Review Article

Collaborative Operation Model with Selling Profit Sharing Strategy for Direct Online Retailing

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This study focuses on the operation mode choice issue on whether brand owners should use the autonomous or the collaborative operation mode in direct online retailing. Results demonstrate that when the service provider’s investment is relatively effective and the selling profit sharing ratio of the service provider is high, the brand owner should choose the collaborative operation mode. Surprisingly, we find that brand owners in a dual-channel scenario have more motivation to use the collaborative operation mode than those in a single e-commerce channel scenario. The e-commerce service market may have much potential in the future.

1. Introduction

Online selling has been rapidly expanding in recent years. As e-commerce grows over time, it increasingly influences brand owners. Many brand owners, e.g., Dell, Sony, Hewlett Packard, Lenovo, Nike, and Estee Lauder, who may have distributed only through traditional offline retail channels, have begun using online channels to sell their products directly to consumers [1–5]. Numerous pure online brand owners, whose products are only sold online and who are without offline physical stores, have become primary players in the market. For example, the Handu Group, a famous e-commerce garment corporation in China, provides more than 70 kinds of pure e-commerce brands (https://www.htcases.com/kw/content/index.html?dbId=6).

Unlike offline channel operation, online operation includes many specified e-commerce operation aspects, such as e-store construction, product image, online marketing and logistics, online customer service, and operation data analysis. Brand owners primarily adopt the autonomous operation mode for online sales, i.e., building an operations department or a branch institution to meet their online operation needs by themselves. However, given their limited capabilities in e-commerce channel operations, some brand owners do not achieve their desired outcome or even damage their business. For example, Metersbonwe Fashion and Accessories, the biggest Chinese home-grown apparel and fashion chain corporation, entered the e-commerce business and launched Bongo.com at the end of 2010. Their YouFan app was then developed to sell directly to customers in 2015. Both attempts at e-commerce failed to achieve the desired goals. In the first half of 2015, the company lost RMB 95 million, and its sales dropped by 7% to RMB 2.8 billion (https://www.forbes.com/sites/russellflannery/2015/08/27/china-retail-billionaires-metersbonwe-posts-loss-as-e-commerce-squeeze-continues/#3589feaa66d79).

In order to improve their direct online selling, many brand owners with online and offline channels, such as Nike, Tissot, and Pandora, have broken away from the traditional autonomous operation mode in direct online channel and embraced the collaborative operation mode, i.e., operating their online channel with supporting from professional e-commerce service providers. This transformation is not achieved overnight, and usually goes through a gradual process. At the beginning, many brand owners only tentatively contacted e-commerce service providers and used them as channels for selling products. When brand owners have a deeper understanding of e-commerce service provider, they may establish cooperative relationships through information sharing and staff exchange. In the end, they may
establish a closer cooperation relationship in the field of supply chain, and carry out collaborative operation in production planning, promotion, logistics, and many other aspects.

Nowadays, brand e-commerce service providers can offer partial or full e-commerce solutions for brand owners, such as IT solutions, e-store operations, digital marketing, online customer services, warehousing, and fulfillment. Brand owners are usually responsible for production and logistics. It is worth noting that the operation of them is not separated, on the contrary, there are close relationship between them. For example, ShangBai e-commerce Company, a golden partner in Taobao.Com (a famous e-commerce platform in China) often optimizes the operation of the supply chain by providing e-commerce operation data and product improvement suggestions to brand owners. With the deepening of cooperation between the two sides, they will hold regular and irregular meetings to discuss various problems in direct online sales, and find solutions to realize the collaborative operation of the supply chain, which greatly improves the performance of the supply chain operation.

The market size of brand e-commerce services in China has maintained a fast growth rate, having exceeded 200 billion yuan in 2020 and is expected to reach 295.09 billion yuan in 2021 (https://www.iimedia.cn/c460/78476.html). The brand e-commerce service industry continues to develop and is becoming mature, which provide a good opportunity for the development of service providers. Baozun, a NASDAQ company that provides e-commerce solutions in China and offers clients IT infrastructure setup and integration and online store design and setup, reached an online selling record of RMB 19.18 billion for its branded customers on November 11, 2021. In 2021, Baozun's total net revenue was RMB 9.396.3 million, up to 6.2% year-on-year (https://www.baozun.com/news-details.html?id=91).

In online channels, selling cost includes information analysis, technical, designing, and product promotion costs [6, 7]. Reducing selling cost has a direct impact on brand owners’ profitability. Some service providers use public e-commerce platforms to provide their service. Some service providers even invest to set up their own service platforms. As a professional brand e-commerce service provider, its effect on cost reduction and efficiency is remarkable. Baozun independently developed an intelligent operation service platform called the Retail Operation Support System (ROSS). The ROSS has three smart tools, namely “designer”, “merchandiser”, and “activist”, which are, respectively, used in picture processing, product management, and activity management, helping the efficiency of brand merchants to increase by 70%. As a service provider serving more than 70 international fashion brands, BUY QUICKLY has successfully doubled the performance of the HUGO BOSS brand on the Tmall platform and increased its fans by 300% through accurate operation management. Based on the data bank, it also precipitates crowd portraits, continuously optimizes crowd strategies, and achieves secondary reach and conversion of target groups (https://www.buyquickly.com/brand-portfolio). Improved efficiency in cost control has led many brand owners to adopt enthusiastically the collaborative operation mode. In practice, brand owners pay for this efficient cost control by paying a fraction of the selling profit as the service fee (https://report.iresearch.cn/report/201707/3026.shtml). Service fees are one of the most common forms of cooperation under the collaborative operation model. In Baozun, for example, service revenue accounts for an increasing share of total revenue, rising from less than 25% in 2015 to over 50%. Compared with the traditional distribution model, the service way can generate greater profit margins, and the size of the service fee has a great impact on the brands’ collaborative operation decisions (https://www.baozun.com/news-details.html?id=91). Nowadays, selling profit sharing strategy is very popular in Chinese brand e-commerce service market.

In the online channel, a key distinction between collaborative operation mode and autonomous operation mode is who sets the investment to reduce selling cost. In the autonomous operation mode, brand owners decide the investment level to reduce selling cost before setting the retail price in the market. However, in the collaborative operation mode, e-commerce service providers decide the investment level, and brand owners set the retail price. Brand owners need to share a certain fraction of the profit with service providers. Service providers need to bear investment costs. Nowadays, although the collaborative operation mode is becoming popular, some brand owners still adopt the autonomous operation mode to operate online channels. Thus, the two kinds of brand owners, pure e-commerce brand owners and dual-channel brand owners, must understand what key factors affect mode choice, and in which conditions the collaborative operation mode should be adopted. Although the existence of different operation modes in online selling is a conspicuous phenomenon, the quantitative studies on this phenomenon are scarce (https://www.cmcct-dut.cn/Cases/Detail/2824).

Specifically, our objective is to study the following research questions in the context of e-commerce supply chains.

(1) How does service providers’ advantage in investment efficiency affect the mode preference of brand owners on collaborative operation in direct online selling?

(2) How does the selling profit sharing ratio affect the profit of brand owners and service providers in two kinds of channel scenarios?

(3) Do pure e-commerce and dual-channel brand owners have the same preference on the collaborative operation mode with selling profit sharing strategy?

To capture the aforementioned issues, we consider two different scenarios. The first is the single-channel scenario, wherein brand owners are pure e-commerce brand owners. The second is the dual-channel scenario, wherein brand
owners sell their products in an online channel and a
cconventional offline reseller channel. In each scenario,
brand owners attempt to use collaborative operation
modes. We assume that the online channel has selling
cost, and supply chain players invest in online channel to
reduce selling cost. To study the influence of the high
investment efficiency of service providers, we assume that
the investment efficiency of brand owners is lower than
that of service providers, and the brand owners and
service providers use a selling profit sharing contract in
the collaborative mode.

The remainder of the paper is organized as follows.
Section 2 reviews the relevant literature. Sections 3 and 4
investigate the single e-commerce channel scenario and the
dual-channel scenario, respectively. Section 5 concludes this
study with a discussion of practical implications. Appendix
contains all proofs to the technical results.

2. Literature Review

Given that our study helps to explain why an e-commerce
channel may not be operated by brand owners, our study is
related to the operations management literature that focuses
on outsourcing. Early works mainly focus on production
outsourcing [8]. For example, Kaya [9] compared an out-
sourcing model and an in-house production model in a two-
facility supply chain in which one of the parties can exert
costly effort to increase demand. The study found that linear,
quantity discount, and simultaneous revenue and cost-
sharing contracts can coordinate the supply chain. Wang
et al. [10] considered two outsourcing structures under push
and pull contracts; the equilibrium production quantity is
higher under control than under delegation for the push
contract, whereas the reverse holds for the pull contract.
Both supply chain members prefer control over delegation
under the push contract. Recent studies have addressed
issues related to service outsourcing in either cost reduction
or demand enhancement. For example, Xia and Gilbert [11]
investigated how investments in demand-enhancing services
for a product line affect the interactions between brand
owners and dealers. They advocated that promotional ser-
VICES can be provided by brand owners or delegated to
dealers. They assumed that the service investments of brand
owners and retailers are equally efficient. Bian et al. [12]
studied the impact of service outsourcing under three supply
chain power structures, namely brand owner-Stackelberg,
vertical-Nash, and retailer-Stackelberg supply chains, while
allowing supply chain members’ service efficiency to be
different. Zhang et al. [13] explored outsourcing strategies
for manufacturer warranty services in a dual-channel supply
chain by which the demand-enhancing service can be un-
ertaken by different supply chain parties, namely retailers,
third-party service providers, and both simultaneously.
From the perspective of a remanufacturing closed-loop
supply chain, Gong et al. [14] studied a manufacturer
outsources second-hand products’ collection and disman-
tling to a specialized dismantling firm. Also, the government
encourages two firms to implement dismantling cooperation
through monetary subsidy.

Our work is not a simple extension of previous studies
on outsourcing. First, existing outsourcing happens between
upstream and downstream enterprises. For example, man-
ufacturers deliver their after-sales services to retailers, or
original equipment manufacturers outsource production to
contract suppliers. The collaborative operation model pro-
posed in our paper refers to a special kind of business
process outsourcing in which brand owners outsource a part
of their online business to professional third-party service
providers. This outsourcing led to collaborative operation of
brand owners’ direct online selling. Second, previous works
have assumed that market demand is a function of the
market price and the effort or service level of the supply
chain members. We adopt a new modeling method in which
the supply chain parties invest in online operation to reduce
selling cost. Moreover, service providers have higher effi-
ciency in investment than brand owners. This cost-reducing
model is in line with the characteristics of an e-commerce
operation.

From the perspective of an e-commerce supply chain,
this study is also closely related to the multiple-channel
management literature. First, when manufacturers sell the
same goods to retailers and directly to consumers, they need
to understand how different product, cost, or service
characteristics influence the equilibrium decisions and
channel choice behavior of such a dual-channel supply chain
[4]. For example, Bernstein et al. [15] showed that the
adoption of e-commerce operations by traditional retailers
arises from strategic necessity. Manufacturers may not
benefit from the adoption of online sales, but consumers do.
Dumrongsiri et al. [16] supposed that consumers choose the
purchase channel on the basis of price and service qualities,
and the results showed that adding a direct channel increases
the overall profit. Yan et al. [6] discussed whether and under
what conditions should the marketplace channel be intro-
duced apart from the reseller channel by measuring the
combined effects of the online spillover, the platform fee,
and brand owners’ retailing inefficiency. Different from the
previous works focusing on dual-channel pricing and
channel strategy, a few new studies have investigated the role
of the e-commerce platform [17]. In the collaborative op-
eration mode, service partners cooperate with brands owner
by using some leading platforms or a self-run platform.

Second, channel competition and coordination in a mul-
tiple-channel e-commerce supply chain are considered in
many works. For example, Ryan et al. [18] analyzed the impact
of competition in an online marketplace system and the op-
timal decisions for the retailer and the marketplace firm; the
study discussed whether the retailer should choose to sell
through the marketplace system, and, if yes, at what price.
Wang et al. [19] analyzed brand owners’ e-channel decision
problem in which brand owners select a direct-sales channel or
a third-party consignment channel to complement their
existing physical retail channel. Yoon [20] extended that the
spillover effect enhances retailers’ profit as long as the en-
croachment does not result in an extreme retail competition by
a certain degree of product differentiation and ultimately
generates Pareto gains in the supply chain. With the devel-
opment of e-commerce platforms, many studies began
investigating platform-based pricing and channel strategies. Biswas and Avittathur [21] considered a single-supplier-multiplesupplier supply chain network with a simultaneous price and inventory competition and showed that the options contract coordinates the supply chain and can eliminate channel conflict stemming from the competition. Matsui and Tsudo [22] examine the feasibility of adopting exclusive sales areas when the manufacturer uses only a single retail channel or distributes products through dual-channel supply chains.

The present study is also closely related to the literature on contract coordination in a multiechelon supply chain. Popular coordination mechanisms include cost-sharing, revenue sharing, profit-sharing, linear two-part tariffs, and quantity discount contracts. For example, Wang et al. [23] discussed a consignment contract with revenue sharing in which retailers deduct a percentage from the selling price of each item sold and remits the balance to suppliers. Fu et al. [24] developed a distributional robust Stackelberg game model to set profit-sharing terms using minimal information on demand and selling price decentralized supply chains. Gong et al. [25] compared revenue-sharing and profit-sharing models using a theoretical framework and investigated market participants’ performance under different schemes. Several studies in the literature analyzed coordination contracts by considering brand owner effort or retailer effort. Cachon [26] stated that coordination with an effort-dependent stochastic demand model is complex when the firms are not allowed to establish a direct contract on the effort level. He et al. [27] claimed that suppliers fail to coordinate the supply chain with buy-back, revenue sharing, quantity flexibility, or sales rebate contracts; various contracts were also considered to coordinate a supply chain with sales effort and price-dependent stochastic demand. They show that a properly designed returns policy with a sales rebate and a penalty contract can achieve channel coordination. Zha et al. [28] investigated the effort of a service platform in cooperation with a hotel and its influence on the hotel’s decision regarding the number of reserved rooms. Ma et al. [29] proposed two-part tariff contracts for the symmetric and asymmetric cases to maximize retailers’ profit and improve contract manufacturers’ commitment to corporate social responsibility. In the present study, the contract in the collaborative operation model is different from the above literature. Following business practice, we assume that brand owners and service providers share the selling profit in the e-commerce direct selling, but service providers still burden all selling investment cost in the online channel. Moreover, we consider this contract both in single-channel and dual-channel scenarios.

### 3. Single e-Commerce Channel Scenario

Pure e-commerce brand owners are those whose products are all sold only in online channels and not in offline brick-and-mortar stores. The most common pure online brands are those that rely on the rapidly developing Internet and are developed through online sales channels such as Taobao. In this section, we consider the single e-commerce channel scenario when brand owners produce a product that they may directly online sell to customers by using their online retail sector or selling products with a collaborative operation mode.

#### 3.1. Model Description

In the single e-commerce channel scenario as shown in Figure 1, the final retail price $P$ is assumed to be an inverse demand function $P = a - q$, where $q$ is the quantity in the channel and $a$ denotes the basic market demand. Without loss of generality, we normalize brand owners’ unit production cost to zero. In the online channel, a basic unit online selling cost exists, which is denoted by $m$. To ensure that the results in the following analysis are meaningful, we suppose $m < a/2$. In a real business, online selling cost includes many aspects. First, information analysis cost exists. The positioning of consumer groups is inseparable from the support of relevant data, the industry positioning, and brand positioning of the e-store. Quarterly and annual plans are made for the online store. Second, technical cost exists, which includes website maintenance or assistance costs for enterprises to enter the platform mall and become professional consultants to communicate with the platform. Third, designing cost exists, which refers to the arrangement of the goods and the packaging design of each product, such as the setting of commodity details on e-commerce platforms. Fourth, product promotion cost exists, which includes the use of online publicity or appropriate promotional means. In line with practice, we assume that the basic online selling cost under the autonomous mode and under collaborative mode are the same.

Moreover, brand owners or service providers can invest in online selling to reduce the selling cost. Similar settings are not uncommon in the relevant literature. Yoon [20] explained encroachment behavior may boost the manufacturer to make a cost-reducing investment about production process improvements and technology innovation. Li and Zhou [30] studied a monopoly upstream manufacturer invests in cost improvement to reduce the initial production costs. Following the setup of Yoon [20]; we denote the investment level as $I$, where $I \in (0, 1)$ and the unit selling cost becomes $m(1 - I)$, which means that a higher investment level leads to lower unit selling cost.

#### 3.2. Benchmark Model: Autonomous Operation Mode

The autonomous operation mode is a benchmark mode, where brand owners sell a product through their online retail sector. In line with the reality, the marginal cost on the investment is increasing with the investment level. We suppose that brand owners’ selling investment cost is $\gamma_b(ml)^{\frac{3}{2}}$, where $\gamma_b$ is the selling investment efficiency indicator in the autonomous operation mode [31, 32]. Higher value of $\gamma_b$ means lower selling investment efficiency. We use superscription ‘as’ to denote the autonomous operation mode in a single e-commerce channel. All the basic notations are listed in Table 1.

In the autonomous operation mode, brand owners first decide the investment level ($I$) and then determine the quantity ($q$) by maximizing profit function $\Pi^a_{ml} = (a - q - m(1 - I))q - \gamma_b(ml)^{\frac{3}{2}}$. After solving the
first-order condition, we obtain the optimal decision \( q^{as} = a - m(1 - I)/2 \). Given the optimal quantity, brand owners determine the optimal investment level \( I \) to maximize their profit \( \Pi^a_B = (a - q^{as} - (c + a(m - 1)))q^{as} - y_b \). The optimal investment level is \( I^{as} = (a - m)/2(2y_b - 1) \). To eliminate the trivial cases \( I^{as} = 1 \), we suppose \( y_b > a/2m \). Correspondingly, the optimal quantity is \( q^{as} = y_b(a - m)/2y_b - 1 \), and the brand owners and supply chains’ maximum profit is \( \Pi^a_B = \Pi^c_B = y_b(a - m)^2/2(2y_b - 1) \).

### 3.3. When to Choose the Collaborative Operation Mode in Direct Online Selling

We now consider the collaborative operation mode in direct online selling in which brand owners sell products through online channels in cooperation with e-commerce service providers. We use the superscription “cs” denotes the case with the collaborative operation mode in a single e-commerce channel. The investment level \( I \) is then decided by service providers, and the investment cost of service providers is \( y_s(mI)^2/2 \), where \( y_s \) is the selling investment efficiency in the collaborative operation mode. Higher value of \( y_s \) means lower selling investment efficiency of the service provider. From the professional capability of online operation, we suppose that service providers have high investment efficiency (i.e., \( y_s < y_b \)). The notations for all thresholds are shown in Table 2.

According to actual practice and some research [25], service providers and brand owners share the total selling profit in the collaborative operation mode. We denote \( R \) as the selling profit sharing ratio of brand owners, and \( R = 1 - R \) is the sharing ratio of service providers. The ratio \( R \) is negotiated between brand owners and service providers before selling products. In line with the real practice, we suppose that brand owners’ share is larger than 1/2 (i.e., \( R > 1/2 \)). However, service providers must burden all the investment cost to reduce the unit selling cost in the supply chain.

Thus, in the collaborative operation mode with selling profit sharing strategy, the brand owners’ profit function is \( \Pi^a_B = R(a - q - m(1 - I))/2 \). We also solve the first-order condition and obtain the optimal decision \( q^{cs} = a - m(1 - I)/2 \). Service providers determine the optimal investment level to reduce selling cost by maximizing the profit function \( \Pi^c_S = R(a - q^{cs} - m(1 - I))q^{cs} - y_s(mI)^2/2 \). The optimal investment level is \( I^{cs} = R(a - m)/m(2y_s - R) \). To eliminate the trivial cases \( I^{cs} = 1 \), we suppose \( y_s > aR/2m \), which equals to \( R > R^{cs}_{low} \), where \( R^{cs}_{low} = 1 - 2my_s/a \). Correspondingly, the optimal quantity is \( q^{cs} = y_s(a - m)/2y_s - R \), the profit of brand owners and service providers are given as

\[
\Pi^a_B = \frac{y^2_s(a - m)^2R}{(2y_s - R)^2}, \quad \Pi^c_S = \frac{y_s(a - m)^2R}{2(2y_s - R)},
\]

and the supply chain’s profit is
Proposition 1. In the single e-commerce channel with collaborative operation mode, there are

\[ \Pi_{cs} = \frac{\gamma_s (2\gamma_s - R)}{2(2\gamma_s - R)} \left( a - m \right)^2. \]  

(2)

Proposition 2. In the single e-commerce channel with collaborative operation mode, there are \( d\Pi_{cs}/d\gamma_s < 0, d\Pi_{cs}/dR < 0, d\Pi_{cs}/d\gamma_s < 0, d\Pi_{cs}/d\gamma_s < 0; \)

(1) \( d\Pi_{cs}/d\gamma_s < 0, d\Pi_{cs}/dR < 0, d\Pi_{cs}/d\gamma_s < 0, d\Pi_{cs}/d\gamma_s < 0; \)

(2) \( d\Pi_{cs}/d\gamma_s < 0, d\Pi_{cs}/dR < 0, d\Pi_{cs}/d\gamma_s < 0, d\Pi_{cs}/d\gamma_s < 0; \)

(3) \( d\Pi_{cs}/d\gamma_s < 0, d\Pi_{cs}/dR < 0, d\Pi_{cs}/d\gamma_s < 0, d\Pi_{cs}/d\gamma_s < 0. \)

\( \gamma_s \) reflects service providers’ investment efficiency. A lower \( \gamma_s \) value means higher efficiency. Proposition 1 (1) confirms that the high efficiency of service providers leads to further investment in selling, additional order quantity, and reduced sale price. Correspondingly, service providers and the supply chain benefit from the high efficiency of service providers. Proposition 1 (2) shows that, when service providers gain further sharing in the total selling profit (i.e., \( R \) becomes smaller), they have an increased incentive to invest further on selling cost reduction. As such, brand owners set additional quantity in the channel, which leads to a reduced retail price. Thus, service providers and the whole supply chain obtain extra profit with the high profit sharing of service providers. As the unit selling cost \( m \) rises, as shown in Proposition 1 (3), the investment level, quantity, profit of service providers, and total profit become small.

Proposition 3. In the single e-commerce channel with collaborative operation mode, there are:

(1) \( \text{when} R/2m < \gamma_s \leq R \gamma_b, \text{there is}, \Pi_{cs} \geq \Pi_{cs}^{\alpha}, q_{cs} \geq q_{cs}^{\alpha}; \text{otherwise, there is} \Pi_{cs}^{1} \leq \Pi_{cs}^{\alpha}, q_{cs}^{1} \geq q_{cs}^{\alpha}; \)

(2) \( \text{when} R/2m < \gamma_s \leq R \gamma_b, \text{there is} \Pi_{cs} \geq \Pi_{cs}^{\alpha}; \text{otherwise, there is} \Pi_{cs}^{1} < \Pi_{cs}^{\alpha}; \)

(3) \( \text{for any} a R/2m < \gamma_s \leq R \gamma_b, \text{there is} \Pi_{cs}^{1} > 0. \)

Proposition 3 (1) shows that, if service providers’ investment efficiency is higher than that of brand owners’ (i.e., \( a R/2m \leq \gamma_s \leq R \gamma_b \)), then service providers will invest more than brand owners to reduce selling cost, and brand owners will set higher order quantity in the collaborative operation mode; otherwise, service providers will invest less and brand owners will set a lower quantity. This result comes from the monotonicity of \( I_{cs}^{\alpha} \) on \( \gamma_s \). Proposition 3 (2) indicates that, if service providers’ investment is effective, then, the total profit in the collaborative operation mode is more than that in the autonomous operation mode. Otherwise, the total profit in the collaborative operation mode is less. This result comes from the monotonicity of \( \Pi_{cs}^{1} \) on \( \gamma_s \). Proposition 3 (3) shows that service providers will consistently obtain positive profit when brand owners use the collaborative operation mode.

Proposition 3 shows that, when the investment efficiency of service providers is high, they will be profitable, and the profit of the whole supply chain will be higher than that when brand owners adopt autonomous operation in direct online selling. Thus, the Tmall platform, which is engaged in e-commerce business under Alibaba, will spare no effort to cultivate high-quality service providers, such as actively helping e-commerce service providers to improve their service capabilities through training, professional counseling, selection of gold partners, and organizing experience sharing meetings.

Proposition 4.

(1) Consider the scenario \( a/2m < \gamma_s < a^2/2m^2 \).

(a) Case \( \gamma_s \geq \gamma_b^{\alpha} \). When \( R \in (R_{low}^{\gamma_b^{\alpha}}, R_{low}^{\gamma_s}), \Pi_{cs} \geq \Pi_{cs}^{\alpha}; \) when \( R \in (R^{\gamma_s}, 1), \Pi_{cs} < \Pi_{cs}^{\alpha}. \)
for brand owners.

operation is a better choice than the autonomous operation mode. Figure 2 shows that the threshold collaborative operation mode, and Region B represents when brand owners should choose the collaborative operation mode. On the one hand, Proposition 4 states when the investment efficiency of service providers is relatively effective, if brand owners' investment efficiency is relatively low; or when the service providers' efficiency is relatively effective, but the sharing

benefit more when they choose the collaborative operation mode. Moreover, if a relatively high profit sharing is allo-

(2) Consider the scenario $\gamma_b \geq a^2/2m^2$.

(a) Case $\gamma_s \leq \bar{\gamma}^{cs}$. When $R \in (R^{cs}_{low}, \bar{R}^{cs})$, $\Pi^c_B \geq \Pi^a_B$; when $R \in (\bar{R}^{cs}, 1)$, $\Pi^c_B < \Pi^a_B$.

(b) Case $\bar{\gamma}^{cs} < \gamma_s < \gamma_b^{cs}$. When $R \in (R^{cs}_{low}, \bar{R}^{cs})$, $\Pi^c_B < \Pi^a_B$; when $R \in (\bar{R}^{cs}, 1)$, $\Pi^c_B < \Pi^a_B$.

(c) Case $\gamma_b^{cs} \leq \gamma_s < \gamma_b$. For any $R \in (R^{cs}_{low}, 1)$, $\Pi^c_B < \Pi^a_B$.

The results in Proposition 2 illustrate the changing rules of $\Pi^c_B$ on $R$ in three kinds of cases of $\gamma_s$. Moreover, $\Pi^c_B$ depends on $\gamma_b$. Thus, the investment efficiency of brand owners $\gamma_b$, the efficiency of service providers $\gamma_s$, and the sharing ratio $R$ influence brand owners’ preference of online operation. On the one hand, Proposition 4 states when brand owners should choose the collaborative operation mode in direct online selling. First, no matter how inefficient brand owners’ investment is, as long as their service providers’ investment efficiency is relatively effective, if brand owners use relatively small sharing $R$, then brand owners can benefit from adopting the collaborative operation mode. Second, when the investment efficiency of service providers is medium, brand owners’ efficiency is low, and the sharing $R$ is in the middle, the brand owners should choose the collaborative operation mode. On the other hand, Proposition 4 states that brand owners should choose to operate by themselves when their service providers’ investment efficiency is relatively low; or when the service providers’ efficiency is relatively effective, but the sharing $R$ is high; or when the service providers’ efficiency is medium, the brand owners’ efficiency is low, and the sharing $R$ is large or small. According to the above analysis, when the investment efficiency of service providers is low, brand owners should not use the collaborative operation mode in all cases. If service providers’ investment efficiency is relatively high and the suitable profit distribution ratio $R$ can be reached with service providers through negotiation, the collaborative operation is a better choice than the autonomous operation for brand owners.

We consider two scenarios in Figure 2 (where $a = 100$, $m = 20$) for a detailed illustration of Proposition 4. We analyze the strategic choice of brand owners from the perspective of investment efficiencies and sharing ratio. Region A represents when brand owners should choose the collaborative operation mode, and Region B represents when brand owners should choose the autonomous operation mode. Figure 2 shows that the threshold $\bar{R}^{cs}$ decreases on $\gamma_s$. Thus, when service providers are relatively efficient, brand owners can require the relatively high sharing of the total profit in the collaborative operation mode, whereas service providers are relatively inefficient and only require relatively low sharing. Therefore, for brand owners, selecting high-quality service providers is a key problem that they must pay attention to, and determining whether potential service providers meet their requirements through careful observation and multiple negotiations is necessary. For example, Midea Company, a well-known home appliance manufacturer in China, had many meetings with its senior managers to understand the operational efficiency and capability of Shanghai e-commerce before making it the company’s service provider (https://www.cmcc-dut.cn/).

Proposition 5.

(1) Consider the scenario $R \leq (a - m^2/a^2)$, $\Pi^c_B < \Pi^a_B$.

(2) Consider the scenario $(a - m^2/a^2 < R < a - m/a)$.

(a) Case $a/2m < \gamma_b \leq \bar{\gamma}^{cs}$. For any $\gamma_s \in (aR/2m, \gamma_b)$, $\Pi^c_B \leq \Pi^a_B$.

(b) Case $\gamma_b > \bar{\gamma}^{cs}$. When $\gamma_s \in (aR/2m, \bar{\gamma}^{cs}]$, $\Pi^c_B \geq \Pi^a_B$; when $\gamma_s \in (\bar{\gamma}^{cs}, \gamma_b)$, $\Pi^c_B < \Pi^a_B$.

(3) Consider the scenario $a - m/a < R < 1$. When $\gamma_s \in (aR/2m, \gamma_b)$, $\Pi^c_B > \Pi^a_B$; when $\gamma_s \in (\gamma_b, \gamma_b^{cs})$, $\Pi^c_B < \Pi^a_B$.

Proposition 5 also analyzes the problem of when brand owners should choose the collaborative operation mode. In contrast to Proposition 4, we consider three scenarios of the sharing ratio $R$. Brand owners should choose collaborative operation in two cases: (1) the sharing $R$ is in the middle, brand owners’ investment is inefficient, and service providers’ investment is efficient; (2) the sharing $R$ is large, and service providers’ investment is efficient. Brand owners should choose to operate by themselves in four cases: (1) the sharing $R$ is small; (2) the sharing $R$ is large, and the investment efficiency of service providers is relatively low; (3) the sharing $R$ is middle, and the efficiency of brand owners is high; (4) the sharing $R$ is middle, and the efficiencies of brand owners and service providers are all relatively low.

The above discussion shows that the choice of autonomous operation or collaborative operation is clear in most situations. However, in the following two cases, the choice of operation mode by brand owners depends on the investment efficiency of service providers. First, when a medium profit sharing is allocated by brand owners to service providers and the investment efficiency of brand owners is low, if the efficiency of service providers is high then brand owners will benefit more when they choose the collaborative operation mode, but if the efficiency of service providers is low then brand owners should choose the autonomous operation mode. Moreover, if a relatively high profit sharing is allocated by brand owners to service providers, the brand owners’ choice of operation mode is similar to the above situation. Thus, it will not be described again.

Figure 3 (where $a = 100$, $m = 20$) presents a detailed illustration of Proposition 5, where Region A stands for the region where brand owners should choose the collaborative operation, and Region B stands for the region where brand owners should operate by themselves. The $\bar{\gamma}^{cs}$ increases on $\gamma_b$, which implies that efficient brand owners must choose equally efficient service providers before choosing profit sharing.
4. Dual Channels Scenario

A growing number of traditional brand owners with offline channels are carrying out online businesses to improve their sales scale. This section considers the operation mode choice problem of brand owners in a dual-channel distribution system, where brand owners directly sell products to the end consumers through the online channel and indirectly sell
products through the offline channel. The brand owners chose the operation mode between autonomous operation mode and collaborative operation mode in the direct online selling.

4.1. Model Description and Autonomous Operation Mode. In the dual-channel scenario as shown in Figure 4, customer demand in the final market is assumed to be captured by inverse demand functions, namely \( P_R = a - q_R - q_B \) and \( P_B = a - q_B - q_R \), where \( P_R \) and \( P_B \) denote the final prices, and \( q_B \) and \( q_R \) are the quantities in the offline and online channels, respectively. We also assume that the unit production cost of brand owners is zero. Following Berger et al. [33], we assume that brand owners and retailers negotiate the wholesale price \( w \) in advance; that is, the wholesale price is exogenous [34–36]. To ensure that the results in the following are meaningful, we suppose \( 2m - 4m - w > 0, a - 4m + w > 0 \).

We now consider the scenario when brand owners chose the autonomous operation mode in the direct channel. Brand owners first decide the investment. Then they decide the online quantity, and retailers simultaneously decide the offline order quantity. We use the superscript ‘a’ to denote the autonomous operation mode in a direct channel. We obtain the profit functions of the offline retailers and the brand owners as

\[
\Pi_R^a = \left( a - q_B - q_R - w \right) q_R, \tag{3}
\]

\[
\Pi_B^a = \left[ a - q_B - q_R - m (1 - l) \right] q_B + w q_B - \frac{m (l)^2}{2}. \tag{4}
\]

Solving the first-order conditions of (3) and (4), we obtain the following profit functions:

\[
\Pi_R^a = \left( a - q_B - q_R - m (1 - l) \right) q_B + w q_B - \frac{m (l)^2}{2} \tag{5}
\]

\[
\Pi_B^a = \left( a - q_B - q_R - w \right) q_R. \tag{6}
\]

4.2. When to Choose the Collaborative Operation Mode in the Direct Online Selling. When brand owners cooperate their e-commerce operation with service providers, service providers decide the investment level. Similar to the previous section, we consider the selling profit-sharing scheme. Thus, the profit function of brand owners and offline retailers are as follows:

\[
\Pi_R^c = \Pi_B^c = \left( a - q_B - q_R - w \right) q_R, \tag{7}
\]

where the ‘c’ denotes the scenario when the brand owner chose collaborative operation mode in the direct online selling channel. After solving the first-order conditions in (6) and (7), the optimal order quantities in the collaborative operation mode have the same expression with that in the autonomous operation mode (i.e., \( q_B^a = q_B^c, q_R^a = q_R^c \)). Then, service providers determine the investment level by maximizing the profit function:

\[
\Pi_S^c = \Pi_B^c = \left( a - q_B - q_R - m (1 - l) \right) q_B + w q_B - \frac{m (l)^2}{2}. \tag{8}
\]

The optimal investment level is

\[
\Pi_B^c = 4R(a - 2m + w) m (9y_B - 8). \tag{9}
\]

To eliminate the trivial cases \( \Pi_B^c = 1 \), we suppose \( y_B > 4(a + w)/9m \). The optimal order quantities are:

\[
q_B^c = 3y_B(a + m - 2w) - 4R(a - w)/9y_B - 8R, \tag{10}
\]

\[
q_R^c = 3y_B(a + m + w)/9y_B - 8R. \tag{11}
\]

The profit of three players is
Proposition 6. In the dual-channel scenario where the brand owner chose collaborative operation mode in the direct online selling channel, there are:

\[ \Pi_R^d = \frac{3y_s(a + m - 2w) - 4R(a - w)}{9y_s - 8R} \]
\[ \Pi_B^d = \frac{9[R(a - 2m + w)^2 + 3w(a + m - 2w)]y_s^2 - 12Rw(5a + 2m - 7w)y_s + 32R^2(a - w)w}{(9y_s - 8R)^2} \]
\[ \Pi_s^d = \frac{(a - 2m + w)^2Ry_s}{9y_s - 8R} \]

Proposition 7. In the dual-channel scenario where the brand owner chose collaborative operation mode in the direct online selling channel, there are, \( \frac{d\Pi_R^d}{dy_s} < 0 \), \( \frac{d\Pi_B^d}{dy_s} < 0 \), \( \frac{d\Pi_s^d}{dy_s} < 0 \), \( \frac{d\Pi_R^d}{dR} > 0 \), \( \frac{d\Pi_B^d}{dR} > 0 \), \( \frac{d\Pi_s^d}{dR} > 0 \).

(1) When \( y_s \leq \bar{y}^d \), for any \( R \in (R^d_{low}, 1) \), \( \frac{d\Pi_B^d}{dR} > 0 \) when \( R \in (R^d_{low}, R^d) \), \( \frac{d\Pi_B^d}{dR} > 0 \) when \( R \in (R^d, 1) \), \( \frac{d\Pi_B^d}{dR} < 0 \);\n
(2) When \( \bar{y}^d < y_s < \bar{y}^d \), when \( R \in (R^d_{low}, R^d) \), \( \frac{d\Pi_B^d}{dR} > 0 \); \n
(3) When \( \bar{y}^d \leq y_s < y_{bo} \), for any \( R \in (R^d_{low}, 1) \), \( \frac{d\Pi_B^d}{dR} > 0 \).

Proposition 8 in dual-channel scenario, comparing the collaborative operation with the autonomous operation mode in the direct online selling channel, when \( 4(a + w)R/9m < y_s \leq \bar{y}^d \), there are \( \Pi_R^d \geq \Pi_B^d, q_R^d \leq q_B^d, \Pi_R^d \leq \Pi_B^d, q_B^d \geq \bar{q}^d \); otherwise, there are \( \Pi_R^d < \Pi_B^d, q_R^d > q_B^d, \Pi_R^d > \Pi_B^d, q_B^d < \bar{q}^d \).

Proposition 8 indicates that, if the investment efficiency of service providers is pronounced, the investment level of service providers will be higher than that of brand owners. The online order quantity of the collaborative operation with profit sharing will be more than the autonomous operation scenario. Proposition 8 shows that, if the investment efficiency of the service providers is only slightly higher than the brand owners (i.e., \( \bar{y}^d < y_s < y_{bo} \)), the offline retailers’ profit in the collaborative operation mode will be more than that in the autonomous operation mode. This claim is different from the results in Yoon [20], where retailers will not benefit from effective investment efficiency. The main reason is that investment only reduces the online marketing cost and makes brand owners more efficient in the retail market. These rules come from the nonmonotonicity of \( \Pi_B^d, q_B^d, \bar{q}^d, \Pi_R^d \) on \( y_s \), which is shown in Proposition 6.

In Figure 5 (where \( a = 100, m = 20, w = 40 \)), with a certain numerical study, we investigate under what circumstances brand owners will adopt the collaborative operation mode and operate by themselves. The threshold \( \bar{R}^d \) is the point \( R \) satisfying \( \Pi_B^d = \Pi_R^d \). Brand owners should choose the collaborative operation mode in Region A and choose the autonomous operation mode in Region B. Figure 5 shows that the threshold \( \bar{R}^d \) decreases on \( y_s \).
Thus, brand owners can require high sharing once service providers have high investment efficiency.

Surprisingly, the area of Region A in Figure 5 is quite larger than that in Figure 2, which means that, in the dual-channel situation, brand owners have more motivation to adopt collaborative operation instead of autonomous operation. The reason is that, under the dual-channel situation, online business is only a part of the revenue source of brand owners, and offline channels with a relatively stable business share the operating pressure of brand owners to a certain extent. Thus, brand owners are daring to outsource online business to service providers and set a high-profit
distribution ratio to enhance their sales profit. In contrast, under the single-channel situation, brand owners can only sell their products through online channels. Given that the investment efficiency of service providers are the same, great business pressure makes brand owners choose the autonomous operation instead of the collaborative operation, and the profit distribution ratio is lower than that of the dual-channel situation.

**Proposition 9.** In dual-channel scenario, comparing the collaborative operation with the autonomous operation mode, when \( R^c_d < R \leq R^d_d \), there are \( f^c_d \leq f^d_d \), \( q^c_R \leq q^d_R \), \( \Pi^c_R \leq \Pi^d_R \), \( q^b_R \geq q^d_R \); otherwise, there are \( f^c_d > f^d_d \), \( q^c_R > q^d_R \), \( \Pi^c_R > \Pi^d_R \), \( q^b_R < q^d_R \).

Proposition 9 explains the change in the investment level and order quantity on the sharing \( R \). When \( R \) is small, service providers obtain a high sharing of the total profit. Moreover, the investment level of service providers and the online order quantity are greater than that of the autonomous operation scenario, and the offline order quantity and the profit of retailers are less than the autonomous operation mode. These rules come from the monotonicity of \( f^c_d \) and \( q^d_R \), \( q^b_R \) on \( R \). This is shown in Proposition 6.

Figure 6 (where \( a = 100, m = 20, \) and \( w = 40 \)) discusses the choice of the autonomous operation or the collaborative operation of brand owners under two different sharing \( R \), where Region A stands for the region where brand owners choose the collaborative operation, and Region B stands for the region where brand owners should operate by themselves. The threshold \( \gamma^c_d \) is the point \( \gamma^a_s \) satisfying \( \Pi^c_B = \Pi^a_B \). The \( \gamma^d \) increases on \( \gamma^a_s \), which means that inefficient brand owners can choose a slightly inefficient service than efficient brand owners.

Comparing Figure 2 with Figure 6, brand owners are more inclined to choose the collaborative operation and accept service providers with low investment efficiency under the dual-channel situation. The reason is that, under the dual-channel situation, offline channels provide some stable profit for brand owners, and brand owners must put some investment to maintain online channels. Thus, even if the efficiency of service providers is relatively low, brand owners are willing to cooperate with them as long as they can bring profit.

**5. Conclusion**

In today’s e-commerce age, brand owners turn to cooperate with professional e-commerce service providers to operate their online business because they have professional technical advantages and data analysis and customer service capabilities. This study analyzes whether brand owners should use the collaborative or autonomous operation mode when they want to sell products through online channels with a selling cost. In the autonomous operation mode, brand owners decide an investment level to reduce selling cost and bear the investment cost. In the collaborative operation mode, brand owners share a certain amount of the selling profit with service providers, and service providers decide an investment level and bear the investment cost. In this study, we investigate two channel scenarios, namely the single e-commerce channel scenario, where brand owners only sell online products, and the dual channels scenario, where brand owners simultaneously carry out their offline and online business.

Our analysis suggests that collaborative operation in online selling is an efficient format, but, whether brand owners should use it depends on the selling profit sharing ratio between brand owners and service providers and the investment efficiency of brand owners and service providers. Specifically, brand owners should choose collaborative operation under two circumstances: first, when the investment efficiency of service providers is much higher than that of brand owners and the sharing ratio of service providers is relatively high; second, when the investment efficiency of service providers is medium and brand owners’ efficiency is low, brand owners and service providers achieve a medium selling profit sharing ratio. Based on the analysis, in the collaborative operation mode, brand owners are willing to transfer a large part of the selling profit ratio to high-efficiency service providers to encourage them to strive to expand the market scale and increase the profit of brand owners and even the whole supply chain. This phenomenon occurs in many famous e-commerce platforms. For example, Tmall exerts all energy to cultivate high-level service providers. The service providers in Tmall always have a high profit-sharing ratio, and their high-efficiency support brand owners to win the market.

Moreover, when the investment effect on product price increases, the difference of products will increase, and consumers will be more willing to pay for the premium of such difference. Therefore, creating differentiated products is a feasible solution for brand owners to increase profit. Furthermore, compared with brand owners who have only one online channel, dual-channel brand owners have a greater incentive to use the collaborative operation than the autonomous operation. The reason is that great commercial pressure forces the former to use autonomous operation, whereas the latter with stable offline channels have more motivation to cooperate with service providers and even accept some service providers with slightly lower investment efficiency.

We present possible directions for future research. First, in the current mode, we assume that the products are homogeneous between online and offline channels. Future research can consider heterogeneous scenarios. Second, in the current mode, the sharing ratio of service providers and brand owners is exogenous. In practice, this ratio is negotiated in the supply chain. Thus, the decision on the sharing ratio is worth considering [37–42].

**Appendix**

**Proof of Proposition 1.** From the optimal decisions and profits in the profit sharing schemes, we know: (i) there are \( \text{d}f^c_s/\text{d}\gamma_s = -2R(a-m)/(2\gamma_s - R)^2 < 0 \), \( \text{d}q^c_R/\text{d}\gamma_s = -(a-m)R/2\gamma_s - R < 0 \), \( \text{d}\Pi^c_R/\text{d}\gamma_s = -(a-m)^2R/2(2\gamma_s - R) < 0 \),
Proof of Proposition 3

(i) The results are obvious, we omit the proof.

(ii) From \( \Pi_{\text{sc}} - \Pi_{\text{sc}} = -(a - m)^2 f(\gamma) / 2 \)
where \( f(\gamma) = 2\gamma R + R(2\gamma R - R - 4\gamma y) + R^2 y \). The quadratic function \( f(\gamma) \) has two roots, \( \gamma_0^c = R[2(1 + R)\gamma y + R - \sqrt{(2\gamma y - 1)[2\gamma y(1 + R^2 - R^3)]}/4, \gamma_0^c = R[2(1 + R)\gamma y + R + \sqrt{(2\gamma y - 1)[2\gamma y(1 + R^2 - R^3)]}] / 4 \). Furthermore, we know \( f(0) = R^2 \gamma y > 0 \), \( f(aR/2m) = -R^2 \left[(a - m)(2\gamma y - a) + amR(2\gamma y - 1)/2m^2\right] < 0 \), \( f(y_b) = 2\gamma y R^2 > 0 \). Thus, the first root \( \gamma_0^c \) of function \( f(\gamma) \) belongs in \((0, aR/2m)\), and the second root \( \gamma_0^b \) belongs in \((aR/2m, y_b)\). Thus, when \( aR/2m < \gamma_b \leq \gamma_0^b \), there is \( f(\gamma_b) = 0 \), and when \( \gamma_0^b \leq \gamma_b < y_b \), there is \( f(\gamma_b) > 0 \). Obviously, \( 2(2\gamma y - 1)(2\gamma y - 1) > 0 \). Then, when \( aR/2m < \gamma_b \leq \gamma_0^b \), there is \( \Pi_{\text{sc}} - \Pi_{\text{sc}} \geq 0 \), and when \( \gamma_0^b \leq \gamma_b < y_b \), there is \( \Pi_{\text{sc}} - \Pi_{\text{sc}} < 0 \).

(iii) The results are obvious, we omit the proof.

Proof of Proposition 4

From the optimal profits in single-channel channel, we have
\( \Pi_{\text{sc}} - \Pi_{\text{sc}} = (a - m)^2 f(R) / 2(2\gamma y - 1)(2\gamma y - R) \),
where \( f(R) = -\gamma y R^2 + 2(2\gamma y - 1)^2 \gamma y \). Obviously, the function \( f(R) \) has two roots: \( R^\ast = a - \beta + \gamma y \sqrt{(2\gamma y - 1)(a - 2\beta)} / \gamma y \),
\( R^\beta = a - \beta + \gamma y \sqrt{(2\gamma y - 1)(2\gamma y - 1)} / (a - 2\beta) / \gamma y \),
where \( a = (2\gamma y - 1)\beta, \beta = (2\gamma y - 1) \).

(1) First, consider the scenario \( a/2m < \gamma_b \leq a^2/2m^2 \).

We have two cases.

(a) Case \( \gamma_b \leq \gamma_1 \). There is \( \gamma_b \leq 2m(a - m) / 2am(2\gamma y - 1) \). Because \( \delta f(R) / dR = (a - m)^2 / am(2\gamma y - 1) > 0 \), from \( \gamma_b < a^2/2m^2 \), we know \( \gamma_1^b < a/a + m = \gamma_3^b \). From Proposition 1 (1), when \( \gamma_b \leq \gamma_1 \), \( \gamma_1^b < \gamma_1^c \), for any \( R \in (R_{\text{low}}^c, 1) \), there is \( \delta f(R) / dR < 0 \).

(2) Second, we consider the scenario \( \gamma_1 < \gamma_b < \gamma_2 \). With the same procedure in above analysis, from \( \gamma_2 < \gamma_b \), we know \( R_{\text{low}}^c < 2\gamma_y - 1 \). Obviously, \( 2\gamma_y - 1 < 1 \). So, the maximum point \( 2\gamma_y - 1 \) of \( \Pi_{\text{sc}} \) is in the feasible region \( (R_{\text{low}}^c, 1) \). Thus, when \( R \in (R_{\text{low}}^c, 2\gamma_y - 1) \), there is \( \delta f(R) / dR > 0 \), and when \( R = 2\gamma_y - 1 \), there is \( \delta f(R) / dR < 0 \).

(iii) \( \gamma_2 < \gamma_b < \gamma_3 \). Similar to case 2 (a) in the above, we know, when \( \gamma_2 < \gamma_b \leq \gamma_3 \), for any \( R \in (R_{\text{low}}^c, 1) \), \( \delta f(R) / dR > 0 \). Also, the function \( h(\gamma) \) decreases in \( \gamma_b \), and \( h(\gamma^c) = 0 \), \( h(\gamma^b) = h((a - m)(2m\gamma y - a) / am < 0 \). Thus, in the region \( \gamma_2 < \gamma_b \leq \gamma_3 \), we have \( h(\gamma_b) < 0 \), and there is \( \Pi_{\text{sc}}^c_{1, R_{\text{low}}^c - \Pi_{\text{sc}}^c_{1, R_{\text{low}}^c}} = h(\gamma_b) / 4(2\gamma y - 1) < 0 \). Since the function \( \Pi_{\text{sc}}^c \) decreases on \( R \), then if \( R \in (R_{\text{low}}^c, 1) \), \( \Pi_{\text{sc}}^c > \Pi_{\text{sc}}^c_{1, R_{\text{low}}^c} \). Otherwise, \( R \in (1, R_{\text{high}}^c) \), \( \Pi_{\text{sc}}^c < \Pi_{\text{sc}}^c_{1, R_{\text{high}}^c} \).

(b) Case \( \gamma_2 < \gamma_b < \gamma_3 \). We discuss it in three situations.

(i) \( \gamma_1 < \gamma_b \leq \gamma_2 \). Similar to case 1 (a) in the above, we know, when \( \gamma_1 < \gamma_b \leq \gamma_2 \), for any \( R \in (R_{\text{low}}^c, 1) \), \( \delta f(R) / dR > 0 \). Also, the function \( h(\gamma) \) decreases in \( \gamma_b \), and \( h(\gamma^c) = 0 \), \( h(\gamma^b) = h((a - m)(2m\gamma y - a) / am < 0 \). Thus, in the region \( \gamma_1 < \gamma_b \leq \gamma_2 \), we have \( h(\gamma_b) < 0 \), and there is \( \Pi_{\text{sc}}^c_{1, R_{\text{low}}^c - \Pi_{\text{sc}}^c_{1, R_{\text{low}}^c}} = h(\gamma_b) / 4(2\gamma y - 1) < 0 \). Since the function \( \Pi_{\text{sc}}^c \) decreases on \( R \), there is \( \Pi_{\text{sc}}^c < \Pi_{\text{sc}}^c_{1, R_{\text{low}}^c} \).

(ii) \( \gamma_3 < \gamma_b < 1 \). From Proposition 1 (2), under the condition \( \gamma_2 < \gamma_b < 1 \), we have: when \( R \in (R_{\text{low}}^c, 2\gamma_y - 1) \), \( \delta f(R) / dR > 0 \); when \( R \in (2\gamma_y - 1, 1) \), \( \delta f(R) / dR < 0 \). When \( R = 2\gamma_y - 1 \), the maximum profit of brand owner is \( \Pi_{\text{sc}}_{1, R_{\text{low}}^c - \Pi_{\text{sc}}_{1, R_{\text{low}}^c}} = h(\gamma_b) / 4(2\gamma y - 1) \), and we get \( \Pi_{\text{sc}}_{1, R_{\text{low}}^c - \Pi_{\text{sc}}_{1, R_{\text{low}}^c}} = (a - m)^2 / 2m^2 \), \( g(\gamma) = (2\gamma y - 1)(4\gamma y - 2\gamma y - 1) \). The function
$g(y_2)$ has two roots: \( y_{21} = x_1 = 2\sqrt{2}y_2 - \sqrt{2}y_2/2y_2 = 1 < 1 \), \( x_2 = 2\sqrt{2}y_2 + \sqrt{2}y_2/2y_2 = 1 > 1 \), it is clear that the root \( x_2 \) is not in the region \( (y_2^*, 1) \). From \( a/2m < y_2 < a^2/2m^2 \), there is \( (m^2/2y_2 - a)/(2y_2 - 1) < 0 \), which equals to \( (2y_2 - 1)m < (2\sqrt{y_2} - 1) \), and \( 2\sqrt{y_2} - 1/2y_2 < 0 \), that is \( 1 - 2\sqrt{y_2} < 1 < 1 < m/a + m \). So, we get \( y_{21} < y_{22}^* \), it means the root \( y_{22}^* \) out of the considering region \( (y_2^*, 1) \), which equals to \( g(y_2) < 0 \), for any \( y_2 < y_{21} < 1 \), \( \Pi_y^{\alpha} \mid_{R=2y_2-1} - \Pi_y^{\alpha} < 0 \), \( \Pi_y^{\alpha} < \Pi_y^{\alpha} \).

(2) Second, we consider the scenario \( y_b \sim a^2/2m^2 \). We have three cases.

(a) Case \( y_2 \leq y_{22}^* \). From Proposition 1 (1), when \( y_2 \leq y_{22}^* \), there is \( y_2 \leq a + m 

(b) Case \( y_{22}^* < y_2 < y_{21} \). From the proof of Proposition 2 (b) (ii), in the above, the threshold \( y_{22}^* = 2\sqrt{2}y_2 - \sqrt{2}y_2/2y_2 = 1 < 1 \), from Proposition 1 (2), in the region \( y_2 < y_{22}^* \), we have when \( R \in (R_{y_2^*}, \sqrt{y_2} \), \( \Pi_y^{\alpha} \mid_{R=2y_2-1} - \Pi_y^{\alpha} < 0 \), \( \Pi_y^{\alpha} < \Pi_y^{\alpha} \), otherwise, \( \Pi_y^{\alpha} \in (\sqrt{y_2}, \Pi_y^{\alpha} \), \( \Pi_y^{\alpha} < \Pi_y^{\alpha} \).

(c) Case \( y_{21} < y_2 \). We discuss it in two situations:

(i) \( y_{21} < y_2 < 1 \). Similar 1(b) (ii) and 2(b) in this proof, because the roots of \( g(y_2) \) have \( y_{21} < 1 < x_2 \), we know when \( y_{21} < y_2 < 1 \), \( g(y_2) < 0 \). There is \( \Pi_y^{\alpha} \mid_{R=2y_2-1} - \Pi_y^{\alpha} < 0 \), \( \Pi_y^{\alpha} < \Pi_y^{\alpha} \).

(ii) \( 1 < y_2 \leq y_b \). From Proposition 1 (3), there is \( \Pi_y^{\alpha} \mid_{R=1} - \Pi_y^{\alpha} < 0 \). We know \( \Pi_y^{\alpha} \mid_{R=1} - \Pi_y^{\alpha} = -(a - m)^2/4(2y_2 - 1) = -4(\sqrt{y_2} - \sqrt{y_2^*})^2 < 0 \). Since \( f(y_2) = -2(2\sqrt{2}y_2 + R)^2 + 4y_2\sqrt{2}y_2 - R^2y_2 \), we know the function \( f(y_2) \) has one zero point: \( y_2^* = R^2/2(2y_2 + \sqrt{2}y_2(2y_2 - 1)^2/2y_2 + R) \). When \( y_2 < y_2^* \), there is \( \Pi_y^{\alpha} \mid_{y_2} - \Pi_y^{\alpha} = -(a - m)^2/4(2y_2 - 1) \), \( y_2^* \in (\sqrt{y_2^*}, R) \), \( h(y_2) = 4\sqrt{y_2} + (6R - 4)y_2 + R^2 \). The discriminant of \( h(y_2) \) is \( \Delta = 4R^2(4R^2 - 3) \). When \( R < 3/4, \Delta < 0 \), there is \( h(y_2) > 0 \). When \( R <= 3/4 \), the symmetry axis of \( h(y_2) \): \( x = 4 - 6R/8R < 0 \), we have \( h(y_2) > 0 \). So, we have \( \Pi_y^{\alpha} \mid_{y_2} - \Pi_y^{\alpha} > 0 \). When \( y_2 = aR^2/2m \), there is \( \Pi_y^{\alpha} \mid_{y_2} - \Pi_y^{\alpha} = 2[aR^2 - (a^2 - m^2)^2]/2R^2(2y_2 - 1) \), and the root is \( y_{21} = aR^2/2[aR^2 - (a^2 - m^2)^2] \).

(i) First, we consider the scenario \( R \leq (a - m^2)/a^2 \), there is \( aR^2 - (a - m^2)^2 \leq 0 \). So, we have \( \Pi_y^{\alpha} \mid_{y_2} - \Pi_y^{\alpha} \leq 0 \). Since \( \Pi_y^{\alpha} \) decreases on \( y_2 \), then for any feasible \( y_2 \), in \( (aR^2/2m, y_2) \), there is \( \Pi_y^{\alpha} \leq \Pi_y^{\alpha} \).

(2) Second, we consider the scenario \( (a - m)^2/2 < R < a/m \). First, when \( R > (a - m)^2/2 \), there is \( aR^2 - (a - m)^2 > 0 \), which means the function \( 2[aR^2 - (a - m)^2]y_2 - a^2R \) increases on \( y_2 

(3) Finally, we consider the scenario \( a - m/a < R < 1 \), there is \( aR^2 < 2R \). We know that \( y_2^* \) out of the feasible area of \( y_2 \), and the function \( 2[aR^2 - (a - m)^2]y_2 - a^2R \) increases on \( y_2 

Proof of Proposition 6. From the optimal decisions and profits in the dual channel with the profit sharing schemes, we know,

\[
\text{(i) } \Pi_y^{\alpha} \mid_{y_2} - \Pi_y^{\alpha} = -36R(\alpha - 2m + w)/(9\gamma_2 - 8R^2) < 0 \quad \text{and} \quad \Pi_y^{\alpha} \mid_{y_2} - \Pi_y^{\alpha} = -24R(\alpha - 2m + w)/(9\gamma_2 - 8R^2) < 0 \quad \text{and} \quad \Pi_y^{\alpha} \mid_{y_2} - \Pi_y^{\alpha} = 12R(\alpha - 2m + w)/(9\gamma_2 - 8R^2) > 0. \quad \text{Obviou}\n
\text{(ii) Similarly, there are } \Pi_y^{\alpha} \mid_{y_2} - \Pi_y^{\alpha} = -36(\alpha - 2m + w)\gamma_2/(9\gamma_2 - 8R^2) < 0 \quad \text{and} \quad \Pi_y^{\alpha} \mid_{y_2} - \Pi_y^{\alpha} = -24(\alpha - 2m + w)\gamma_2/(9\gamma_2 - 8R^2) < 0 \quad \text{and} \quad \Pi_y^{\alpha} \mid_{y_2} - \Pi_y^{\alpha} = 12(\alpha - 2m + w)\gamma_2/(9\gamma_2 - 8R^2) > 0. \quad \text{Obviou}\n
\text{Proof of Proposition 7. From the optimal decisions and profits in the dual channel with the profit sharing schemes, we know, } \Pi_y^{\alpha} \mid_{y_2} - \Pi_y^{\alpha} = -12R(\alpha - 2m + w)/(12R(\alpha - 2m + w) - 9\gamma_2) + 8R/(9\gamma_2 - 8R^2). \quad \text{With the} \n
condition $R > 1/2$, we have $12R(2 - m + w) - 9w > 3(2a - 4m - w) - 9w > 3(2a - 4m - w) > 0$, there is $dI_{B}^d/dy_{B} > 0$.

Now, we consider the property of $I_{B}^d$ on $R$. We know $dI_{B}^d/dR = 3\gamma_{y}(2 - m + w)f(R)/(9\gamma_{y} - 8R)$, where $f(R) = 8(4a - 3y_{y}(2 - m + w))R + 3\gamma_{y}(2 - m + w) + 4u(9\gamma_{y} - 8)$, obviously, when $R = R = [3\gamma_{y}(2 - m + w) + 4u(9\gamma_{y} - 8)]$, $f(R) = 0$. Furthermore, $f(0) = 0\gamma_{y} = 8, f(1) = 3\gamma_{y}[9a(2 - m + w)\gamma_{y} - 4(4a - 8m + w)].$ We now consider the value of $\gamma_{y}$.

Firstly, from the condition $2a - 4m - w > 0$, which equals to $2a - 4m - 2w < 4a + w$, that is $8/9 < 4(4a + w)/9(2a + m + w)$. Finally, from the condition $2a - 4m - w > 0$, which equals to $4a + w > 2(2a + m + w) = 8m(2a + m + w)$, and we get $4(4a + w)/9(2a + m + w) = 4(4a - 8m + w)/9(2a - 2m + w)$. Thus, for $\gamma_{y} = 4(4a - 8m + w)/9(2a - 2m + w)$. There are two cases:

(i) When $\gamma_{y} = 4(4a - 8m + w)/9(2a - 2m + w)$, that is $\gamma_{y} = 4(4a - 8m + w)/9(2a + m + w)$, we have $f(1) > 0$. Therefore, $f(R)$ is an increasing function, $f(R_{l_{B}^d}) < f(1) < 0$, that is for any $R \in (R_{l_{B}^d}, 1)$, $f(R) < 0$. Obviously, $\gamma_{y} = 4(4a - 8m + w)/9(2a + m + w), (9\gamma_{y} - 8R) > 0$, thus, for any $R \in (R_{l_{B}^d}, 1)$, there is $dI_{B}^d/dR > 0$.

(ii) When $4a + w > 2a + m + w$, we have $4a - 3\gamma_{y}(2 - m + w) < 0$, $f(R_{l_{B}^d}) < 0$. Therefore, $f(R)$ is a decreasing function, $f(1) < f(R_{l_{B}^d}) < 0$, that is for any $R \in (R_{l_{B}^d}, 1)$, $f(R) > 0$. Thus, for any $R \in (R_{l_{B}^d}, 1)$, there is $dI_{B}^d/dR < 0$.

(2) Second, we consider the scenario $\gamma_{y} < 4(4a - 8m + w)/9(2a - 2m + w)$, that is $\gamma_{y} > 4(4a + w)/9(2a + m + w)$, we have $f(1) < 0$. Since $\gamma_{y} > 4(4a + w)/9(2a + m + w)$, there is $4w - 3\gamma_{y}(2 - m + w) < 0$. Therefore, $f(R)$ is a decreasing function, thus, when $R \in (R_{l_{B}^d}, R_{l_{B}^d} + 1), f(R) > 0$; when $R \in (R_{l_{B}^d}, 1), f(R) < 0$. Thus, when $R \in (R_{l_{B}^d}, R_{l_{B}^d} + 1), dI_{B}^d/dR > 0$; when $R \in (R, 1), dI_{B}^d/dR < 0$.

(3) Finally, we consider the scenario $\gamma_{y} \geq 4(4a - 8m + w)/9(2a - 2m + w)$, we have $f(1) > 0$. Since $\gamma_{y} > 4(4a - 8m + w)/9(2a - 2m + w)$, that is $\gamma_{y} > 4\gamma_{y}(2 - m + w) > 0$. Therefore, $f(R)$ is a decreasing function, $f(R_{l_{B}^d}) > f(1) > 0$, thus, for any $R \in (R_{l_{B}^d}, 1), f(R) > 0$. Thus, for any $R \in (R_{l_{B}^d}, 1), dI_{B}^d/dR > 0$.

Proof of Proposition 9. From the optimal decisions in the collaborative operation and autonomous operation, we know, $I_{c}^d - I_{a}^d = -3f(R)(m/9\gamma_{y} - 8)(9\gamma_{y} - 8R)$, where $f(R) = 4[3(2a + m + w) - 2w]R + 1, 3 - a + m(1 - I_{c}^d - I_{a}^d)/3, 9\gamma_{y} - d_{a} - d_{c} = a + w - 2m(1 - I_{c}^d)/3, 3 - a + w - 2m(1 - I_{a}^d)/3, f^2 = 3 - 3a + m(1 - I_{c}^d)/3, (\Pi_{c}^d - \Pi_{a}^d)(f_{a}^d - f_{c}^d) = (d_{a}^d - d_{c}^d)(d_{c}^d - d_{a}^d).$ Thus, if $4(4a + w)/9m(y_{y} - \gamma_{y}), there is $d_{a}^d < d_{c}^d \geq 0$; if $d_{a}^d < d_{c}^d \geq 0$, there is $\gamma_{y} < y_{B}, there is $d_{a}^d < d_{c}^d \geq 0$.

Data Availability
The data used to support the findings of this study are included within the article.

Conflicts of Interest
The authors declare no conflicts of interest.

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