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Research Article

An Optimization Model and Computer Simulation for Allocation Planning of Hospital Bed Resources

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Aim. This study intends to improve the arrangement of hospital beds, make rational use of limited beds to minimize the time for patients to seek medical care, improve service quality, and reduce hospital operating costs. *Methods*. This study determines the reasonable evaluation indices, established a reasonable evaluation index system, and assesses the merits and demerits of the bed arrangement model for this problem. After observing and dealing with various types of patients' conditions in the ophthalmology outpatient department of the hospital, some effective hospitalization rules are proposed. The coefficients of each evaluation index under the FCFS (first-come first-service) and hospitalization rules are obtained, and a mathematical model of evaluation indices is constructed. The merits and demerits of the evaluation index system under each rule are found with MATLAB programming. At the same time, considering that the five types of patients are in a certain serial sequence during the queuing process for medical treatments, a random planning model with the least average stay time of patient is established via introducing the corresponding joint probability density function for the stay time of each patient. The proportional allocation of beds is obtained by genetic algorithms with the speed and convenience of computer calculations. *Results*. The proportions of beds allocated to the five categories of patients with trauma, cataract in one eye, cataract in both the eyes, glaucoma, and retina are 15%, 16%, 28%, 11%, and 29%, respectively. *Conclusion*. The results are in good agreement with the actual situation, and the optimization model has a certain degree of reference significance for the reasonable allocation of beds.

1. Problem Statement

Queuing in hospitals for medical treatments is a common phenomenon in the society today. It is a prominent manifestation of the contradiction between patient needs and insufficient resources in hospitals and also an important fuse for the tension between doctors and patients. How to rationally arrange limited beds and make use of relatively insufficient resources to reduce patients' queuing time for medical treatments to the maximum degree and extent has always been a concern of scholars and hospital management. Queuing theory is widely used in the design, operation, and analysis of medical systems [1], such as to determine the number of medical personnel [2], hospital bed allocation [3, 4], medical staff scheduling [5–7], and patient waiting time [8–10]. In response to issues related to inpatient bed planning, research studies mainly focused on increasing the utilization rate of inpatient beds and reducing the waiting time of patients. Beds are one of the most important resources in a hospital. Berry Jaeker and Tucker found an inverted U-shaped relationship between utilization and throughput time. Meanwhile, patient' length of stay (LOS) increased as occupancy increases, until a tipping point, after which patients were discharged early to alleviate congestion. More interestingly, the two years of inpatient data from 203 California hospitals revealed the second tipping point of 93% occupancy, beyond which additional occupancy leads to a longer LOS. Thomas Schneider et al. employed discrete event simulation (DES) to evaluate the impact of allocating beds in inpatient wards to accommodate emergency admissions and analyze the impact of pooling the number of beds allocated for emergency admissions in inpatient wards. Kao and Tung adopted a queuing model to approximate the patient population dynamics for each service, with admission rates provided by forecasts, the expected overflows under each configuration are computed via a normal loss integral. Bed allocation was done in two stages. First, they established a baseline requirement for each service, so that it can handle a prescribed amount of patient load based on a yearly projection of demand. They then employed marginal analysis to distribute the remaining beds to minimize the expected total average overflows while taking month-to-month demand fluctuations into account. Xie et al. proposed the bed shortage index (BSI) to capture more facets of bed shortage risk than traditional metrics such as the occupancy rate, the probability of shortages, and expected shortages; they also proposed optimization models to plan for bed capacity via this metric. Meanwhile, [15, 16] took an eye hospital as an example; they established comparatively perfect decision-making models of the patient admission scheduling and based on overall analysis on waiting times for hospital admission, waiting times for surgery, and the length of waiting list for hospital admission, and they also introduced the genetic algorithm and overall satisfaction evaluation model to optimize patient admission scheduling.

Considering all above, in this study, we take the same eye hospital used by [15, 16] to collect the data, and we adopt a reasonable mathematical model to solve the problem of reasonably arranging its beds and maximizing its resource benefits.

The specific situation of the ophthalmology hospital is as follows. The ophthalmology clinic of this hospital is open every day, and there are 79 beds in the inpatient department. The ophthalmological operations of the hospital can only perform the following four types of surgery: cataract, retinal disease, glaucoma, and trauma. The specific conditions of various ophthalmic operations in the ophthalmology department of the hospital are as follows. Cataract surgery is relatively simple, and there is no emergency. At present, the hospital offers cataract surgeries every Monday and Wednesday. The preparation time for such patients is only one or two days. There are more patients with surgery for two eyes than those with one eye, accounting for about 60%. If a person needs surgeries for both the eyes, the surgery on one eye is on Monday and then the other is on Wednesday. Trauma disease is usually an emergency, so hospital admission will be arranged immediately when the bed is available, and surgery will be arranged the next day after admission. The surgeries for other eye diseases are more complicated, and there are various conditions, but surgery can be performed within 2-3 days after admission, and usually, the observation time after surgery is longer. The operation time for this type of disease can be arranged according to needs and generally not on Monday or Wednesday. Considering the rationality of FCFS basis for patient admission scheduling, we assume the following: patients with the same type of disease will be admitted according to the order in which they arrive and patients with the same type of disease will have surgery in the order of their admissions [17].

According to the specific conditions of the various types of patients in the hospital, this study intends to solve the following three problems: determine a reasonable evaluation index system, establish a reasonable bed arrangement model, and establish the proportional hospital bed allocation model that keep all patients in the system with the shortest average stay time.

2. Model Assumptions and Symbol Description

2.1. Model Assumptions

(1) It is assumed that the impact of seasons and weather changes on the condition of the disease is not considered

(2) It is assumed that other ophthalmic diseases other than traumas are not considered for emergencies

(3) It is assumed that the serious illness caused by delay in treatment is not considered

(4) It is assumed that the referral of patients due to too long waiting time or waiting team is not considered within a certain time frame

(5) It is assumed that there will be no sporadic large-scale sudden eye surgery

(6) It is assumed that the ophthalmic surgery technique has not been significantly improved, resulting in a significant reduction in the length of stay (LOS)

2.2. Symbol Description. n is the evaluation index unit, m is the evaluation index exponent, a_{ij} is the raw data of the i^{th} evaluation index and the j^{th} evaluation exponent, r_{ij} is the rated value of a_{ij}, T_G is the expected LOS of glaucoma patients, T_R is the expected LOS of retinal patients, T_{C1} is the expected LOS of patients with cataract in one eye, T_{C2} is the expected LOS for cataract patients with cataract in both the eyes, and T_T is the expected LOS for trauma patients.

3. Mathematical Model Establishment and Solution

First, it is required to determine reasonable evaluation indices and then establish a reasonable evaluation index system to evaluate the pros and cons of the bed arrangement model for this problem. Second, observe and deal with the various types of patients' conditions in the ophthalmology outpatient department of the hospital, propose some effective hospitalization rules, and obtain the coefficients of each evaluation index under the FCFS rules and these hospitalization rules. Based on the evaluation index mathematical model, MATLAB programming is employed to find the pros and cons of the evaluation index system under each rule. Determine which patients should be hospitalized the next day using this model according to the known number of patients to be discharged on the second day. The recursive method could assist in predicting the admission time interval of outpatients according to the specific situation of the patients who have been discharged from the hospital and the patients' hospitalization. At the same time, considering that the five types of patients are in a certain serial sequence during the queuing process for medical treatment, a random planning model with the least average stay time of patient can be established via introducing the corresponding joint probability density function for the stay time of each patient, and the speed and convenience of computer calculations can be employed to solve the proportional allocation of beds by genetic algorithms.

3.1. Evaluation Index System

3.1.1. Selection of Evaluation Indices. In order to effectively evaluate the pros and cons of the ophthalmological bed arrangement model of the hospital, this study selects the following four important evaluation indices: number of bed turnovers, number of discharges, average bed working days, and the average length of stay (LOS) of discharged patients.

3.1.2. Evaluation Index Model Based on the TOPSIS Method. It is found from the literature review that the average LOS has an inverse effect on the number of bed turnovers [18, 19]. Therefore, shortening the average LOS of discharged patients and accelerating bed turnover can reduce the patient's medical care expenses, expand the capacity of hospitals to admit and treat patients and improve social and economic benefits, and improve the current shortage of beds to a certain extent.

Evaluation index mathematical model: based on the TOPSIS method [20], through the same trending and normalization of the original data, the closeness of the index is reflected in the calculation of the ratio of the distance between the actual value and the optimal value. The sorting result makes full use of the original data and can be applied to multilevel evaluation of multiple indicators.

(1) Build the original data matrix

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & a_{n4} \end{bmatrix}. \tag{1}$$

(2) Same trending to establish high-quality indices and low-quality indices:

Among them, the number of bed turnovers, the number of discharged patients, and the average bed working days are high-quality indices, and the average LOS of discharged patients is a low-quality index. The average LOS of discharged patients is an absolute number index, using the reciprocal method, that is, $(1/a_{i4})$ is transformed into a high-quality index.

(3) Standardized data matrix

$$A' = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \dfrac{1}{a_{14}} \\ a_{21} & a_{22} & a_{23} & \dfrac{1}{a_{24}} \\ a_{31} & a_{32} & a_{33} & \dfrac{1}{a_{34}} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \dfrac{1}{a_{n4}} \end{bmatrix}. \tag{2}$$

(4) Normalization processing to normalize the matrix after the same trending

$$r_{ij} = \frac{a_{ij}}{\sqrt{\sum_{i=1}^{n} a_{ij}^2}} \quad i = 1, 2, \ldots, n; j = 1, 2, \ldots, m. \tag{3}$$

(5) Define the "optimal value" and "worst value" of each evaluation index

The optimal value is the maximum rated value of each evaluation index:

$$B_j = \max_{1\le i\le n}\{r_{ij}\} \quad i = 1, 2, \ldots, n; j = 1, 2, \ldots, m. \tag{4}$$

The worst value is the minimum rated value of each evaluation index:

$$S_j = \min_{1\le i\le n}\{r_{ij}\} \quad i = 1, 2, \ldots n; j = 1, 2, \ldots, m. \tag{5}$$

The optimal value and the worst value of each evaluation index are as follows: The optimal value: $B_j = \{B_1, B_2, B_3, \ldots, B_m\}$. The worst value: $S_j = \{S_1, S_2, S_3, \ldots, S_m\}$.

Calculate the distance between each evaluation index and the optimal value and the worst value:

$$\begin{aligned} d_i^+ &= \sqrt{\sum_{j=1}^{m} \left(r_{ij} - B_j\right)^2}, \\ d_i^- &= \sqrt{\sum_{j=1}^{m} \left(r_{ij} - S_j\right)^2}. \end{aligned} \tag{6}$$

The close value is zero-dimensional, in which the maximum distance and the minimum distance between the optimal value and the worst value of each unit are mentioned. Then, it makes its own comparison and comprehensive evaluation of the closeness of the optimal value and the worst value of this problem.

$$\begin{aligned} \text{Denoted by}: d^+ &= \min_{1\le i\le n}\{d_i^+\}, \\ d^- &= \max_{1\le i\le n}\{d_i^-\}, \\ \text{The close value: } C_i &= \frac{d_i^-}{d^- + d^+}. \end{aligned} \tag{7}$$

When the value of c_i becomes larger and larger, the evaluation unit gets closer and closer to the optimal point and farther and farther away from the worst point. Therefore, C can be adopted to measure the pros and cons of the bed arrangement model.

3.2. Hospital Bed Arrangement Model

3.2.1. Queuing System Model ([21, 22])

(1) Average patient arrival rate per day:

$$\lambda = \frac{\sum n_i f_i}{\sum f_i}. \tag{8}$$

(2) Average interval of patient arrival time:

$$\frac{1}{\lambda} = \frac{\sum f_i}{\sum n_i f_i}. \tag{9}$$

(3) Average number of people completing inspections per day:

$$\mu = \frac{\sum f_v}{\sum \mathrm{vf}_v}. \tag{10}$$

(4) Average time for each patient to see a doctor:

$$\frac{1}{\mu} = \frac{\sum \mathrm{vf}_v}{\sum f_v}. \tag{11}$$

Perform a statistical test (χ^2 inspection) for λ, μ. First, assume that the arrival law conforms to the parameter λ Poisson distribution:

$$\chi^2 = \sum_{0}^{12} \frac{(f_i - \mathrm{np}_i)^2}{\mathrm{np}_i}, p_i = \frac{e^{-\lambda}\lambda^{n_i}}{n_i!}. \tag{12}$$

(5) Service intensity and idle probability:

$$\begin{aligned} \rho &= \frac{\lambda}{\mu}, \\ P_0 &= 1 - \rho, \\ P &= 1 - P_0. \end{aligned} \tag{13}$$

(6) Operational indicators:

(a) Average number of patients in the system (expected value of queue length):

$$L_s = \frac{\lambda}{\mu - \lambda}. \tag{14}$$

(b) Average number of patients waiting in the queue (expected value of queue length):

$$L_q = \frac{\rho\lambda}{\mu - \lambda}. \tag{15}$$

(c) Expected value of patient stay time in the system:

$$W_s = \frac{1}{\mu - \lambda}. \tag{16}$$

(d) Expected waiting time of patients in the queue:

$$W_q = \frac{\rho}{\mu - \lambda}. \tag{17}$$

(e) Patient loss of time:

$$R = \frac{W_q}{1/\mu} = W_q \times \mu. \tag{18}$$

3.2.2. Information on Different Types of Patients Visiting the Ophthalmology Clinic of the Hospital. Based on Tables 1–5, it is noted that each type of patients visiting the ophthalmology clinic of the hospital conformed to the Poisson distribution, through which, the expected LOS could be calculated for each kind of patients in the ophthalmology department of the hospital.

$$\begin{aligned} &T_G = 10\,(days),\ T_T = 7\,(days),\ T_{c1} = 5\,(days), T_{c2} \\ &\quad = 9\,(\mathrm{days}),\ T_T = 7\,(days). \end{aligned} \tag{19}$$

Based on this, the following hospitalization rules are proposed:

(1) Priority queuing rules based on waiting time: for traumatic diseases, hospital admission will be arranged immediately when the bed is available, and surgery will be arranged the next day after admission; patients of cataract (one eye) are only admitted on Saturday, Sunday, Monday, or Tuesday, patients of cataract (both eyes) are only admitted on Saturday or Sunday; patients of cataract (both eyes) have a higher priority between patients of cataract (one eye) and cataract (both eyes); for patients with glaucoma and retinal diseases, if there is a cataract surgery on Monday or Wednesday on the 4th and 5th day after their admission, the admission will be postponed, and the admission is made if not. Glaucoma has a higher priority between glaucoma and retinal diseases.

This rule is abbreviated as Rule 1.

(2) First-come, first-served rule for classification discussion: divide all kinds of patients into 5 situations according to their conditions and then arrange the beds according to the principle of first-come, first-served for all kinds of patients.

This rule is abbreviated as Rule 2.

(3) Rules based on the principle of priority of each illness using the expert scoring method: the priority of each illness condition is set according to the expert scoring method. Among them, trauma is the highest priority, cataract is the second priority, retina is the third priority, and glaucoma is the lowest priority. Hospital beds are arranged for all types of patients according to this priority.

Table 1: Information on trauma patients visiting the ophthalmology clinic of the hospital ($\lambda = 64 \div 61 = 1.0492$).

Number of trauma patients arriving each day, n	Actual frequency, f_n	$n \times f_n$	Probability, p_n	Theoretical frequency, T	$(T - f_n)^2/T$
0	21	0	0.3445	21.0163	1.2700e − 005
1	20	20	0.3671	22.3945	0.2560
2	16	32	0.1956	11.9315	1.3873
3	4	12	0.0695	4.2380	0.0134
Total	61	64			1.6567

Table 2: Information on retina patients visiting the ophthalmology clinic of the hospital ($\lambda = 170 \div 61 = 2.7869$).

Number of retina patients arriving each day, n	Actual frequency, f_n	$n \times f_n$	Probability, p_n	Theoretical frequency, T	$(T - f_n)^2/T$
0	2	0	0.0616	3.7584	0.8227
1	16	16	0.1717	10.4742	2.9152
2	11	22	0.2393	14.5952	0.8856
3	11	33	0.2223	13.5584	0.4827
4	9	36	0.1549	9.4464	0.0211
5	10	50	0.0863	5.2652	4.2578
6	1	6	0.0401	2.4456	0.8545
7	1	7	0.0160	0.9737	7.1297e − 004
Total	61	170			10.2403

Table 3: Information on patients with cataract in one eye visiting the ophthalmology clinic of the hospital ($\lambda = 100 \div 61 = 1.6393$).

Number of patients with cataract in one eye arriving each day, n	Actual frequency, f_n	$n \times f_n$	Probability, p_n	Theoretical frequency, T	$(T - f_n)^2/T$
0	9	0	0.1973	12.0363	0.7659
1	24	24	0.3202	19.5342	1.0209
2	15	30	0.2599	15.8516	0.0457
3	7	21	0.1406	8.5754	0.2894
4	5	20	0.0570	3.4794	0.6646
5	1	5	0.0185	1.1294	0.0148
Total	61	100			2.8013

Table 4: Information on patients with cataract in both the eyes visiting the ophthalmology clinic of the hospital ($\lambda = 128 \div 61 = 2.0984$).

Number of patients with cataract in both the eyes arriving each day, n	Actual frequency, f_n	$n \times f_n$	Probability, p_n	Theoretical frequency, T	$(T - f_n)^2/T$
0	8	0	0.1130	6.8933	0.1777
1	20	20	0.2464	15.0296	1.6438
2	10	20	0.2686	16.3847	2.4880
3	12	36	0.1952	11.9080	7.1044e − 004
4	6	24	0.1064	6.4908	0.0371
5	3	15	0.0464	2.8304	0.0102
6	1	6	0.0169	1.0285	7.9227e − 004
7	1	7	0.0053	0.3204	1.4418
Total	61	128			5.8001

Table 5: Information on glaucoma patients visiting the ophthalmology clinic of the hospital ($\lambda = 63 \div 61 = 1.0328$).

Number of glaucoma patients arriving each day, n	Actual frequency, f_n	$n \times f_n$	Probability, p_n	Theoretical frequency, T	$(T - f_n)^2/T$
0	20	0	0.3502	21.3637	0.0870
1	25	25	0.3674	22.4144	0.2983
2	13	26	0.1928	11.7584	0.1311
3	0	0	0.0674	4.1122	4.1122
4	3	12	0.0177	1.0786	3.4227
Total	61	63			8.0513

This rule is abbreviated as Rule 3.

According to Table 6, the coefficients of each evaluation index are found under this rule as follows.

The number of bed turnovers is 5.4 (persons/piece); the number of discharged patients is 428 (persons); the average number of bed working days is 53 (days); the average LOS of discharged patients is 7.3 (days); in the same way, the coefficients of various evaluation indices based on other rules can be obtained, as given in Table 6.

According to the mathematical model of the evaluation index, the evaluation index system under various rules is calculated via programming with MATLAB:

$$A' = \begin{bmatrix} 4.8 & 378 & 54 & 11.3636 \\ 5.4 & 428 & 53 & 13.6986 \\ 5.1 & 403 & 53 & 13.1579 \\ 4.9 & 387 & 54 & 12.3457 \end{bmatrix}. \tag{20}$$

From this calculation,

$$r_{\mathrm{ij}} == \begin{bmatrix} 0.0060 & 0.4760 & 0.0680 & 0.0143 \\ 0.0065 & 0.5188 & 0.0667 & 0.0173 \\ 0.0063 & 0.4974 & 0.0667 & 0.0166 \\ 0.0062 & 0.4873 & 0.0680 & 0.0155 \end{bmatrix}.$$

From this calculation, $B_j = [\,0.0065 \;\; 0.5188 \;\; 0.0680 \;\; 0.0173\,]$.

$$S_j = [\,0.0060 \;\; 0.4760 \;\; 0.0667 \;\; 0.0143\,]. \tag{21}$$

From this calculation, $d_i^+ = \begin{bmatrix} 0.0429 \\ 0.0013 \\ 0.0215 \\ 0.0315 \end{bmatrix}$, $d_i^- = \begin{bmatrix} 0.0013 \\ 0.0429 \\ 0.0215 \\ 0.0115 \end{bmatrix}$.

From this calculation, $C_i = \begin{bmatrix} 0.0285 \\ 0.9715 \\ 0.5008 \\ 0.2668 \end{bmatrix}$.

Combine the data obtained above into a table, as given in Table 7.

Based on this study, it is concluded that the bed arrangement model of Rule 1 is the best. According to this model, which patients should be arranged for hospitalization on the next day could be determined based on the known number of patients to be discharged on the next day.

3.3. The Proportional Hospital Bed Allocation Model. The establishment of the model of $f(u, v, w, l, m)$ is to express the combined probability density function [23] of the stay time of the five types of patients: trauma, cataract in one eye, cataract in both the eyes, glaucoma, and retina. The five types of patients have a certain priority in the process of queuing for medical treatment, so the process is sequential and belongs to a series of activities. The stay time of the five types of patients is equal to the sum of their stay time; thus,

$$\begin{aligned} &E(T) = E(T_1) + E(T_2) + E(T_3) + E(T_4) + E(T_5), \\ &\int_0^\infty t \times f_{l1}(t)\mathrm{d}t + \int_0^\infty t \times f_{l2}(t)\mathrm{d}t + \int_0^\infty t \times f_{l3}(t)\mathrm{d}t + \int_0^\infty t \times f_{l4}(t)\mathrm{d}t + \int_0^\infty t \times f_{l5}(t)\mathrm{d}t = \int_0^x t \times f(t)\mathrm{d}t, \\ &\text{At the same time}: \int_0^x t \times f(t)\mathrm{d}t = \iiint\limits_{u+v+w \le x} (u, v, w) \times f(u, v, w)\mathrm{d}w\,\mathrm{d}v\,\mathrm{d}u \\ &= \int_0^x \int_0^{x-u} \int_0^{x-u-v} (u + v + w) \times f_{13}(w) f_{12}(v) f_{11}(u)\mathrm{d}w\,\mathrm{d}v\,\mathrm{d}u, \end{aligned} \tag{22}$$

where $f_{l1}(t), f_{l2}(t), f_{l3}(t), f_{l4}(t)$, and $f_{l5}(t)$ are, respectively, the probability density functions of the stay time of the five types of patients, T_1, T_2, T_3, T_4, and T_5 are the stay time of those five types of patients, and $E(T)$ means the average stay time of the five types of patients.

The goal of this article is to find how the hospital should provide a matrix of bed numbers $[m_1, m_2, m_3, m_4, m_5]$ for the five types of patients in order to minimize the average stay time of patients.

Suppose that when a certain patient (glaucoma) arrives, the number of beds used for glaucoma in the hospital is n. When $n < m_4$, the patient does not need to wait and can be directly prepared for admission, and the admission time is $t_\lambda \ge 2$. When $n \ge m_4$, the patient can choose the shortest time to wait, and the stay time is subject to the round$(n/m_4) + 1$ order of Erlang distribution. It is easy to calculate the probability of the patient's stay time as

$$P_0 = \left[\sum_{K=0}^{4} \frac{1}{K!}\left(\frac{\lambda}{\mu}\right)^K + \frac{1}{5!}\frac{1}{1-e}\left(\frac{\lambda}{\mu}\right)^5\right]^{-1}. \tag{23}$$

To minimize the average stay time of patients, it is necessary to make

$$\begin{gathered} \min T \\ \text{s.t. } P\left\{\sum_{i=1}^{5} T_i \le E(T)\right\}. \end{gathered} \tag{24}$$

4. Solve the Model

Due to the growing complexity of the structure of the stochastic programming model, the traditional precise algorithms become unworkable, leaving only computer simulation algorithms available. This study employs genetic

Table 6: Coefficients of evaluation indices under various rules.

Hospital bed arrangement index	Number of bed turnovers	Number of discharged patients	Average bed working days	Average LOS of discharged patients
FCFS	4.8	378	54	8.8
Rule 1	5.4	428	53	7.3
Rule 2	5.1	403	53	7.6
Rule 3	4.9	387	54	8.1

Table 7: Results of the evaluation index system under various rules.

Bed arrangement	d_i^+	d^-	C_i	Ranking results
FCFS	0.0429	0.0013	0.0285	4
Rule 1	0.0013	0.0429	0.9715	1
Rule 2	0.0215	0.0215	0.5008	2
Rule 3	0.0315	0.0115	0.2668	3

algorithms to solve the problem. The steps are given in the following sections.

4.1. Data Distribution. The selected samples are divided into two parts: one part is used as a training sample set to train the model and the other part is used to test whether the model met the basic skills required to implement the model. In the research, 3–8 training samples are selected for each weight in a small sample, whereas 5–10 training samples are selected for each weight in a large sample. In practical applications, to ensure that the subspace of the test sample is included in the training, there are roughly two methods for selecting the number of nodes in the middle hidden layer at present:

One type is the static construction method, that is, the structure does not change during the process of updating and learning of the weight and threshold of the network [24]. In this method, the appropriate number of hidden layer nodes for the network model at the beginning is not known, and the appropriate weights under the optimized network structure are not known either. So, the network structure can only be determined based on past experiences. The constructed network space is large and the training time is long.

The other is the dynamic construction method, that is, the number of input and output nodes is unchanged, while the number of hidden layer nodes is variable. Initially put enough hidden layer nodes and then gradually delete those inoperative nodes until the nodes cannot be contracted. This study employs the second method. Substitute the SCG and COB algorithms [25] with the best BP network iteration effect into the MATLAB program to run 10 times, take the average, and finally get the best number of hidden layer nodes of the network model as 5. According to this, the optimal network topology based on the BP neural network bed allocation model is $6 \times 5 \times 1$.

4.2. Establishment of the Mathematical Model of the BP Neural Network. First, based on the following most basic mathematical formulas along the direction of information propagation, the state equation of the network is given. Using $\mathrm{in}_j^{(i)}$ and $\mathrm{out}_j^{(i)}$, the input and output relationships of the various layers of the network [26] can be described as follows.

Layer 1 (input layer): introduce the input to the neural network.

$$\mathrm{out}_i^{(1)} = \mathrm{in}_i^{(1)} = x \quad i = 1, 2, \ldots, n. \tag{25}$$

Layer 2 (hidden layer): different forms can be taken for $g(x)$, such as

$$S\,\text{function: } g(x) = \frac{1}{1 + e^{-x}}. \tag{26}$$

Gaussian function $g(x) = e^{(x-a)^2/b^2}$, as well as radial basis function, spline basis function, and wavelet function. Gaussian function is adopted for the convenience and effectiveness of research.

Layer 3 (output layer):

$$y_k = \mathrm{out}_k^{(3)} = \mathrm{in}_k^{(3)} = \sum_{j=1}^{l} w_{\mathrm{jk}}^{(2)} * \mathrm{out}_j^{(2)}; \quad k = 1, 2, 3, \ldots, m. \tag{27}$$

Based on the conditions of the model in this article, $i = 6, l = 4, k = 1$, the basic network construction of the neural network is completed above.

4.3. Determining the Learning Algorithm of the Network. The basic idea of learning is adjust the weight of the network through a certain algorithm, so that the actual output of the network is as close as possible to the expected output. In this network, the error back propagation (BP) algorithm is adopted to adjust the weight (that is, the BP neural network). The basic principle is when the input of the network, that is, the corresponding influence factor $X = (x_1, x_2, x_3, x_4, x_5)$, the actual output of the network is $Y = (y_1, y_2, y_3, y_4, y_5)$, the expected output of the network (that is, the actual number of wards allocated) $D = (d_1, d_2, d_3, d_4, d_5)$ [26].

The objective function to define learning is (mean square error method used by [27])

$$\delta = \frac{1}{2}\sum_{i=1}^{m}(d_i - y_i)^2 = \frac{1}{2}\sum_{i=1}^{m} e_i^2. \tag{28}$$

At the same time, the BP algorithm employs the following formula to adjust the weight to minimize the objective function (steepest descent method used by [26]):

$$w(t+1) = w(t) - \eta \frac{\partial \delta}{\partial w}, \tag{29}$$

Table 8: Results of bed allocation for five types of patients.

Trauma	Cataract in one eye	Cataract in both the eyes	Glaucoma	Retina
12	13	22	9	23

where η is the learning rate. By specifically analyzing each layer of the neural network, the following equation is obtained:

$$w_{ij}^{(1)}(t+1) = w_{ij}^{(1)}(t) + \eta_1 \frac{\partial \delta}{\partial w_{ij}^{(1)}},$$
$$w_{jk}^{(2)}(t+1) = w_{ij}^{(2)}(t) + \eta_1 \frac{\partial \delta}{\partial w_{jk}^{(2)}}. \quad (30)$$

Based on the following formula, the deviation is gradually reversed:

$$\frac{\partial \delta}{\partial w_{jk}^{(2)}} = \frac{\partial \delta}{\partial y_k} \cdot \frac{\partial y_k}{\partial o\ ut_k^{(3)}} \cdot \frac{\partial o\ ut_k^{(3)}}{\partial i\ n_k^{(3)}} \cdot \frac{\partial i\ n_k^{(3)}}{\partial w_{jk}^{(2)}},$$
$$\frac{\partial \delta}{\partial w_{ij}^{(1)}} = \sum_{k=1}^{m} \left(\frac{\partial \delta}{\partial y_k} \cdot \frac{\partial y_k}{\partial \mathrm{in}_k^{(3)}} \cdot \frac{\partial \mathrm{in}_k^{(3)}}{\partial \mathrm{out}_j^{(2)}} \right) \cdot \frac{\partial \mathrm{out}_k^{(2)}}{\partial \mathrm{in}_j^{(2)}} \cdot \frac{\partial \mathrm{in}_{ij}^{(2)}}{\partial w_{ij}^{(1)}}. \quad (31)$$

Subsequently, the matric for the number of beds equipped for five types of patients is obtained, and the number of bed arrangements is given in Table 8.

Through the MATLAB software, the number of beds allocated to the five types of patients with trauma, cataract in one eye, cataract in both eyes, glaucoma, and retina is 12, 13, 22, 9, and 23, respectively; the allocated bed ratios are 15%, 16%, 28%, 11%, and 29%, and the results obtained are in good agreement with the actual situation.

From the perspective of models, the models of [15, 16] are poor in taking the fairness among the patients with different types of diseases into account, and the results are different from those in this study. It can be visually seen that the model of this study is more in line with the reality through the different rules and has a high degree of matching and consistency with the actual situation. It also has a high reference value.

5. Conclusion

The bed is an indispensable factor of hospital medical resources. With the gradual deepening of public hospital reforms and the accelerated development of the medical market, the competition among major hospitals has become increasingly fierce. At the same time, the contradiction between the needs of patients for medical treatment and the shortage of beds has become increasingly prominent, which has aggravated the tension between doctors and patients to a certain extent. According to the latest statistics, the utilization efficiency of hospital beds in our country is generally low, with an average utilization rate of less than 85% [28]. It can be seen that on the one hand, there is a shortage of beds and a serious shortage of medical resources; on the other hand, there is a serious waste of health resources. Therefore, it has been a topic worthy of exploration and research for the scientific, reasonable, and effective allocation of hospital beds to improve the efficiency of the use of hospital beds and alleviate the excessive demand for medical treatment. This study established a reasonable bed arrangement model by determining a reasonable evaluation index system, established a bed proportional allocation model for the hospital by constructing a bed proportional allocation model that minimizes the average stay time of all patients in the system, and adopted the MATLAB programming algorithm to get the calculation results. The results obtained are in good agreement with the actual situation, which has a certain significance for reference. This model can be widely applied in related fields of hospital management. For general hospitals, the above methods can be adopted to build a bed allocation system suitable for the hospital according to the specific conditions of the beds for each hospital, the waiting period of the disease and other factors to improve service capabilities, reduce patient waiting time, and save health resources.

Data Availability

The data used to support the findings of this study can be accessed in https://www.mcm.edu.cn/html_cn/node/50a7a9fc36c5ce6fd242dbbc1da5878e.html. (The Chinese college mathematical modeling contest organizing committee, 2009, Chinese national college mathematical modeling contest B topic-ophthalmic hospital of reasonable arrangement (Chinese)) and are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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References

[1] S. Fomundam and J. W. Herrmann, "A Survey of Queuing Theory Applications in healthcare[J]," UNIVERSITY OF MARYLAND, ISR Technical Report, 2007.

[2] R. Liu and X. Xie, "Physician staffing for emergency departments with time-varying demand," *INFORMS Journal on Computing*, vol. 30, no. 3, pp. 588–607, 2018.
[3] J. K. Cochran and K. Roche, "A queuing-based decision support methodology to estimate hospital inpatient bed demand," *Journal of the Operational Research Society*, vol. 59, no. 11, pp. 1471–1482, 2008.
[4] A. M. De Bruin, A. C. Van Rossum, M. C. Visser, and G. M. Koole, "Modeling the emergency cardiac in-patient flow: an application of queuing theory," *Health Care Management Science*, vol. 10, no. 2, pp. 125–137, 2007.
[5] L. V. Green, J. Soares, J. F. Giglio, and R. A. Green, "Using queueing theory to increase the effectiveness of emergency department provider staffing," *Academic Emergency Medicine*, vol. 13, no. 1, pp. 61–68, 2006.
[6] C. Lakshmi and S. A. Iyer, "Application of queueing theory in health care: a literature review," *Operations research for health care*, vol. 2, no. 1-2, pp. 25–39, 2013.
[7] X. Wang, "Emergency Department Staffing: A Separated Continuous Linear Programming Approach," *Mathematical Problems in Engineering*, vol. 2013, Article ID 680152, 8 pages, 2013.
[8] R. A. Adeleke, O. D. Ogunwale, and O. Y. Halid, "Application of queuing theory to waiting time of out-patients in hospitals," *Journal of Science and Technology*, vol. 10, no. 2, pp. 2070–2274, 2009.
[9] F. P. Mardiah and M. H. Basri, "The analysis of appointment system to reduce outpatient waiting time at Indonesia's public hospital," *Human Resource Management Research*, vol. 3, no. 1, pp. 27–33, 2013.
[10] S. A. Zenios, "Modeling the transplant waiting list: a queueing model with reneging," *Queueing Systems*, vol. 31, no. 3, pp. 239–251, 1999.
[11] J. A. Berry Jaeker and A. L. Tucker, "Past the point of speeding up: the negative effects of workload saturation on efficiency and patient severity," *Management Science*, vol. 63, no. 4, pp. 1042–1062, 2017.
[12] A. J. Thomas Schneider, P. Luuk Besselink, M. E. Zonderland et al., "Allocating emergency beds improves the emergency admission flow," *Interfaces*, vol. 48, no. 4, pp. 384–394, 2018.
[13] E. P. C. Kao and G. G. Tung, "Bed allocation in a public health care delivery system," *Management Science*, vol. 27, no. 5, pp. 507–520, 1981.
[14] J. Xie, G. G. Loke, M. Sim et al., "The Analytics of Bed Shortages: Coherent Metric, Prediction, and Optimization," *Operations Research*, 2022.
[15] L. S. Cao, Y. B. Zhang, C. Chang et al., "Evaluation model of the lay-out of sickbeds that based on the overall satisfaction [C]//2010 2nd international conference on advanced computer control," *IEEE*, vol. 4, pp. 307–311, 2010.
[16] L. M. Zhang, H. Y. Chang, and R. T. Xu, "The patient Admission scheduling of an ophthalmic hospital using genetic algorithm," *Advanced Materials Research*, vol. 756-759, pp. 1423–1432, 2013.
[17] The Chinese college mathematical modeling contest organizing committee, "Chinese National College Mathematical Modeling Contest B Topic-Ophthalmic Hospital of Reasonable Arrangement(Chinese)[EB/OL]," 2009, http://www.mcm.edu.cn/mcm09/Problems2009c. asp . 2009 - 9 - 11.
[18] D. I. Giokas, "Greek hospitals: how well their resources are used," *Omega*, vol. 29, no. 1, pp. 73–83, 2001.
[19] M. Mehrtak, H. Yusefzadeh, and E. Jaafaripooyan, "Pabon Lasso and Data Envelopment Analysis: a complementary approach to hospital performance measurement," *Global Journal of Health Science*, vol. 6, no. 4, pp. 107–16, 2014.
[20] Z. Pavić and V. Novoselac, "Notes on TOPSIS method," *International Journal of Research in Engineering and Science*, vol. 1, no. 2, pp. 5–12, 2013.
[21] S. Belciug and F. Gorunescu, "Improving hospital bed occupancy and resource utilization through queuing modeling and evolutionary computation," *Journal of Biomedical Informatics*, vol. 53, pp. 261–269, 2015.
[22] X. Li, P. Beullens, D. Jones, and M. Tamiz, "An integrated queuing and multi-objective bed allocation model with application to a hospital in China," *Journal of the Operational Research Society*, vol. 60, no. 3, pp. 330–338, 2009.
[23] G. W. Harrison, A. Shafer, and M. Mackay, "Modelling variability in hospital bed occupancy," *Health Care Management Science*, vol. 8, no. 4, pp. 325–334, 2005.
[24] S. Xie, Z. Hu, D. Zhou et al., "Multi-objective active distribution networks expansion planning by scenario-based stochastic programming considering uncertain and random weight of network," *Applied Energy*, vol. 219, pp. 207–225, 2018.
[25] S. Ding and Q. H. Wu, "A MATLAB-based study on approximation performances of improved algorithms of typical BP neural networks," *Applied Mechanics and Materials*, vol. 313-314, pp. 1353–1356, 2013.
[26] J. Li, J.-h. Cheng, J.-y. Shi, and F. Huang, "Brief introduction of back propagation (BP) neural network algorithm and its improvement," *Advances in Computer Science and Information Engineering*, Springer, Berlin, Heidelberg, pp. 553–558, 2012.
[27] N. Kim and H. Park, "Modified UMP-BP decoding algorithm based on mean square error," *Electronics Letters*, vol. 40, no. 13, pp. 816-817, 2004.
[28] J. Sun and H. Luo, "Evaluation on equality and efficiency of health resources allocation and health services utilization in China," *International Journal for Equity in Health*, vol. 16, no. 1, pp. 127-128, 2017.