Hindawi
Mathematical Problems in Engineering
Volume 2022, Article ID 3196608, 11 pages
https://doi.org/10.1155/2022/3196608

Research Article

Study of a Fractional System of Predator-Prey with Uncertain Initial Conditions

Shabir Ahmad ,[1] Aman Ullah,[1] and Ali Akgul[2]

[1]*Department of Mathematics, University of Malakand Chakdara Dir Lower, Khyber, Pakhtunkhwa, Pakistan*
[2]*Department of Mathematics, Art and Science Faculty, Siirt University, 56100 Siirt, Turkey*

Correspondence should be addressed to Shabir Ahmad; shabirahmad2232@gmail.com

Received 9 March 2022; Accepted 20 May 2022; Published 13 June 2022

Academic Editor: Harish Garg

In this manuscript, we study a nonlinear fractional-order predator-prey system while considering uncertainty in initial values. We derive the feasibility region and the boundness of the solution. The suggested model's equilibrium points and the basic reproduction number are calculated. The stability of equilibrium points is presented. We use the metric fixed point theory to study the existence and uniqueness results concerning the solution of the model. We use the notion of UH-stability to show that the model is Ulam–Hyres type stable. To attain the approximate solution of the proposed model, we construct a method that uses the fuzzy Laplace transform in collaboration with the ADM (Adomian decomposition method). Finally, we simulate our theoretical results using MATLAB to show the dynamics of the considered model.

1. Introduction

A mathematical model is a mathematical representation of a process. The different phenomenon of the real world has been modelled by DEs or by a system of DEs [1]. Various techniques such Taylor series method, variational iteration method, modified variational iteration algorithms, and homotopy perturbation method have been discussed to solve different models [2–5]. Here, we consider the simplest model named as "Lotka–Volterra" or Predator-Prey, presented independently by "Lotka" in (1925) and "Volterra" in (1926) [6]. It refers to the relationship between predator and prey controls, as well as the population development of both species. The model describes the evolution of a biological Predator-Prey relationship through a system of two nonlinear DEs, generalized to a more complex and realistic phenomenon [6–9]. The general system for the dynamic phenomenon is as follows:

$$\begin{cases} \dfrac{\mathrm{d}P}{\mathrm{d}t} = P\theta(x, y), \\ \\ \dfrac{\mathrm{d}Q}{\mathrm{d}t} = P\theta(x, y), \end{cases} \tag{1}$$

where $P(t)$ is the prey (rabbits), $Q(t)$ is the predator (foxes) population at a time t, $\theta(x, y)$ and $\theta(x, y)$ are its functions, respectively. The growth rate and the density at time t have a direct relationship. This is presumed that the populations live in a community, where the age factor is not considered. The prey is abundant in natural resources, and the only threat is the specialist predator, whose growth depends on prey catches. For the prey model, the rate of growth of prey is presumed to be constant, and the particular rate of growth is reduced by an amount equal to the population of the predator [6]. The proposed model of the classical order is governed by

$$\begin{cases} \dfrac{dP}{dt} = aP(t) - bP(t)Q(t); \quad a, b > 0, \\ \\ \dfrac{dQ}{dt} = -cQ(t) + dP(t)Q(t); \quad c, d > 0. \end{cases} \tag{2}$$

From the 20th century, the subject of fractional-order integrals and derivatives has a substantial effect on modelling and simulation due to its nonlocal nature [10, 11]. It is seen that models involving integrals and derivatives of fractional order are more accurate, rather than classical models because fractional operators have long memory and heredity properties [12]. Das and Gupta [13] studied system (2) under fractional differential equations with constant coefficients [13]. Ahmad et al. [14] analyzed the human liver model by using hybrid fractional operators [15]. Alderremy et al. [16] analyzed multi space-fractional Gardner equation in [16]. For more applications of different fractional operators, see [17–20]. It should be noted that, in many branches of mathematics, DEs have been expanded to fill the study gape. Since we live an uncertain environment, therefore, we cannot model any phenomenon accurately. So, fuzzy operators are the best tools to handle the uncertain situation. Therefore, classical calculus is generalized to fractional and fuzzy Calculus, while DEs are generalized to fractional and fuzzy DEs [21–23]. Fuzzy DEs can model a physical phenomenon more accurately by considering fuzziness or uncertainty in initial conditions. To solve such problems, they utilized different techniques [24, 25]. Recently, fuzzy DEs have been utilized to analyze various models that occur in biology and physics [15, 26, 27]. As in the classical model, the coefficients are constant; therefore, we extend the model given in (2) to fuzzy fractional-order operator with uncertain initial data. For order, $0 < \varpi \leq 1$ and $0 \leq \varrho \leq 1$.

$$\begin{cases} D_t^{\varpi}[\widetilde{p}(t,\varrho)] = a\widetilde{p}(t,\varrho) - b\widetilde{p}(t,\varrho)\widetilde{Q}(t,\varrho), \\ D_t^{\varpi}[\widetilde{Q}(t,\varrho)] = -c\widetilde{Q}(t,\varrho) + d\widetilde{p}(t,\varrho)\widetilde{Q}(t,\varrho), \end{cases} \tag{3}$$

where

$$\begin{aligned} \widetilde{p}(t,\varrho) &= \widetilde{x}_0, \\ \widetilde{Q}(t,\varrho) &= \widetilde{y}_0. \end{aligned} \tag{4}$$

Here, D_t^{ϖ} represents fractional derivative in the Caputo sense, $\widetilde{p}(t,\varrho)$ and $\widetilde{Q}(t,\varrho)$ are fuzzy valued functions, and $\widetilde{x}_0$ and $\widetilde{y}_0$ are fuzzy numbers.

The goal of this research is to look at the model under consideration by including fuzziness in the initial values. The model's basic dynamical features are studied. Fixed point theory and nonlinear analysis are used to derive the existence and stability conclusions of model (3). LADM is used to find the solution to the presented model. To demonstrate the model's dynamics, the obtained results are simulated for various fractional-order values.

The structure of the paper is as follows: Section 1 deals with the introduction and motivation part of the paper. Section 2 provides the basic concepts of fuzzy fractional calculus. Section 3 deals with stability and feasibility of the equilibrium points. Section 4 provides the existence and uniqueness theory of the proposed model. Section 5 discusses the Ulam–Hyres stability via nonlinear functional analysis. Section 6 gives the general procedure for the solution of the considered model via fuzzy Laplace transform. Section 7 demonstrates the model through 2D and 3D simulations. Section 8 includes conclusion of the manuscript.

2. Preliminaries

Definition 1 (see [28]). Consider a fuzzy set of real line, i.e., $u\colon \mathbb{R} \longrightarrow [0,1]$. Then, u is called a fuzzy number if it fulfills the following features:

(1) The closure of the set $\{a \in \mathbb{R}, u(a) > 0\}$ is compact

(2) u is upper semicontinuous on $\mathbb{R}$

(3) u is convex $(u(qa + (1-q)b) \geq (u(a) \wedge u(b))\forall q \in [0,1], a, b \in \mathbb{R})$

(4) u is normal for some $a_0 \in \mathbb{R}$; $u(a_0) = 1$)

Definition 2 (see [28]). If u is a fuzzy number, then the parametric form is represented by $[\underline{u}(\varrho), \overline{u}(\varrho)]$ such that $0 \leq \varrho \leq 1$ and fulfills the properties which are given as follows:

(1) $\underline{u}(\varrho)$ is bounded, left continuous, and increasing function in $(0,1]$ and right continuous at 0

(2) $\overline{u}(\varrho)$ is bounded, right continuous, and decreasing in $(0,1]$ and right continuous at 0

(3) $\underline{u}(\varrho) \leq \overline{u}(\varrho)$

Also, for crisp case, we have $\underline{u}(\varrho) \leq \overline{u}(\varrho) = \varrho$.

Definition 3 (see [25]). Let $\mathbb{E}$ contain all fuzzy numbers. We take into consideration a mapping $\widetilde{\mho}\colon \mathbb{E} \times \mathbb{E} \longrightarrow \mathbb{R}$ and take $\underline{g}(\varrho)[\underline{g}(\varrho), \overline{g}(\varrho)]$ and $b = (\underline{b}(\varrho), \overline{b}(\varrho))$ as two fuzzy numbers. The Hausdorff distance between g and b is presented as follows:

$$\widetilde{\mho}(g,b) = \sup_{\varrho \in [0,1]} \left[\max\left\{ \left| \underline{g}(\varrho) - \underline{b}(\varrho) \right|, \left| \overline{g}(\varrho) - \overline{b}(\varrho) \right| \right\} \right]. \tag{5}$$

In $\mathbb{E}$, the metric $\widetilde{\mho}$ satisfies the properties which is given as follows:

(1) $\widetilde{\mho}(g + v, b + v) = \widetilde{\mho}(g,b) \forall g, v, b \in \mathbb{E}$

(2) $\widetilde{\mho}(g\varrho, b\varrho) = |\varrho| \widetilde{\mho}(g,b) \forall g, b \in \mathbb{E}, \varrho \in \mathbb{R}$

(3) $\widetilde{\mho}(g + \xi, b + \varsigma) \leq \rho(g,b) + \widetilde{\mho}(\xi, \varsigma) \forall g, b, \xi, \varsigma \in \mathbb{E}$

(4) $(\mathbb{E}, \widetilde{\mho})$ is a complete metric space

Definition 4 (see [28]). Let $\widetilde{\mho}\colon \mathbb{R} \longrightarrow \mathbb{E}$ be a fuzzy set valued function. Then, $\widetilde{\mho}$ is called continuous, if for each $\varepsilon > 0 \exists \delta > 0$ and fixed value of $\wp_0 \in [\zeta_1, \zeta_2]$, the following relation holds:

$$\rho\left(\widetilde{\mho}(\wp), \widetilde{\mho}(\wp_0)\right) < \varepsilon; \quad \text{whenever} \left|\wp - \wp_0\right| < \delta. \tag{6}$$

Definition 5 (see [28]). Let $\mho$ be a continuous fuzzy function on $[0, q] \subseteq \mathbb{R}$, then for $0 < \varpi \leq 1$, a fuzzy Riemann–Liouville fractional integral is given by

$$I^{\varpi}\mho(t) = \frac{1}{\mho(\varpi)}\int_0^t (t - \mathfrak{I})^{\varpi-1}\mho(\varpi)\mathrm{d}\mathfrak{I}, \quad \text{where } \mathfrak{I} \in (0, \infty). \tag{7}$$

Additionally, if $\mho \in L^F[0, q] \cap C^F[0, q]$, where $L^F[0, q]$ and $C^F[0, q]$ are the spaces that contains fuzzy Lebesgue integrable functions and fuzzy continuous, respectively, then

$$\left[I^{\varpi}\mho(t)\right]_{\varrho} = \left[I^{\varpi}\underline{\mho}_{\varrho}(t), I^{\varpi}\overline{\mho}_{\varrho}(t)\right], \quad 0 \leq \varrho \leq 1, \tag{8}$$

where

$$I^{\varpi}\underline{\mho}_{\varrho}(t) = \frac{1}{\Gamma(\varpi)}\int_0^t (t - \mathfrak{I})^{\varpi-1}\underline{\mho}_{\varrho}(t)\mathrm{d}\mathfrak{I}, \quad \mathfrak{I} \in (0, \infty),$$

$$I^{\varpi}\overline{\Phi}_{\varrho}(t) = \frac{1}{\Gamma(\varpi)}\int_0^t (t - \mathfrak{I})^{\varpi-1}\overline{\mho}_{\varrho}(t)\mathrm{d}\mathfrak{I}, \quad \mathfrak{I} \in (0, \infty). \tag{9}$$

Definition 6 (see [28]). Consider a fuzzy function $\mho \in L^F[0, b] \cap C^F[0, b]$ with parametric form $\mho = [\underline{\mho}_{\varrho}(t), \overline{\mho}_{\varrho}(t)], 0 \leq \varrho \leq 1$, then the Caputo fuzzy fractional-order derivative is presented by

$$\left[D^{\varpi}\mho(t_0)\right]_{\varrho} = \left[D^{\varpi}\underline{\mho}_{\varrho}(t_0), D^{\varpi}\overline{\mho}_{\varrho}(t_0)\right], \quad 0 < \varpi \leq 1, \tag{10}$$

where

$$D^{\varpi}\underline{\mho}_{\varrho}(t_0) = \frac{1}{\Gamma(j - \varpi)}\left[\int_0^t (t - \mathfrak{I})^{j-\varpi-1}\frac{\mathrm{d}^j}{\mathrm{d}\mathfrak{I}^j}\underline{\mho}_{\varrho}(\mathfrak{I})\mathrm{d}\mathfrak{I}\right]_{t=t_0},$$

$$D^{\varpi}\overline{\mho}_{\varrho}(t_0) = \frac{1}{\Gamma(j - \varpi)}\left[\int_0^t (t - \mathfrak{I})^{j-\varpi-1}\frac{\mathrm{d}^j}{\mathrm{d}\mathfrak{I}^j}\overline{\mho}_{\varrho}(\mathfrak{I})\mathrm{d}\mathfrak{I}\right]_{t=t_0}, \tag{11}$$

so that the right side integration converges and j is bracket function of ϖ.

Definition 7 (see [28]). The fuzzy Laplace transform (LT) for continuous and fuzzy Reimann-integrable function $\mho$ on $[0, \infty)$ is defined as follows:

$$\mathbf{L}[\mho(\varsigma)] = \int_0^{\infty}\mho(\varsigma)\cdot e^{-s\varsigma}\mathrm{d}\varsigma. \tag{12}$$

Using parametric form of $\mho(\varsigma)$, we have for $0 \leq \varrho \leq 1$:

$$\int_0^{\infty}\mho(\varsigma, \varrho)\cdot e^{-s\varsigma}\mathrm{d}\varsigma = \left[\int_0^{\infty}\underline{\mho}(\varsigma, \varrho)\cdot e^{-s\varsigma}\mathrm{d}\varsigma, \int_0^{\infty}\overline{\mho}(\varsigma, \varrho)\cdot e^{-s\varsigma}\mathrm{d}\varsigma\right]. \tag{13}$$

Hence,

$$\mathbf{L}[\mho(\varsigma, \varrho)] = \left[\mathbf{L}\,\underline{\mho}(\varsigma, \varrho), \mathbf{L}\overline{\mho}(\varsigma, \varrho)\right]. \tag{14}$$

Theorem 1 (see [21]). *The LT of Caputo fractional derivative of $\mho(t)$ is presented as follows:*

$$\mathbf{L}\left[\left(D^{\varpi}\mho(t)\right)\right] = s^{\varpi}\mathbf{L}[\mho(t)] - s^{\varpi-1}[\mho(0)]. \tag{15}$$

3. Feasibility and Stability Analysis

In this part, the feasibility and stability of the points of equilibrium will be discussed. To discuss the model's boundedness and feasibility, we have the following theorem.

Theorem 2. *The solution of the proposed model is bound to the feasible region given by*

$$\mathbb{T} = \left\{(\widetilde{P}, \widetilde{Q}) \in \mathbf{R}_+^2 : 0 \leq G(t) \leq \frac{a - c}{b - d}\right\}. \tag{16}$$

Proof. By adding both equation of (2) and considering $G(t) = \widetilde{P}(t) + \widetilde{Q}(t)$, one gets

$$\begin{aligned}\frac{\mathrm{d}G(t)}{\mathrm{d}t} &= a\widetilde{P}(t) - c\widetilde{Q}(t) - b\widetilde{P}(t)\widetilde{Q}(t) + d\widetilde{P}(t)\widetilde{Q}(t) \\ &\leq a - c - (b - d)(\widetilde{P} + \widetilde{Q}) \\ &\leq a - c - (b - d(G(t))), \\ \frac{\mathrm{d}G(t)}{\mathrm{d}t} &+ (b - d)G(t) \leq a - c.\end{aligned} \tag{17}$$

On using simple integration, we have

$$G(t) \leq \frac{a - c}{b - d} + C\exp(-(b - d)t). \tag{18}$$

This implies that $t \longrightarrow \infty, G(t) \leq (a - c/b - d)$. Consequently, the result is obtained. □

For stability, the equilibrium points for (3) must be found as

$$\begin{aligned}D_t^{\varpi}(\widetilde{P} = 0(t)) &= 0, \\ D_t^{\varpi}(\widetilde{Q} = 0(t)) &= 0.\end{aligned} \tag{19}$$

We have two points of equilibrium which are as follows: $E_0 = ((c/d), 0)$ if $\widetilde{Q} = 0$ and $E_0 = (0, (a/b))$ if $\widetilde{P} = 0$.

Theorem 3. *The reproduction number for (1) is $R_0 = -1$.*

Proof. To find the reproduction number, consider 2^{nd} equation of (1) as $X = I$,

$$\begin{aligned}D_t^{\varpi}(X) &= D_t^{\varpi}(I) = -c\widetilde{Q}(t) + d\widetilde{P}(t)\widetilde{Q}(t); \quad c, d > 0, \\ D_t^{\varpi}(X) &= F - V.\end{aligned} \tag{20}$$

such that $F = d\widetilde{P}(t)\widetilde{Q}(t)$, $V = -c\widetilde{Q}(t)$ where V and F represent the linear and nonlinear terms, respectively. The next step is to determine the next generation matrix $\mathscr{F}\mathscr{V}^{-1}$, where

$$\mathcal{F} = \left[\frac{\partial}{\partial y}\left(d\widetilde{P}(t)\widetilde{Q}(t)\right)\right] = [d\widetilde{P}(t)],$$

$$\mathcal{V} = \left[\frac{\partial}{\partial \widetilde{Q}}\left(-c\widetilde{Q}(t)\right)\right] = [-c], \tag{21}$$

$$\mathcal{V}^{-1} = \left[\frac{-1}{c}\right],$$

then

$$\mathcal{F}\mathcal{V}^{-1} = \left[\frac{d\widetilde{P}(t))}{-c}\right]. \tag{22}$$

The leading eigen value is equal to R_0 at free equilibrium point $E_0 = (c/d, 0)$ in $\mathcal{F}\mathcal{V}^{-1}$, which can be computed as

$$\rho\left(\mathcal{F}\mathcal{V}^{-1}\right)_{E_0} = \left[\frac{\mathrm{dc}}{-\mathrm{dc}}\right]. \tag{23}$$

Similarly, we can compute the reproduction number for 1^{st} equation of (1). Hence, $R_0 = -1$. □ □

Theorem 4. *If $R_0 < 1$, then free equilibrium points of (1) are locally asymptotically stable.*

Proof. Proof is obvious. □

4. Existence Theory

Here, we will establish the existence theory for the model by using fixed point theory. Let us take into consideration the right hand sides of the given system as follows:

$$\begin{aligned}\Psi(t, \widetilde{P}(t,\varrho), \widetilde{Q}(t,\varrho)) &= a\widetilde{P}(t,\varrho) - \widetilde{P}(t,\varrho)\widetilde{Q}(t,\varrho),\\ \Xi(t, \widetilde{P}(t,\varrho), \widetilde{Q}(t,\varrho)) &= -c\widetilde{Q}(t,\varrho) + d\widetilde{P}(t,\varrho)\widetilde{Q}(t,\varrho).\end{aligned} \tag{24}$$

Thus, the given system (3) can be written as follows: for $0 < \varpi \le 1$ and $0 \le \varrho \le 1$,

$$\begin{cases} D_t^{\varpi}[\widetilde{P}(t,\varrho)] = \Psi(t, \widetilde{P}(t,\varrho), \widetilde{Q}(t,\varrho)),\\ D_t^{\varpi}[\widetilde{Q}(t,\varrho)] = \Xi(t, \widetilde{P}(t,\varrho), \widetilde{Q}(t,\varrho)),\end{cases} \tag{25}$$

where

$$\begin{aligned}\widetilde{P}(0,\varrho) &= \widetilde{x}_0,\\ \widetilde{Q}(0,\varrho) &= \widetilde{y}_0.\end{aligned} \tag{26}$$

On using the corresponding integral I^r, we have

$$\begin{cases}\widetilde{P}(t,\varrho) = \widetilde{P}(0,\varrho) + \dfrac{1}{\Gamma(\varpi)}\displaystyle\int_0^t (t-\mathfrak{J})^{\varpi-1}\Psi(\mathfrak{J}, \widetilde{P}(\mathfrak{J},\varrho), \widetilde{Q}(\mathfrak{J},\varrho))\mathrm{d}\mathfrak{J},\\ \widetilde{Q}(t,\varrho) = \widetilde{Q}(0,\varrho) + \dfrac{1}{\Gamma(\varpi)}\displaystyle\int_0^t (t-\mathfrak{J})^{\varpi-1}\Xi(\mathfrak{J}, \widetilde{P}(\mathfrak{J},\varrho), \widetilde{Q}(\mathfrak{J},\varrho))\mathrm{d}\mathfrak{J}.\end{cases} \tag{27}$$

We denote Banach space as $\mathbb{B} = \mathbb{B}_1 \times \mathbb{B}_2$ under the fuzzy norm as follows:

$$\|(\widetilde{P}(t,\varrho), \widetilde{Q}(t,\varrho))\| = \max_{t\in[0,T]}[|\widetilde{P}(t,\varrho) + \widetilde{Q}(t,\varrho)|]. \tag{28}$$

Then, we reach

$$\widetilde{\aleph}(t,\varrho) = \widetilde{\aleph}_0(t,\varrho) + \frac{1}{\Gamma\varpi}\int_0^t (t-\mathfrak{J})^{\varpi-1}\Theta(\mathfrak{J}, \widetilde{S}(\mathfrak{J},\varrho))\mathrm{d}\mathfrak{J}, \tag{29}$$

where $\widetilde{S}(t,\varrho) = \begin{cases}\widetilde{P}(t,\varrho)\\ \widetilde{Q}(t,\varrho)\end{cases}$ $\widetilde{S}_0(t,\varrho) = \begin{cases}\widetilde{P}(0,\varrho)\\ \widetilde{Q}(0,\varrho)\end{cases}$ and $\Theta(t, \widetilde{S}(t,\varrho)) = \begin{cases}\Psi(s, \widetilde{P}(s,\varrho), \widetilde{Q}(s,\varrho))\\ \Xi(s, \widetilde{P}(s,\varrho), \widetilde{Q}(s,\varrho))\end{cases}$.

We take some assumption on nonlinear function Θ: $\mathbb{B} \longrightarrow \mathbb{B}$ as follows:

C-1 . For each $\widetilde{S}_1(t,\varrho), \widetilde{S}_2(t,\varrho) \in \mathbb{B}\exists K_S > 0$ such that

$$\left|\Theta\left(t, \widetilde{S}_1(t,\varrho) - \Theta\left(t, \widetilde{S}_2(t,\varrho)\right)\right)\right| \le K_S\left|\widetilde{S}_1(t,\varrho) - \widetilde{S}_2(t,\varrho)\right|. \tag{30}$$

C-2 . There exist constants $M_S > 0$ and $N_S > 0$ such that

$$|\Theta(t, \widetilde{S}(t,\varrho))| \le M_S|\widetilde{S}(t,\varrho)| + N_S. \tag{31}$$

Theorem 5. *Let assumption **C**-2 holds, then model (25) possesses at least one solution.*

Proof. Let us consider a closed and convex fuzzy set $\mathcal{A} = \left\{\widetilde{S}(t,\varrho) \in \mathbb{B}: \|\widetilde{S}(t,\varrho)\| \le r\right\} \subset \mathbb{B}$. Let us define a mapping ψ: $\mathcal{A} \longrightarrow \mathcal{A}$ such that

$$\psi(\widetilde{S}(t,\varrho)) = \widetilde{S}_0(t,\varrho) + \frac{1}{\Gamma(\varpi)}\int_0^t (t-\mathfrak{J})^{\varpi-1}\Theta(\mathfrak{J}, \widetilde{S}(\mathfrak{J},\varrho))\mathrm{d}\mathfrak{J}. \tag{32}$$

For any $\widetilde{S}(t,\varrho) \in \mathcal{U}$, we have

$$\begin{aligned}\|\psi(\widetilde{S}(t,\varrho))\| &= \max_{t\in[0,T]}\left|\widetilde{S}_0(t,\varrho) + \frac{1}{\Gamma(\varpi)}\int_0^t (t-\mathfrak{J})^{\varpi-1}\Theta(\mathfrak{J}, \widetilde{S}(\mathfrak{J},\varrho))\mathrm{d}\mathfrak{J}\right|\\ &\le \left|\widetilde{S}_0(t,\varrho)\right| + \frac{1}{\Gamma(\varpi)}\int_0^t (t-\mathfrak{J})^{\varpi-1}|\Theta(\mathfrak{J}, \widetilde{S}(\mathfrak{J},\varrho))|\mathrm{d}\mathfrak{J}\\ &\le \left|\widetilde{S}_0(t,\varrho)\right| + \frac{1}{\Gamma(\varpi)}\int_0^t (t-\mathfrak{J})^{\varpi-1}\left[M_S|\widetilde{S}(t,\varrho)| + N_S\right]\mathrm{d}\mathfrak{J}\\ &\le \left|\widetilde{S}_0(t,\varrho)\right| + \frac{\varrho^{\varpi}}{\Gamma(\varpi+1)}\left[M_S|\widetilde{S}(t,\varrho)| + N_S\right].\end{aligned} \tag{33}$$

The last relation shows that $\|\psi(\widetilde{S}(t,\varrho))\| \le r$. From the last inequality, we have $\psi(\mathcal{U}) \subset \mathcal{U}$. It follows that ψ is bounded operator. Next, the next step is to show that the operator ψ is completely continuous. For this let $\phi_1, \phi_2 \in [0,T]$ such that $\phi_1 < \phi_2$, then

$$\begin{aligned}\|\psi(\widetilde{S}(t,\varrho))(\phi_2)-\psi(\widetilde{S}(t,\varrho))(\phi_1)\| &= |\frac{1}{\Gamma(\varpi)}\int_0^{\phi_2}(\phi_2-\mathfrak{I})^{\varpi-1}\Theta(\mathfrak{I},\widetilde{S}(\mathfrak{I},\varrho))d\mathfrak{I}\\ &-\frac{1}{\Gamma(\varpi)}\int_0^{\phi_1}(\phi_1-\mathfrak{I})^{\varpi-1}\Theta(\mathfrak{I},\widetilde{S}(\mathfrak{I},\varrho))d\mathfrak{I}|\le\left[\phi_2^{\varpi}-\phi_1^{\varpi}\right]\frac{\left[M_S|\widetilde{S,\varrho}(t,\varrho)|+N_S\right]}{\Gamma(\varpi+1)}.\end{aligned}\tag{34}$$

The last inequality implies that

$$\|\psi(\widetilde{S}(t,\varrho))(\phi_2)-\psi(\widetilde{S}(t,\varrho))(\phi_1)\|\longrightarrow 0\text{ as }\phi_2\longrightarrow\phi_1.\tag{35}$$

□

Theorem 6. *Suppose that **C**-1 holds, then the considered model (25) possesses the unique solution if* $\varrho^{\varpi}K_S<\Gamma(\varpi+1)$.

Proof. Let $\widetilde{S_1}(t,\varrho),\widetilde{S_2}(t,\varrho)\in\mathbb{B}$, then

$$\begin{aligned}\left\|\psi\left(\widetilde{S_1}(t,\varrho)\right)-\psi\left(\widetilde{S_2}(t,\varrho)\right)\right\| &= \max_{t\in[0,T]}|\frac{1}{\Gamma(\varpi)}\int_0^t(t-\mathfrak{I})^{\varpi-1}\Theta\left(\mathfrak{I},\widetilde{S}_1(\mathfrak{I},\varrho)\right)d\mathfrak{I}\\ &-\frac{1}{\Gamma(\varpi)}\int_0^t(t-\mathfrak{I})^{\varpi-1}\Theta\left(\mathfrak{I},\widetilde{S}_2(\mathfrak{I},\varrho)\right)d\mathfrak{I}|\le\frac{\varrho^{\varpi}}{\Gamma(\varpi+1)}K_S|\widetilde{S_1}(t,\varrho)-\widetilde{S_2}(t,\varrho)|.\end{aligned}\tag{36}$$

This implies that $\|\psi(\widetilde{S_1}(t,\varrho))-\psi(\widetilde{S_2}(t,\varrho))\|\le(\varrho^{\varpi}/\Gamma(\varpi+1))K_S|\widetilde{S_1(t,\varrho)}-\widetilde{S_2(t,\varrho)}|$. Hence, ψ fulfills the contraction condition. Thus, by "Banach fixed point theorem" system (25) possesses the unique solution. □

5. Ulam–Hyres Stability

We take a small alteration $\phi\in C[0,T]$ such that $\phi(0)=0$ depends only on the solution $\psi(S_1\widetilde{(t,\varrho)})$ as follows:

(i) $|\phi(t)|\le\nu$, for $\nu>0$

(ii) $D_t^{\varpi}\psi(\widetilde{S}(t,\varrho))=\Psi(t,\psi(\widetilde{S}(t,\varrho))+\phi(t))$

Lemma 1. *The solution of the perturbed problem*

$$\begin{aligned}D_t^{\varpi}\psi(\widetilde{S}(t,\varrho))&=\Psi(t,\psi(\widetilde{S}(t,\varrho))+\phi(t)),\\ \psi(\widetilde{S}(0,\varrho))&=\psi\left(\widetilde{S}_{(0,\varrho)}\right),\end{aligned}\tag{37}$$

fulfills the relation which is given as follows:

$$\begin{aligned}&\left|(\widetilde{S}(t,\varrho))-\left((\widetilde{S}(0)(t,\varrho))+\frac{1}{\Gamma(\varpi)}\int_0^t(t-\mathfrak{I})^{\varpi-1}\Psi(\mathfrak{I},(\widetilde{S}(t,\mathfrak{I})))d\mathfrak{I}\right)\right|\\ &\le\frac{T^{\varpi}}{\Gamma(\varpi+1)}\nu=\Omega_{T,\varpi}\nu.\end{aligned}\tag{38}$$

Proof. The proof is straightaway. □

Theorem 7. *Assuming* $(C-2)$ *along with (38), the solution of equation (37) is UH-stable. Thus, the numerical results of the model being considered are UH-stable if* $\Delta=(T^{\theta}/\Gamma(\varpi+1))K_{\Psi}<1$.

Proof. Let $\widetilde{S}(t,\varrho)\in\mathscr{C}$ be the solution of equation (37) and $\overline{\widetilde{S}}(t,\varrho)\in\mathscr{C}$ be the unique solution of equation (3), then

$$\begin{aligned}|\widetilde{S}(t.\varrho)-\overline{\widetilde{S}}(t,\varrho)| &= \left|\widetilde{S}(t,\varrho)-\left(\widetilde{S}_0(t,\varrho)+\frac{1}{\Gamma(\varpi)}\int_0^t(t-\mathfrak{I})^{\varpi-1}\Psi(\mathfrak{I},\overline{\widetilde{S}}(\mathfrak{I},\varrho)d\mathfrak{I}\right)\right|\\ &\le\left|\widetilde{S}(t,\varrho)-\left(\widetilde{S}_0(t,\varrho)+\frac{1}{\Gamma(\varpi)}\int_0^t(t-\mathfrak{I})^{\varpi-1}\Psi(\mathfrak{I},\widetilde{S}(\mathfrak{I},\varrho))d\mathfrak{I}\right)\right|\\ &+\left|\frac{1}{\Gamma(\varpi)}\int_0^t(t-\mathfrak{I})^{\varpi-1}\Psi(\mathfrak{I},\overline{\widetilde{S}}(\mathfrak{I}))d\mathfrak{I}-\frac{1}{\Gamma(\varpi)}\int_0^t(t-\mathfrak{I})^{\varpi-1}\Psi(\mathfrak{I},\overline{\widetilde{S}}(\mathfrak{I},\varrho))ds\right|\\ &\le\Omega_{T,\varpi}\nu+\Delta\|\widetilde{S}(t,\varrho)-\overline{\widetilde{S}}(t,\varrho)\|.\end{aligned}\tag{39}$$

From (39), we get

$$\|\widetilde{S}(t,\varrho)-\overline{\widetilde{S}}(t,\varrho)\|\le\frac{\Omega_{T,\varpi}}{1-\Delta}\nu.\tag{40}$$

Hence, we infer from (39) that the solution of (38) is UH-stable. Consequently, the model (3) is UH-stable. □

6. Derivation of General Procedure for Solution

Here, we derive an algorithm for the solution of the proposed model by fuzzy LADM. Consider system (3) with initial conditions as follows:

$$\begin{cases} D_t^{\varpi}[\widetilde{P}(t,\varrho)] = \Psi(t,\widetilde{P}(t,\varrho),\widetilde{Q}(t,\varrho)), \\ D_t^{\varpi}[\widetilde{Q}(t,\varrho)] = \Xi(t,\widetilde{P}(t,\varrho),\widetilde{Q}(t,\varrho)), \end{cases} \tag{41}$$

where

$$\begin{aligned} \widetilde{P}(0,\varrho) &= \widetilde{x_0} = (\underline{P}(0,\varrho),\overline{P}(0,\varrho)), \\ \widetilde{Q}(0,\varrho) &= \widetilde{y_0} = (\underline{Q}(0,\varrho),\overline{Q}(0,\varrho)). \end{aligned} \tag{42}$$

Then, we reach

$$\begin{aligned} \mathbf{L}\left[D_t^{\varpi}[\widetilde{P}(t,\varrho)]\right] &= \mathbf{L}[\Psi(t,\widetilde{P}(t,\varrho),\widetilde{Q}(t,\varrho))], \\ \mathbf{L}\left[D_t^{\varpi}[\widetilde{Q}(t,\varrho)]\right] &= \mathbf{L}[\Xi(t,\widetilde{P}(t,\varrho),\widetilde{Q}(t,\varrho))], \\ s^{\varpi}\mathbf{L}[\widetilde{P}(t,\varrho)] &= s^{\varpi-1}\widetilde{P}(0,\varrho) + \mathbf{L}[\Psi(t,\widetilde{P}(t,\varrho),\widetilde{Q}(t,\varrho))], \\ s^{\varpi}\mathbf{L}[\widetilde{Q}(t,\varrho)] &= s^{\varpi-1}\widetilde{Q}(0,\varrho) + \mathbf{L}[\Xi(t,\widetilde{P}(t,\varrho),\widetilde{Q}(t,\varrho))], \\ \mathbf{L}[\widetilde{P}(t,\varrho)] &= \frac{1}{s}\widetilde{P}(0,\varrho) + \frac{1}{s^{\varpi}}\mathbf{L}[\Psi(t,\widetilde{P}(t,\varrho),\widetilde{Q}(t,\varrho))], \\ \mathbf{L}[\widetilde{Q}(t,\varrho)] &= \frac{1}{s}\widetilde{Q}(0,\varrho) + \frac{1}{s^{\varpi}}\mathbf{L}[\Xi(t,\widetilde{P}(t,\varrho),\widetilde{Q}(t,\varrho))]. \end{aligned} \tag{43}$$

The infinite series solution is presented as follows:

$$\begin{aligned} \widetilde{P}(t,\varrho) &= \sum_{k=0}^{\infty}\widetilde{P}_k(t,\varrho), \\ \widetilde{Q}(t,\varrho) &= \sum_{k=0}^{\infty}\widetilde{Q}_k(t,\varrho), \\ \widetilde{P}(t,\varrho)\widetilde{Q}(t,\varrho) &= \sum_{k=0}^{\infty}\widetilde{P}_k(t,\varrho)\sum_{k=0}^{\infty}\widetilde{Q}_k(t,\varrho). \end{aligned} \tag{44}$$

Thus, we reach

$$\begin{aligned} \mathbf{L}[\widetilde{P}(t,\varrho)] &= \frac{1}{s}\widetilde{P}(0,\varrho) + \frac{1}{s^{\varpi}}\mathbf{L}\left[\Psi\left(t,\sum_{k=0}^{\infty}\widetilde{P}_k(t,\varrho),\sum_{k=0}^{\infty}\widetilde{Q}_k(t,\varrho)\right)\right], \\ \mathbf{L}[\widetilde{Q}(t,\varrho)] &= \frac{1}{s}\widetilde{Q}(0,\varrho) + \frac{1}{s^{\varpi}}\mathbf{L}\left[\Xi\left(t,\sum_{k=0}^{\infty}\widetilde{P}_k(t,\varrho),\sum_{k=0}^{\infty}\widetilde{Q}_k(t,\varrho)\right)\right]. \end{aligned} \tag{45}$$

Comparison of terms on both sides gives

$$\begin{aligned} \mathbf{L}[\underline{P}_0(t,\varrho)] &= \frac{1}{s}\underline{P}(0,\varrho), \\ \mathbf{L}[\overline{P}_0(t,\varrho)] &= \frac{1}{s}\overline{P}(0,\varrho), \\ \mathbf{L}\left[\underline{Q}_0(t,\varrho)\right] &= \frac{1}{s}\underline{Q}(0,\varrho), \\ \mathbf{L}\left[\overline{Q}_0(t,\varrho)\right] &= \frac{1}{s}\overline{Q}(0,\varrho), \\ \mathbf{L}[\underline{P}_1(t,\varrho)] &= \frac{1}{s^{\varpi}}\mathbf{L}\left[\Psi\left(t,\underline{P}_0(t,\varrho),\underline{Q}_0(t,\varrho)\right)\right], \\ \mathbf{L}[\overline{P}_1(t,\varrho)] &= \frac{1}{s^{\varpi}}\mathbf{L}\left[\Psi(t,\overline{P}_0(t,\varrho),\overline{Q}_0(t,\varrho))\right], \\ \mathbf{L}\left[\underline{Q}_1(t,\varrho)\right] &= \frac{1}{s^{\varpi}}\mathbf{L}\left[\Xi\left(t,\underline{P}_0(t,\varrho),\underline{Q}_0(t,\varrho)\right)\right], \\ \mathbf{L}\left[\overline{Q}_1(t,\varrho)\right] &= \frac{1}{s^{\varpi}}\mathbf{L}\left[\Xi(t,\overline{P}_0(t,\varrho),\overline{Q}_0(t,\varrho))\right], \\ &\vdots \\ \mathbf{L}[\underline{P}_{n+1}(t,\varrho)] &= \frac{1}{s^{\varpi}}\mathbf{L}\left[\Psi\left(t,\underline{P}_n(t,\varrho),\underline{Q}_n(t,\varrho)\right)\right], \\ \mathbf{L}[\overline{P}_{n+1}(t,\varrho)] &= \frac{1}{s^{\varpi}}\mathbf{L}\left[\Psi(t,\overline{P}_n(t,\varrho),\overline{Q}_n(t,\varrho))\right], \\ \mathbf{L}\left[\underline{Q}_{n+1}(t,\varrho)\right] &= \frac{1}{s^{\varpi}}\mathbf{L}\left[\Xi\left(t,\underline{P}_n(t,\varrho),\underline{Q}_n(t,\varrho)\right)\right], \\ \mathbf{L}\left[\overline{Q}_{n+1}(t,\varrho)\right] &= \frac{1}{s^{\varpi}}\mathbf{L}\left[\Xi(t,\overline{P}_n(t,\varrho),\overline{Q}_n(t,\varrho))\right]. \end{aligned} \tag{46}$$

Taking inverse Laplace transform, we get

$$
\begin{aligned}
&\underline{P}_0(t,\varrho)=\underline{P}(0,\varrho),\\
&\overline{P}_0(t,\varrho)=\overline{P}(0,\varrho),\\
&\underline{Q}_0(t,\varrho)=\underline{Q}(0,\varrho),\\
&\overline{Q}_0(t,\varrho)=\overline{Q}(0,\varrho),\\
&\underline{P}_1(t,\varrho)=\mathbf{L}^{-1}\left[\frac{1}{s^{\varpi}}\mathbf{L}\left[\Psi\left(t,\underline{P}_0(t,\varrho),\underline{Q}_0(t,\varrho)\right)\right]\right],\\
&\overline{P}_1(t,\varrho)=\mathbf{L}^{-1}\left[\frac{1}{s^{\varpi}}\mathbf{L}\left[\Psi\left(t,\overline{P}_0(t,\varrho),\overline{Q}_0(t,\varrho)\right)\right]\right],\\
&\underline{Q}_1(t,\varrho)=\mathbf{L}^{-1}\left[\frac{1}{s^{\varpi}}\mathbf{L}\left[\Xi\left(t,\underline{P}_0(t,\varrho),\underline{Q}_0(t,\varrho)\right)\right]\right],\\
&\overline{Q}_1(t,\varrho)=\mathbf{L}^{-1}\left[\frac{1}{s^{\varpi}}\mathbf{L}\left[\Xi\left(t,\overline{P}_0(t,\varrho),\overline{Q}_0(t,\varrho)\right)\right]\right],\\
&\quad\vdots\\
&\underline{P}_{n+1}(t,\varrho)=\mathbf{L}^{-1}\left[\frac{1}{s^{\varpi}}\mathbf{L}\left[\Psi\left(t,\underline{P}_n(t,\varrho),\underline{Q}_n(t,\varrho)\right)\right]\right],\\
&\overline{P}_{n+1}(t,\varrho)=\mathbf{L}^{-1}\left[\frac{1}{s^{\varpi}}\mathbf{L}\left[\Psi\left(t,\overline{P}_n(t,\varrho),\overline{Q}_n(t,\varrho)\right)\right]\right],\\
&\underline{Q}_{n+1}(t,\varrho)=\mathbf{L}^{-1}\left[\frac{1}{s^{\varpi}}\mathbf{L}\left[\Xi\left(t,\underline{P}_n(t,\varrho),\underline{Q}_n(t,\varrho)\right)\right]\right],\\
&\overline{Q}_{n+1}(t,\varrho)=\mathbf{L}^{-1}\left[\frac{1}{s^{\varpi}}\mathbf{L}\left[\Xi\left(t,\overline{P}_n(t,\varrho),\overline{Q}_n(t,\varrho)\right)\right]\right].
\end{aligned}
\tag{47}
$$

Therefore, we get

$$
\begin{aligned}
\underline{P}(t,\varrho)&=\underline{P}_0(t,\varrho)+\underline{x_1(t,\varrho)+\cdots},\\
\overline{P}(t,\varrho)&=\overline{P}_0(t,\varrho)+\overline{P}_1(t,\varrho)+\cdots,\\
\underline{Q}(t,\varrho)&=\underline{Q}_0(t,\varrho)+\underline{Q}_1(t,\varrho)+\cdots,\\
\overline{Q}(t,\varrho)&=\overline{Q}_0(t,\varrho)+\overline{Q}_1(t,\varrho)+\cdots.
\end{aligned}
\tag{48}
$$

7. Results and Simulations

In this part, we illustrate the proposed method through simulations; consider the system above under some specific values of parameters as follows:

$$
\begin{cases}
D_t^{\varpi}[\widetilde{P}(t,\varrho)]=a\widetilde{P}(t,\varrho)-b\widetilde{P}(t,\varrho)\widetilde{Q}(t,\varrho),\\
D_t^{\varpi}[\widetilde{Q}(t,\varrho)]=\Xi(t,\widetilde{P}(t,\varrho),\widetilde{Q}(t,\varrho)),
\end{cases}
\tag{49}
$$

where

$$
\begin{aligned}
\widetilde{P}(0,\varrho)&=(\varrho-1,1-\varrho),\\
\widetilde{Q}(0,\varrho)&=(\varrho-1,1-\varrho).
\end{aligned}
\tag{50}
$$

After applying the proposed method, we get

$$
\begin{aligned}
\underline{P}_0(t,\varrho)&=\underline{P}(0,\varrho)=\varrho-1,\\
\overline{P}_0(t,\varrho)&=\overline{P}(0,\varrho)=1-\varrho,\\
\underline{Q}_0(t,\varrho)&=\underline{Q}(0,\varrho)=\varrho-1,\\
\overline{Q}_0(t,\varrho)&=\overline{Q}(0,\varrho)=1-\varrho.
\end{aligned}
\tag{51}
$$

Similarly, the next terms are computed as

$$
\begin{aligned}
\underline{P}_1(t,\varrho)&=\mathbf{L}^{-1}\left[\frac{1}{s^{\varpi}}\mathbf{L}\left[\Psi\left(t,\underline{P}_0(t,\varrho),\underline{Q}_0(t,\varrho)\right)\right]\right]=\mathbf{L}^{-1}\left[\frac{1}{s^{\varpi}}\mathbf{L}\left[a\underline{P}_0(t,\varrho)-b\underline{P}_0(t,\varrho)\underline{Q}_0(t,\varrho)\right]\right.\\
&=\mathbf{L}^{-1}\left[\frac{1}{s^{\varpi}}\mathbf{L}\left[a(\varrho-1)-b(\varrho-1)^2\right]=\left[a(\varrho-1)-b(\varrho-1)^2\right]\frac{t^{\varpi}}{\Gamma(\varpi+1)}\right..
\end{aligned}
\tag{52}
$$

We can obtain other terms of the series, i.e.,

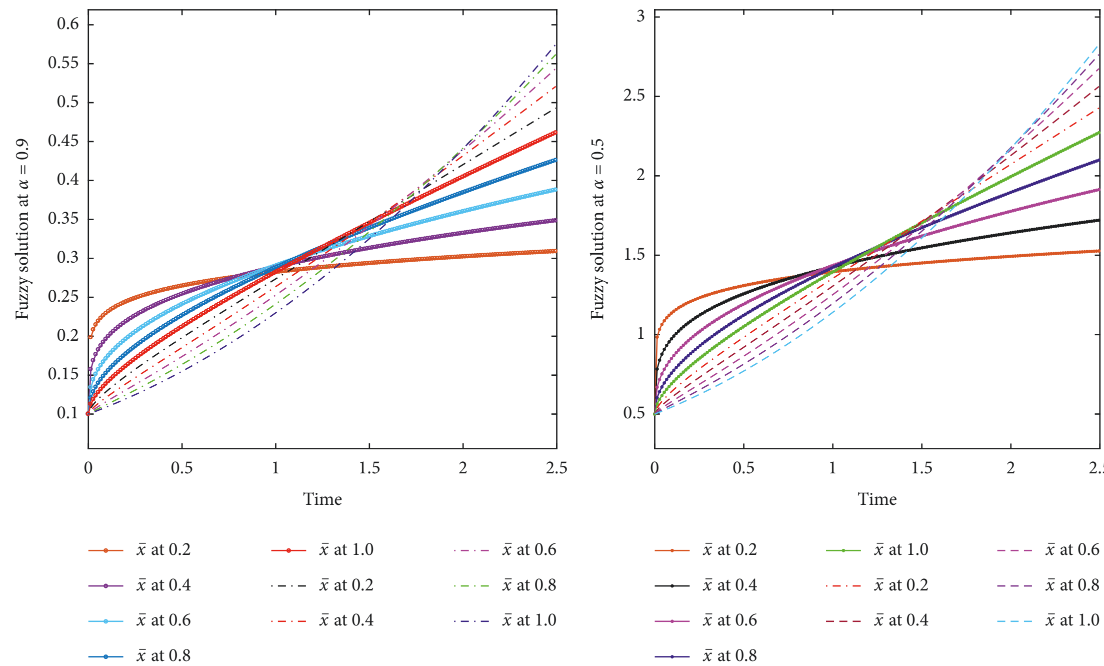

FIGURE 1: Graphical presentation of approximate fuzzy solution for specie $\widetilde{P(t)}$ up to three terms at various uncertainty and fractional orders for problem (49). Here, the lower and upper cut of x represents the lower and upper solution of $\widetilde{P}(t)$.

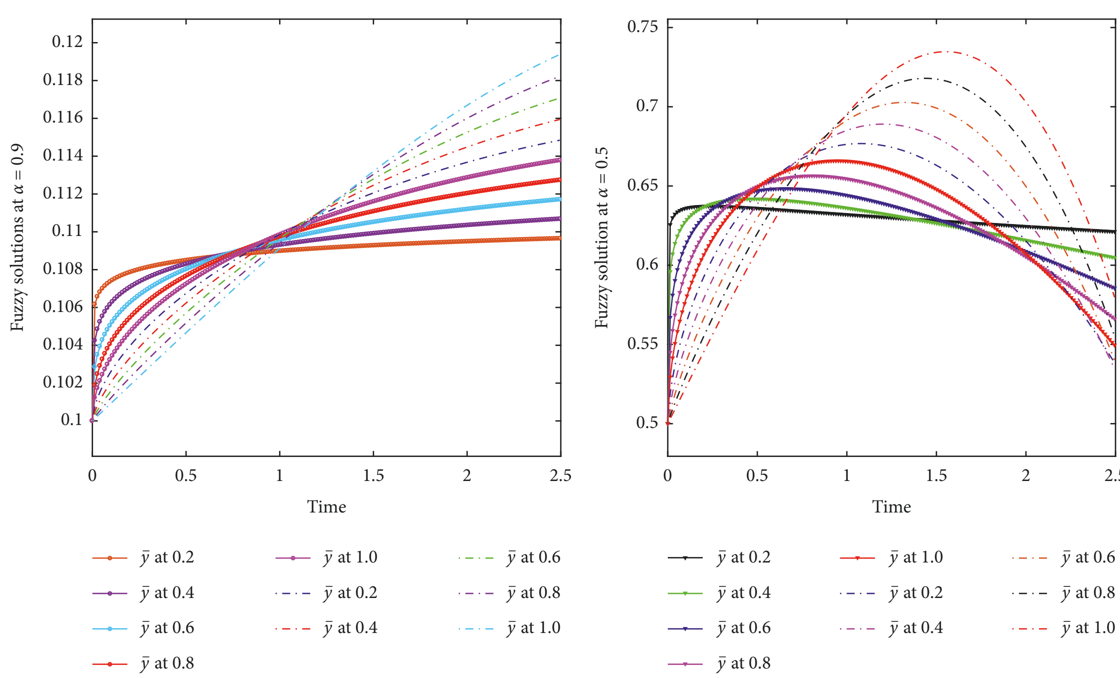

FIGURE 2: Graphical presentation of approximate fuzzy solution for specie $\widetilde{Q}(t)$ up to three terms at various uncertainty and fractional orders for problem (49). Here, the lower and upper cut of y represents the lower and upper solution of $\widetilde{Q}(t)$.

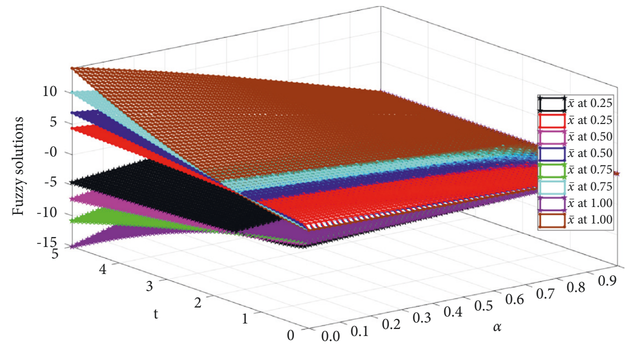

Figure 3: 3D presentation of approximate fuzzy solution for specie $\widetilde{P}(t)$ up to three terms at various uncertainty and fractional orders for problem (49). Here, the lower and upper cut of x represents the lower and upper solution of $\widetilde{P}(t)$.

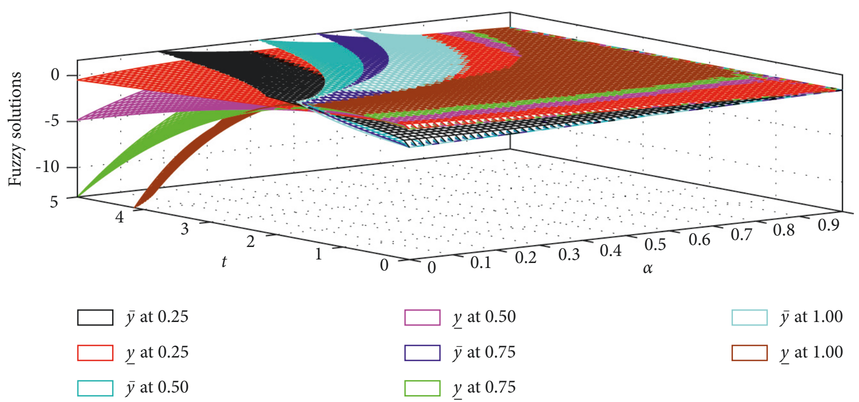

Figure 4: 3D presentation of approximate fuzzy solution for specie $\widetilde{Q}(t)$ up to three terms at various uncertainty and fractional orders for problem (49). Here, the lower and upper cut of y represents the lower and upper solution of $\widetilde{Q}(t)$.

$$
\begin{aligned}
\overline{P}_1(t,\varrho) &= \left[a(1-\varrho) - b(1-\varrho)^2\right]\frac{t^{\varpi}}{\Gamma(\varpi+1)},\\
\underline{Q}_1(t,\varrho) &= \left[d(\varrho-1)^2 - c(\varrho-1)\right]\frac{t^{\varpi}}{\Gamma(\varpi+1)},\\
\overline{Q}_1(t,\varrho) &= \left[d(1-\varrho)^2 - c(1-\varrho)\right]\frac{t^{\varpi}}{\Gamma(\varpi+1)},\\
\underline{P}_2(t,\varrho) &= av_1\frac{t^{2\varpi}}{\Gamma(2\varpi+1)} - bv_1v_3\frac{\Gamma(2\varpi+1)}{\Gamma^2(\varpi+1)}\frac{t^{3\varpi}}{\Gamma(3\varpi+1)},\\
\overline{P}_2(t,\varrho) &= av_2\frac{t^{2\varpi}}{\Gamma(2\varpi+1)} - bv_2v_4\frac{\Gamma(2\varpi+1)}{\Gamma^2(\varpi+1)}\frac{t^{3\varpi}}{\Gamma(3\varpi+1)},\\
\underline{Q}_2(t,\varrho) &= -cv_3\frac{t^{2\varpi}}{\Gamma(2\varpi+1)} + dv_1v_3\frac{\Gamma(2\varpi+1)}{\Gamma^2(\varpi+1)}\frac{t^{3\varpi}}{\Gamma(3\varpi+1)},\\
\overline{Q}_2(t,\varrho) &= -cv_4\frac{t^{2\varpi}}{\Gamma(2\varpi+1)} + dv_2v_4\frac{\Gamma(2\varpi+1)}{\Gamma^2(\varpi+1)}\frac{t^{3\varpi}}{\Gamma(3\varpi+1)},
\end{aligned}
\tag{53}
$$

and so on. The unknown terms in the above equations are given as

$$
\begin{aligned}
v_1 &= a(\varrho-1) - b(\varrho-1)^2,\\
v_2 &= a(1-\varrho) - b(1-\varrho)^2,\\
v_3 &= d(\varrho-1)^2 - c(\varrho-1),\\
v_4 &= d(1-\varrho)^2 - c(1-\varrho).
\end{aligned}
\tag{54}
$$

Now, we simulate our obtained results to investigate the dynamics of the proposed model for the uncertain initial conditions. In Figures 1 and 2, we plot the fuzzy approximate results up to three terms for the given examples under two different uncertainty values as follows:

We displayed the approximate fuzzy solutions for the considered model for particular fuzzy initial conditions and at the given uncertainty against different fractional orders in Figures 1 and 2. The fuzzy fractional derivative produced global dynamics of the interaction of the two species under fuzzy concept that under taking some values for uncertainty $\varrho = 0.5, 0.9$. Increasing the uncertainty, the dynamics of interaction is also affected as shown in Figures 1 and 2. Such situation is natural and may be observed in daily life. Next, in

Figure 3 and Figure 4, we attempt on 3D plots of fuzzy solutions for first three terms as follows: by surfaces plots given in Figures 3 and 4, we have graphed the solutions for different fractional order over the interval $[0, 1]$ of uncertainty for the given problem 15.

8. Conclusion

In this research, we have used a fuzzy fractional derivative in Caputo sense to extend the fractional predator-prey model. We have discussed the equilibrium points and their stability. Using metric fixed point theory, we were able to derive the existence and uniqueness results for the nonlinear fuzzy fractional predator-prey model. We have shown that the proposed model is HU stable through nonlinear functional analysis. Besides, we have developed a general algorithm for obtaining an approximate solution to the proposed model using an efficient method (fuzzy LADM). If we replace $[\varrho - 1, 1 - \varrho] = 1$ in the obtained numerical results, then we recover the results obtained in the fractional-order model. Thus, our proposed model is the generalization of the fractional model of the predator-prey system. Finally, we simulated numerical results for different fractional order and 0.5 and 0.9 uncertainty values via MATLAB. From figures, we have found that fractional calculus can be combined with the fuzzy theory and thus glorify the global dynamics of species interaction.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding this research work.

References

[1] E. Leah, *Mathematical Models in Biology*, Society For Industrial And Applied Mathematics, Philadelphia, PA, USA, 2005.

[2] J. H. He, “Taylor series solution for a third order boundary value problem arising in architectural engineering,” *Ain Shams Engineering Journal*, vol. 11, no. 4, pp. 1411–1414, 2020.

[3] J. H. He and H. Latifizadeh, “A general numerical algorithm for nonlinear differential equations by the variational iteration method,” *International Journal of Numerical Methods for Heat and Fluid Flow*, vol. 30, no. 11, pp. 4797–4810, 2020.

[4] H. Ahmad, A. R. Seadawy, and T. A. Khan, “Numerical solution of Korteweg–de Vries-Burgers equation by the modified variational iteration algorithm-II arising in shallow water waves,” *Physica Scripta*, vol. 95, no. 4, Article ID 045210, 2020.

[5] J. H. He, “The simpler, the better: analytical methods for nonlinear oscillators and fractional oscillators,” *Journal of Low Frequency Noise, Vibration and Active Control*, vol. 38, no. 3-4, pp. 1252–1260, 2019.

[6] M. Pescitelli, “Lotka-Volterra predator-prey model with a predating scavenger,” Master dissertation, Georgia College, Milledgeville, Georgia, 2013.

[7] M. Rafei, H. Daniali, and D. D. Ganji, “Variational iteration method for solving the epidemic model and the prey and predator problem,” *Applied Mathematics and Computation*, vol. 186, no. 2, pp. 1701–1709, 2007.

[8] D. Baleanu, A. Jajarmi, H. Mohammadi, and S. Rezapour, “A new study on the mathematical modelling of human liver with Caputo–Fabrizio fractional derivative,” *Chaos, Solitons & Fractals*, vol. 134, Article ID 109705, 2020.

[9] H. Ahmad, T. A. Khan, I. Ahmad, P. S. Stanimirović, and Y. M. Chu, “A new analyzing technique for nonlinear time fractional Cauchy reaction-diffusion model equations,” *Results in Physics*, vol. 173, no. 2, pp. 1101–1106, 2006.

[10] D. Baleanu, K. Diethelm, E. Scalas, and J. J. Trujillo, *Fractional Calculus: Models and Numerical Methods*World Scientific, Singapore, 2012.

[11] S. Z. Rida, A. Abdel Rady, A. A. M. Arafa, and M. Khalil, “Approximate analytical solution of the fractional epidemic model,” *International Journal of Applied Mathematical Research*, vol. 1, pp. 17–19, 2012.

[12] K. S. Nisar, S. Ahmad, A. Ullah, K. Shah, H. Alrabaiah, and M. Arfan, “Mathematical analysis of SIRD model of COVID-19 with Caputo fractional derivative based on real data,” *Results in Physics*, vol. 21, Article ID 103772, 2021.

[13] S. Das and P. k. Gupta, “A mathematical model on fractional Lotka-Volterra equations,” *Journal of Theoretical Biology*, vol. 277, pp. 1–6, 2011.

[14] S. Ahmad, A. Ullah, A. Akgul, and D. Baleanu, “On solution of fractional model of human liver under hybrid fractional derivative,” *The interdisciplinary journal of Discontinuity Nonlinearity and Complexity*, vol. 11, no. 1, pp. 183–190, 2022.

[15] S. Ahmad, A. Ullah, A. Akgül, and T. Abdeljawad, “Numerical analysis of fractional human liver model in fuzzy environment,” *Journal of Taibah University for Science*, vol. 15, no. 1, pp. 840–851, 2021.

[16] A. A. Alderremy, K. M. Saad, P. Agarwal, S. Aly, and S. Jain, “Certain new models of the multi space-fractional Gardner equation,” *Physica A: Statistical Mechanics and its Applications*, vol. 545, Article ID 123806, 2020.

[17] W. Gao and H. M. Baskonus, “Deeper investigation of modified epidemiological computer virus model containing the Caputo operator,” *Chaos, Solitons & Fractals*, vol. 158, Article ID 112050, 2022.

[18] S. Saifullah, A. Ali, M. Irfan, and K. Shah, “Time-fractional klein–gordon equation with solitary/shock waves solutions,” *Mathematical Problems in Engineering*, vol. 202115 pages, Article ID 6858592, 2021.

[19] F. Rahman, A. Ali, and S. Saifullah, “Analysis of time-fractional $$\phi ^{4}$$-Equation with singular and non-singular kernels4-equation with singular and non-singular kernels,” *Int. J. Appl. Comput. Math.*vol. 7, no. 5, p. 192, 2021.

[20] W. Gao, P. Veeresha, D. G. Prakasha, and H. M. Baskonus, “Regarding new numerical results for the dynamical model of romantic relationships with fractional derivative,” *Fractals*, vol. 30, no. 1, Article ID 2240009, 2022.

[21] A. A. Kilbas, H. Srivastava, and J. Trunjillo, *Theory and Applications of Fractional Differential Equations*, Elseveir, Amsterdam, Netherlands, 2006.

[22] O. Kaleva, “Fuzzy differential equations,” *Fuzzy Sets and Systems*, vol. 24, no. 3, pp. 301–317, 1987.

[23] S. Arshad and V. Luplescu, “Fractional differential equation with fuzzy initial conditon,” *The Electronic Journal of Differential Equations*, vol. 34, pp. 1–8, 2011.

[24] T. Allahviranloo and M. B. Ahmadi, “Fuzzy Laplace transforms,” *Soft Computing*, vol. 14, no. 3, pp. 235–243, 2010.

[25] S. Salahshour, T. Allahviranloo, and S. Abbasbandy, “Solving fuzzy fractional differential equations by fuzzy Laplace

transforms," *Communications in Nonlinear Science and Numerical Simulation*, vol. 17, no. 3, pp. 1372–1381, 2012.

[26] A. Hanif, A. I. K. Butt, S. Ahmad, R. U. Din, and M. Inc, "A new fuzzy fractional order model of transmission of Covid-19 with quarantine class," *Eur. Phys. J. Plus*, vol. 136, no. 11, p. 1179, 2021.

[27] Y. Wu, S. Ahmad, A. Ullah, and K. Shah, "Study of the fractional-order HIV-1 infection model with uncertainty in initial data," *Mathematical Problems in Engineering*, vol. 2022, Article ID 7286460, 16 pages, 2022.

[28] T. Allahviranloo, *Fuzzy Fractional Differential Operators and Equations*, Springer, Berlin, Germany, 2021.