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Research Article

Dynamics Analysis for a Stochastic SIRI Model Incorporating Media Coverage

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We investigate a stochastic SIRI model incorporating media coverage. Firstly, existence and uniqueness of global positive solution of the model are established. By using the Hasminskii theory, we study stationary distribution of the model. Then, we prove extinction of the disease. Moreover, the asymptotic properties of solutions are studied by using Lyapunov method. Finally, numerical simulations are presented. In addition, it shows that media coverage can strain the spread of infectious diseases.

1. Introduction

Although human society is constantly developing and progressing, various infectious diseases still plague humans (such as COVID-19 and mixed influenza A and B). Governments around the world have taken a series of measures to prevent the spread of infectious disease and protect human life and health. Furthermore, with the acceleration of globalization, the convenient transportation and the frequent movement of population have accelerated the spread of infectious diseases, so as to study the infectious law of disease, predict its development trend, and seek for prevention and treatment strategies, scholars have established various epidemic models and studied their dynamic behaviors (see [1–8]).

Media coverage is an effective measure to prevent the spread of disease (see [9, 10]). Through media reports (such as TV, Internet, radio, advertising, Weibo, WeChat, etc.), the public can timely understand the latest information of diseases in order to take preventive measures and avoid bad behaviors caused by panic. Many mathematical models (see [11–16]) have been established to research the impact of media reports on communication dynamics with the different incidence rates, for example, $\beta e^{-mI}SI\,(m>0)$ in [13], $(\beta_1-\beta_2 I/(m+I))SI/(S+I)$ in [11], $(\beta_1-\beta_2 f(I))SI/(S+I)$ in [15], and $(\beta_1-\beta_2 I(t)/(m_1+I(t)))S(t)I(t)/(1+m_2 I(t))$ in [16].

As can be seen from the above discussion, in order to study the influence of media coverage on the spread of infectious disease, previous models only considered the influence of the number of infected individuals on media coverage. When infected individuals appear in an area, the public are conscious of the potential danger of infected individuals, they will voluntarily reduce their contact with the outside world for fear of infection. The more the infected individuals are, the less contact they have with the outside world. That is, media coverage can minimize the contact between susceptible and infected individuals. In addition, the number of recovered individuals also partly reflects the severity of the epidemic. Hence, the number of infected and recovered individuals is larger, and the impact of media coverage is larger on the exposure rate. Thus, the reduction of contact rate is related to the number of infected and recovered individuals. Additionally, due to the influence of psychological and social factors, susceptible individuals reduce their contact with infected individuals and strengthen protective measures, so that the effective contact rate between infected and susceptible individuals tends to saturation state. Based on this, a deterministic SIRI model with saturated incidence rate is established

$$\begin{cases} \dot{S}(t) = \Lambda - \left(\beta_1 - \dfrac{\beta_2 (I(t) + \alpha R(t))}{m_1 + I(t) + \alpha R(t)}\right) \dfrac{S(t)I(t)}{1 + m_2 I(t)} - \mu S(t), \\ \dot{I}(t) = \left(\beta_1 - \dfrac{\beta_2 (I(t) + \alpha R(t))}{m_1 + I(t) + \alpha R(t)}\right) \dfrac{S(t)I(t)}{1 + m_2 I(t)} - (\gamma + \mu) I(t) + \delta R(t), \\ \dot{R}(t) = \gamma I(t) - (\mu + \delta) R(t). \end{cases} \tag{1}$$

Here, $S(t), I(t), R(t)$ are the susceptible, infected, and recovered individuals, respectively; Λ is the birth rate or migration rate of susceptible individuals; γ is the recovery rate of infected individuals; μ is the natural mortality; δ denotes the recurrence rate of the recovered; β_1 is the maximum effective contact rate; and $\beta_1 - \beta_2 (I + \alpha R)/(m_1 + I + \alpha R)$ denotes the contact rate under media coverage. $m_1 > 0$, $m_2 > 0$, $0 < \alpha \leq 1$. Since media coverage will not completely prevent the spread of diseases, then $\beta_1 > \beta_2$. For model (1), the basic reproduction number is easily obtained as

$$R_0 = \frac{\beta_1 \Lambda}{\mu(\mu + \gamma - \gamma\delta/(\mu + \delta))}. \tag{2}$$

In reality, infectious diseases are always influenced by environmental noise, which makes the related parameters (such as contact rate, mortality rate, and recovery rate) show random fluctuations. Thus, it is more realistic to research the dynamic behaviors of stochastic epidemic model (see [17–23]). At present, a lot of academics have studied stochastic epidemic models with media coverage and obtained some results (see [14, 16, 24]). In [24], a stochastic SIS model with media coverage on two patches was established, and almost sure exponential stability of disease-free equilibrium was obtained. In [14], asymptotic behaviors around the equilibria of a stochastic SIRI model were considered. In [16], ergodic stationary distribution of a stochastic SIRS model was investigated.

Considering the influence of environmental random factors, we establish the stochastic SIRI epidemic model

$$\begin{cases} \mathrm{d}S = \left[\Lambda - \left(\beta_1 - \dfrac{\beta_2 (I + \alpha R)}{m_1 + I + \alpha R}\right) \dfrac{SI}{1 + m_2 I} - \mu S\right] \mathrm{d}t + \sigma_1 S \mathrm{d}B_1(t), \\ \mathrm{d}I = \left[\left(\beta_1 - \dfrac{\beta_2 (I + \alpha R)}{m_1 + I + \alpha R}\right) \dfrac{SI}{1 + m_2 I} - (\gamma + \mu) I + \delta R\right] \mathrm{d}t + \sigma_2 I \mathrm{d}B_2(t), \\ \mathrm{d}R = [\gamma I - (\delta + \mu) R] \mathrm{d}t + \sigma_3 R \mathrm{d}B_3(t), \end{cases} \tag{3}$$

where $B_i(t)$ is the Brownian motion on $(\Omega, \mathcal{F}, \mathbb{P})$, which is a complete probability space; σ_i^2 is the intensity of $B_i(t)$, $i = 1, 2, 3$.

In the following, we shall study existence of the stationary distribution and sufficient conditions for extinction of the disease. The dynamic properties of solutions near equilibria are to be considered. The results reveal the influence of noise intensity on disease transmission. In addition, the numerical simulation shows that increasing the media coverage can reduce the number of infected individuals, so media coverage can reduce the spread of infectious diseases. Besides, the less impact the number of recovered individuals has on media coverage, the lower the transmission rate is, which in turn leads to fewer infected individuals. Thus, it provides a scientific theoretical basis for the prevention and control of infectious diseases.

2. Existence and Uniqueness of Global Positive Solution

Theorem 1. *For every* $(S(0), I(0), R(0)) \in \mathbb{R}_+^3$, *there is a unique global positive solution to (3) on* $[0, \infty)$, *namely,* $(S(t), I(t), R(t)) \in \mathbb{R}_+^3$, *for any* $t \geq 0$ *almost surely.*

Proof. Obviously, the coefficients of (3) are locally Lipschitz continuous. Hence, for every $(S(0), I(0), R(0)) \in \mathbb{R}_+^3$, there exists a unique maximal local solution $(S(t), I(t), R(t))$ to model (3), where $t \in [0, t\tau_e)$ and τ_e denotes the explosion time. In the following, it is proved that $\tau_e = \infty$ a.s. Let $n_0 > 0$ such that $S(0), I(0)$ and $R(0)$ all belong to $(1/n_0, n_0)$. Define

$$\tau_n = \inf\left\{t \in [0, \tau_e) : \max\{S(t), I(t), R(t)\} \geq n \,\text{or}\, \min\{S(t), I(t), R(t)\} \leq \frac{1}{n}\right\}, \tag{4}$$

for any integer $n \geq n_0$. Obviously, $\{\tau_n\}_{n\geq n_0}$ is increasing. Set $\tau_\infty = \lim_{n\longrightarrow\infty}\tau_n$, then $\tau_\infty \leq \tau_e$ a.s. Let us show $\tau_\infty = \infty$ a.s. Or else there are $\varepsilon \in (0,1)$ and $T>0$ satisfying $\mathbb{P}\{\tau_\infty \leq T\} > \varepsilon$. Then, for any $n \geq n_1$, there exists $n_1 \geq n_0$ satisfying $\mathbb{P}\{\tau_n \leq T\} > \varepsilon$. Define a C^2-function $V\colon (0,\infty)^3 \longrightarrow [0, t\infty)$ by

$$V_1(S,I,R) = (S-1-\ln S) + (R-1-\ln R) + (I-1-\ln I). \tag{5}$$

For $n \geq n_1, T>0$ and $0 \leq t \leq \tau_n \wedge T$, It $\hat{o}'s$ formula shows that

$$\begin{aligned} dV_1(S,I,R) &= LV_1(S,I,R)dt + \sigma_1(S-1)dB_1(t) \\ &\quad + \sigma_2(I-1)dB_2(t) + \sigma_3(R-1)dB_3(t), \end{aligned} \tag{6}$$

where

$$\begin{aligned} LV_1(S,I,R) &= \left(1-\frac{1}{S}\right)\left[\Lambda - \left(\beta_1 - \frac{\beta_2(I+\alpha R)}{m_1+I+\alpha R}\right)\frac{SI}{1+m_2 I} - \mu S\right] \\ &\quad + \left(1-\frac{1}{I}\right)\left[\left(\beta_1 - \frac{\beta_2(I+\alpha R)}{m_1+I+\alpha R}\right)\frac{SI}{1+m_2 I} + \delta R - (\gamma+\mu)I\right] \\ &\quad + \left(1-\frac{1}{R}\right)[-(\delta+\mu)R + \gamma I] + \frac{1}{2}\left(\sigma_1^2+\sigma_2^2+\sigma_3^2\right) \\ &= \Lambda - \mu S - \frac{\Lambda}{S} + \left(\beta_1 - \frac{\beta_2(I+\alpha R)}{m_1+I+\alpha R}\right)\frac{I}{1+m_2 I} - \mu I + \gamma - \frac{\delta R}{I} - \mu R - \frac{\gamma I}{R} \\ &\quad - \left(\beta_1 - \frac{\beta_2(I+\alpha R)}{m_1+I+\alpha R}\right)\frac{S}{1+m_2 I} + 3\mu + \delta + \frac{1}{2}\left(\sigma_1^2+\sigma_2^2+\sigma_3^2\right) \\ &\leq \Lambda + \frac{\beta_1}{m_2} + 3\mu + \delta + \gamma + \frac{1}{2}\left(\sigma_1^2+\sigma_2^2+\sigma_3^2\right). \end{aligned} \tag{7}$$

The remaining is similar to that of Theorem 2.1 in [23], so it is omitted. □

3. Stationary Distribution and Ergodicity

Let $X(t)$ be a Markov process in $\mathbb{R}^d$ and satisfy

$$dX(t) = b(X)dt + \sum_{k=1}^{n}\sigma_k(X)dB_k(t), \tag{8}$$

where $\sigma_k(X) = (\sigma_k^1(X), \ldots, \sigma_k^n(X))^T$. Then, the diffusion matrix is

$$\overline{A}(X) = \left(\overline{a}_{ij}(X)\right) \text{and}\, \overline{a}_{ij}(X) = \sum_{k=1}^{n}\sigma_k^i(X)\sigma_k^j(X). \tag{9}$$

Lemma 1 (see [25]). *For a bounded domain $U \subset \mathbb{R}^d$ with regular boundary ∂U, if*

(i) There exists $\omega > 0$ such that $\sum_{i,j=1}^{d}\overline{a}_{ij}(X)\xi_i\xi_j \geq \omega|\xi|^2$, $X \in U$, $\xi \in \mathbb{R}^d$;

(ii) There exist $\overline{C} > 0$ and a C^2-function $V \geq 0$ such that $LV(x) < -\overline{C}$ for every $x \in \mathbb{R}^d \backslash U$, then $X(t)$ has a unique ergodic stationary distribution $\mu(\cdot)$. Further, if $\overline{f}(\cdot)$ is an integrable function with respect to the measure μ, then for any $x \in \mathbb{R}^d$,

$$\mathbb{P}\left\{\lim_{Q\longrightarrow\infty}\frac{1}{Q}\int_0^Q \overline{f}(X(t))dt = \int_{\mathbb{R}^d}\overline{f}(x)\mu(dx)\right\} = 1. \tag{10}$$

Denote

$$\mathbb{R}_s = \frac{\beta_1 - \beta_2}{\left(\mu + (2\beta_1 t - n\beta_2)/m_2 + \sigma_1^2/2\right)\left(\mu + \gamma + \sigma_2^2/2\right)}. \tag{11}$$

Theorem 2. *If $\mathbb{R}_s > 1$, then model (3) exists a unique ergodic stationary distribution.*

Proof. For every $(S(0), I(0), R(0)) \in (0,\infty)^3$, it follows from Theorem 1 that there exists a unique global positive solution $(S(t), I(t), R(t))$. Let $\varepsilon > 0$ be sufficiently small. Denote

$$\mathcal{U}_\varepsilon = \left\{(S,I,R) \in \mathbb{R}_+^3\colon S, I \in \left[\varepsilon, \frac{1}{\varepsilon}\right], R \in \left[\varepsilon^2, \frac{1}{\varepsilon^2}\right]\right\}, \tag{12}$$

and $A(X) = \operatorname{diag}(\sigma_1^2 S^2, \sigma_2^2 I^2, \sigma_3^2 R^2)$. Let

$$M = \min_{(S,I,R)\in\mathcal{U}_\varepsilon}\left\{\sigma_1^2 S^2, \sigma_2^2 I^2, \sigma_3^2 R^2\right\}. \tag{13}$$

For every $(S,I,R) \in \overline{\mathcal{U}}_\varepsilon$, $\xi \in (0,\infty)^3$,

$$\sum_{i,j=1}^{3} \overline{a}_{ij}\xi_i\xi_j = \sigma_1^2 S^2\xi_1^2 + \sigma_2^2 I^2\xi_2^2 + \sigma_3^2 R^2\xi_3^2 \geq M|\xi|^2. \quad (14)$$

Thus, (i) in Lemma 1 holds. Define $\widetilde{V}(S,I,R)\colon (0,\infty)^3 \longrightarrow (-\infty,\infty)$ by

$$\begin{aligned}\widetilde{V}(S,I,R) &= N\left(S + \frac{m_2}{\mu+\gamma}I + 2R - c_1\ln S - c_2\ln I\right)\\ &\quad - \ln R - \ln S + \frac{1}{m+1}(R+S+I)^{m+1} \quad (15)\\ &=: NW_1 + W_2 + W_3 + W_4,\end{aligned}$$

where $c_1 = \Lambda/(\mu + (2\beta_1-\beta_2)/m_2 + \sigma_1^2/2)$, $c_2 = \Lambda/(\mu+\gamma+\sigma_2^2/2)$. Positive constants m and N satisfy

$$\mu - \frac{m}{2}\left(\sigma_1^2\vee\sigma_2^2\vee\sigma_3^2\right) > 0, \quad -3N\Lambda\left(\mathbb{R}_s^{1/3}-1\right) + 2N\gamma\varepsilon + G < -1. \quad (16)$$

Here,

$$\begin{aligned}B &= \sup_{(S,I,R)\in(0,\infty)^3}\left\{\Lambda(R+S+I)^m - \frac{1}{4}\left[2\mu - m\left(\sigma_1^2\vee\sigma_2^2\vee\sigma_3^2\right)\right](S+I+R)^{m+1}\right\},\\ G &= \sup_{(S,I,R)\in(0,\infty)^3}\left\{\begin{array}{c}-\frac{1}{2}\left[\mu - \frac{m}{2}\left(\sigma_1^2\vee\sigma_2^2\vee\sigma_3^2\right)\right]\left(S^{m+1}+I^{m+1}+R^{m+1}\right) + \frac{N(2\beta_1-\beta_2)}{\mu+\gamma}S\\ -N\left(2\mu+2\delta-\frac{m_2\delta}{\mu+\gamma}\right)R + N + 2\mu + \delta + \frac{2\beta_1-\beta_2}{m_2} + \frac{1}{2}\left(\sigma_1^2+\sigma_3^2\right) + B\end{array}\right\}.\end{aligned} \quad (17)$$

Obviously,

$$\liminf_{l\to\infty,(S,I,R)\in(0,\infty)^3} \backslash\mathcal{U}_l\widetilde{V}(S,I,R) = +\infty, \quad (18)$$

where $\mathcal{U}_l = (1/l, l)^3$. In addition, $\widetilde{V}(S,I,R)$ has a minimum point $(\overline{S}_0,\overline{I}_0,\overline{R}_0)$ in $(0,\infty)^3$. Define $V_2\colon (0,\infty)^3 \longrightarrow (-\infty,\infty)$ by

$$V_2(S,I,R) = -\widetilde{V}(\overline{S}_0,\overline{I}_0,\overline{R}_0) + \widetilde{V}(S,I,R). \quad (19)$$

The It $\hat{o}$ formula shows that

$$\begin{aligned}LW_1 &= \Lambda - \left(\beta_1 - \frac{\beta_2(I+\alpha R)}{m_1+I+\alpha R}\right)\frac{SI}{1+m_2I} - \mu S - \frac{c_1\Lambda}{S} + \left(\beta_1 - \frac{\beta_2(I+\alpha R)}{m_1+I+\alpha R}\right)\frac{c_1 I}{1+m_2I}\\ &\quad + \frac{m_2}{\mu+\gamma}\left(\beta_1 - \frac{\beta_2(I+\alpha R)}{m_1+I+\alpha R}\right)\frac{SI}{1+m_2I} - m_2 I - \left(\beta_1 - \frac{\beta_2(I+\alpha R)}{m_1+I+\alpha R}\right)\frac{c_2 S}{1+m_2I}\\ &\quad + c_1\mu + \frac{m_2\delta}{\mu+\gamma}R + c_2(\mu+\gamma) - \frac{c_2\delta R}{I} + 2\gamma I - 2(\mu+\delta)R + \frac{1}{2}c_1\sigma_1^2 + \frac{1}{2}c_2\sigma_2^2\\ &\leq \left[-(m_2I+1) - \frac{c_1\Lambda}{S} - \frac{c_2(\beta_1-\beta_2)S}{1+m_2I}\right] + 1 + \Lambda - \mu S - 2(\mu+\delta)R\\ &\quad - \left(\beta_1 - \frac{\beta_2(I+\alpha R)}{m_1+I+\alpha R}\right)\frac{SI}{1+m_2I} + \left(\beta_1 - \frac{\beta_2(I+\alpha R)}{m_1+I+\alpha R}\right)\frac{c_1 I}{1+m_2I} + c_1\mu\\ &\quad + \frac{m_2\delta}{\mu+\gamma}R + c_2(\mu+\gamma) - \frac{c_2\delta R}{I} + 2\gamma I + \frac{1}{2}c_1\sigma_1^2 + \frac{1}{2}c_2\sigma_2^2 + \frac{m_2(2\beta_1-\beta_2)SI}{(\mu+\gamma)(1+m_2I)}\end{aligned}$$

$$
\begin{aligned}
&\leq -3\left[\Lambda(\beta_1-\beta_2)c_1c_2\right]^{\frac{1}{3}}+3\Lambda+1+2\gamma I-\mu S-\left(2\mu+2\delta-\frac{m_2\delta}{\mu+\gamma}\right)R \\
&\quad+\frac{m_2(2\beta_1-\beta_2)SI}{(\mu+\gamma)(1+m_2I)}, \\
\mathrm{L}W_2 &= -\frac{\Lambda}{S}+\left(\beta_1-\frac{\beta_2(I+\alpha R)}{m_1+I+\alpha R}\right)\frac{I}{1+m_2I}+\mu+\frac{\sigma_1^2}{2}, \\
\mathrm{L}W_3 &= -\frac{\gamma I}{R}+\mu+\delta+\frac{\sigma_3^2}{2}, \\
\mathrm{L}W_4 &= (S+I+R)^m(\Lambda-\mu S-\mu I-\mu R)+\frac{1}{2}m(S+I+R)^{m-1}\left(\sigma_1^2S^2+\sigma_2^2I^2+\sigma_3^2R^2\right) \\
&\leq \Lambda(R+S+I)^m-\left[\mu-\frac{m}{2}\left(\sigma_3^2\vee\sigma_2^2\vee\sigma_1^2\right)\right](R+S+I)^{m+1} \\
&\leq B-\frac{1}{4}\left[2\mu-m\left(\sigma_1^2\vee\sigma_2^2\vee\sigma_3^2\right)\right]\left(S^{m+1}+I^{m+1}+R^{m+1}\right).
\end{aligned} \tag{20}
$$

Hence,

$$
\begin{aligned}
LV_2(S,I,R) \leq{}& -3N\Lambda\left(\mathbb{R}_s^{1/3}-1\right)+N+2N\gamma I-N\mu S-N\left(2\mu+2\delta-\frac{m_2\delta}{\mu+\gamma}\right)R+\delta+2\mu \\
&+\frac{m_2N(2\beta_1-\beta_2)}{\mu+\gamma}\frac{SI}{1+m_2I}-\frac{\Lambda}{S}+\left(\beta_1-\frac{\beta_2(I+\alpha R)}{m_1+I+\alpha R}\right)\frac{I}{1+m_2I}-\frac{\gamma I}{R} \\
&+\frac{1}{2}\left(\sigma_1^2+\sigma_3^2\right)+B-\frac{1}{2}\left[\mu-\frac{m}{2}\left(\sigma_1^2\vee\sigma_2^2\vee\sigma_3^2\right)\right]\left(S^{m+1}+I^{m+1}+R^{m+1}\right).
\end{aligned} \tag{21}
$$

Denote

$$
F=\sup_{(S,I,R)\in\mathbb{R}_+^3}\left\{\begin{aligned}&-\frac{1}{2}\left[\mu-\frac{m}{2}\left(\sigma_1^2\vee\sigma_2^2\vee\sigma_3^2\right)\right]\left(S^{m+1}+I^{m+1}+R^{m+1}\right)+\frac{N(2\beta_1-\beta_2)}{\mu+\gamma}S+2N\gamma I\\&-N\left(2\mu+2\delta-\frac{m_2\delta}{\mu+\gamma}\right)R+N+2\mu+\delta+\frac{2\beta_1-\beta_2}{m_2}+\frac{1}{2}\left(\sigma_1^2+\sigma_3^2\right)+B\end{aligned}\right\},
$$

$$
H=\sup_{(S,I,R)\in\mathbb{R}_+^3}\left\{\begin{aligned}&-\frac{1}{4}\left[\mu-\frac{m}{2}\left(\sigma_1^2\vee\sigma_2^2\vee\sigma_3^2\right)\right]S^{m+1}-\frac{1}{4}\left[2\mu-m\left(\sigma_1^2\vee\sigma_2^2\vee\sigma_3^2\right)\right]\left(I^{m+1}+R^{m+1}\right)+\frac{2\beta_1-\beta_2}{m_2}\\&+\frac{N(2\beta_1-\beta_2)}{\mu+\gamma}S-N\left(2\mu+2\delta-\frac{m_2\delta}{\mu+\gamma}\right)R+N+2\mu+\delta+2N\gamma I+\frac{1}{2}\left(\sigma_1^2+\sigma_3^2\right)+B\end{aligned}\right\},
$$

$$
J=\sup_{(S,I,R)\in\mathbb{R}_+^3}\left\{\begin{aligned}&-\frac{1}{4}\left[\mu-\frac{m}{2}\left(\sigma_1^2\vee\sigma_2^2\vee\sigma_3^2\right)\right]I^{m+1}-\frac{1}{4}\left[2\mu-m\left(\sigma_1^2\vee\sigma_2^2\vee\sigma_3^2\right)\right]\left(S^{m+1}+R^{m+1}\right)+\frac{2\beta_1-\beta_2}{m_2}\\&+\frac{N(2\beta_1-\beta_2)}{\mu+\gamma}S-N\left(2\mu+2\delta-\frac{m_2\delta}{\mu+\gamma}\right)R+N+2\mu+\delta+2N\gamma I+\frac{1}{2}\left(\sigma_1^2+\sigma_3^2\right)+B\end{aligned}\right\},
$$

$$L=\sup_{(S,I,R)\in\mathbb{R}_+^3}\left\{\begin{array}{l}-\frac{1}{4}\left[\mu-\frac{m}{2}\left(\sigma_1^2\vee\sigma_2^2\vee\sigma_3^2\right)\right]R^{m+1}-\frac{1}{4}\left[2\mu-m\left(\sigma_1^2\vee\sigma_2^2\vee\sigma_3^2\right)\right]\left(S^{m+1}+I^{m+1}\right)+\frac{2\beta_1-\beta_2}{m_2}\\ +\frac{N(2\beta_1-\beta_2)}{\mu+\gamma}S-N\left(2\mu+2\delta-\frac{m_2\delta}{\mu+\gamma}\right)R+N+2\mu+\delta+2N\gamma I+\frac{1}{2}\left(\sigma_1^2+\sigma_3^2\right)+B\end{array}\right\}. \tag{22}$$

Let $\varepsilon>0$ be sufficiently small for

$$\begin{aligned}&-\frac{\Lambda}{\varepsilon}+F<-1,\quad -\frac{\gamma}{\varepsilon}+F<-1,\\&-\frac{1}{4}\left[\mu-\frac{m}{2}\left(\sigma_1^2\vee\sigma_2^2\vee\sigma_3^2\right)\right]\frac{1}{\varepsilon^{m+1}}+H<-1,\\&-\frac{1}{4}\left[\mu-\frac{m}{2}\left(\sigma_1^2\vee\sigma_2^2\vee\sigma_3^2\right)\right]\frac{1}{\varepsilon^{m+1}}+J<-1,\\&-\frac{1}{4}\left[\mu-\frac{m}{2}\left(\sigma_1^2\vee\sigma_2^2\vee\sigma_3^2\right)\right]\frac{1}{\varepsilon^{2m+2}}+L<-1.\end{aligned} \tag{23}$$

Next, $\mathbb{R}_+^3\backslash\mathscr{U}_\varepsilon$ is divided into six areas:

$$\begin{aligned}&\mathscr{U}_1=\left\{(S,I,R)\in(0,\infty)^3:\ S\in(0,\varepsilon)\right\},\\&\mathscr{U}_2=\left\{(S,I,R)\in(0,\infty)^3:\ S\in\left[\varepsilon,\frac{1}{\varepsilon}\right],I\in(0,\varepsilon)\right\},\\&\mathscr{U}_3=\left\{(S,I,R)\in(0,\infty)^3:\ S,I\in\left[\varepsilon,\frac{1}{\varepsilon}\right],R\in\left(0,\varepsilon^2\right)\right\},\\&\mathscr{U}_4=\left\{(S,I,R)\in(0,\infty)^3:\ S\in\left(\frac{1}{\varepsilon},\infty\right)\right\},\\&\mathscr{U}_5=\left\{(S,I,R)\in(0,\infty)^3:\ S\in\left[\varepsilon,\frac{1}{\varepsilon}\right],I\in\left(\frac{1}{\varepsilon},\infty\right)\right\},\\&\mathscr{U}_6=\left\{(S,I,R)\in(0,\infty)^3:\ S,I\in\left[\varepsilon,\frac{1}{\varepsilon}\right],R\in\left(\frac{1}{\varepsilon^2},\infty\right)\right\}.\end{aligned} \tag{24}$$

We shall prove $LV_2(S,I,R)<-1$ in $(0,\infty)^3\backslash\mathscr{U}_\varepsilon$. □

Case 1. Let $(S,I,R)\in\mathscr{U}_1$. Then,

$$\begin{aligned}LV_2\leq&-\frac{\Lambda}{S}-\frac{1}{4}\left[2\mu-m\left(\sigma_1^2\vee\sigma_2^2\vee\sigma_3^2\right)\right]\left(S^{m+1}+I^{m+1}+R^{m+1}\right)\\&+\frac{N(2\beta_1-\beta_2)}{\gamma+\mu}S+2N\gamma I\\&-N\left(2\mu+2\delta-\frac{m_2\delta}{\mu+\gamma}\right)R+N+2\mu+\delta\\&+\frac{2\beta_1-\beta_2}{m_2}+\frac{1}{2}\left(\sigma_1^2+\sigma_3^2\right)+B\\\leq&F-\frac{\Lambda}{S}\\<&F-\frac{\Lambda}{\varepsilon}\\<&-1.\end{aligned} \tag{25}$$

Case 2. Let $(S,I,R)\in\mathscr{U}_2$. Then,

$$\begin{aligned}LV_2\leq&-3N\Lambda\left(\mathbb{R}_s^{1/3}-1\right)+2N\gamma I-\frac{1}{2}\left[\mu-\frac{m}{2}\left(\sigma_1^2\vee\sigma_2^2\vee\sigma_3^2\right)\right]\left(S^{m+1}+I^{m+1}+R^{m+1}\right)+N+\delta\\&-N\left(2\mu+2\delta-\frac{m_2\delta}{\mu+\gamma}\right)R+2\mu+\frac{N(2\beta_1-\beta_2)}{\mu+\gamma}S+\frac{2\beta_1-\beta_2}{m_2}+\frac{1}{2}\left(\sigma_1^2+\sigma_3^2\right)+B\\\leq&-3N\Lambda\left(\mathbb{R}_s^{1/3}-1\right)+2N\gamma I+G\\\leq&-3N\Lambda\left(\mathbb{R}_s^{1/3}-1\right)+2N\gamma\varepsilon+G\\<&1.\end{aligned} \tag{26}$$

Case 3. Let $(S, I, R) \in \mathcal{U}_3$. Then,

$$LV_2 \leq -\frac{\gamma I}{R} + F \leq -\frac{\gamma}{\varepsilon} + F < -1. \tag{27}$$

Case 4. Let $(S, I, R) \in \mathcal{U}_4$. Then,

$$\begin{aligned} LV_2 \leq & -\frac{1}{8}\left[2\mu - m\left(\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2\right)\right]S^{m+1} - \frac{1}{8}\left[2\mu - m\left(\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2\right)\right]S^{m+1} + \frac{N(2\beta_1 - \beta_2)}{\mu + \gamma}S \\ & -\frac{1}{4}\left[2\mu - m\left(\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2\right)\right]\left(I^{m+1} + R^{m+1}\right) - N\left(2\mu + 2\delta - \frac{m_2\delta}{\mu + \gamma}\right)R \\ & + 2N\gamma I + N + 2\mu + \delta + \frac{2\beta_1 - \beta_2}{m_2} + \frac{1}{2}\left(\sigma_1^2 + \sigma_3^2\right) + B \\ \leq & -\frac{1}{8}\left[2\mu - m\left(\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2\right)\right]S^{m+1} + H \\ \leq & -\frac{1}{8}\left[2\mu - m\left(\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2\right)\right]\frac{1}{\varepsilon^{m+1}} + H \\ < & -1. \end{aligned} \tag{28}$$

Case 5. Let $(S, I, R) \in \mathcal{U}_5$. Then,

$$\begin{aligned} LV_2 \leq & -\frac{1}{8}\left[2\mu - m\left(\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2\right)\right]I^{m+1} - \frac{1}{8}\left[2\mu - m\left(\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2\right)\right]I^{m+1} + \frac{N(2\beta_1 - \beta_2)}{\mu + \gamma}S \\ & -\frac{1}{4}\left[2\mu - m\left(\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2\right)\right]\left(S^{m+1} + R^{m+1}\right) - N\left(2\mu + 2\delta - \frac{m_2\delta}{\mu + \gamma}\right)R \\ & + 2N\gamma I + N + 2\mu + \delta + \frac{2\beta_1 - \beta_2}{m_2} + \frac{1}{2}\left(\sigma_1^2 + \sigma_3^2\right) + B \\ \leq & -\frac{1}{8}\left[2\mu - m\left(\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2\right)\right]I^{m+1} + J \\ \leq & -\frac{1}{8}\left[2\mu - m\left(\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2\right)\right]\frac{1}{\varepsilon^{m+1}} + J \\ < & -1. \end{aligned} \tag{29}$$

Case 6. Let $(S, I, R) \in \mathcal{U}_6$. Then,

$$\begin{aligned} LV_2 \leq & -\frac{1}{8}\left[2\mu - m\left(\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2\right)\right]R^{m+1} - \frac{1}{8}\left[2\mu - m\left(\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2\right)\right]R^{m+1} + \frac{N(2\beta_1 - \beta_2)}{\mu + \gamma}S \\ & -\frac{1}{4}\left[2\mu - m\left(\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2\right)\right]\left(S^{m+1} + I^{m+1}\right) - N\left(2\mu + 2\delta - \frac{m_2\delta}{\mu + \gamma}\right)R \\ \leq & + 2N\gamma I + N + 2\mu + \delta + \frac{2\beta_1 - \beta_2}{m_2} + \frac{1}{2}\left(\sigma_1^2 + \sigma_3^2\right) + B \end{aligned}$$

$$
\begin{aligned}
&-\frac{1}{8}\left[2\mu - m\left(\sigma_1^2\vee\sigma_2^2\vee\sigma_3^2\right)\right]R^{m+1} + L\\
&\leq -\frac{1}{8}\left[2\mu - m\left(\sigma_1^2\vee\sigma_2^2\vee\sigma_3^2\right)\right]\frac{1}{\varepsilon^{2m+2}} + L\\
&< -1.
\end{aligned}
\tag{30}
$$

From (25)–(30), for every $(S, I, R) \in (0,\infty)^3\backslash\mathcal{U}_\varepsilon$, $LV(S, I, R) < -1$. From Lemma 1, Theorem 2 holds.

4. Extinction

For convenience, denote

$$
\widehat{\mathbb{R}}_s = \frac{2\Lambda(2\beta_1 - \beta_2)(\gamma+\mu)^2}{m_2\mu\left[\sigma_2^2\gamma^2/2\wedge(\gamma+\mu)^2\left(\mu+\sigma_3^2/2\right)\right]}. \tag{31}
$$

Theorem 3. *For any $(S_0, I_0, R_0) \in (0,\infty)^3$, let $(S(t), I(t), R(t))$ be the solution of (2). If $\widehat{\mathbb{R}}_s < 1$, then*

$$
\lim_{t\longrightarrow\infty} I(t) = 0,\quad \lim_{t\longrightarrow\infty} R(t) = 0, \text{ a.s.} \tag{32}
$$

Furthermore,

$$
\limsup_{t\longrightarrow\infty}\frac{1}{t}\int_0^t S(u)du = \frac{\Lambda}{\mu} \text{ a.s.} \tag{33}
$$

Proof. Define $V_3 = \ln(\gamma I + (\mu+\gamma)R)$. Then,

$$
\begin{aligned}
dV_3 &= \frac{1}{\gamma I + (\gamma+\mu)R}\left\{\left(\beta_1 - \frac{\beta_2(I+\alpha R)}{m_1 + I + \alpha R}\right)\frac{\gamma SI}{1+m_2 I} - \mu(\delta+\gamma+\mu)R - \frac{\sigma_2^2\gamma^2 I^2 + \sigma_3^2(\gamma+\mu)^2R^2}{2(\gamma I + (\gamma+\mu)R)}\right\}dt\\
&\quad + \frac{1}{\gamma I + (\gamma+\mu)R}\left(\gamma\sigma_2 I dB_2 + (\gamma+\mu)\sigma_3 R dB_3\right)\\
&\leq \frac{(2\beta_1-\beta_2)S}{m_2}dt - \left(\frac{\mu(\gamma+\mu+\delta)R}{\gamma I + (\gamma+\mu)R} + \frac{\sigma_2^2\gamma^2 I^2 + \sigma_3^2(\gamma+\mu)^2R^2}{2(\gamma I + (\gamma+\mu)R)^2}\right)dt\\
&\quad + \frac{1}{\gamma I + (\gamma+\mu)R}\left(\gamma\sigma_2 I dB_2 + (\gamma+\mu)\sigma_3 R dB_3\right)\\
&\leq \frac{(2\beta_1-\beta_2)S}{m_2}dt - \left[\frac{\sigma_2^2\gamma^2}{2}\wedge(\gamma+\mu)^2\left(\mu+\frac{\sigma_3^2}{2}\right)\right]\frac{I^2+R^2}{[\gamma I + (\gamma+\mu)R]^2}dt\\
&\quad + \frac{1}{\gamma I + (\gamma+\mu)R}\left(\gamma\sigma_2 I dB_2 + (\gamma+\mu)\sigma_3 R dB_3\right)\\
&\leq \frac{(2\beta_1-\beta_2)S}{m_2}dt - \frac{\sigma_2^2\gamma^2/2\wedge\left[(\gamma+\mu)^2\left(\mu+\sigma_3^2/2\right)\right]}{2(\gamma+\mu)^2}dt + \frac{1}{\gamma I + (\gamma+\mu)R}\left(\gamma\sigma_2 I dB_2 + (\gamma+\mu)\sigma_3 R dB_3\right).
\end{aligned}
\tag{34}
$$

From (3),

$$
\begin{aligned}
d(S+R+I) &= [-\mu(S+R+I) + \Lambda]dt\\
&\quad + \sigma_1 S dB_1 + \sigma_2 I dB_2 + \sigma_3 R dB_3.
\end{aligned}
\tag{35}
$$

Furthermore, it is similar to the proof in [26] and obtained that

$$
\begin{aligned}
&\lim_{t\longrightarrow\infty}\frac{I(t)}{t} = 0,\ \lim_{t\longrightarrow\infty}\frac{S(t)}{t} = 0,\ \lim_{t\longrightarrow\infty}\frac{R(t)}{t} = 0, \text{ a.s.}\\
&\lim_{t\longrightarrow\infty}\frac{1}{t}\int_0^t I(v)dB_2(v) = 0,\ \lim_{t\longrightarrow\infty}\frac{1}{t}\int_0^t R(v)dB_3(v) = 0,\\
&\lim_{t\longrightarrow\infty}\frac{1}{t}\int_0^t S(v)dB_1(v) = 0, \text{ a.s.}
\end{aligned}
\tag{36}
$$

From (35) and (36),

$$\limsup_{t\longrightarrow\infty}\frac{\int_0^t (S(u)+R(u)+I(u))du}{t}\leq\frac{\Lambda}{\mu}\ \text{a.s.} \tag{37}$$

From (34) and (37) and $\widehat{\mathbb{R}}_s<1$, we obtain that

$$\limsup_{t\longrightarrow\infty}\frac{\ln(\gamma I+(\mu+\gamma)R)}{t}\leq\frac{(2\beta_1-\beta_2)\Lambda}{m_2\mu} - \frac{\sigma_2^2\gamma^2/2\wedge\left[\left(\mu+\sigma_3^2/2\right)(\mu+\gamma)^2\right]}{2(\mu+\gamma)^2}<0\ \text{a.s.} \tag{38}$$

This implies

$$\lim_{t\longrightarrow\infty} R(t)=0 \text{ and } \lim_{t\longrightarrow\infty} I(t)=0,\ \text{a.s.} \tag{39}$$

Further, it follows from (35) that

$$\limsup_{t\longrightarrow\infty}\frac{\int_0^t S(u)du}{t}=\frac{\Lambda}{\mu}\ \text{a.s.} \tag{40}$$

□

5. Asymptotic Property around the Disease-Free Equilibrium

Theorem 4. *For every $(S_0,I_0,R_0)\in(0,\infty)^3$, let $(S(t),I(t),R(t))$ be the solution to (3). If $\mathbb{R}_0<1$, $\sigma_1^2<\mu$, $\sigma_2^2<2\mu$, and $\sigma_3^2<2\mu$, then*

$$\limsup_{t\longrightarrow\infty}\frac{1}{t}\mathbb{E}\int_0^t\left[\left(S(v)-\frac{\Lambda}{\mu}\right)^2+I^2(v)+R^2(v)\right]dv\leq\frac{K_1\sigma_1^2}{l_1}, \tag{41}$$

where $K_1=2(1+2\mu/\delta)(\Lambda/\mu)^2$, $l_1=\min\{2(1+2\mu/\delta)(\mu-\sigma_1^2),(1+2\mu/\delta)(2\mu-\sigma_2^2),-(\sigma_3^2-2\mu)\}$.

Proof. Define

$$V_4(S,I,R)=v_1+\frac{2\mu}{\delta}v_2+\frac{2\mu}{\beta_1\delta}(2\delta+2\mu+\gamma)v_3, \tag{42}$$

where

$$v_1=\left(S+R+I-\frac{\Lambda}{\mu}\right)^2,\quad v_2=\left(I+S-\frac{\Lambda}{\mu}\right)^2,\quad v_3 \tag{43}$$

where $k>0$ is a constant to be chosen. Then,

$$\begin{aligned}
Lv_1 &= -2\mu\left(S-\frac{\Lambda}{\mu}\right)^2-2\mu\left(I^2+R^2\right)-4\mu(R+I)\left(S-\frac{\Lambda}{\mu}\right)-4\mu IR+\sigma_1^2S^2+\sigma_2^2I^2+\sigma_3^2R^2\\
&\leq -2\left(\mu-\sigma_1^2\right)\left(S-\frac{\Lambda}{\mu}\right)^2-4\mu(R+I)\left(S-\frac{\Lambda}{\mu}\right)-\left(2\mu-\sigma_2^2\right)I^2\\
&\quad-\left(2\mu-\sigma_3^2\right)R^2-4\mu IR+2\left(\frac{\Lambda\sigma_1}{\mu}\right)^2,\\
Lv_2 &= -2\mu\left(-\frac{\Lambda}{\mu}+S\right)^2-\left(2(\mu+\gamma)-\sigma_2^2\right)I^2+2\delta\left(S-\frac{\Lambda}{\mu}\right)R\\
&\quad-2(2\mu+\gamma)\left(-\frac{\Lambda}{\mu}+S\right)I+2\delta IR+\sigma_1^2S^2\\
&\leq -2\left(-\sigma_1^2+\mu\right)\left(-\frac{\Lambda}{\mu}+S\right)^2-2(\gamma+2\mu)\left(S-\frac{\Lambda}{\mu}\right)I-\left(2(\gamma+\mu)-\sigma_2^2\right)I^2\\
&\quad+2\delta\left(S-\frac{\Lambda}{\mu}\right)R+2\delta RI+2\left(\frac{\Lambda\sigma_1}{\mu}\right)^2,\\
Lv_3 &= \left(\beta_1-\frac{\beta_2(I+\alpha R)}{m_1+I+\alpha R}\right)\frac{IS}{1+m_2I}+[-\gamma(1-k)-\mu]I+[\delta(1-k)-k\mu]R\\
&\leq\beta_1SI+[-\mu-\gamma+k\gamma]I+[\delta(1-k)-k\mu]R\\
&=\beta_1\left(-\frac{\Lambda}{\mu}+S\right)I+\left[\frac{\Lambda\beta_1}{\mu}-\mu-\gamma(1-k)\right]I+[\delta(1-k)-k\mu]R.
\end{aligned} \tag{44}$$

Take $\delta\gamma/(\mu+\delta)\le k\gamma\le \gamma+\mu-\Lambda\beta_1/\mu$. From $\mathbb{R}_0<1$,

$$\mathrm{L}v_3\le \beta_1 I\left(S-\frac{\Lambda}{\mu}\right). \tag{45}$$

It follows from (44) and (45) that

$$\begin{aligned}\mathrm{L}V_4\le & -2\left(\mu-\sigma_1^2\right)\left(\frac{2\mu}{\delta}+1\right)\left(-\frac{\Lambda}{\mu}+S\right)^2-\left(\frac{2\mu}{\delta}+1\right)\left(-\sigma_2^2+2\mu\right)I^2\\ &+2\left(\frac{2\mu}{\delta}+1\right)\left(\frac{\sigma_1\Lambda}{\mu}\right)^2-\left(-\sigma_3^2+2\mu\right)R^2.\end{aligned} \tag{46}$$

Hence,

$$\begin{aligned}\mathrm{d}V_4 =& \mathrm{L}V_4\mathrm{d}t+2\sigma_1^2\left[\left(S-\frac{\Lambda}{\mu}+I+R\right)+\frac{2\mu}{\delta}\left(S-\frac{\Lambda}{\mu}+I\right)\right]S^2\mathrm{d}B_1(t)\\ &+\left[2\sigma_2^2\left(\left(1+\frac{2\mu}{\delta}\right)\left(S-\frac{\Lambda}{\mu}+I\right)+R\right)I^2+\frac{2\mu\sigma_2}{\beta_1\delta}(2\delta+2\mu+\gamma)I\right]\mathrm{d}B_2(t)\\ &+\left[2\sigma_3^2\left(S-\frac{\Lambda}{\mu}+I+R\right)R^2+\frac{2\mu\sigma_3 k}{\beta_1\delta}(2\delta+2\mu+\gamma)R\right]\mathrm{d}B_3(t)\\ \le& \left\{-l_1\left[R^2+I^2+\left(-\frac{\Lambda}{\mu}+S\right)^2\right]+2\left(\frac{2\mu}{\delta}+1\right)\left(\frac{\sigma_1\Lambda}{\mu}\right)^2\right\}\mathrm{d}t\\ &+2\sigma_1^2\left[\left(S-\frac{\Lambda}{\mu}+I+R\right)+\frac{2\mu}{\delta}\left(S-\frac{\Lambda}{\mu}+I\right)\right]S^2\mathrm{d}B_1(t)\\ &+\left[2\sigma_2^2\left(\left(1+\frac{2\mu}{\delta}\right)\left(S-\frac{\Lambda}{\mu}+I\right)+R\right)I^2+\frac{2\mu\sigma_2}{\beta_1\delta}(2\delta+2\mu+\gamma)I\right]\mathrm{d}B_2(t)\\ &+\left[2\sigma_3^2\left(S-\frac{\Lambda}{\mu}+I+R\right)R^2+\frac{2\mu\sigma_3 k}{\beta_1\delta}(2\delta+2\mu+\gamma)R\right]\mathrm{d}B_3(t).\end{aligned} \tag{47}$$

Then,

$$\begin{aligned}0\le V_4(t)\le& V_4(0)-l_1\int_0^t\left(\left(S(v)-\frac{\Lambda}{\mu}\right)^2+I^2(v)+R^2(v)\right)\mathrm{d}v\\ &+2\left(1+\frac{2\mu}{\delta}\right)\left(\frac{\Lambda\sigma_1}{\mu}\right)^2t+\widehat{M}(t),\end{aligned} \tag{48}$$

where

$$\begin{aligned}\widehat{M}(t)=&\\ 2\sigma_1^2&\int_0^t\left[\left(S-\frac{\Lambda}{\mu}+I+R\right)+\frac{2\mu}{\delta}\left(S-\frac{\Lambda}{\mu}+I\right)\right]S^2\mathrm{d}B_1(v)\\ &+\int_0^t\left[2\sigma_2^2\left(\left(1+\frac{2\mu}{\delta}\right)\left(S-\frac{\Lambda}{\mu}+I\right)+R\right)I^2\right.\\ &\left.+\frac{2\mu\sigma_2}{\beta_1\delta}(2\delta+2\mu+\gamma)I\right]\mathrm{d}B_2(v)\\ &+\int_0^t\left[2\sigma_3^2\left(S-\frac{\Lambda}{\mu}+I+R\right)R^2\right.\\ &\left.+\frac{2\mu\sigma_3 k}{\beta_1\delta}(2\delta+2\mu+\gamma)R\right]\mathrm{d}B_3(v).\end{aligned} \tag{49}$$

Hence,

$$\limsup_{t\longrightarrow\infty}\frac{1}{t}\mathbb{E}\int_0^t\left[\left(S(v)-\frac{\Lambda}{\mu}\right)^2+I^2(v)+R^2(v)\right]dv\le\frac{K_1\sigma_1^2}{l_1}. \tag{50}$$

□

Remark 1. By Theorem 4, if $\mathbb{R}_0 < 1$, $\sigma_1^2 < \mu, \sigma_2^2 < 2\mu, \sigma_3^2 < 2\mu$, then the solution of (3) fluctuates around the disease-free equilibrium. In addition, if $\sigma_1 = 0$, then $E_0 = (\Lambda/\mu, 0, 0)$ is the disease-free equilibrium of (3). Then, (46) becomes

$$LV_4 \le -2\mu\left(\frac{2\mu}{\delta}+1\right)\left(-\frac{\Lambda}{\mu}+S\right)^2 - \left(\frac{2\mu}{\delta}+1\right) \cdot \left(-\sigma_2^2+2\mu\right)I^2 - \left(2\mu-\sigma_3^2\right)R^2. \tag{51}$$

Thus, E_0 is globally asymptotically stable if $\mathbb{R}_0 < 1$, $\sigma_2^2 < 2\mu, \sigma_3^2 < 2\mu$.

6. Asymptotic Property around the Endemic Equilibrium

Assume $\mathbb{R}_0 > 1$. $E^*(S_*, I_*, R_*)$ is the endemic equilibrium of (1). From

$$\begin{cases} \Lambda - \left(\beta_1 - \dfrac{\beta_2(I_*+\alpha R_*)}{m_1+I_*+\alpha R_*}\right)\dfrac{S_* I_*}{1+m_2 I_*} - \mu S_* = 0, \\ \left(\beta_1 - \dfrac{\beta_2(I_*+\alpha R_*)}{m_1+I_*+\alpha R_*}\right)\dfrac{S_* I_*}{1+m_2 I_*} - (\mu+\gamma)I_* + \delta R_* = 0, \\ \gamma I_* - (\mu+\delta)R_* = 0, \end{cases} \tag{52}$$

we can easily show

$$S_* = \frac{(\mu+\gamma-\delta\gamma/(\mu+\delta))\left[m_1(\mu+\delta)+(\mu+\delta+\alpha\gamma)I*\right](1+m_2 I_*)}{\beta_2(\mu+\delta+\alpha\gamma)I_*},\ R_* = \frac{\gamma I_*}{\delta+\mu}. \tag{53}$$

Substituting S_*, R_* into (52) yields

$$A_1 I_*^2 + A_2 I_* + A_3 = 0, \tag{54}$$

where

$$\begin{aligned} A_1 &= -(\delta+\mu+\alpha\gamma)\left(\gamma+\mu-\frac{\delta\gamma}{\mu+\delta}\right)(\beta_1-\beta_2+\mu m_2), \\ A_2 &= (\mu+\alpha\gamma+\delta)\left[-\mu\left(\mu+\gamma-\frac{\delta\gamma}{\delta+\mu}\right)+\Lambda(-\beta_2+\beta_1)\right] \\ &\quad - m_1(\mu+\delta)(\beta_1-\mu m_2)\left(-\frac{\delta\gamma}{\delta+\mu}+\gamma+\mu\right), \\ A_3 &= m_1(\mu+\delta)\left[\Lambda\beta_1-\mu\left(-\frac{\delta\gamma}{\delta+\mu}+\gamma+\mu\right)\right]. \end{aligned} \tag{55}$$

From $\beta_1 \ge \beta_2$ and $\mathbb{R}_0 > 1$, we have $A_1 < 0$ and $A_3 > 0$. So, $E^*(S_*, I_*, R_*)$ is a unique endemic equilibrium of (1). Denote

$$\begin{aligned} p &= \frac{(2\mu+\gamma)(1+m_2 I_*)}{\beta_1-\beta_2(I_*+\alpha R_*)/(m_1+I_*+\alpha R_*)}, \\ q &= \frac{\delta(2\mu+\gamma)(1+m_2 I_*)R_*}{\gamma I_*(\beta_1-\beta_2(I_*+\alpha R_*)/(m_1+I_*+\alpha R_*))}. \end{aligned} \tag{56}$$

Theorem 5. *For any $(S_0, I_0, R_0) \in \mathbb{R}_+^3$, let $(S(t), I(t), R(t))$ be the solution of (3). If $\mathbb{R}_0 > 1$, $\sigma_1^2 < \mu/2 - p\beta_2\alpha, \sigma_2^2 < \mu/2 + \gamma - \gamma^2/2\delta, \sigma_3^2 < \mu + \delta/2 - \delta^2/\mu$, then*

$$\limsup_{t\to\infty}\frac{1}{t}\mathbb{E}\int_0^t\left[(S(v)-S_*)^2+(R(v)-R_*)^2 + (I(v)-I_*)^2\right]dv \le \frac{K_2}{l_2}, \tag{57}$$

where

$$\begin{aligned} K_2 &= \left(\sigma_1^2+p\beta_2\alpha\right)S_*^2+\sigma_2^2 I_*^2+\sigma_3^2 R_*^2+\frac{1}{2}p\sigma_2^2 I_* \\ &\quad +\frac{1}{2}q\sigma_3^2 R_*+\frac{1}{2}p\beta_2\alpha\left(R_*+\frac{I_*}{\alpha}\right)^2, \\ l_2 &= \min\left\{\frac{\mu}{2}-\sigma_1^2-p\beta_2\alpha, \frac{\mu}{2}+\gamma-\sigma_2^2-\frac{\gamma^2}{2\delta}, \mu+\frac{\delta}{2}-\frac{\delta^2}{\mu}-\sigma_3^2\right\}. \end{aligned} \tag{58}$$

Proof. Define $V_5(S, I, R)$: $\mathbb{R}_+^3 \longrightarrow \mathbb{R}$ by

$$V_5(S,I,R) = w_1(S,I) + w_2(R) + pw_3(I) + qw_4(R), \tag{59}$$

where

$$\begin{aligned} w_1(S,I) &= \frac{(I-I_*+S-S_*)^2}{2}, w_2(R) = \frac{(-R_*+R)^2}{2}, \\ w_3(I) &= -I_*\ln\frac{I}{I_*}+(I-I_*), w_4(R) = -R_*\ln\frac{R}{R_*}+(R-R_*). \end{aligned} \tag{60}$$

Using the It $\hat{o}$ formula and (52),

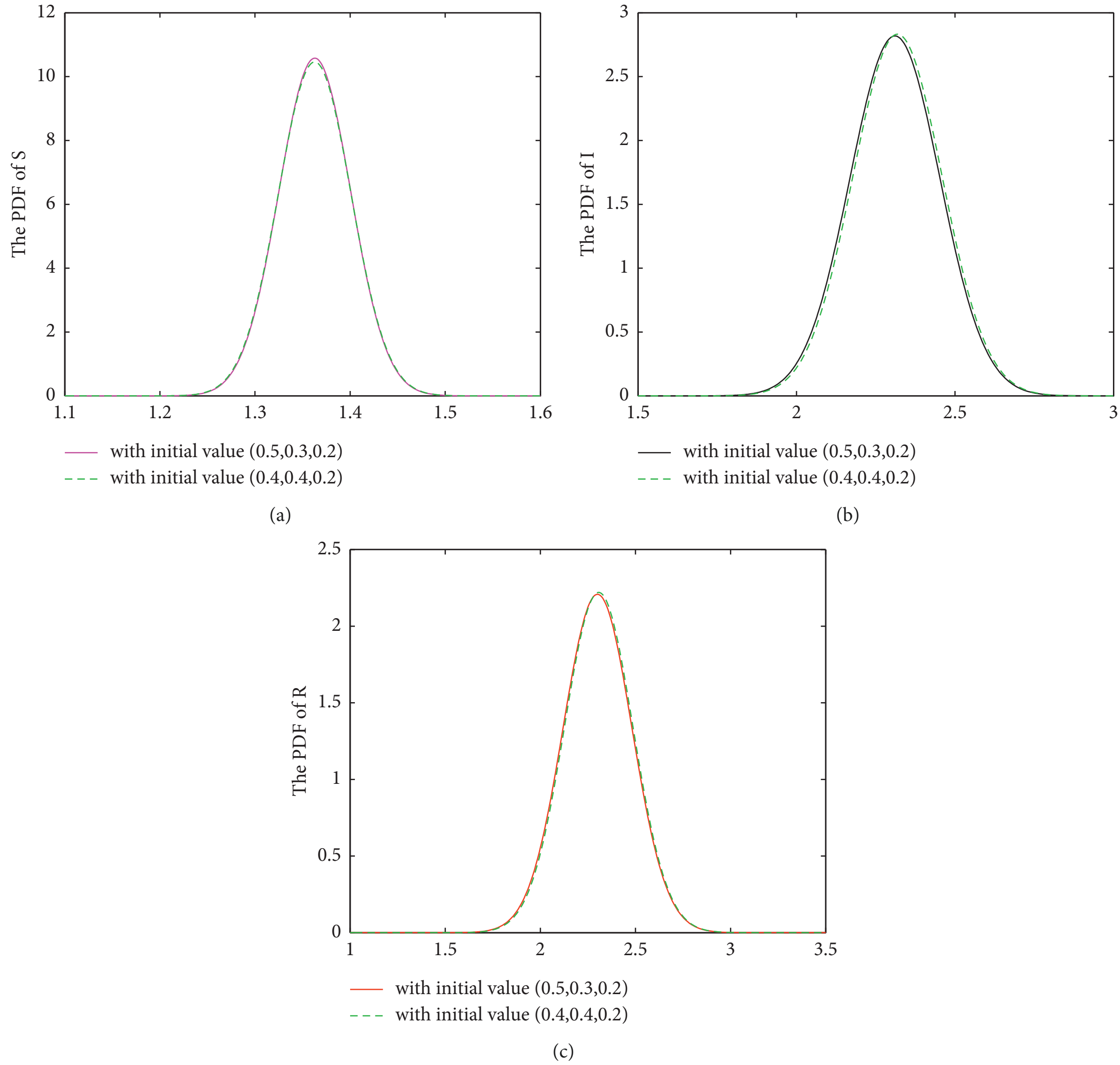

Figure 1: The probability density function trajectories of susceptible, infected, and recovered individuals of (3) with $(S(0), I(0), R(0)) = (0.5, 0.3, 0.2)$ or $(0.4, 0.4, 0.2)$.

$$\begin{aligned}
\mathrm{L}w_1 &= (I - I_* + S - S_*)[-\mu(I - I_* + S - S_*) + \delta(R - R_*) - \gamma(I - I_*)] + \frac{1}{2}\sigma_1^2 S^2 + \frac{1}{2}\sigma_2^2 I^2 \\
&\leq -(\mu - \sigma_1^2)(S - S_*)^2 - (I - I_*)^2(\gamma + \mu - \sigma_2^2) + \sqrt{\mu}(S - S_*)\frac{\delta}{\sqrt{\mu}}(R - R_*) \\
&\quad - (\gamma + 2\mu)(I - I_*)(S - S_*) + \sqrt{\mu}(I - I_*)\frac{\delta}{\sqrt{\mu}}(R - R_*) + \sigma_1^2 S_*^2 + \sigma_2^2 I_*^2 \\
&\leq \frac{\delta^2}{\mu}(-R_* + R)^2 - \left(\frac{\mu}{2} - \sigma_1^2\right)(-S_* + S)^2 - \left(\gamma + \frac{\mu}{2} - \sigma_2^2\right)(-I_* + I)^2 \\
&\quad - (2\mu + \gamma)(-I_* + I)(-S_* + S) + \sigma_1^2 S_*^2 + \sigma_2^2 I_*^2,
\end{aligned} \tag{61}$$

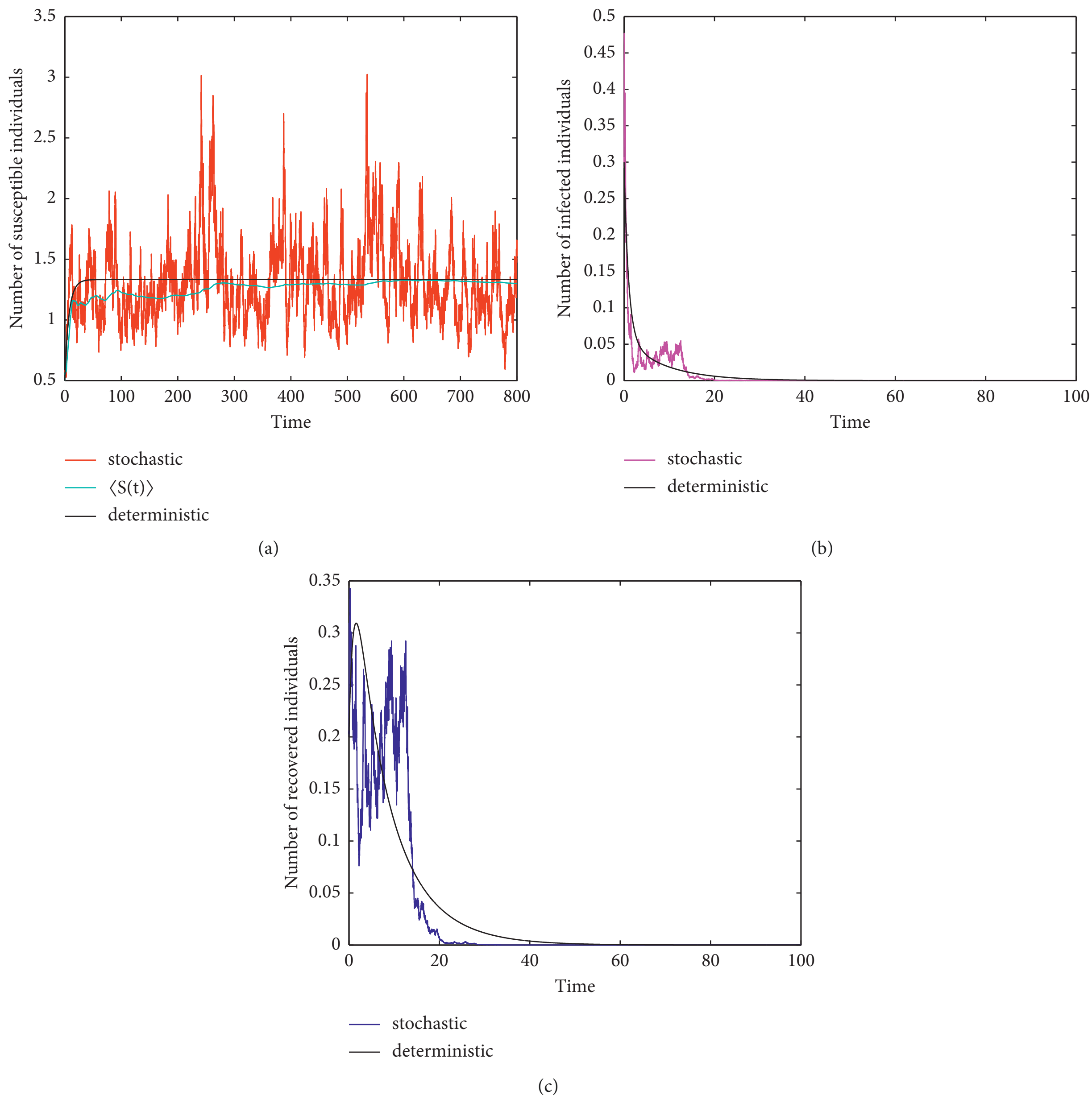

Figure 2: The trajectories of (1) and sample path of (3).

$$\begin{aligned} Lw_2 &= \left[-(\delta+\mu)(R-R_*)+\gamma(I-I_*)\right](R-R_*)+\frac{\sigma_3^2R^2}{2} \\ &\leq \gamma(R-R_*)(I-I_*)-\left(\delta+\mu-\sigma_3^2\right)(R-R_*)^2+\sigma_3^2R_*^2 \\ &\leq \frac{\gamma^2}{2\delta}(I-I_*)^2+\sigma_3^2R_*^2-\left(\mu+\frac{\delta}{2}-\sigma_3^2\right)(R-R_*)^2, \end{aligned} \tag{62}$$

$$\begin{aligned} Lw_3 &= (I-I_*)\left[\left(\beta_1-\frac{\beta_2(I+\alpha R)}{m_1+I+\alpha R}\right)\frac{S}{1+m_2I}-\left(\beta_1-\frac{\beta_2(I_*+\alpha R_*)}{m_1+I_*+\alpha R_*}\right)\frac{S_*}{1+m_2I_*}-\frac{\delta R_*}{I_*}+\frac{\delta R}{I}\right]+\frac{1}{2}\sigma_2^2I_* \\ &= -\frac{\beta_2S(I-I_*)}{1+m_2I}\left(\frac{\alpha R+I}{m_1+I+\alpha R}-\frac{I_*+\alpha R_*}{m_1+\alpha R_*+I_*}\right)-\frac{\alpha R_*+I_*}{m_1+\alpha R_*+I_*}-\frac{\beta_2S(I-I_*)}{1+m_2I} \\ &\quad+\frac{\beta_1S(I-I_*)}{1+m_2I}-\left(\beta_1-\frac{\beta_2(\alpha R_*+I_*)}{m_1+\alpha R_*+I_*}\right)\frac{(I-I_*)S_*}{1+m_2I_*}+\delta(I-I_*)\left(-\frac{R_*}{I_*}+\frac{R}{I}\right)+\frac{1}{2}\sigma_2^2I_* \end{aligned}$$

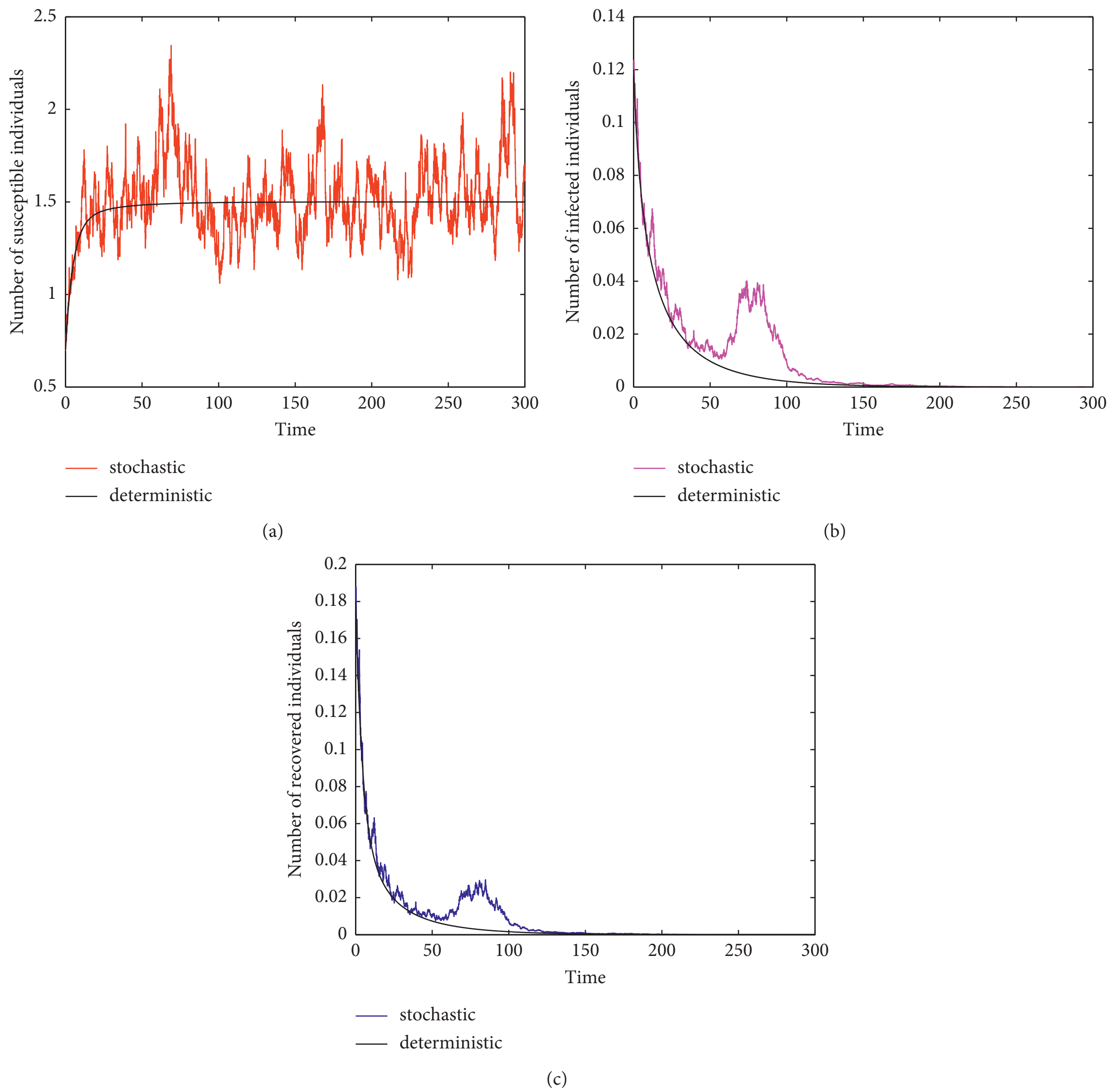

Figure 3: The trajectories of (1) and sample path of (3).

$$
\begin{aligned}
&= -\frac{\beta_2 m_1 (I-I_*)^2 S}{(1+m_2 I)(m_1+\alpha R+I)(m_1+I_*+\alpha R_*)} + \delta R_* \left(\frac{R}{R_*} - \frac{I}{I_*} - \frac{I_* R}{I R_*} + 1\right) \\
&\quad - \frac{\beta_2 \alpha m_1 S (I-I_*)(R-R_*)}{(m_1+I_*+\alpha R_*)(1+m_2 I)(m_1+I_*+\alpha R)} + \frac{1}{2}\sigma_2^2 I_* \\
&\quad + (I-I_*)\left(\frac{S}{1-m_2 I} - \frac{S_*}{1+m_2 I_*}\right)\left(\beta_1 - \frac{\beta_2 (I_*+\alpha R_*)}{m_1+I_*+\alpha R_*}\right) \\
&\le \beta_2 \alpha (S-S_*)^2 + \beta_2 \alpha S_*^2 + \frac{(S-S_*)(I-I_*)}{1+m_2 I_*}\left(\beta_1 - \frac{\beta_2 (I_*+\alpha R_*)}{m_1+I_*+\alpha R_*}\right) \\
&\quad + \frac{1}{2}\beta_2 \alpha \left(R_* + \frac{I_*}{\alpha}\right)^2 + \delta R_* \left(\frac{R}{R_*} - \frac{I}{I_*} - \frac{R I_*}{I R_*} + 1\right) + \frac{1}{2}\sigma_2^2 I_*,
\end{aligned}
\tag{63}
$$

$$
Lw_4 = \gamma\left(-\frac{R_*}{R} + 1\right)\left(-\frac{R I_*}{R_*} + I\right) + \frac{1}{2}\sigma_3^2 R_* = \gamma I_* \left(1 - \frac{I R_*}{I_* R} - \frac{R}{R_*} + \frac{I}{I_*}\right) + \frac{1}{2}\sigma_3^2 R_*. \tag{64}
$$

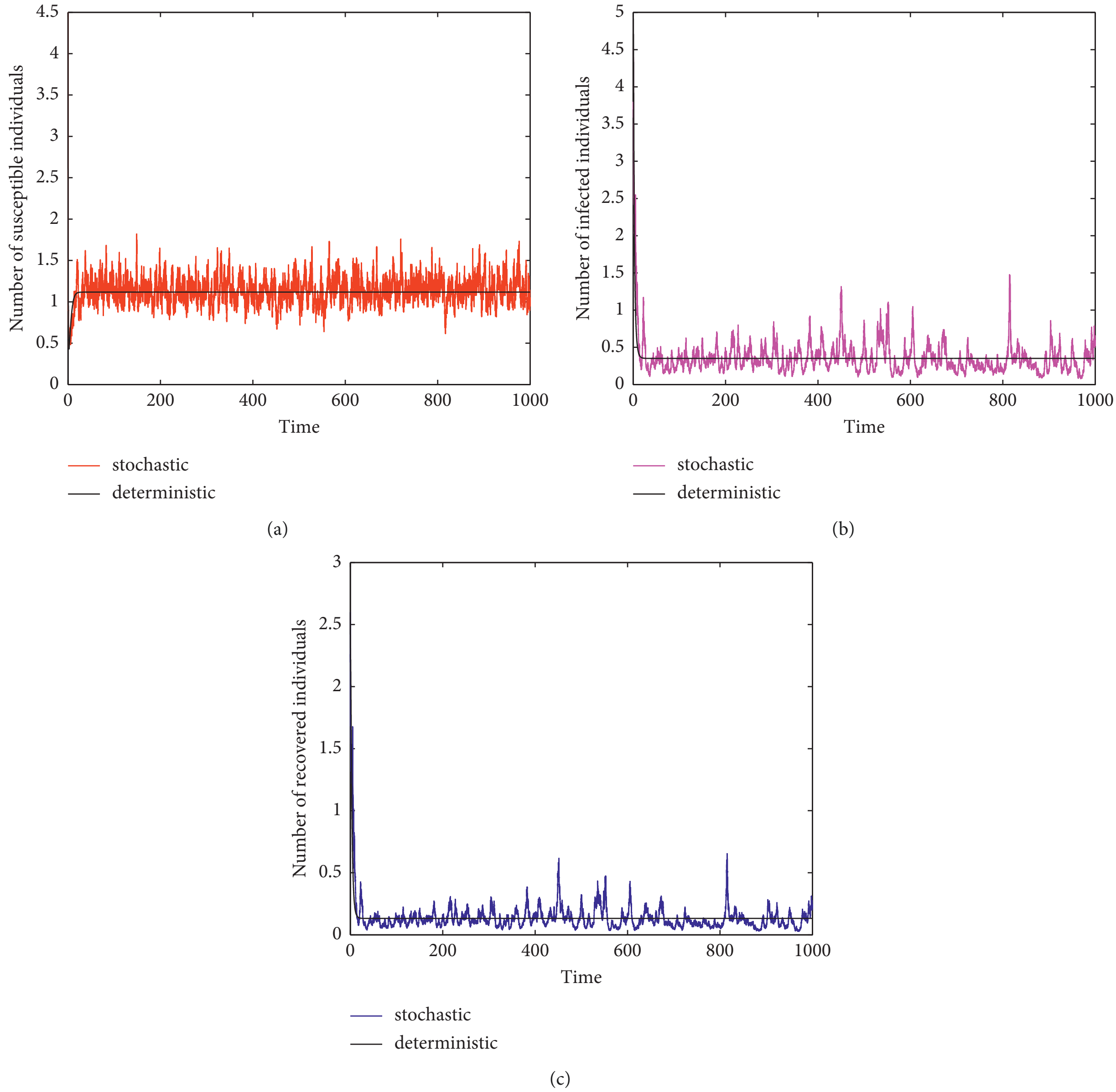

Figure 4: The trajectories of (1) and sample path of (3).

It follows from (61)–(64) that

$$\begin{aligned}
LV_5 \leq & -\left(\frac{\mu}{2} - \sigma_1^2 - p\beta_2\alpha\right)(-S_* + S)^2 - \left(\frac{\mu}{2} + \gamma - \sigma_2^2 - \frac{\gamma^2}{2\delta}\right)(-I_* + I)^2 - \left(\frac{\delta}{2} + \mu - \frac{\delta^2}{\mu} - \sigma_3^2\right)(-R_* + R)^2 \\
& + \sigma_1^2 S_*^2 + \sigma_2^2 I_*^2 + \sigma_3^2 R_*^2 + p\beta_2\alpha S_*^2 + \frac{1}{2} p\beta_2\alpha\left(R_* + \frac{I_*}{\alpha}\right)^2 + \frac{p\sigma_2^2 I_*}{2} + \frac{q\sigma_3^2 R_*}{2} \\
\leq & -l_2\left[(-S_* + S)^2 + (-I_* + I)^2 + (-R_* + R)^2\right] + K_2.
\end{aligned} \tag{65}$$

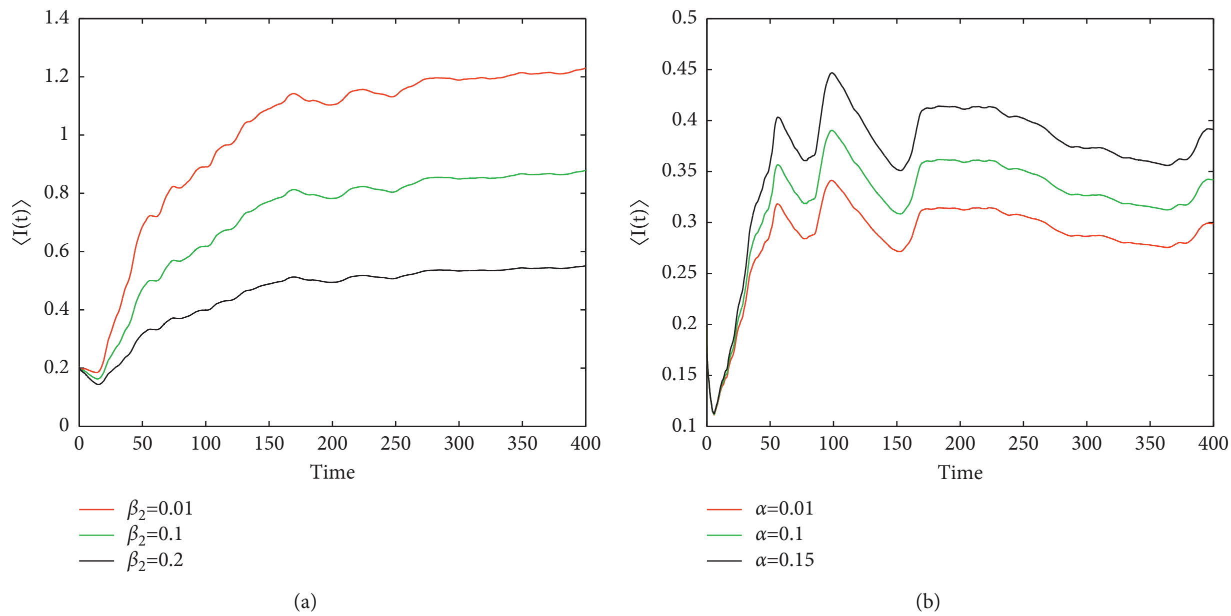

FIGURE 5: The sample path of the time mean of infected individuals in (3).

Hence,

$$\begin{aligned} \mathrm{d}V_5 \le & -l_2\left[(-S_* + S)^2 + (-R_* + R)^2 + (-I_* + I)^2\right] \\ & + K_2 + p\sigma_2(-I_* + I)\mathrm{d}B_2(t) \\ & + (-S_* + S - I_* + I)(\sigma_1 S\mathrm{d}B_1(t) + \sigma_2 I\mathrm{d}B_2(t)) \\ & + (R + q)\sigma_3(R - R_*)\mathrm{d}B_3(t). \end{aligned} \tag{66}$$

Then,

$$\begin{aligned} \mathbb{E}V_5(S(t), I(t), R(t)) \le & V_5(S(0), I(0), R(0)) + K_2 t \\ & - l_2\mathbb{E}\int_0^t \left(S(v) - S_*^2 + (R(v) - R_*)^2 + I(v) - I_*\right)^2 \mathrm{d}v. \end{aligned} \tag{67}$$

Therefore,

$$\begin{aligned} & \limsup_{t\longrightarrow\infty} \frac{1}{t}\mathbb{E}\int_0^t\left[\left((S(v) - S_*)^2 + (R(v) - R_*)^2 + (I(v) - I_*)\right)^2\right]dv \\ & \le \frac{K_2}{l_2}. \end{aligned} \tag{68}$$

The proof is complete. □

Remark 2. By Theorem 5, if the noise intensity is small and $\mathbb{R}_0 > 1$, the solution of (3) fluctuates around the endemic equilibrium. That is, if the intensity of random disturbance is small enough, the disease will persist.

7. Numerical Simulations

Example 1. Let $\beta_1 = 0.5, \beta_2 = 0.01, \mu = 0.1, \Lambda = 0.6, \gamma = 0.2, \delta = 0.1, m_1 = m_2 = 1, \alpha = 0.5,$ $\sigma_1^2 = 0.02, \sigma_2^2 = 0.01,$ $(S(0), I(0), R(0)) = (0.7, 0.12, 0.18)$. Then $\mathbb{R}_s \approx 1.4605 > 1$, and model (3) has a stationary distribution by Theorem 2. As can be seen from Figure 1, when $(S(0), I(0), R(0)) = (0.5, 0.3, 0.2)$ or $(0.4, 0.4, 0.2)$, there is a stable distribution in model (3).

Example 2. Let $\beta_1 = 0.2, \beta_2 = 0.18, \mu = 0.15, \Lambda = 0.2, \gamma = 0.8, \delta = 0.1, m_1 = 1, m_2 = 2.7, \alpha = 0.5, \sigma_2^2 = 0.4, \sigma_3^2 = 0.2,$ $(S(0), I(0), R(0)) = (0.5, 0.3, 0.2)$. Then, $\widehat{\mathbb{R}}_s \approx 0.6566 < 1$. By Theorem 3, the disease of (3) becomes extinct (see Figure 2).

Example 3. Let $\beta_1 = 0.2, \beta_2 = 0.1, \mu = 0.2, \Lambda = 0.3, \gamma = 0.2, \delta = 0.1, m_1 = m_2 = 1, \alpha = 0.5,$ $\sigma_1^2 = 0.1, \sigma_2^2 = 0.1, \sigma_3^2 = 0.2,$ $(S(0), I(0), R(0)) = (0.7, 0.12, 0.18)$. We obtain $\mathbb{R}_0 \approx 0.8996 < 1,$ $0.1 = \sigma_1^2 < \mu = 0.2, 0.1 = \sigma_2^2 < 2\mu = 0.4,$ $0.2 = \sigma_3^2 < 2\mu = 0.4$. It is known from Theorem 4 that the solution of (3) fluctuates around the curve of (1). Figure 3 is consistent with this result.

Example 4. Let $\beta_1 = 0.65, \beta_2 = 0.05, \mu = 0.5, \Lambda = 0.8, \gamma = 0.3, \delta = 0.3, m_1 = 1, m_2 = 0.1, \alpha = 0.01, \sigma_1^2 = 0.02,$ $\sigma_2^2 = 0.03, \sigma_3^2 = 0.04,$ $(S(0), I(0), R(0)) = (4.5, 3.8, 2.6)$. Then, $\mathbb{R}_0 \approx 1.5125 > 1, p \approx 2.1242,$ $0.02 = \sigma_1^2 < (\mu/2) - p\beta_2\alpha \approx 0.2489, 0.03 = \sigma_2^2 < (\mu/2) + \gamma - (\gamma^2/2\delta) = 0.4, 0.04 = \sigma_3^2 < \mu + (\delta/2) - (\delta^2/\mu) = 0.47$. Solution of (3) fluctuates around the endemic equilibrium of (1) by Theorem 5. Figure 4 is consistent with this result.

Example 5. To show the impact of media coverage on the spread of diseases, different media coverage parameters are taken as $\beta_2 = 0.01, 0.1, 0.2$. Let $\beta_1 = 0.18, \mu = 0.25, \Lambda = 0.8, \gamma = 0.2, \delta = 0.2, m_1 = 1, m_2 = 0.1, \alpha = 0.01, \sigma_1^2 = 0.01, \sigma_2^2 = 0.04, \sigma_3^2 = 0.03,$ $(S(0), I(0), R(0)) = (0.5, 0.4, 0.1)$. As can be seen from Figure 5(a), the larger the media coverage parameter β_2 is, the smaller the time mean $1/t\int_0^t I(s)\mathrm{d}s$ of the infected individuals is. This suggests that media coverage can decrease the spread of disease.

Example 6. To illustrate the influence of the number of recovered individuals on the spread of infectious diseases, we take respectively $\alpha = 0.01, 0.1, 0.15$. $\beta_1 = 0.18, \beta_2 = 0.15$, $\mu = 0.25, \Lambda = 0.8, \gamma = 0.2, \delta = 0.2, m_1 = 1, m_2 = 0.1$, $\sigma_1^2 = 0.01, \sigma_2^2 = 0.03, \sigma_3^2 = 0.02$, $(S(0), I(0), R(0)) = (0.5, 0.4, 0.1)$. From Figure 5(b), the time mean of the infected individuals decreases with the decrease of the parameter α. This shows that the less impact the number of recovered individuals has on media coverage, the lower the transmission rate is, which in turn leads to fewer infected individuals.

8. Conclusions

We analyze a stochastic SIRI model with media coverage. To begin with, existence and uniqueness of global positive solution of the model are established. Using the Hasminskii theory, we prove existence of a stationary distribution. Then, conditions of disease extinction are proved. By constructing the suitable Lyapunov functions, the dynamic properties of solutions near equilibria are studied. If $\mathbb{R}_0 < 1$, when the intensity of random disturbance is sufficiently small, infectious diseases will be extinct; if $\mathbb{R}_0 > 1$ and the random disturbance is small, infectious diseases will persist. In addition, the less impact the number of recovered individual has on media coverage, the lower the transmission rate is, which in turn leads to fewer infected individuals.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors' Contributions

All the authors contributed equally and significantly in writing this paper. All authors read and approved the final manuscript.

Acknowledgments

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