

## Research Article

# Fruit Production Planning in Semiarid Zones: A Novel Triangular Intuitionistic Fuzzy Linear Programming Approach

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Received 13 November 2021; Accepted 11 January 2022; Published 12 February 2022

Academic Editor: Cristiano Fragassa

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A triangular intuitionistic fuzzy linear programming (TIFLP) model is formulated for the planning of sustainable fruit production system for hyperarid regions while assuming the availability of resources and existing knowledge. A remarkable advancement is achieved through the composition of intuitionistic fuzzy concept with the linear programming by considering all parameters and variables in the form of triangular intuitionistic fuzzy numbers, which provides a planning or strategic tool for handling uncertain situations with more control and in a realistic way. This fuzzy optimization model is redesigning the feasible region obtained by linear programming which is presented in graphical form. Moreover, the practical application and implementation of this fruit production system for planning in real-life scenarios are accomplished considering the case study of fruit orchards of Baluchistan, Pakistan.

## 1. Introduction

Have you ever imagined experiencing the world without agriculture? In that case, most of the world's population could not outlive hunger, and the remaining ones would be hunting for food. In fact, you would no longer be here to read this paper because the path of modern civilization would be lost forever with the absence of agriculture. Agriculture is art, science, and business of all types of crop production which flourished into seven major branches named as agronomy, horticulture, forestry, animal husbandry, agricultural engineering, fishery, and home science [1]. The beginning of human civilization started with agricultural development referred to as first agricultural revolution. Later on, agriculture and farming spread into different regions around the world and broadened with livestock, industrial agriculture, agronomy, and much more. The history of human civilization is reflected by the inventions, methods, and techniques used to enhance the

agriculture and its different branches in a productive manner. Throughout modification in agricultural field, it has been improved and transformed into much more ultra-modern form known as “sustainable agriculture” which equally impacts the environment, society, and economy [2].

The ultimate motive of sustainable agriculture is the satisfaction of all human needs and necessities with the major contribution to economy in healthy environmental conditions. The improvement of our food security system is the mostly targeted goal for the betterment of present and future generation. The sustainable development goal is the eradication of hunger by accomplishing food security and improving the nutrition intake by 2030 [3]. A thorough analysis was carried out about the achievement of “zero hunger” goal by studying all the existing scientific literature to assess their contribution to the achievement of the sustainable development goal [4]. According to a latest study, the fourth agricultural revolution demands the balance between the agricultural production and world's population

together with the environment [5]. To eradicate the undernourishment of the world, fruit consumption rate of the world per capita should be according to diverging health conditions. The low intake of fruit and vegetable increases the worldwide burden of disease, which can be controlled through the ample amount of fruit consumption and production [6]. Analytical study reveals that approximately 22% of difference exists between the demand and supply of fruit production, whereas this percentage increases to 58% for the underdeveloped countries, which is increasing with the passage of time [7].

Pakistan, being a middle-income developing country, produces five major crops, wheat, rice, sugarcane, maize, and cotton, along with the most importantly fruits and vegetables with pulses and oilseeds [8]. The production of fruits and vegetables is approximately 12 million tons per year. More precisely, fruits contribute 2.48% to agricultural gross domestic product of Pakistan, producing apples, mangoes, grapes, dates, citrus, peaches, cherries, plums, loquat, pears, and guava. According to a rough analysis, Pakistan earned \$730 million by exporting 1.165 million tons of fruits and vegetables in a year [9]. The study of Pakistan recommends investing in research and development to find innovative strategies to enhance production and quality and reduce postharvest losses in order to boost fruit and vegetable export competitiveness [10]. The global horticultural products trade for the past two decades was maximized by four times by making earnings of USD 51 billion in 2001 to USD 200 billion in 2018 [11]. The international trade competitiveness of Pakistan is evaluated through the analysis of competitive and comparative demand and supply of vegetables and fruits [12]. The overwhelming pressure on the demand of food security caused by population increase and global development results in the destruction of natural resources and food crises [13]. Additionally, COVID-19 and intense climate changes severely escalate the demand of food by decreasing the average agricultural production [14].

Real-life situations can be assessed mathematically. For modeling and management of certain scenarios, mathematical analysis of real-life occurrences utilized quantitative and qualitative methodologies. Linear programming is a generalized and renowned technique presented by Kantorovich [15] to optimize agricultural aims and objectives by allocation and restriction of certain demand and availability constraints [16]. In light of our current agricultural requirements, our objective is not only food supply but also the ample amount and quality of food provision around the world. Therefore, agricultural planning is carried out for this goal using operational mathematical approaches in the most efficient way in order to eliminate food security issues [17, 18]. It is used as a single objective as well as multiple objectives to minimize and maximize the cost and profit by the utilization and management of natural resources, labor, techniques, research, capital regarding land allocation, cropping patterns, optimization of water resources, raising livestock, and production maximization with cost minimization [19].

Food production system must be thoroughly modified and armed with resilience and adaptivity and have high

diversity against different situations and factors (climate change, pest attacks and diseases, governmental policies at national and international level, social and cultural stability factors) [20]. For perfection in the precision of goals regarding planning, this area still needs much more modifications in terms of changing environmental, ecological, and social factors [21]. Globally, agricultural output continuously confronts drastic fluctuations due to which sustainable agriculture is constantly evolving with the passage of time and demand of the world is changing continuously regarding various aspects. These factors generate uncertainly and vagueness in environment, which is assessed by using the concept of fuzzy sets introduced by Zadeh [22]. Indeed, fuzzy set and its generalizations such as intuitionistic fuzzy sets [23] are utilized to present data that is fuzzy in nature. Eventually, fuzzy optimization theory was initiated by Zimmermann for effective decision making in fuzzy environment [24].

Fuzzy linear programming approach was further investigated through meticulous application to decision making and management problems considered in uncertain environment, and it obtained much more precise and feasible output [25]. Under unpredictable circumstances in energy-water nexus, an integrated fuzzy optimization approach was proposed for agricultural water and land resource management [26]. Multiobjective fuzzy methodology having three goals was considered as maximization of net benefits, agricultural output, and labor employment for Pune city of Maharashtra State, India [27]. Another study was conducted by applying intuitionistic fuzzy optimization technique in agricultural production planning, with a focus on smallholder farmers in north Bihar, India [28].

Specifically, fruit production planning by using linear programming is done, which is generalized for production maximization in hyperarid regions with available resources, labor, capital, etc. Further, in order to evaluate a targeted objective function that stays valid and optimal under the influence of climatic, social, and economic conditions, triangular intuitionistic fuzzy linear programming has been constructed more accurately and meticulously. The article is divided into five sections, where all the basic and essential information is provided in Preliminaries section. The objective function and constraints for optimal fruit production in crisp and intuitionistic fuzzy environment are defined in Methodology section. The model is then applied to a real-life example by considering fruit production data from Baluchistan province of Pakistan. The superiority of the proposed methodology is supported by comparative and postoptimal analysis.

## 2. Preliminaries

**2.1. Fuzzy Set.** Let  $X$  be the universal set. A fuzzy set  $\tilde{A}$  [22] consists of a pair defined as  $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)), x \in X\}$ , in which the first element  $x$  of  $(x, \mu_{\tilde{A}}(x))$  belongs to classical set and the second element defined as  $\mu_{\tilde{A}}(x): X \rightarrow [0, 1]$  refers to the membership degree of  $x$  in  $\tilde{A}$ , called the membership function of  $\tilde{A}$ .

2.2. *Fuzzy Intuitionistic Sets.* Let  $X$  be denoted as a universal set. An intuitionistic fuzzy set (IFS)  $\tilde{A}^I$  [23] is defined as set of ordered triplets  $\tilde{A}^I = \left\{ (x, \mu_A^L(x), \nu_A^L(x)); x \in X \right\}$ , in which the functions  $\mu_A^L(x): X \rightarrow [0, 1]$  and  $\nu_A^L(x): X \rightarrow [0, 1]$  represent membership and nonmembership degree of  $x$  in  $\tilde{A}$ , respectively, for each element  $x \in X$  satisfying  $0 \leq \mu_A^L(x) + \nu_A^L(x) \leq 1$ .

2.3. *Triangular Intuitionistic Fuzzy Number.* A triangular intuitionistic fuzzy number (TIFN) [29]  $\tilde{S}$  is an especial IFN with the membership function and nonmembership function defined as follows:

$$\mu_{\tilde{S}}^{-I}(x) = \begin{cases} 0, & \text{if } x < a, \\ \frac{x-a}{b-a}, & \text{if } a \leq x \leq b, \\ 1, & \text{if } x = b, \\ \frac{c-x}{c-b}, & \text{if } b \leq x \leq c, \\ 0, & \text{if } x > c, \end{cases} \quad (1)$$

$$\nu_{\tilde{S}}^{-I}(x) = \begin{cases} 1, & \text{if } x < d; \\ \frac{b-x}{b-d}, & \text{if } d \leq x \leq b; \\ 0, & \text{if } x = s_2, \\ \frac{x-b}{e-b}, & \text{if } b \leq x \leq e, \\ 1, & \text{if } x > e, \end{cases}$$

where  $d \leq a \leq b \leq c \leq e$ , denoted by  $\tilde{S}^I = (a, b, c; d, b, e)$  or TIFN. Membership and nonmembership functions of TIFN are presented in Figure 1.

2.4. *Accuracy Function.* The accuracy function [30] for triangular intuitionistic fuzzy numbers  $A = (a_1, a_2, a_3; a'_1, a'_2, a'_3)$  is defined as

$$H(\tilde{A}^I) = \frac{(a_1 + 2a_2 + a_3) + (a'_1 + 2a'_2 + a'_3)}{8}. \quad (2)$$

### 3. Operations on Triangular Intuitionistic Fuzzy Number

A triangular intuitionistic fuzzy number  $\tilde{S}^I = (s_1, s_2, s_3; s'_1, s'_2, s'_3)$  is said to be nonnegative if and only if  $s'_i \geq 0$ .

The arithmetic operations of triangular intuitionistic fuzzy number [29], i.e., addition, subtraction,

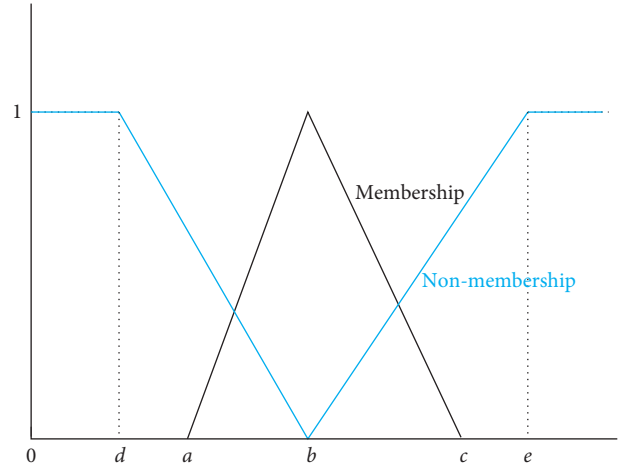


FIGURE 1: Triangular intuitionistic fuzzy number.

multiplications, and division, are defined by considering two nonnegative triangular intuitionistic fuzzy numbers  $\tilde{S}^I = (s_1, s_2, s_3; s'_1, s'_2, s'_3)$  and  $\tilde{R}^I = (r_1, r_2, r_3; r'_1, r'_2, r'_3)$ . Two triangular intuitionistic fuzzy numbers are equal,  $\tilde{S}^I = \tilde{R}^I$ , if and only if  $s_1 = r_1, s_2 = r_2, s_3 = r_3, s'_1 = r'_1,$  and  $s'_3 = r'_3$ .

#### 3.1. Addition

$$\begin{aligned} \tilde{S}^I \oplus \tilde{R}^I &= (s_1, s_2, s_3; s'_1, s'_2, s'_3) \oplus (r_1, r_2, r_3; r'_1, r'_2, r'_3) \\ &= (s_1 + r_1, s_2 + r_2, s_3 + r_3; s'_1 + r'_1, s'_2 + r'_2, s'_3 + r'_3). \end{aligned} \quad (3)$$

#### 3.2. Subtraction

$$\begin{aligned} \tilde{S}^I \ominus \tilde{R}^I &= (s_1, s_2, s_3; s'_1, s'_2, s'_3) \ominus (r_1, r_2, r_3; r'_1, r'_2, r'_3), \\ &= (s_1, s_2, s_3; s'_1, s'_2, s'_3) \ominus (r_1, r_2, r_3; r'_1, r'_2, r'_3). \end{aligned} \quad (4)$$

#### 3.3. Symmetric Property

$$-\left(\tilde{S}^I\right) = (-s_1, -s_2, -s_3; -s'_1, -s'_2, -s'_3). \quad (5)$$

#### 3.4. Scalar Multiplication.

Let  $\alpha$  be any scalar; then,

$$\begin{aligned} \alpha(\tilde{S}^I) &= (\alpha s_1, \alpha s_2, \alpha s_3; \alpha s'_1, \alpha s'_2, \alpha s'_3), \alpha \geq 0, \\ \alpha(\tilde{S}^I) &= (\alpha s_3, \alpha s_2, \alpha s_1; \alpha s'_3, \alpha s'_2, \alpha s'_1), \alpha < 0. \end{aligned} \quad (6)$$

#### 3.5. Multiplication

$$\begin{aligned} \tilde{S}^I \otimes \tilde{R}^I &= (s_1, s_2, s_3; s'_1, s'_2, s'_3) \otimes (r_1, r_2, r_3; r'_1, r'_2, r'_3) \\ &\cong (s_1 r_1, s_2 r_2, s_3 r_3; s'_1 r'_1, s'_2 r'_2, s'_3 r'_3). \end{aligned} \quad (7)$$

*Remark 1.* If  $\tilde{S}$  and  $\tilde{R}$  are not nonnegative triangular fuzzy numbers, then their multiplication will be performed as

$$\tilde{S} \otimes \tilde{R} = (a, b, c; a', b', c'), \quad (8)$$

where

$$\begin{aligned} a &= \min(s_1 r_1, s_1 r_3, s_3 r_1, s_3 r_3), \\ a' &= \min(s'_1 r'_1, s'_1 r'_3, s'_3 r'_1, s'_3 r'_3), \\ b &= s_2 r_2, \\ b' &= s_2 r_2, \\ c &= \max(s_1 r_1, s_1 r_3, s_3 r_1, s_3 r_3), \\ c' &= \max(s'_1 r'_1, s'_1 r'_3, s'_3 r'_1, s'_3 r'_3). \end{aligned} \quad (9)$$

#### 4. Linear Programming Model

General linear programming [16] is defined as

$$(\text{Max})Z(x) = \sum_{i=1}^p \sum_{j=1}^q c_{ij} x_{ij}, \quad (10)$$

subject to the following constraints:

$$\sum_{i=1}^p \sum_{j=1}^q a_{ij} x_{ij} = u_i, \quad (11)$$

Condition of nonnegativity is as follows:

$$x_{ij} \geq 0 \text{ for all } i = 1, 2, \dots, p; j = 1, 2, \dots, q, \quad (12)$$

where  $x_{ij}$ ,  $c_{ij}$ ,  $a_{ij}$ , and  $u_i$  are the decision variables, coefficients of quantity which we have to maximize or minimize, constraints coefficients, and constants, respectively. This represents the crisp modeling of the problem, but for the most beneficial implementation of this model in our daily life problems, we used its modified form “triangular intuitionistic fuzzy linear programming” which is endowed with the generalized techniques for the absorption of fuzziness due to unpredictable and unfortunate scenario.

#### 5. Triangular Intuitionistic Fuzzy Linear Programming Model

Triangular intuitionistic fuzzy linear programming enhances the targeted requirements by evaluating the problem specifications meticulously using the generalization of fuzzy logics intuitionistic fuzzy sets. A triangular intuitionistic fuzzy linear programming [25] can be formulated as follows:

$$(\text{Max})\tilde{Z} = \sum_{i=1}^p \sum_{j=1}^q \tilde{c}_{ij} \otimes \tilde{x}_{ij}, \quad (13)$$

subject to the following constraints:

$$\sum_{i=1}^p \sum_{j=1}^q \tilde{a}_{ij} \otimes \tilde{x}_{ij} = \tilde{u}_i, \quad (14)$$

Condition of nonnegativity is as follows:

$$\tilde{x}_{ij}^I \geq 0 \text{ for all } i = 1, 2, \dots, p; j = 1, 2, \dots, q, \quad (15)$$

where the model contains all coefficients, variables, and constants in the form of triangular intuitionistic fuzzy numbers; for example,  $\tilde{c}_{ij}^I = (c_{ij,1}, c_{ij,2}, c_{ij,3}; c'_{ij,1}, c'_{ij,2}, c'_{ij,3})$ ,  $\tilde{a}_{ij}^I = (a_{ij,1}, a_{ij,2}, a_{ij,3}; a'_{ij,1}, a'_{ij,2}, a'_{ij,3})$ , and  $\tilde{u}_i^I = (u_{i,1}, u_{i,2}, u_{i,3}; u'_{i,1}, u'_{i,2}, u'_{i,3})$  are triangular intuitionistic fuzzy cost coefficients, triangular intuitionistic fuzzy constraints coefficients, and constants, respectively, with  $\tilde{x}_{ij}^I = (x_{ij,1}, x_{ij,2}, x_{ij,3}; x'_{ij,1}, x'_{ij,2}, x'_{ij,3})$  being triangular intuitionistic fuzzy decision variables. Ultimately,  $\tilde{Z}$  is the maximum triangular intuitionistic fuzzy objective value.

#### 6. Methodology

The linear programming for fruit production maximization is developed as

$$(\text{Max})\text{FP}_M(x) = \sum_{i=1}^p \sum_{j=1}^q P_{ij} x_{ij}, \quad (16)$$

subject to the following constraints:

$$\sum_{i=1}^p \sum_{j=1}^q A_{ij} x_{ij} = v_i, \quad (17)$$

Condition of nonnegativity is as follows:

$$x_{ij} \geq 0 \text{ for all } i = 1, 2, \dots, p; j = 1, 2, \dots, q, \quad (18)$$

where  $\text{FP}_M$  is maximized fruit production;  $x_{ij}$  refers to activities (cutting, pruning, harvesting, thinning, leveling, sales, etc.);  $P_{ij}$  indicates objective coefficients (market prices of variables, product profit, etc.);  $A_{ij}$  denotes constraints coefficients (utilized resources and capital per unit of fruit production); and  $v_i$  is the total available amount/units/volume of supplies per hectare.

Generally defined constraints for major fruit production are further written as follows:

$$\begin{aligned} \text{total land availability constraints: } & \sum_{i=1}^h G_i^I \leq \text{TL}, \\ \text{maximum sowing area constraints: } & G_1^I \leq \text{TL}^G, \\ & : G_2^I \leq \text{TL}^A Z, \\ & : G_3^I \leq \text{TL}^C, \\ & : G_4^I \leq \text{TL}^{\text{AL}}, \\ & : G_5^I \leq \text{TL}^{\text{PL}}, \end{aligned} \quad (19)$$

availability of labor units constraints:  $\sum H_i R_i \leq \text{TH}_i$ ,

balanced fertilizers input constraints:  $\sum F_i R_i = 0$ ,

pesticide input constraints:  $\sum S_i R_i = 0$ ,

cost constraints:  $\sum B_i R_i = 0$ ,

average yield constraints:  $\sum Y_i R_i - M_i = 0$ ,

where  $h$  is the total number of fruit crops, TL is the total cultivated land,  $G_i^I$  is the total available area for each fruit,

$TL^G$  is the total area for grapes,  $TL^A$  is the total area for apples,  $TL^C$  is the total area for cherry,  $TL^{AL}$  is the total area for almond,  $TL^{PL}$  is the total area for plum,  $TH_i$  is the total available hours or man-days for labor,  $R_i$  is the area for each fruit crop,  $H_i$  is the required working hours or man-days for each  $i$ th crop,  $F_i$  represents the required amount of fertilizer per hecter,  $S_i$  represents the required amount of pesticide per hecter,  $B_i$  is the total cost per hecter,  $Y_i$  is the amount of yields in kg per hecter, and  $M_i$  is the market selling price of yield per kg.

Then, we need much more precision regarding data and situation analysis because of changing factors and circumstances in our universe. The world we are living in is not like before; it is constantly changing, which makes it more challenging for us to change ourselves and our methods according to that change. The simple linear programming is not enough for our environment changes like climate changes, economic downfall, fluctuation of prices and demand, unsuitability of resources, pest and diseases, governmental policies, international trade agreements, topography, and political and social factors. We made a conscious effort regarding this issue especially for the hyperarid zones of Pakistan to improve our food security and GDP. Here, a triangular fuzzy linear programming is formulated according to the present situation analysis of fruit production of Pakistan for improvement.

The triangular intuitionistic fuzzy linear programming for fruit production maximization is developed as

$$(\text{Max})\check{FP}_M^I = \sum_{i=1}^p \sum_{j=1}^q \check{P}_{ij}^I \otimes \check{x}_{ij}, \quad (20)$$

subject to the following constraints:

$$\sum_{i=1}^p \sum_{j=1}^q \check{K}_{ij}^I \otimes \check{x}_{ij} = \check{v}_i^I. \quad (21)$$

Condition of nonnegativity is as follows:

$$\check{x}_{ij}^I \geq 0^I \text{ for all } i = 1, 2, \dots, p; j = 1, 2, \dots, q, \quad (22)$$

where  $\check{FP}_M^I$  is the triangular intuitionistic fuzzy maximized fruit production;  $\check{x}_{ij}^I$  refers to the triangular intuitionistic fuzzy activities (cutting, pruning, harvesting, thinning, leveling, sales, etc.);  $\check{P}_{ij}^I$  indicates the objective triangular intuitionistic fuzzy coefficients (market prices of variables, product profit, etc.);  $\check{K}_{ij}^I$  represents the triangular intuitionistic fuzzy constraints coefficients (utilized resources and capital per unit of fruit production); and  $\check{v}_i^I$  is the total available triangular intuitionistic fuzzy amount/units/volume of supplies per hecter.

The objective function and constraints equations will be written as

$$\begin{aligned} (\text{Max})\check{FP}_M^I &= \sum_{i=1}^p \sum_{j=1}^q (P_{ij,1}, P_{ij,2}, P_{ij,3}; P_{ij,1}', P_{ij,2}', P_{ij,3}') \otimes (x_{ij,1}, x_{ij,2}, x_{ij,3}; x_{ij,1}', x_{ij,2}', x_{ij,3}'), \\ &\cdot \sum_{i=1}^p \sum_{j=1}^q (K_{ij,1}, K_{ij,2}, K_{ij,3}; K_{ij,1}', K_{ij,2}', K_{ij,3}') \otimes (x_{ij,1}, x_{ij,2}, x_{ij,3}; x_{ij,1}', x_{ij,2}', x_{ij,3}') = (u_{i,1}, u_{i,2}, u_{i,3}; u_{i,1}', u_{i,2}', u_{i,3}'), \\ &\cdot (x_{ij,1}, x_{ij,2}, x_{ij,3}; x_{ij,1}', x_{ij,2}', x_{ij,3}') \geq 0^I. \end{aligned} \quad (23)$$

By using the operations of triangular fuzzy numbers,

$$\begin{aligned} (\text{max})\check{FP}_M^I &= \sum_{i=1}^p \sum_{j=1}^q (P_{ij,1}x_{ij,1}, P_{ij,2}x_{ij,2}, P_{ij,3}x_{ij,3}; P_{ij,1}'x_{ij,1}', P_{ij,2}'x_{ij,2}', P_{ij,3}'x_{ij,3}'), \\ &\cdot \sum_{i=1}^p \sum_{j=1}^q (K_{ij,1}x_{ij,1}, K_{ij,2}x_{ij,2}, K_{ij,3}x_{ij,3}; K_{ij,1}'x_{ij,1}', K_{ij,2}'x_{ij,2}', K_{ij,3}'x_{ij,3}') = (u_{i,1}, u_{i,2}, u_{i,3}; u_{i,1}', u_{i,2}', u_{i,3}'), \\ &x_{ij,3} \geq 0, x_{ij,3} - x_{ij,3}' \geq 0, x_{ij,2} - x_{ij,3} \geq 0, x_{ij,1} - x_{ij,2} \geq 0, x_{ij,1}' - x_{ij,1} \geq 0. \end{aligned} \quad (24)$$

Further simplification was carried out using accuracy function on the triangular intuitionistic fuzzy objective function.

$$\begin{aligned}
 & \text{W max} \left( \sum_{i=1}^p \sum_{j=1}^q P_{ij,1} x_{ij,1}, \sum_{i=1}^p \sum_{j=1}^q P_{ij,2} x_{ij,2}, \sum_{i=1}^p \sum_{j=1}^q P_{ij,3} x_{ij,3}; \sum_{i=1}^p \sum_{j=1}^q P_{ij,1}' x_{ij,1}', \right. \\
 & \left. \sum_{i=1}^p \sum_{j=1}^q P_{ij,2}' x_{ij,2}', \sum_{i=1}^p \sum_{j=1}^q P_{ij,3}' x_{ij,3}' \right) \\
 & = \frac{1}{8} \left( \sum_{i=1}^p \sum_{j=1}^q P_{ij,1} x_{ij,1} + 2 \sum_{i=1}^p \sum_{j=1}^q P_{ij,2} x_{ij,2} + \sum_{i=1}^p \sum_{j=1}^q P_{ij,3} x_{ij,3} \right) \\
 & \quad + \frac{1}{8} \left( \sum_{i=1}^p \sum_{j=1}^q P_{ij,1}' x_{ij,1}' + 2 \sum_{i=1}^p \sum_{j=1}^q P_{ij,2}' x_{ij,2}' + \sum_{i=1}^p \sum_{j=1}^q P_{ij,3}' x_{ij,3}' \right).
 \end{aligned} \tag{25}$$

Ultimately, triangular intuitionistic fuzzy objective function is transmuted into linear objective function by

accuracy function, and regarding that reference, the constraints are thoroughly modified into

$$\begin{aligned}
 & \left( \sum_{i=1}^p \sum_{j=1}^q K_{ij,1} x_{ij,1}, \sum_{i=1}^p \sum_{j=1}^q K_{ij,2} x_{ij,2}, \sum_{i=1}^p \sum_{j=1}^q K_{ij,3} x_{ij,3}; \sum_{i=1}^p \sum_{j=1}^q K_{ij,1}' x_{ij,1}', \sum_{i=1}^p \sum_{j=1}^q K_{ij,2}' x_{ij,2}', \sum_{i=1}^p \sum_{j=1}^q K_{ij,3}' x_{ij,3}' \right) \\
 & = (u_{i,1}, u_{i,2}, u_{i,3}; u_{i,1}', u_{i,2}', u_{i,3}').
 \end{aligned} \tag{26}$$

Using the equality condition of triangular intuitionistic fuzzy number, we have

$$\begin{aligned}
 & \sum_{i=1}^p \sum_{j=1}^q K_{ij,1} x_{ij,1} = u_{i,1}, \\
 & \sum_{i=1}^p \sum_{j=1}^q K_{ij,2} x_{ij,2} = u_{i,2}, \\
 & \sum_{i=1}^p \sum_{j=1}^q K_{ij,3} x_{ij,3} = u_{i,3}, \\
 & \sum_{i=1}^p \sum_{j=1}^q K_{ij,1}' x_{ij,1}' = u_{i,1}', \\
 & \sum_{i=1}^p \sum_{j=1}^q K_{ij,2}' x_{ij,2}' = u_{i,2}', \\
 & \sum_{i=1}^p \sum_{j=1}^q K_{ij,3}' x_{ij,3}' = u_{i,3}'.
 \end{aligned} \tag{27}$$

Now, the model is converted into simple linear problem which can be easily solved through LP algorithm

or Excel Solver. Then, we get the values of unknowns (decision variables) that are substituted into the triangular intuitionistic fuzzy objective function to get the maximized result in the form of triangular intuitionistic fuzzy number.

### 7. Application

The provinces of Punjab and Baluchistan produce abundant amount of fruit where Baluchistan lies in the arid regions of Pakistan. Baluchistan is the largest province on the basis of area occupying 347,190 square kilometres and located in southwest direction. The climatic conditions of Baluchistan region are characterized by very cold winter and very hot summer with maximum of 50°C to 53°C [31]. Moreover, strong windstorms and temperature make the area very hot arid zone, which is referred to as hyperarid zone. Baluchistan contributes nearly 4.9% to GDP which is far less than other provinces. Recently, water availability for the expansion of sustainable agricultural land is achieved by making Mirani Dam on the Dasht River which irrigates 35,000 km<sup>2</sup> of area [32]. For practical application of our formulated models, data for fruit production is collected from Baluchistan and is arranged in tabular form for easy further use.

## 8. Mathematical Model Formulation

The practical formulation of the model is carried out through the application of the above statistics that are

specifically gathered from the Baluchistan province based on the data given in Tables 1–3.

Objective function is as follows:

$$\text{Max } Z_{\text{FP}} = -110x_5 + X_7 + 160x_9 + 120x_{10} + 150x_{11} + 150x_{12} + 200x_{13}, \quad (28)$$

subject to the following constraints:

$$\begin{aligned} x_1 + x_2 + x_3 + x_4 + x_5 &= 120, \\ x_1 &\leq 30, \\ x_2 &\leq 30, \\ x_3 + x_4 + x_5 &\leq 60, \\ -300x_1 - 250x_2 - 200x_3 - 210x_4 - 230x_5 + x_6 &= 0, \\ 3000x_1 + 2850x_2 + 2900x_3 + 3100x_4 + 3050x_5 - x_7 &= 0, \\ -6.1x_1 - 3.5x_2 - 4.5x_3 - 3.2x_4 - 3.5x_5 + x_8 &\leq 20.5, \\ 13700x_1 - x_9 &= 0, \\ 17100x_1 - x_{10} &= 0, \\ 25000x_1 - x_{11} &= 0, \\ 81600x_1 - x_{12} &= 0, \\ 52800x_1 - x_{13} &= 0, \\ 52800x_1 - x_{13} &= 0. \end{aligned} \quad (29)$$

In this model, we used fertilizers, all types of cost, available labor hours, and average fruit yield as constraints to find the optimal fruit production. After the above developments, we used Excel Solver for the maximum yield which gives objective value  $Z_{\text{FP}} = 858880500$  kg. Afterwards, fuzzy modification of model is carried out to figure out more optimal way of modeling the existing methodology. The triangular fuzzy intuitionistic linear programming is given as follows.

Intuitionistic fuzzy objective function is as follows:

$$\begin{aligned} (\text{Max}) \tilde{Z}_{\text{FP}}^I &= \left( -(120, 110, 100; 130, 110, 90) \otimes \tilde{x}_6^I \right) + \left( (1.3, 1, 0.7; 1.6, 1, 0.4) \otimes \tilde{x}_7^I \right) + \left( (180, 160, 140; 200, 160, 120) \otimes \tilde{x}_9^I \right) \\ &+ \left( (140, 120, 100; 160, 120, 180) \otimes \tilde{x}_{10}^I \right) + \left( (175, 150, 125; 200, 150, 100) \otimes \tilde{x}_{11}^I \right) \\ &+ \left( (160, 150, 140; 170, 150, 130) \otimes \tilde{x}_{12}^I \right) + \left( (220, 200, 180; 240, 200, 160) \otimes \tilde{x}_{13}^I \right), \end{aligned} \quad (30)$$

TABLE 1: Orchard area statistics.

Specifications	Occupied area (ha)	Percentage of average cultivated land (%)	Number of trees (\ha)
Apple	30	25	900
Grapes	30	25	1000
Apricot	20	16.66	455
Peach	20	16.66	450
Plum	20	16.66	430
Total	120	100	3235

TABLE 2: Orchard production statistics.

Specifications	Yield (kg\ha)	Price (Rs\kg)
Apple	13700	160
Grapes	17100	120
Apricot	25000	150
Peach	81600	150
Plum	52800	200

TABLE 3: Material consumption statistics.

Specifications	Available units (kg, hrs, Rs\ha)		
	Fertilizers (kg\ha)	Cost (Rs\ha)	Labor (hrs\ha)
Apple	300	3000	6.1
Grapes	250	2850	3.5
Apricot	200	2900	4.5
Peach	210	3100	3.2
Plum	230	3050	3.5

Price of the fertilizer is Rs 110/kg.

subject to the following intuitionistic fuzzy constraints:

$$\begin{aligned}
 & (1.4, 1, 0.6; 1.8, 1, 0.2)\tilde{x}_1^I + (1.3, 1, 0.7; 1.6, 1, 0.4)\tilde{x}_2^I + (1.2, 1, 0.8; 1.4, 1, 0.6)\tilde{x}_3^I + (1.1, 1, 0.9; 1.2, 1, 0.8) \\
 & \cdot \tilde{x}_4^I + (1.4, 1, 0.6; 1.8, 1, 0.2)\tilde{x}_5^I = (125, 120, 115; 130, 120, 110), \\
 & \cdot (1.3, 1, 0.7; 1.6, 1, 0.4)\tilde{x}_1^I \leq (35, 30, 25; 40, 30, 20), \\
 & \cdot (1.3, 1, 0.7; 1.6, 1, 0.4)\tilde{x}_2^I \leq (35, 30, 25; 40, 30, 20), \\
 & \cdot (1.4, 1, 0.6; 1.8, 1, 0.2)\tilde{x}_3^I + (1.2, 1, 0.8; 1.4, 1, 0.6)\tilde{x}_4^I + (1.1, 1, 0.9; 1.2, 1, 0.8)\tilde{x}_5^I \leq (65, 60, 55; 70, 60, 50), \\
 & - (310, 300, 290; 320, 300, 280)\tilde{x}_1^I - (260, 250, 240; 270, 250, 230)\tilde{x}_2^I - (205, 200, 195; 210, 200, 190)\tilde{x}_3^I - \\
 & \cdot (220, 210, 200; 230, 210, 190)\tilde{x}_4^I - (240, 230, 220; 250, 230, 210)\tilde{x}_5^I + (1.4, 1, 0.6; 1.8, 1, 0.2)\tilde{x}_6^I = \tilde{0}^I, \\
 & \cdot (3050, 3000, 2950; 3100, 3000, 2900)\tilde{x}_1^I + (2900, 2850, 2800; 2950, 2850, 2750)\tilde{x}_2^I + \\
 & \cdot (2925, 2900, 2875; 2950, 2900, 2850)\tilde{x}_3^I + (3200, 3100, 3000; 3300, 3100, 2900)\tilde{x}_4^I + \\
 & \cdot (3100, 3050, 3000; 3150, 3050, 2950)\tilde{x}_5^I - (1.4, 1, 0.6; 1.8, 1, 0.2)\tilde{x}_7^I = \tilde{0}^I, \\
 & - (6.4, 6.1, 5.8; 6.7, 6.1, 5.5)\tilde{x}_1^I - (4, 3.5, 3; 4.4, 3.5, 2.5)\tilde{x}_2^I - (4.8, 4.5, 4.2; 5.1, 4.5, 3.9)\tilde{x}_3^I - (3.4, 3.2, 3; \\
 & \cdot 3.6, 3.2, 2.8)\tilde{x}_4^I - (3.75, 3.5, 3.25; 4, 3.5, 3)\tilde{x}_5^I + (1.4, 1, 0.6; 1.8, 1, 0.2)\tilde{x}_8^I \leq (21, 20.5, 20; 21.5, 20.5, 19.5), \\
 & \cdot (13800, 13700, 13600; 13900, 13700, 13500)\tilde{x}_1^I - (1.4, 1, 0.6; 1.8, 1, 0.2)\tilde{x}_9^I = \tilde{0}^I, \\
 & \cdot (17200, 17100, 17000; 17300, 17100, 16900)\tilde{x}_2^I - (1.4, 1, 0.6; 1.8, 1, 0.2)\tilde{x}_{10}^I = \tilde{0}^I, \\
 & \cdot (25100, 25000, 24900; 25200, 2500, 24800)\tilde{x}_3^I - (1.4, 1, 0.6; 1.8, 1, 0.2)\tilde{x}_{11}^I = \tilde{0}^I, \\
 & \cdot (81700, 81600, 81500; 81800, 81600, 81400)\tilde{x}_4^I - (1.4, 1, 0.6; 1.8, 1, 0.2)\tilde{x}_{12}^I = \tilde{0}^I, \\
 & \cdot (52850, 52800, 52750; 52900, 52800, 52700)\tilde{x}_5^I - (1.4, 1, 0.6; 1.8, 1, 0.2)\tilde{x}_{13}^I = \tilde{0}^I.
 \end{aligned} \tag{31}$$

This is the mathematical formulation of triangular intuitionistic fuzzy linear programming in which all the decision variables and the regarding coefficients are

triangular intuitionistic fuzzy numbers. As stated above, we cannot directly solve this model. Ultimately, we convert this model into crisp linear programming by using the accuracy



function and arithmetic operations of triangular intuitionistic fuzzy numbers accordingly.

$$\begin{aligned} & \left( \left( -(120, 110, 100; 130, 110, 90) \otimes \tilde{x}_6^I \right) + \left( (1.3, 1, 0.7; 1.6, 1, 0.4) \otimes \tilde{x}_7^I \right) + \left( (180, 160, 140; 200, 160, 120) \right. \right. \\ & \left. \left. \otimes \tilde{x}_9^I \right) + \left( (140, 120, 100; 160, 120, 180) \otimes \tilde{x}_{10}^I \right) + \left( (175, 150, 125; 200, 150, 100) \otimes \tilde{x}_{11}^I \right) + \left( (160, 150, 140; \right. \right. \\ & \left. \left. 170, 150, 130) \otimes \tilde{x}_{12}^I \right) + \left( (220, 200, 180; 240, 200, 160) \otimes \tilde{x}_{13}^I \right) \right), \\ (\text{Max}) \tilde{Z}_{\text{FP}}^I &= \frac{1}{8} \left\{ -120x_{6,1} + 1.3x_{7,1} + 180x_{9,1} - 140x_{10,1} - 175x_{11,1} + 160x_{12,1} + 220x_{13,1} \right\} + \frac{4}{8} \left\{ -110x_{6,2} \right. \\ & \left. + 1x_{7,2} + 160x_{9,2} - 120x_{10,2} - 150x_{11,2} + 150x_{12,2} + 200x_{13,2} \right\} + \frac{1}{8} \left\{ -100x_{6,3} + 0.7x_{7,3} \right. \\ & \left. + 140x_{9,3} - 100x_{10,3} - 125x_{11,3} + 140x_{12,3} + 180x_{13,3} \right\} + \frac{1}{8} \left\{ -130x'_{6,1} + 1.6x'_{7,4} + 200x'_{9,4} \right. \\ & \left. - 160x'_{10,1} - 200x'_{11,1} + 170x'_{12,1} + 240x'_{13,1} \right\} + \frac{1}{8} \left\{ -90x'_{6,3} + 0.4x'_{7,3} + 120x'_{9,3} - 180x'_{10,3} \right. \\ & \left. - 100x'_{11,3} + 130x'_{12,3} + 160x'_{13,3} \right\}. \end{aligned} \tag{32}$$

Along with the linear constraints simplification, which is carried out using the arithmetic operations of multiplication and equality of triangular intuitionistic fuzzy numbers according to the methodology, we have the crisp LP model

which is simply solved through Excel Solver to find the values of decision variables. The values of decision variables obtained are

$$\begin{aligned} \tilde{x}_1^I &= (21.72619048, 30, 35.7142871; 16.66666667, 30, 50), \\ \tilde{x}_2^I &= (26.92307692, 30, 35.71428571; 25, 30, 50), \\ \tilde{x}_3^I &= (0, 0, 53.57142857; 0, 0, 40), \\ \tilde{x}_4^I &= (54.16666667, 60, 28.57142857; 50, 60, 70), \\ \tilde{x}_5^I &= (0, 0, 0; 0, 0, 0), \\ \tilde{x}_6^I &= (18322.70408, 29100, 58482.14286; 13101.85185, 29100, 232000), \\ \tilde{x}_7^I &= (226910.8124, 361500, 741815.4762; 161342.5926, 361500, 2997500), \\ \tilde{x}_8^I &= (0, 0, 0; 0, 0, 0), \\ \tilde{x}_9^I &= (214158.1633, 411000, 809523.8095; 128703.7037, 411000, 3375000), \\ \tilde{x}_{10}^I &= (330769.2308, 513000, 1011904.762; 240277.7778, 513000, 4225000), \\ \tilde{x}_{11}^I &= (0, 0, 2223214.286; 0, 0, 4960000), \\ \tilde{x}_{12}^I &= (3161011.905, 4896000, 3880952.381; 2272222.222, 4896000, 28490000), \\ \tilde{x}_{13}^I &= (0, 0, 0; 0, 0, 0). \end{aligned} \tag{33}$$

The triangular intuitionistic fuzzy objective value is obtained by putting the values of decision variables

$\tilde{x}_1^I, \tilde{x}_2^I, \tilde{x}_3^I, \tilde{x}_4^I, \tilde{x}_5^I, \dots, \tilde{x}_{13}^I$  into the triangular intuitionistic fuzzy objective function as follows:

$$\tilde{Z}_{\text{FP}}^I = (588714344, 859026000, 1042552710; 449017835.6, 859026000, 5345519000), \tag{34}$$

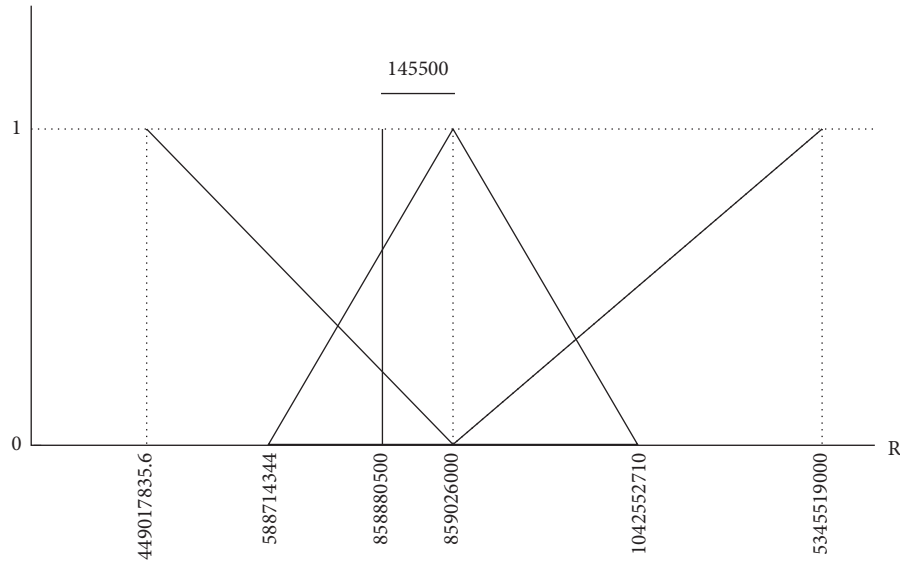


FIGURE 2: Graphical comparison of optimal solution.

TABLE 4: Sensitivity report (variables).

Variable name	Final value	Objective coefficient	Allowable increase	Allowable decrease	Variable name	Final value	Objective coefficient	Allowable increase	Allowable decrease
$x_{1,1}$	21.72619048	0	10082.65797	333727.8781	$x'_{7,1}$	161342.5926	0.2	1E + 30	50.32943262
$x_{1,2}$	30	0	1.00E + 30	67325	$x'_{7,3}$	2997500	0.05	264.5900754	157.5387006
$x_{1,3}$	35.71428571	0	1E + 30	5840117.708	$x_{8,1}$	0	0	0	1E + 30
$x'_{1,1}$	16.66666667	0	23365.45139	65707.87037	$x_{8,2}$	0	0	0	1E + 30
$x'_{1,3}$	50	0	1E + 30	1394217.5	$x_{8,3}$	0	0	0	1E + 30
$x_{2,1}$	26.92307692	0	1E + 30	9362468112	$x'_{8,1}$	0	0	0	1E + 30
$x_{2,2}$	30	0	67325	1E + 30	$x'_{8,3}$	0	0	0	1E + 30
$x_{2,3}$	35.71428571	0	1E + 30	6706814.583	$x_{9,1}$	214158.1633	22.5	1.022878344	33.8564514
$x'_{2,1}$	25	0	1E + 30	20769.29012	$x_{9,2}$	411000	80	1E + 30	4.914233577
$x'_{2,3}$	50	0	1E + 30	1626235	$x_{9,3}$	809523.8095	17.5	1E + 30	257.6522518
$x_{3,1}$	0	0	955994.5206	7.45517E + 21	$x_{9,1}$	128703.7037	25	3.025741906	8.508932854
$x_{3,2}$	0	0	4244550	2.29518E + 22	$x'_{9,3}$	3375000	15	1E + 30	20.65507407
$x_{3,3}$	53.57142857	0	1197645.461	1E + 30	$x_{10,1}$	330769.2308	17.5	1E + 30	0.762061358
$x'_{3,1}$	7.105E-15	0	875857.0547	1.16921E + 20	$x_{10,2}$	513000	60	3.937134503	1E + 30
$x'_{3,3}$	40	0	1355195.833	1E + 30	$x_{10,3}$	1011904.762	12.5	1E + 30	236.7111029
$x_{4,1}$	54.16666667	0	6.08437E + 19	101852.0147	$x'_{10,1}$	240277.7778	20	1E + 30	2.160966602
$x_{4,2}$	60	0	1E + 30	841125	$x'_{10,3}$	4225000	10	1E + 30	19.24538462
$x_{4,3}$	28.57142857	0	1E + 30	1233965.205	$x_{11,1}$	0	21.875	53.32240354	1E + 30
$x'_{4,1}$	50	0	1E + 30	32853.93519	$x_{11,2}$	0	75	169.782	1E + 30
$x'_{4,3}$	70	0	1E + 30	2393337.5	$x_{11,3}$	2223214.286	15.625	28.85892678	1E + 30
$x_{5,1}$	0	0	9364.34684	1E + 30	$x'_{11,1}$	0	25	62.56121819	1E + 30
$x_{5,2}$	0	0	841125	1E + 30	$x'_{11,3}$	4960000	12.5	10.92899866	1E + 30
$x_{5,3}$	0	0	4442274.739	1E + 30	$x_{12,1}$	3161011.905	20	1.04261E + 15	1.745322162
$x'_{5,1}$	0	0	28160.51587	1E + 30	$x_{12,2}$	4896000	75	1E + 30	10.30790441
$x'_{5,3}$	3.552E-15	0	5265342.5	1E + 30	$x_{12,3}$	3880952.381	17.5	1E + 30	9.08440642
$x_{6,1}$	18322.70408	-15	470.5240384	2701.020218	$x'_{12,1}$	2272222.222	21.25	1E + 30	0.722947229
$x_{6,2}$	29100	-55	42056.25	1346.5	$x'_{12,3}$	28490000	16.25	1E + 30	5.880436118
$x_{6,3}$	58482.1486	-12.5	18573.97104	47352.30574	$x_{13,1}$	0	27.5	2.473227731	1E + 30
$x'_{6,1}$	13101.85185	-16.25	2588.173077	601.3940678	$x_{13,2}$	0	100	15.93039773	1E + 30
$x'_{6,3}$	232000	-11.25	3682.057692	1366.879902	$x_{13,3}$	0	22.5	50.52824348	1E + 30
$x_{7,1}$	226910.8124	0.1625	1.00E + 30	190.3709959	$x'_{13,1}$	0	30	0.958202809	1E + 30
$x_{7,2}$	361500	0.5	1E + 30	448.8333333	$x_{13,3}$	0	20	19.98232448	1E + 30
$x_{7,3}$	741815.4762	0.0875	1579.475463	1E + 30					

TABLE 5: Sensitivity report (constraints).

Constraints (LHS)	Final value	Shadow price	Constraints (RHS)	Allowable increase	Allowable decrease
Total land 1 LHS	125	156298.7883	125	7.275641026	30.41666667
Total land 2 LHS	120	1013675	120	0	30
Total land 3 LHS	115	-9081770.833	115	4.761904762	6.696428571
Total land 4 LHS	130	105839.506 2	130	15	30
Total land 5 LHS	110	-1983712.5	110	70	13.33333333
Land for apple 1 LHS	28.24404762	0	35	1E + 30	6.755952381
Land for apple 2 LHS	30	67325	30	30	0
Land for apple 3 LHS	25	8343025.297	25	7.812 5	5.555555556
Land for apple 4 LHS	26.66666667	0	40	1E + 30	13.33333333
Land for apple 5 LHS	20	3485543.75	20	26.66666667	20
Land for grapes 1 LHS	35	7201.898548	35	30.41666667	7.275641026
Land for grapes 2 LHS	30	0	30	1E + 30	-7.10543E-15
Land for grapes 3 LHS	25	9581163.69	25	0	4.761904762
Land for grapes 4 LHS	40	12 980.80633	40	30	15
Land for grapes 5 LHS	20	4065587.5	20	13.33333333	20
Land for drupes 1 LHS	65	827690.3965	65	33.18181818	0
Land for drupes 2 LHS	60	5096325	60	30	0
Land for drupes 3 LHS	55	13183684.9	55	5.952380952	3.571428571
Land for drupes 4 LHS	70	597840.9392	70	35	17.5
Land for drupes 5 LHS	50	13651262.5	50	10	23.33333333
Fertilizers 1 LHS	-1.66619E-09	-10.71428571	0	1E + 30	25651.78571
Fertilizers 2 LHS	-3.63798E-12	-55	0	1E + 30	29100
Fertilizers 3 LHS	5.31873E-09	-20.83333333	0	1E + 30	35089.28571
Fertilizers 4 LHS	5.96629E-10	-9.027777778	0	1E + 30	23583.33333
Fertilizers 5 LHS	-1.05501E-08	-56.25	0	1E + 30	46400
Costs 1 LHS	-8.24803E-08	-0.116071429	0	317675.1374	1E + 30
Costs 2 LHS	0	-0.5	0	361500	1E + 30
Costs 3 LHS	2.69793E-07	-0.145 833 333	0	445089.2857	1E + 30
Costs 4 LHS	-1.17405E-07	-0.111 111 111	0	290416.6667	1E + 30
Costs 5 LHS	2.18092E-06	-0.25	0	599500	1E + 30
Labor 1 LHS	-430.9065934	0	21	1E + 30	451.9065934
Labor 2 LHS	-480	0	20.5	1E + 30	500.5
Labor 3 LHS	-625	0	20	1E + 30	645
Labor 4 LHS	-404.166 666 7	0	21.5	1E + 30	425.6666667
Labor 5 LHS	-752	0	19.5	1E + 30	771.5
Apple yield 1 LHS	-7.78819E-08	-16.07142857	0	299821.4286	1E + 30
Apple yield 2 LHS	5.82077E-11	-80	0	411000	1E + 30
Apple yield 3 LHS	2.94473E-07	-29.166 666 67	0	485 714.2857	1E + 30
Apple yield 4 LHS	-9.3627E-08	-13.88888889	0	231666.6667	1E + 30
Apple yield 5 LHS	2.45555E-06	-75	0	675000	1E + 30
Grapes yield 1 LHS	4.81319E-07	-12.5	0	463076.9231	1E + 30
Grapes yield 2 LHS	0	-60	0	513000	1E + 30
Grapes yield 3 LHS	-1.47265E-06	-20.83333333	0	607142.8571	1E + 30
Grapes yield 4 LHS	-1.74856E-07	-11.11111111	0	432500	1E + 30
Grapes yield 5 LHS	3.07418E-06	-50	0	845000	1E + 30
Apricot yield 1 LHS	0	-53.7124311	0	1165357.143	0
Apricot yield 2 LHS	0	-244.782	0	1500000	0
Apricot yield 3 LHS	-3.23541E-06	-26.04166667	0	1333928.571	1E + 30
Apricot yield 4 LHS	1.79057E-10	-48.64512122	0	980000	1.79057E-10
Apricot yield 5 LHS	3.60981E-06	-62.5	0	992000	1E + 30
Peach yield 1 LHS	-1.83983E-05	-14.28571429	0	4425416.667	1E + 30
Peach yield 2 LHS	0	-75	0	4896000	1E + 30
Peach yield 3 LHS	2.25897E-05	-29.16666667	0	2328571.429	1E + 30
Peach yield 4 LHS	6.61286E-06	-11.80555556	0	4090000	1E + 30
Peach yield 5 LHS	-8.29175E-05	-81.25	0	5698000	1E + 30
Plum yield 1 LHS	0	-21.40944838	0	3122 954.545	0
Plum yield 2 LHS	0	-115.9303977	0	3168000	0
Plum yield 3 LHS	0	-121.7137391	0	418650.7937	0
Plum yield 4 LHS	0	-17.19900156	0	2057222.222	0
Plum yield 5 LHS	1.87228E-10	-199.9116224	0	1676818.182	1.87228E-10

with membership and nonmembership degree as follows:

$$\mu_{Z_{FP}}^{-I}(x) = \begin{cases} 0, & x < 588714344, \\ \frac{x - 588714344}{270311656}, & 588714344 \leq x \leq 859026000, \\ 1, & x = 859026000, \\ \frac{1042552710 - x}{183526710}, & 859026000 \leq x \leq 1042552710, \\ 0, & x > 1042552710, \end{cases}$$

$$\nu_{Z_{FP}}^{-I}(x) = \begin{cases} 1, & x < 449017835.6, \\ \frac{859026000 - x}{410008165}, & 449017835.6 \leq x \leq 859026000, \\ 0, & x = 859026000, \\ \frac{x - 859026000}{4486493000}, & 859026000 \leq x \leq 5345519000, \\ 1, & x > 5345519000. \end{cases} \quad (35)$$

**8.1. Interpretation and Comparison of Results.** For comparison, the results obtained by optimization model considered in fuzzy environment should be compared with the linear programming in crisp environment. The general linear programming specifically designed for fruit production gives the output of 858880500 kg which is maximum fruit yield by consuming the available resources and inputs. The modified triangular intuitionistic fuzzy linear programming yields the result of

$$\tilde{Z}_{FP}^{-I} = (588714344, 859026000, 1042552710; 449017835.6, 859026000, 5345519000), \quad (36)$$

which is clearly maximum fruit production output in the form of triangular intuitionistic fuzzy number. These results are further explained and demonstrated through detailed analysis in the form of graphical representation in Figure 2 which shows the output of both techniques. The level of satisfaction increases with the production increase from 588 714 344 to 859 026 000, reaches the maximum over 859 026 000 with membership degree 1, and then decreases afterwards to 1042 552 710. It is obvious that degree of nonmembership decreases with the increase in membership degree simultaneously. The vertical line in the graph at 858 880 500 represents the results of linear programming. In comparison, the graph already shows that 145 500 kg of yield increased by triangular intuitionistic fuzzy linear programming and the optimal region obtained from this technique is much more acceptable due to the feasibility levels at certain situations.

**8.2. Postoptimality (Sensitivity) Analysis.** Sensitivity analysis (postoptimality analysis) is the process of determining how changes in the optimal solution influence it, within certain limits. The sensitivity analysis is carried out by changing the coefficients of objective function and the right-hand side (RHS) values of constraints. Here, postoptimality (sensitivity) analysis of triangular intuitionistic fuzzy linear programming is assessed using the Tables 4 and 5. The solution remains optimal and feasible within the specified limits of variables and parameters. Range of optimality is dependent on the coefficients of objective function, which means that change in the coefficients of objective function affects the optimality of solution, which is represented by Table 4. This table contains the limits for the coefficients of each variable in the form of allowable increase and decrease. For example, the limit of coefficient of  $x_{1,1}$  having original value 0 is between 10082.65797 and 333727.8781, and the solution remains optimal for this range. The cell containing value  $1E + 30$  in the form of allowable increase or decrease means that there is no limit for the increase or decrease of that specific variable.

In Table 5, the range of each constraint is presented with the shadow increase in objective value, which is only valid for given ranges. A change in the right-hand side of a constraint directly changes the feasible region which perhaps influences the optimal solution. From Table 5, it is clear that our feasibility region remains feasible and the same if the constraints change within the allowable range. As observed from Table 5, the total land constraint 1 has a range between 7.275641026 and 30.41666667 in which feasibility region of the model remains unchanged. Moreover, shadow price is also given per unit increase in the right-hand side of the constraint providing improvement in the value of the optimal solution. The above analysis indicated that this technique is providing flexible optimal solution with the original data.

## 9. Conclusion

The comparison of methodologies, postoptimality (sensitivity) analysis, and compiled statistics stated that the triangular intuitionistic fuzzy linear programming is providing best results for management of real-life problems. The feasible region for optimal production in fuzzy environment remains feasible and optimal within sufficient range. In future, we can consider this model in different fuzzy environments to optimize production and observe the optimality and feasibility levels more accurately. To maintain the level of food security nationally or internationally, we can design a multilevel model in fuzzy environment for the achievement of best optimal agricultural production with least cost by consuming available resources.

## Data Availability

Fruit production data were collected from local farmers to support this study.

## Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this article.

## Authors' Contributions

All authors contributed equally to the preparation of this manuscript.

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