Research Article

Supply Chain Coordination in Industrial Symbiosis Networks with Uncertain Waste Demand and Supply

Xiaoying Tang and Yong He

School of Economics and Management, Southeast University, Nanjing 210096, China

Correspondence should be addressed to Yong He; hy@seu.edu.cn

Received 18 January 2022; Revised 16 April 2022; Accepted 4 May 2022; Published 1 July 2022

Academic Editor: Ardashir Mohammadzadeh

Copyright © 2022 Xiaoying Tang and Yong He. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

By-product synergy (BPS) is considered an effective measure to support a circular economy and sustainable development as it can reduce waste disposal and natural resource consumption. The adoption of BPS can create new supplier-buyer relations that are different from traditional ones because the traded products are usually outside of the core business of the supplier, so the new relations need to be coordinated. This paper considers a two-echelon supply chain consisting of a manufacturer and a retailer. The by-product is randomly generated along with the production of the core product. The manufacturer sells its core product directly to the market with uncertain demand and sells the by-product to the retailer facing stochastic demand. To coordinate both the manufacturer’s production and retailer’s order decisions and achieve efficient performance, we analyze several contracts and find that a combined contract with an ABD contract and a revenue sharing contract could coordinate the supply chain and realize Pareto-improvement. A numerical experiment is also carried out to explore the influences of fluctuations in demand and supply on the performance of the manufacturer, the retailer, and the total supply chain.

1. Introduction

Since the industrial revolution began in the 1840s, industrial development has brought technological progress and changed our way of life. At the same time, it also brought negative impacts such as environmental pollution, global warming, and resource depletion. With the continuous deterioration of the environment and the unsustainable pattern of consumption and production, environmental sustainability has received more and more attention and has become an issue that cannot be ignored in industrial development. Therefore, to achieve sustainable development, it is necessary to promote innovative and efficient production processes and encourage the optimization of resource utilization in the context of a circular economy [1]. Converting the waste streams that accompany the production of the core products into useful and marketable by-products is an important means for industrial companies to realize sustainable development.

Turning wastes into useable and saleable by-products to promote sustainability is referred to as “by-product synergy (BPS)” or “industrial symbiosis (IS)” [2]. There are many application cases of BPS in the industry, for example, Terra upgraded its production facilities to purify the carbon dioxide produced with the production of nitrogenous fertilizers and piped it to Air Liquide as a by-product. Air Liquide then sold the carbon dioxide to beverage manufacturers for use in soft drinks and fizzy water. Suncor produced ethanol from corn and adopted the BPS process to capture carbon dioxide for Praxair to produce soft drinks and dried and transported the distiller’s grain to farms in southern Ontario for cattle breeding [3]. The implementation of BPS can bring significant economic, social, and environmental benefits, e.g., Dow Chemical Company collaborated with the U.S. Business Council for Sustainable Development to pilot BPS. Ultimately, it was found that the implementation of BPS cannot only reduce fuel consumption by 900 thousand MMBtu and carbon dioxide emissions by 108 million pounds per year but can also save 15 million dollars in annual costs [4].

Despite the great benefits of BPS, it has not yet been popularized in the fields of the chemical industry,
agriculture, and manufacturing because of the enormous challenges it poses to companies [5]. From the perspective of operations and supply chain management, the implementation of BPS introduces new supply-demand relations that are different from the traditional ones. Taking Terra for an example, the company adopted BPS to turn its own generated waste into by-products, then by-products exchanged between the supply, and demand parties are not within the core business scope of the supplier, that is, the quantity of by-products is not generated according to the buyer’s demand but is driven by the production volume of the core business product. Therefore, BPS emphasizes collaboration to improve the efficient use of resources and solve the mismatch between demand and supply of by-products to ensure the performance of the total supply chain and its members [6, 7]. Besides, since by-products are generated with the production of core products, the quality of by-products can be compromised as companies focus most of their attention on the core products. Natural variations in product quality lead to variability in by-products quantity [8]. For example, the carbon dioxide that accompanies the production of nitrogen-based fertilizer, which is Terra’s core product, is impure because it is blended with air, resulting in fluctuations in the output of purified carbon dioxide. Fluctuations in the output of by-products can also lead to the mismatch, so the stochastic supply of by-products cannot be ignored [9].

This paper fills a niche in the literature by studying the supply chain coordination mechanisms to cope with the mismatch between supply and demand of by-products brought by the implementation of BPS, to achieve the long-term and stable development of the supply chain. Specifically, this paper considers a two-echelon supply chain consisting of a manufacturer and a retailer. The manufacturer adopts BPS and randomly produces the by-product during the production of the core product. The core product is sold to the market with uncertain demand, while the by-product is sold to the retailer facing stochastic demand. The research in this paper mainly addresses the following two issues: (i) Is there a coordination mechanism to coordinate the supply-demand relations arising from the implementation of BPS? (ii) How do fluctuations in demand and supply affect the performance of the manufacturer, the retailer, and the supply chain?

The remainder of this paper is organized as follows: In Section 2, we review the related literature. Section 3 gives the model description and analyzes the centralized supply chain. In Section 4, we investigate supply chain performance under an ABD contract and an ABD with a revenue sharing contract. Section 5 examines the impact of demand and supply fluctuations on the supply chain efficiency, and Section 6 contains concluding remarks.

2. Literature Review

Our work is related to the literature on by-product synergy, coproduction, and supply chain collaboration.

By-product synergy has attracted the attention of many scholars in terms of operations and supply chain management. The problem of producing the core product and converting useful by-product within a company is first studied by Lee [4]. He examines the optimal operation and licensing strategies for the manufacturer adopting BPS. The market conditions under which the manufacturer can gain the most benefit are also analyzed, then Lee and Tongarlak [10] study the benefits of implementing BPS from a retailer’s perspective, and the demand for both products is considered uncertain. Both papers assessed the environmental impact of BPS. Another common phenomenon of BPS is outsourcing by-product processing to other companies. In this case, about optimal decision-making, Cao et al. [11] and Zhou et al. [12] study the optimal production decisions and waste discharge levels. Zhu et al. [13] examine the impact of implementing BPS on company’s operations based on a case of paper-sugar symbiosis and analyze the company’s willingness to implement BPS. Fraccasia and Yanzan [14] design an agent-based model to highlight the important role of information-sharing in BPS. Herczeg et al. [15] and Parlar et al. [7] investigate BPS from the perspective of supply chain collaboration. Specifically, Herczeg et al. [15] develop a theoretical framework to analyze the main factors that influence the collaboration and performance of supply chain members. Parlar et al. [7] assume that demand is linearly related to time and propose a nucleolus-based cooperative game theory method to share the benefits among companies to minimize members’ dissatisfaction from an ecosystem perspective. A comprehensive review can be found in the study by Liu et al. [16], Yeo et al. [17], and Neves et al. [18]. Considering the manufacturer converts its own generated waste into by-products by implementing BPS and the demand for the core product and by-product is uncertain, this paper differs from Lee and Tongarlak [10] in which we extend the relationship to two companies to investigate the coordination mechanism and take into account the uncertainty of the by-product output.

Another related stream of research is coproduction. A coproduction system uses a common input and produces multiple products simultaneously in a single production process [19]. Focusing on a vertical market model in which the co-products are substitutable, Bitran and Dasu [20]; Bitran and Gilbert [19]; Gerchak et al. [21]; Hsu and Bassok [22]; and Bansal and Transchel [23] study the production decision under random yields. In addition, Chen et al. [24]; Transchel et al. [25]; and Lin et al. [26] analyze product design and production issues. Research on horizontal coproduct setting in operations management includes the following: Dong et al. [27] examine the drivers of conversion flexibility and the impact of input and output market conditions on the profit of petroleum refinery; Boyabatli et al. [28] study the periodic production and capacity investment decisions; moreover, Xu et al. [29] investigate the optimal production decision in multiple periods considering that the demand for multiple products is stochastic. For both vertical and horizontal settings, Chen et al. [30] examine the optimal pricing and production decisions with deterministic demand. In the horizontal setting, the coproducts differ in their applications and are not substitutable and so does under BPS. However, BPS is different from the horizontal coproduct setting. In terms of product characteristics, horizontal co-products are
two or more major products with a higher economic value produced from the same raw material and through the same production process, while by-products are non-major products produced from the same raw material and need to be further processed through other processes by using the waste generated in production. In addition, the cost calculation method is inconsistent. The production costs of coproducts are allocated according to the quantity of each product, while by-products only bear the cost of further processing and other production costs belong to the core product. Furthermore, we expand the scope of research related to coproduction by exploring coordination mechanisms.

Supply chain researchers have recognized the importance of collaboration among supply chain partners. From the perspective of operations management, collaboration refers to the cooperation of production and delivery, which aims to match products with customer demand and minimize uncertainties [31]. Jagdev and Thoben [32] mention that making contracts and agreements is one of the ways to help partners achieve coordination. Scholars have studied coordination from various contract perspectives, such as wholesale price contracts, buyback contracts [33], revenue sharing contracts [34], options contracts [35], consignment contracts [36], VMI contracts [37], quantity flexibility contracts [38], APD contracts [39], ABD contracts [40], and SRP contracts [41]. For a detailed review of this literature, readers can refer to Shen et al. [42]; Ma et al. [43]; and Vosooghidizaji et al. [44]. Considering both the uncertainty of demand and supply, He and Zhao [45] study a three-echelon supply chain and find that the combination of returns policy and wholesale price contract can perfectly coordinate the supply chain. Furthermore, He and Zhao [46] find that the VMI and APD contract can coordinate the two-echelon supply chain with uncertain demand and supply. Unlike the study of He and Zhao [46], Xie et al. [47] argue that the uncertain yield is handled by the retailer rather than the supplier and analyze the performance of buyback contracts in the supply chain. Different from the research of He and Zhao [46], this paper considers that the manufacturer produces two types of product, and the output of the core product is deterministic, while the output of the by-product is uncertain because it accompanies the production of the core product and therefore experiences yield uncertainty. Moreover, both products face uncertain demand. Then, we focus on finding coordination mechanisms to achieve the best performance of the supply chain.

3. Preliminaries

3.1. Model Description. We consider a two-echelon supply chain consisting of a manufacturer and a retailer. The manufacturer produces core product A and randomly produces by-product B during the production process. The manufacturer sells the core product directly to the market and sells the by-product through a retailer. Both the manufacturer and the retailer face uncertain demand.

We summarize the notations of the paper as follows:

**Exogenous parameters** are as follows:

- \( c_A \): the production cost of the core product
- \( c_B \): the production cost of the by-product
- \( p_A \): the exogenous retail price of the core product
- \( p_B \): the exogenous retail price of the by-product
- \( D_A \): demand for the core product, which is a random variable characterized by cumulative distribution function (CDF) \( F(\cdot) \) and probability density function (PDF) \( f(\cdot) \) with mean \( \mu_A > 0 \) and standard deviation \( \sigma_A > 0 \)
- \( D_B \): demand for the by-product, characterized by CDF \( H(\cdot) \) and PDF \( h(\cdot) \) with mean \( \mu_B > 0 \) and standard deviation \( \sigma_B > 0 \)
- \( \epsilon \): quantity of by-product accompanying each unit of core product output, which is a random variable characterized by PDF \( g(\cdot) \) and CDF \( G(\cdot) \) with mean \( \mu > 0 \) and standard deviation \( \sigma > 0 \), and has a support of \([A, B]\) where \( B > A > 0 \)

**Contract terms under negotiation are as follows:**

- \( w \): the wholesale price of the by-product
- \( w_m \): the wholesale price of the by-product for the late orders
- \( \phi \): the revenue sharing earned by the retailer

**Decision variables** are as follows:

- \( Q \): quantity of the by-product ordered by the retailer before production begins, which is a decision variable
- \( R \): production quantity of the core product determined by the manufacturer, which is a decision variable

We use superscript \( C \) to denote the centralized supply chain, \( d \) to denote the decentralized supply chain, \( \lambda \) to denote the ABD contract, and \( \lambda, \phi \) to denote the ABD contract with revenue sharing.

3.2. The Centralized Supply Chain. We first analyze the centralized case in which the total expected profit of the entire supply chain \( \Pi^C_T(R^C) \) is

\[
\Pi^C_T(R^C) = E[p_A \min(R^C, D_A) + p_B \min(R^C, \epsilon, DB) - c_AR^C - c_B R^C \epsilon],
\]

where the first term is the revenue from the core product, the second term is the revenue from the by-product, and the last two terms are the production costs of the core product and by-product, respectively.

Then, the first-derivation and second-derivation of the total expected profit with respect to \( R^C \) are

\[
\frac{d\Pi^C_T(R^C)}{dR^C} = p_A \left(1 - F(R^C)\right) + p_B \mu
\]

\[
- p_B \int_A^B yH(R^C, y)g(y)dy - c_A - c_B \mu,
\]

\[
\frac{d^2\Pi^C_T(R^C)}{dR^{C2}} = -p_A f(R^C) - p_B \int_A^B y^2 h(R^C, y)g(y)dy < 0.
\]
Therefore, the optimal production quantity of the core product $R^*$ can be derived from the equation $d\Pi_C^*(R^*)/dR^* = 0$, i.e.,

$$p_A F(R^*) + p_B \int_0^B y f(y) dR^* = p_A - c_A + (p_B - c_B)\mu. \tag{4}$$

Substituting (4) into (1), we can get

$$\Pi_C^*(R^*) = p_A \int_0^R x f(x) dx + p_B \int_0^B \int_0^y z h(z) dz g(y) dy. \tag{5}$$

4. The Decentralized Supply Chain

In this subsection, we try to find a suitable contract to help supply chain members achieve coordination. Before examining the contract, we first present the following lemma.

Lemma 1. The optimal profit of the decentralized supply chain is equal to that of the centralized supply chain when the following conditions are satisfied: (i) $R^d = R^*$ and (ii) $Q^d \geq R^* B$.

The optimal solutions and related proofs of Section 4 are given in Appendix A.

4.1. ABD Contract. A large number of contracts (e.g., buy-back contracts and revenue sharing contracts) have been studied to achieve supply chain coordination in the context of news-vendor models. However, we have checked that the wholesale price contract, buy-back contract, and revenue sharing contract cannot coordinate the supply chain. Given the uncertainty in the supply and demand of by-product, we try to examine whether an advance booking discount (ABD) contract can coordinate the supply chain. Under the ABD contract, there are two types of retail orders. The first one is the "prebook" order, which is submitted to the manufacturer before production begins and received before the start of the selling season. The manufacturer charges $w$ per unit in the prebook order. During the selling season, the retailer can submit a "real-time" order if the supply of by-products is higher than the retailer’s prebook order, and at the same time, the retailer has a shortage. The retailer pays the manufacturer $w_a = (1 + \lambda)w$ per unit in the real-time order, where $0 \leq \lambda \leq ((p_B - w)/w)$.

Under the ABD contract, we describe the sequence of the events as follows:

1. The manufacturer and the retailer negotiate contract terms $w$ and $w_a$. 
2. The manufacturer determines the production capacity of the core product and generates a random proportion of the by-product.
3. The retailer determines the "prebook" order quantity of the by-product.
4. The manufacturer produces both products and sends the by-product to the retailer.
5. The demand for both products is realized. If both the realized demand and output of by-product are greater than the retailer’s "prebook" order quantity, a real-time order can be submitted. Finally, both products are sold at exogenous market prices.

The retailer’s expected profit $\Pi_m^\lambda(Q^\lambda)$ is

$$\Pi_m^\lambda(Q^\lambda) = E\left[p_A \min\left(R^\lambda \epsilon, D_B\right) - w \min\left(R^\lambda \epsilon, Q^\lambda\right) - w_a \left[\min\left(R^\lambda \epsilon, D_B\right) - Q^\lambda\right]\right], \tag{6}$$

where the first term is the total revenue of the retailer, the second term is the procurement cost of "prebook" order, and the last term is the procurement cost of the "real-time" order.

The manufacturer’s expected profit $\Pi_m^\lambda(R^\lambda)$ is

$$\Pi_m^\lambda(R^\lambda) = E\left[p_A \min\left(R^\lambda \epsilon, D_A\right) + w \min\left(R^\lambda \epsilon, Q^\lambda\right) + w_a \left[\min\left(R^\lambda \epsilon, D_B\right) - Q^\lambda\right] - c_A R^\lambda - c_B R^\lambda \epsilon\right], \tag{7}$$

where the first term is the sales revenue of the core product, the second term is the sales revenue from the "prebook" order of the by-product, the third term is the sales revenue from the "real-time" order of the by-product, and the last two terms are the production costs of the core product and by-product, respectively.

Taken the first-derivation of the retailer’s expected profit with respect to $Q^\lambda$, we can get that the optimal "prebook" order quantity $Q^\lambda_{\ast}$ satisfies the following:

$$Q^\lambda_{\ast} = H^{-1}\left(\frac{\lambda}{1 + \lambda}\right). \tag{8}$$
Based on the analysis in Appendix A, in anticipating the retailer’s best response, we derive that the optimal production quantity $R^\lambda$ satisfies the following equation:

$$p_A(1 - F(R^\lambda)) + w \int_0^{Q_R^{\lambda}} yg(y)dy + \int_{Q_R^{\lambda}}^B y(1 - H(R^\lambda y))g(y)dy = w_B \int_0^{B_R^\lambda} y(1 - H(R^\lambda y))g(y)dy > 0,$$

or $\lambda = \lambda^*$. The ABD contract cannot coordinate the supply chain.

4.2. ABD with Revenue Sharing Contract. Based on the above analysis, we deduce that the ABD contract cannot coordinate the channel if the supply and demand of the by-product are uncertain. In this subsection, we check whether a combined contract that includes an ABD contract and a revenue sharing contract can achieve coordination and if so, how to develop the contract terms. Specifically, in this combined contract, the retailer can also submit two types of retail orders but needs to give $1 - \phi$ portion of its revenue to the manufacturer. Then, the retailer’s expected profit $\mathbb{E}_\lambda(Q^{\lambda,\phi})$ can be expressed as follows:

$$\mathbb{E}_\lambda(Q^{\lambda,\phi}) = \mathbb{E}\left[ p_B \min(Q^{\lambda,\phi}e, D_B) - w \min(Q^{\lambda,\phi}e, Q^{\lambda,\phi}) - w_B \min(Q^{\lambda,\phi}e, D_B) - Q^{\lambda,\phi} \right],$$

where the first term is the sales revenue of the by-product, the second term is the procurement cost of “prebook” order, and the third term is the procurement cost of “real-time” order.

The manufacturer’s expected profit $\mathbb{E}_m(Q^{\lambda,\phi})$ can be expressed as:

$$\mathbb{E}_m(Q^{\lambda,\phi}) = \mathbb{E}\left[ p_A(1 - F(R^\lambda)) + w_B \int_0^{B_R^\lambda} yg(y)dy + (1 - \phi)p_B \min(Q^{\lambda,\phi}e, D_B) - c_B R^{\lambda,\phi} - c_B R^{\lambda,\phi} \right],$$

where the first term is the sales revenue of the core product, the second term is the sales revenue from the “prebook” order of the by-product, the third term is the sales revenue from the “real-time” order of the by-product, the fourth term is the revenue of the by-product when a sale occurs, and the last two terms are the production cost of the core product and by-product, respectively.

Similar to the analysis of ABD contract, we can get that the optimal order quantity $Q^{\lambda,\phi}$ satisfies the following:

$$H(Q^{\lambda,\phi}) = \frac{\lambda}{1 + \lambda}.$$  

Similarly, in anticipating the retailer’s best response, we derive that the optimal $R^{\lambda,\phi}$ satisfies the following equation:

Assuming $\lambda^* = \lambda$, we can get $\lambda^* = \lambda^*$. The ABD contract cannot coordinate the supply chain.

Proposition 1. When the sets of contract terms $(w^*, \lambda^*, \phi^*)$ satisfy
choosing contract terms

\[ \begin{align*}
\sigma_A = 400 & \quad 4.01 \quad 626 \quad 1228 \quad 6711.04 \\
\sigma_A = 600 & \quad 4.01 \quad 627 \quad 1224 \quad 5964.71 \\
\sigma_A = 800 & \quad 4.01 \quad 627 \quad 1254 \quad 5352.2 \\
\sigma_B = 200 & \quad 5.05 \quad 600 \quad 1250 \quad 6460.83 \\
\sigma_B = 400 & \quad 4.01 \quad 627 \quad 1241 \quad 5964.71 \\
\sigma_B = 600 & \quad 3.37 \quad 707 \quad 1236 \quad 5804.9 \\
\sigma = 0.2 & \quad 3.62 \quad 694 \quad 1172 \quad 6160.57 \\
\sigma = 0.5 & \quad 4.01 \quad 627 \quad 1241 \quad 5964.71 \\
\sigma = 0.6 & \quad 4.01 \quad 627 \quad 1234 \quad 5808.09 \\
\end{align*} \]

\[ \begin{align*}
\text{Efficiency} & = 740.12 \quad 6548.21 \quad 7459.91 \\
\text{Efficiency} & = 87.78 \quad 87.78 \\
\end{align*} \]

Table 1: Solutions for standard deviations under the basic parameter values.

In this section, we explore the impact of uncertain demands and stochastic supply on supply chain efficiency, which is characterized by \( \Pi_c(R^2, Q^2)/\Pi_T(R^2) \). The basic parameter values are as follows: \( \rho_A = 10 \), \( \rho_B = 6 \), \( \epsilon_A = 3 \), and \( \epsilon_B = 2 \); the demand for the core product \( D_1 \) follows a normal distribution with a mean \( \mu_1 = 1200 \) and a standard deviation \( \sigma_1 = 650 \), the demand for the by-product \( D_2 \) follows a normal distribution with a mean \( \mu_2 = 800 \) and \( \sigma_2 = 400 \), and the quantity of by-product produced along with each unit of core product output \( e \) follows a uniform distribution with \( \mu = 1 \) and \( \sigma = 0.5 \). Furthermore, \( \sigma_A = [400, 600, 800] \), \( \sigma_B = [200, 400, 600] \), and \( \sigma = [0.2, 0.5, 0.6] \) are adopted to analyze the influence of supply and demand variations on the performance of supply chain members and supply chain efficiency, and a simple wholesale price contract is considered [46]. Table A gives the solutions for different standard deviations under the basic parameter values.

It can be seen from Table 1 that when the demand for the core product becomes more uncertain, i.e., with a larger \( \sigma_A \), the manufacturer tends to increase the production quantity of the core product and remain the wholesale price of the by-product to maintain the retailer’s order quantity. Therefore, the retailer’s profit remains almost unchanged, while the manufacturer’s profit decreases with demand uncertainty, with a corresponding decrease in total supply chain profit and supply chain efficiency.

When the demand for the by-product becomes more uncertain, i.e., with a larger \( \sigma_B \), the manufacturer’s production quantity of the core product keeps almost unchanged and charges a lower price to induce the retailer to increase the order volume. The positive effect of increased order for by-product cannot offset the negative impact of a lower wholesale price, so the manufacturer’s profit decreases. Conversely, increased order for by-product and lower wholesale price improves the retailer’s profit. Since the combination of increased profit for the retailer and decreased profit for the manufacturer affects both the total supply chain profit and the supply chain efficiency, we find that when the

\[ w^* \int_A^{Q_i(w^*, \phi)} \int_B^{R_i(w^*, \phi)} y g(y) dy - \phi B_p \int_A^{R_i(w^*, \phi)} y [1 - H(R_i w^*)] g(y) dy + (1 + \lambda^*) w^* \int_A^{Q_i(w^*, \phi)} \int_B^{R_i(w^*, \phi)} y [1 - H(R_i w^*)] g(y) dy = 0, \]

\[ (15) \]

the optimal total profit of the supply chain under the combined contract equals that in the centralized case.

Proposition 1 implies that to achieve the best total profit of the supply chain, the contract term \( w^* \) should be chosen from the range \([0, \sigma B_p - w^*/w^*]\), \( \lambda^* \) should be chosen from the range \([0, \sigma B_p - w^*/w^*]\), and \( \phi^* \) should be calculated by equation (15) where \( R_i \) can be derived from equation (4). Moreover, \( R_i(\lambda^*, \phi^*) \) and \( Q_i(\lambda^*, \phi^*) \) can be derived from equation (13).

Proposition 2. Varying \( (w^*, \lambda^*, \phi^*) \) allows for an arbitrary allocation of the best total profit of the supply chain. Specifically, the manufacturer’s profit is \( \Pi_m^{Q_i(\lambda^*, \phi^*)} (R_i(\lambda^*, \phi^*)) = \delta (\Pi_T^{C(R_i)} - A) \) and the retailer’s profit is \( \Pi_r^{Q_i(\lambda^*, \phi^*)} (Q_i(\lambda^*, \phi^*)) = (1 - \delta) \Pi_T^{C(R_i)} + A \delta \), where \( \delta = 1 - \phi^* + \bar{\lambda} \) \( \times \) \( w^* \beta / p_B \). \( \Pi_T^{C(R_i)} \) can be derived from equation (4). Moreover, \( R_i(\lambda^*, \phi^*) \) and \( Q_i(\lambda^*, \phi^*) \) can be derived from equation (13).

Then, given an existing state \( (\Pi_m^{Q_i(\lambda^*, \phi^*)}, \Pi_r^{Q_i(\lambda^*, \phi^*)}) \), we can deduce the Nash bargaining solution to achieve Pareto-improvement, i.e., \( \Pi_m^{Q_i(\lambda^*, \phi^*)} (R_i(\lambda^*, \phi^*)) \geq \Pi_m^{Q_i(\lambda^*, \phi^*)} (R_i(\lambda^*, \phi^*)) \) and \( \Pi_r^{Q_i(\lambda^*, \phi^*)} (Q_i(\lambda^*, \phi^*)) \geq \Pi_r^{Q_i(\lambda^*, \phi^*)} (Q_i(\lambda^*, \phi^*)) \).

Proposition 3. The Nash bargaining solution to the combined contract is

\[ \delta^* = \frac{1}{2} \left[ \frac{\Pi_T^{Q_i(R_i)}}{\Pi_T^{R_i} - A} + \frac{\Pi_m^{Q_i(R_i)} - \Pi_m^{Q_i(Q_i)}}{\Pi_T^{R_i} - A} \right]. \]

Based on Propositions 1–3, we find that supply chain coordination and Pareto-improvement can be achieved by choosing contract terms \( (w^*, \lambda^*, \phi^*) \) that satisfy (15) and (16). Furthermore, from (16), we can see that if \( \Pi_m^{Q_i(R_i)} > \Pi_m^{Q_i(Q_i)} \), then \( \delta^* > (1/2)[\Pi_T^{Q_i(R_i)}/(\Pi_T^{C(R_i)} - A)] \), and the manufacturer can get more profit under coordination \( ((1 - \delta) \Pi_T^{C(R_i)} + A \delta > \Pi_T^{C(R_i)})/2 \). Conversely, if \( \Pi_m^{Q_i(R_i)} < \Pi_m^{Q_i(Q_i)} \), the retailer gains more profits. Therefore, we deduce that the respective bargaining power of the manufacturer and the retailer determines their profits under the Nash bargaining scheme.

5. Numerical Experiment

In this section, we explore the impact of uncertain demands and stochastic supply on supply chain efficiency, which is characterized by \( \Pi_c(R^2, Q^2)/\Pi_T(R^2) \). The basic parameter values are as follows: \( \rho_A = 10 \), \( \rho_B = 6 \), \( \epsilon_A = 3 \), and \( \epsilon_B = 2 \); the demand for the core product \( D_1 \) follows a normal distribution with a mean \( \mu_1 = 1200 \) and a standard deviation \( \sigma_1 = 650 \), the demand for the by-product \( D_2 \) follows a normal distribution with a mean \( \mu_2 = 800 \) and \( \sigma_2 = 400 \), and the quantity of by-product produced along with each unit of core product output \( e \) follows a uniform distribution with \( \mu = 1 \) and \( \sigma = 0.5 \). Furthermore, \( \sigma_A = [400, 600, 800] \), \( \sigma_B = [200, 400, 600] \), and \( \sigma = [0.2, 0.5, 0.6] \) are adopted to analyze the influence of supply and demand variations on the performance of supply chain members and supply chain efficiency, and a simple wholesale price contract is considered [46]. Table A gives the solutions for different standard deviations under the basic parameter values.
demand uncertainty for the by-product is small, the total supply chain profit and supply chain efficiency decrease with demand uncertainty because the change in manufacturer's profit has a greater impact on the total supply chain profit, while when the demand uncertainty for the by-product is high, the retailer has a greater impact on the total supply chain profit; thus, the total supply chain profit and supply chain efficiency increases with demand uncertainty.

When the supply of the by-product becomes more uncertain, i.e., with a larger \( \sigma \), the manufacturer tends to increase the production quantity of the core product to generate a reliable amount of by-product and charges a higher wholesale price for the by-product when \( \sigma \) is in a lower range. However, when \( \sigma \) is large enough, the manufacturer is reluctant to produce many by-products after weighing the revenue gained from selling the core product and by-product because overproduction of the core product would harm his profit, and then the manufacturer charges a lower wholesale price for the by-product to attract more orders from the retailer. The higher wholesale price reduces the retailer's order volume, and vice versa. In addition, we find that the retailer's profit decreases with supply uncertainty due to the decreased order or wholesale price for the by-product; thus, the total supply chain profit and supply chain efficiency decreases with supply uncertainty.

\[
\text{Table 2: Solutions for standard derivation } \sigma_A \text{ under different values of } \sigma_B \text{ and } \sigma.
\]

<table>
<thead>
<tr>
<th>( \sigma_B ) = 200</th>
<th>( \sigma_A ) = 400</th>
<th>( w )</th>
<th>( Q_w^\sigma )</th>
<th>( R^\sigma )</th>
<th>( \Pi_m^\sigma (R^\sigma) )</th>
<th>( \Pi_m^\sigma (Q_w^\sigma) )</th>
<th>( \Pi_r^\sigma )</th>
<th>( \Pi_T^\sigma )</th>
<th>Efficiency (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.05</td>
<td>600</td>
<td>1242</td>
<td>7205.32</td>
<td>444.66</td>
<td>7649.98</td>
<td>8580.32</td>
<td>89.16</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \sigma_B ) = 400</td>
<td>( \sigma_A ) = 600</td>
<td>4.01</td>
<td>626</td>
<td>1228</td>
<td>6711.04</td>
<td>786.16</td>
<td>7497.2</td>
<td>8447.61</td>
<td>88.75</td>
</tr>
<tr>
<td>( \sigma_A ) = 800</td>
<td>3.37</td>
<td>707</td>
<td>1236</td>
<td>6551.02</td>
<td>1141.82</td>
<td>7983.78</td>
<td>8388.09</td>
<td>92.15</td>
<td></td>
</tr>
<tr>
<td>( \sigma_B ) = 600</td>
<td>( \sigma_A ) = 400</td>
<td>3.36</td>
<td>691</td>
<td>1168</td>
<td>6903.97</td>
<td>1079.81</td>
<td>7200.45</td>
<td>8281.88</td>
<td>87.96</td>
</tr>
<tr>
<td>( \sigma_B ) = 600</td>
<td>( \sigma_A ) = 600</td>
<td>3.64</td>
<td>692</td>
<td>1172</td>
<td>7160.57</td>
<td>1079.95</td>
<td>7420.52</td>
<td>8281.88</td>
<td>87.96</td>
</tr>
<tr>
<td>( \sigma_A ) = 800</td>
<td>3.64</td>
<td>692</td>
<td>1172</td>
<td>7546.91</td>
<td>1079.95</td>
<td>7662.86</td>
<td>7624.64</td>
<td>86.89</td>
<td></td>
</tr>
</tbody>
</table>

\[
\text{Table 3: Solutions for standard derivation } \sigma_B \text{ under different values of } \sigma_A \text{ and } \sigma.
\]

<table>
<thead>
<tr>
<th>( \sigma_B ) = 200</th>
<th>( \sigma_A ) = 400</th>
<th>( w )</th>
<th>( Q_w^\sigma )</th>
<th>( R^\sigma )</th>
<th>( \Pi_m^\sigma (R^\sigma) )</th>
<th>( \Pi_m^\sigma (Q_w^\sigma) )</th>
<th>( \Pi_r^\sigma )</th>
<th>( \Pi_T^\sigma )</th>
<th>Efficiency (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.05</td>
<td>600</td>
<td>1242</td>
<td>7205.32</td>
<td>444.66</td>
<td>7649.98</td>
<td>8580.32</td>
<td>89.16</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \sigma_B ) = 400</td>
<td>( \sigma_A ) = 400</td>
<td>3.37</td>
<td>707</td>
<td>1236</td>
<td>6551.02</td>
<td>1141.82</td>
<td>7983.78</td>
<td>8388.09</td>
<td>92.15</td>
</tr>
<tr>
<td>( \sigma_A ) = 600</td>
<td>3.64</td>
<td>691</td>
<td>1168</td>
<td>6903.97</td>
<td>1079.81</td>
<td>7200.45</td>
<td>8281.88</td>
<td>87.96</td>
<td></td>
</tr>
<tr>
<td>( \sigma_B ) = 600</td>
<td>( \sigma_A ) = 800</td>
<td>3.64</td>
<td>692</td>
<td>1172</td>
<td>7160.57</td>
<td>1079.95</td>
<td>7420.52</td>
<td>8281.88</td>
<td>87.96</td>
</tr>
<tr>
<td>( \sigma_B ) = 600</td>
<td>( \sigma_A ) = 600</td>
<td>3.64</td>
<td>692</td>
<td>1172</td>
<td>7546.91</td>
<td>1079.95</td>
<td>7662.86</td>
<td>7624.64</td>
<td>86.89</td>
</tr>
</tbody>
</table>

When the supply of the by-product becomes more uncertain, i.e., with a larger \( \sigma \), the manufacturer tends to increase the production quantity of the core product to generate a reliable amount of by-product and charges a higher wholesale price for the by-product when \( \sigma \) is in a lower range. However, when \( \sigma \) is large enough, the manufacturer is reluctant to produce many by-products after weighing the revenue gained from selling the core product and by-product because overproduction of the core product would harm his profit, and then the manufacturer charges a lower wholesale price for the by-product to attract more orders from the retailer. The higher wholesale price reduces the retailer's order volume, and vice versa. In addition, we find that the retailer's profit decreases with supply uncertainty due to the decreased order or wholesale price for the by-product; thus, the total supply chain profit and supply chain efficiency decreases with supply uncertainty.
Table 4: Solutions for standard derivation \( \sigma \) under different values of \( \sigma_A \) and \( \sigma_B \).

<table>
<thead>
<tr>
<th>( \sigma )</th>
<th>( w )</th>
<th>( Q_w )</th>
<th>( R_w )</th>
<th>( \Pi_m^{\sigma} )</th>
<th>( \Pi_{\sigma}^{(Q_w)} )</th>
<th>( \Pi_{\sigma}^{w} )</th>
<th>( \Pi_{\sigma}^{*} )</th>
<th>Efficiency (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>4</td>
<td>3.64</td>
<td>691</td>
<td>1168</td>
<td>6903.97</td>
<td>1079.81</td>
<td>7983.78</td>
<td>8958.57</td>
</tr>
<tr>
<td>0.5</td>
<td>4</td>
<td>4.01</td>
<td>626</td>
<td>1228</td>
<td>6711.04</td>
<td>786.16</td>
<td>7497.2</td>
<td>8447.61</td>
</tr>
<tr>
<td>0.6</td>
<td>4</td>
<td>628</td>
<td>1220</td>
<td>6554.7</td>
<td>737.56</td>
<td>7292.26</td>
<td>8204.86</td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>4</td>
<td>3.64</td>
<td>692</td>
<td>1172</td>
<td>6160.57</td>
<td>1079.95</td>
<td>7240.52</td>
<td>8231.88</td>
</tr>
<tr>
<td>0.5</td>
<td>4</td>
<td>4.01</td>
<td>627</td>
<td>1241</td>
<td>5964.71</td>
<td>788.69</td>
<td>6753.4</td>
<td>7721.64</td>
</tr>
<tr>
<td>0.6</td>
<td>4</td>
<td>628</td>
<td>1234</td>
<td>5808.09</td>
<td>740.12</td>
<td>6548.21</td>
<td>7459.91</td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>4</td>
<td>3.64</td>
<td>692</td>
<td>1172</td>
<td>5546.91</td>
<td>1079.95</td>
<td>6626.86</td>
<td>7624.44</td>
</tr>
<tr>
<td>0.5</td>
<td>4</td>
<td>4</td>
<td>627</td>
<td>1254</td>
<td>5352.2</td>
<td>791.2</td>
<td>6143.4</td>
<td>7117.67</td>
</tr>
<tr>
<td>0.6</td>
<td>4</td>
<td>629</td>
<td>1250</td>
<td>5195.3</td>
<td>742.79</td>
<td>5938.09</td>
<td>6862.85</td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td></td>
<td>4.75</td>
<td>638</td>
<td>1150</td>
<td>6638.98</td>
<td>660.24</td>
<td>7299.22</td>
<td>8392.36</td>
</tr>
<tr>
<td>0.5</td>
<td></td>
<td>5.05</td>
<td>600</td>
<td>1247</td>
<td>6460.08</td>
<td>445.19</td>
<td>6905.28</td>
<td>7854.21</td>
</tr>
<tr>
<td>0.6</td>
<td></td>
<td>5.05</td>
<td>600</td>
<td>1233</td>
<td>6268.79</td>
<td>416.13</td>
<td>6684.91</td>
<td>7578.91</td>
</tr>
<tr>
<td>0.2</td>
<td></td>
<td>3.64</td>
<td>692</td>
<td>1172</td>
<td>6160.57</td>
<td>1079.95</td>
<td>7240.52</td>
<td>8231.88</td>
</tr>
<tr>
<td>0.5</td>
<td></td>
<td>4.01</td>
<td>627</td>
<td>1241</td>
<td>5964.71</td>
<td>788.69</td>
<td>6753.4</td>
<td>7721.64</td>
</tr>
<tr>
<td>0.6</td>
<td></td>
<td>4</td>
<td>628</td>
<td>1234</td>
<td>5808.09</td>
<td>740.12</td>
<td>6548.21</td>
<td>7459.91</td>
</tr>
<tr>
<td>0.2</td>
<td></td>
<td>3.1</td>
<td>776</td>
<td>1187</td>
<td>6059.23</td>
<td>1465.82</td>
<td>7525.04</td>
<td>8090.42</td>
</tr>
<tr>
<td>0.5</td>
<td></td>
<td>3.37</td>
<td>708</td>
<td>1245</td>
<td>5805.05</td>
<td>1144.79</td>
<td>6949.84</td>
<td>7614.42</td>
</tr>
<tr>
<td>0.6</td>
<td></td>
<td>3.36</td>
<td>711</td>
<td>1233</td>
<td>5661.08</td>
<td>1075.93</td>
<td>6737.02</td>
<td>7387.67</td>
</tr>
</tbody>
</table>

Table 5: Solutions for standard derivations when \( p_A = 20 \).

<table>
<thead>
<tr>
<th>( \sigma )</th>
<th>( w )</th>
<th>( Q_w )</th>
<th>( R_w )</th>
<th>( \Pi_m^{(Q_w)} )</th>
<th>( \Pi_{\sigma}^{w} )</th>
<th>( \Pi_{\sigma}^{*} )</th>
<th>Efficiency (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>4</td>
<td>3.64</td>
<td>692</td>
<td>1172</td>
<td>6160.57</td>
<td>1079.95</td>
<td>7240.52</td>
</tr>
<tr>
<td>0.5</td>
<td>4</td>
<td>4.01</td>
<td>627</td>
<td>1241</td>
<td>5964.71</td>
<td>788.69</td>
<td>6753.4</td>
</tr>
<tr>
<td>0.6</td>
<td>4</td>
<td>628</td>
<td>1234</td>
<td>5808.09</td>
<td>740.12</td>
<td>6548.21</td>
<td>7459.91</td>
</tr>
<tr>
<td>0.2</td>
<td>4</td>
<td>3.64</td>
<td>692</td>
<td>1172</td>
<td>6160.57</td>
<td>1079.95</td>
<td>7240.52</td>
</tr>
<tr>
<td>0.5</td>
<td>4</td>
<td>4.01</td>
<td>627</td>
<td>1241</td>
<td>5964.71</td>
<td>788.69</td>
<td>6753.4</td>
</tr>
<tr>
<td>0.6</td>
<td>4</td>
<td>628</td>
<td>1234</td>
<td>5808.09</td>
<td>740.12</td>
<td>6548.21</td>
<td>7459.91</td>
</tr>
<tr>
<td>0.2</td>
<td>4</td>
<td>3.64</td>
<td>692</td>
<td>1172</td>
<td>6160.57</td>
<td>1079.95</td>
<td>7240.52</td>
</tr>
<tr>
<td>0.5</td>
<td>4</td>
<td>4.01</td>
<td>627</td>
<td>1241</td>
<td>5964.71</td>
<td>788.69</td>
<td>6753.4</td>
</tr>
<tr>
<td>0.6</td>
<td>4</td>
<td>628</td>
<td>1234</td>
<td>5808.09</td>
<td>740.12</td>
<td>6548.21</td>
<td>7459.91</td>
</tr>
</tbody>
</table>

Table 6: Solutions for standard derivations when \( p_B = 15 \).

<table>
<thead>
<tr>
<th>( \sigma )</th>
<th>( w )</th>
<th>( Q_w )</th>
<th>( R_w )</th>
<th>( \Pi_m^{(Q_w)} )</th>
<th>( \Pi_{\sigma}^{w} )</th>
<th>( \Pi_{\sigma}^{*} )</th>
<th>Efficiency (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>4</td>
<td>3.64</td>
<td>692</td>
<td>1172</td>
<td>6160.57</td>
<td>1079.95</td>
<td>7240.52</td>
</tr>
<tr>
<td>0.5</td>
<td>4</td>
<td>4.01</td>
<td>627</td>
<td>1241</td>
<td>5964.71</td>
<td>788.69</td>
<td>6753.4</td>
</tr>
<tr>
<td>0.6</td>
<td>4</td>
<td>628</td>
<td>1234</td>
<td>5808.09</td>
<td>740.12</td>
<td>6548.21</td>
<td>7459.91</td>
</tr>
</tbody>
</table>

Furthermore, we also use values other than the basic parameter values to verify the validity of the results; see Tables 2–6, in Appendix B for details. After analysis, we find that the results are the same as in the case of basic parameter values.

6. Conclusions

This paper examines the coordination problem among supply chain members who adopt BPS and face uncertain demand and supply. After analysis, we find that the ABD contract cannot achieve supply chain synergy, while a combined contract consisting of ABD and revenue sharing contract can achieve the best supply chain performance, support the arbitrary distribution of the optimal profit, and achieve Pareto-improvement.
To study the impact of the randomness of demand and supply on the supply chain efficiency, our numerical experiment indicates that the supply chain efficiency decreases with the demand uncertainty for the core product but first decreases and then increases with the supply and demand uncertainty for the by-product. This is because when the supply for the by-product fluctuates, the manufacturer focuses his main attention on the sale of the core product when the standard derivation is large enough, thereby reducing the impact of fluctuations on supply chain performance; moreover, when the demand for the by-product fluctuates, the retailer’s profit increases with the uncertainty, and the greater the fluctuation, the greater the effect of his profit change on the whole supply chain.

As a natural extension of our work, future research could be done by considering the endogenous retail price; meanwhile, it would be interesting to extend our model by having one manufacturer selling by-products to multiple retailers. Asymmetric information also exists in the BPS, e.g., the lack of information about by-product quantities, by-product quality, potential costs of treatment, and so on, which makes it harder to coordinate the supply chain.

Appendix

A. The Decentralized Supply Chain

A.1. ABD Contract

After simplification, the retailer’s expected profit is

\[
\frac{d\Pi^1_m(Q^1)}{dQ^1} = \left(1 - Q^1 \right) \left(1 + \lambda \right) \left(1 - H(Q^1) \right) w - w. \quad (A.2)
\]

Furthermore, the supplier’s expected profit is

\[
\frac{d\Pi^1_m(R^1)}{dR^1} = p_A \left(1 - F(R^1) \right) + w \int_{Q^1 - R^1} g(y)dy + w_a \int_{Q^1 - R^1} y \left[1 - H(Q^1) \right] g(y)dy - c_A - c_B \mu. \quad (A.3)
\]

Then, the authors can derive that

\[
\frac{d^2\Pi^1_m(R^1)}{dR^{12}} = -p_A f(R^1) - \frac{w(Q^1)^2}{R^1} g \left(\frac{Q^1}{R^1} \right) + w_a \left(\frac{Q^1}{R^1} \right) \left(1 - H(Q^1) \right) g \left(\frac{Q^1}{R^1} \right) - w_a \int_{Q^1 - R^1} y^2 h(R^1 y) g(y)dy. \quad (A.4)
\]
Mathematical Problems in Engineering

\[
A_x \quad d^2 \prod_m^{\lambda} (R^y)/d(R^y)^2 \mid_{H(Q^{\lambda})=\lambda/(1+\lambda)} = -p_A f(R) - w_A \int_{\lambda}^{Q^{\lambda}/R^y} y^2 h(R^y) g(y)dy < 0, \] quasi-concave in \( R^y \), and the optimal production quantity \( R^y \) satisfies the following equation:

\[
p_A (1 - F(R^y)) + w \int_A^{Q^{\lambda}/R^y} y g(y)dy + w_A \int_{\lambda}^{Q^{\lambda}/R^y} y \left[1 - H(R^y)\right] g(y)dy - c_A - c_B \mu = 0. \tag{A.7}
\]

A.2. ABD with Revenue Sharing Contract

After simplification, the retailer’s expected profit is

\[
\prod_{\lambda}^{Q^{\lambda}} (R^y) = (\phi f y - w_0) \int_A^{R^y} \int_0^{\lambda} y h(z)dz + \int_0^{\lambda} y f R^y h(z)dz \mid_{Q^{\lambda}/R^y} g(y)dy
\]

+ \( w \int_{\lambda}^{Q^{\lambda}/R^y} R^y g(y)dy + \int_{Q^{\lambda}/R^y} R^y h(z)dz \mid_{Q^{\lambda}/R^y} g(y)dy \)

+ \( w_A \int_{R^y}^{Q^{\lambda}/R^y} \int_0^{R^y} y h(z)dz + \int_0^{R^y} y f R^y h(z)dz \mid_{Q^{\lambda}/R^y} g(y)dy \)

+ \( w_A \int_{Q^{\lambda}/R^y} R^y h(z)dz + \int_{Q^{\lambda}/R^y} R^y h(z)dz \mid_{Q^{\lambda}/R^y} g(y)dy \)

- \( c_A R^y - c_B R^y \mu \). \tag{A.8}

Similar to the analysis of ABD contract, the retailer’s expected profit \( \prod_{\lambda}^{Q^{\lambda}} (R^y) \) is quasi-concave in \( Q^{\lambda} \) and the optimal \( Q^{\lambda} \) satisfies the following equation:

\[
\prod_{\lambda}^{Q^{\lambda}} (R^y) = p_A \left\{ \int_0^{R^y} x f(x)dx + R^y - R^y \mu \right\} + w \left\{ \int_A^{R^y} y f(y)dy + \int_{Q^{\lambda}/R^y} R^y g(y)dy \right\}
\]

+ \( (w_A + (1 - \phi) p_B) \int_{Q^{\lambda}/R^y} R^y h(z)dz + \int_{Q^{\lambda}/R^y} R^y h(z)dz \mid_{Q^{\lambda}/R^y} g(y)dy \)

- \( c_A R^y - c_B R^y \mu \). \tag{A.10}

Then, the authors can derive that

\[
\frac{d \prod_{\lambda}^{Q^{\lambda}} (R^y)}{d(R^y)} = p_A (1 - F(R^y)) + w \int_A^{Q^{\lambda}/R^y} y g(y)dy + (1 - \phi)p_B \int_{Q^{\lambda}/R^y} y \left[1 - H(R^y)\right] g(y)dy \tag{A.11}
\]

+ \( w_A \int_{Q^{\lambda}/R^y} y \left[1 - H(R^y)\right] g(y)dy - c_A - c_B \mu \),

\[
\frac{d^2 \prod_{\lambda}^{Q^{\lambda}} (R^y)}{d(R^y)^3} = -p_A f(R) - \frac{w_A (Q^{\lambda} - R^y)^2}{(Q^{\lambda}/R^y)^3} \left[ \frac{Q^{\lambda} - R^y}{R^y} \right] - (1 - \phi)p_B \int_A^{Q^{\lambda}/R^y} y^2 h(R^y) g(y)dy
\]

+ \( w_A (Q^{\lambda}/R^y)^2 \left[ \frac{Q^{\lambda} - R^y}{R^y} \right] - (1 - \phi)p_B \int_{Q^{\lambda}/R^y} y^2 h(R^y) g(y)dy \). \tag{A.12}
As $d^2 \prod_{m}^{\lambda} (R^{\lambda})^{\gamma} / d (R^{\lambda})^{\gamma} |_{H(Q^{\lambda})=\lambda} = -p_A f (R^{\lambda}) - w_a \int_{Q^{\lambda}}^{\hat{Q}^{\lambda}} y^2 h (R^{\lambda}) g (y) dy - (1 - \phi) p_B \int_{A}^{B} y^2 h (R^{\lambda}) g (y) dy < 0$. $\prod_{m}^{\lambda} (R^{\lambda})$ is quasi-concave in $R^{\lambda}$, and the optimal production quantity of the core product $R^{\lambda,*}$ satisfies the following equation:

$$p_A (1 - F (R^{\lambda,*})) + w \int_{A}^{B} y g (y) dy + (1 - \phi) p_B \int_{A}^{B} y [1 - H (R^{\lambda,*})] g (y) dy$$

(A.13)

Proof of Lemma 1. The total expected profit of the decentralized supply chain is

$$\Pi_t (R^d, Q^d) = E [p_A \min (R^d, D_A) + p_B \min (R^d, D_B) - c_A R^d - c_B R^d]$$

$$= E [p_A \min (R^d, D_A) + p_B \min (R^d, D_B) - p_B \min (R^d, D_B) - Q^d] - c_A R^d - c_B R^d$$

(A.14)

$$\leq E [p_A \min (R^d, D_A) + p_B \min (R^d, D_B) - c_A R^d - c_B R^d] \leq \Pi_t (R^*)$$

then $\Pi_t (R^d, Q^d)$ equals to $\Pi_t (R^*)$ unless $R^d = R^*$ and $Q^d \geq R^d B$.

Proof of Proposition 1. Substituting the optimal solutions $R^{\lambda,*}$ and $Q^{\lambda,*}$ into the total expected profit of the supply chain under the combined contract, the authors can get

$$\prod_{m}^{\lambda} (R^{\lambda,*}) = E [p_A \min (R^{\lambda,*}, D_A) + p_B \min (R^{\lambda,*}, D_B) - c_A R^{\lambda,*} - c_B R^{\lambda,*}]$$

(A.15)

Thus, if and only if $R^* = R^*$, the authors can get that $\prod_{m}^{\lambda} (R^{\lambda,*}) = \prod_{m}^{\lambda} (R^*)$. Then, from (4) and (14), to coordinate the channel, the contract terms should satisfy the following equation:

$$w \int_{A}^{B} y g (y) dy - \varphi (1 - H (R^*) g (y) dy$$

$$+ w_a \int_{Q^{\lambda,*}}^{\hat{Q}^{\lambda,*}} y [1 - H (R^*)] g (y) dy = 0$$

(A.16)

Using (A.16), the authors can derive that

$$\varphi (\lambda) = \frac{w \int_{A}^{B} y g (y) dy + (1 + \lambda) w \int_{Q^{\lambda,*}}^{\hat{Q}^{\lambda,*}} y [1 - H (R^*)] g (y) dy}{p_B \int_{A}^{B} y [1 - H (R^*)] g (y) dy}$$

(A.17)

Taking the first-derivation with respect to $\lambda$, the authors can get
\[ \frac{d\varphi(\lambda)}{d\lambda} = \left( \frac{w^{Q^{\varphi}\ast}/(R^{\varphi\ast})^2}{dQ^{\varphi}\ast/d\lambda} - \left( 1 + \lambda \right) w^{Q^{\varphi}\ast}/(R^{\varphi\ast})^2 \right) \frac{dQ^{\varphi}\ast/d\lambda}{1 - H(Q^{\varphi\ast})} + \frac{w^{Q^{\varphi}\ast}/R^{\varphi\ast}}{dQ^{\varphi}\ast/d\lambda} \frac{y[1 - H(R^{\varphi\ast})]g(y)dy}{P_B \int_A y[1 - H(R^{\varphi\ast})]g(y)dy} \]

(A.18)

As \( H(Q^{\varphi\ast}) = \lambda/1 + \lambda, \) \( \frac{d\varphi(\lambda)}{d\lambda} = (w \int_B^{Q^{\varphi\ast}/R^{\varphi\ast}} y[1 - H(R^{\varphi\ast})]g(y)dy/P_B \int_A y[1 - H(R^{\varphi\ast})]g(y)dy) \), \( H \) is a function of \( \lambda \) with \( \lambda \).

Considering the combined contract, the authors have \( w < w_a < \varphi P_B \), so the sets of contract terms should be in the range of \( w/P_B < \varphi < 1 \) and \( 0 < \lambda < (\varphi P_B - w)/w < (P_B - w)/w \). Then, when \( \lambda = (P_B - w)/w, \) the authors have

\[ \varphi \left( \frac{P_B - w}{w} \right) = \frac{w \int_A^{Q^{\varphi\ast}/R^{\varphi\ast}} yg(y)dy + \int_B^{Q^{\varphi\ast}/R^{\varphi\ast}} y[1 - H(R^{\varphi\ast})]g(y)dy}{P_B \int_A y[1 - H(R^{\varphi\ast})]g(y)dy} \]

(A.19)

When \( \lambda = 0, \)

\[ \varphi(0) = \frac{w \int_A^{Q^{\varphi\ast}/R^{\varphi\ast}} yg(y)dy + \int_B^{Q^{\varphi\ast}/R^{\varphi\ast}} y[1 - H(R^{\varphi\ast})]g(y)dy}{P_B \int_A y[1 - H(R^{\varphi\ast})]g(y)dy} \]

(A.20)

From the above analysis, the authors can derive that \( w/P_B < \varphi(0) < \varphi((\varphi P_B - w)/w) < \varphi((P_B - w)/w) < 1 \). Therefore, the authors can derive that when \( 0 < \lambda < (\varphi P_B - w)/w, \varphi \) is in the interval \((w/P_B, 1)\), and the contract terms that satisfy equation (A.16) can achieve the optimal total supply chain’s profit.

\[ \square \]

**Proof of Proposition 2.** Substituting (A.16) into \( \prod_m^{l \ast P} (R^{l \ast P}) \), the authors can derive

\[ \prod_m^{l \ast P} (R^{l \ast P}) = P_A \int_0^{K^\ast} xf(x)dx + \left( 1 - \varphi^\ast \right) P_B \int_A^{R^\ast} \int_B^{y \in R^\ast} zH(z)dz \right) g(y)dy + \left( 1 + \lambda \right) w\int_B^{Q^{\varphi\ast}/R^{\varphi\ast}} \int_A^{R^\ast} zH(z)dz \right) g(y)dy \]

\[ = \left[ \prod_T (R^\ast) - P_A \int_0^{K^\ast} xf(x)dx \right] \left( 1 - \varphi^\ast + \left( 1 + \lambda \right) w\frac{\beta}{P_B} \frac{P_A}{P_B} \right) \]

\[ = \delta \left[ \prod_T (R^\ast) - A \right] \]

(A.21)
where \( \delta = 1 - \varphi^* + (1 + \lambda^*) \omega^* \frac{\beta}{p_B} + \xi \frac{p_A}{p_B} \), \( A = p_A \int_0^R x f(x)dx, \beta = \int_{Q^*}^* x^* R^B [\int_{Q^*}^* dx] z \frac{h(z)dz}{\int_0^R x f(x)dx/\int_{Q^*}^* dz} g(y)dy \), and \( \xi = \int_0^R x f(x)dx/\int_{Q^*}^* dz \frac{h(z)dz}{\int_0^R x f(x)dx} g(y)dy \).

Further, \( \Pi^V (Q^{(1 ')\varphi^{* '}}) = \Pi^T (R^*) - \Pi^T (Q^{(1 ')\varphi^*}) = (1 - \delta) \Pi^C (R^*) + A \delta. \)

\[ \text{Proof of Proposition 3.} \text{ Set } \Phi(\delta) = \left[ \prod_{n=0}^m \left( \prod_{i=1}^V (R^*) - \prod_{i=1}^V (R^i) \right) \right] \left[ \prod_{i=1}^V (Q^*) - \prod_{i=1}^V (Q^i) \right] = \left[ \delta (1 - \varphi^*) \varphi^* C^T (R^*) - A \right] - \delta^2 \prod_{n=0}^m (R^*) - A^2 \geq 0 \right. \)

\( (1 - \delta) \prod_{n=0}^m (R^*) + A \delta \geq \prod_{n=0}^m (Q^*) \), the authors identify the range of \( \delta \) which lead to Pareto-improvement \( \Pi^*_m (R^*) \geq \Pi^*_m (Q^*) \). As \( \delta^* = \prod_{n=0}^m (R^*) - \prod_{n=0}^m (Q^*) \geq 0 \), the authors can identify that \( \delta^* \) is in the Pareto-improving range.

B. Numerical Analysis

Solutions for standard deviation under different values are shown in Tables 2–6.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

The research was supported by the National Natural Science Foundation of China (Nos. 72171047 and 71771053), the Natural Science Foundation of Jiangsu Province (No. BK20201144), and the Key Project of Social Science Foundation of Jiangsu Province (No. 21GLA002).

References


