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Research Article

Improved Estimation of Finite Population Variance Using Dual Supplementary Information under Stratified Random Sampling

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In this article, we propose an improved estimator for finite population variance based on stratified sampling by using the auxiliary variable as well as the rank of the auxiliary variable. Expressions for the bias and the mean square error of the estimators are derived up to the first order of approximation. Four real data sets are used to measure the performances of estimators. Moreover, a simulation study is also conducted to observe the efficiency of the proposed variance estimator. The theoretical and numerical results show that the proposed estimator under stratified random sampling is more efficient as compared to the existing estimators.

1. Introduction

The need for supplementary information in survey sampling has long been acknowledged as producing efficient estimators of population parameters such as the mean, median, mode, quartiles, interquartile, percentile, coefficient of variation, and proportion. A wide amendment of methods for retaining supplementary information is described in the literature on survey sampling. The ratio, product, and regression type estimators take advantage of the correlation between the study variable and the auxiliary variable. These estimators perform better when the correlation between the study variable and the auxiliary variable exists and is often used to increase the precision of estimators. When the correlation between the study variable and the supplementary information occurs, the rank of the supplementary information is also correlated with the study variable, and therefore, this rank can be used as an essential factor for increasing the accuracy of an estimator. When the population variance of the supplementary information is known in advance, researchers have developed ratio and product-type estimators for variance estimation. In trades such as agriculture, medicine, biology, and industry, where we encounter populations that are likely to be skewed, a variance estimate for finite population parameters is considered. Variations can occur in reality in our daily lives in various fields such as environmental, genetic, and economic studies. For example, an agriculturist needs a suitable understanding of the variation in weather factors, particularly from time to time or from place to place, to be able to plan on where, how and when to plant his crop. A physician needs a full understanding of variation in the degree of human blood pressure, body temperature, and pulse rate for an adequate prescription. An industrialist desires continual knowledge of the level of variation in people's response to his product to be capable of knowing whether to reduce or increase his price or improve the quality of his product.

The issue of estimating the population variance has been broadly argued by various authors. Das and Tripathi [1] considered the use of the auxiliary variable in estimating the finite population variance. Isaki [2] discussed the variance estimation using ratio and regression-type estimators. Some important references regarding variance estimation include Garcia and Cebrian [3]; Arcos et al. [4]; Shabbir and Gupta [5]; Singh and Solanki [6]; Adichwal et al. [7, 8], Ahmad et al. [9]; Shabbir and Gupta [10]; Singh and Khalid [11, 12] and Zaman and Bulut [13].

In stratified random sampling, a population is divided into a number of nonoverlapping groups or subgroups called strata. These groups are completely homogeneous, and the sample is taken independently from each stratum. Stratification increases precision when the variance among the strata is much larger than the variances within the strata. Some important references under stratified random sampling are Rao and Shao [14], Kadilar and Cingi [15, 16], Singh and Vishwakarma [17], Koyuncu and Kadilar [18], Shabbir and Gupta [19], Ozel et al. [20], Sidelel et al. [21], Ahmad and Shabbir [22], Hussain et al. [23], Shehzad et al. [24], Singh et al. [25], Zaman [26], Zaman and Bulut [27], and Ahmad et al. [28].

In Section 2, we discuss some notations and symbols of population variance under stratified random sampling. In Section 3, we review some adopted existing estimators. A proposed estimator is given in Section 4. The efficiency comparisons are given in Section 5. In Section 6, we discuss the numerical investigation. The simulation study is given in Section 7. The discussion of the article is discussed in Section 8. The conclusion of the paper is given in Section 9.

2. Notations and Symbols

Consider a finite population $\psi = \{\psi_1, \psi_2, \ldots, \psi_N\}$, having N units into L strata. Let y_{hi}, x_{hi}, and $r_{x_{\mathrm{hi}}}$ be the characteristics of the study variable (y), auxiliary variable (x), and rank of the auxiliary variable (r_x), respectively, in stratum h such that $\sum_{h=1}^{L} N_h = N$.

We draw a random sample of size n_h from a population N_h such that $\sum_{h=1}^{L} n_h = n$. Let $\overline{Y}_h$, $\overline{X}_h$, and $\overline{R}_{xh}$ be the population means corresponding to the sample means $\overline{y}_h$, $\overline{x}_h$, and $\overline{r}_{xh}$, respectively, in each stratum.

For y, x, and the rank of x, we take the values of y_{hi}, x_{hi}, and $r_{x_{\mathrm{hi}}}$ for the i^{th} unit of the h^{th} stratum.

Let $s_{yh}^2 = 1/n_h - 1\sum_{i=1}^{n_h} (y_{\mathrm{hi}} - \overline{y}_h)^2$, $s_{xh}^2 = 1/n_h - 1\sum_{i=1}^{n_h} (x_{hi} - \overline{x}_h)^2$ and $s_{rxh}^2 = 1/n_h - 1\sum_{i=1}^{n_h} (r_{x_{hi}} - \overline{r}_{xh})^2$ be the sample variance, corresponding to the population variances $S_{yh}^2 = 1/N_h - 1\sum_{i=1}^{N_h} (y_{hi} - \overline{Y}_h)^2$, $S_{xh}^2 = 1/N_h - 1\sum_{i=1}^{N_h} (x_{hi} - \overline{X}_h)^2$, and $S_{rxh}^2 = 1/N_h - 1\sum_{i=1}^{N_h} (r_{x_{hi}} - \overline{R}_{xh})^2$.

To derive the bias and mean square error, we define the following error terms:

$$\xi_{oh} = \left(\frac{s_{yh}^2 - S_{yh}^2}{S_{yh}^2}\right), \xi_{1h} = \left(\frac{s_{xh}^2 - S_{xh}^2}{S_{xh}^2}\right), \xi_{2h} = \left(\frac{s_{rxh}^2 - S_{rxh}^2}{S_{rxh}^2}\right), \tag{1}$$

$E(\xi_{ih}) = 0$,

where $i = 0, 1, 2$.

$$\begin{aligned} &E\left(\xi_{oh}^2\right) = \Lambda_h \lambda_{400h}^*, E\left(\xi_{1h}^2\right) = \Lambda_h \lambda_{040h}^*, E\left(\xi_{2h}^2\right) = \Lambda_h \lambda_{004h}^*, \\ &E(\xi_{oh}\xi_{1h}) = \Lambda_h \lambda_{220h}^*, E(\xi_{0h}\xi_{2h}) = \Lambda_h \lambda_{202h}^*, \\ &E(\xi_{1h}\xi_{2h}) = \Lambda_h \lambda_{022h}^*, \end{aligned} \tag{2}$$

where

$$\begin{aligned} \lambda_{abch} &= \frac{\upsilon_{abch}}{\upsilon_{200h}^{a/2}\upsilon_{020h}^{b/2}\upsilon_{002h}^{c/2}}, \\ \upsilon_{abch} &= \frac{\sum_{i=1}^{N} (y_{hi} - \overline{Y}_h)^a (x_{hi} - \overline{X}_h)^b \left(r_{x_{hi}} - \overline{R}_{xh}\right)^c}{N_h - 1} \\ \lambda_{abch}^* &= (\lambda_{abch} - 1) \text{ and } \Lambda_h = \left(\frac{1}{n_h} - \frac{1}{N_h}\right). \end{aligned} \tag{3}$$

3. Existing Estimators

In this section, we adopt some variance estimators under stratified random sampling, which are available in the literature.

(i) The traditional unbiased estimator is given by

$$G_{0(st)} = \sum_{h=1}^{l} W_h^2 \Lambda_h S_{yh}^2. \tag{4}$$

The variance of $G_{0(st)}$ is given by

$$\mathrm{Var}\left(G_{0(st)}\right) \cong \sum_{h=1}^{l} W_h^4 \Lambda_h^3 S_{yh}^4 \lambda_{400h}^*. \tag{5}$$

(ii) Isaki [2] suggested a ratio estimator which is given by

$$G_{1(st)} = \sum_{h=1}^{l} W_h^2 \Lambda_h \left\{ s_{yh}^2 \left(\frac{S_{xh}^2}{s_{xh}^2}\right)\right\}. \tag{6}$$

The bias and MSE of $G_{1(st)}$ are given by

$$\begin{aligned} \mathrm{Bias}\left(G_{1(st)}\right) &\cong \sum_{h=1}^{l} W_h^2 \Lambda_h^2 S_{yh}^2 \{\lambda_{040h}^* - \lambda_{220h}^*\}, \\ &\text{and,} \\ \mathrm{MSE}\left(G_{1(st)}\right) &\cong \sum_{h=1}^{l} W_h^4 \Lambda_h^3 S_{yh}^4 \{\lambda_{400h}^* + \lambda_{040h}^* - 2\lambda_{220h}^*\}. \end{aligned} \tag{7}$$

(iii) The usual difference-type estimator is given by

$$Ϭ_{2(st)} = \sum_{h=1}^{l} W_h^2 \Lambda_h \left\{ s_{yh}^2 + \Psi_h \left(S_{xh}^2 - s_{xh}^2 \right) \right\}. \quad (8)$$

Where Ψ_h is the unknown constant.

The optimum value of Ψ_h is given by

$$\Psi_h = \left\{ \frac{S_{yh}^2 \lambda_{220h}^*}{S_{xh}^2 \lambda_{040h}^*} \right\}. \quad (9)$$

The minimum variance at the optimal value of Ψ_h is given by

$$\mathrm{Var}\left(Ϭ_{2(st)}\right)_{\min} = \sum_{h=1}^{l} W_h^4 \Lambda_h^3 S_{yh}^4 \lambda_{400h}^* \left(1 - \rho_h^2\right). \quad (10)$$

Where

$$\rho_h = \frac{\lambda_{220h}^*}{\sqrt{\lambda_{400h}^*}\sqrt{\lambda_{040h}^*}}. \quad (11)$$

(iv) Rao [29] proposed a difference-type estimator which is given by

$$Ϭ_{3(st)} = \sum_{h=1}^{l} W_h^2 \Lambda_h \left\{ Ш_{1h} s_{yh}^2 + Ш_{2h} \left(S_{xh}^2 - s_{xh}^2 \right) \right\}, \quad (12)$$

where, $Ш_{1h}$ and $Ш_{2h}$ are the unknown constants and are given by

$$Ш_{1h} = \frac{\lambda_{040h}^*}{\Lambda_h \left\{ \lambda_{400h}^* \lambda_{040h}^* - \lambda_{220h}^{*2} \right\} + \lambda_{040h}^*},$$

and, (13)

$$Ш_{2h} = \frac{S_{yh}^2 \lambda_{220h}^*}{S_{xh}^2 \left\{ \Lambda_h \left(\lambda_{400h}^* \lambda_{040h}^* - \lambda_{220h}^{*2} \right) + \lambda_{040h}^* \right\}}.$$

The minimum bias and minimum MSE at the optimal values of $Ш_{1h}$ and $Ш_{2h}$ are given by

$$\mathrm{Bias}\left(Ϭ_{3(st)}\right)_{\min} = \sum_{h=1}^{l} W_h^2 \Lambda_h^2 S_{yh}^2 \cdot \left\{ \frac{\lambda_{040h}^*}{\Lambda_h \left\{ \lambda_{400h}^* \lambda_{040h}^* - \lambda_{220h}^{*2} \right\}} + \lambda_{040h}^* - 1 \right\},$$

and, (14)

$$\mathrm{MSE}\left(Ϭ_{3(st)}\right)_{\min} \cong \sum_{h=1}^{l} W_h^4 \Lambda_h^3 S_{yh}^4 \cdot \left(\lambda_{400h}^* \lambda_{040h}^* - \frac{\lambda_{220h}^{*2}}{\Lambda_h \left[\lambda_{400h}^* \lambda_{040h}^* - \lambda_{220h}^{*2} \right]} + \lambda_{040h}^* \right).$$

(v) Singh et al. [30] proposed exponential ratio and product-type estimators which are given by

$$Ϭ_{4(st)} = \sum_{h=1}^{l} W_h^2 \Lambda_h^2 S_{yh}^2 \exp \left\{ \frac{S_{xh}^2 - s_{xh}^2}{S_{xh}^2 + s_{xh}^2} \right\}, \quad (15)$$

$$Ϭ_{5(st)} = \sum_{h=1}^{l} W_h^2 \Lambda_h^2 S_{yh}^2 \exp \left\{ \frac{S_{xh}^2 - s_{xh}^2}{S_{xh}^2 + s_{xh}^2} \right\}. \quad (16)$$

The bias and MSE of $Ϭ_{4(st)}$ and $Ϭ_{5(st)}$ are given by

$$\mathrm{Bias}\left(Ϭ_{4(st)}\right) = \sum_{h=1}^{l} W_h^2 \Lambda_h^2 S_{yh}^2 \left\{ \frac{3}{8} \lambda_{040h}^* - \frac{1}{2} \lambda_{220h}^* \right\}$$

$$\mathrm{MSE}\left(Ϭ_{4(st)}\right) = \sum_{h=1}^{l} W_h^4 \Lambda_h^3 S_{yh}^4 \left[\lambda_{400h}^* + \frac{1}{4} \lambda_{040h}^* - \lambda_{220h}^* \right], \quad (17)$$

and,

$$\mathrm{Bias}\left(Ϭ_{5(st)}\right) = \sum_{h=1}^{l} W_h^2 \Lambda_h^2 S_{yh}^2 \left\{ \frac{1}{2} \lambda_{220h}^* - \frac{1}{8} \lambda_{040h}^* \right\},$$

$$\mathrm{MSE}\left(Ϭ_{5(st)}\right) = \sum_{h=1}^{l} W_h^4 \Lambda_h^3 S_{yh}^4 \left[\lambda_{400h}^* + \frac{1}{4} \lambda_{040h}^* + \lambda_{220h}^* \right]. \quad (18)$$

(vi) Shabbir and Gupta [5] proposed an estimator in stratified random sampling which is given by

$$\mathsf{G}_{6(st)} = \sum_{h=1}^{l} W_h^2 \Lambda_h \left[Ɯ_{3h} s_{yh}^2 + Ɯ_{4h}\left(S_{xh}^2 - s_{xh}^2\right)\right] \exp\left(\frac{S_{xh}^2 - s_{xh}^2}{S_{xh}^2 + s_{xh}^2}\right), \quad (19)$$

where $Ɯ_{3h}$ and $Ɯ_{4h}$ are suitably chosen constants having optimum values

$$Ɯ_{3h(opt)} = \frac{\Lambda_h \lambda^*_{040h}}{8}\left[\frac{8 - \Lambda_h \lambda^*_{040h}}{\Lambda_h\left(\lambda^*_{040h} + \lambda^*_{004h}\lambda^*_{040h} - \lambda^{*2}_{220h}\right)}\right],$$

and,

$$Ɯ_{4h(opt)} = \frac{S_{yh}^2}{8S_{xh}^2}\left[\frac{-3\lambda^*_{040h} + 8\lambda^{*2}_{220h} - \lambda^*_{040h}\lambda^{*2}_{220h} + 4\lambda^*_{004h}\lambda^*_{040h} - 4\lambda^{*2}_{220h}}{\left(\lambda^*_{040h} + \lambda^*_{004h}\lambda^*_{040h} - \lambda^{*2}_{220h}\right)}\right], \quad (20)$$

$$\text{Bias}\left(\mathsf{G}_{6(st)}\right) \cong \sum_{h=1}^{l} W_h^2 \Lambda_h^2 \left[S_{yh}^2 + Ɯ_{3h} S_{yh}^2 \left\{1 + \frac{3}{8}\lambda^*_{040h} - \frac{1}{2}\lambda^*_{220h}\right\} + \frac{1}{2} Ɯ_{4h} S_{xh}^2 \Lambda_h \lambda^*_{040h}\right].$$

The minimum MSE of $\mathsf{G}_{6(st)}$ at the optimum values of $Ɯ_{3h(\text{opt})}$ and $Ɯ_{4h(\text{opt})}$ is given by

$$\text{MSE}\left(\mathsf{G}_{6(st)}\right)_{\min} \cong \sum_{h=1}^{l} W_h^4 \Lambda_h^3 S_{yh}^4 \left[\left(\frac{64\left\{\lambda^*_{400h}\lambda^*_{040h} - \lambda^{*2}_{220h}\right\} - \Lambda_h \lambda^{*3}_{040h} - 16\Lambda_h \lambda^*_{040h}\left\{\lambda^*_{400h}\lambda^*_{040h} - \lambda^{*2}_{220h}\right\}}{64\left\{\Lambda_h\left(\lambda^*_{400h}\lambda^*_{040h} - \lambda^{*2}_{220h}\right) + \lambda^*_{040h}\right\}}\right)\right]. \quad (21)$$

4. Proposed Estimator in Stratified Random Sampling

In survey sampling, the estimation of the finite population variance under stratified random sampling has customary very little consideration. The use of supplementary information can increase the performance of the estimator in survey sampling. When the study variable and the supplementary information are correlated with each other, then the rank of the supplementary information is also correlated with the study variable. As a result, the rank supplementary information can be observed as new supplementary information, and this information may aid in improving the efficiency of estimators. The main advantage of our proposed variance estimator under stratified random sampling is that it is more flexible and efficient than the existing estimators. Taking motivation from Shabbir and Gupta [10], we propose an improved variance estimator under stratified random sampling given by

$$\mathsf{G}_{\text{prop}(st)} = \sum_{h=1}^{l} W_h^2 \Lambda_h \left[\mathcal{K}_{1h} s_{yh}^2 + \mathcal{K}_{2h}\left(S_{xh}^2 - s_{xh}^2\right) + \mathcal{K}_{3h}\left(S_{rxh}^2 - s_{rxh}^2\right)\right] \exp\left(\frac{S_{xh}^2 - s_{xh}^2}{S_{xh}^2 + s_{xh}^2}\right), \quad (22)$$

$$\mathsf{G}_{\text{prop}(st)} = \sum_{h=1}^{l} W_h^2 \Lambda_h \left[\mathcal{K}_{1h} S_{yh}^2 (1 + \xi_{oh}) - \mathcal{K}_{2h} S_{xh}^2 \xi_{1h} - \mathcal{K}_{3h} S_{rxh}^2 \xi_{2h}\right]\left(1 - \frac{1}{2}\xi_{1h} + \frac{3}{8}\xi_{1h}^2\right). \quad (23)$$

After expanding (23), we have

$$\left(\mathsf{G}_{prop(st)} - \sum_{h=1}^{L} W_h^2 \Lambda_h^2 S_{yh}^2\right) \cong \sum_{h=1}^{l} W_h^2 \Lambda_h \Big[-S_{yh}^2 + \mathcal{K}_{1h} S_{yh}^2 + \mathcal{K}_{1h} S_{yh}^2 \xi_{oh} - \frac{1}{2}\mathcal{K}_{1h} S_{yh}^2 \xi_{1h} - \mathcal{K}_{2h} S_{xh}^2 \xi_{1h} - \mathcal{K}_{3h} S_{rxh}^2 \xi_{2h}$$
$$-\frac{1}{2}\mathcal{K}_{1h} S_{yh}^2 \xi_{oh}\xi_{1h} + \frac{3}{8}\mathcal{K}_{1h} S_{yh}^2 \xi_{1h}^2 + \frac{1}{2}\mathcal{K}_{2h} S_{xh}^2 \xi_{1h}^2 + \frac{1}{2}\mathcal{K}_{3h} S_{rxh}^2 \xi_{1h}\xi_{2h}\Big].$$
$$\cong \sum_{h=1}^{l} W_h^2 \Lambda_h \Big[-S_{yh}^2 + S_{yh}^2 \mathcal{K}_{1h} - \frac{1}{2} S_{yh}^2 \mathcal{K}_{1h} \lambda_{220h}^* + \frac{3}{8}\mathcal{K}_{1h} S_{yh}^2 \lambda_{040h}^* + \frac{1}{2} S_{xh}^2 \mathcal{K}_{2h} \lambda_{040h}^{*2} + \frac{1}{2}\mathcal{K}_{3h} S_{rxh}^2 \lambda_{022h}^*\Big]. \tag{24}$$

The bias of $\mathsf{G}_{\text{prop}(st)}$ is given by

$$\text{Bias}\left(\mathsf{G}_{\text{prop}(st)}\right) \cong \sum_{h=1}^{l} W_h^2 \Lambda_h^2 \left\{-S_{yh}^2 + \mathcal{K}_{1h} S_{yh}^2 \left(1 + \frac{3}{8}\lambda_{040h}^* - \frac{1}{2}\lambda_{220h}^*\right) + \frac{1}{2}\Lambda_h \left(\mathcal{K}_{2h} S_{xh}^2 \lambda_{040h}^* + \mathcal{K}_{3h} S_{rxh}^2 \lambda_{022h}^*\right)\right\}, \tag{25}$$

where $\mathcal{K}_{1h}$, $\mathcal{K}_{2h}$, and $\mathcal{K}_{3h}$ are the unknown constants. The optimum values are

$$\mathcal{K}_{1h(\text{opt})} = \frac{8 - \Lambda_h \lambda_{040h}^*}{\lambda_{400h}^* \left[8\{1/\lambda_{400h}^* + \Lambda_h (\gamma_h^* + 1)\}\right]}$$
$$\mathcal{K}_{2h(\text{opt})} = \frac{S_{yh}^2 \left(\dfrac{\Lambda_h \lambda_{040h}^* \left(\lambda_{040h}^* \lambda_{004h}^* - \lambda_{220h}^{*2}\right) + \left(\lambda_{004h}^* \lambda_{220h}^* - \lambda_{202h}^* \lambda_{022h}^*\right)\left(8 - \Lambda_h \lambda_{040h}^*\right) + 4\lambda_{400h}^* \left(\lambda_{040h}^* \lambda_{004h-}^* \lambda_{022h}^{*2}\right)}{\{(-1/\lambda_{400h}^*) + \Lambda_h (\gamma_h^* + 1)\}}\right)}{8 S_{xh}^2 \lambda_{400h}^* \left\{\lambda_{040h}^* \lambda_{004h}^* - \lambda_{220h}^{*2}\right\}\left[(-1/\lambda_{400h}^*) + \Lambda_h (\gamma_h^* + 1)\right]}, \tag{26}$$

and,

$$\mathcal{K}_{3h(\text{opt})} = \frac{S_{yh}^2 \left[8 - \Lambda_h \lambda_{040h}^*\right]\left(\lambda_{220h}^* \lambda_{002h}^* - \lambda_{040h}^* \lambda_{202h}^*\right)}{8 S_{rxh}^2 \lambda_{400h}^* \left[\lambda_{040h}^* \lambda_{004h}^* - \lambda_{220h}^{*2}\right]\{((-1/\lambda_{400h}^*) + \Lambda_h (\gamma_h^* + 1)\}}.$$

The minimum mean square error at the optimum values of $\mathcal{K}_{1h(\text{opt})}$, $\mathcal{K}_{2h(\text{opt})}$, and $\mathcal{K}_{3h(\text{opt})}$ is given by

$$\text{MSE}\left(\mathsf{G}_{\text{prop}(st)}\right)_{\min} = \sum_{h=1}^{l} W_h^4 \Lambda_h^3 S_{yh}^4 \left(\frac{64(\gamma_h^* + 1) - \Lambda_h \left(\lambda_{040h}^{*2}/\lambda_{400h}^*\right) - 16\Lambda_h \lambda_{040h}^* (\gamma_h^* + 1)}{64\{1/\lambda_{400h}^* + \Lambda_h (\gamma_h^* + 1)\}}\right), \tag{27}$$

where

$$\gamma_h^* = \frac{2\lambda_{220h}^* \lambda_{202h}^* \lambda_{022h}^* - \lambda_{040h}^* \lambda_{202h}^{*2} - \lambda_{004h}^* \lambda_{220h}^{*2}}{\lambda_{400h}^* \left(\lambda_{040h}^* \lambda_{004h-}^* \lambda_{022h}^{*2}\right)}. \tag{28}$$

5. Efficiency Comparison

We compare the proposed estimator with its existing counterparts.

(1) By taking equations (5) and (27), we get

$$\begin{aligned}
&\operatorname{Var}\left(\mathcal{G}_{0(st)}\right) - \operatorname{MSE}\left(\mathcal{G}_{\text{prop}(st)}\right)_{\min} > 0,\\
&\sum_{h=1}^{l} W_h^4 \Lambda_h^3 S_{yh}^4\left(\lambda_{400h}^* - \frac{64\left(\gamma_h^* + 1\right) - \Lambda_h\left(\lambda_{040h}^*/\lambda_{400h}^*\right) - 16\Lambda_h \lambda_{040h}^*\left(\gamma_h^* + 1\right)}{64\left\{1/\lambda_{400h}^* + \Lambda_h\left(\gamma_h^* + 1\right)\right\}}\right) > 0.
\end{aligned} \tag{29}$$

(2) By taking (7) and (27), we get

$$\begin{aligned}
&\operatorname{MSE}\left(\mathcal{G}_{1(st)}\right) - \operatorname{MSE}\left(\mathcal{G}_{\text{prop}(st)}\right)_{\min} > 0,\\
&\sum_{h=1}^{l} W_h^4 \Lambda_h^3 S_{yh}^4\left(\left\{\lambda_{400h}^* + \lambda_{040h}^* - 2\lambda_{220h}^*\right\} - \frac{64\left(\gamma_h^* + 1\right) - \Lambda_h\left(\lambda_{040h}^*/\lambda_{400h}^*\right) - 16\Lambda_h \lambda_{040h}^*\left(\gamma_h^* + 1\right)}{64\left\{1/\lambda_{400h}^* + \Lambda_h\left(\gamma_h^* + 1\right)\right\}}\right) > 0.
\end{aligned} \tag{30}$$

(3) By taking (10) and (27), we obtain

$$\begin{aligned}
&\operatorname{Var}\left(\mathcal{G}_{2(st)}\right)_{\min} - \operatorname{MSE}\left(\mathcal{G}_{\text{prop}(st)}\right)_{\min} > 0,\\
&\sum_{h=1}^{l} W_h^4 \Lambda_h^3 S_{yh}^4\left(\lambda_{400h}^*\left(1 - \rho_h^2\right) - \frac{64\left(\gamma_h^* + 1\right) - \Lambda_h\left(\lambda_{040h}^*/\lambda_{400h}^*\right) - 16\Lambda_h \lambda_{040h}^*\left(\gamma_h^* + 1\right)}{64\left\{1/\lambda_{400h}^* + \Lambda_h\left(\gamma_h^* + 1\right)\right\}}\right) > 0.
\end{aligned} \tag{31}$$

(4) By taking (14) and (27), we obtain

$$\begin{aligned}
&\operatorname{MSE}\left(\mathcal{G}_{3(st)}\right)_{\min} - \operatorname{MSE}\left(\mathcal{G}_{\text{prop}(st)}\right)_{\min} > 0,\\
&\sum_{h=1}^{l} W_h^4 \Lambda_h^3 S_{yh}^4\left(\left[\lambda_{400h}^*\lambda_{040h}^* - \lambda_{220h}^{*2}\right]/\Lambda_h\left[\lambda_{400h}^*\lambda_{040h}^* - \lambda_{220h}^{*2}\right] + \lambda_{040h}^* - 64\left(\gamma_h^* + 1\right) - \Lambda_h\left(\lambda_{040h}^*/\lambda_{400h}^*\right) - 16\Lambda_h \lambda_{040h}^*\left(\gamma_h^* + 1\right)/64\left\{1/\lambda_{400h}^* + \Lambda_h\left(\gamma_h^* + 1\right)\right\}\right) > 0.
\end{aligned} \tag{32}$$

(5) By taking (17) and (27), we obtain

$$\begin{aligned}
&\operatorname{MSE}\left(\mathcal{G}_{4(\text{st})}\right) - \operatorname{MSE}\left(\mathcal{G}_{\text{prop}(\text{st})}\right)_{\min} > 0,\\
&\sum_{h=1}^{l} W_h^4 \Lambda_h^3 S_{yh}^4\left(\left[\lambda_{400h}^* + 1/4\lambda_{040h}^* - \lambda_{220h}^*\right] - 64\left(\gamma_h^* + 1\right) - \Lambda_h\left(\lambda_{040h}^*/\lambda_{400h}^*\right) - 16\Lambda_h \lambda_{040h}^*\left(\gamma_h^* + 1\right)/64\left\{1/\lambda_{400h}^* + \Lambda_h\left(\gamma_h^* + 1\right)\right\}\right) > 0.
\end{aligned} \tag{33}$$

(6) By taking (18) and (27), we obtain

$$\begin{aligned}
&\operatorname{MSE}\left(\mathcal{G}_{5(st)}\right) - \operatorname{MSE}\left(\mathcal{G}_{\text{prop}(st)}\right)_{\min} > 0,\\
&\sum_{h=1}^{l} W_h^4 \Lambda_h^3 S_{yh}^4\left(\left[\lambda_{400h}^* + 1/4\lambda_{040h}^* - \lambda_{220h}^*\right] - 64\left(\gamma_h^* + 1\right) - \Lambda_h\left(\lambda_{040h}^*/\lambda_{400h}^*\right) - 16\Lambda_h \lambda_{040h}^*\left(\gamma_h^* + 1\right)/64\left\{1/\lambda_{400h}^* + \Lambda_h\left(\gamma_h^* + 1\right)\right\}\right) > 0.
\end{aligned} \tag{34}$$

(7) By taking (21) and (27), we obtain

$$\begin{aligned}&\mathrm{MSE}\left(G_{6(st)}\right)_{\min} - \mathrm{MSE}\left(G_{prop(st)}\right)_{\min} > 0,\\ &\sum_{h=1}^{l} W_h^4 \Lambda_h^3 S_{yh}^4 \Big(64\left\{\lambda_{400h}^{*}\lambda_{040h}^{*} - \lambda_{220h}^{*2}\right\} - \Lambda_h \lambda_{040h}^{*3} - 16\Lambda_h \lambda_{040h}^{*}\left\{\lambda_{400h}^{*}\lambda_{040h}^{*} - \lambda_{220h}^{*2}\right\}/64\left\{\Lambda_h\left(\lambda_{400h}^{*}\lambda_{040h}^{*} - \lambda_{220h}^{*2}\right) + \lambda_{040h}^{*}\right\}\\ &\quad - 64(\gamma_h^{*} + 1) - \Lambda_h\left(\lambda_{040h}^{*}/\lambda_{400h}^{*}\right) - 16\Lambda_h \lambda_{040h}^{*}\left(\gamma_h^{*} + 1\right)/64\left\{1/\lambda_{400h}^{*} + \Lambda_h\left(\gamma_h^{*} + 1\right)\right\}\Big) > 0.\end{aligned} \tag{35}$$

6. Numerical Study

To show the performance of our proposed estimator, we conduct a numerical study using four real data sets. We compare the performances of our proposed variance estimator with existing counterparts in terms of percentage relative efficiency. The summary statistics are given in Tables 1–8. The conditional values are given in Table 9. The biases are given in Table 10. The MSEs and efficiency values are given in Tables 11 and 12. To obtain the percentage relative efficiency (PRE), we used the following expression:

$$\mathrm{PRE} = \frac{\mathrm{Var}\left(G_{0(st)}\right)}{\mathrm{MSE}\left(G_{u(st)}\right)} \times 100, \tag{36}$$

where $(u) = (0, 1, 2, 3, 4, 5, 6, \mathrm{prop})$.

Population 1: (Source: Murthy [31])

Y: Production of a factory
X: Number of employees
R_x = Rank of the X variable

Population 2: (Source: Singh and Chaudhary [32])

Y: Area under wheat in the region in 1974
X: Area under wheat in the region in 1973
R_x = Rank of the X variable

Population 3: (Source: Turkey [33])

Y = Apple production in 1999,
X = Number of apple trees in 1999,
R_x = Rank of the X variable

Population 4: (Source: Koyuncu and Kadilar [18])

Y = The number of teachers
X = The number of classes in both primary and secondary schools in Turkey in 2007 for 923 districts in six regions
R_x = Rank of the X variable

7. Simulation Study

A simulation study is performed to determine the efficiency of the estimators for variance stratified random sampling. Three populations are generated from normal distribution by using the R language program. The first population is generated for equal strata and the second one is generated for unequal strata, and the third one is generated for equal strata of a small sample size. The population details are given as follows.

7.1. Population I.

$N_1 = 1000, N_2 = 1000, N_3 = 1000, N_4 = 1000, N_5 = 1000, N_6 = 1000,$

$n_1 = 200, n_2 = 200, n_3 = 200, n_4 = 200, n_5 = 200, n_6 = 200,$

$X_1 = rnorm\ (1000, 5, 10), U_1 = X_1 + rnorm\ (1000, 0, 1), Y_1 = U_1 + rnorm\ (1000, 1, 3),$

$X_2 = rnorm\ (1000, 4, 8), U_2 = X_2 + rnorm\ (1000, 0, 1), Y_2 = U_2 + rnorm\ (1000, 1, 3),$

$X_3 = rnorm\ (1000, 4, 9), U_3 = X_3 + rnorm\ (1000, 0, 1), Y_3 = U_3 + rnorm\ (1000, 1, 3),$

$X_4 = rnorm\ (1000, 3, 7), U_4 = X_4 + rnorm\ (1000, 0, 1), Y_5 = U_5 + rnorm\ (1000, 1, 3),$

$X_5 = rnorm\ (1000, 3, 8), U_5 = X_5 + rnorm\ (1000, 0, 1), Y_5 = U_5 + rnorm\ (1000, 1, 3),$

$X_6 = rnorm\ (1000, 2, 5), U_6 = X_6 + rnorm\ (1000, 0, 1), Y_6 = U_6 + rnorm\ (1000, 1, 3).$

7.2. Population II.

$N_1 = 1200, N_2 = 800, N_3 = 1300, N_4 = 900, N_5 = 1100, N_6 = 700,$

$n_1 = 240, n_2 = 160, n_3 = 260, n_4 = 180, n_5 = 220, n_6 = 140,$

$X_1 = \mathrm{rnorm}\ (1200, 5, 10), U_1 = X_1 + \mathrm{rnorm}\ (1200, 0, 1), Y_1 = U_1 + \mathrm{rnorm}\ (1200, 1, 3),$

$X_2 = \mathrm{rnorm}\ (800, 4, 8), U_2 = X_2 + \mathrm{rnorm}\ (800, 0, 1), Y_2 = U_2 + \mathrm{rnorm}\ (800, 1, 3),$

$X_3 = \mathrm{rnorm}\ (1300, 4, 9), U_3 = X_3 + \mathrm{rnorm}\ (1300, 0, 1), Y_3 = U_3 + \mathrm{rnorm}\ (1300, 1, 3),$

$X_4 = \mathrm{rnorm}\ (900, 3, 7), U_4 = X_4 + \mathrm{rnorm}\ (900, 0, 1), Y_5 = U_5 + \mathrm{rnorm}\ (900, 1, 3),$

$X_5 = \mathrm{rnorm}\ (1100, 3, 8), U_5 = X_5 + \mathrm{rnorm}\ (1100, 0, 1), Y_5 = U_5 + \mathrm{rnorm}\ (1100, 1, 3),$

$X_6 = \mathrm{rnorm}\ (700, 2, 5), U_6 = X_6 + \mathrm{rnorm}\ (700, 0, 1), Y_6 = U_6 + \mathrm{rnorm}\ (700, 1, 3).$

7.3. Population III.

$N_1 = 300, N_2 = 300, N_3 = 300, N_4 = 300, N_5 = 300, N_6 = 300,$

$n_1 = 60, n_2 = 60, n_3 = 60, n_4 = 60, n_5 = 60, n_6 = 60,$

$X_1 = \mathrm{rnorm}\ (300, 5, 10), U_1 = X_1 + \mathrm{rnorm}\ (300, 0, 1), Y_1 = U_1 + \mathrm{rnorm}\ (300, 1, 3),$

Table 1: Summary statistics using population 1.

h	N_h	n_h	λ_h	W_h	$\overline{Y}_h$	$\overline{X}_h$	$\overline{R}_{xh}$	S^2_{yh}
1	19	10	0.048	0.2375	2975.26	64.89	10	583977.5
2	32	16	0.032	0.4	4653.28	140.12	16	456563.3
3	14	7	0.080	0.4375	6553.28	402.08	7.5	195208.8
4	15	8	0.060	0.1875	782.66	764.27	8	437923.5

Table 2: Summary statistics using population 1.

λ^*_{400h}	λ^*_{040h}	λ^*_{004h}	λ^*_{220h}	λ^*_{202h}	λ^*_{022h}
2.28	0.45	0.69	0.46	0.78	0.53
0.56	2.09	0.75	0.74	0.58	0.98
0.62	0.62	0.66	0.80	0.78	0.63
1.22	0.90	0.67	1.02	0.70	0.69

Table 3: Summary statistics using population 2.

h	N_h	n_h	λ_h	W_h	$\overline{Y}_h$	$\overline{X}_h$	$\overline{R}_{xh}$	S^2_{yh}
1	9	3	0.222	0.2647	253	253.4	5	31978.25
2	10	3	0.23	0.2941	213.5	226.8	5.5	37629.39
3	15	4	0.18	0.4411	157.9	170.2	8	6893.067

Table 4: Summary statistics using population 2.

λ^*_{400h}	λ^*_{040h}	λ^*_{004h}	λ^*_{220h}	λ^*_{202h}	λ^*_{022h}
1.9286	1.07	0.57	1.38	0.77	0.64
0.511	0.42	0.59	0.42	0.41	0.40
1.20	1.42	0.67	1.28	0.65	0.76

Table 5: Summary statistics using population 3.

h	N_h	n_h	λ_h	W_h	$\overline{Y}_h$	$\overline{X}_h$	$\overline{R}_{xh}$	S^2_{yh}
1	106	9	0.10167	0.12412	1536.77	24375.5	53.45	41281746
2	106	17	0.04938	0.12412	2212.59	27421.7	53.33	133437791
3	94	38	0.01567	0.11007	9384.30	72409.9	47.44	894457433
4	171	67	0.00907	0.20023	5588.01	74364.6	85.88	820445636
5	204	7	0.13795	0.23887	966.95	26441.7	102.30	5710999
6	173	2	0.49421	0.20257	404.39	9843.8	86.80	89440.3

Table 6: Summary statistics using population 3.

λ^*_{400h}	λ^*_{040h}	λ^*_{004h}	λ^*_{220h}	λ^*_{202h}	λ^*_{022h}
76.8505	25.9404	0.7658	31.9862	1.7359	1.580836
93.2709	34.2325	0.7665	55.8554	1.8755	1.582854
24.2633	26.1169	0.7629	19.5822	1.6395	1.569098
99.7898	95.5953	0.7802	97.9452	1.8603	1.827503
53.5128	28.4838	0.7835	19.9886	1.6095	1.603712
28.7724	28.9154	0.7812	21.9465	1.5207	1.579165

$X_2 = r\text{norm}\ (300, 4, 8), U_2 = X_2 + r\text{norm}\ (300, 0, 1), Y_2 = U_2 + r\text{norm}\ (300, 1, 3),$

$X_3 = r\text{norm}\ (300, 4, 9), U_3 = X_3 + r\text{norm}\ (300, 0, 1), Y_3 = U_3 + r\text{norm}\ (300, 1, 3),$

$X_4 = r\text{norm}\ (300, 3, 7), U_4 = X_4 + r\text{norm}\ (300, 0, 1), Y_5 = U_5 + r\text{norm}\ (300, 1, 3),$

$X_5 = r\text{norm}\ (300, 3, 8), U_5 = X_5 + r\text{norm}\ (300, 0, 1), Y_5 = U_5 + r\text{norm}\ (300, 1, 3),$

Table 7: Summary statistics using population 4.

h	N_h	n_h	λ_h	W_h	$\overline{Y}_h$	$\overline{X}_h$	$\overline{R}_{xh}$	S^2_{yh}
1	127	31	0.02438	0.13759	703.740	20804.59	64.00	781163.9
2	117	21	0.03907	0.12676	413.000	9211.79	59.00	415924.8
3	103	29	0.02477	0.11159	573.174	14309.30	52.00	1068054.0
4	170	38	0.02043	0.18418	424.664	9478.85	85.48	657047.8
5	205	22	0.04057	0.22210	267.029	5569.94	102.99	162936.9
6	201	39	0.20066	0.21177	393.840	12997.59	100.99	506549

Table 8: Summary statistics using population 4.

λ^*_{400h}	λ^*_{040h}	λ^*_{004h}	λ^*_{220h}	λ^*_{202h}	λ^*_{022h}
3.94783	6.251589	0.771618	3.720488	0.9587947	1.188102
17.33181	19.35622	0.769189	18.35209	1.503609	1.517015
15.87136	16.3073	0.764996	16.09088	1.515692	1.525051
13.60375	11.67999	0.778862	11.65605	1.44632	1.431276
22.31908	23.14865	0.782227	22.30021	1.51224	1.526917
21.49882	24.26014	0.781766	21.79386	1.572395	1.601076

Table 9: Conditional values of different estimators using real data sets.

Conditional values	Population-I	Population-II	Population-III	Population-IV
Condition I	308077.037	170053.88236	185082878990	268317.021
Condition II	288895.337	24619.25236	40763973959	27349.521
Condition III	144739.537	2754.81436	31444528127	22541.981
Condition IV	127762.337	2238.92116	34911528799	9365.981
Condition V	293935.637	55621.42578	83019573416	102341.721
Condition VI	649319.337	351276.68236	88294916731	146686.721
Condition VII	125488.0445	1040.469061	392550613	4265.953

Table 10: Biases values of different estimators using real data sets.

Estimators	Population-I	Population-II	Population-III	Population-IV
$G_{0(st)}$	—	—	—	—
$G_{1(st)}$	83.9204	−30.89047	−137147.8	34.49139
$G_{2(st)}$	—	—	—	—
$G_{3(st)}$	254.9	−1354.096	19.74803	7.871472
$G_{4(st)}$	11.00481	−54.05082	−152436.6	−44.53858
$G_{5(st)}$	181.1137	317.4172	917899.3	595.1742
$G_{6(st)}$	182588.8	169013.42	$1.8469903e-11$	264051.068
$G_{prop(st)}$	308077.037	170053.89	163411187990	268317.021

Table 11: Mean square error (MSE) of estimators using populations I–IV.

Estimators	Population-I	Population-II	Population-III	Population-IV
$G_{0(st)}$	425574	188538	187490767203	272821.8
$G_{1(st)}$	406392.3	26463.37	43171862172	31854.3
$G_{2(st)}$	262236.5	21238.9320	33852416340	27046.76
$G_{3(st)}$	245259.3	20723.0388	35152317012	13870.76
$G_{4(st)}$	305432.6	74105.54342	85427461629	106846.5
$G_{5(st)}$	766816.3	369760.8	90702804944	151191.5
$G_{6(st)}$	242985.20075	19524.586701	2800438826	8359.251
$G_{prop(st)}$	117496.963	18484.11764	2407888213	4504.779

Table 12: Percentage relative efficiency (PRE) of estimators using populations I–IV.

Estimators	Population-I	Population-II	Population-III	Population-IV
$G_{0(st)}$	100	100	100	100
$G_{1(st)}$	104.724	712.430	434.2893	856.4677
$G_{2(st)}$	162.280	887.700	553.8475	1008.704
$G_{3(st)}$	173.523	909.799	219.4736	1966.885
$G_{4(st)}$	139.333	254.412	206.7089	255.34
$G_{5(st)}$	55.49881	208.246	6695.05	180.4478
$G_{6(st)}$	175.144	965.644	5293.644	3263.711
$G_{prop(st)}$	362.200	1020.00	7786.523	6056.276

Table 13: Mean square error (MSE) of estimators based on the simulation study.

Estimators	Population-I	Population-II	Population-III
$G_{0(st)}$	$5.167166e-06$	$7.274847e-06$	0.0001922386
$G_{1(st)}$	$3.510773e-06$	$4.860487e-06$	0.0001276158
$G_{2(st)}$	$2.723914e-06$	$3.727381e-06$	0.0001027585
$G_{3(st)}$	$1.750417e-06$	$2.244754e-06$	$6.313865e-05$
$G_{4(st)}$	$3.667552e-06$	$4.874891e-06$	0.000141089
$G_{5(st)}$	$9.072873e-06$	$1.1539e-05$	0.0003547078
$G_{6(st)}$	$3.005942e-07$	$3.781101e-07$	$1.031059e-05$
$G_{prop(st)}$	$7.132623e-08$	$7.55517e-08$	$2.195092e-06$

Table 14: Percentage relative efficiency (PRE) of estimators based on the simulation study.

Estimators	Population-I	Population-II	Population-III
$G_{0(st)}$	100	100	100
$G_{1(st)}$	147.1803	149.6732	150.6385
$G_{2(st)}$	189.6963	195.1732	187.0781
$G_{3(st)}$	295.1963	324.0821	304.4705
$G_{4(st)}$	140.8887	149.231	136.2534
$G_{5(st)}$	56.95181	63.04572	54.19633
$G_{6(st)}$	1718.984	1924.002	1864.477
$G_{\text{prop}(st)}$	7244.411	9628.964	8757.656

$X_6 = r\text{norm }(300, 2, 5), U_6 = X_6 + r\text{norm }(300, 0, 1), Y_6 = U_6 + r\text{norm }(300, 1, 3)$.

The percentage relative efficiency (PRE) is calculated as follows:

$$\text{PRE} = \frac{\text{Var}\left(G_{0(st)}\right)}{\text{MSE}\left(G_{j(st)}\right)} * 100, \quad (37)$$

where $j = 0, 1, 2, 3, 4, 5, 6$, prop.

The mean square error and percentage relative efficiency values are given in Tables 13 and 14.

8. Discussion

We used four real data sets to obtain the percentage relative efficiency of all estimators under variance stratified random sampling. The data descriptions of these populations are presented in Tables 1–8. All the conditional values of different estimators using four real data sets are given in Table 9. Bias values of the proposed and existing estimators using real data sets are presented in Table 10. The mean square error and percentage relative efficiency of estimators based on real data sets are given in Tables 11 and 12. Similarly, MSE and PRE of the simulation study are given in Tables 13 and 14. From the numerical results, it is observed that the proposed estimator is appreciable in terms of the minimum mean square error and higher percentage relative efficiency (PRE) as compared to existing estimators. The larger gain in efficiency is observed by using the proposed estimator over some existing estimators under stratified random sampling. The results integrated with this study are very sound and quite illuminating. Thus, it is recommended that the proposed estimator is useful in practice.

9. Conclusion

Several estimators for estimating the finite population variances are constructed on the basis of the auxiliary variable. We proposed an improved estimator using the dual supplementary information for a variance estimator in stratified sampling. The proposed estimator is the particular class of estimators given by Shabbir and Gupta [10]. The bias and MSE of the existing and proposed estimator are derived up to the first order of approximation. The performance of the proposed estimators is the best as compared to existing counterparts in terms of efficiency. Four real data sets are used to assess the efficiency of the proposed estimator over its existing counterparts. Moreover, a simulation study is also carried out to check the robustness and generalizability of the proposed variance estimator. Based on the numerical findings, the proposed estimator may be preferable for use in practical situations. The possible extension of this current work is to develop an improved class of estimators under nonresponse, distribution function, and measurement error for estimating the finite population variance under stratified random sampling.

Data Availability

All the data used for this study can be found inside the manuscript.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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