Research Article

Kinematics Analysis and Simulation of a Novel 3T Parallel Mechanism

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Three translational (3T) degree-of-freedom (DOF) parallel mechanism is widely used in the industrial field because of its compact structure and excellent dynamic performance. Combined with the 3T parallel mechanism studied at this stage, a novel 3T parallel mechanism is proposed. Firstly, the DOF of the mechanism is analyzed by using the screw theory. Secondly, the position, velocity, and singularity of the mechanism are analyzed, and the workspace of the mechanism is analyzed through the positive position solution. Then, the trajectory planning in joint space and coordinate space is completed, and a common mathematical model of grasping trajectory is established. Thirdly, the kinematics simulation analysis of the mechanism is carried out. Finally, the prototype platform was built and the grasping experiment was carried out, which verified the rationality of the mechanism design and the correctness of the theoretical analysis.

1. Introduction

Compared with the traditional series mechanism, the parallel mechanism has the advantages of large rigidity, compact structure, fast dynamic response, and high precision [1, 2]. In recent years, it has become a research hot spot in industry and academia. The parallel mechanism with three translational degrees of freedom has attracted much attention because of its simple structure and can meet the requirements of industrial robots to complete various pick-and-place operations in space. In 1985, the three translational delta parallel mechanism proposed by Dr. Clavel [3] has been widely used in industry [4–7]. After that, Hervé [8] proposed a 3T star mechanism. In 1996, Tsai [9, 10] used Hooke hinges instead of spherical hinges to simplify the delta mechanism and proposed a 3T Tsai mechanism. In 2006, Liu [11–13] replaced the rotation pairs and input components of the delta parallel mechanism and Tsai mechanism with moving amplitude and slider, respectively,

to obtain two linearly driven 3T parallel mechanisms. In 2020, Zou [14, 15] listed a series of three DOF parallel mechanisms with parallelogram branches and mentioned that when more planar joints (moving and rotating joints) are used, higher performance can be obtained.

All of the above mechanism subchains contain parallelograms with fixed side lengths, and the parallelograms are used to limit the DOFs of rotation or enhance the mechanism performance. In recent years, some scholars have proposed and studied the parallelogram closed-loop subchain with variable rod length and thus designed a variety of new parallel mechanisms. Gogu [16] mentioned a variety of parallelogram subchains with variable rod lengths in his work and pointed out that such chains can increase the working space or improve the accuracy of the mechanism under different circumstances. In 2018, Yang [17, 18] proposed a new type of retractable parallelogram subchain, analyzed 3T, 3T1R, and 3R mechanisms compounded by different combinations of this type of subchains, verified

their feasibility, and piloted a planar 2T mechanism for automatic docking devices. In 2019, Yang [19] proposed a scalable parallelogram branched chain composed of collapsible kinematic chains, and based on the collapsibility of the branched chain, the authors discussed the potential application of the mechanism on UAVs, but its mechanical analysis and singularity analysis were complicated by the fact that the branched chain consisted of kinematic chains. In the same year, Wang [20] proposed a 3T parallel mechanism with a parallelogram synchronous telescopic branch chain, using a motor and a synchronous belt to make a variable-length parallelogram as the drive. However, since the drive is mounted on the branch chain, the mechanism has high rotational inertia and poor dynamics. In response to the problems arising from the above variable rod length parallelogram closed-loop subchain, this paper designs a novel 3T parallel mechanism in combination with the delta parallel mechanism [21, 22].

2. Introduction to Parallel Mechanism

In this paper, the 3T parallel mechanism consists of a static platform, a moving platform, and three composite branches of the same structure containing parallelogram closed-loop subchains with variable rod lengths. The three composite branches are distributed on the static platform at an angle of 120° . The three rotation pairs of the branch chain connected to the static platform are the active pairs of the parallel mechanism, and the motor drives the compound branch chain through the active pair to realize the three translational movement of the moving platform. The mechanism is mainly composed of lightweight rods. The composite branches are symmetrically distributed and isotropic. The driving motors are fixed on the static platform, which makes the machine have good motion performance. The structure diagram of the parallel mechanism is shown in Figure 1.

As shown in Figure 2, each composite branch chain is composed of an active arm and a variable-length parallelogram closed-loop subchain connected by a rotating pair, and the variable-length parallelogram closed-loop subchain is formed by a connecting piece and a sliding rod connected by a moving pair. The active arm and the variable-length parallelogram closed-loop subchain are always maintained at 90° , and the parallelogram closed-loop subchain is connected with the moving platform by a spherical pair. ADEF is a parallelogram, and ABCD is a parallelogram with variable rod length. The lengths of rods AB and rod CD vary. M_i and N_i are the midpoints of AD and BC, respectively. From the geometric relationship, we can see that $AB = M_i N_i = CD$. The composite branch chain is connected to the static platform through the rotating pair L_i and connected to the moving platform through two spherical hinges B and C. $L_i M_i$ is perpendicular to AD.

3. Degree of Freedom Analysis

As shown in Figure 3, the parallelogram closed-loop subchain of variable rod length contains two RPS branches (R represents rotation pair, whose axis is parallel to axis X, and

FIGURE 1: The structure diagram of parallel mechanism.

passes through point A, P represents moving pair, whose direction is AB, S represents spherical hinge, whose center is located at point B), and a parallel mechanism composed of a parallelogram closed-loop subchain ADEF where the parallelogram closed-loop subchain is to provide constraints such that AB and CD are parallel. $AB = l_1$, $CD = l_2$, $AE = DF = l_3$, and $BC = AD = EF = l_4$. Given initial conditions $l_1 = l_2$, $\beta_i = \pi/2$, and the mechanism is in a singular position.

Because the rods AB and CD are telescopic, when the angle of the closed-loop branch reaches a singularity of the mechanism, the motion bifurcation may occur when passing through the singularity, resulting in two configurations.

As shown in Figure 4, when in configuration 1, the mechanism is a parallelogram mechanism, and the two groups of opposite edges are parallel and equal, so the rod AB and rod CD are synchronous telescopic parts.

To study the degree of freedom of this mechanism, as shown in Figure 5, when it is a fixed value, the 4-rod mechanism is regarded as a parallel mechanism with two branches, and the relative degree of freedom between its upper and lower links is studied. This closed loop can be regarded as a parallel mechanism consisting of a moving platform, a static platform, and two RS branches, with member BC as the "moving platform".

Component AD is the relative "static platform," AB is the first branch, and CD is the second branch. Point A is located on the axis of the rotation pair of the first branch. Take point A as the origin of the coordinate system. The axis X is perpendicular to the parallelogram plane, the axis Y is along the direction of AD, and the axis Z follows the right-hand rule.

Branch 1 includes a spherical hinge and a rotation pair, and a spherical hinge is equivalent to three degrees of freedom of rotation perpendicular to each other, so its motion spiral system is given by

$$\begin{aligned} \mathcal{S}_A &= [1 \ 0 \ 0 \ 0 \ 0 \ 0]^T, \\ \mathcal{S}_B: \begin{cases} \mathcal{S}_1 = [1 \ 0 \ 0 \ 0 \ f \ -e]^T, \\ \mathcal{S}_2 = [0 \ 1 \ 0 \ -f \ 0 \ 0]^T, \\ \mathcal{S}_3 = [0 \ 0 \ 1 \ e \ 0 \ 0]^T. \end{cases} \end{aligned} \quad (1)$$

FIGURE 2: Composite branch structure diagram of (a) compound branch diagram and of (b) partial view.

FIGURE 3: Composite branched chain mechanism diagram.

FIGURE 4: Configuration 1, parallelogram configuration.

For the kinematic spiral system above, the inverse spiral can be obtained:

$$\begin{aligned} \mathcal{S}_1^r &= [1 \ 0 \ 0 \ 0 \ f \ -e], \\ \mathcal{S}_2^r &= [0 \ e \ f \ 0 \ 0 \ 0]^T. \end{aligned} \quad (2)$$

The two antihelices represent two constraints that limit movement parallel to the axis X and the rod AB direction, respectively.

Because the two branches around have the same structure and configuration, as a result, they have the same antihelix and have the same two binding forces. For the BC rod of the moving platform, it bears a total of four binding forces, analyzing the correlation of its spiral, since the four binding forces are all line vectors, and the two line vectors of coplanar parallel are linearly independent. The two line vectors intersecting the plane are also linearly independent. This indicates that there are four independent constraints in the mechanism, which, respectively, constrain rotation about axis X and axis Z , and movement along axis X and rod AB direction. In this way, the moving

FIGURE 5: Parallelogram closed-loop subchain mechanism diagram.

platform has only the freedom of movement in the direction perpendicular to the rod AB and the freedom of rotation around the bar BC .

To sum up, as shown in Figure 4, configuration 1 is a synchronous telescopic parallelogram mechanism, so when l is a variable, only one translational DOF along the AB direction is added for the moving platform. Therefore, in configuration 1, the DOF of the moving platform BC can be expressed by the basic motion screw system as follows:

$$\begin{aligned}\$1 &= [0 \ 1 \ 0 \ 0 \ 0 \ 0]^T, \\ \$2 &= [0 \ 0 \ 0 \ 0 \ 1 \ 0]^T, \\ \$3 &= [0 \ 0 \ 0 \ 0 \ 0 \ 1]^T.\end{aligned}\quad (3)$$

As shown in Figure 6, when the mechanism is in configuration 2, there always have ${}^1B^1C = A^1D$, ${}^1BA//{}^1CD$. This moving platform only has one translational DOF in the 1BA direction, one in the direction parallel to the axis X, but rotational DOFs at the origin, and a local rotational DOF around the ${}^1B^1C$ direction. The motion spiral system $\$_{sp}$ can be expressed as (6).

When the three branch chains are in configuration 1 of Figure 4, the DOF of the moving platform of each branch can be expressed by the moving screw system:

$$\begin{aligned}\$1 &= [0 \ 1 \ 0 \ 0 \ 0 \ 0]^T, \\ \$2 &= [0 \ 0 \ 0 \ 0 \ 1 \ 0]^T, \\ \$3 &= [0 \ 0 \ 0 \ 0 \ 0 \ 1]^T.\end{aligned}\quad (4)$$

Therefore, the dynamic platform BC of the branch closed-loop subchain can be expressed as one rotational and two translational DOFs. Thus, the 2RPS parallelogram mechanism can be regarded as a generalized pair ([PPR]) with 3 DOFs.

As shown in Figure 7, after replacing each closed-loop subchain with a generalized motion pair, it can be regarded as composed of three connecting rods and four motion pairs (R [PPR]), among which the first R pair is connected with the frame in the mechanism, and the R pair in the generalized motion pair is connected with the moving platform. When the branch coordinate system is taken on the equivalent mechanism of the mechanism, the axis X can be along the normal surface of the closed-loop of the mechanism, and the axis Y can be along the axis of the rotation pair on the fixator of the mechanism.

$$\begin{cases} \$_{sp1} = [0 \ 0 \ 0 \ 0 \ \cos \beta_1 \ \sin \beta_1]^T, \\ \$_{sp2} = \left[1 \ 0 \ 0 \ 0 \ \frac{l_1 \sin \beta_1}{2} \ \frac{l_1 \cos \beta_1}{2} \right]^T, \\ \$_{sp3} = [0 \ l_4 + (l_2 - l_1) \cos \beta_1 \ (l_2 - l_1) \sin \beta_1 \ -l_1 l_4 \sin \beta_1 \ 0 \ 0]^T. \end{cases}\quad (5)$$

The branch motion spiral system of this mechanism is

$$\begin{aligned}\$1 &= [0 \ 1 \ 0 \ 0 \ 0 \ 0]^T, \\ \$2 &= [0 \ 1 \ 0 \ d_{i2} \ 0 \ f_{i2}]^T, \\ \$3 &= [0 \ 0 \ 0 \ 0 \ 1 \ 0]^T, \\ \$4 &= [0 \ 0 \ 0 \ 0 \ 0 \ 1]^T.\end{aligned}\quad (6)$$

FIGURE 6: Configuration 2, isosceles trapezoid.

FIGURE 7: Mechanism equivalent diagram.

Its branch constraint antihelix system is

$$\begin{aligned}\$_{i1}^r &= [0 \ 0 \ 0 \ 1 \ 0 \ 0]^T, \\ \$_{i2}^r &= [0 \ 0 \ 0 \ 0 \ 0 \ 1]^T.\end{aligned}\quad (7)$$

Because these two antihelices are mutually perpendicular constraint pairs, their directions are perpendicular to the direction of the corresponding rotation axis on the fixator of the mechanism. Therefore, the moving platform of the mechanism bears six constraint pairs from three branches. Three of them are perpendicular to the fixed platform, forming a common constraint, namely, $\lambda = 1$; the other three pairs are parallel to the fixed platform and are not parallel to each other, but they are linearly related to each other, and the maximum linearly independent number is 2, so there is a redundant constraint, namely, $\nu = 1$.

Thus, the final freedom of the agency is

$$M = d(n - g - 1) + \sum_{i=1}^g f_i + \nu - \delta = 5(11 - 12 - 1) + 12 + 1 = 3.\quad (8)$$

Considering that the moving platform of the mechanism is subjected to six constraint pairs from three branches, and their maximum linearly independent number is 3, the three rotational DOFs of the moving platform are constrained. Therefore, when the branches are in singular configuration

and configuration 1, the mechanism is a 3T parallel mechanism.

When the three branched chains are located in the singular configuration in Figure 3, each branched chain relative to configuration 1 has added an instantaneous rotational DOF around the axis X, three branched chains, and a total of three instantaneous rotational DOFs of moving platform because these three degrees of freedom are in different directions and space concurrent, thus linearly independent and mutual constraints.

Therefore, it can be known from the above analysis that, under the mutual constraints of the three branched chains, the moving platform of the mechanism will not rotate and not be in configuration 2, and each branch always keeps configuration 1. Therefore, rod AB and rod CD are synchronous telescopic components, and the whole mechanism is a 3T parallel mechanism.

4. Location Analysis

4.1. Coordinate System Establishment. Each slave arm of the parallel mechanism is a parallelogram, and the distance between the two sliding rods is unchanged, so the structure can be simplified, as shown in Figure 8. The three points L_1 , L_2 , and L_3 are the vertices of the equilateral triangle, and the radius of its circumscribed circle is R . With the center of the circle as the origin, the direction from which the center of the circle points to L_1 as the axis X, and the normal direction of the plane where the equilateral triangle is located as the axis Z, we establish the basic coordinate system O-XYZ according to the right-hand rule. The three points $N_1N_2N_3$ are the vertices of an equilateral triangle whose circumscribed circle radius is r . Taking the center of the circle as the origin, and the three coordinate axes at the initial position are parallel to the basic coordinate system, we establish a dynamic coordinate system O'-X'Y'Z'. The length of the active arm L_iM_i is l_b , and the length of the slave arm M_iN_i is l_{ai} , where l_{ai} is a variable. The angle between the static platform and the active arm is θ_i , and θ_i is the input parameter, where $i = 1, 2, 3$. The coordinate of the center O' of the moving platform is O'(x, y, z), where x, y, and z are output parameters.

4.2. Position Inverse Solution. According to the geometric relationship that always exists in the simplified mechanism: LiMi⊥MiNi, from the Pythagorean theorem

$$\overline{LN}^2 = \overline{LM}^2 + \overline{MN}^2, \quad (9)$$

where the coordinate of point L_i : $L_i = R \begin{bmatrix} \cos\alpha_i \\ \sin\alpha_i \\ 0 \end{bmatrix}$; the co-

ordinate of point N_i : $N_i = \begin{bmatrix} x + r\cos\alpha_i \\ y + r\sin\alpha_i \\ z \end{bmatrix}$; the coordinate of

point M_i : $M_i = \begin{bmatrix} (l_b\cos\theta_i + R)\cos\alpha_i \\ (l_b\cos\theta_i + R)\sin\alpha_i \\ l_b\sin\theta_i \end{bmatrix}$; and the rotation angle of the active arm is θ_i , where $i = 1, 2, 3$.

FIGURE 8: Coordinate system establishment.

Known from the coordinates,

$$\begin{aligned} \overline{LN}^2 &= [(x + r\cos\alpha_i) - R\cos\alpha_i]^2 \\ &\quad + [(x + r\sin\alpha_i) - R\sin\alpha_i]^2 + z^2, \\ \overline{LM}^2 &= l_b^2, \\ \overline{MN}^2 &= [(x + r\cos\alpha_i) - (l_b\cos\theta_i + R)\cos\alpha_i]^2 \\ &\quad + [(x + r\sin\alpha_i) - (l_b\cos\theta_i + R)\sin\alpha_i]^2 \\ &\quad + (z - l_b\sin\theta_i)^2. \end{aligned} \quad (10)$$

Substituting $\overline{LN}^2$, $\overline{LM}^2$, and $\overline{MN}^2$ into (9), after simplification, the equation of the rotation angle θ_i of the active arm and the center coordinates O'(x, y, z) of the moving platform can be obtained as

$$[x\cos\alpha_i + y\sin\alpha_i + (r - R)]\cos\theta_i + z\sin\theta_i - l_b = 0. \quad (11)$$

The above equation can be abbreviated as

$$Q_i\cos\theta_i + z\sin\theta_i - l_b = 0, \quad (12)$$

where

$$Q_i = [x\cos\alpha_i + y\sin\alpha_i + (r - R)]. \quad (13)$$

Let $t_i = \tan(\theta_i/2)$, according to the universal equation, substituting (12) and sorting it out yield

$$(Q_i + l_b)t_i^2 - 2zt_i + (l_b - Q_i) = 0. \quad (14)$$

The above equation can be solved by the one-dimensional quadratic equation:

$$t_i = \frac{z \pm \sqrt{Q_i^2 + z^2 - l_b^2}}{Q_i + l_b}. \quad (15)$$

Get the inverse solution of the mechanism

$$\theta_i = 2\arctan(t_i). \quad (16)$$

4.3. Position Positive Solution. On the basis of the position inverse solution, according to (11), the simplified solution is

$$\cos\alpha_i \cos\theta_i x + \sin\alpha_i \cos\theta_i y + \sin\theta_i z + \cos\theta_i (r - R) - l_b = 0, \quad (17)$$

where

$$i = 1, 2, 3. \quad (18)$$

The above formula is simplified to

$$A_i x + B_i y + C_i z + D_i = 0 \quad (i = 1, 2, 3), \quad (19)$$

where

$$\begin{aligned} A_i &= \cos\alpha_i \cos\theta_i, \\ B_i &= \sin\alpha_i \cos\theta_i, \\ C_i &= \sin\theta_i, \\ D_i &= \cos\theta_i (r - R) - l_b. \end{aligned} \quad (20)$$

Equation (19) is eliminated and sorted out, and its positive solution is

$$\begin{cases} x = -(B_1 C_2 D_3 - B_1 C_3 D_2 - B_2 C_1 D_3 + B_2 C_3 D_1 + B_3 C_1 D_2 - B_3 C_2 D_1) / (A_1 B_2 C_3 - A_1 B_3 C_2 - A_2 B_1 C_3 + A_2 B_3 C_1 + A_3 B_1 C_2 - A_3 B_2 C_1), \\ y = -(A_1 C_2 D_3 - A_1 C_3 D_2 - A_2 C_1 D_3 + A_2 C_3 D_1 + A_3 C_1 D_2 - A_3 C_2 D_1) / (A_1 B_2 C_3 - A_1 B_3 C_2 - A_2 B_1 C_3 + A_2 B_3 C_1 + A_3 B_1 C_2 - A_3 B_2 C_1), \\ z = -(A_1 B_2 D_3 - A_1 B_3 D_2 - A_2 B_1 D_3 + A_2 B_3 D_1 + A_3 B_1 D_2 - A_3 B_2 D_1) / (A_1 B_2 C_3 - A_1 B_3 C_2 - A_2 B_1 C_3 + A_2 B_3 C_1 + A_3 B_1 C_2 - A_3 B_2 C_1). \end{cases} \quad (21)$$

5. Velocity Analysis and Jacobian Matrix

The Jacobian matrix, also known as the first-order motion influence coefficient, is the mapping between the operating speed of the machine and the joint speed and can also be regarded as the transmission ratio of the motion speed from the joint space to the operating space. The end speed of the parallel mechanism and the input angular speed are expressed as

$$\dot{X} = J \dot{\theta}, \quad (22)$$

where $\dot{X} = [\dot{x} \ \dot{y} \ \dot{z}]^T$ represents the end velocity vector, $\dot{\theta} = [\dot{\theta}_1 \ \dot{\theta}_2 \ \dot{\theta}_3]^T$ represents the driving speed vector, and J represents the Jacobian matrix of the parallel mechanism.

By equation (11), let

$$F_i = [x \cos\alpha_i + y \sin\alpha_i + (r - R)] \cos\theta_i + z \sin\theta_i - l_b = 0. \quad (23)$$

Solve the derivative of the above equation with respect to time

$$\frac{\partial F_i}{\partial x} \dot{x} + \frac{\partial F_i}{\partial y} \dot{y} + \frac{\partial F_i}{\partial z} \dot{z} + \frac{\partial F_i}{\partial \theta_i} \dot{\theta}_i = 0. \quad (24)$$

Sorted it out

$$A \dot{X} = B \dot{\theta}. \quad (25)$$

It can be concluded that the Jacobian matrix of the parallel mechanism is

$$J = A^{-1} B. \quad (26)$$

Here, in (24)

$$\begin{aligned} \frac{\partial F_i}{\partial x} &= \cos\alpha_i \cos\theta_i, \\ \frac{\partial F_i}{\partial y} &= \sin\alpha_i \cos\theta_i, \\ \frac{\partial F_i}{\partial z} &= \sin\theta_i, \\ \frac{\partial F_i}{\partial \theta_i} &= z \cos\theta_i - (x \cos\alpha_i + y \sin\alpha_i + r - R) \sin\theta_i. \end{aligned} \quad (27)$$

Here, in (25)

$$A = \begin{bmatrix} \frac{\partial F_1}{\partial x} & \frac{\partial F_1}{\partial y} & \frac{\partial F_1}{\partial z} \\ \frac{\partial F_2}{\partial x} & \frac{\partial F_2}{\partial y} & \frac{\partial F_2}{\partial z} \\ \frac{\partial F_3}{\partial x} & \frac{\partial F_3}{\partial y} & \frac{\partial F_3}{\partial z} \end{bmatrix}, B = \begin{bmatrix} \frac{\partial F_1}{\partial \theta_1} & 0 & 0 \\ 0 & \frac{\partial F_2}{\partial \theta_2} & 0 \\ 0 & 0 & \frac{\partial F_3}{\partial \theta_3} \end{bmatrix}. \quad (28)$$

6. Singular Configuration Analysis

The singularity of the mechanism is related to the stability of the mechanism's performance. When the mechanism is in a singular position, there will be one or more degrees of freedom lost, causing the moving platform to be uncontrollable in certain directions. The methods to analyze the singular configuration of the mechanism mainly include kinematics, geometry, and algebra. Among them, the algebraic method to solve the singular configuration of the parallel mechanism is to find whether the Jacobian matrix is full rank. When $J = A^{-1} B = 0$, the parallel mechanism will show singular configuration.

According to the Jacobian matrix of the parallel mechanism, the singular configuration of the mechanism in this paper is analyzed, and (26) is solved. When the angle between the static platform and the active arm is $\pi/2$, we can get $J = 0$. The mechanism has a singular configuration. As shown in Figure 9, at this time, the branch chain constraint of the parallel mechanism increases, and the degree of freedom of the moving platform decreases.

7. Workspace Analysis

The workspace of the parallel mechanism refers to the collection of all points that the reference point of the end-effector can reach in the workspace. This paper mainly adopts the Monte Carlo method that a large number of random numbers are used to determine enough random joints. The reference points of all end actuators of the

FIGURE 9: Singularity diagram.

mechanism are drawn to form the workspace of the mechanism.

The position equation is combined with the known parameters of the parallel mechanism: the radius of the circumscribed circle of the static platform is $R = 120$ mm, the radius of the circumscribed circle of the movable platform is $r = 34.64$ mm, and the length of the active arm member is $l_b = 99.75$ mm, and the angle θ_i between the static platform and the active arm is defined as $[20^\circ, 75^\circ]$. The maximum length of the variable-length links is 350 mm. In this range, by setting a large number of random variables to substitute the position forward solution, through MATLAB programming, the workspace diagram of the parallel mechanism is obtained, as shown in Figure 10. Figures 10(a)–10(c) show three views of different workspaces, and Figure 10(d) shows a three-dimensional view of the workspace.

8. Trajectory Planning

Trajectory planning can be summarized as follows: according to the needs of the robot in actual working conditions, plan a trajectory or path that can complete the work task within the reach of its end-effector. Reasonable trajectory planning can not only shorten the motion cycle and improve work efficiency but also reduce structural vibration and extend the service life.

8.1. Joint Space Trajectory Planning. To plan the trajectory of the parallel mechanism, in order to ensure the smooth movement of each joint and prevent sudden changes in acceleration, this article uses a fifth-degree polynomial for interpolation

$$\begin{cases} s(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5, \\ v(t) = a_1 + 2a_2 t + 3a_3 t^2 + 4a_4 t^3 + 5a_5 t^4, \\ a(t) = 2a_2 + 6a_3 t + 12a_4 t^2 + 20a_5 t^3, \end{cases} \quad (29)$$

where s , v , and a are the path length, velocity, and acceleration, respectively. $a_0, a_1, \dots, a_5$ are the undetermined coefficients of the polynomial, and t is the time.

In order to ensure the smooth operation of the parallel mechanism, the following boundary constraints are specified:

$$\begin{aligned} s(0) &= 0, v(0) = 0, a(0) = 0, \\ s(t_f) &= s_f, v(t_f) = 0, a(t_f) = 0, \end{aligned} \quad (30)$$

where t_f represents the terminal time of the trajectory and s_f is the total length of the planned path of the trajectory.

Substituting the above boundary conditions, the undetermined coefficients can be solved, and the trajectory planning equation is obtained as follows:

$$\begin{cases} s(t) = \frac{10s_f}{t_f^3}t^3 - \frac{15s_f}{t_f^4}t^4 + \frac{6s_f}{t_f^5}t^5, \\ v(t) = \frac{30s_f}{t_f^3}t^2 - \frac{60s_f}{t_f^4}t^3 + \frac{30s_f}{t_f^5}t^4, \\ a(t) = \frac{60s_f}{t_f^3}t - \frac{180s_f}{t_f^4}t^2 + \frac{30s_f}{t_f^5}t^3. \end{cases} \quad (31)$$

8.2. Coordinate Space Trajectory Planning. The trajectory planning of the parallel mechanism needs to involve the relationship of the path and the time of its motion. This paper mainly introduces a kind of door-shaped trajectory, which can meet the needs of picking and placing operations in the industry.

As shown in Figure 11, P_0 is the picking point and P_5 is the placing point. The trajectory is composed of three straight lines and two arcs. The two transition arcs can effectively avoid sudden changes in speed and acceleration.

Assuming that the door-shaped trajectory is in the XOZ plane, the initial position $P_0(x_0, 0, z_0)$ is given. Interpolate the straight line and circular arc in Figure 11, respectively, to obtain the door-shaped trajectory motion (32).

$\varphi_1 = (s(t_i) - h/r)$ and $\varphi_2 = (s(t_i) - h - (1/2)\pi r_p - l/r_p)$. $t_1 \dots t_5$ correspond to the time of arrival at $P_1 \dots P_5$, respectively. t_i is any time in the cycle.

9. Simulation Analysis

According to the analysis of trajectory planning, the trajectory of the end-effector of the parallel mechanism is divided into three straight sections and two circular arcs. The picking position is $P_0(-45, 0, 240)$, and the placing position is $P_5(45, 0, 240)$. $h = 30$ mm, $l = 70$ mm, $r_p = 10$ mm, and $t_f = 3$ s.

In the initial position, the default initial angle θ_i , in SolidWorks, between the static platform and the active arm is 0. Sampling points in these five trajectories, and then performing motion analysis in SolidWorks, let the end-effector move along the door-shaped trajectory, and draw the curve of the angle θ_i on the three composite branched chains with time t , as shown in Figure 12.

FIGURE 10: Schematic diagram of workspace. (a) y-o-z view, (b) x-o-y view, (c) x-o-z view, and (d) workspace diagram.

FIGURE 11: Door-shaped trajectory diagram.

FIGURE 12: Variation curve of different branch chain angles 1. (a) Branched chain 1, (b) branched chain 2, and (c) branched chain 3.

FIGURE 13: Continued.

FIGURE 13: Variation curve of different branch chain angles 2. (a) Branched chain 1, (b) branched chain 2, and (c) branched chain 3.

FIGURE 14: 3T parallel mechanism. (a) Overall picture of the mechanism and (b) operation interface and control system.

$$\left\{ \begin{array}{ll} (x, y, z) = (x_0, 0, z_0 - s(t_i)) & t_0 < t_i \leq t_1 \\ (x, y, z) = (x_0 + r_p - r_p \cos \varphi_1, 0, z_0 - h - r_p \sin \varphi_1) & t_1 < t_i \leq t_2 \\ (x, y, z) = (x_0 + r_p + s(t_i) - 0.5\pi r_p - h, 0, z_0 - h - r_p) & t_2 < t_i \leq t_3 \\ (x, y, z) = (x_0 + r_p + l + r_p \sin \varphi_2, 0, z_0 - h - r_p \cos \varphi_2) & t_3 < t_i \leq t_4 \\ (x, y, z) = (x_0 + 2r_p + l, 0, z_0 - s_f + s(t_i)) & t_4 < t_i \leq t_5 \end{array} \right. \quad (32)$$

We simulate the gate trajectory in MATLAB, using the door-shaped trajectory equation in trajectory planning. Then, through the inverse solution of position, we program to obtain the change curve graph of the upper angle θ_i of the three composite branch chains with time t , as shown in Figure 13. By comparing with Figure 12, it can be seen that the angle change rates of the three branched chains are the same.

10. Prototype and Experiments

The 3T parallel mechanism is composed of a mechanical body and a control system. The mechanical body is mainly composed of structural parts, motors, and suction cups, as shown in Figure 14(a). The control system is mainly composed of industrial computer, motion controller, servo driver, and servo part, as shown in Figure 14(b). The servo

FIGURE 15: Prototype door trajectory movement diagram.

TABLE 1: Parameters of trajectory point.

Tracing point	Coordinate (mm)
P0	(−45, 0, 240)
P1	(−45, 0, 210)
P2	(−35, 0, 200)
P3	(35, 0, 200)
P4	(45, 0, 210)
P5	(45, 0, 240)

systems of the three branch chains coordinate with each other to determine the movement of the moving platform of the parallel mechanism. The industrial computer communicates with the controller through the PCIe interface and sends control information to the controller. The controller outputs pulses to control the speed of the servo motor, thereby realizing the motion control of the moving platform.

Running the experiment according to the path in the trajectory planning, a sample is tested from $P0 \cdots P5$ according to the trajectory of door shape. The door-shaped trajectory motion diagram of the whole experiment is shown in Figure 15, and the red dots represent the endpoints of the effector of the parallel mechanism. The relevant parameters of all trajectory points in the basic coordinate system are shown in Table 1.

11. Conclusion

- (1) A novel 3T parallel mechanism with variable-length parallelogram closed-loop subchain was designed. Based on the screw theory, the degree of freedom of the mechanism was analyzed, and the number and nature of the mechanism were 3T.

- (2) The kinematics analysis of the mechanism was carried out, including the positive and inverse solution of position, speed, workspace, and singularity. The trajectory planning of the mechanism was completed, and the rationality of the mechanism design and the correctness of the theoretical analysis were verified through simulation analysis and prototype testing.
- (3) Compared with other mechanisms with variable-length parallelogram closed-loop subchains, the mechanism described in this article has the advantages of simple structure and the drive can be installed on a static platform. Therefore, the mechanism has a good application prospect in light load and fast picking occasions.

Data Availability

All the data in this paper are obtained by calculation. The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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