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Research Article

MULTIMOORA Method for Addressing Security Algorithms Evaluation Problem under *q*-Rung Orthopair Fuzzy Environment

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How to determine a suitable security algorithm for a special application scenario is a complex problem. In this paper, this complex problem is formulated as a multicriteria decision-making (MCDM) problem, and we propose a novel MULTIMOORA (multiobjective optimization on the basis of a ratio analysis plus the full MULTIpevaluation information in the security algorithms evaluation problem. The MULTIMOORA method is an excellent decision method, which owns strong robustness. However, it has not been used to process the complex information structure of q-rung orthopair fuzzy sets. Moreover, it cannot solve the problem that the extreme values negatively influence the ranking results, and it also cannot capture the interrelationship hiding behind the criteria. To overcome the above challenges, we propose novel q-rung orthopair fuzzy **Dombi power Heronian mean** (DPHM) operator and q-rung orthopair fuzzy **Dombi power geometric Heronian mean** (DPGHM) operator. Based on these two operators, the MULTIMOORA method is improved for solving the security algorithms' evaluation problem. Finally, a practical example for evaluating five security algorithms is used to illustrate the decision process of the proposed q-rung orthopair fuzzy MULTIMOORA method.

1. Introduction

With the quick development of multiple information technologies including cloud computing, Internet of Things, and edge computing [1], more and more companies and personals choose to upload their private data to the network [2]. However, as the scale of network becomes larger, the whole network becomes more complicated [3]. The network shows massive security loophole [4]. The companies and personals also own the special software to improve their business. The software also has massive security loophole and risks. To ensure the reliability of software and network, researchers and scholars have provided some solutions. For example, Abdel-Basset et al. [5] have put forward a neutrosophic decision-making model for evaluating the e-government website according to the quality, security, and accessibility. Wang et al. [6] have combined the TOPSIS (technique for order of preference by similarity to ideal solution) approach with the 0-1 integer programming method to choose an intelligent web service for improving the reliability of network.

Researchers and scholars also designed a number of efficient security algorithms to ensure the security requirements of network [7–9]. However, these security algorithms usually own different characteristics and advantages [10]. For a special application scenario, a suitable security algorithm should be selected for satisfying the requirements of this application scenario. How to choose the most suitable security algorithm for a special application scenario is a big challenge. To address this problem, Ning et al. [11] formulated this problem as a multicriteria decision-making (MCDM) problem and proposed a hybrid model for selecting the best encryption algorithm according to several requirements such as the performance, physical, and security. However, the study [11] still has some shortcomings.

(1) In the study [11], crisp values are used to evaluate security algorithms. Since the security algorithm evaluation problem becomes more and more complex, it is not easy for decision makers to use accurate crisp values for evaluating security algorithms [13]. The birth of fuzzy sets (FSs) [14] provides decision makers with a new way to express uncertain evaluation information. However, FSs only describe the membership degree (MD) information. To enhance the uncertain information modeling capability, intuitionistic fuzzy sets (IFSs) [15] were proposed to express the MD and nonmembership degree (NMD) information. In IFSs, the sum of MD and NMD values is not larger than 1. To provide the decision makers with more freedom for expressing the evaluation information, the concept of Pythagorean fuzzy sets (PFSs) was proposed by Yager and Abbasov [16], where the square sum of MD and NMD is not larger than 1. To generalize the concepts of IFSs and PFSs, a generic version, called q-rung orthopair fuzzy set (q-ROFS), was proposed by Yager [17]. In this study, we intend to use q-ROFSs to express the uncertain information. The significance of q-ROFSs is that this information representation way is flexible, and it provides the decision makers with more freedom than PFSs and IFSs.

(2) q-ROFSs have attracted many researchers since its birth. For example, linguistic q-ROFSs [18–20] and interval-valued q-ROFSs [21] are the qualitative and uncertain versions of q-ROFSs. To fuse q-ROFSs' information, various aggregation operators have been proposed [22–28], such as Archimedean Bonferroni mean operators [22], partitioned Bonferroni mean operators [23], Heronian mean operators [24], Maclaurin symmetric mean operators [25, 26], Hamy mean operators [27], and Choquet integral operators [28]. They are the value measurement MCDM methods [29–31], which do not consider the distance between each criterion value and maximum criterion value.

(3) As one of the efficient decision methods, the MULTIMOORA (multiobjective optimization on the basis of a ratio analysis plus the full MULTIplicative form) method [32] consists of three submodels for comprehensively determining the decision results. As shown in Table 1, the decision results that are obtained from the MULTIMOORA method are robust and the MULTIMOORA method outperforms than some other decision methods [12]. Because of its excellent characteristics, the MULTIMOORA method has been used to process various evaluation information, such as interval numbers [33], IFSs [34], picture fuzzy sets [35], and probabilistic linguistic term sets [36]. To the best of our knowledge, there have been no research results on the combination of q-ROFSs and MULTIMOORA method to date. In this paper, we intend to extend the MULTIMOORA method for processing the q-ROFS information in the MCDM problems. Nevertheless, the MULTIMOORA method cannot handle the case that extreme values influence the reliability of the decision results. Moreover, it is incapable of processing the complex interrelationships hiding behind criteria values.

Hence, the motivations of this study are summarized as

(1) A more flexible way of q-ROFSs is used to express the uncertain and vague evaluation information for the security algorithms evaluation problems

(2) A novel decision-making method is developed to solve the security algorithms evaluation problems and select an appropriate algorithm for a special application scenario

To overcome the challenges, a novel q-rung orthopair fuzzy MULTIMOORA method based on Dombi power Heronian mean aggregation operators is proposed in this paper, and it is applied to solve the security algorithms' evaluation problem.

(1) The Dombi operational laws, special forms of t-norms and t-conorms, show strong flexibility when computing input values. The power average (PA) operator has the ability of alleviating negative influences of extreme input values on the decision results. The Heronian mean (HM) acts as a mapping function that can capture the complex interrelationships among input values. Considering the excellent characteristics, in this paper, some Dombi power Heronian mean aggregation operators are proposed to fuse q-rung orthopair fuzzy numbers (q-ROFNs), which are q-rung orthopair fuzzy Dombi power Heronian mean (q-ROFDPHM) operator and q-rung orthopair fuzzy Dombi power geometric Heronian mean (q-ROFDPGHM) operator, as well as their weighted forms. Afterwards, their features are discussed.

(2) The weighted forms of the q-ROFDPHM and q-ROFDPGHM operators are applied to improve the MULTIMOORA method so that a novel q-rung orthopair fuzzy MULTIMOORA method is put forward for handling the security algorithms' evaluation problem. After that, the detailed decision-making procedure of the proposed q-rung orthopair fuzzy MULTIMOORA method is provided.

(3) A case concerning the evaluation of five security algorithms is provided to show the implementation processes of the proposed q-rung orthopair fuzzy MULTIMOORA method. Afterwards, the influences of the parameters on the ranking results are analyzed. Then, the q-rung orthopair fuzzy MULTIMOORA method is compared with the existing decision methods that handle the q-ROFS information.

Table 1: Performance comparison among decision-making methods [12].

Methods	Computational time	Simplicity	Mathematical calculations	Stability
MULTIMOORA	Very less	Very simple	Minimum	Good
TOPSIS	Moderate	Moderately critical	Moderate	Poor
VIKOR	Less	Simple	Moderate	Medium
ELECTRE	High	Moderately critical	Moderate	Medium

The rest content of this paper is organized as follows. The basic knowledge of q-rung orthopair fuzzy sets, PA, Dombi T-conorm and T-norm, HM operator, and MULTIMOORA method is provided in Section 2. In Section 3, the q-ROFDPHM operator and its weighted form are put forward. Section 4 puts forward the q-ROFDPGHM operator and its weighted form. In Section 5, we apply the proposed operators to propose a novel q-rung orthopair fuzzy MULTIMOORA method and also present the decision procedure. In Section 6, an illustrative example of evaluating of security algorithms is provided to show the implementation process of the proposed q-rung orthopair fuzzy MULTIMOORA method. In Section 7, some valuable conclusions are listed.

2. Preliminaries

In this paper, the basic information of q-ROFSs, PA, Dombi T-conorm and T-norm, HM operator, and MULTIMOORA method is provided.

2.1. q-Rung Orthopair Fuzzy Sets. The concept of q-ROFSs was proposed based on IFSs and PFSs. The q-ROFSs show higher flexibility and larger value range than IFSs and PFSs [37–39].

Definition 1 (see [17]). Let $X = \{x_1, x_2, \ldots, x_n\}$ be a finite universe of discourse (UoD); then, a q-ROFS A on X is mathematically expressed as

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in X\}, \tag{1}$$

where $\mu_A: X \longrightarrow [0,1]$ and $\nu_A: X \longrightarrow [0,1]$ are the membership degree (MD) and nonmembership degree (NMD) of the element x belonging to the q-ROFS A, respectively. The constraint conditions for q-ROFS are $0 \le \mu_A(x) \le 1$, $0 \le \nu_A(x) \le 1$, and $0 \le \mu_A^q(x) + \nu_A^q(x) \le 1$, for all $q \ge 1$. The parameter q is a positive integer. The value of $\pi_A(x) = \sqrt[q]{(1 - (\mu_A(x))^q - (\nu_A(x))^q)}$ is defined to be the hesitant degree (HD) of the element x belonging to the q-ROFS A. For convenience, the two-tuple $(\mu_A(x), \nu_A(x))$ is simplified as (μ_A, ν_A), which is also called q-rung orthopair fuzzy number (q-ROFN) by Liu and Wang [40].

For comparing q-ROFNs, the definitions of score function and accuracy function were given by Liu and Wang [40] for q-ROFNs as follows.

Definition 2 (see [40]). Given a q-ROFN $o = (\mu, \nu)$, then its score function and accuracy function are defined as $s(o) = \mu^q - \nu^q$ and $h(o) = \mu^q + \nu^q$, in which $s(o) \in [-1, 1]$ denotes the score function and $h(o) \in [0, 1]$ is the accuracy function.

Based on the above score function and accuracy function presented in Definition 2, Liu and Wang [40] gave a method for comparing two q-ROFNs as follows.

Definition 3 (see [40]). Given two q-ROFNs $o_1 = (\mu_1, \nu_1)$ and $o_2 = (\mu_2, \nu_2)$, $s(o_1)$ and $s(o_2)$ are their score function values, and $h(o_1)$ and $h(o_2)$ are their accuracy function values,

(1) If $s(o_1) > s(o_2)$, then it can be considered that $o_1 > o_2$.

(2) If $s(o_1) = s(o_2)$, then their accuracy function values should be further compared as follows:

  (1) If $h(o_1) > h(o_2)$, then $o_1 > o_2$.

  (2) If $h(o_1) = h(o_2)$, then $o_1 = o_2$.

To measure the deviation degree between any two q-ROFNs, Liu et al. [41] provided the definition of distance between them as follows.

Definition 4 (see [41]). For q-ROFNs, $o_1 = (\mu_1, \nu_1)$ and $o_2 = (\mu_2, \nu_2)$, the distance between them is computed as

$$d(o_1, o_2) = \frac{1}{2}\left(\left|\mu_1^q - \mu_2^q\right| + \left|\nu_1^q - \nu_2^q\right| + \left|\pi_1^q - \pi_2^q\right|\right), \tag{2}$$

where π_1 and π_2 are the HD values of q-ROFNs o_1 and o_2, respectively.

2.2. Power Average Operator. The power average (PA) is a useful aggregation operator that was put forward by Yager [42]. The PA operator has the ability of alleviating negative influences of extreme input values on the calculation results. The original PA operator was devised to process crisp values. Its mathematical definition is given as follows:

Definition 5 (see [42]). Let $o_i\,(i = 1, 2, \ldots, n)$ be a series of nonnegative crisp values; then, the PA operator really acts as a function that

$$\mathrm{PA}(o_1, o_2, \ldots, o_n) = \sum_{i=1}^{n}\left(\frac{(1 + S(o_i))}{\sum_{k=1}^{n}(1 + S(o_k))} o_i\right), \tag{3}$$

where $S(o_i) = \sum_{j=1, j\ne i}^{n} \mathrm{Sup}(o_i, o_j)$ is the support degree for o_i from o_j and $\mathrm{Sup}(o_i, o_j) = 1 - d(o_i, o_j)$.

The support degree satisfies the following features:

(1) $\mathrm{Sup}(o_i, o_j) \in [0, 1]$

(2) $\mathrm{Sup}(o_i, o_j) = \mathrm{Sup}(o_j, o_i)$

(3) If $d(o_i, o_j) < d(o_r, o_k)$, then $\mathrm{Sup}(o_i, o_j) > \mathrm{Sup}(o_r, o_k)$, where $d(o_i, o_j)$ denotes the distance between o_i and o_j

2.3. Dombi T-Norm and T-Conorm. The Dombi T-norm (TNM) and T-conorm (TCNM), which were proposed by Dombi [43], are referred to as special forms of t-norms and t-conorms. Their mathematical expressions are provided as follows.

Definition 6 (see [43]). Given any two real values, m and n, then the Dombi TNM and Dombi TCNM act as two functions, which are mathematically defined as

$$D(m,n) = \frac{1}{1+\left(((1-m)/m)^{\aleph}+((1-n)/n)^{\aleph}\right)^{(1/\aleph)}},$$
$$D^{*}(m,n) = 1-\frac{1}{1+\left((m/(1-m))^{\aleph}+(n/(1-n))^{\aleph}\right)^{(1/\aleph)}}, \tag{4}$$

where $\aleph > 0$, $m, n \in [0,1]$.

Based on the above Dombi TNM and Dombi TCNM, Jana et al. [44] gave the Dombi operational laws for computing q-ROFNs as follows.

Definition 7 (see [44]). Given two q-ROFNs $o_1 = (\mu_1, \nu_1)$ and $o_2 = (\mu_2, \nu_2)$, then the Dombi operational laws of q-ROFNs are defined as

(1) $o_1 \oplus o_2 = ((1-(1/(1+((\mu_1^q/(1-\mu_1^q))^{\aleph}+(\mu_2^q/(1-\mu_2^q))^{\aleph})^{(1/\aleph)})))^{(1/q)}, (1/(1+(((1-\nu_1^q)/\nu_1^q)^{\aleph}+((1-\nu_2^q)/\nu_2^q)^{\aleph})^{(1/\aleph)}))^{(1/q)})$

(2) $o_1 \otimes o_2 = ((1/(1+(((1-\mu_1^q)/\mu_1^q)^{\aleph}+((1-\mu_2^q)/\mu_2^q)^{\aleph})^{(1/\aleph)}))(1/q), (1-(1/(1+((\nu_1^q/(1-\nu_1^q))^{\aleph}+(\nu_2^q/(1-\nu_2^q))^{\aleph})^{(1/\aleph)})))^{(1/q)})$

(3) $lo_1 = ((1-(1/(1+(l(\mu_1^q/(1-\mu_1^q))^{\aleph})^{(1/\aleph)})))^{(1/q)}, (1/1+(l((1-\nu_1^q)/\nu_1^q)^{\aleph})^{(1/\aleph)})^{(1/q)})$

(4) $o_1^l = ((1/(1+(l((1-\mu_1^q)/\mu_1^q)^{\aleph})^{(1/\aleph)}))^{(1/q)}, (1-(1/(1+(l(\nu_1^q/(1-\nu_1^q))^{\aleph})^{(1/\aleph)})))^{(1/q)})$, where $\aleph > 0$

2.4. Heronian Mean and Geometric Heronian Mean Operators. The aggregation operators (AOs) [45–47] are value measurement MCDM methods. It is very simple and easy to perform AOs. The AOs are the processes, which fuse given input values into a single value [48]. For aggregating the complicated information structures of various fuzzy sets, researchers have put forward various AOs. The Heronian mean (HM) operator [49], an excellent and useful AO, is capable of processing the complicated interrelationships among input values, which are common in the MCDM contexts. The HM operators can be divided into two categories: arithmetic HM (AHM) and geometric HM (GHM) operators, which are mathematically defined as follows.

Definition 8 (see [49]). Let $o_i\,(i = 1, 2, \ldots, n)$ be a series of nonnegative real values; the parameters γ, $\eta \geq 0$; then, the AHM operator can aggregate the nonnegative real values as

$$\mathrm{HM}^{\gamma,\eta}(o_1, o_2, \ldots, o_n) = \left(\frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=i}^{n} o_i^{\gamma} o_j^{\eta}\right)^{(1/(\gamma+\eta))}. \tag{5}$$

Definition 9 (see [49]). Let $o_i\,(i = 1, 2, \ldots, n)$ be a series of nonnegative real values; the parameters γ, $\eta \geq 0$; then, the GHM operator can aggregate the nonnegative real values as

$$\mathrm{GHM}^{\gamma,\eta}(o_1, o_2, \ldots, o_n) = \frac{1}{\gamma+\eta}\prod_{i=1}^{n}\prod_{j=i}^{n}\left(\gamma o_i + \eta o_j\right)^{(2/n(n+1))}. \tag{6}$$

For the AHM operator, its aggregated values are greatly influenced by extreme values [50]. The GHM operator is capable of balancing the big differences among input values [51]. Therefore, the GHM operator performs better than the AHM operator in some cases.

2.5. MULTIMOORA. To obtain more robust decision results, the full multiplicative form (FMF) was applied by Brauers and Zavadskas [32] to extend the initial MOORA (multiobjective optimization on the basis of ratio analysis) method. Thus, the MULTIMOORA method has three components: ratio system (RS) component, reference point (RP) component, and FMF component, respectively [52]. These three components derive the decision results independently. For the purpose of determining the final decision result, the decision results obtained from these three components are processed by the dominance theory [32]. In the following part, the process for implementing the MULTIMOORA method is listed as follows.

Let us suppose that there exists an MCDM problem consisting of m alternatives $\{x_1, x_2, \ldots, x_m\}$ and n criteria $\{a_1, a_2, \ldots, a_n\}$. The weight vector of criteria is denoted as $[\omega_1, \omega_2, \ldots, \omega_n]$, where $\sum_{j=1}^{n}\omega_j = 1$ and $0 \leq \omega_j \leq 1$. The decision matrix (DM) $R = (o_{ij})_{m\times n}$ corresponding to the MCDM problem contains the evaluation information from experts. The element o_{ij} represents the evaluation information of alternative x_i with respect to criterion a_j.

The evaluation information of alternatives across multiple criteria usually shows different dimensions, so the evaluation information in the DM $R = (o_{ij})_{m\times n}$ is suggested to be normalized as

$$\widetilde{o}_{ij} = \frac{o_{ij}}{\sqrt{\sum_{i=1}^{m}\left(o_{ij}\right)^2}}. \tag{7}$$

After that, the normalized DM $\widetilde{R} = (\widetilde{o}_{ij})_{m\times n}$ can be derived.

2.5.1. RS Component. In this component, the criteria should be divided into two categories: benefit-type (BT) criteria and cost-type (CT) criteria. For BT criteria, the larger the evaluation information of alternative, the better the alternative. For CT criteria, the larger the evaluation information of alternative, the worse the alternative. The weighted arithmetic aggregation operator (AAO) is used to calculate the ranking value Y_i of alternative x_i as

$$Y_i = \sum_{j=1}^{k} \omega_j \tilde{o}_{ij} - \sum_{j=k+1}^{n} \omega_j \tilde{o}_{ij}, \tag{8}$$

where k represents the number of benefit-type criteria and $n-k$ means the number of cost-type criteria. From the above equation, it is noted that the alternative in the RS component having the maximum ranking value is considered as the best one. Therefore, the alternatives can be ranked based on the descending order of their ranking values.

2.5.2. RP Component. For the RP component, the worst criterion value of each alternative that is farthest from the reference point of the corresponding criterion should be first derived, and then, the alternative with the smallest worst criterion value is considered as the optimal one.

In this component, the reference point of each criterion is first determined as

$$o_j = \left\{ \max_i \tilde{o}_{ij}, j \le k; \min_i \tilde{o}_{ij}, j > k \right\}, \tag{9}$$

where o_j denotes the reference point of alternatives with respect to criterion a_j.

Then, the weighted distance between the normalized evaluation information of the alternative x_i with respect to each criterion and the reference point of the same criterion is computed as $d_{ij} = \omega_j |o_j - \tilde{o}_{ij}|$.

Finally, the ranking value D_i of alternative x_i is computed as $D_i = \max_j d_{ij}$.

According to the RP component, the optimal alternative should have the smallest ranking value. Thus, the alternatives can be ranked based on the ascending order of their ranking values.

2.5.3. FMF Component. The design idea of FMF component is the same as that of RS component. In the FMF component, the better alternative should have higher values for benefit-type criteria and lower values for cost-type criteria. The weighted geometric aggregation operator (GGO) is used to determine the ranking value U_i of alternative x_i as $U_i = \prod_{j=1}^{k} (\tilde{o}_{ij})^{\omega_j} / \prod_{j=k+1}^{n} (\tilde{o}_{ij})^{\omega_j}$.

According to the design idea, the alternative having the largest ranking value should be considered as the best one in the FMF component. Hence, the alternatives can be ranked based on the descending order of their ranking values.

To aggregate the ranking orders of alternatives obtained from these three components, the dominance theory was suggested by Brauers and Zavadskas [32] to be used for deriving the final decision results.

3. *q*-Rung Orthopair Fuzzy Dombi Power Heronian Mean Operators

In this section, we use the PA operator, Dombi operational laws for q-ROFNs, and arithmetic HM operator to propose q-rung orthopair fuzzy Dombi power HM (q-ROFDPHM) operator and its weighted form. Then, the features are discussed.

Definition 10. Given a set of q-ROFNs $o_i = (\mu_i, \nu_i)\,(i = 1, 2, \ldots, n)$ and three parameters $\gamma, \eta \ge 0$ and $\aleph > 0$, then the q-ROFDPHM operator is defined as

$$q-\mathrm{ROFDPHM}(o_1, o_2, \ldots, o_n) = \left(\frac{2}{n(n+1)} \mathop{\oplus}_{i=1}^{n} \mathop{\oplus}_{j=i}^{n} \left(\left(\frac{n(1+S(o_i))}{\sum_{k=1}^{n}(1+S(o_k))} o_i \right)^{\gamma} \otimes \left(\frac{n(1+S(o_j))}{\sum_{k=1}^{n}(1+S(o_k))} o_j \right)^{\eta} \right) \right)^{(1/(\gamma+\eta))}. \tag{10}$$

Based on the Dombi operational laws of q-ROFNs [36] and HM operator, the following theorems can be derived.

Theorem 1. *Given n q-ROFNs $o_i = (\mu_i, \nu_i)\,(i = 1, 2, \ldots, n)$ and the parameters $\gamma, \eta \ge 0$ and $\aleph > 0$, then the aggregated result derived from equation (10) is still an q-ROFN, which is*

$$\begin{aligned} q-\mathrm{ROFDPHM}(o_1, o_2, \ldots, o_n) = \Bigg(&\left(\frac{1}{1 + (n(n+1)/2(\gamma+\eta))^{(1/\aleph)} \times \left(1/\left(\sum_{i=1,j=i}^{n} \left(1/\left(\gamma t_i a_i^{\aleph} + \eta e_j a_j^{\aleph} \right) \right) \right) \right)^{(1/\aleph)}} \right)^{(1/q)}, \\ &\left(1 - \frac{1}{1 + (n(n+1)/2(\gamma+\eta))^{(1/\aleph)} \times \left(1/\left(\sum_{i=1,j=i}^{n} \left(1/\left(\gamma t_i b_i^{\aleph} + \eta e_j b_j^{\aleph} \right) \right) \right) \right)^{(1/\aleph)}} \right)^{(1/q)} \Bigg), \end{aligned} \tag{11}$$

where $\xi_i = ((1+S(o_i))/\sum_{k=1}^{n}(1+S(o_k)))$, $\psi_j = ((1+S(o_j))/\sum_{k=1}^{n}(1+S(o_k)))$, $(\mu_i^q/(1-\mu_i^q)) = (1/a_i)$, $(\mu_j^q/(1-\mu_j^q)) = (1/a_j)$, $((1-\nu_i^q)/\nu_i^q) = (1/b_i)$, $((1-\nu_j^q)/\nu_j^q) = (1/b_j)$, $(1/n\xi_i) = t_i$, *and* $(1/n\psi_j) = e_j$.

Proof. According to Definition 10, we have

$$q-\mathrm{ROFDPHM}(o_1,o_2,\ldots,o_n) = \left(\frac{2}{n(n+1)}\overset{n}{\underset{i=1}{\oplus}}\overset{n}{\underset{j=i}{\oplus}}\left(\left(\frac{n(1+S(o_i))}{\sum_{k=1}^{n}(1+S(o_k))}o_i\right)^{\gamma}\otimes\left(\frac{n(1+S(o_j))}{\sum_{k=1}^{n}(1+S(o_k))}o_j\right)^{\eta}\right)\right)^{(1/(\gamma+\eta))}. \quad (12)$$

Let $\xi_i = ((1+S(o_i))/\sum_{k=1}^{n}(1+S(o_k)))$ and $\psi_j = ((1+S(o_j))/\sum_{k=1}^{n}(1+S(o_k)))$; then, we can derive

$$q-\mathrm{ROFDPHM}(o_1,o_2,\ldots,o_n) = \left(\frac{2}{n(n+1)}\overset{n}{\underset{i=1}{\oplus}}\overset{n}{\underset{j=i}{\oplus}}\left((n\xi_i o_i)^{\gamma}\otimes(n\psi_j o_j)^{\eta}\right)\right)^{(1/(\gamma+\eta))}. \quad (13)$$

According to Definition 7, it can be derived that

$$\begin{aligned}
n\xi_i o_i &= \left(\left(1-\frac{1}{1+\left(n\xi_i(\mu_i^q/(1-\mu_i^q))^{\aleph}\right)^{(1/\aleph)}}\right)^{(1/q)}, \left(\frac{1}{1+\left(n\xi_i((1-\nu_i^q)/\nu_i^q)^{\aleph}\right)^{(1/\aleph)}}\right)^{(1/q)}\right),\\
n\psi_j o_j &= \left(\left(1-\frac{1}{1+\left(n\psi_j(\mu_j^q/(1-\mu_j^q))^{\aleph}\right)^{(1/\aleph)}}\right)^{(1/q)}, \left(\frac{1}{1+\left(n\psi_j((1-\nu_j^q)/\nu_j^q)^{\aleph}\right)^{(1/\aleph)}}\right)^{(1/q)}\right).
\end{aligned} \quad (14)$$

Let $(\mu_i^q/(1-\mu_i^q)) = (1/a_i)$, $(\mu_j^q/(1-\mu_j^q)) = (1/a_j)$, $((1-\nu_i^q)/\nu_i^q) = (1/b_i)$, and $((1-\nu_j^q)/\nu_j^q) = (1/b_j)$; then, we have

$$\begin{aligned}
n\xi_i o_i &= \left(\left(1-\frac{1}{1+\left(n\xi_i(1/a_i^{\aleph})\right)^{(1/\aleph)}}\right)^{(1/q)}, \left(\frac{1}{1+\left(n\xi_i(1/b_i^{\aleph})\right)^{(1/\aleph)}}\right)^{(1/q)}\right),\\
n\psi_j o_j &= \left(\left(1-\frac{1}{1+\left(n\psi_j(1/a_j^{\aleph})\right)^{(1/\aleph)}}\right)^{(1/q)}, \left(\frac{1}{1+\left(n\psi_j(1/b_j^{\aleph})\right)^{(1/\aleph)}}\right)^{(1/q)}\right),\\
(n\xi_i o_i)^{\gamma} &= \left(\left(\frac{1}{1+\left(\gamma(1/n\xi_i)a_i^{\aleph}\right)^{(1/\aleph)}}\right)^{(1/q)}, \left(1-\frac{1}{1+\left(\gamma(1/n\xi_i)b_i^{\aleph}\right)^{(1/\aleph)}}\right)^{(1/q)}\right),\\
(n\psi_j o_j)^{\eta} &= \left(\left(\frac{1}{1+\left(\eta(1/n\psi_j)a_j^{\aleph}\right)^{(1/\aleph)}}\right)^{(1/q)}, \left(1-\frac{1}{1+\left(\eta(1/n\psi_j)b_j^{\aleph}\right)^{(1/\aleph)}}\right)^{(1/q)}\right).
\end{aligned} \quad (15)$$

Let $(1/n\xi_i) = t_i$ and $(1/n\psi_j) = e_j$; then, the above equations can be transformed into

$$(n\xi_i o_i)^{\gamma} = \left(\left(\frac{1}{1+\left(\gamma t_i a_i^{\aleph}\right)^{(1/\aleph)}}\right)^{(1/q)}, \left(1-\frac{1}{1+\left(\gamma t_i b_i^{\aleph}\right)^{(1/\aleph)}}\right)^{(1/q)}\right),$$
$$\left(n\psi_j o_j\right)^{\eta} = \left(\left(\frac{1}{1+\left(\eta e_j a_j^{\aleph}\right)^{(1/\aleph)}}\right)^{(1/q)}, \left(1-\frac{1}{1+\left(\eta e_j b_j^{\aleph}\right)^{(1/\aleph)}}\right)^{(1/q)}\right). \tag{16}$$

Thus, $(n\xi_i o_i)^{\gamma} \otimes (n\psi_j o_j)^{\eta} = (((1/(1+(\gamma t_i a_i^{\aleph} + \eta e_j a_j^{\aleph})^{(1/\aleph)}))^{(1/q)}, (1-(1/(1+(\gamma t_i b_i^{\aleph} + \eta e_j b_j^{\aleph})^{(1/\aleph)})))^{(1/q)}),$

$$\bigoplus_{i=1}^{n}\bigoplus_{j=i}^{n}\left(\left(n\xi_i o_i\right)^{\gamma} \otimes \left(n\psi_j o_j\right)^{\eta}\right) = \left(\left(1-\frac{1}{1+\left(\sum_{i=1,j=i}^{n}\left(1/\left(\gamma t_i a_i^{\aleph} + \eta e_j a_j^{\aleph}\right)\right)\right)^{(1/\aleph)}}\right)^{(1/q)}, \left(\frac{1}{1+\left(\sum_{i=1,j=i}^{n}\left(1/\left(\gamma t_i b_i^{\aleph} + \eta e_j b_j^{\aleph}\right)\right)\right)^{(1/\aleph)}}\right)^{(1/q)}\right),$$

$$\frac{2}{n(n+1)}\bigoplus_{i=1}^{n}\bigoplus_{j=i}^{n}\left(\left(n\xi_i o_i\right)^{\gamma} \otimes \left(n\psi_j o_j\right)^{\eta}\right) = \left(\left(1-\frac{1}{1+\left((2/n(n+1))\sum_{i=1,j=i}^{n}\left(1/\left(\gamma t_i a_i^{\aleph} + \eta e_j a_j^{\aleph}\right)\right)\right)^{(1/\aleph)}}\right)^{(1/q)}, \left(\frac{1}{1+\left((2/n(n+1))\sum_{i=1,j=i}^{n}\left(1/\left(\gamma t_i b_i^{\aleph} + \eta e_j b_j^{\aleph}\right)\right)\right)^{(1/\aleph)}}\right)^{(1/q)}\right),$$

$$\left(\frac{2}{n(n+1)}\bigoplus_{i=1}^{n}\bigoplus_{j=i}^{n}\left(\left(n\xi_i o_i\right)^{\gamma} \otimes \left(n\psi_j o_j\right)^{\eta}\right)\right)^{(1/(\gamma+\eta))} = \left(\begin{array}{c}\left(\frac{1}{1+\left((1/(\gamma+\eta))\left(1/(2/n(n+1))\sum_{i=1,j=i}^{n}\left(1/\left(\gamma t_i a_i^{\aleph} + \eta e_j a_j^{\aleph}\right)\right)\right)\right)^{(1/\aleph)}}\right)^{(1/q)}, \\ \left(1-\frac{1}{1+\left(((1/(\gamma+\eta))\left(1/(2/n(n+1))\sum_{i=1,j=i}^{n}\left(1/\left(\gamma t_i b_i^{\aleph} + \eta e_j b_j^{\aleph}\right)\right)\right)\right)^{(1/\aleph)}}\right)^{(1/q)}\end{array}\right). \tag{17}$$

Then, the final result can be determined as

$$\left(\frac{2}{n(n+1)}\bigoplus_{i=1}^{n}\bigoplus_{j=i}^{n}\left(\left(n\xi_i o_i\right)^{\gamma} \otimes \left(n\psi_j o_j\right)^{\eta}\right)\right)^{(1/(\gamma+\eta))} = \left(\begin{array}{c}\left(\frac{1}{1+(n(n+1)/2(\gamma+\eta))^{(1/\aleph)} \times \left(1/\left(\sum_{i=1,j=i}^{n}\left(1/\left(\gamma t_i a_i^{\aleph} + \eta e_j a_j^{\aleph}\right)\right)\right)^{(1/\aleph)}\right)}\right)^{(1/q)}, \\ \left(1-\frac{1}{1+(n(n+1)/2(\gamma+\eta))^{(1/\aleph)} \times \left(1/\left(\sum_{i=1,j=i}^{n}\left(1/\left(\gamma t_i b_i^{\aleph} + \eta e_j b_j^{\aleph}\right)\right)\right)^{(1/\aleph)}\right)}\right)^{(1/q)}\end{array}\right). \tag{18}$$

Then, we need to prove that the aggregated result from the q-ROFDPHM operator is still a q-ROFN.

Let $\hat{\mu} = ((1/(1 + (n(n+1)/2(\gamma+\eta))^{(1/\aleph)} \times (1/(\sum_{i=1,j=i}^{n} (1/(\gamma t_i a_i^{\aleph} + \eta e_j a_j^{\aleph})))^{(1/\aleph)}))))^{(1/q)}$ and $\hat{\nu} = (1-(1/(1+ (n(n+1)/2(\gamma+\eta))^{(1/\aleph)} \times (1/(\sum_{i=1,j=i}^{n} (1/(\gamma t_i b_i^{\aleph} + \eta e_j b_j^{\aleph})))^{(1/\aleph)}))))^{(1/q)}$; then, we need to prove that (1) $0 \le \hat{\mu}^q \le 1$ and $0 \le \hat{\nu}^q \le 1$ and (2) $0 \le \hat{\mu}^q + \hat{\nu}^q \le 1$.

Since $(n(n+1)/2(\gamma+\eta))^{(1/\aleph)} \ge 0$ and $(\sum_{i=1,j=i}^{n} (1/(\gamma t_i a_i^{\aleph} + \eta e_j a_j^{\aleph})))^{(1/\aleph)} > 0$, then $(n(n+1)/2(\gamma+\eta))^{(1/\aleph)} \times (1/(\sum_{i=1,\, j=i}^{n} (1/(\gamma t_i a_i^{\aleph} + \eta e_j a_j^{\aleph})))^{(1/\aleph)}) \ge 0$.

Then, we can have

$$
\begin{aligned}
&1 + \left(\frac{n(n+1)}{2(\gamma+\eta)}\right)^{(1/\aleph)} \times \frac{1}{\left(\sum_{i=1,j=i}^{n}\left(1/\left(\gamma t_i a_i^{\aleph} + \eta e_j a_j^{\aleph}\right)\right)\right)^{(1/\aleph)}} \ge 1 \Longrightarrow 0 \\
&\le \frac{1}{1 + (n(n+1)/2(\gamma+\eta))^{(1/\aleph)} \times \left(1/\left(\sum_{i=1,j=i}^{n}\left(1/\left(\gamma t_i a_i^{\aleph} + \eta e_j a_j^{\aleph}\right)\right)\right)^{(1/\aleph)}\right)} \le 1.
\end{aligned}
\tag{19}
$$

Thus, $0 \le \hat{\mu}^q \le 1$. Similarly, it can be proven that $0 \le \hat{\nu}^q \le 1$. It can be derived that $0 \le \hat{\mu}^q + \hat{\nu}^q$.

Because $\mu_i^q + \nu_i^q \le 1$ and $\mu_j^q + \nu_j^q \le 1$, then $\mu_i^q \le 1 - \nu_i^q$ and $\mu_j^q \le 1 - \nu_j^q$. Thus, $((1-\mu_i^q)/(\mu_i^q)) \ge (\nu_i^q/(1-\nu_i^q))$ and $((1-\mu_j^q)/(\mu_j^q)) \ge (\nu_j^q/(1-\nu_j^q))$. It can be derived that $a_i \ge b_i$ and $a_j \ge b_j$. Then, we have

$$
\begin{aligned}
&\left(\sum_{i=1,j=i}^{n} \frac{1}{\gamma t_i a_i^{\aleph} + \eta e_j a_j^{\aleph}}\right)^{(1/\aleph)} \le \left(\sum_{i=1,j=i}^{n} \frac{1}{\gamma t_i b_i^{\aleph} + \eta e_j b_j^{\aleph}}\right)^{(1/\aleph)} \Longrightarrow \frac{1}{\left(\sum_{i=1,j=i}^{n}\left(1/\left(\gamma t_i a_i^{\aleph} + \eta e_j a_j^{\aleph}\right)\right)\right)^{(1/\aleph)}} \\
&\ge \frac{1}{\left(\sum_{i=1,j=i}^{n}\left(1/\left(\gamma t_i b_i^{\aleph} + \eta e_j b_j^{\aleph}\right)\right)\right)^{(1/\aleph)}} \\
&\Longrightarrow 1 + \left(\frac{n(n+1)}{2(\gamma+\eta)}\right)^{(1/\aleph)} \times \frac{1}{\left(\sum_{i=1,j=i}^{n}\left(1/\left(\gamma t_i a_i^{\aleph} + \eta e_j a_j^{\aleph}\right)\right)\right)^{(1/\aleph)}} \\
&\ge 1 + \left(\frac{n(n+1)}{2(\gamma+\eta)}\right)^{(1/\aleph)} \times \frac{1}{\left(\sum_{i=1,j=i}^{n}\left(1/\left(\gamma t_i b_i^{\aleph} + \eta e_j b_j^{\aleph}\right)\right)\right)^{(1/\aleph)}}.
\end{aligned}
\tag{20}
$$

Thus,

$$
\begin{aligned}
&\frac{1}{1 + (n(n+1)/2(\gamma+\eta))^{(1/\aleph)} \times \left(1/\left(\sum_{i=1,j=i}^{n}\left(1/\left(\gamma t_i a_i^{\aleph} + \eta e_j a_j^{\aleph}\right)\right)\right)^{(1/\aleph)}\right)} \\
&\le \frac{1}{1 + (n(n+1)/2(\gamma+\eta))^{(1/\aleph)} \times \left(1/\left(\sum_{i=1,j=i}^{n}\left(1/\left(\gamma t_i b_i^{\aleph} + \eta e_j b_j^{\aleph}\right)\right)\right)^{(1/\aleph)}\right)}.
\end{aligned}
\tag{21}
$$

Then, we can have

$$
\begin{aligned}
\hat{\mu}^{q}+\hat{\nu}^{q}={}&\left(\frac{1}{1+(n(n+1)/2(\gamma+\eta))^{(1/\aleph)}\times\left(1/\left(\sum_{i=1,j=i}^{n}\left(1/\left(\gamma t_i a_i^{\aleph}+\eta e_j a_j^{\aleph}\right)\right)\right)^{(1/\aleph)}\right)}\right)^{(1/q)\times q}\\
&+\left(1-\frac{1}{1+(n(n+1)/2(\gamma+\eta))^{(1/\aleph)}\times\left(1/\left(\sum_{i=1,j=i}^{n}\left(1/\left(\gamma t_i b_i^{\aleph}+\eta e_j b_j^{\aleph}\right)\right)\right)^{(1/\aleph)}\right)}\right)^{(1/q)\times q}\\
={}&1+\frac{1}{1+(n(n+1)/2(\gamma+\eta))^{(1/\aleph)}\times\left(1/\left(\sum_{i=1,j=i}^{n}\left(1/\left(\gamma t_i a_i^{\aleph}+\eta e_j a_j^{\aleph}\right)\right)\right)^{(1/\aleph)}\right)}\\
&-\frac{1}{1+(n(n+1)/2(\gamma+\eta))^{(1/\aleph)}\times\left(1/\left(\sum_{i=1,j=i}^{n}\left(1/\left(\gamma t_i b_i^{\aleph}+\eta e_j b_j^{\aleph}\right)\right)\right)^{(1/\aleph)}\right)}\leq 1,
\end{aligned}
\tag{22}
$$

which completes the proof of Theorem 1. □

Theorem 2 (idempotency). *Suppose that there are a group of q-ROFNs $o_i=(\mu_i,\nu_i)\,(i=1,2,\ldots,n)$ and the parameters γ, $\eta\geq 0$ and $\aleph>0$. If $o_i=o_j=o=(\mu,\nu)$, for $i,j=1,2,\ldots,n$, then we have*

$$
\begin{aligned}
q-\mathrm{ROFDPHM}(o_1,o_2,\ldots,o_n)&=\begin{pmatrix}\left(\dfrac{1}{1+(n(n+1)/2(\gamma+\eta))^{(1/\aleph)}\times\left(1/\left(\sum_{i=1,j=i}^{n}\left(1/\left(\gamma t_i a_i^{\aleph}+\eta e_j a_j^{\aleph}\right)\right)\right)^{(1/\aleph)}\right)}\right)^{(1/q)},\\[2ex]
\left(1-\dfrac{1}{1+(n(n+1)/2(\gamma+\eta))^{(1/\aleph)}\times\left(1/\left(\sum_{i=1,j=i}^{n}\left(1/\left(\gamma t_i b_i^{\aleph}+\eta e_j b_j^{\aleph}\right)\right)\right)^{(1/\aleph)}\right)}\right)^{(1/q)}\end{pmatrix}\\
&=(\mu,\nu)=o,
\end{aligned}
\tag{23}
$$

where $\xi_i=((1+S(o_i))/\sum_{k=1}^{n}(1+S(o_k)))$, $\psi_j=((1+S(o_j))/\sum_{k=1}^{n}(1+S(o_k)))$, $(\mu_i^q/(1-\mu_i^q))=(1/a_i)$, $(\mu_j^q/(1-\mu_j^q))=(1/a_j)$, $((1-\nu_i^q)/\nu_i^q)=(1/b_i)$, $((1-\nu_j^q)/\nu_j^q)=(1/b_j)$, $(1/n\xi_i)=t_i$, *and* $(1/n\psi_j)=e_j$.

Proof. Since $o_i=o_j=o=(\mu,\nu)$, for $i,j=1,2,\ldots,n$, then $\mathrm{Sup}(o_i,o_j)=1$.

Thus, we have

$$
\begin{aligned}
&\xi_i=\frac{(1+S(o_i))}{\sum_{k=1}^{n}(1+S(o_k))}=\psi_j=\frac{\left(1+S(o_j)\right)}{\sum_{k=1}^{n}(1+S(o_k))}=\frac{1}{n},\\
&t_i=e_j=1,\\
&a_i^{\aleph}=a_j^{\aleph}=\left(\frac{1-\mu_i^q}{\mu_i^q}\right)^{\aleph}=\left(\frac{1-\mu_j^q}{\mu_j^q}\right)^{\aleph}=\left(\frac{1-\mu^q}{\mu^q}\right)^{\aleph}.
\end{aligned}
\tag{24}
$$

Then, it can be derived that

$$\left(\sum_{i=1,j=i}^{n} \frac{1}{\gamma t_i a_i^{\aleph} + \eta e_j a_j^{\aleph}}\right)^{(1/\aleph)} = \left(\sum_{i=1,j=i}^{n} \frac{1}{\gamma\left(\left(1-\mu^q\right)/\mu^q\right)^{\aleph} + \eta\left(\left(1-\mu^q\right)/\mu^q\right)^{\aleph}}\right)^{(1/\aleph)} = \left(\sum_{i=1,j=i}^{n} \frac{1}{(\gamma+\eta)\times\left(\left(1-\mu^q\right)/\mu^q\right)^{\aleph}}\right)^{(1/\aleph)}$$

$$= \left(\frac{1}{(\gamma+\eta)} \times \frac{n(n+1)}{2} \times \left(\frac{\mu^q}{1-\mu^q}\right)^{\aleph}\right)^{(1/\aleph)} \Rightarrow \frac{1}{\left(\sum_{i=1,j=i}^{n}\left(1/\left(\gamma t_i a_i^{\aleph} + \eta e_j a_j^{\aleph}\right)\right)\right)^{(1/\aleph)}}$$

$$= \frac{1}{\left((1/(\gamma+\eta)) \times (n(n+1)/2) \times \left(\mu^q/\left(1-\mu^q\right)\right)^{\aleph}\right)^{(1/\aleph)}}$$

$$= \frac{1}{\left((n(n+1)/2(\gamma+\eta)) \times \left(\mu^q/\left(1-\mu^q\right)\right)^{\aleph}\right)^{(1/\aleph)}} = \left(\frac{2(\gamma+\eta)}{n(n+1)} \times \left(\frac{1-\mu^q}{\mu^q}\right)^{\aleph}\right)^{(1/\aleph)}$$

$$\Rightarrow \left(\frac{1}{1 + (n(n+1)/2(\gamma+\eta))^{(1/\aleph)} \times \left(1/\left(\sum_{i=1,j=i}^{n}\left(1/\left(\gamma t_i a_i^{\aleph} + \eta e_j a_j^{\aleph}\right)\right)\right)^{(1/\aleph)}\right)}\right)^{(1/q)}$$

$$= \left(\frac{1}{1 + (n(n+1)/2(\gamma+\eta))^{(1/\aleph)} \times \left(2(\gamma+\eta)/n(n+1) \times \left(\left(1-\mu^q\right)/\mu^q\right)^{\aleph}\right)^{(1/\aleph)}}\right)^{(1/q)}$$

$$= \left(\frac{1}{1 + \left(\left(1-\mu^q\right)/\mu^q\right)}\right)^{(1/q)} = \mu. \tag{25}$$

Similarly, $(1/b_i) = (1/b_j) = \left(\left(1-\nu_i^q\right)/\nu_i^q\right) = \left(\left(1-\nu_j^q\right)/\nu_j^q\right) = \left(\left(1-\nu^q\right)/\nu^q\right)$. Then, the NMD value of the q-ROFDPHM operator can be derived as

$$\left(1 - \frac{1}{1 + (n(n+1)/2(\gamma+\eta))^{(1/\aleph)} \times \left(1/\left(\sum_{i=1,j=i}^{n}\left(1/\left(\gamma t_i b_i^{\aleph} + \eta e_j b_j^{\aleph}\right)\right)\right)^{(1/\aleph)}\right)}\right)^{(1/q)}$$

$$= \left(1 - \frac{1}{1 + (n(n+1)/2(\gamma+\eta))^{(1/\aleph)} \times \left(1/\left(\sum_{i=1,j=i}^{n}\left(1/(\gamma+\eta)\left(\nu^q/\left(1-\nu^q\right)\right)^{\aleph}\right)\right)^{(1/\aleph)}\right)}\right)^{(1/q)}$$

$$= \left(1 - \frac{1}{1 + (n(n+1)/2(\gamma+\eta))^{(1/\aleph)} \times \left(1/\left((n(n+1)/2(\gamma+\eta))\left((1-\nu^q)/\nu^q\right)^{\aleph}\right)^{(1/\aleph)}\right)}\right)^{(1/q)}$$
$$= \left(1 - \frac{1}{1 + (\nu^q/(1-\nu^q))}\right)^{(1/q)} = \nu. \tag{26}$$

Thus, we have $q-\mathrm{ROFDPHM}(o_1, o_2, \ldots, o_n) = (\mu, \nu) = o$, which completes the proof of Theorem 2. □

Theorem 3 (boundedness). *Suppose that there are a group of q-ROFNs $o_i = (\mu_i, \nu_i)\,(i = 1, 2, \ldots, n)$ and the parameters γ, $\eta \geq 0$ and $\aleph > 0$. If $o_l = \min(o_1, o_2, \ldots, o_n) = (\mu_l, \nu_l)$ and $o_h = \max(o_1, o_2, \ldots, o_n) = (\mu_h, \nu_h)$, then we have*

$$o_l \leq q-\mathrm{ROFDPHM}(o_1, o_2, \ldots, o_n) \leq o_h, \tag{27}$$

where

$$o_l = \left(\begin{array}{c} \left(\dfrac{1}{1 + (n(n+1)/2(\gamma+\eta))^{(1/\aleph)} \times \left(1/\left(\sum_{i=1, j=i}^{n}\left(1/\left(\gamma t_i a_i^{\aleph} + \eta e_j a_j^{\aleph}\right)\right)\right)^{(1/\aleph)}\right)}\right)^{(1/q)}, \\ \left(1 - \dfrac{1}{1 + (n(n+1)/2(\gamma+\eta))^{(1/\aleph)} \times \left(1/\left(\sum_{i=1, j=i}^{n}\left(1/\left(\gamma t_i b_i^{\aleph} + \eta e_j b_j^{\aleph}\right)\right)\right)^{(1/\aleph)}\right)}\right)^{(1/q)} \end{array} \right),$$

$$o_h = \left(\begin{array}{c} \left(\dfrac{1}{1 + (n(n+1)/2(\gamma+\eta))^{(1/\aleph)} \times \left(1/\left(\sum_{i=1, j=i}^{n}\left(1/\left(\gamma t_i a_i^{\aleph} + \eta e_j a_j^{\aleph}\right)\right)\right)^{(1/\aleph)}\right)}\right)^{(1/q)}, \\ \left(1 - \dfrac{1}{1 + (n(n+1)/2(\gamma+\eta))^{(1/\aleph)} \times \left(1/\left(\sum_{i=1, j=i}^{n}\left(1/\left(\gamma t_i b_i^{\aleph} + \eta e_j b_j^{\aleph}\right)\right)\right)^{(1/\aleph)}\right)}\right)^{(1/q)} \end{array} \right), \tag{28}$$

$$\frac{1}{n\xi_i} = t_i,$$

$$\frac{1}{n\psi_j} = e_j,$$

$$\xi_i = \frac{(1 + S(o_i))}{\sum_{k=1}^{n}(1 + S(o_k))},$$

$$\psi_j = \frac{\left(1 + S\left(o_j\right)\right)}{\sum_{k=1}^{n}(1 + S(o_k))},$$

$$\frac{\mu_i^q}{1 - \mu_i^q} = \frac{1}{a_i},$$

$$\frac{1 - \nu_i^q}{\nu_i^q} = \frac{1}{b_i}.$$

Proof. According to Definition 7, we have

$$
\begin{aligned}
n\xi_i o_i &= \left(\left(1 - \frac{1}{1 + \left(n\xi_i \left(\mu_i^q/\left(1-\mu_i^q\right)\right)^{\aleph}\right)^{(1/\aleph)}} \right)^{(1/q)}, \left(\frac{1}{1 + \left(n\xi_i \left(\left(1-\nu_i^q\right)/\nu_i^q\right)^{\aleph}\right)^{(1/\aleph)}} \right)^{(1/q)} \right), \\
n\psi_j o_j &= \left(\left(1 - \frac{1}{1 + \left(n\psi_j \left(\mu_j^q/\left(1-\mu_j^q\right)\right)^{\aleph}\right)^{(1/\aleph)}} \right)^{(1/q)}, \left(\frac{1}{1 + \left(n\psi_j \left(\left(1-\nu_i^q\right)/\nu_i^q\right)^{\aleph}\right)^{(1/\aleph)}} \right)^{(1/q)} \right).
\end{aligned}
\tag{29}
$$

Since $o_l \leq o_i$, then we have $\mu_i \geq \mu_l$ and $\nu_i \leq \nu_l$. Thus, it can be derived that

$$
\begin{aligned}
&\mu_i^q \geq \mu_l^q \Longrightarrow \frac{1}{1-\mu_i^q} \geq \frac{1}{1-\mu_l^q} \Longrightarrow \frac{\mu_i^q}{1-\mu_i^q} \geq \frac{\mu_l^q}{1-\mu_l^q} \Longrightarrow \frac{1}{a_i} \geq \frac{1}{a_l} \Longrightarrow a_i \leq a_l, \\
&\nu_i \leq \nu_l \Longrightarrow 1-\nu_i^q \geq 1-\nu_l^q, \ \frac{1}{\nu_i^q} \geq \frac{1}{\nu_l^q} \Longrightarrow \frac{1}{b_i} \geq \frac{1}{b_l} \Longrightarrow b_i \leq b_l.
\end{aligned}
\tag{30}
$$

Then, we can have

$$
\begin{aligned}
n\xi_i o_i &= \left(\left(1 - \frac{1}{1 + \left(n\xi_i \left(1/a_i^{\aleph}\right)\right)^{(1/\aleph)}} \right)^{(1/q)}, \left(\frac{1}{1 + \left(n\xi_i \left(1/b_i^{\aleph}\right)\right)^{(1/\aleph)}} \right)^{(1/q)} \right) \\
&\geq \left(\left(1 - \frac{1}{1 + \left(n\xi_i \left(1/a_l^{\aleph}\right)\right)^{(1/\aleph)}} \right)^{(1/q)}, \left(\frac{1}{1 + \left(n\xi_i \left(1/b_l^{\aleph}\right)\right)^{(1/\aleph)}} \right)^{(1/q)} \right) \\
\Longrightarrow \left(n\xi_i o_i\right)^{\gamma} &= \left(\left(\frac{1}{1 + \left(\gamma \left(1/n\xi_i\right) a_i^{\aleph}\right)^{(1/\aleph)}} \right)^{(1/q)}, \left(1 - \frac{1}{1 + \left(\gamma \left(1/n\xi_i\right) b_i^{\aleph}\right)^{(1/\aleph)}} \right)^{(1/q)} \right) \\
&\geq \left(\left(\frac{1}{1 + \left(\gamma \left(1/n\xi_i\right) a_l^{\aleph}\right)^{(1/\aleph)}} \right)^{(1/q)}, \left(1 - \frac{1}{1 + \left(\gamma \left(1/n\xi_i\right) b_l^{\aleph}\right)^{(1/\aleph)}} \right)^{(1/q)} \right).
\end{aligned}
\tag{31}
$$

Similarly, we have

$$
\begin{aligned}
\left(n\psi_j o_j\right)^{\eta} &= \left(\left(\frac{1}{1 + \left(\eta \left(1/n\psi_j\right) a_j^{\aleph}\right)^{(1/\aleph)}} \right)^{(1/q)}, \left(1 - \frac{1}{1 + \left(\eta \left(1/n\psi_j\right) b_j^{\aleph}\right)^{(1/\aleph)}} \right)^{(1/q)} \right) \\
&\geq \left(\left(\frac{1}{1 + \left(\eta \left(1/n\psi_j\right) a_l^{\aleph}\right)^{(1/\aleph)}} \right)^{(1/q)}, \left(1 - \frac{1}{1 + \left(\eta \left(1/n\psi_j\right) b_l^{\aleph}\right)^{(1/\aleph)}} \right)^{(1/q)} \right).
\end{aligned}
\tag{32}
$$

Since $(1/n\xi_i) = t_i$ and $(1/n\psi_j) = e_j$, then we have

$$\left(n\xi_i o_i\right)^{\gamma} = \left(\left(\frac{1}{1+\left(\gamma t_i a_i^{\aleph}\right)^{(1/\aleph)}}\right)^{(1/q)}, \left(1-\frac{1}{1+\left(\gamma t_i b_i^{\aleph}\right)^{1/\aleph}}\right)^{(1/q)}\right)$$

$$\geq \left(\left(\frac{1}{1+\left(\gamma t_i a_l^{\aleph}\right)^{1/\aleph}}\right)^{(1/q)}, \left(1-\frac{1}{1+\left(\gamma t_i b_l^{\aleph}\right)^{1/\aleph}}\right)^{(1/q)}\right)$$

$$\Longrightarrow \left(n\psi_j o_j\right)^{\eta} = \left(\left(\frac{1}{1+\left(\eta e_j a_j^{\aleph}\right)^{(1/\aleph)}}\right)^{(1/q)}, \left(1-\frac{1}{1+\left(\eta e_j b_j^{\aleph}\right)^{(1/\aleph)}}\right)^{(1/q)}\right)$$

$$\geq \left(\left(\frac{1}{1+\left(\eta e_j a_l^{\aleph}\right)^{(1/\aleph)}}\right)^{(1/q)}, \left(1-\frac{1}{1+\left(\eta e_j b_l^{\aleph}\right)^{(1/\aleph)}}\right)^{(1/q)}\right),$$

$$\left(n\xi_i o_i\right)^{\gamma} \otimes \left(n\psi_j o_j\right)^{\eta} = \left(\left(\frac{1}{1+\left(\gamma t_i a_i^{\aleph} + \eta e_j a_j^{\aleph}\right)^{(1/\aleph)}}\right)^{(1/q)}, \left(1-\frac{1}{1+\left(\gamma t_i b_i^{\aleph} + \eta e_j b_j^{\aleph}\right)^{(1/\aleph)}}\right)^{(1/q)}\right)$$

$$\geq \left(\left(\frac{1}{1+\left(\gamma t_i a_l^{\aleph} + \eta e_j a_l^{\aleph}\right)^{(1/\aleph)}}\right)^{(1/q)}, \left(1-\frac{1}{1+\left(\gamma t_i b_l^{\aleph} + \eta e_j b_l^{\aleph}\right)^{(1/\aleph)}}\right)^{(1/q)}\right),$$

$$\underset{i=1}{\overset{n}{\oplus}}\underset{j=i}{\overset{n}{\oplus}}\left(\left(n\xi_i o_i\right)^{\gamma} \otimes \left(n\psi_j o_j\right)^{\eta}\right) = \begin{pmatrix} \left(1-\dfrac{1}{1+\left(\sum_{i=1,j=i}^{n}\left(1/\left(\gamma t_i a_i^{\aleph} + \eta e_j a_j^{\aleph}\right)\right)\right)^{(1/\aleph)}}\right)^{(1/q)}, \\ \left(\dfrac{1}{1+\left(\sum_{i=1,j=i}^{n}\left(1/\left(\gamma t_i b_i^{\aleph} + \eta e_j b_j^{\aleph}\right)\right)\right)^{(1/\aleph)}}\right)^{(1/q)} \end{pmatrix}$$

$$\geq \begin{pmatrix} \left(1-\dfrac{1}{1+\left(\sum_{i=1,j=i}^{n}\left(1/\left(\gamma t_i a_i^{\aleph} + \eta e_j a_l^{\aleph}\right)\right)\right)^{(1/\aleph)}}\right)^{(1/q)}, \\ \left(\dfrac{1}{1+\left(\sum_{i=1,j=i}^{n}\left(1/\left(\gamma t_i b_i^{\aleph} + \eta e_j b_l^{\aleph}\right)\right)\right)^{(1/\aleph)}}\right)^{(1/q)} \end{pmatrix},$$

$$\frac{2}{n(n+1)}\underset{i=1}{\overset{n}{\oplus}}\underset{j=i}{\overset{n}{\oplus}}\left(\left(n\xi_i o_i\right)^{\gamma} \otimes \left(n\psi_j o_j\right)^{\eta}\right) = \begin{pmatrix} \left(1-\dfrac{1}{1+\left((2/n(n+1))\sum_{i=1,j=i}^{n}\left(1/\left(\gamma t_i a_i^{\aleph} + \eta e_j a_j^{\aleph}\right)\right)\right)^{(1/\aleph)}}\right)^{(1/q)}, \\ \left(\dfrac{1}{1+\left((2/n(n+1))\sum_{i=1,j=i}^{n}\left(1/\left(\gamma t_i b_i^{\aleph} + \eta e_j b_j^{\aleph}\right)\right)\right)^{(1/\aleph)}}\right)^{(1/q)} \end{pmatrix}$$

$$\geq \left(\begin{array}{c} \left(1-\frac{1}{1+\left((2/n(n+1))\sum_{i=1,j=i}^{n}\left(1/\left(\gamma t_i a_i^{\aleph}+\eta e_j a_l^{\aleph}\right)\right)\right)^{(1/\aleph)}}\right)^{(1/q)}, \\ \left(\frac{1}{1+\left((2/n(n+1))\sum_{i=1,j=i}^{n}\left(1/\left(\gamma t_i b_i^{\aleph}+\eta e_j b_l^{\aleph}\right)\right)\right)^{(1/\aleph)}}\right)^{(1/q)} \end{array} \right),$$

$$\left(\frac{2}{n(n+1)}\overset{n}{\underset{i=1}{\oplus}}\overset{n}{\underset{j=i}{\oplus}}\left(\left(n\xi_i o_i\right)^{\gamma}\otimes\left(n\psi_j o_j\right)^{\eta}\right)\right)^{(1/(\gamma+\eta))}$$

$$= \left(\begin{array}{c} \left(\frac{1}{1+\left((1/(\gamma+\eta))\left(1/(2/n(n+1))\sum_{i=1,j=i}^{n}\left(1/\left(\gamma t_i a_i^{\aleph}+\eta e_j a_j^{\aleph}\right)\right)\right)\right)^{(1/\aleph)}}\right)^{(1/q)}, \\ \left(1-\frac{1}{1+\left((1/(\gamma+\eta))\left(1/(2/n(n+1))\sum_{i=1,j=i}^{n}\left(1/\left(\gamma t_i b_i^{\aleph}+\eta e_j b_j^{\aleph}\right)\right)\right)\right)^{(1/\aleph)}}\right)^{(1/q)} \end{array} \right) \tag{33}$$

$$\geq \left(\begin{array}{c} \left(\frac{1}{1+\left((1/(\gamma+\eta))\left(1/(2/n(n+1))\sum_{i=1,j=i}^{n}\left(1/\left(\gamma t_i a_l^{\aleph}+\eta e_j a_l^{\aleph}\right)\right)\right)\right)^{(1/\aleph)}}\right)^{(1/q)}, \\ \left(1-\frac{1}{1+\left((1/(\gamma+\eta))\left(1/(2/n(n+1))\sum_{i=1,j=i}^{n}\left(1/\left(\gamma t_i b_l^{\aleph}+\eta e_j b_l^{\aleph}\right)\right)\right)\right)^{(1/\aleph)}}\right)^{(1/q)} \end{array} \right) = o_l.$$

Similarly, it can be proven that $q-\text{ROFDPHM}(o_1, o_2, \ldots, o_n) \leq o_h$ in the same way, which completes the proof of Theorem 3.

It can be noted that the proposed q-ROFDPHM operator uses the PA operator and Dombi operational laws to optimize the HM operator. Its significance can be listed as follows. (1) It can alleviate negative influences of extreme input values on the calculation results. (2) It shows strong flexibility for computing input values. (3) It is capable of capturing the complex interrelationships among criteria values. (4) It can process the complex information structure of q-ROFSs. Nevertheless, during the aggregation processes, it does not consider the weight values of criteria, which is very important in the MCDM contexts. To tackle this deficiency, a novel q-rung orthopair fuzzy weighted Dombi power Heronian mean (q-ROFWDPHM) operator is put forward in the following part. □

Definition 11. Given a set of q-ROFNs $o_i = (\mu_i, \nu_i)\,(i = 1, 2, \ldots, n)$, three parameters $\gamma, \eta \geq 0$ and $\aleph > 0$, and the weight values $[\omega_1, \omega_2, \ldots, \omega_n]$ of q-ROFNs, then the q-ROFWDPHM operator is defined as

$$q-\text{ROFWDPHM}(o_1, o_2, \ldots, o_n) = \left(\frac{2}{n(n+1)}\overset{n}{\underset{i=1}{\oplus}}\overset{n}{\underset{j=i}{\oplus}}\left(\frac{n\omega_i(1+S(o_i))}{\sum_{k=1}^{n}\omega_k(1+S(o_k))}o_i\right)^{\gamma}\otimes\left(\frac{n\omega_j(1+S(o_j))}{\sum_{k=1}^{n}\omega_k(1+S(o_k))}o_j\right)^{\eta}\right)^{(1/(\gamma+\eta))}. \tag{34}$$

Theorem 4. *Given a set of q-ROFNs* $o_i = (\mu_i, \nu_i)\,(i = 1, 2, \ldots, n)$*, three parameters* $\gamma, \eta \geq 0$ *and* $\aleph > 0$*, and the weight values* $[\omega_1, \omega_2, \ldots, \omega_n]$ *of q-ROFNs, then the aggregated result obtained from the q-ROFWDPHM operator is still a q-ROFN, which is*

$$q-\mathrm{ROFWDPHM}(o_1, o_2, \ldots, o_n) = \left(\begin{array}{c} \left(\dfrac{1}{1 + (n(n+1)/2(\gamma+\eta))^{(1/\aleph)} \times \left(1/\left(\sum_{i=1, j=i}^{n}\left(1/\left(\gamma t_i a_i^{\aleph} + \eta e_j a_j^{\aleph}\right)\right)\right)^{(1/\aleph)}\right)} \right)^{(1/q)}, \\ \left(1 - \dfrac{1}{1 + (n(n+1)/2(\gamma+\eta))^{(1/\aleph)} \times \left(1/\left(\sum_{i=1, j=i}^{n}\left(1/\left(\gamma t_i b_i^{\aleph} + \eta e_j b_j^{\aleph}\right)\right)\right)^{(1/\aleph)}\right)} \right)^{(1/q)} \end{array} \right), \tag{35}$$

where $\xi_i = ((1 + S(o_i))/\sum_{k=1}^{n}\omega_k(1 + S(o_k)))$, $\psi_j = ((1 + S(o_j))/\sum_{k=1}^{n}\omega_k(1 + S(o_k)))$, $(\mu_i^q/(1-\mu_i^q)) = (1/a_i)$, $(\mu_j^q/1-\mu_j^q) = (1/a_j)$, $((1-\nu_i^q)/\nu_i^q) = (1/b_i)$, $((1-\nu_j^q)/\nu_j^q) = (1/b_j)$, $t_i = (1/n\omega_i\xi_i)$, *and* $e_j = (1/n\omega_j\psi_j)$.

Proof. Let $\xi_i = ((1 + S(o_i))/\sum_{k=1}^{n}\omega_k(1 + S(o_k)))$ and $\psi_j = ((1 + S(o_j))/\sum_{k=1}^{n}\omega_k(1 + S(o_k)))$; then, equation (34) can be transformed into

$$q-\mathrm{ROFWDPHM}(o_1, o_2, \ldots, o_n) = \left(\frac{2}{n(n+1)} \mathop{\oplus}_{i=1}^{n} \mathop{\oplus}_{j=i}^{n} (n\omega_i\xi_i o_i)^{\gamma} \otimes \left(n\omega_j\psi_j o_j\right)^{\eta} \right)^{(1/(\gamma+\eta))}. \tag{36}$$

According to Definition 7, we have

$$\begin{aligned} n\omega_i\xi_i o_i &= \left(\left(1 - \frac{1}{1 + \left(n\omega_i\xi_i\left(\mu_i^q/(1-\mu_i^q)\right)^{\aleph}\right)^{(1/\aleph)}} \right)^{(1/q)}, \left(\frac{1}{1 + \left(n\omega_i\xi_i\left((1-\nu_i^q)/\nu_i^q\right)^{\aleph}\right)^{(1/\aleph)}} \right)^{(1/q)} \right), \\ n\omega_j\psi_j o_j &= \left(\left(1 - \frac{1}{1 + \left(n\omega_j\psi_j\left(\mu_j^q/(1-\mu_j^q)\right)^{\aleph}\right)^{(1/\aleph)}} \right)^{(1/q)}, \left(\frac{1}{1 + \left(n\omega_j\psi_j\left((1-\nu_j^q)/\nu_j^q\right)^{\aleph}\right)^{(1/\aleph)}} \right)^{(1/q)} \right). \end{aligned} \tag{37}$$

Let $(\mu_i^q/(1-\mu_i^q)) = (1/a_i)$, $(\mu_j^q/1-\mu_j^q) = (1/a_j)$, $((1-\nu_i^q)/\nu_i^q) = (1/b_i)$, and $((1-\nu_j^q)/\nu_j^q) = (1/b_j)$; then, these equations can be transformed into

$$\begin{aligned} n\omega_i\xi_i o_i &= \left(\left(1 - \frac{1}{1 + (n\omega_i\xi_i(1/a_i^{\aleph}))^{(1/\aleph)}} \right)^{(1/q)}, \left(\frac{1}{1 + (n\omega_i\xi_i(1/b_i^{\aleph}))^{(1/\aleph)}} \right)^{(1/q)} \right), \\ n\omega_j\psi_j o_j &= \left(\left(1 - \frac{1}{1 + \left(n\omega_j\psi_j\left(1/a_j^{\aleph}\right)\right)^{(1/\aleph)}} \right)^{(1/q)}, \left(\frac{1}{1 + \left(n\omega_j\psi_j\left(1/b_j^{\aleph}\right)\right)^{(1/\aleph)}} \right)^{(1/q)} \right). \end{aligned} \tag{38}$$

According to Definition 7, then we have

$$\begin{aligned}
\left(n\omega_i\xi_i o_i\right)^{\gamma} &= \left(\left(\frac{1}{1+\left(\gamma\left(1/n\omega_i\xi_i\right)a_i^{\aleph}\right)^{(1/\aleph)}}\right)^{(1/q)}, \left(1-\frac{1}{1+\left(\gamma\left(1/n\omega_i\xi_i\right)b_i^{\aleph}\right)^{(1/\aleph)}}\right)^{(1/q)}\right),\\
\left(n\omega_j\psi_j o_j\right)^{\eta} &= \left(\left(\frac{1}{1+\left(\eta\left(1/n\omega_j\psi_j\right)a_j^{\aleph}\right)^{(1/\aleph)}}\right)^{(1/q)}, \left(1-\frac{1}{1+\left(\eta\left(1/n\omega_j\psi_j\right)b_j^{\aleph}\right)^{(1/\aleph)}}\right)^{(1/q)}\right).
\end{aligned} \tag{39}$$

Let $t_i = (1/n\omega_i\xi_i)$ and $e_j = (1/n\omega_j\psi_j)$; then, the above two equations can be transformed into

$$\begin{aligned}
\left(n\omega_i\xi_i o_i\right)^{\gamma} &= \left(\left(\frac{1}{1+\left(\gamma t_i a_i^{\aleph}\right)^{(1/\aleph)}}\right)^{(1/q)}, \left(1-\frac{1}{1+\left(\gamma t_i b_i^{\aleph}\right)^{(1/\aleph)}}\right)^{(1/q)}\right),\\
\left(n\omega_j\psi_j o_j\right)^{\eta} &= \left(\left(\frac{1}{1+\left(\eta e_j a_j^{\aleph}\right)^{(1/\aleph)}}\right)^{(1/q)}, \left(1-\frac{1}{1+\left(\eta e_j b_j^{\aleph}\right)^{(1/\aleph)}}\right)^{(1/q)}\right),\\
\left(n\omega_i\xi_i o_i\right)^{\gamma} \otimes \left(n\omega_j\psi_j o_j\right)^{\eta} &= \left(\left(\frac{1}{1+\left(\gamma t_i a_i^{\aleph}+\eta e_j a_j^{\aleph}\right)^{(1/\aleph)}}\right)^{(1/q)}, \left(1-\frac{1}{1+\left(\gamma t_i b_i^{\aleph}+\eta e_j b_j^{\aleph}\right)^{(1/\aleph)}}\right)^{(1/q)}\right),\\
\mathop{\oplus}_{i=1}^{n}\mathop{\oplus}_{j=i}^{n}\left(n\omega_i\xi_i o_i\right)^{\gamma} \otimes \left(n\omega_j\psi_j o_j\right)^{\eta} &= \left(\begin{array}{c}\left(1-\dfrac{1}{1+\left(\sum_{i=1,j=i}^{n}\left(1/\left(\gamma t_i a_i^{\aleph}+\eta e_j a_j^{\aleph}\right)\right)\right)^{(1/\aleph)}}\right)^{(1/q)},\\[2ex] \left(\dfrac{1}{1+\left(\sum_{i=1,j=i}^{n}\left(1/\left(\gamma t_i b_i^{\aleph}+\eta e_j b_j^{\aleph}\right)\right)\right)^{(1/\aleph)}}\right)^{(1/q)}\end{array}\right),\\
\frac{2}{n(n+1)}\mathop{\oplus}_{i=1}^{n}\mathop{\oplus}_{j=i}^{n}\left(n\omega_i\xi_i o_i\right)^{\gamma} \otimes \left(n\omega_j\psi_j o_j\right)^{\eta} &= \left(\begin{array}{c}\left(1-\dfrac{1}{1+\left((2/n(n+1))\sum_{i=1,j=i}^{n}\left(1/\left(\gamma t_i a_i^{\aleph}+\eta e_j a_j^{\aleph}\right)\right)\right)^{(1/\aleph)}}\right)^{(1/q)},\\[2ex] \left(\dfrac{1}{1+\left((2/n(n+1))\sum_{i=1,j=i}^{n}\left(1/\left(\gamma t_i b_i^{\aleph}+\eta e_j b_j^{\aleph}\right)\right)\right)^{(1/\aleph)}}\right)^{(1/q)}\end{array}\right),
\end{aligned}$$

$$\left(\frac{2}{n(n+1)}\underset{i=1}{\overset{n}{\oplus}}\underset{j=i}{\overset{n}{\oplus}}\left(n\omega_i\xi_i o_i\right)^{\gamma}\otimes\left(n\omega_j\psi_j o_j\right)^{\eta}\right)^{(1/(\gamma+\eta))}$$

$$=\left(\begin{array}{c}\left(\dfrac{1}{1+\left((1/(\gamma+\eta))\left(1/\left((2/n(n+1))\sum_{i=1,j=i}^{n}\left(1/\left(\gamma t_i a_i^{\aleph}+\eta e_j a_j^{\aleph}\right)\right)\right)\right)\right)^{(1/\aleph)}}\right)^{(1/q)},\\ \left(1-\dfrac{1}{1+\left((1/(\gamma+\eta))\left(1/(2/n(n+1))\sum_{i=1,j=i}^{n}\left(1/\left(\gamma t_i b_i^{\aleph}+\eta e_j b_j^{\aleph}\right)\right)\right)\right)^{(1/\aleph)}}\right)^{(1/q)}\end{array}\right),$$

$$q-\mathrm{ROFWDPHM}\left(o_1,o_2,\ldots,o_n\right)=\left(\frac{2}{n(n+1)}\underset{i=1}{\overset{n}{\oplus}}\underset{j=i}{\overset{n}{\oplus}}\left(n\omega_i\xi_i o_i\right)^{\gamma}\otimes\left(n\omega_j\psi_j o_j\right)^{\eta}\right)^{(1/(\gamma+\eta))}=$$

$$\left(\begin{array}{c}\left(\dfrac{1}{1+(n(n+1)/2(\gamma+\eta))^{(1/\aleph)}\times\left(1/\left(\sum_{i=1,j=i}^{n}\left(1/\left(\gamma t_i a_i^{\aleph}+\eta e_j a_j^{\aleph}\right)\right)\right)^{(1/\aleph)}\right)}\right)^{(1/q)},\\ \left(1-\dfrac{1}{1+(n(n+1)/2(\gamma+\eta))^{(1/\aleph)}\times\left(1/\left(\sum_{i=1,j=i}^{n}\left(1/\left(\gamma t_i b_i^{\aleph}+\eta e_j b_j^{\aleph}\right)\right)\right)^{(1/\aleph)}\right)}\right)^{(1/q)}\end{array}\right). \tag{40}$$

The process for proving that the aggregation result of the q-ROFWDPHM operator is a q-ROFN is the same as that of Theorem 1. Thus, it is omitted here.

The proposed q-ROFWDPHM operator also owns the features of idempotency and boundedness as the proposed q-ROFDPHM operator. Their proof processes are similar to those of Theorems 2 and 3. Due to the limited space, the proof processes are omitted here. □

4. q-Rung Orthopair Fuzzy Dombi Power Geometric Heronian Mean Operators

In this section, we use the PA operator, Dombi operational laws for q-ROFNs, and geometric HM operator to develop a novel q-ROFDPGHM operator and its weighted form. Then, the features are discussed.

Definition 12. Given a set of q-ROFNs $o_i=(\mu_i,\nu_i)\,(i=1,2,\ldots,n)$ and three parameters $\gamma,\eta\geq 0$ and $\aleph>0$, then the q-ROFDPGHM operator is defined as

$$q-\mathrm{ROFDPGHM}\left(o_1,o_2,\ldots,o_n\right)=\frac{1}{\gamma+\eta}\left(\underset{i=1}{\overset{n}{\otimes}}\underset{j=i}{\overset{n}{\otimes}}\left(\gamma o_i^{\left(n\left(1+S\left(o_i\right)\right)/\sum_{k=1}^{n}\left(1+S\left(o_k\right)\right)\right)}\oplus\eta o_j^{\left(n\left(1+S\left(o_j\right)\right)/\sum_{k=1}^{n}\left(1+S\left(o_k\right)\right)\right)}\right)\right)^{(2/n(n+1))}. \tag{41}$$

Based on the Dombi operational laws of q-ROFNs and GHM operator, a theorem is derived.

Theorem 5. *Given n q-ROFNs $o_i=(\mu_i,\nu_i)\,(i=1,2,\ldots,n)$ and the parameters $\gamma,\eta\geq 0$ and $\aleph>0$, then the* *aggregated result derived from equation (41) is still a q-ROFN, which is*

$$q-\mathrm{ROFDPGHM}(o_1,o_2,\ldots,o_n)=\left(\begin{array}{l}\left(1-\dfrac{1}{1+(n(n+1)/2(\gamma+\eta))^{(1/\aleph)}\times\left(1/\left(\sum_{i=1,j=i}^{n}\left(1/\gamma t_i\left(1/a_i^{\aleph}\right)+\eta e_j\left(1/a_j^{\aleph}\right)\right)\right)^{(1/\aleph)}\right)}\right)^{(1/q)},\\ \left(\dfrac{1}{1+(n(n+1)/2(\gamma+\eta))^{(1/\aleph)}\times\left(1/\left(\sum_{i=1,j=i}^{n}\left(1/\gamma t_i\left(1/b_i^{\aleph}\right)+\eta e_j\left(1/b_j^{\aleph}\right)\right)\right)^{(1/\aleph)}\right)}\right)^{(1/q)}\end{array}\right), \tag{42}$$

where $\xi_i=((1+S(o_i))/\sum_{k=1}^{n}(1+S(o_k)))$, $\psi_j=((1+S(o_j))/\sum_{k=1}^{n}(1+S(o_k)))$, $a_i=((1-\mu_i^q)/\mu_i^q)$, $a_j=((1-\mu_j^q)/\mu_j^q)$, $b_i=(\nu_i^q/(1-\nu_i^q))$, $b_j=(\nu_j^q/(1-\nu_j^q))$, $t_i=(1/n\xi_i)$, *and* $e_j=(1/n\psi_j)$.

The proof process of this theorem is similar to that of Theorem 1. Thus, it is omitted here.

The proposed q-ROFDPGHM operator also owns the features of idempotency and boundedness as the proposed q-ROFDPHM operator. Their proof processes are similar to those of Theorems 2 and 3. Due to the limited space, the proof processes are omitted here.

Similar to the proposed q-ROFDPHM operator, the q-ROFDPGHM also does not consider the weight values of criteria. To tackle this deficiency, a new q-rung orthopair fuzzy weighted Dombi power geometric Heronian mean (q-ROFWDPGHM) operator is put forward in the following part.

Definition 13. Given a set of q-ROFNs $o_i=(\mu_i,\nu_i)(i=1,2,\ldots,n)$, three parameters $\gamma,\eta\geq 0$ and $\aleph>0$, and the weight values $[\omega_1,\omega_2,\ldots,\omega_n]$ of q-ROFNs, then the q-ROFWDPGHM operator is defined as

$$q-\mathrm{ROFWDPGHM}(o_1,o_2,\ldots,o_n)=\frac{1}{\gamma+\eta}\left(\bigotimes_{i=1}^{n}\bigotimes_{j=i}^{n}\left(\gamma o_i^{\left(n\omega_i(1+S(o_i))/\sum_{k=1}^{n}\omega_k(1+S(o_k))\right)}\oplus\eta o_j^{\left(n\omega_j(1+S(o_j))/\sum_{k=1}^{n}\omega_k(1+S(o_k))\right)}\right)\right)^{(2/n(n+1))}. \tag{43}$$

Based on the Dombi operational laws of q-ROFNs and GHM operator, a theorem is derived.

Theorem 6. *Given a set of q-ROFNs* $o_i=(\mu_i,\nu_i)(i=1,2,\ldots,n)$, *the parameters* $\gamma,\eta\geq 0$ *and* $\aleph>0$, *and the weight values* $[\omega_1,\omega_2,\ldots,\omega_n]$ *of q-ROFNs, then the aggregated result that is obtained from equation (43) is still a q-ROFN, which is*

$$q-\mathrm{ROFWDPGHM}(o_1,o_2,\ldots,o_n)=\left(\begin{array}{l}\left(1-\dfrac{1}{1+(n(n+1)/2(\gamma+\eta))^{(1/\aleph)}\times\left(1/\left(\sum_{i=1,j=i}^{n}\left(1/\gamma t_i\left(1/a_i^{\aleph}\right)+\eta e_j\left(1/a_j^{\aleph}\right)\right)\right)^{(1/\aleph)}\right)}\right)^{(1/q)},\\ \left(\dfrac{1}{1+(n(n+1)/2(\gamma+\eta))^{(1/\aleph)}\times\left(1/\left(\sum_{i=1,j=i}^{n}\left(1/\gamma t_i\left(1/b_i^{\aleph}\right)+\eta e_j\left(1/b_j^{\aleph}\right)\right)\right)^{(1/\aleph)}\right)}\right)^{(1/q)}\end{array}\right), \tag{44}$$

where $\xi_i = ((1+S(o_i))/\sum_{k=1}^{n}\omega_k(1+S(o_k)))$, $\psi_j = ((1+S(o_j))/\sum_{k=1}^{n}\omega_k(1+S(o_k)))$, $t_i = (1/n\omega_i\xi_i)$, $e_j = (1/n\omega_j\psi_j)$, $a_i = ((1-\mu_i^q)/\mu_i^q)$, $a_j = ((1-\mu_j^q)/\mu_j^q)$, $b_i = (\nu_i^q/(1-\nu_i^q))$, *and* $b_j = (\nu_j^q/(1-\nu_j^q))$.

Proof. The proof process is similar to that of Theorem 1. Thus, it is omitted here.

The proposed q-ROFWDPGHM operator also has the features of idempotency and boundedness as the proposed q-ROFDPHM operator. Their proof processes are similar to those of Theorems 2 and 3. Due to the limited space, the proof processes are omitted here. □

5. MULTIMOORA Method for q-Rung Orthopair Fuzzy Sets

In this section, the MULTIMOORA method is improved for processing the MCDM problems with the q-ROFS information. There usually exist the interrelationships among the criteria in the MCDM problems. Moreover, there may be extreme criteria values in the MCDM problems. To tackle these two problems, we use the proposed q-ROFWDPHM and q-ROFWDPGHM operators to modify the MULTIMOORA method.

5.1. Problem Description. Let us suppose that there exists an MCDM problem consisting of m alternatives $\{x_1, x_2, \ldots, x_m\}$ and n criteria $\{a_1, a_2, \ldots, a_n\}$. The weight values of criteria are denoted as $[\omega_1, \omega_2, \ldots, \omega_n]$, where $\sum_{j=1}^{n}\omega_j = 1$ and $0 \le \omega_j \le 1$. The decision matrix (DM) $R = (o_{ij})_{m\times n}$ corresponding to this MCDM problem consists of the evaluation information from experts. The element o_{ij} denotes the evaluation information of alternative x_i with respect to criterion a_j. In this MCDM problem, experts use the flexible q-ROFNs for expressing the evaluation information of alternative x_i with respect to criterion a_j, namely, $o_{ij} = (\mu_{ij}, \nu_{ij})$. Here, the criteria are divided into two different categories: benefit-type criteria and cost-type criteria.

Before processing DM $R = (o_{ij})_{m\times n}$, equation (45) is used to transform the values of cost-type criteria for deriving the transformed DM $\widetilde{R} = (\widetilde{o}_{ij})_{m\times n}$:

$$\widetilde{o}_{ij} = \begin{cases} (\mu_{ij}, \nu_{ij}), & \text{for benefit} - \text{type criterion}\, a, \\ (\nu_{ij}, \mu_{ij}), & \text{for cost} - \text{type criterion}\, a_j. \end{cases} \tag{45}$$

5.2. q-Rung Orthopair Fuzzy MULTIMOORA Method. According to the above problem description, we introduce the q-ROFWDPHM and q-ROFWDPGHM operators to improve the original MULTIMOORA method so as to propose a novel q-rung orthopair fuzzy MULTIMOORA (q-ROF-MULTIMOORA) method. Similar to the original MULTIMOORA method [53], the q-ROF-MULTMOORA method is also composed of three components, which are the q-rung orthopair fuzzy RS (q-ROF-RS) component, q-rung orthopair fuzzy RP (q-ROF-RP) component, and q-rung orthopair fuzzy FMF (q-ROF-FMF) component, respectively. Based on the transformed DM $\widetilde{R} = (\widetilde{o}_{ij})_{m\times n}$, these three components compute the ranking values of alternatives as follows.

5.2.1. q-ROF-RS Component. In this component, the q-ROFWDPHM operator is applied to aggregate the evaluation information of each alternative x_i with respect to its n criteria. Therefore, using (34), the aggregated criteria value of alternative x_i can be computed as

$$f_{i1} = \left(\frac{2}{n(n+1)}\bigoplus_{h=1}^{n}\bigoplus_{g=h}^{n}\left(\frac{n\omega_h(1+S(\widetilde{o}_{ih}))}{\sum_{k=1}^{n}\omega_k(1+S(\widetilde{o}_{ik}))}\widetilde{o}_{ih}\right)^{\gamma}\otimes\left(\frac{n\omega_g(1+S(\widetilde{o}_{ig}))}{\sum_{k=1}^{n}\omega_k(1+S(\widetilde{o}_{ik}))}\widetilde{o}_{ig}\right)^{\eta}\right)^{(1/\gamma+\eta)}, \tag{46}$$

where $S(\widetilde{o}_{ik}) = \sum_{l=1,l\ne k}^{n}\mathrm{Sup}(\widetilde{o}_{ik}, \widetilde{o}_{il})$ and $\mathrm{Sup}(\widetilde{o}_{ik}, \widetilde{o}_{il}) = 1 - d(\widetilde{o}_{ik}, \widetilde{o}_{il})$.

Since the aggregated value is a q-ROFN, then the score function in Definition 2 is used to derive the crisp ranking value of alternative x_i as

$$\begin{aligned} f_{i1} &= s(f_{i1}) = s\left(\left(\frac{2}{n(n+1)}\bigoplus_{h=1}^{n}\bigoplus_{g=h}^{n}\left(\frac{n\omega_h(1+S(\widetilde{o}_{ih}))}{\sum_{k=1}^{n}\omega_k(1+S(\widetilde{o}_{ik}))}\widetilde{o}_{ih}\right)^{\gamma}\otimes\left(\frac{n\omega_g(1+S(\widetilde{o}_{ig}))}{\sum_{k=1}^{n}\omega_k(1+S(\widetilde{o}_{ik}))}\widetilde{o}_{ig}\right)^{\eta}\right)^{(1/\gamma+\eta)}\right) \\ &= s\left(\begin{matrix} \left(\dfrac{1}{1+(n(n+1)/2(\gamma+\eta))^{(1/\aleph)}\times\left(1/\left(\sum_{h=1,g=h}^{n}\left(1/\left(\gamma t_{ih}a_{ih}^{\aleph}+\eta e_{ig}a_{ig}^{\aleph}\right)\right)\right)^{(1/\aleph)}\right)}\right)^{(1/q)}, \\ \left(1-\dfrac{1}{1+(n(n+1)/2(\gamma+\eta))^{(1/\aleph)}\times\left(1/\left(\sum_{h=1,g=h}^{n}\left(1/\left(\gamma t_{ih}b_{ih}^{\aleph}+\eta e_{ig}b_{ig}^{\aleph}\right)\right)\right)^{(1/\aleph)}\right)}\right)^{(1/q)} \end{matrix}\right), \end{aligned} \tag{47}$$

where $\xi_{ih} = ((1+S(\tilde{o}_{ih}))/\sum_{k=1}^{n}\omega_k(1+S(\tilde{o}_{ik})))$, $\psi_{ig} = ((1+S(\tilde{o}_{ig}))/\sum_{k=1}^{n}\omega_k(1+S(\tilde{o}_{ik})))$, $(\mu_{ih}^{q}/(1-\mu_{ih}^{q})) = (1/a_{ih})$, $(\mu_{ig}^{q}/1-\mu_{ig}^{q}) = (1/a_{ig})$, $(1-\nu_{ih}^{q}/\nu_{ih}^{q}) = (1/b_{ih})$, $((1-\nu_{ig}^{q})/\nu_{ig}^{q}) = (1/b_{ig})$, $t_{ih} = (1/n\omega_h\xi_{ih})$, and $e_{ig} = (1/n\omega_g\psi_{ig})$.

The alternative with larger ranking value is better. Hence, all the alternatives can be ranked according to the descending order of their ranking values.

5.2.2. q-ROF-RP Component. In this component, the reference point of each criterion is first derived as

$$\rho_j = \operatorname{argmax}_{i=1}^{m} s\left(\tilde{o}_{ij}\right), \quad (j = 1, 2, \ldots, n). \tag{48}$$

In the second step, Definition 4 is applied to compute the distance between the evaluation information of alternative x_i with respect to each criterion and the reference point of the same criterion as

$$\varepsilon_{ij} = d\left(\tilde{o}_{ij}, \rho_j\right). \tag{49}$$

It can be known that ε_{ij} is a real value and $\varepsilon_{ij} \geq 0$. Considering the interrelationships among criteria, the ranking value of alternative x_i is computed by aggregating the criteria distances of alternative x_i as

$$f_{i2} = \left(\frac{2}{n(n+1)}\sum_{h=1}^{n}\sum_{g=h}^{n}\left(\frac{n\omega_h\left(1+S\left(\varepsilon_{ih}\right)\right)}{\sum_{k=1}^{n}\omega_k\left(1+S\left(\varepsilon_{ik}\right)\right)}\varepsilon_{ih}\right)^{\gamma} \times \left(\frac{n\omega_g\left(1+S\left(\varepsilon_{ig}\right)\right)}{\sum_{k=1}^{n}\omega_k\left(1+S\left(\varepsilon_{ik}\right)\right)}\varepsilon_{ig}\right)^{\eta}\right)^{(1/(\gamma+\eta))}, \tag{50}$$

where $S(\varepsilon_{ik}) = \sum_{l=1, l\neq k}^{n}\operatorname{Sup}(\varepsilon_{ik}, \varepsilon_{il})$ and $\operatorname{Sup}(\varepsilon_{ik}, \varepsilon_{il}) = 1-|\varepsilon_{ik}-\varepsilon_{il}|$.

In this component, the alternative with smaller ranking value is better. Thus, all the alternatives should be ranked according to the ascending order of their ranking values.

5.2.3. q-ROF-FMF Component. In this component, the proposed q-ROFWDPGHM operator is applied to aggregate the evaluation information of each alternative x_i with respect to its n criteria. Thus, using equation (43), the aggregated criteria value of alternative x_i can be computed as

$$f_{i3} = \frac{1}{\gamma+\eta}\left(\bigotimes_{h=1}^{n}\bigotimes_{g=h}^{n}\left(\gamma\tilde{o}_{ih}^{\left(n\omega_h\left(1+S\left(\tilde{o}_{ih}\right)\right)/\sum_{k=1}^{n}\omega_k\left(1+S\left(\tilde{o}_{ik}\right)\right)\right)} \oplus \eta\tilde{o}_{ig}^{\left(n\omega_g\left(1+S\left(\tilde{o}_{ig}\right)\right)/\sum_{k=1}^{n}\omega_k\left(1+S\left(\tilde{o}_{ik}\right)\right)\right)}\right)\right)^{(2/n(n+1))}, \tag{51}$$

where $S(\tilde{o}_{ik}) = \sum_{l=1, l\neq k}^{n}\operatorname{Sup}(\tilde{o}_{ik}, \tilde{o}_{il})$ and $\operatorname{Sup}(\tilde{o}_{ik}, \tilde{o}_{il}) = 1-d(\tilde{o}_{ik}, \tilde{o}_{il})$.

Since the aggregated value is a q-ROFN, then the score function in Definition 2 is used to derive the crisp ranking value of alternative x_i as

$$f_{i3} = s\left(f_{i3}\right) = s\left(\begin{array}{c}\left(1-\dfrac{1}{1+\left(n(n+1)/2(\gamma+\eta)\right)^{(1/\aleph)} \times \left(1/\left(\sum_{h=1,g=h}^{n}\left(1/\gamma t_{ih}\left(1/a_{ih}^{\aleph}\right)+\eta e_{ig}\left(1/a_{ig}^{\aleph}\right)\right)\right)^{(1/\aleph)}\right)}\right)^{(1/q)}, \\ \left(\dfrac{1}{1+\left(n(n+1)/2(\gamma+\eta)\right)^{(1/\aleph)} \times \left(1/\left(\sum_{h=1,g=h}^{n}\left(1/\gamma t_{ih}\left(1/b_{ih}^{\aleph}\right)+\eta e_{ig}\left(1/b_{ig}^{\aleph}\right)\right)\right)^{(1/\aleph)}\right)}\right)^{(1/q)}\end{array}\right), \tag{52}$$

where $\xi_{ih} = ((1+S(\tilde{o}_{ih}))/\sum_{k=1}^{n}\omega_k(1+S(\tilde{o}_{ik})))$, $\psi_{ig} = ((1+S(\tilde{o}_{ig}))/\sum_{k=1}^{n}\omega_k(1+S(\tilde{o}_{ik})))$, $a_{ih} = ((1-\mu_{ih}^{q})/\mu_{ih}^{q})$, $a_{ig} = (1-\mu_{ig}^{q}/\mu_{ig}^{q})$, $b_{ih} = (\nu_{ih}^{q}/(1-\nu_{ih}^{q}))$, $b_{ig} = (\nu_{ig}^{q}/(1-\nu_{ig}^{q}))$, $t_{ih} = (1/n\omega_h\xi_{ih})$, and $e_{ig} = (1/n\omega_g\psi_{ig})$.

In this component, the alternative with larger ranking value is better. Hence, all the alternatives can be ranked according to the descending order of their ranking values.

After obtaining the ranking values of all the alternatives from these three components, we need to fuse them for deriving the final ranking values. In the original MULTIMOORA method, the dominance theory is usually used to aggregate three ranking orders for deriving the final ranking order. However, it is incapable of handling massive operations resulting from its cumbersome pairwise comparison processes [54]. For the purpose of overcoming the deficiency of dominancy theory, the HM operator is put forward for integrating the ranking values of alternatives obtained from three components of the proposed q-ROF-MULTIMOORA method. The HM operator owns the advantage of capturing the interrelationships hiding behind input values. Afterwards, by using the ranking values obtained from equations (47)–(52), here a new DM is constructed, where the three components of the q-ROF-MULTIMOORA method are regarded as criteria of alternatives: q-ROF-RS component (c_1), q-ROF-RP component (c_2), and q-ROF-FMF component (c_3). Hence, the new DM M is constructed as

$$M = \begin{array}{c} \\ x_1 \\ x_2 \\ \vdots \\ x_m \end{array} \begin{array}{c} \begin{array}{ccc} c_1 & c_2 & c_3 \end{array} \\ \begin{pmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ \vdots & \vdots & \vdots \\ f_{m1} & f_{m2} & f_{m3} \end{pmatrix} \end{array}, \quad (53)$$

where $\{x_1, x_2, \ldots, x_m\}$ denotes the set of alternatives and $\{c_1, c_2, c_3\}$ denotes the set of criteria. The element $f_{iy}\,(y = 1, 2, 3)$ in the DM M denotes the ranking value of the alternative x_i with respect to the criterion c_y. Let $\chi = \{\chi_1, \chi_2, \chi_3\}$ be the weight values of criteria $\{c_1, c_2, c_3\}$, satisfying $0 \le \chi_y \le 1$ and $\sum_{y=1}^{3} \chi_y = 1$. In general, the weight values of criteria are set to $\chi_1 = \chi_2 = \chi_3 = (1/3)$. For the DM M, the ranking values of each alternative with respect to three criteria should be aggregated for determining the final ranking values. However, the ranking values $f_{iy}\,(y = 1, 2, 3)$ show different dimensions because they are obtained from the different components. For the purpose of making them dimensionless, all the ranking values $f_{iy}\,(y = 1, 2, 3)$ are normalized as

$$\tilde{f}_{iy} = \begin{cases} \dfrac{f_{iy} - \min_{1\le i\le m} f_{iy}}{\max_{1\le i\le m} f_{iy} - \min_{1\le i\le m} f_{iy}}, & \text{for benefit} - \text{type criterion } c_y, \\[2ex] \dfrac{\max_{1\le i\le m} f_{iy} - f_{iy}}{\max_{1\le i\le m} f_{iy} - \min_{1\le i\le m} f_{iy}}, & \text{for cost} - \text{type criterion } c_y, \end{cases} \quad (54)$$

where $1 \le i \le m$ and $1 \le y \le 3$.

Afterwards, the weighted HM operator [55] is used to aggregate the normalized ranking values $\tilde{f}_{iy}$ of each alternative x_i with respect to three criteria for deriving the final ranking value of this alternative as

$$F_i = \left(\frac{2}{3(3+1)} \sum_{h=1}^{3} \sum_{g=h}^{3} \left(\chi_h \tilde{f}_{ih}\right)^{\gamma} \left(\chi_g \tilde{f}_{ig}\right)^{\eta} \right)^{(1/(\gamma+\eta))}. \quad (55)$$

The alternative with larger final ranking value is better. Hence, all the alternatives can be ranked based on the descending order of their final ranking values.

5.3. Decision-Making Procedure. Based on the discussion and results in Section 5.2, the decision-making procedure of the proposed q-ROF-MULTIMOORA method is summarized using the following 7 steps.

Step 1: all the evaluation information is collected for constructing DM $R = (o_{ij})_{m\times n} = (\mu_{ij}, \nu_{ij})_{m\times n}$. At the same time, the values of the parameters q, γ, η, and $\aleph$ should be provided.

Step 2: to transform the criteria values of each alternative with respect to cost-type criteria, (45) is used to transform DM $R = (o_{ij})_{m\times n}$ into DM $\tilde{R} = (\tilde{o}_{ij})_{m\times n}$.

Step 3: for the transformed DM $\tilde{R} = (\tilde{o}_{ij})_{m\times n}$, (47) is applied to compute the ranking value f_{i1} of each alternative x_i with respect to the q-ROF-RS component. The alternatives can be ranked according to the descending order of their ranking values.

Step 4: for the transformed DM $\tilde{R} = (\tilde{o}_{ij})_{m\times n}$, (50) is applied to compute the ranking value f_{i2} of each alternative x_i with respect to the q-ROF-RP component. The alternatives can be ranked according to the ascending order of their ranking values.

Step 5: for the transformed DM $\tilde{R} = (\tilde{o}_{ij})_{m\times n}$, (52) is applied to compute the ranking value f_{i3} of each alternative x_i with respect to the q-ROF-FMF component. The alternatives can be ranked according to the descending order of their ranking values.

Step 6: based on the ranking values of alternatives obtained from three components in Steps 3–5, a new DM $M = (f_{iy})_{m\times 3}$ is constructed. Afterwards, (54) is applied to transform DM $M = (f_{iy})_{m\times 3}$ into $\widetilde{M} = (\widetilde{f}_{iy})_{m\times 3}$.

Step 7: in the final step, (55) is used to aggregate the ranking values of alternatives with respect to three components of the q-ROF-MULTIMOORA method for deriving the final ranking values. Then, all the alternatives are ranked according to the descending order of their final ranking values.

The above steps are also shown in Figure 1.

The q-ROF-MULTIMOORA method is a combination of PA operator, Dombi operational laws, AHM, GHM, and MULTIMOORA. It shows the following advantages:

(1) It has the ability of alleviating negative influences of extreme criteria values on the decision results, which makes the decision results more stable and robust.

(2) It shows strong flexibility when computing the criteria values due to the Dombi operational laws of q-ROFNs.

(3) The HM and GHM operators are capable of capturing the complex interrelationships hiding behind the criteria values. Moreover, the MULTIMOORA method integrates the ranking values obtained from three components for deriving the final ranking values. Thus, the decision results of the q-ROF-MULTIMOORA method are more reasonable and effective.

6. Illustrative Example and Comparison Analysis

In this section, a practical case concerning the evaluation of security algorithm is shown to illustrate the decision-making procedure of the proposed q-ROF-MULTIMOORA method. Afterwards, the influences of the parameters on the decision results are analyzed. Finally, the proposed q-ROF-MULTIMOORA method is compared with the original MULTIMOORA method for processing q-ROFNs.

6.1. Decision Process Using the q-ROF-MULTIMOORA Method. In this section, a real case concerning the evaluation of security algorithms is provided to illustrate the decision procedure of the proposed q-ROF-MULTIMOORA method.

Example 1. With the quick development of Internet applications, more and more user data are stored online. Hackers frequently attack the Internet applications for obtaining the privacy data. To protect users' privacy data, various security algorithms have been designed and implemented. However, these security algorithms show different features. How to choose the suitable security algorithm is a big challenge for organizations since multiple criteria should be considered. Here, we try to formulate the process of evaluating the security algorithms and selecting a suitable one as a classical MCDM problem. Suppose organization plans to evaluate 5 candidates of security algorithms and select the suitable one by considering 6 criteria: function (c_1), reliability (c_2), usability (c_3), performance (c_4), portability (c_5), and complexity (c_6). Hence, an MCDM problem composed of 5 security algorithms $\{x_1, x_2, x_3, x_4, x_5\}$ and 6 criteria $\{c_1, c_2, c_3, c_4, c_5, c_6\}$ can be constructed. According to the real requirements for building the security system, the organization sets the weights of criteria as $\omega = (0.10, 0.15, 0.35, 0.20, 0.10, 0.10)$. The technical panel of this organization uses the q-ROFNs to evaluate these five security algorithms with respect to their criteria. All the q-ROFNs are collected to form the DM $R = (o_{ij})_{5\times 6} = (\mu_{ij}, \nu_{ij})_{5\times 6}$, as shown in Table 2.

Step 1: the values of the parameters γ, η, and $\aleph$ are set to 1 and the value of the parameter q is set to 3.

Step 2: the first five criteria are benefit-type criteria, while the maintenance cost is cost-type criteria. Hence, (45) is used to transform DM $R = (o_{ij})_{5\times 6}$ in Table 2 into DM $\widetilde{R} = (\widetilde{o}_{ij})_{5\times 6}$ as depicted in Table 3.

Step 3: for the transformed DM $\widetilde{R} = (\widetilde{o}_{ij})_{5\times 6}$, (47) is applied to compute the ranking value f_{i1} of each security algorithm x_i with respect to the q-ROF-RS component as

$$\begin{aligned} f_{11} &= 0.036,\\ f_{21} &= 0.027,\\ f_{31} &= 0.033,\\ f_{41} &= 0.111,\\ f_{51} &= 0.047. \end{aligned} \tag{56}$$

Hence, these security algorithms can be ranked as $x_4 \succ x_5 \succ x_1 \succ x_3 \succ x_2$.

Step 4: for the transformed DM $\widetilde{R} = (\widetilde{o}_{ij})_{5\times 6}$, (50) is applied to compute the ranking value f_{i2} of each security algorithm x_i with respect to the q-ROF-RP component as

$$\begin{aligned} f_{12} &= 0.179,\\ f_{22} &= 0.225,\\ f_{32} &= 0.213,\\ f_{42} &= 0.031,\\ f_{52} &= 0.229. \end{aligned} \tag{57}$$

Hence, these security algorithms can be ranked as $x_4 \succ x_1 \succ x_3 \succ x_2 \succ x_5$.

Step 5: for the transformed DM $\widetilde{R} = (\widetilde{o}_{ij})_{5\times 6}$, (52) is applied to compute the ranking value f_{i3} of each security algorithm x_i with respect to the q-ROF-FMF component as

$$\begin{aligned} f_{13} &= -0.004,\\ f_{23} &= -0.018,\\ f_{33} &= 0.007,\\ f_{43} &= -0.001,\\ f_{53} &= -0.035. \end{aligned} \tag{58}$$

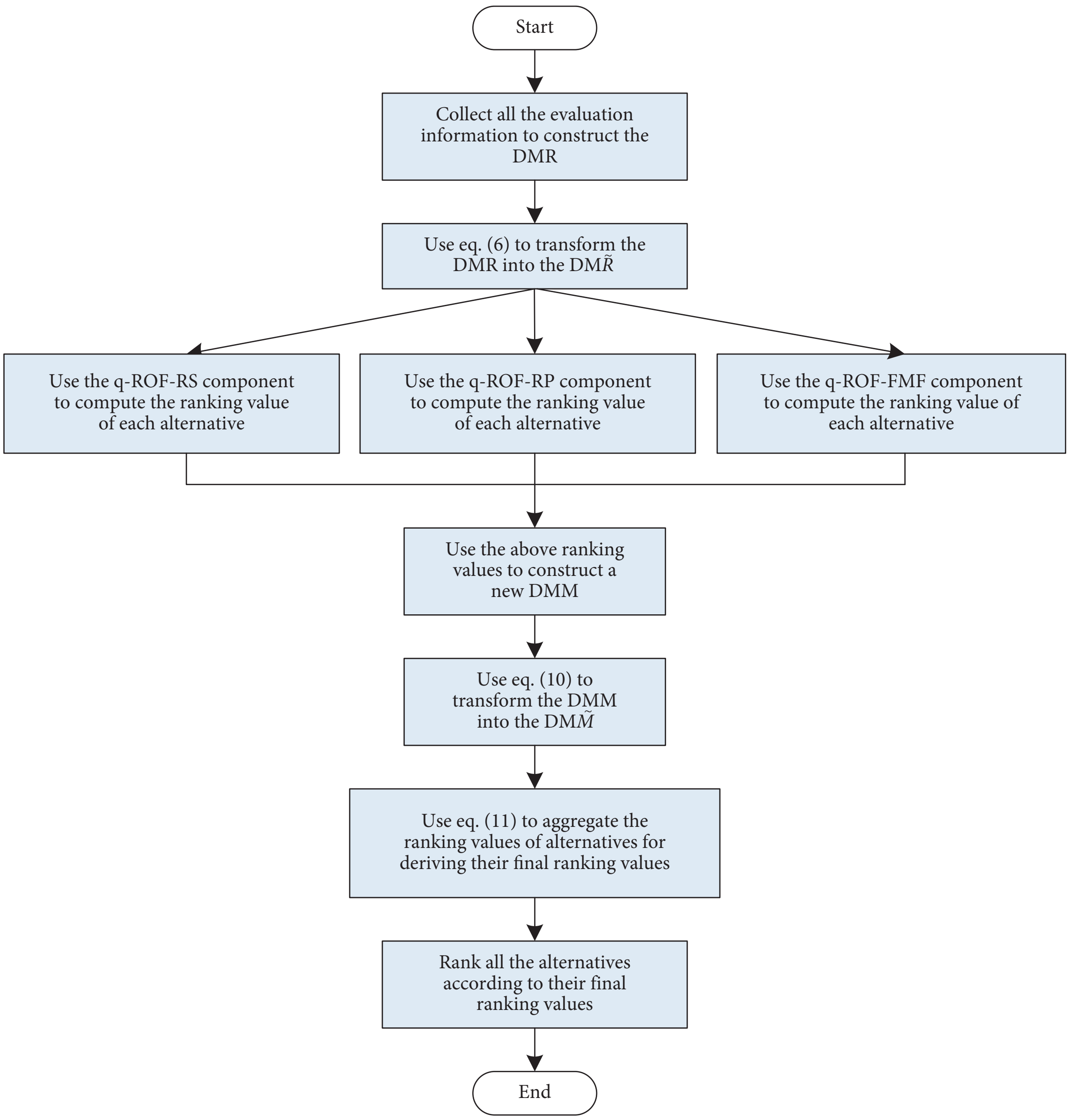

Figure 1: The frame diagram of the proposed method.

Hence, these security algorithms can be ranked as $x_3 \succ x_4 \succ x_1 \succ x_2 \succ x_5$.

Step 6: based on the ranking values of security algorithms obtained from three components in Steps 3–5, a new DM $M = (f_{iy})_{m\times 3}$ is formed. Afterwards, (54) is applied to transform the DM $M = (f_{iy})_{m\times 3}$ into $\widetilde{M} = (\widetilde{f}_{iy})_{m\times 3}$ as

$$\widetilde{M} = \begin{pmatrix} 0.107 & 0.253 & 0.738 \\ 0.000 & 0.020 & 0.405 \\ 0.071 & 0.081 & 1.000 \\ 1.000 & 1.000 & 0.810 \\ 0.238 & 0.000 & 0.000 \end{pmatrix}. \quad (59)$$

Step 7: in the final step, equation (55) is used to aggregate the ranking values of five security algorithms with respect to three components for deriving the final ranking values as

$$\begin{aligned} F_1 &= 0.130, \\ F_2 &= 0.057, \\ F_3 &= 0.147, \\ F_4 &= 0.313, \\ F_5 &= 0.032. \end{aligned} \quad (60)$$

Then, the final ranking order of these security algorithms is $x_4 \succ x_3 \succ x_1 \succ x_2 \succ x_5$. Thus, the security algorithm x_4 is the suitable one for the organization when building the security system.

6.2. Influences of the Parameters on the Ranking Results. In this section, the influences of the parameters on the ranking results are discussed.

6.2.1. Influence of the Parameter q on the Final Ranking Results. The influence of the parameter q on the final ranking results of the q-ROF-MULTIMOORA method is first discussed. In this case, the parameters $\gamma = \eta = \aleph = 1$. For the transformed DM $R = (o_{ij})_{5\times 6}$ in Table 2, the ranking results of security algorithms are shown in Table 4 and Figure 2 when the value of the parameter q varies.

From Table 4, it can be known that the ranking results of security algorithms are different when the value of q varies. When $q = 1$, the ranking result of security algorithms is $x_4 \succ x_1 \succ x_3 \succ x_2 \succ x_5$. When $q = 2$, the ranking result of security algorithms is $x_4 \succ x_2 \succ x_1 \succ x_3 \succ x_5$. When $q = 3$ or $q = 5$, the ranking results of security algorithms are $x_4 \succ x_3 \succ x_1 \succ x_2 \succ x_5$. Although the ranking result of security algorithms changes when the value of the parameter q varies, the most suitable security algorithm keeps unchanged, namely, x_4. When $q = 1$, then q-ROFNs reduce to IFNs. When $q = 2$, then q-ROFNs reduce to PFNs. How to determine the reasonable value of q depends on the evaluation information provided by the expert. The smallest value of the parameter q should satisfy $\mu^q + \nu^q \leq 1$. For instance, if the evaluation information given by the expert is (0.9, 0.9), then the smallest value of the parameter q should be 7 so that $0.9^7 + 0.9^7 < 1$.

6.2.2. Influences of the Parameters γ and η on the Ranking Results. The influences of the parameters γ and η on the ranking results of the q-ROF-MULTIMOORA method are analyzed in this part. In this case, the parameters $\aleph = 1$ and $q = 3$. For DM $R = (o_{ij})_{5\times 6}$ in Table 2, the ranking results of security algorithms are shown in Table 5 and Figure 3 when the values of the parameters γ and η vary.

From Table 5, it can be seen that the ranking result obtained from the q-ROF-MULTIMOORA method is always $x_4 \succ x_3 \succ x_1 \succ x_2 \succ x_5$ except when $\gamma = 0$ and $\eta = 1$. Nevertheless, the most suitable security algorithm is always x_4. When $\gamma = 0$ and $\eta = 1$, the ranking result of security algorithms changes into $x_4 \succ x_1 \succ x_3 \succ x_5 \succ x_2$. Thus, the ranking result obtained from the q-ROF-MULTIMOORA method is not sensitive to the values of these two parameters. In other words, the q-ROF-MULTIMOORA method is robust and effective.

6.2.3. Influence of the Parameter $\aleph$ on the Ranking Results. The influence of the parameter $\aleph$ on the ranking results of the q-ROF-MULTIMOORA method is analyzed in this part. In this case, the parameters $\gamma = \eta = 1$ and $q = 3$. For DM $R = (o_{ij})_{5\times 6}$ in Table 2, the ranking results of security algorithms are listed in Table 6 and Figure 4 when the value of the parameter $\aleph$ varies.

Table 2: The q-rung orthopair fuzzy DM.

	c_1	c_2	c_3	c_4	c_5	c_6
x_1	(0.1,0.2)	(0.3,0.3)	(0.5,0.3)	(0.6,0.3)	(0.2,0.1)	(0.1,0.1)
x_2	(0.3,0.2)	(0.1,0.1)	(0.5,0.1)	(0.4,0.5)	(0.3,0.2)	(0.6,0.1)
x_3	(0.6,0.1)	(0.2,0.2)	(0.4,0.3)	(0.2,0.4)	(0.3,0.2)	(0.1,0.2)
x_4	(0.2,0.5)	(0.1,0.3)	(0.7,0.1)	(0.8,0.1)	(0.4,0.3)	(0.3,0.3)
x_5	(0.4,0.4)	(0.4,0.5)	(0.5,0.2)	(0.1,0.4)	(0.3,0.4)	(0.4,0.4)

Table 3: The transformed q-rung orthopair fuzzy DM.

	c_1	c_2	c_3	c_4	c_5	c_6
x_1	(0.1,.02)	(0.3,0.3)	(0.5,0.3)	(0.6,0.3)	(0.2,0.1)	(0.1,0.1)
x_2	(0.3,0.2)	(0.1,0.1)	(0.5,0.1)	(0.4,0.5)	(0.3,0.2)	(0.1,0.6)
x_3	(0.6,0.1)	(0.2,0.2)	(0.4,0.3)	(0.2,0.4)	(0.3,0.2)	(0.2,0.1)
x_4	(0.2,0.5)	(0.1,0.3)	(0.7,0.1)	(0.8,0.1)	(0.4,0.3)	(0.3,0.3)
x_5	(0.4,0.4)	(0.4,0.5)	(0.5,0.2)	(0.1,0.4)	(0.3,0.4)	(0.4,0.4)

From Table 6, it can be seen that the ranking result obtained from the q-ROF-MULTIMOORA method slightly changes when the value of the parameter $\aleph$ varies. When the value of the parameter $\aleph$ is set to $\aleph = 1$, then the ranking result of security algorithms is $x_4 \succ x_3 \succ x_1 \succ x_2 \succ x_5$. When the value of the parameter $\aleph$ is set to a value in the integer set $\{2, 3, \ldots, 10\}$, then the ranking result of security algorithms is changed into $x_4 \succ x_1 \succ x_3 \succ x_5 \succ x_2$. However, the most suitable security algorithm always keeps unchanged, namely, x_4 no matter how the value of the parameter $\aleph$ varies. Hence, the ranking result that is obtained from the q-ROF-MULTIMOORA method is relatively stable. Because of the Dombi operational laws for q-ROFNs, the q-ROF-MULTIMOORA method has high flexibility by providing the parameter $\aleph$. Experts can adjust the value of the parameter $\aleph$ according to the actual situation of MCDM problems.

6.3. Comparative Analysis. For the proposed q-ROF-MULTIMOORA method, it applies the PA operator to alleviate the negative influence of extreme values on the ranking results and integrates the AHM and GHM operators to handle the interrelationships hiding behind criteria values. For the purpose of verifying the effectiveness of the q-ROF-MULTIMOORA method, it is compared with the original MULTIMOORA method [32, 56] for handling the q-ROFNs. Different from the q-ROF-MULTIMOORA method, the original MULTIMOORA method does not contain the PA operator to solve the problem of extreme values and also does not integrate the AHM and GHM operators to handle the interrelationships among criteria values. Hence, it is a suitable way for comparing the q-ROF-MULTIMOORA method with the original MULTIMOORA method. For the purpose of conducting this comparative analysis, an example of evaluating blockchain platforms is given.

Example 2. The blockchain technology has the ability to solve the problems resulting from our increasingly connected society and tackle real-world business concerns. It has been broadly applied to many fields such as distributed

Table 4: Ranking results of the q-ROF-MULTIMOORA method when the value of q varies.

	Final ranking values of security algorithms	Ranking results
$q = 1$	$F_1 = 0.210$, $F_2 = 0.096$, $F_3 = 0.192$, $F_4 = 0.333$, and $F_5 = 0.000$	$x_4 \succ x_1 \succ x_3 \succ x_2 \succ x_5$
$q = 2$	$F_1 = 0.093$, $F_2 = 0.191$, $F_3 = 0.069$, $F_4 = 0.312$, and $F_5 = 0.043$	$x_4 \succ x_2 \succ x_1 \succ x_3 \succ x_5$
$q = 3$	$F_1 = 0.130$, $F_2 = 0.057$, $F_3 = 0.147$, $F_4 = 0.313$, and $F_5 = 0.032$	$x_4 \succ x_3 \succ x_1 \succ x_2 \succ x_5$
$q = 5$	$F_1 = 0.136$, $F_2 = 0.019$, $F_3 = 0.142$, $F_4 = 0.302$, and $F_5 = 0.014$	$x_4 \succ x_3 \succ x_1 \succ x_2 \succ x_5$

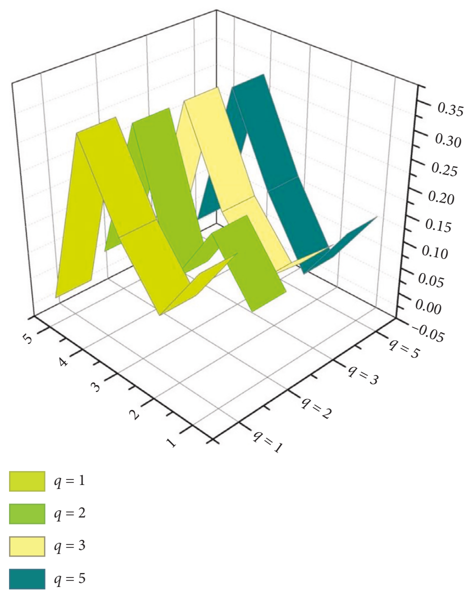

Figure 2: Ranking results of the q-ROF-MULTIMOORA method when the value of q varies.

Table 5: Ranking results of the q-ROF-MULTIMOORA method when the values of γ and η vary.

	Final ranking values of security algorithms	Ranking results
$\gamma = 1$ and $\eta = 0$	$F_1 = 0.094$, $F_2 = 0.038$, $F_3 = 0.118$, $F_4 = 0.307$, and $F_5 = 0.021$	$x_4 \succ x_3 \succ x_1 \succ x_2 \succ x_5$
$\gamma = 1$ and $\eta = 0.5$	$F_1 = 0.112$, $F_2 = 0.041$, $F_3 = 0.131$, $F_4 = 0.312$, and $F_5 = 0.024$	$x_4 \succ x_3 \succ x_1 \succ x_2 \succ x_5$
$\gamma = 1$ and $\eta = 1$	$F_1 = 0.130$, $F_2 = 0.057$, $F_3 = 0.147$, $F_4 = 0.313$, and $F_5 = 0.032$	$x_4 \succ x_3 \succ x_1 \succ x_2 \succ x_5$
$\gamma = 1$ and $\eta = 2$	$F_1 = 0.151$, $F_2 = 0.070$, $F_3 = 0.187$, $F_4 = 0.310$, and $F_5 = 0.044$	$x_4 \succ x_3 \succ x_1 \succ x_2 \succ x_5$
$\gamma = 1$ and $\eta = 3$	$F_1 = 0.174$, $F_2 = 0.082$, $F_3 = 0.215$, $F_4 = 0.312$, and $F_5 = 0.051$	$x_4 \succ x_3 \succ x_1 \succ x_2 \succ x_5$
$\gamma = 1$ and $\eta = 4$	$F_1 = 0.185$, $F_2 = 0.086$, $F_3 = 0.235$, $F_4 = 0.315$, and $F_5 = 0.055$	$x_4 \succ x_3 \succ x_1 \succ x_2 \succ x_5$
$\gamma = 0$ and $\eta = 1$	$F_1 = 0.197$, $F_2 = 0.003$, $F_3 = 0.170$, $F_4 = 0.333$, and $F_5 = 0.070$	$x_4 \succ x_1 \succ x_3 \succ x_5 \succ x_2$
$\gamma = 0.5$ and $\eta = 1$	$F_1 = 0.139$, $F_2 = 0.043$, $F_3 = 0.155$, $F_4 = 0.308$, and $F_5 = 0.024$	$x_4 \succ x_3 \succ x_1 \succ x_2 \succ x_5$
$\gamma = 2$ and $\eta = 1$	$F_1 = 0.144$, $F_2 = 0.073$, $F_3 = 0.184$, $F_4 = 0.313$, $F_5 = 0.047$	$x_4 \succ x_3 \succ x_1 \succ x_2 \succ x_5$
$\gamma = 3$ and $\eta = 1$	$F_1 = 0.162$, $F_2 = 0.093$, $F_3 = 0.213$, $F_4 = 0.317$, and $F_5 = 0.051$	$x_4 \succ x_3 \succ x_1 \succ x_2 \succ x_5$
$\gamma = 4$ and $\eta = 1$	$F_1 = 0.181$, $F_2 = 0.106$, $F_3 = 0.233$, $F_4 = 0.318$, and $F_5 = 0.054$	$x_4 \succ x_3 \succ x_1 \succ x_2 \succ x_5$

cloud storage and health care, as well as payment and transfers. To benefit from the blockchain technology, organizations across the globe try to introduce the blockchain technology for building their own decentralized systems. The blockchain platforms are a good solution, which help organizations to facilitate the development and deployment of decentralized systems. Suppose that an organization wants to evaluate five blockchain platforms with respect to five criteria: usability, performance, scalability, security, and cost. Therefore, evaluating these five blockchain platforms can be formulated as an MCDM composed of blockchain platforms $\{x_1, x_2, x_3, x_4, x_5\}$ with respect to criteria $\{c_1, c_2, c_3, c_4, c_5\}$. The weights of criteria are set to $\omega = (0.10, 0.25, 0.35, 0.2, 0.1)$. The transformed DM $R = (o_{ij})_{5\times 5} = (\mu_{ij}, \nu_{ij})_{5\times 5}$ is given in Table 7.

The original MULTIMOORA method and q-ROF-MULTIMOORA method are applied to process the transformed q-rung orthopair fuzzy DM in Table 7. Because of the limited space, the computation processes are omitted here and the ranking results of different methods are provided in Table 8.

In Table 8, the ranking results obtained from the q-ROF-MULTIMOORA method and original method are provided. Moreover, the ranking results obtained from the three components of q-ROF-MULTIMOORA and original method are also given. From Table 8, it can be noted that the

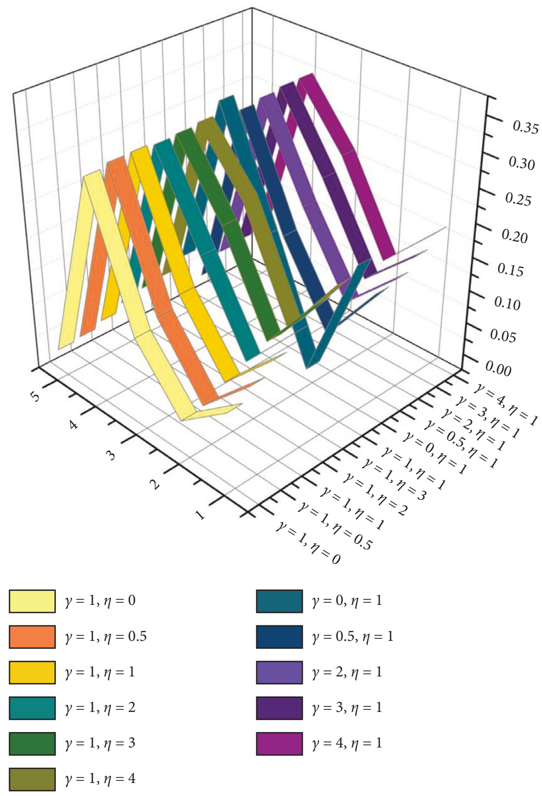

Figure 3: Ranking results of the q-ROF-MULTIMOORA method when the values of γ and η vary.

Table 6: Ranking results of the q-ROF-MULTIMOORA method when the value of $\aleph$ varies.

	Final ranking values of security algorithms	Ranking results
$\aleph = 1$	$F_1 = 0.130$, $F_2 = 0.057$, $F_3 = 0.147$, $F_4 = 0.313$, and $F_5 = 0.032$	$x_4 \succ x_3 \succ x_1 \succ x_2 \succ x_5$
$\aleph = 2$	$F_1 = 0.153$, $F_2 = 0.003$, $F_3 = 0.150$, $F_4 = 0.297$, and $F_5 = 0.027$	$x_4 \succ x_1 \succ x_3 \succ x_5 \succ x_2$
$\aleph = 3$	$F_1 = 0.170$, $F_2 = 0.003$, $F_3 = 0.154$, $F_4 = 0.296$, and $F_5 = 0.054$	$x_4 \succ x_1 \succ x_3 \succ x_5 \succ x_2$
$\aleph = 4$	$F_1 = 0.172$, $F_2 = 0.003$, $F_3 = 0.151$, $F_4 = 0.291$, and $F_5 = 0.063$	$x_4 \succ x_1 \succ x_3 \succ x_5 \succ x_2$
$\aleph = 5$	$F_1 = 0.173$, $F_2 = 0.003$, $F_3 = 0.149$, $F_4 = 0.286$, and $F_5 = 0.065$	$x_4 \succ x_1 \succ x_3 \succ x_5 \succ x_2$
$\aleph = 6$	$F_1 = 0.173$, $F_2 = 0.003$, $F_3 = 0.147$, $F_4 = 0.285$, and $F_5 = 0.067$	$x_4 \succ x_1 \succ x_3 \succ x_5 \succ x_2$
$\aleph = 7$	$F_1 = 0.173$, $F_2 = 0.003$, $F_3 = 0.145$, $F_4 = 0.283$, and $F_5 = 0.066$	$x_4 \succ x_1 \succ x_3 \succ x_5 \succ x_2$
$\aleph = 8$	$F_1 = 0.174$, $F_2 = 0.003$, $F_3 = 0.145$, $F_4 = 0.283$, and $F_5 = 0.066$	$x_4 \succ x_1 \succ x_3 \succ x_5 \succ x_2$
$\aleph = 9$	$F_1 = 0.174$, $F_2 = 0.003$, $F_3 = 0.144$, $F_4 = 0.282$, and $F_5 = 0.067$	$x_4 \succ x_1 \succ x_3 \succ x_5 \succ x_2$
$\aleph = 10$	$F_1 = 0.174$, $F_2 = 0.003$, $F_3 = 0.142$, $F_4 = 0.282$, and $F_5 = 0.067$	$x_4 \succ x_1 \succ x_3 \succ x_5 \succ x_2$

ranking results obtained from the q-ROF-MULTIMOORA method and original MULTIMOORA method are different. Moreover, the ranking result obtained from the q-ROF-RS component of the q-ROF-MULTIMOORA method is different from that obtained from the RS component of the original MULTIMOORA method, so do the ranking results of other two components in the q-ROF-MULTIMOORA method and original MULTIMOORA method. The reasons are analyzed as follows:

(1) In the process of evaluating blockchain platforms, there are the interrelationships hiding behind the criteria values. The q-ROF-MULTIMOORA method has been equipped with the AHM and GHM operators to process the interrelationships, while the original MULTIMOORA method is unable to process the hiding interrelationships.

(2) For the q-rung orthopair fuzzy DM R, there exists relatively great difference among criteria values. The q-ROF-MULTIMOORA method is integrated with the PA operator to alleviate the negative impact of extreme criteria values on the ranking results, while the original MULTIMOORA method ignores this case.

From the above analysis, it can be noted that the q-ROF-MULTIMOORA method performs better than the original MUTLIMOORA method because the q-ROF-MULTIMOORA method derives more robust and reasonable ranking results.

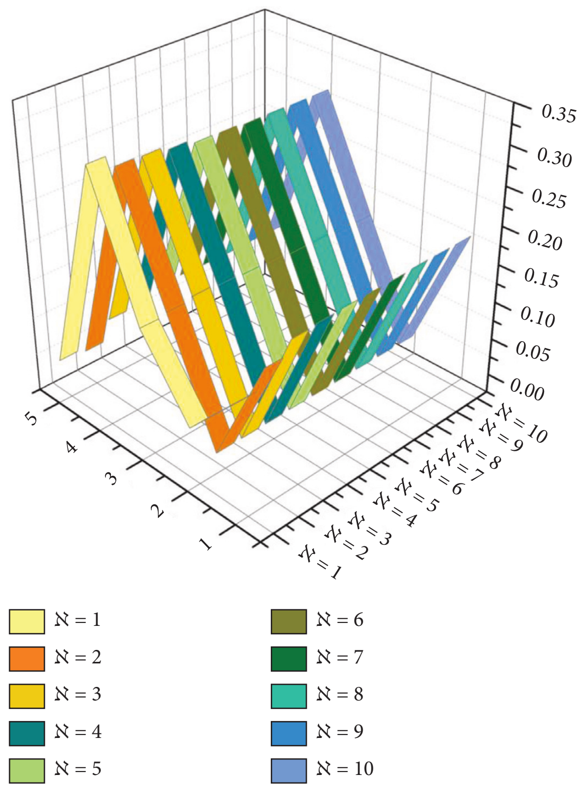

Figure 4: Ranking results of the q-ROF-MULTIMOORA method when the value of $\aleph$ varies.

Table 7: The transformed q-rung orthopair fuzzy DM for evaluating blockchain platforms.

	c_1	c_2	c_3	c_4	c_5
x_1	(0.5,0.1)	(0.5,0.1)	(0.6,0.1)	(0.8,0.1)	(0.5,0.3)
x_2	(0.1,0.2)	(0.2,0.2)	(0.3,0.4)	(0.4,0.1)	(0.2,0.2)
x_3	(0.3,0.1)	(0.2,0.4)	(0.5,0.3)	(0.5,0.4)	(0.3,0.1)
x_4	(0.4,0.2)	(0.7,0.2)	(0.6,0.1)	(0.5,0.1)	(0.7,0.2)
x_5	(0.5,0.4)	(0.2,0.1)	(0.2,0.4)	(0.3,0.5)	(0.2,0.4)

Table 8: Ranking results of five blockchain platforms when using different methods.

	Decision method	Ranking results
1	RS component [32]	$x_1 \succ x_4 \succ x_3 \succ x_2 \succ x_5$
2	RP component [32]	$x_1 \succ x_4 \succ x_3 \succ x_2 \succ x_5$
3	FMF component [32]	$x_4 \succ x_1 \succ x_3 \succ x_2 \succ x_5$
4	The original MULTIMOORA [32]	$x_1 \succ x_4 \succ x_3 \succ x_2 \succ x_5$
5	q-ROF-RS component	$x_4 \succ x_1 \succ x_3 \succ x_5 \succ x_2$
6	q-ROF-RP component	$x_4 \succ x_1 \succ x_3 \succ x_2 \succ x_5$
7	q-ROF-FMF component	$x_1 \succ x_4 \succ x_3 \succ x_2 \succ x_5$
8	q-ROF-MULTIMOORA	$x_4 \succ x_1 \succ x_3 \succ x_2 \succ x_5$

7. Conclusions

To solve the security algorithms' evaluation problem, we propose an efficient q-ROF-MULTIMOORA method in this paper. Our contributions are listed as follows:

(1) We combine the PA operator, Dombi operational laws, and AHM and GHM operators to design the q-ROFDPHM, q-ROFWDPHM, q-ROFDPGHM, and q-ROFWDPGHM operators to aggregate q-ROFNs.

(2) The proposed q-ROFWDPHM and q-ROFWDPGHM operators are applied to modify the original MULTIMOORA method for proposing a novel q-ROF-MULTIMOORA method.

(3) A practical case of evaluating five security algorithms is given to show the decision procedure of the q-ROF-MULTIMOORA method. The influences of the parameters on the ranking results are analyzed.

(4) To validate the effectiveness of the proposed q-ROF-MULTIMOORA method, a new example of evaluating blockchain platforms is given.

The proposed methods also have some limitations:

(1) the q-ROFSs model, the uncertain information uses only three characteristic functions and does not have the characteristic function that denotes the degree of abstinence. This limitation can be removed by introducing the concept of T-spherical fuzzy sets, which was proposed by Mahmood et al. [57]. It has been studied by many scholars [58,59].

(2) The weights of attributes are directly given in this study. It ignores the objective significance. The method combining the objective weights and subjective weights should be considered in the future.

(3) In the proposed q-ROF-MULTIMOORA method, the q-ROFDPHM and q-ROFDPGHM operators do not consider the interaction between the membership degree and the nonmembership degree of q-ROFSs, which will produce unreasonable aggregated results.

The proposed q-ROF-MULTIMOORA method has some potential applications. In the future research plan, we intend to apply the proposed method into the sustainable supplier selection [60]. According to the third limitation mentioned in the above paragraph, the idea of interaction operational rules [61] will be used to improve the proposed method.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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