Research Article

Analytical Method of Calculating Reliability Sensitivity for Space Capsule Life Support Systems

Emad Kareem Mutar

Department of Mathematics, Directorate of Education Babylon, Babylon 51005, Iraq

Correspondence should be addressed to Emad Kareem Mutar; emad77math@gmail.com

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Several fields within the social and technical sciences have identified the importance of theoretical and practical research into complex systems. The sensitivity of these systems is one of the primary focuses. The capsule’s high-pressure oxygen supply system (HPOSS) prepares a complex system with complex interconnections. The reliability of HPOSS cannot be reduced to a single equivalent component or block by combining series and parallel reductions. The aim of this paper is to demonstrate the application of mathematical detection tests to HPOSS in order to assess the sensitivity of the reliability of complex systems. Using the matrix-based minimal cut methodology and the linear sensitivity model of system reliability, the critical system illustrates the suggested methodologies and their relevance to investigating the sensitivity of complex interconnected systems.

1. Introduction

Complex systems analysis has become highly relevant in many areas of technological research. Engineering and technology systems design and operation experts have a substantial effect on these developments. A combination of series and parallel reductions cannot reduce these systems to single equivalent components or blocks. The summing of the probability of all operating states of the system can be used to get the optimal reliability parameters for the system with complex interconnections. The matrix-based minimal cut method is used to transform a complex system into a parallel-series system consisting of all minimal cut sets to facilitate the calculation of the reliability and uncertainty of the system [1–4].

There are several study books and articles in the engineering literature that analyze complex systems, networks, and their reliability theories from both theoretical and practical perspectives. For illustrate, Tillman et al. [5] based on the reliability of a space capsule’s life support system, which serves as a high-pressure oxygen delivery system for a spacecraft (HPOSS). High-pressure oxygen is delivered to the cabin via a series of regulators and valves from a high-pressure oxygen tank. Two pairs of check valves, shut-off valves, and nonreturn automatic shut-off valves comprise the system. These valves are meant to limit the reverse passage of air from the cabin to the gas tank in the case of low pressure in the headline or cabin, thus reducing gas waste. Aggarwal [1] illustrates a general system that results in a nonseries-parallel logic diagram with a complicated arrangement of components.

Myers [6] concentrated on modeling the reliability of complex multichannel systems such as the digital fly-by-wire aircraft control system. In addition, Horvath [7] detailed several important measurements of the properties of complex networks using the Cxnet Complex Network Analyzer Software, as well as measures he gathered through scientific work. Mi et al. [8] offered a conventional reliability analysis, such as the truth table technique, based on the premise that occurrences are binary, i.e., success or failure. Hassan and Mutar [9] studied the design of electrical device reliability models (from a geometry perspective) used within spacecraft, namely, the high-pressure oxygen supply system (HPOSS). With the exception of proven methodologies for assessing system reliability, complexity is very variable. The difficulty of the equations is important because the efficiency
of the approach is calculated using standard matrix computations. Kumar et al. [10] analyzed the reliability cost optimization of a space capsule’s life support system using a multiobjective gray wolf optimizer algorithm. Negi et al. [11] use a hybrid PSO-GWO algorithm (HPSOOGWO) to solve the reliability allocation and optimization problems of the complex bridge system and the life support system in the space capsule.

A system under study can be represented as a graph with its components (vertices and edges) considered binary objects, and its success or failure can be determined. The connection between any two vertices in a binary system can be expressed as a Boolean function [1, 2, 12, 13]. A complicated system will be modeled as a directed graph [13]. This system requires efficiently and methodically calculating the probability that at least one path exists between any two nodes, which is known as the source of terminal reliability [4]. Two-terminal reliability is a critical aspect of system design and maintenance. Consider communication networks as an example [14].

The probability of correctly sending data from source to sink may be thought of as the two-terminal reliability. Earlier techniques relied heavily on counting minimal paths (or minimal cuts) or on decomposition theory. Mutar [3] described a method for deriving minimal cut sets from minimal path sets in order to generate the Incidence Matrix, which was then compared to the system’s truth table. This comparison, which is based on some algebraic principles, results in the minimal-cut sets for the complex system. By algebraically transposing an incidence matrix with the truth matrix, the system’s matrix-based minimal cut structure can be produced. It is frequently utilized to determine the system’s exact reliability.

A sensitivity analysis is used to determine the sensitivity of a reliability model to changes in input parameters, including component reliability [15]. When small changes in component reliability result in big changes in system reliability, the model is said to have been sensitive to that parameter. Changes can be actual, anticipated, or hypothetical. Sensitivity analyses are often done to find out how the outputs of a calculation or evaluation depend on the inputs and to guide future experimental research on how to improve the input values to improve the output values [16].

Pokorádi [17] made Linear Fault Tree Sensitivity Models (LFTSM) and Linear Sensitivity Models of System Reliability (LSMoSR) by using a linear mathematical analytical modeling method. These are tools that approach linearized data using a matrix-algebraic method. Daneshkhah et al. [18] provide a novel method for analyzing the availability sensitivity analysis. Presented is an alternate sensitivity analysis of the quantities of interest in the reliability study, such as the availability/unavailability function, with regard to the modifications of unknown parameters. Oakkley [19] first developed this approach to analyze the sensitivity analysis of a complicated model with regard to changes in its inputs using an emulator that approximates the model. Pokorádi and Seebauer [20] use the True Table Method and the Linear Sensitivity Model of System Reliability to compute and analyze the reliabilities and sensitivities of bridge structure systems. Zhang et al. [21] presented an analytical calculation approach for the reliability sensitivity indexes of distribution systems to explicitly quantify the effect of numerous influencing variables on system reliability.

The primary aim of this paper is to modify mathematical diagnostic procedures for a life support system installed in a space capsule in order to establish the system’s reliability and sensitivity as a complex interconnected system. The matrix-based minimal cut approach, which is based on certain algebraic principles, generates minimal cut sets for the complex system using a Mathematica algorithm. In addition, the minimal cut sets properly represent a system’s operational state and are equal to the knowledge of the structure and function of a complex system, such as a Wheatstone-like complex system. The study discusses the theoretical foundations of a suggested approach and its relevance to the investigation of the reliability of (HPOSS) through the use of many cases. The remainder of the paper is structured as follows: it illustrates how system reliability is determined in three scenarios and discusses sensitivity analysis in general and in the critical system cases. Finally, we analyze the findings about the suggested methodologies and the sensitivity of the examined system’s reliability.

2. Reliability of Life Support System with Complex Interconnections

The reliability block diagram of a system is a graph whose edges represent the components of the system, whereas the standby supply diagram comprises a pair of nodes called terminal nodes. This defines the functional relationship between the components and indicates if there exists a path between the terminal nodes that is wholly composed of functional component edges (making, consequently, the entire system functional; in the contrary case, this is nonfunctional). It shows the functional connection between the components and implies the availability of a path between the terminal nodes that is completely formed of functional edges. Otherwise, it will be rendered inoperable. The graphic model shows the structure of the system’s reliability, which can be described as either series, parallel, or complex [1, 12].

System S is made of n components X1, X2, X3, . . . , Xn, each of which is operational or failing in just one of two states. After that, we can define Boolean (binary) indicator variables for each component or system. The structure and connecting pathways of the system define the system’s reliability structure, that might or might not correlate to the system’s functional block diagram. Modules can then be joined in a complicated structure, as illustrated in Figure 1 [1, 5, 9, 12, 22].

Consider a complex system in Figure 1 with the assumption the high-pressure oxygen tank as the source and the cabin as the sink.

The system is mathematically represented as a graph G = (V, E), where V = {a, b, . . . , g} and E = {1, 2, . . . , 9, α1, α2}. Here, edges α1 and α2 are not permanent and do not represent system components where a and g are the two-terminal graph. The graph G in Figure 2 shows the reliability block diagram and is a simple, connected and all edges are
directed and has all of its edges pointing in the same direction. To discover the structural function between the source and sink components.

Let us first analyze the system’s reliability characteristics, as shown in Figure 2, to demonstrate the proposed method for evaluating sensitivity. The Complex system (HPOSS) has nine components \( X_1, X_2, \ldots, X_9 \) are independent random variables with the probability of each component \( \Pr(X_i) = R_i \), then \( R_s \) is a function of the reliabilities of the components \( R_1, R_2, \ldots, R_9 \). In this case, letting \( R_s = (R_1, R_2, \ldots, R_9) \), the system’s reliability can be represented as follows:

\[
R_s = h(R) \quad \text{if} \quad R = R_1 = R_2 = \cdots = R_9. \tag{1}
\]

If \( (R_1, R_2, R_3, \ldots, R_9) \) are independent identical. The system’s unreliability \( Q_i \) is given by the following equation:

\[
Q_i = 1 - R_i. \tag{2}
\]

The components’ reliability and failure probability should be structured as vectors for simplicity of use in further application:

\[
R^T = [R_1, R_2, R_3, R_4, R_5, R_6, R_7, R_8, R_9]. \tag{3}
\]

\[
Q^T = [Q_1, Q_2, Q_3, Q_4, Q_5, Q_6, Q_7, Q_8, Q_9]. \tag{4}
\]

2.1. Minimal Paths of System Reliability. The Incidence matrix (IM) represents all of the potential states of the system, with each component either a good or a fault condition. Assume that n-minimal paths are a special case of

\[
\begin{align*}
\text{Figure 1: Space capsule’s life support system.} \\
\text{Figure 2: A graph G of space capsule’s life support system.}
\end{align*}
\]
the paths problem in row. The minimal paths are defined by $P_1, P_2, \ldots, P_n$, the $P_i$ is the shortest path between two-terminal systems [3, 13]. Each system state’s probability is represented by the component $X_i$. Given that the (IM) matrix covers all possible minimal path choices, We calculate the HPOSS incidence matrix (IM), which has the following form:

$$
IM = \begin{bmatrix}
P_1 & X_1 & X_2 & X_3 & X_4 & X_5 & X_6 & X_7 & X_8 & X_9 \\
P_2 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
P_3 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\
P_4 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
P_5 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
P_6 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\
P_7 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\
P_8 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\
\end{bmatrix}. \tag{5}
$$

The components $X_i$ can be in operation (defined as the 1 state) or inactive (defined as the 0 state). As a result, the n-component system has a mapping $\phi: \{0, 1\}^n \rightarrow \{0, 1\}$ of potential states termed order of the system (in our example, $2^9 = 512$). We only evaluated at the minimal paths because they indicate the true status of the system’s operation and the absence of energy. The system’s all-minimal path sets are

$$
P_1 = \{X_1, X_5\}, \\
P_2 = \{X_3, X_7\}, \\
P_3 = \{X_2, X_4\}, \\
P_4 = \{X_1, X_8\}, \\
P_5 = \{X_3, X_6, X_9\}, \\
P_6 = \{X_2, X_4, X_8\}, \\
P_7 = \{X_2, X_6, X_9\}. \tag{6}
$$

The operating system’s state probabilities are shown in the rows of incidence matrix in equation (5) the reliability of the system can be estimated using the following formula:

$$
R_{system} = 1 - \sum_{j=1}^{8} (1 - P_j). \tag{7}
$$

The reliability of the system is calculated by equation (7) as a parallel configuration of all minimal paths included in these rows. As shown in Figure 3, the system’s reliability is determined by the component’s reliability.

If a single component fails in a critical state, the system becomes inoperable [20, 22–24]. There is no redundancy in the most critical system. Use the formula to determine if your system is in critical condition.

$$
R_{critical} = \prod_{j=1}^{8} (1 - P_j). \tag{8}
$$

The critical system has variable component reliability. As component reliabilities increase, so does the asymptotic probability of failures. Figure 4 illustrates the reliability $R_{critical}$ of the system by equation (8). Improvements in HPOSS reliability estimate the probability of redundancy.

2.2. Minimal Cuts of System Reliability. System failure or unreliability is the probability that component 1 fails, component 2 fails, and all of the other components fail in a system with $n$ independent random parallel components [3, 14, 25]. The matrix can be used to represent the failure conditions of all components because it comprises all minimal cuts in rows and the components are sorted sequentially according to the columns. As a result, the (HPOSS) CM matrix has the following form:

$$
MC = \begin{bmatrix}
X_1 & X_2 & X_3 & X_4 & X_5 & X_6 & X_7 & X_8 & X_9 \\
C_1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\
C_2 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\
C_3 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 \\
C_4 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\
C_5 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\
C_6 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\
C_7 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\
C_8 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\
C_9 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}. \tag{9}
$$

There are nine minimal cuts can be obtained.
5: Unreliability of the system in the case of different component’s unreliability.

\[ C_1 = \{X_2X_7X_8X_9\}, \]
\[ C_2 = \{X_2X_6X_8\}, \]
\[ C_3 = \{X_3X_5X_9\}, \]
\[ C_4 = \{X_4X_5X_7\}, \]
\[ C_5 = \{X_2X_5X_8\}, \]
\[ C_6 = \{X_2X_3X_4X_5\}, \]
\[ C_7 = \{X_1X_3X_5\}, \]
\[ C_8 = \{X_1X_2X_4X_7\}, \]
\[ C_9 = \{X_1X_2X_3\}. \]

As a result, the unreliability system state can be estimated using the following formula:

\[ Q_{\text{system}} = \prod_{j=1}^{9} (1 - C_j). \]  

7: Critical unreliability of the system in the case of different component’s unreliability.

\[ R_{\text{system}} = R_1R_8 + R_2R_5 - R_1R_5R_6 + R_3R_7 + R_1R_4R_7 \]
\[ - R_1R_5R_6R_7 - R_1R_5R_6R_8 + R_1R_2R_3R_7R_8 + R_1R_4R_8 \]
\[ + R_2R_3R_5R_6 - R_1R_2R_3R_6R_8 - R_1R_2R_3R_8 - R_1R_2R_4R_6 \]
\[ + R_1R_2R_5R_6R_7R_8 + R_1R_2R_5R_6R_8 - R_1R_2R_5R_7R_8 \]
\[ + R_1R_2R_5R_6R_7R_8R_9 + R_1R_2R_5R_6R_7R_8R_9 - R_1R_2R_5R_6R_7R_8R_9 \]
\[ + R_1R_2R_5R_6R_7R_8R_9R_8 - R_1R_2R_5R_6R_7R_8R_9R_9 \]
\[ - R_1R_2R_5R_6R_7R_8R_9R_9R_9. \]  

As a result, the critical system unreliability in equation (12)

\[ Q_{\text{critical}} = 1 - \prod_{j=1}^{9} (1 - C_j). \]

3. Sensitivity Analysis

The theoretical technique for creating LSMoSU is discussed in detail in references [14, 17, 20]. Following that, the sensitivity coefficients must be determined. The probability of possible system states of all minimal paths in rows in equation (5) can be expressed in the following manner:

\[ P_j = \prod_{i=1}^{9} U_i(R_i). \]
Using the minimal cuts matrix (8), defines the probability of possible system states.

\[ C_j = \prod_{i=1}^{9} U_i(R_i) \tag{16} \]

where \( U_i \) is the inner function can take one of two forms. The state of a component and the state of the system can be the same or inverse i.e., the states are identical. If the state of the component is identical to the state of the examined system, the sensitivity \( U_i = R_i \) coefficient is as follows:

\[ K_{ji} = 1. \tag{17} \]

If the component’s state is complementary to the analyzed system’s states, the inner function, as defined in equation (1), \( U_i = 1 - R_i \), then the sensitivity coefficient is as follows:

\[ K_{ji} = \frac{R_i}{P_j} \prod_{k=1/k\neq i}^{n} U_k, \tag{18} \]

\[ K_{ji} = \frac{R_i}{C_j} \prod_{k=1/k\neq i}^{n} U_k. \tag{19} \]

These coefficients can be obtained by applying equations (7) and (11) to functions that explicitly define probabilistic system parameters.

\[ K_j = \frac{P_j}{R_{\text{system}}}, \tag{20} \]

\[ K_j = \frac{C_j}{Q_{\text{system}}}. \tag{21} \]

Our analysis functions as dependent variables in both probabilistic system parameters and theoretical system states. The probabilistic parameters of the components are known as independent parameters [15, 17]. The relationship between the independent variable and the dependent variable can then be described by an equation.

\[ A\delta Y = B\delta X, \tag{22} \]

where the coefficient matrices \( A \in \mathbb{R}^{mxn} \) and \( B \in \mathbb{R}^{mxn} \) represent the independent and dependent parameters, respectively, and the vectors \( \delta X \in \mathbb{R}^{n} \), \( \delta Y \in \mathbb{R}^{m} \) represent the relative change in the independent and dependent parameters, and \( n \) and \( m \in \mathbb{N} \) are the number of dependent parameters, respectively. Utilize

\[ D = A^{-1} B, \quad D \in \mathbb{R}^{mxn}. \tag{23} \]

As a result, the matrix of the system’s relative sensitivity coefficients, the equation

\[ \delta Y = D\delta X. \tag{24} \]

Useful for relative sensitivity [21]. Component reliabilities are included in the independent parameter vector.

\[ \delta X^T = [ \delta R_1 \delta R_2 \delta R_3 \delta R_4 \delta R_5 \delta R_6 \delta R_7 \delta R_8 ] \tag{25} \]

And, the vector representing the dependent parameters’ relative changes as given in equation (7):

\[ \delta Y^T = [ \delta R_{\text{sys}} \delta P_1 \delta P_2 \delta P_3 \delta P_4 \delta P_5 \delta P_6 \delta P_7 \delta P_8 ]. \tag{26} \]

3.1. Sensitivity Analysis of Critical System Reliability States of HPOSS. The probabilities of system reliability and Critical System Reliability states can make up the dependent parameter vector.

\[ Y_{\text{remal}}^T = \begin{bmatrix} R_{\text{sys}} & P_1 & P_2 & P_3 & P_4 & P_5 & P_6 & P_7 & P_8 \end{bmatrix}. \tag{27} \]

The matrix of dependent parameters’ coefficients transposed:

\[ A_{\text{remal}}^T = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -K_{P_1} & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -K_{P_2} & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -K_{P_3} & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ -K_{P_4} & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ -K_{P_5} & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ -K_{P_6} & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ -K_{P_7} & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ -K_{P_8} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \tag{28} \]
Depending on the reliability of components, the sensitivity of critical system reliability changes [16]. Increased component reliability will minimize reliability’s sensitivity to component reliability (see Figure 7).

### 3.2. Sensitivity Analysis of Critical System Unreliability States of HPOSS.

The deterministic probabilities of system unreliability and Critical System Unreliability States make up the dependent parameter vector.

\[ Y_{Q_{\text{critical}}}^T = \begin{bmatrix} Q_{\text{sys}} & C_1 & C_2 & C_3 & C_4 & C_5 & C_6 & C_7 & C_8 & C_9 \end{bmatrix} \]  

The matrix of dependent parameters’ coefficients transposed:

\[ A_{Q_{\text{critical}}}^T = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-K_{C_1} & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-K_{C_2} & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
-K_{C_3} & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
-K_{C_4} & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
-K_{C_5} & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
-K_{C_6} & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
-K_{C_7} & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
-K_{C_8} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
-K_{C_9} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \]
The sensitivity of critical system’s unreliability changes depending on the unreliability of components. The crucial unreliability’s sensitivity to component reliability will increase as a result of increased component unreliability. As seen in Figure 8.

The system was equally sensitive to the reliability parameters of its components. Elements’ reliability parameters were identical in all situations evaluated. This statement may be erroneous if the element parameters change.

4. Conclusions

(1) Create a matrix of all minimal paths (IM) for a complex system and use it to determine the system’s reliability. They also determine the system’s critical reliability (failure status). On the other hand, the generation of the (CM) matrix is used to determine the system’s unreliability, which indicates system failure, as well as the critical unreliability, which is used to determine the system’s sensitivity.

(2) System reliability approaches 1 as component reliability increases (see Figure 3). The probability of critical system reliability decreases asymptotically as the relative sensitivity of system reliability to component reliability increases.

(3) System reliability is less sensitive to component reliability as component reliability increases (see Figure 6). The number of possible system states increases exponentially with element complexity. Adding one component increases the range of possible system states.

(4) The sensitivity of a critical unreliability system increases with the increasing unreliability of system components. Critical unreliability determines the sensitivity of the components responsible for the device failure (see Figure 7).

(5) The structural sensitivity coefficient shows a component’s function from a systems perspective. Its location in the system determines its value.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The author declares that there are no conflicts of interest regarding the publication of this paper.
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