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Research Article

A Distribution-Free THWMA Control Chart under Ranked Set Sampling

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The basic assumption of the parametric control charts is that the underlying process follows the specific distribution. The appropriateness of parametric control charts is questionable when different industrial applications do not support this desired assumption. Recently, nonparametric control charts have been introduced to overcome this deficiency of parametric control charts. Nonparametric control charts have the same in-control run-length characteristics for all continuous distributions and are in-control robust. On the other hand, the ranked set sampling technique is preferred over the simple random sampling technique with control charts because it reduces variability and improves the control chart's performance. So, this study aims to propose a nonparametric triple homogenously weighted moving average control chart under Wilcoxon signed-rank test with a ranked set sampling technique (regarded as $\text{NPTHWMA}_{\text{RSS}}$) to further enhance the process location monitoring. Monte Carlo simulations are used for computational purposes. The proposed control chart's run-length performance is compared with competing control charts, like TEWMA-SR, TEWMA-$\overline{X}$, TEWMA-SN, TEWMA-SR_{RSS}, and $\text{NPDHWMA}_{\text{RSS}}$ control charts. The comparison revealed that the proposed $\text{NPTHWMA}_{\text{RSS}}$ control chart outperformed the other competing control charts, particularly for small to moderate shifts in process location. Finally, a real-life application is presented for quality practitioners to illustrate the effectiveness of the proposed control chart.

1. Introduction

The main concern in the manufacturing process is the product's quality, and the variations are the primary source that influences the quality of the product. The variations are generally categorized into the natural cause of variations and unnatural causes of variations. Natural variations are harmless while unnatural variations seriously impact the quality of the ultimate product. The process remains in-control (IC) with natural variations, whereas the process is regarded as an out-of-control (OOC) process in the presence of unnatural variations. The existence of unnatural variations causes the shift in process parameters (location and/or dispersion). Control charts are used to monitor these shifts in the process parameters. Control charts are typically divided into memoryless and memory-type control charts based on their structural system. Shewhart and Van Nostrand [1] presented the first memoryless control chart, regarded as the Shewhart control chart, whereas Page [2] and Roberts [3] introduced the concept of memory-type control charts, like cumulative sum (CUSUM) and exponentially weighted moving average (EWMA) control charts, respectively.

Several extensions to the main structure of classical control charts have already been made in order to improve their ability to detect earlier shifts. For example, Alevizakos et al. [4] offered a triple EWMA (TEWMA) control chart that outperformed the EWMA and DEWMA control charts. Similarly, Chatterjee et al. [5] investigated the use of a new S^2 TEWMA control chart for detecting shifts in process dispersion. Likewise, Alevizakos et al. [6] and Alevizakos et al. [7] introduced the TEWMA-SR and the TEWMA-SN control charts, respectively, to enhance their shift detection ability in the process location. Also, Mahmood et al. [8] suggested a control chart for joint monitoring of process location and dispersion. Later, Alevizakos et al. [9] presented one and two-sided TEWMA control chart, observing the time between $k \geq 1$ events using the gamma distribution. After that, Rasheed et al. [10] and Rasheed et al. [11] presented two control charts for efficiently monitoring the process mean.

Conventional parametric control charts are often used when an ongoing process follows a specified probability distribution. In many cases, the ongoing process may deviate from a particular distribution, or the distribution of the current process may be in question. Nonparametric (NP) control charts are a good alternative to parametric control charts. The IC run-length (RL) features of the NP control charts remain the same for all symmetric continuous distributions. The sign (SN), as well as Wilcoxon signed-rank (SR), are quite well NP approaches used in statistical process monitoring (SPM). Likewise, simple random sampling (SRS) and ranked set sampling (RSS) techniques are quite often used in SPM with both parametric and NP control charts to observe the ongoing process data. The SPM literature supports RSS because it reduces variability and enhances the efficiency of the associated control charts (Haq et al. [12] and Noor-ul-Amin and Tayyab [13]). Many researchers, including Tsai et al. [14], Abid et al. [15], Abbasi [16], Chakraborti and Graham [17], Rasheed et al. [18], and Abbasi et al. [19], also used these sampling techniques along with NP control charts.

Hunter [20] pointed out that the EWMA control chart gives more weight to recent observations and less weight to former observations. Abbas [21] introduced the homogeneously weighted moving average (HWMA) control chart to monitor shifts in process location effectively. It gives a specific weight to current observations and the remaining weight to all previous observations equally. The distribution of weights in such a way enhances the HWMA control chart's capacity to detect early shifts. Similarly, Adeoti and Koleoso [22] developed a hybrid HWMA (HHWMA) control chart to monitor the process location's shifts. Furthermore, Anwar et al. [23] suggested a double HWMA (DHWMA) control chart for observing shifts in process location. They reported that the DHWMA control chart detects shifts better than the HWMA control chart. Also, Riaz et al. [24] designed a triple HWMA (THWMA) control chart to identify the small shifts in the process location. The THWMA control chart outperforms the HWMA and DHWMA control charts for monitoring process location.

Takahasi and Wakimoto [25] suggested that an RSS-based mean estimator is more unbiased and efficient than an SRS-based estimator. Similarly, the THWMA control chart is more effective for monitoring process location parameters. A thorough literature review observed that no one had explored an NP THWMA control chart based on RSS to date. To fill this gap in this study, we are aimed to propose an NP THWMA-SR control chart under RSS ($\text{NPTHWMA}_{\text{RSS}}$) for monitoring shifts in process location for continuous and symmetric distribution. The proposed control chart's RL characteristics, including average RL (ARL), median RL (MDRL), and standard deviation of RL (SDRL), are measured using various distributions like normal, student's t, contaminated normal (CN), Laplace, and logistic distributions. These RL characteristics of the proposed $\text{NPTHWMA}_{\text{RSS}}$ control chart is compared to the other competing control charts, like TEWMA-SR, TEWMA-$\overline{X}$, TEWMA-SN, TEWMA-SR_{RSS}, and $\text{NPDHWMA}_{\text{RSS}}$ control charts.

The remainder of the paper is ordered as follows: Section 2 offers the design structure of the competing and the proposed control chart. Section 3 defines the proposed control chart's IC and out-of-control (OOC) performance. Section 4 covers a comparative study of the proposed control chart, whereas Section 5 provides a real-life application. Section 6 concludes with closing remarks.

2. Competing and Proposed Control Charts

This section provides the design structure of the competing and the proposed $\text{NPTHWMA}_{\text{RSS}}$ control chart. More details can be found in the subsequent subsections.

2.1. $\text{NPHWMA}_{\text{RSS}}$ Control Chart. Various researchers like Kim and Kim [26] and Abid et al. [15] used the following RSS-based Wilcoxon signed-rank statistic to monitor the shift in process location.

$$SR_{RSS_t} = \sum_{j=1}^{n}\sum_{h=1}^{m} \text{sign}\left(X_{tj(h)} - \theta_0\right)R^{+}_{tj(h)}, \tag{1}$$

where θ_0 symbolizes the process median and t, j, and h denote the number of samples, observations, and cycles used in the RSS approach, respectively. The mean and variance of SR_{RSS_t} statistic are $E(SR_{RSS_t}) = 0$ and $\text{Var}(SR_{RSS_t}) = (r(r+1)(2r+1)/6)\varpi_0^2$., respectively, where r denotes the number of replications and can be defined as $r = nm$. In this study, we assume $m = 1$ for a valid assessment of the proposed control chart to the competitors, so $r = nm$, which is $r = n$. The quantity ϖ_0^2 is used to improve the efficiency of the control chart and can be defined as $\varpi_0^2 = 1 - (4/n)\sum_{j=1}^{n}(F_k(0) - (1/2))^2$. The values of $F_k(0)$ can be obtained by solving the following mathematical expression $F_k(0) = (r!/(j-1)\ !(r-j)!)\int_{-\infty}^{0} F(t)^{j-1}(1-F(t))^{r-j} f(t)dt$. The plotting statistic of the NP HWMA under RSS ($\text{NPHWMA}_{\text{RSS}}$) control chart can be defined as follows:

$$NPH_t = \eta SR_{(RSS)_t} + (1-\eta)\overline{SR}_{(RSS)t-1}, \tag{2}$$

where η is a smoothing parameter and meets the condition $\eta \epsilon (0,1]$. Also, the term $\overline{SR}_{(RSS)t-1}$ is the mean of $SR_{(RSS)}$ of $t-1$ samples. The mean and variance of the statistic NPH_t are $E(NPH_t) = \mu_0$ and

$$\operatorname{Var}(NPH_t) = \begin{cases} \eta^2\left(\dfrac{r(r+1)(2r+1)}{6}\right)\varpi_0^2, & \text{if } t = 1 \\ \left(\eta^2 + \dfrac{(1-\eta)^2}{t-1}\right)\left(\dfrac{r(r+1)(2r+1)}{6}\right)\varpi_0^2, & \text{if } t > 1 \end{cases}, \tag{3}$$

respectively. The control limits of the NPHWMA$_{\mathrm{RSS}}$ control chart are as follows:

$$\left.\begin{aligned} LCL_{(NPHWMA_{RSS})t} &= \begin{cases} \mu_0 - L\sqrt{\eta^2\left(\dfrac{r(r+1)(2r+1)}{6}\right)\varpi_0^2}, & \text{if } t = 1 \\ \mu_0 - L\sqrt{\left(\eta^2 + \dfrac{(1-\eta)^2}{t-1}\right)\left(\dfrac{r(r+1)(2r+1)}{6}\right)\varpi_0^2}, & \text{if } t > 1 \end{cases} \\ CL_{(NPHWMA_{RSS})t} &= \mu_0 \\ UCL_{(NPHWMA_{RSS})t} &= \begin{cases} \mu_0 + L\sqrt{\eta^2\left(\dfrac{r(r+1)(2r+1)}{6}\right)\varpi_0^2}, & \text{if } t = 1 \\ \mu_0 + L\sqrt{\left(\eta^2 + \dfrac{(1-\eta)^2}{t-1}\right)\left(\dfrac{r(r+1)(2r+1)}{6}\right)\varpi_0^2}, & \text{if } t > 1 \end{cases} \end{aligned}\right\}. \tag{4}$$

If $NPH_t > UCL_{(NPHWMA_{RSS})t}$ or $NPH_t < LCL_{(NPHWMA_{RSS})t}$, the process is considered OOC; otherwise, it remains IC.

2.2. NPDHWMA$_{RSS}$ Control Chart. The plotting statistic of the NPDHWMA$_{\mathrm{RSS}}$ control chart is defined as follows:

$$\left.\begin{aligned} NPH_t &= \eta SR_{RSS_{(t)}} + (1-\eta)\overline{SR}_{(RSS)t-1}, \\ NP\,DH_t &= \eta NPH_t + (1-\eta)\overline{SR}_{(RSS)t-1}, \end{aligned}\right\}. \tag{5}$$

The simplified version of the plotting statistic of the proposed NPDHWMA$_{\mathrm{RSS}}$ control chart is expressed as follows:

$$NP\,DH_t = \eta^2 SR_{RSS_t} + \left(1-\eta^2\right)\overline{SR}_{(RSS)t-1}. \tag{6}$$

The initialvalueofSR_{RSS_t} is 0, whereas the IC mean and variance of the plotting statistic $NP\,DH_t$ are

$$\left.\begin{aligned} E(DH_t) &= \mu_0 \\ \operatorname{Var}(DH_t) &= \eta^4\left(\frac{r(r+1)(2r+1)}{6}\right)\varpi_0^2 \end{aligned}\right\} \text{if } t = 1, \quad (7)$$

and

$$\left.\begin{aligned} E(DH_t) &= \mu_0 \\ \operatorname{Var}(DH_t) &= \left(\eta^4 + \frac{(1-\eta)^2(1+\eta)^2}{t-1}\right)\left(\frac{r(r+1)(2r+1)}{6}\right)\varpi_0^2 \end{aligned}\right\} \text{if } t > 1. \quad (8)$$

The control limits of the proposed $\mathrm{NPDHWMA_{RSS}}$ control chart are defined as follows:

$$\left.\begin{aligned} \mathrm{LCL}_{(\mathrm{NPDHWMA_{RSS}})t} &= \begin{cases} \mu_0 - L\sqrt{\eta^4\left(\dfrac{r(r+1)(2r+1)}{6}\right)\varpi_0^2}, & \text{if } t = 1 \\ \mu_0 - L\sqrt{\left(\eta^4 + \dfrac{(1-\eta)^2(1+\eta)^2}{t-1}\right)\left(\dfrac{r(r+1)(2r+1)}{6}\right)\varpi_0^2}, & \text{if } t > 1 \end{cases} \\ CL_{(\mathrm{NPDHWMA_{RSS}})t} &= \mu_0 \\ UCL_{(\mathrm{NPDHWMA_{RSS}})t} &= \begin{cases} \mu_0 + L\sqrt{\eta^4\left(\dfrac{r(r+1)(2r+1)}{6}\right)\varpi_0^2}, & \text{if } t = 1 \\ \mu_0 + L\sqrt{\left(\eta^4 + \dfrac{(1-\eta)^2(1+\eta)^2}{t-1}\right)\left(\dfrac{r(r+1)(2r+1)}{6}\right)\varpi_0^2}, & \text{if } t > 1 \end{cases} \end{aligned}\right\}. \quad (9)$$

Plot $NP\ DH_t$ against $\mathrm{LCLD}_{(\mathrm{NPDHWMA_{RSS}})t}$ and $\mathrm{UCLD}_{(\mathrm{NPDHWMA_{RSS}})t}$. If $\mathrm{NPDH}_t > \mathrm{UCLD}_{(\mathrm{NPDHWMA_{RSS}})t}$ or $\mathrm{NPDH}_t < \mathrm{LCLD}_{(\mathrm{NPDHWMA_{RSS}})t}$, the underlying process is OOC; else, it is IC.

2.3. Proposed $NPTHWMA_{RSS}$ Control Chart. This subsection describes the design structure of the proposed $\mathrm{NPTHWMA_{RSS}}$ control chart to monitor shifts in process location. The plotting statistic of the $\mathrm{NPTHWMA_{RSS}}$ control chart based on RSS is defined as follows:

$$\left.\begin{aligned} NPH_t &= \eta SR_{(RSS)_t} + (1-\eta)\overline{SR}_{(RSS)t-1}, \\ NP\ DH_t &= \eta NPH_t + (1-\eta)\overline{SR}_{(RSS)t-1}, \\ NPTH_t &= \eta NP\ DH_t + (1-\eta)\overline{SR}_{(RSS)t-1}, \end{aligned}\right\}. \quad (10)$$

The simplified version of (10) is

Table 1: PDFs of the continuous distributions used for this study.

	Distributions	PDF
(i)	Standard normal	$f(X) = (e^{(-X^2/2)}/\sqrt{2\pi})$, where $M_0 = 0$ and $\sigma^2 = 1$
(ii)	Student's t_v	$f(X) = (\Gamma((v+1)/2)/(\Gamma(v/2)\sqrt{v\pi}))(1+(X^2/v))^{((v+1)/2)}$, where $M_0 = 0$, $\sigma^2 = (v/(v-2))$ and $v = 4$ and 8 are taken
(iii)	Logistic	$f(X) = e^{(-\pi X)/\sqrt{3}}/((\sqrt{3}/\pi)(1+e^{(-\pi X)/\sqrt{3}})^2)$ where $M_0 = 0$ and $\sigma^2 = 3/\pi^2$
(iv)	Laplace	$f(X) = (1/2)e^{-\lvert X\rvert}$, where $M_0 = 0$ and $\sigma^2 = 1/2$
(v)	Contaminated normal (CN)	$f(X) = (0.95e^{(-X^2/2)}/\sqrt{2\pi}) + (0.05e^{-(X^2/2\sigma_0^2)}/\sigma_0\sqrt{2\pi})$, where $M_0 = 0$ and $\sigma^2 = 0.95 + 0.05\sigma_0^2$

$$NPTH_t = \eta^3 SR_{(RSS)_t} + \left(1-\eta^3\right)\overline{SR}_{(RSS)t-1}. \quad (11)$$

The mean and variance of the statistic $NPTH_t$ are $E(NPTH_t) = \mu$ and

$$\operatorname{Var}(TH_t) = \left[\eta^6 + \frac{\left(1-\eta^3\right)^2}{(t-1)}\right]\left(\frac{r(r+1)(2r+1)}{6}\right)\varpi_0^2. \quad (12)$$

Based on this mean and variance, the control limits for the $\text{NPTHWMA}_{\text{RSS}}$ control chart are as follows:

$$\left.\begin{aligned}
LCL_{(\text{NPTHWMA})t} &= \begin{cases} \mu - L\sqrt{\eta^6\left(\dfrac{r(r+1)(2r+1)}{6}\right)\varpi_0^2}, & t = 1\\[2ex] \mu - L\sqrt{\left[\eta^6 + \dfrac{\left(1-\eta^3\right)^2}{(t-1)}\right]\left(\dfrac{r(r+1)(2r+1)}{6}\right)\varpi_0^2}, & t > 1\end{cases}\\
CL_{(THWMA)t} &= \mu\\
UCL_{(\text{NPTHWMA})t} &= \begin{cases} \mu + L\sqrt{\eta^6\left(\dfrac{r(r+1)(2r+1)}{6}\right)\varpi_0^2}, & t = 1\\[2ex] \mu + L\sqrt{\left[\eta^6 + \dfrac{\left(1-\eta^3\right)^2}{(t-1)}\right]\left(\dfrac{r(r+1)(2r+1)}{6}\right)\varpi_0^2}, & t > 1\end{cases}
\end{aligned}\right\}. \quad (13)$$

When $LCL_{(\text{NPTHWMA})t} < NPTH_t < UCL_{(\text{NPTHWMA})t}$, the process is in IC state; otherwise, it is OOC.

3. Implementation of the Proposed Control Chart

The performance metrics of the proposed $\text{NPTHWMA}_{\text{RSS}}$ control chart for monitoring shifts in process location is given in this section. Subsection 3.1 offers the performance metrics of the proposed $\text{NPTHWMA}_{\text{RSS}}$ control chart. Similarly, the proposed control chart's robustness, IC, and OOC performance are presented in Subsection 3.2.

3.1. Performance Metrics. The ARL is commonly used to evaluate the control chart's performance. The ARL is the expected number of sample points before the first OOC signal from the control chart. In this study, we set $\text{ARL}_0 = 370$, with sample sizes $n = 5$ and 10. Monte Carlo simulations with 50,000 simulations in the R program are used to determine the RL characteristics. To examine the performance behaviour of the proposed $\text{NPTHWMA}_{\text{RSS}}$ control chart, various values of $\eta \in$ (0.10, 0.25, 0.50, and 0.75) and $\delta \in$ (0.025, 0.05, 0.075, 0.10, 0.25, 0.50, 0.75, 1.00, 1.50, 2.00, 2.50, 3.00, and 5.00) are used. The following steps are used to obtain the nominal (prespecified) ARL_0 value:

(i) To select a random sample from the evaluated distribution.

(ii) Specify the process parameters (η and L).

(iii) Determine the $NPTH_t$ plotting statistics using equation (8).

(iv) Find $LCL_{(\text{NPTHWMA}_{\text{RSS}})t}$ and $UCL_{(\text{NPTHWMA}_{\text{RSS}})t}$ from equation (9).

Table 2: RL characteristics of the proposed $NPTHWMA_{RSS}$ control chart under different distributions with nominal $ARL_0 = 370$ and $n = 5$.

(η, L)	Distributions	Metrics	δ = 0	0.025	0.05	0.075	0.1	0.25	0.5	0.75	1	1.5	2	2.5	3	5
0.10, 1.26	Normal	ARL	371.46	50.94	23.15	15.13	11.27	3.98	1.93	1.26	1.06	1.00	1.00	1.00	1.00	1.00
		MDRL	8.00	9.00	7.00	7.00	5.00	3.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	2361.91	132.30	41.95	22.77	14.61	3.23	1.31	0.71	0.35	0.06	0.00	0.00	0.00	0.00
	$t(4)$	ARL	371.51	59.55	28.23	17.93	13.26	4.71	2.29	1.56	1.26	1.07	1.03	1.02	1.01	1.00
		MDRL	9.00	8.00	8.00	7.00	6.00	4.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	2348.39	164.04	55.17	28.17	18.29	4.08	1.57	1.01	0.69	0.38	0.24	0.18	0.13	0.05
	$t(8)$	ARL	372.01	52.21	25.43	15.33	11.03	4.10	1.96	1.32	1.09	1.01	1.00	1.00	1.00	1.00
		MDRL	9.23	7.05	6.73	6.22	5.87	3.41	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	2375.02	139.31	50.07	24.17	15.00	2.79	1.11	0.81	0.49	0.15	0.09	0.02	0.00	0.00
	CN	ARL	372.17	52.38	24.74	15.95	11.18	4.15	2.01	1.34	1.09	1.01	1.00	1.00	1.00	1.00
		MDRL	9.00	8.00	8.00	7.00	5.00	4.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	2337.53	137.50	45.11	24.19	14.63	3.46	1.37	0.79	0.43	0.15	0.07	0.04	0.00	0.00
	Laplace	ARL	371.09	38.05	17.93	11.14	8.17	3.12	1.67	1.25	1.10	1.02	1.01	1.00	1.00	1.00
		MDRL	9.00	9.00	7.00	5.00	5.00	3.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	2433.95	82.52	28.75	14.66	8.91	2.31	1.09	0.68	0.44	0.21	0.10	0.05	0.03	0.00
	Logistic	ARL	371.58	46.21	21.13	13.06	9.20	3.23	1.41	1.09	1.02	1.00	1.00	1.00	1.00	1.00
		MDRL	8.77	7.81	6.79	5.90	4.72	2.75	0.83	0.53	0.21	0.12	0.01	0.00	0.00	0.00
		SDRL	2464.11	113.09	35.80	18.51	11.47	2.16	1.01	0.53	0.28	0.08	0.01	0.00	0.00	0.00
0.25, 1.615	Normal	ARL	370.10	161.69	66.15	35.89	22.95	5.83	2.40	1.50	1.15	1.01	1.00	1.00	1.00	1.00
		MDRL	126.00	70.50	35.00	22.00	15.00	5.00	3.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	481.04	210.92	80.73	39.85	24.58	4.45	1.50	0.92	0.53	0.13	0.00	0.00	0.00	0.00
	$t(4)$	ARL	371.23	177.19	78.26	43.01	28.02	6.40	2.21	1.30	1.11	1.01	1.00	1.00	1.00	1.00
		MDRL	137.03	76.11	40.31	23.19	17.53	5.49	2.33	1.31	1.09	1.01	1.00	1.00	1.00	1.00
		SDRL	494.57	242.89	96.61	50.39	30.17	5.08	1.20	1.08	0.82	0.31	0.19	0.07	0.01	0.00
	$t(8)$	ARL	371.60	169.22	70.29	38.90	25.03	5.11	2.10	1.19	1.09	1.03	1.01	1.00	1.00	1.00
		MDRL	134.02	73.92	35.08	22.71	16.01	4.29	2.01	1.20	1.06	1.01	1.00	1.00	1.00	1.00
		SDRL	493.61	227.49	87.50	44.13	26.05	4.63	1.39	1.02	0.62	0.21	0.14	0.05	0.01	0.00
	CN	ARL	371.25	160.18	69.05	37.60	24.39	6.01	2.19	1.47	1.10	1.01	1.00	1.00	1.00	1.00
		MDRL	138.03	73.05	35.10	22.01	14.51	4.31	2.10	1.03	1.01	1.00	1.00	1.00	1.00	1.00
		SDRL	496.32	210.21	84.13	42.01	25.12	4.10	1.19	1.01	0.70	0.19	0.08	0.05	0.01	0.00
	Laplace	ARL	371.83	109.31	39.81	21.47	14.08	3.51	1.85	1.46	1.07	1.01	1.00	1.00	1.00	1.00
		MDRL	134.63	52.31	23.57	14.01	9.02	3.65	2.01	1.37	1.02	1.01	1.00	1.00	1.00	1.00
		SDRL	496.02	141.32	46.59	23.11	13.69	2.03	1.11	0.82	0.59	0.20	0.09	0.01	0.00	0.00
	Logistic	ARL	370.99	139.20	57.11	31.01	19.70	4.89	2.05	1.10	1.07	1.01	1.00	1.00	1.00	1.00
		MDRL	134.01	64.11	29.30	18.04	12.11	3.61	2.33	1.03	1.01	1.00	1.00	1.00	1.00	1.00
		SDRL	490.10	184.17	67.89	33.68	20.41	3.59	1.18	0.69	0.44	0.18	0.09	0.01	0.00	0.00

Table 2: Continued.

(η, L)	Distributions	Metrics	δ = 0	0.025	0.05	0.075	0.1	0.25	0.5	0.75	1	1.5	2	2.5	3	5
0.50, 2.67	Normal	ARL	370.77	248.78	133.08	78.50	51.22	12.29	4.45	2.82	2.11	1.39	1.12	1.03	1.01	1.00
		MDRL	307.00	201.00	108.00	66.00	44.00	11.00	4.00	3.00	2.00	1.00	1.00	1.00	1.00	1.00
		SDRL	284.49	197.20	101.51	55.61	34.87	6.87	1.95	1.13	0.93	0.62	0.35	0.17	0.08	0.00
	$t(4)$	ARL	370.85	269.93	155.20	94.06	63.10	15.37	5.45	3.35	2.55	1.74	1.38	1.21	1.12	1.02
		MDRL	310.00	221.00	125.00	78.00	53.00	14.00	5.00	3.00	3.00	1.00	1.00	1.00	1.00	1.00
		SDRL	282.03	211.07	119.14	68.69	43.76	9.02	2.65	1.43	1.13	0.87	0.67	0.50	0.38	0.17
	$t(8)$	ARL	369.03	264.34	143.18	86.11	57.48	13.86	4.90	3.07	2.32	1.56	1.22	1.10	1.04	1.00
		MDRL	302.00	214.00	117.00	72.00	49.00	13.00	5.00	3.00	3.00	1.00	1.00	1.00	1.00	1.00
		SDRL	285.75	207.19	108.67	62.91	39.21	7.96	2.26	1.28	1.02	0.76	0.49	0.32	0.22	0.06
	CN	ARL	368.81	258.94	138.49	82.54	54.41	12.99	4.65	2.95	2.21	1.47	1.16	1.06	1.02	1.00
		MDRL	304.00	210.00	113.00	70.00	47.00	12.00	4.00	3.00	2.00	1.00	1.00	1.00	1.00	1.00
		SDRL	280.42	206.50	104.54	58.49	36.77	7.39	2.10	1.18	0.97	0.69	0.43	0.26	0.16	0.04
	Laplace	ARL	369.34	195.55	90.98	51.04	33.04	8.45	3.57	2.45	1.91	1.39	1.17	1.09	1.04	1.00
		MDRL	307.00	161.00	76.00	44.00	29.00	8.00	3.00	3.00	2.00	1.00	1.00	1.00	1.00	1.00
		SDRL	281.59	152.85	65.44	34.61	21.15	4.57	1.61	1.13	0.94	0.65	0.44	0.30	0.20	0.04
	Logistic	ARL	370.13	237.57	121.49	70.03	45.85	10.96	4.09	2.66	2.03	1.39	1.14	1.06	1.02	1.00
		MDRL	307.00	192.00	100.00	59.00	39.00	10.00	4.00	3.00	2.00	1.00	1.00	1.00	1.00	1.00
		SDRL	281.59	186.55	90.12	49.64	30.86	6.10	1.81	1.13	0.94	0.64	0.39	0.25	0.15	0.02
0.75, 2.859	Normal	ARL	371.01	353.40	272.81	191.02	110.85	20.74	5.01	2.66	1.89	1.34	1.12	1.03	1.01	1.00
		MDRL	286.00	256.00	188.00	133.00	83.50	16.00	4.00	3.00	2.00	1.00	1.00	1.00	1.00	1.00
		SDRL	327.29	341.61	281.16	205.18	114.83	16.92	2.94	1.23	0.77	0.49	0.33	0.16	0.09	0.00
	$t(4)$	ARL	372.03	355.04	291.69	213.52	144.03	26.49	6.51	3.39	2.34	1.61	1.32	1.18	1.11	1.02
		MDRL	286.00	261.00	207.00	147.00	100.00	21.00	6.00	3.00	2.00	1.00	1.00	1.00	1.00	1.00
		SDRL	327.18	337.98	286.71	223.50	152.11	21.83	4.19	1.78	1.13	0.72	0.53	0.42	0.33	0.14
	$t(8)$	ARL	369.36	355.52	288.97	207.82	130.42	23.25	5.72	3.00	2.10	1.45	1.21	1.09	1.04	1.00
		MDRL	275.02	263.10	201.00	138.00	91.00	18.00	5.00	3.00	2.00	1.00	1.00	1.00	1.00	1.00
		SDRL	329.72	335.93	286.60	213.56	130.83	18.73	3.55	1.50	0.94	0.58	0.42	0.30	0.20	0.06
	CN	ARL	372.01	358.82	287.96	198.80	125.90	22.20	5.36	2.83	1.99	1.39	1.15	1.06	1.02	1.00
		MDRL	285.00	256.00	201.00	129.50	87.00	17.00	5.00	3.00	2.00	1.00	1.00	1.00	1.00	1.00
		SDRL	329.53	342.61	281.79	207.94	125.36	18.01	3.23	1.34	0.85	0.54	0.38	0.24	0.15	0.04
	Laplace	ARL	368.77	307.89	196.55	103.52	63.29	11.61	3.65	2.29	1.73	1.32	1.16	1.08	1.04	1.00
		MDRL	281.00	221.00	133.00	75.00	48.00	9.00	3.00	2.00	2.00	1.00	1.00	1.00	1.00	1.00
		SDRL	323.25	294.27	199.75	96.29	55.28	8.51	2.05	1.14	0.81	0.54	0.38	0.27	0.20	0.04
	Logistic	ARL	369.51	343.92	259.29	172.78	98.22	17.08	4.45	2.50	1.82	1.32	1.13	1.05	1.02	1.00
		MDRL	283.00	245.00	179.00	112.00	73.00	13.00	4.00	2.00	2.00	1.00	1.00	1.00	1.00	1.00
		SDRL	325.98	329.09	256.64	181.86	90.14	13.26	2.50	1.17	0.79	0.50	0.34	0.22	0.15	0.02

Table 3: RL characteristics of the proposed $NPTHWMA_{RSS}$ control chart under different distributions at nominal $ARL_0 = 370$ and $n = 10$.

(η, L)	Distributions	Metrics	δ = 0	0.025	0.05	0.075	0.1	0.25	0.5	0.75	1	1.5	2	2.5	3	5
0.25, 1.362	Normal	ARL	369.95	66.05	23.75	12.59	8.40	2.52	1.14	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		MDRL	124.00	35.00	15.00	9.00	6.00	3.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	479.18	79.87	24.77	11.63	7.06	1.56	0.52	0.08	0.00	0.00	0.00	0.00	0.00	0.00
	$t(4)$	ARL	370.89	84.15	29.79	16.01	10.26	3.06	1.34	1.03	1.00	1.00	1.00	1.00	1.00	1.00
		MDRL	128.00	44.00	18.00	11.00	8.00	3.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	478.00	101.26	32.09	15.39	9.15	1.91	0.77	0.25	0.08	0.01	0.00	0.00	0.00	0.00
	$t(8)$	ARL	369.66	75.03	26.32	14.15	9.24	2.77	1.23	1.01	1.00	1.00	1.00	1.00	1.00	1.00
		MDRL	129.00	38.00	16.00	10.00	7.00	3.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	485.37	92.18	28.39	13.59	7.93	1.74	0.64	0.15	0.02	0.00	0.00	0.00	0.00	0.00
	CN	ARL	371.42	70.35	24.94	13.41	8.77	2.61	1.17	1.01	1.00	1.00	1.00	1.00	1.00	1.00
		MDRL	121.00	36.00	16.00	9.00	7.00	3.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	479.26	86.88	26.81	12.66	7.49	1.63	0.57	0.10	0.00	0.00	0.00	0.00	0.00	0.00
	Laplace	ARL	370.07	45.69	16.18	8.75	5.81	1.95	1.07	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		MDRL	126.00	27.00	11.00	7.00	5.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	478.42	51.94	15.73	7.49	4.42	1.23	0.36	0.06	0.00	0.00	0.00	0.00	0.00	0.00
	Logistic	ARL	371.42	61.32	21.09	11.43	7.48	2.31	1.10	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		MDRL	129.00	34.00	13.00	8.00	6.00	3.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	480.37	71.43	22.02	10.16	6.16	1.44	0.44	0.05	0.00	0.00	0.00	0.00	0.00	0.00
0.50, 2.22	Normal	ARL	371.17	127.15	49.54	26.48	16.84	4.15	1.66	1.05	1.00	1.00	1.00	1.00	1.00	1.00
		MDRL	298.00	102.00	42.00	23.00	15.00	4.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	306.27	98.00	34.15	16.67	10.21	1.93	0.87	0.26	0.03	0.00	0.00	0.00	0.00	0.00
	$t(4)$	ARL	368.97	152.05	60.96	32.98	21.35	5.10	2.09	1.27	1.05	1.00	1.00	1.00	1.00	1.00
		MDRL	292.00	123.00	52.00	29.00	19.00	5.00	2.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	306.34	118.72	43.55	21.86	13.29	2.51	1.04	0.59	0.24	0.04	0.01	0.00	0.00	0.00
	$t(8)$	ARL	370.66	139.50	55.34	29.74	19.04	4.60	1.87	1.13	1.01	1.00	1.00	1.00	1.00	1.00
		MDRL	291.00	112.00	47.00	26.00	17.00	4.00	2.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	309.72	110.19	38.86	19.14	11.81	2.24	0.95	0.42	0.12	0.00	0.00	0.00	0.00	0.00
	CN	ARL	369.50	134.07	51.71	28.07	17.88	4.35	1.75	1.09	1.00	1.00	1.00	1.00	1.00	1.00
		MDRL	292.00	107.00	44.00	25.00	16.00	4.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	307.98	106.00	36.26	18.23	10.87	2.05	0.91	0.35	0.06	0.01	0.00	0.00	0.00	0.00
	Laplace	ARL	369.36	90.87	33.18	17.73	11.21	3.12	1.38	1.04	1.00	1.00	1.00	1.00	1.00	1.00
		MDRL	287.00	76.00	29.00	16.00	10.00	3.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	308.29	67.54	21.63	10.79	6.41	1.45	0.70	0.24	0.07	0.00	0.00	0.00	0.00	0.00
	Logistic	ARL	370.26	117.05	43.95	24.02	14.91	3.80	1.54	1.05	1.00	1.00	1.00	1.00	1.00	1.00
		MDRL	290.00	95.00	37.00	21.00	13.00	4.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	309.29	90.56	29.54	14.98	8.97	1.75	0.80	0.25	0.07	0.00	0.00	0.00	0.00	0.00

Table 3: Continued.

(η, L)	Distributions	Metrics	δ = 0	0.025	0.05	0.075	0.1	0.25	0.5	0.75	1	1.5	2	2.5	3	5
0.75, 2.319	Normal	ARL	371.13	228.89	100.43	47.22	26.67	4.14	1.50	1.04	1.00	1.00	1.00	1.00	1.00	1.00
		MDRL	262.00	160.00	72.00	34.00	20.00	4.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	361.86	228.87	95.92	42.78	22.45	2.38	0.64	0.20	0.04	0.00	0.00	0.00	0.00	0.00
	$t(4)$	ARL	370.19	255.05	120.96	62.38	35.39	5.46	1.86	1.20	1.03	1.00	1.00	1.00	1.00	1.00
		MDRL	260.00	175.00	87.00	46.00	27.00	5.00	2.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	362.84	257.07	114.87	57.16	30.69	3.42	0.88	0.42	0.18	0.04	0.00	0.00	0.00	0.00
	$t(8)$	ARL	371.31	246.14	109.42	54.56	30.50	4.76	1.65	1.11	1.01	1.00	1.00	1.00	1.00	1.00
		MDRL	259.00	169.00	78.00	40.00	23.00	4.00	2.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	368.61	250.95	104.57	49.40	26.31	2.90	0.75	0.32	0.10	0.01	0.00	0.00	0.00	0.00
	CN	ARL	369.71	237.43	104.05	50.21	28.55	4.40	1.55	1.07	1.00	1.00	1.00	1.00	1.00	1.00
		MDRL	258.00	164.00	75.00	37.00	21.00	4.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	359.39	236.67	99.19	45.25	24.93	2.56	0.68	0.25	0.06	0.00	0.00	0.00	0.00	0.00
	Laplace	ARL	371.00	174.13	62.14	27.47	15.27	2.93	1.28	1.04	1.00	1.00	1.00	1.00	1.00	1.00
		MDRL	260.00	123.00	46.00	21.00	12.00	3.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	364.84	167.98	56.34	23.27	11.82	1.56	0.50	0.19	0.06	0.01	0.00	0.00	0.00	0.00
	Logistic	ARL	369.71	217.79	88.66	40.95	22.37	3.67	1.40	1.04	1.00	1.00	1.00	1.00	1.00	1.00
		MDRL	259.00	153.00	63.00	31.00	17.00	3.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	362.01	217.27	85.10	35.74	18.45	2.06	0.57	0.19	0.05	0.00	0.00	0.00	0.00	0.00

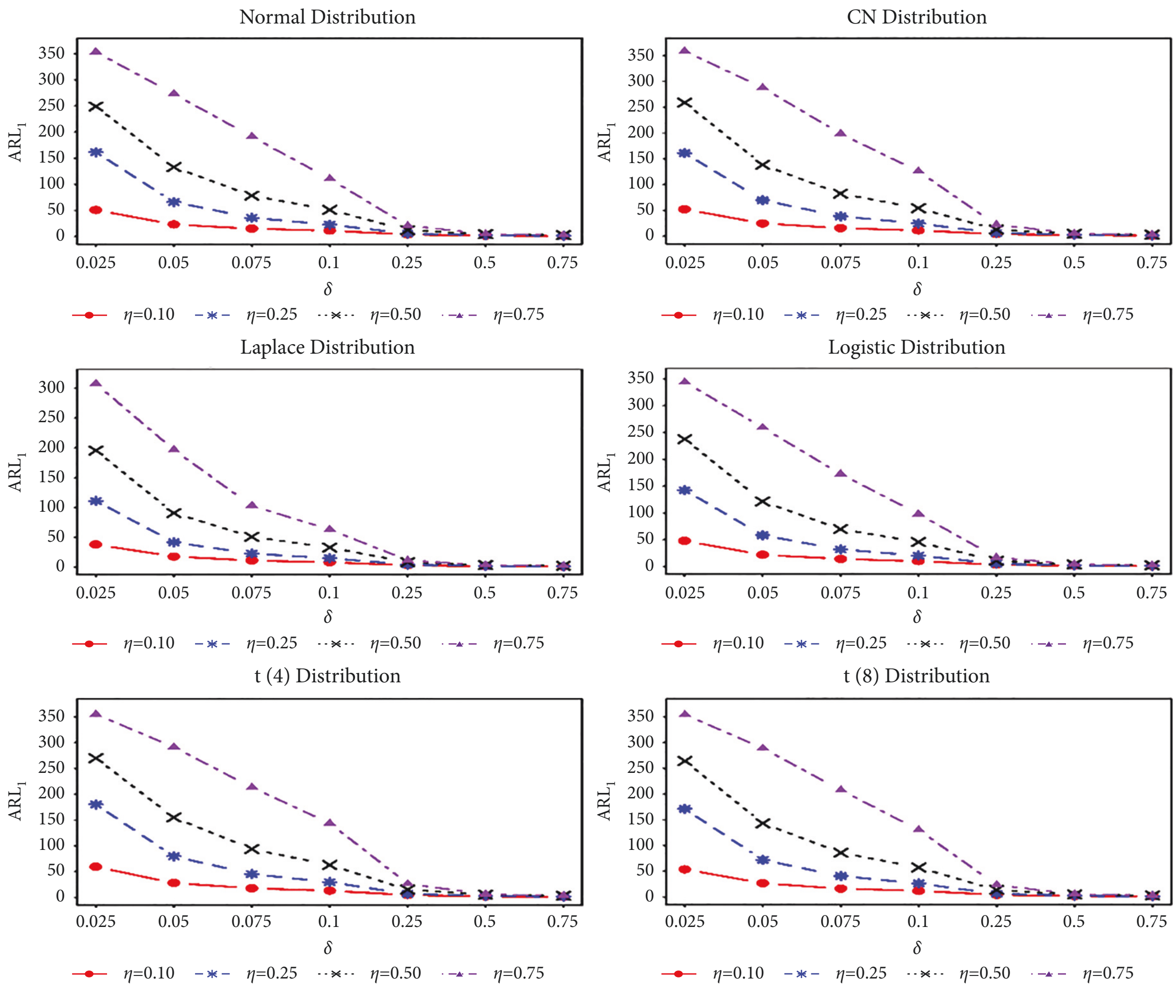

FIGURE 1: ARL characteristic of the proposed $\text{NPTHWMA}_{\text{RSS}}$ control chart under various distributions for different values of η when $n = 5$ and $\text{ARL}_0 = 370$.

(v) Plot the plotting statistic $NPTH_t$ against $LCL_{(\text{NPTHWMA}_{\text{RSS}})t}$ and $UCL_{(\text{NPTHWMA}_{\text{RSS}})t}$ over t.

(vi) If $NPTH_t > UCL_{(\text{NPTHWMA}_{\text{RSS}})t}$ or $NPTH_t < LCL_{(\text{NPTHWMA}_{\text{RSS}})t}$, note this sample of $NPTH_t$ statistic as an RL. For instance, at $t = 195$, if $NPTH_t > UCL_{(\text{NPTHWMA}_{\text{RSS}})t}$ or $NPTH_t < LCL_{(\text{NPTHWMA}_{\text{RSS}})t}$, record 195 as a first RL.

(vii) Repeat steps (ii) to (vi) 50,000 times and record RLs.

(viii) Using 50,000 simulations, calculate ARL_0 and record RLs.

(ix) If $\text{ARL}_0 = 300$; otherwise, adjust the constant in step (ii) as needed, and then, repeat steps (ii) to (ix) to achieve $\text{ARL}_0 = 300$.

(x) To acquire ARL_1 values, draw a shifted sample from the considered distribution again and repeat steps (ii) to (ix).

3.2. Robustness, IC, and OOC Performances of the NPTHWMA_{RSS} Control Chart. This subsection highlights the proposed $\text{NPTHWMA}_{\text{RSS}}$ control chart's robustness, IC, and OOC behaviour when a process location is shifted. Normal and nonnormal continuous and symmetrical distributions are used to evaluate such properties. For this study, the distributions used were the standard normal distribution, i.e., $N(0,1)$, Laplace or double exponential, i.e., $D.E(0.1/\sqrt{2})$, student's t distribution, i.e., $t(v)$, logistic distribution i.e., $(0, (\sqrt{3}/\pi))$, contaminated normal (CN) distribution, which is the mixture of $N(0,\sigma_0^2)$ and $N(0,\sigma_1^2)$ (see Table 1). Tables 2 and 3 demonstrate the RL characteristics of the proposed $\text{NPTHWMA}_{\text{RSS}}$ control chart for location shift. For comparison purposes, all these distributions were reparametrized with zero mean/median and unit variance. For all symmetric continuous distributions, the IC RL characteristics of the NP control chart remain constant.

The same parameters are used for comparison purposes as reported in numerous relevant articles. The ARL measures are used to compare the proposed and competing control charts. Table 2 and Figures 1 and 2 provide the following outcomes:

(i) The proposed control chart's IC RL distribution looks remarkably similar to all distributions examined in this study. For example, at $\eta = (0.10, 0.25, 0.50, 0.75)$, $n = 5$ and 10 and the $\text{ARL}_0 = 370$ for all investigated distributions (see Tables 2 and 3).

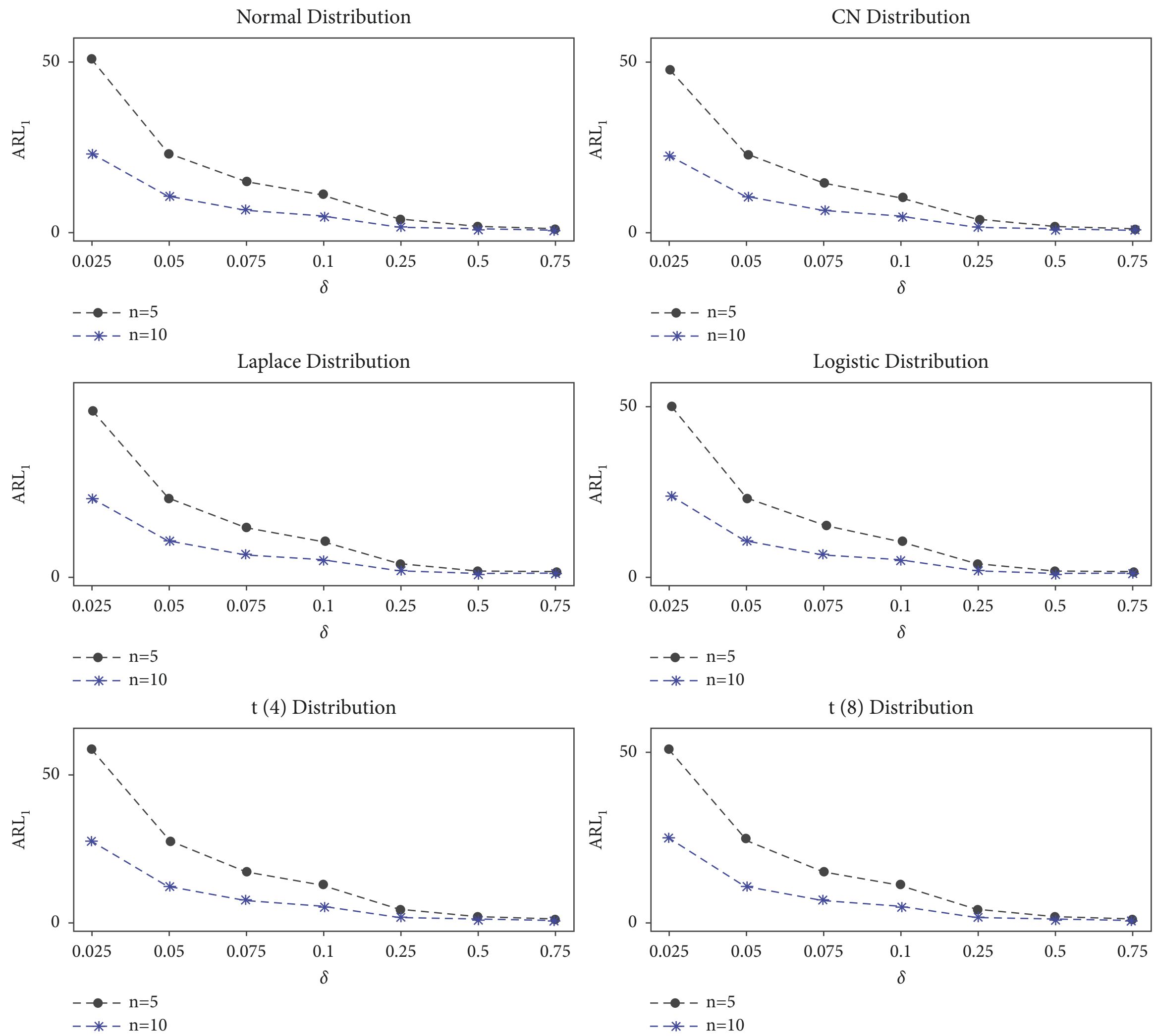

FIGURE 2: ARL characteristic of the proposed $NPTHWMA_{RSS}$ control chart under various distributions when n = 5 and 10 and $ARL_0 = 370$.

(ii) As the smoothing parameter reduces, the proposed control chart becomes more effective in detecting shifts. This illustrates that the proposed $NPTHWMA_{RSS}$ control chart is more sensitive to small smoothing parameters (see Figure 1).

(iii) As the sample size increases, the proposed $NPTHWMA_{RSS}$ control chart's performance increases.

(iv) The Laplace distribution outperforms the other distributions regarding OOC RL performance (see Figure 2).

(v) The proposed $NPTHWMA_{RSS}$ control chart's ARL_1 values increase as η increases at a certain size of the shift. For instance, under normal distribution at $\eta = 0.25, n = 5$, and $\delta = 0.025$, the $ARL_1 = 161.69$, whereas when $\eta = 0.50$, $n = 5$, and $\delta = 0.025$, the $ARL_1 = 248.78$ (see Tables 2 and 3).

(vi) The distribution of RL values is positively skewed, i.e., ARL > MRDL (see Tables 2 and 3).

(vii) The ARL_1 values of the proposed $NPTHWMA_{RSS}$ control chart are smaller than competing control charts with different shift sizes in process location (see Figure 3).

4. Comparative Study

This section compares the proposed $NPTHWMA_{RSS}$ control chart with other competing control charts, including TEWMA-SR, TEWMA-$\overline{X}$, TEWMA-SN, TEWMA-SR_{RSS}, and $NPDHWMA_{RSS}$.

4.1. Proposed versus TEWMA-SR Control Chart. The ARL profile demonstrates that the proposed $NPTHWMA_{RSS}$ control chart outperforms the TEWMA-SR control chart. For example, under normal distribution, at $\eta = 0.10, n = 5$, and $\delta = 0.05, 0.10, 0.25, 0.50$, and 0.75, the ARL_1 values of the proposed $NPTHWMA_{RSS}$ control chart are 23.15, 11.27, 3.98, 1.93, and 1.26, whereas the ARL_1 values of the TEWMA-SR control charts are 201.99, 87.06, 21.39, 7.37, and 4.12 (see Tables 2 and 4). Similarly, when we consider Laplace distribution for comparison, we observed the same behaviour of the proposed $NPTHWMA_{RSS}$ control chart. For instance, at $\eta = 0.25, n = 5$, and $\delta = 0.25$ and 0.50 the ARL_1 values of the

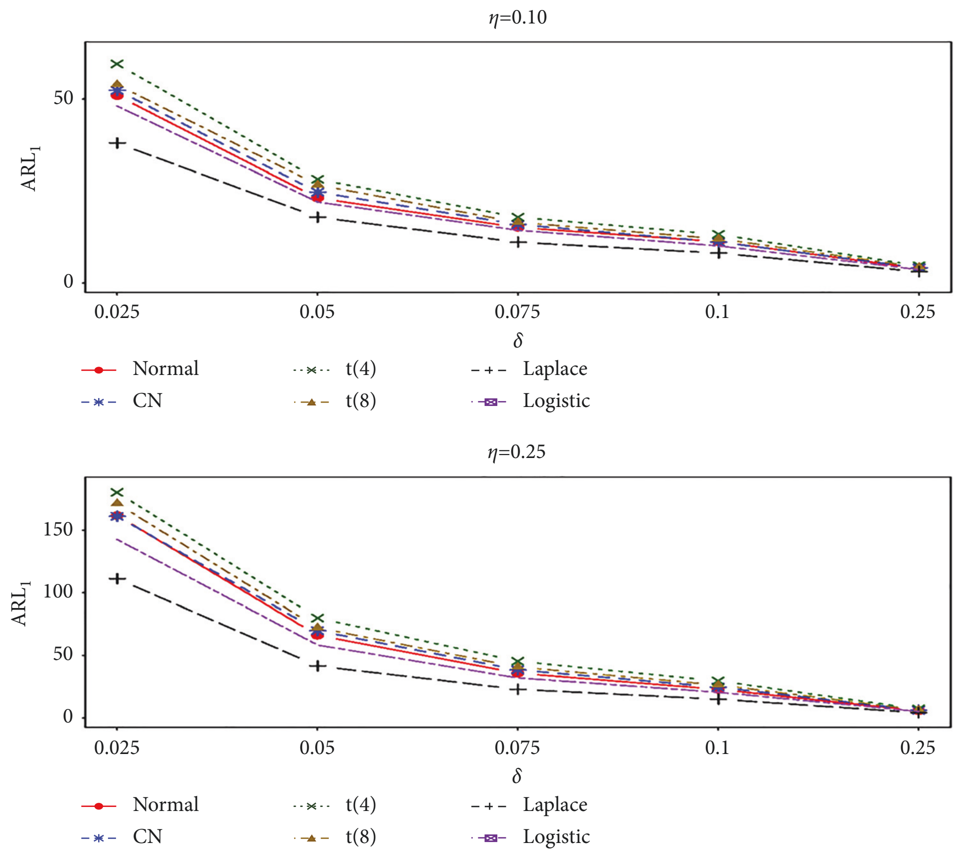

Figure 3: ARL characteristic of the proposed $NPTHWMA_{RSS}$ control chart under various distributions when $\eta = 0.10$ and 0.25, $n = 5$, and $ARL_0 = 370$.

proposed $NPTHWMA_{RSS}$ control chart are 4.34 and 2.10 while the ARL_1 values of the TEWMA-SR control chart are 16.59 and 6.25 (see Tables 2 and 4). Figure 4 depicts the efficacy of the proposed control chart over the TEWMA-SR control chart. These findings demonstrate that the proposed $NPTHWMA_{RSS}$ control chart performs better than the TEWMA-SR control chart for detecting shifts in process location.

4.2. Proposed versus TEWMA-SN Control Chart. The proposed $NPTHWMA_{RSS}$ control chart performs better than the TEWMA-SN control chart. The RL profiles of $NPTHWMA_{RSS}$ and TEWMA-SN control charts are shown in Tables 2 and 5 for a nominal ARL_0 of 370. It is preferable to use the control chart with the smaller ARL_1 value in a given shift because it detects the shift more quickly. The proposed $NPTHWMA_{RSS}$ control charts' efficiency can be observed under the normal distribution. For example, at $n = 5, \eta = 0.25$, and $\delta = 0.05, 0.10$, and 0.5, the ARL_1 values for the proposed $NPTHWMA_{RSS}$ control charts are 66.15, 22.95, and 2.40, whereas the ARL_1 values for the TEWMA-SN control charts are 238.00, 141.00, and 26.00 (see Tables 2 and 5). Figure 4 shows the supremacy of the proposed control chart over the TEWMA-SN control chart.

4.3. Proposed Versus TEWMA-$\overline{X}$ Control Chart. The proposed $NPTHWMA_{RSS}$ control chart's ARL values are more sensitive at all combinations of δ and η than the TEWMA-$\overline{X}$ control chart. For instance, in case of logistic distribution, when $n = 5, \eta = 0.10$, and $\delta = 0.05, 0.10, 0.25, 0.50$, and 0.75, the ARL_1 values of the proposed $NPTHWMA_{RSS}$ and TEWMA-$\overline{X}$ control charts are 22.05, 10.14, 3.69, 1.81, and 1.26 and 182.82, 76.01, 18.40, 5.95, and 2.94, respectively (see Tables 2 and 6). The supremacy of the proposed $NPTHWMA_{RSS}$ control chart to the TEWMA-$\overline{X}$ can also be seen in Figure 4. When compared to the normal distribution, the $NPTHWMA_{RSS}$ and TEWMA-$\overline{X}$ control charts reduce ARL 82.12% and 8.80%, respectively, when $\eta = 25\%$ and $\delta = 5\%$, (see Tables 2 and 6). In all investigated distributions, the proposed $NPTHWMA_{RSS}$ control chart is more responsive to identifying shifts than the TEWMA-$\overline{X}$ control chart.

4.4. Proposed versus TEWMA-SR_{RSS} Control Chart. The ARL study reveals that the proposed $NPTHWMA_{RSS}$ control chart outperforms the TEWMA-SR_{RSS} control chart for all possible values of η and δ (see Tables 2 and 7). As an illustration, for CN distribution, at $\eta = 0.10$ and $\delta = 0.05, 0.075, 0.10, 0.5$, and 0.75, the ARL_1 values of the $NPTHWMA_{RSS}$ and TEWMA-SR_{RSS} control charts are 24.74, 15.95, 11.18, 2.01, and 1.34 and 110.71, 64.19, 40.58, 3.28, and 1.79, respectively (see Tables 2 and 7). Under normal distribution, at $\eta = 25\%$, a 2.5% increase in process location parameter reduces the ARL by 23.31% for the TEWMA-SR_{RSS} control chart while the $NPTHWMA_{RSS}$ reduces ARL by 56.31% (see Tables 2 and

Table 4: RL characteristics of the TEWMA-SR control chart under different distribution at $ARL_0 = 370$ and $n = 5$.

Distri	Normal			CN			Laplace			$t(4)$			$t(8)$			Logistic		
δ	ARL	MDRL	SDRL	ARL	MDRL	SDRL	ARL	MDRL	SDRL	ARL	MDRL	SDRL	ARL	MDRL	SDRL	ARL	MDRL	SDRL
(0.10, 2.04)																		
0.00	370.38	251.00	386.98	370.01	251.00	388.09	369.82	251.00	390.04	371.41	254.00	390.11	370.69	249.00	398.13	370.92	251.00	390.27
0.05	201.99	140.00	206.95	196.07	136.00	201.10	143.21	101.00	142.05	161.05	113.00	163.04	186.01	129.00	190.51	183.86	126.00	186.31
0.10	87.06	65.00	82.05	84.05	63.00	78.13	56.14	45.00	49.25	62.51	51.00	55.61	77.02	59.00	71.92	77.01	58.00	71.63
0.25	21.39	20.00	16.02	20.38	19.00	15.02	14.61	14.00	10.98	15.72	15.00	10.99	19.05	19.00	14.87	19.13	17.00	13.80
0.50	7.37	7.00	5.02	7.07	7.00	4.81	5.57	6.00	3.79	5.61	6.00	3.72	6.72	8.00	4.99	6.65	7.00	4.55
0.75	4.12	4.00	2.51	4.01	4.00	1.96	3.41	3.00	1.95	3.34	3.00	1.86	3.76	3.00	2.25	3.75	4.00	2.24
1.00	2.81	3.00	1.29	2.89	3.00	1.23	2.64	3.00	1.19	2.56	2.00	1.05	2.70	3.00	1.23	2.71	3.00	1.22
1.25	2.29	2.00	0.82	2.27	3.00	0.81	2.31	2.00	0.73	2.27	2.00	0.66	2.31	2.00	0.74	2.30	2.00	0.71
1.50	2.07	2.00	0.42	2.05	2.00	0.43	2.05	2.00	0.48	2.04	2.00	0.39	2.09	1.00	0.53	2.08	2.00	0.40
(0.25, 2.435)																		
0.00	370.41	256.00	372.81	371.12	256.00	371.25	370.89	255.00	375.36	368.89	257.00	369.99	371.25	256.00	370.99	372.04	256.00	375.42
0.05	250.98	174.00	250.11	245.11	169.00	245.36	192.03	136.00	189.21	209.02	146.00	208.11	235.56	167.00	235.23	233.89	163.00	231.59
0.10	124.03	89.00	121.23	118.51	86.00	113.32	78.11	57.00	73.89	89.05	66.00	106.35	111.20	80.00	106.21	110.25	76.00	105.36
0.25	26.21	21.00	21.09	25.01	21.00	20.63	16.59	15.00	12.03	17.92	16.00	13.51	22.84	19.00	18.02	23.02	18.00	18.52
0.50	8.23	8.00	4.70	7.96	8.00	4.71	6.25	8.00	3.39	6.31	7.00	3.41	7.53	8.00	4.23	7.51	8.00	4.17
0.75	4.72	5.00	2.30	4.55	5.00	2.16	4.01	5.00	1.89	3.92	4.00	1.81	4.41	4.00	2.14	4.39	3.00	2.09
1.00	3.39	4.00	1.36	3.27	4.00	1.32	3.14	3.00	1.26	3.01	3.00	1.15	3.22	3.00	1.29	3.23	3.00	1.31
1.25	2.75	2.00	0.90	2.68	3.00	0.92	2.67	2.00	0.90	2.59	2.00	0.83	2.69	3.00	0.88	2.70	2.00	0.88
1.50	2.41	1.00	0.63	2.36	2.00	0.60	2.41	2.00	0.70	2.32	2.00	0.65	2.41	2.00	0.65	2.41	2.00	0.63

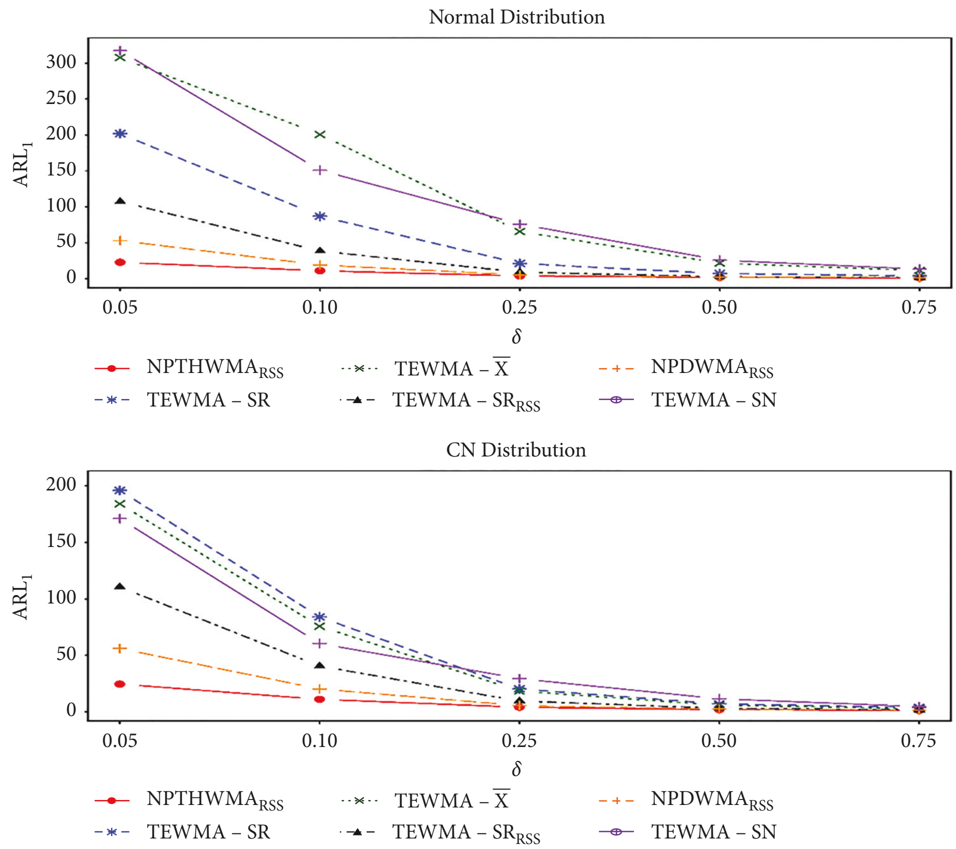

FIGURE 4: ARL characteristic of the proposed NPTHWMA$_{RSS}$ and competing control charts under various distributions when $n = 5$ and $ARL_0 = 370$.

7). Figure 4 also shows the superiority of the NPTHWMA$_{RSS}$ control chart over the TEWMA-SR$_{RSS}$ control chart. The results show that the NPTHWMA$_{RSS}$ control chart is superior to the TEWMA-SR$_{RSS}$ control chart in process location's shift detection.

4.5. Proposed versus NPDHWMA$_{RSS}$ Control Chart. The NPTHWMA$_{RSS}$ control chart outperforms the NPDHWMA$_{RSS}$ control chart in detecting shifts in process location. For example, using $t(8)$ distribution at $n = 5, \eta = 0.10$, and $\delta = 0.025, 0.10$, and 0.25, the ARL_1 values of the proposed NPTHWMA$_{RSS}$ and NPDHWMA$_{RSS}$ control charts are $53.91, 12.06$, and 4.37 and $142.35, 21.43$, and 5.93, respectively (see Tables 2 and 8). Similarly, under Laplace distribution, when $n = 5, \eta = 0.25$, and $\delta = 0.05$ and 0.50, the ARL_1 values of the proposed NPTHWMA$_{RSS}$ control chart are 41.74 and 2.10 while the ARL_1 values for NPDHWMA$_{RSS}$ control chart are 77.27 and 3.20, respectively (see Tables 2 and 8). As illustrated in Figure 4, the NPTHWMA$_{RSS}$ control chart outperforms the NPDHWMA$_{RSS}$ control chart.

5. Real-Life Application

This section illustrates a real-life application of the non-isothermal continuous stirred tank reactor (CSTR) process to demonstrate the applicability of the proposed NPTHWMA$_{RSS}$ control chart. Adegoke et al. [27] recently considered this real-life data. The CSTR process has nine different variables, one of which we choose as the variable of interest (X), which represents the output temperature, and this variable is used in real-life application with parameters $\mu_X = 369.88$ and $\sigma_X^2 = 0.32$. It started with 1000 observations, 600 of which were made while the process was in an IC state. After the 24th observation, a shift in the process location is introduced following Anwar et al. [28] and Anwar et al. [29]. The estimation is carried out with the help of the mentioned parameters, and the control limits are obtained. In order to use the proposed control chart and the existing (TEWMA-SR$_{RSS}$) control chart in practise, the variable of interest (X) is used. We used the RSS approach to generate 40 paired observations of size $n = 5$ and $m = 1$ from a normal distribution. The parameters of the proposed NPTHWMA$_{RSS}$ and TEWMA-SR$_{RSS}$ control charts used for real-life analysis are $L = 1.26, \eta = 0.10, ARL_0 = 370$ and $L = 1.837, \eta = 0.10, ARL_0 = 370$, respectively. Figure 5 indicates that the proposed NPTHWMA$_{RSS}$ control chart triggers the first OOC signal at sample number 29 while the TEWMA-SR$_{RSS}$ control chart detects the first OOC point at sample number 35. Similarly, the proposed NPTHWMA$_{RSS}$ control chart detects overall 12 OOC points, whereas the TEWMA-SR$_{RSS}$ control chart detects 06 OOC points (see Figure 5).

Table 5: RL characteristics of the TEWMA-SN control chart under normal distribution at $ARL_0 = 370$ and $n = 5$.

δ	(η, L) 0.05, 1.755			(η, L) 0.10, 2.057			(η, L) 0.25, 2.444			(η, L) 0.50, 2.744			(η, L) 0.75, 2.807		
	ARL	MDRL	SDRL	ARL	MDRL	SDRL	ARL	MDRL	SDRL	ARL	MDRL	SDRL	ARL	MDRL	SDRL
0.00	370.19	228.00	434.01	370.91	245.00	404.01	371.21	256.00	377.14	370.90	257.00	374.52	370.25	262.00	368.25
0.05	300.18	190.00	344.05	317.22	211.00	344.23	342.63	238.00	345.36	354.63	245.00	357.31	362.71	250.00	359.81
0.10	127.39	91.00	133.11	151.00	105.00	155.12	202.21	141.00	199.58	255.01	178.00	253.69	295.36	207.00	392.11
0.25	65.41	53.00	62.00	76.03	57.00	71.51	108.25	78.00	104.23	161.54	114.00	159.01	215.14	150.00	214.56
0.50	23.65	20.00	21.01	26.11	23.00	20.39	33.01	26.00	28.14	53.26	38.00	50.41	88.48	64.00	86.61
0.75	12.30	11.00	10.52	13.57	13.00	11.21	15.67	14.00	11.18	22.04	10.00	19.03	39.31	28.00	37.01
1.00	7.39	7.00	6.24	8.35	8.00	6.27	9.45	9.00	5.85	11.68	10.00	8.90	19.47	15.00	17.62
1.25	3.36	3.00	2.65	3.92	4.00	2.89	4.72	5.00	2.53	5.17	4.00	3.00	6.78	6.00	5.01
1.50	1.77	2.00	1.21	2.09	2.00	1.51	2.88	2.00	1.20	3.12	3.00	2.00	3.37	3.45	2.01

Table 6: RL characteristics of the TEWMA-$\overline{X}$ control chart under different distributions at $ARL_0 = 370$ and $n = 5$.

Shift	Normal			CN			Laplace			$t(4)$			$t(8)$			Logistic		
	ARL	MDRL	SDRL	ARL	MDRL	SDRL	ARL	MDRL	SDRL	ARL	MDRL	SDRL	ARL	MDRL	SDRL	ARL	MDRL	SDRL
(0.10, 2.052)																		
0.00	369.03	250.00	395.95	370.86	238.00	406.48	369.16	253.00	396.30	369.29	242.00	400.55	369.13	241.50	401.65	369.25	251.00	395.31
0.05	307.06	202.00	328.82	184.21	127.50	187.56	181.96	130.00	191.07	186.99	124.00	194.09	187.84	129.50	197.86	182.82	129.00	190.55
0.10	200.72	136.00	211.07	75.84	57.00	70.40	76.63	57.00	72.13	77.45	56.00	74.04	75.73	56.00	71.48	76.01	57.00	71.15
0.25	65.87	49.00	60.94	18.45	16.00	13.78	18.55	16.00	14.25	18.30	16.00	13.24	18.58	17.00	13.83	18.40	16.00	14.08
0.50	21.83	19.00	16.99	5.88	5.00	4.48	5.95	5.00	4.47	5.80	5.00	4.36	6.07	5.00	4.56	5.95	5.00	4.51
0.75	11.54	10.00	8.57	2.84	2.00	2.11	2.85	2.00	2.07	2.86	2.00	2.15	2.91	2.00	2.18	2.94	2.00	2.17
1.00	7.17	6.00	5.37	1.76	1.00	1.14	1.78	1.00	1.17	1.75	1.00	1.18	1.78	1.00	1.16	1.80	1.00	1.20
1.25	4.79	4.00	3.60	1.30	1.00	0.65	1.31	1.00	0.64	1.26	1.00	0.60	1.31	1.00	0.65	1.28	1.00	0.63
1.50	3.55	3.00	2.72	1.10	1.00	0.35	1.10	1.00	0.33	1.10	1.00	0.35	1.10	1.00	0.34	1.10	1.00	0.35
(0.25, 2.456)																		
0.00	369.58	255.00	371.62	370.73	246.50	382.48	370.35	259.00	385.77	369.93	259.00	374.25	369.15	252.00	378.00	370.81	259.50	379.06
0.05	337.05	237.00	344.14	240.89	171.00	237.76	238.73	165.00	241.88	244.15	163.00	244.05	245.69	170.00	244.08	235.39	165.00	236.63
0.10	253.31	180.00	249.89	111.72	79.00	108.46	113.35	79.00	109.29	112.68	80.00	106.57	113.57	79.00	113.25	109.97	76.00	106.03
0.25	92.03	63.00	90.27	21.68	17.00	17.44	22.41	18.00	17.63	22.79	18.00	18.02	22.10	18.00	17.96	22.42	18.00	17.58
0.50	26.45	21.00	21.86	6.62	6.00	4.19	6.52	6.00	3.99	6.56	6.00	4.02	6.60	6.00	4.07	6.72	6.00	4.19
0.75	12.90	11.00	9.25	3.29	3.00	2.04	3.39	3.00	2.11	3.35	3.00	2.03	3.42	3.00	2.08	3.44	3.00	2.15
1.00	8.04	7.00	5.23	2.11	2.00	1.25	2.11	2.00	1.24	2.09	2.00	1.24	2.12	2.00	1.26	2.10	2.00	1.21
1.25	5.49	5.00	3.44	1.52	1.00	0.79	1.50	1.00	0.77	1.50	1.00	0.79	1.50	1.00	0.77	1.50	1.00	0.77
1.50	4.12	4.00	2.57	1.21	1.00	0.48	1.21	1.00	0.49	1.20	1.00	0.47	1.22	1.00	0.48	1.22	1.00	0.48

Table 7: RL characteristics of the TEWMA-SR_{RSS} control chart under different distributions at $n = 5$ and $ARL_0 = 370$.

	Distributions	Metrics	δ 0.00	0.025	0.03	0.05	0.075	0.10	0.15	0.20	0.25	0.50	0.75	1.00	1.50	2.00
(0.1, 1.837)	*N*(0, 1)	ARL	371.18	240.21	201.32	108.08	58.01	39.00	22.10	13.34	9.22	3.11	1.81	1.30	1.04	1.01
		MDRL	242.01	163.90	142.99	76.12	44.13	31.89	20.02	11.93	7.99	2.01	1.02	1.10	1.01	1.00
		SDRL	401.88	254.19	201.14	107.01	53.39	32.87	17.01	10.00	7.05	2.20	0.98	0.57	0.19	0.04
		ARL	369.87	223.16	190.11	111.67	64.28	40.65	20.96	14.02	10.00	3.30	1.81	1.36	1.08	1.00
	CN	MDRL	241.10	154.82	135.98	76.01	47.96	33.90	17.99	11.95	7.97	2.97	1.98	1.02	1.01	1.01
		SDRL	403.28	234.26	190.11	110.01	59.88	31.99	17.21	10.14	7.05	2.41	1.05	0.58	0.30	0.10
		ARL	368.99	219.80	180.21	98.05	51.92	36.01	17.89	11.19	7.93	2.75	1.71	1.32	1.10	1.04
	Logistic	MDRL	260.00	151.01	125.11	73.07	41.21	30.10	15.80	9.95	6.87	2.02	1.01	1.01	1.01	1.00
		SDRL	381.31	231.52	178.03	91.25	49.39	32.67	14.13	7.99	6.08	1.88	0.93	0.61	0.30	0.09
		ARL	370.01	180.11	146.49	71.98	38.93	26.04	13.99	9.00	6.38	2.52	1.62	1.31	1.11	1.09
	Laplace	MDRL	260.10	120.00	104.00	56.20	32.82	21.96	12.90	7.91	4.93	2.10	1.08	1.07	1.05	1.01
		SDRL	380.32	193.11	143.63	67.71	35.29	20.47	11.53	5.98	4.89	1.73	0.91	0.61	0.29	0.03
		ARL	371.01	258.90	230.50	143.10	80.95	49.02	27.00	17.03	11.98	4.01	2.32	1.63	1.18	1.07
	t(4)	MDRL	240.38	168.61	156.10	103.00	61.53	39.02	23.61	14.31	12.01	3.13	2.09	1.21	1.08	1.01
		SDRL	384.00	291.02	242.26	136.35	76.41	43.52	20.00	13.21	9.03	2.91	1.63	1.00	0.63	0.30
		ARL	369.92	241.83	204.59	126.83	68.05	45.01	23.00	16.20	10.61	3.53	1.99	1.40	1.09	1.02
	t(8)	MDRL	251.23	170.25	140.01	90.30	53.21	37.61	20.11	14.35	9.22	3.05	2.02	1.02	1.001	1.00
		SDRL	418.75	261.49	203.81	130.21	60.36	36.89	17.28	11.32	7.63	2.58	1.25	0.68	0.30	0.09
(0.25, 2.204)		ARL	370.74	284.31	253.51	151.84	88.78	54.57	24.92	15.57	10.38	3.59	2.09	1.44	1.07	1.01
	N(0, 1)	MDRL	267.50	199.50	167.00	108.50	64.00	38.50	19.00	13.00	9.00	3.00	2.00	1.00	1.00	1.00
		SDRL	369.16	291.85	267.11	151.35	88.66	50.80	20.17	11.01	6.99	2.13	1.06	0.63	0.26	0.08
		ARL	371.10	293.39	250.56	154.48	88.79	57.94	27.45	15.74	11.22	3.78	2.12	1.58	1.13	1.02
	CN	MDRL	273.00	214.50	172.50	109.50	64.00	42.00	21.00	13.00	10.00	3.50	2.00	1.00	1.00	1.00
		SDRL	366.10	279.85	252.76	141.01	83.90	53.54	23.59	11.44	7.55	2.10	1.08	0.75	0.37	0.15
		ARL	370.38	271.57	242.25	137.58	77.87	47.20	22.34	13.59	9.28	3.23	1.95	1.44	1.08	1.02
	Logistic	MDRL	265.50	180.50	169.50	103.00	58.00	36.00	18.50	12.00	8.00	3.00	2.00	1.00	1.00	1.00
		SDRL	360.62	282.24	242.01	135.38	69.56	40.44	16.29	9.51	5.85	1.87	1.02	0.65	0.28	0.13
		ARL	370.38	230.11	199.04	105.76	52.44	30.72	15.08	10.52	7.07	2.82	1.83	1.35	1.10	1.02
	Laplace	MDRL	265.50	156.50	144.00	75.00	38.50	23.00	13.00	9.00	6.00	3.00	2.00	1.00	1.00	1.00
		SDRL	360.62	223.94	196.34	100.81	47.67	26.97	11.19	7.14	4.41	1.68	0.97	0.65	0.33	0.17
		ARL	371.04	304.19	282.63	193.67	113.38	72.96	32.78	19.37	13.04	4.52	2.52	1.85	1.28	1.11
	t(4)	MDRL	251.00	208.50	200.00	136.50	80.00	51.00	25.00	15.00	11.00	4.00	2.00	2.00	1.00	1.00
		SDRL	367.97	313.54	280.19	186.75	112.53	70.92	27.16	16.71	8.95	2.58	1.48	0.98	0.55	0.36
		ARL	370.07	116.22	108.76	81.73	54.05	36.08	19.70	12.12	8.85	3.08	1.89	1.35	1.07	1.01
	t(8)	MDRL	261.50	82.50	74.00	60.50	39.00	26.00	16.00	11.00	8.00	3.00	2.00	1.00	1.00	1.00
		SDRL	366.37	115.99	114.08	80.49	53.32	36.90	16.95	9.09	6.20	1.92	1.04	0.59	0.26	0.11

Table 8: RL characteristics of the $NPDHWMA_{RSS}$ control chart under various distributions at nominal $ARL_0 = 370$ and n = 5.

			δ													
(η, L)	Distributions	Metrics	0	0.025	0.05	0.075	0.1	0.25	0.5	0.75	1	1.5	2	2.5	3	5
(0.10, 1.449)	Normal	ARL	370.46	130.33	53.14	29.55	19.28	5.41	2.41	1.54	1.17	1.01	1.00	1.00	1.00	1.00
		MDRL	50.00	37.00	22.00	15.00	11.00	4.00	3.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	631.89	202.98	72.06	36.33	21.47	4.05	1.47	0.95	0.57	0.12	0.01	0.00	0.00	0.00
	CN	ARL	371.89	135.05	56.18	31.29	20.34	5.66	2.52	1.62	1.23	1.03	1.01	1.00	1.00	1.00
		MDRL	50.00	36.00	23.00	16.00	12.00	4.00	3.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	629.45	211.06	76.72	38.76	22.85	4.31	1.54	1.01	0.65	0.23	0.11	0.06	0.03	0.00
	$t(4)$	ARL	371.56	152.05	65.24	36.22	23.72	6.52	2.89	1.92	1.47	1.14	1.05	1.03	1.01	1.00
		MDRL	49.00	37.50	26.00	18.00	13.00	5.00	3.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	628.70	241.61	90.21	45.80	27.69	5.27	1.80	1.20	0.89	0.52	0.32	0.23	0.16	0.08
	$t(8)$	ARL	372.17	142.35	59.16	33.06	21.43	5.93	2.64	1.73	1.31	1.05	1.01	1.00	1.00	1.00
		MDRL	47.00	38.00	24.00	16.00	12.00	5.00	3.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	628.03	223.96	81.03	41.17	24.61	4.61	1.62	1.08	0.74	0.32	0.15	0.09	0.05	0.01
	Laplace	ARL	370.58	90.41	34.86	18.83	12.75	4.10	2.10	1.48	1.23	1.05	1.01	1.00	1.00	1.00
		MDRL	49.00	31.00	17.00	11.00	9.00	4.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	631.32	133.73	43.28	21.12	12.78	2.75	1.29	0.90	0.65	0.30	0.15	0.08	0.04	0.00
	Logistic	ARL	370.48	118.87	47.92	26.21	16.97	4.94	2.26	1.51	1.18	1.02	1.00	1.00	1.00	1.00
		MDRL	49.00	36.00	21.00	14.00	10.00	4.00	3.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	631.32	182.11	63.79	31.65	18.29	3.58	1.39	0.92	0.58	0.22	0.06	0.03	0.00	0.00
(0.25, 2.365)	Normal	ARL	368.05	227.94	114.94	67.51	44.59	10.56	3.97	2.51	1.82	1.15	1.01	1.00	1.00	1.00
		MDRL	333.00	191.00	98.00	59.00	39.00	9.00	4.00	3.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	257.53	174.14	85.11	47.88	30.83	6.47	1.89	1.24	1.02	0.53	0.15	0.03	0.01	0.00
	CN	ARL	367.90	235.00	120.52	70.80	46.80	11.17	4.16	2.62	1.90	1.22	1.04	1.01	1.00	1.00
		MDRL	331.00	198.00	102.00	61.00	41.00	10.00	4.00	3.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	255.79	178.68	90.06	50.87	32.52	6.87	2.03	1.29	1.06	0.62	0.29	0.14	0.08	0.00
	$t(4)$	ARL	369.09	250.46	134.32	81.33	54.38	13.18	4.82	3.00	2.23	1.48	1.19	1.07	1.04	1.00
		MDRL	330.00	212.00	113.00	70.00	47.00	12.00	4.00	3.00	3.00	1.00	1.00	1.00	1.00	1.00
		SDRL	257.37	188.46	101.04	59.21	38.32	8.38	2.51	1.51	1.22	0.87	0.59	0.38	0.26	0.08
	$t(8)$	ARL	368.82	240.57	124.20	74.38	49.35	11.85	4.37	2.75	2.01	1.30	1.07	1.02	1.01	1.00
		MDRL	333.00	203.00	105.00	64.00	43.00	10.00	4.00	3.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	256.51	181.39	93.03	53.19	34.51	7.39	2.19	1.36	1.11	0.72	0.38	0.19	0.10	0.00
	Laplace	ARL	368.23	174.66	77.27	44.43	29.23	7.30	3.20	2.15	1.63	1.17	1.04	1.01	1.00	1.00
		MDRL	334.00	146.00	67.00	39.00	26.00	7.00	3.00	3.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	256.71	132.27	55.54	31.21	19.79	4.26	1.63	1.21	0.97	0.57	0.29	0.15	0.06	0.00
	Logistic	ARL	368.23	212.61	104.47	60.21	39.34	9.33	3.65	2.35	1.72	1.17	1.03	1.01	1.00	1.00
		MDRL	334.00	179.00	89.00	52.00	34.00	8.00	4.00	3.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	256.71	162.25	78.14	42.44	27.06	5.63	1.78	1.22	1.00	0.56	0.24	0.10	0.04	0.00

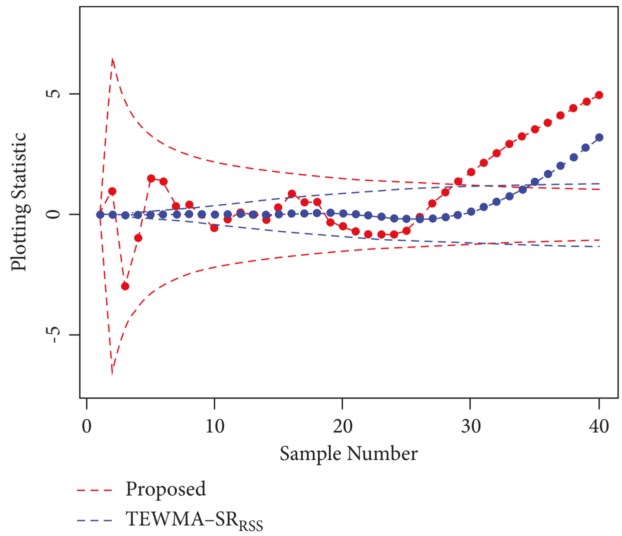

FIGURE 5: Real-life application of the proposed $NPTHWMA_{RSS}$ control chart against the $TEWMA\text{-}SR_{RSS}$ control chart.

6. Conclusions and Recommendations

The basic assumption of the parametric control charts is that the underlying process follows the specific distribution. When the features do not follow a particular distribution, using nonparametric (NP) control charts to monitor the quality characteristics is recommended. The triple homogeneously weighted moving average (THWMA) is the advanced version of the triple exponentially weighted moving average (TEWMA) control chart for enhanced process location monitoring. Similarly, the ranked set sampling (RSS) technique is preferred over simple random sampling (SRS) when the sampling procedure is costly and complicated. To this end, this study uses the NP THWMA control chart and the RSS technique, which presents an NP THWMA Wilcoxon signed-rank control chart (denoted by $NPTHWMA_{RSS}$) for enhanced monitoring of process location shifts. The proposed control chart's performance is evaluated in ARL, MDRL, and SDRL. The findings demonstrated that the proposed control chart outperforms competitor control charts such as TEWMA-SR, TEWMA-$\overline{X}$, TEWMA-SN, $TEWMA\text{-}SR_{RSS}$, and $NPDHWMA_{RSS}$. Furthermore, a real-life application is presented to illustrate the proposed control chart's application in practice. The proposed study can be extended to the more ranked set sampling techniques and multivariate scenarios.where $X\epsilon\mathfrak{R}$.

Data Availability

The [nonisothermal continuous stirred tank reactor] data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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