is the buoyancy parameter, and Rd is the radiation parameter. Measure two dissimilar kinds of nanoparticles, Cu and Al_2O_3. The boundary-layer viscosity rises with the rise in velocity gradient and the temperature gradient. Performances of resistance force such as density and thickness of the hybrid nanofluid; as a result, the particle viscosity and speed rise by the growth in other production follow. The volume of nanoparticles increases the thermal conductivity rises, as the case of this is increases temperature.

Figure 2 shows the influence of ϕ_2 on mix nanoparticles $Cu - Al_2O_3/H_2O$ in the velocity profile.

At that time, velocity decreases as the volume fraction increases. The presence of dense nanoparticles info to other weakening the velocity boundary-layer thickness.

Figure 3 represents the velocity profile which the magnetic parameter effect of hybrid nanofluid. Because of the description, velocity of hybrid nanofluid decreases with the varying of M. It obviously proves that right angles' attractive field plays with the transportation occurrences. It is main indication that the great oppositions continuously particles of fluid, which effect to viscosity produced in hybrid nanofluid. Figure 4 presents the velocity profile; the increase of Rd is an increase in the velocity of hybrid nanofluid. Because the viscosity and density decrease in the cause of this is increase the velocity of hybrid nanofluid. Figure 4 shows that the velocity of hybrid nanoparticles increases as the Rd vary. We can say that if varying Rd, then the stretching sheet of the nanoparticles and velocity is abtained. Figures 5 and 6 show that the varying λ and θ_w increase the velocity of hybrid nanofluid $Cu - Al_2O_3/H_2O$. The boundary-layer thickness will be increased.

Table 1: Thermophysical possessions of water hybrid nanofluids in addition nanoparticles [3, 5, 6].

Properties	Pure water H_2O	Alumina (Al_2O_3)	Copper (Cu)
$\rho(kg/m^3)$	997.1	3970	8933
$C_p\,(J/kgk)$	4179	765	385
k $(Wm^{-1}k^{-1})$	0.6130	40	401
$\beta \times 10^5 k^{-1}$	21	0.850	1.670
$\sigma(s/m)$	5.5×10^{-6}	3.5×10^{6}	59.6×10^{6}

Figures 7 and 8 appear that the temperature profiles θ_w and ϕ_2 are varying; then, the heat transfer increases. $\theta_w > 1$ Then, the given solution is nonlinear and plot will nonlinear plot. In this case, the solution of equation is nonlinear. Figure 8 calculates the thermal conductivity increases, and therefore, the thermal boundary-layer thickness rises, as the nanoparticle volume fraction increases. This case is in submission with the main proposes of using hybrid nanofluid and furthermore approves by physical performance; after the size increase of nanoparticles, then k_{hnf} and thermal boundary-layer viscosity rise. Figure 9 shows temperature circulation, and the heat transfer rises with rise in the attractive factor due to description; the temperature boundary-layer thickness rises. It observably shows that the right angles attractive field be pitted against the moving occurrences.

Figure 10 shows that decreasing of buoyancy parameter changes the hotness transfer of the hybrid nanoparticles which represent the temperature profile; however, increasing λ, the heat decreasing because the thermal conductivity is reduction. If thermal conductivity reduced, then thermal boundary layer reduced as well. Figure 11 clearly describes that the radiation Rd increases as well as increasing the heat transfer. Table 1 presents the thermophysical values of the base liquid and nanoparticles.

5. Conclusions

This work reported the study of heat transport phenomena in Cu-Al_2O_3/H_2O hybrid nanofluid by taking the effects of thermal radiations, magnetic field, and varying volumetric fraction. The flow situation is modeled over a stretched surface via similarity equations. The resultant hybrid model is accommodated via numerical technique and furnished the results against the parameters. Form the analysis, it is examined that [18]

(i) The velocity of Cu-Al_2O_3/H_2O declines against higher volumetric fraction of Cu-Al_2O_3 and stronger Lorentz forces

(ii) The fluid temperature significantly augmented for θ_w, and it vanishes asymptotically far from the surface

(iii) The velocity rises due to mixed convection effects λ, and it opposes the fluid temperature.

(iv) Thermal radiations worked as catalysis in the study regarding thermal transport which prominently played positive role

Data Availability

The study is based on numerical technique and no data were used in findings of the study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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