Research Article

Microstructure Characterization of High Explosives by Wavelet Transform

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The key macro properties of high explosives including sensitivity to shock, the possibility of initiation, and the subsequent chemical reaction are known to be controlled by processes occurring at their microstructure level. However, there is a lack of an easy, effective, and accurate method to quantify the microstructure, termed as fabric, of high explosives despite an abundance of evidence regarding its importance. This study proposes a rotational Haar wavelet transform (RHWT) method to characterize the fabric of high explosives from two-dimensional images, yielding key fabric parameters including rose diagram, fabric direction, and degree of fabric anisotropy. The fabric tensor commonly used in numerical simulations and constitutive models can also be determined by RHWT. The RHWT was implemented on microscopic images of six high explosives captured by various imaging techniques including scanning electrical microscopy, polarized light microscopy, and micro X-ray computed tomography. Despite these variables, the proposed RHWT successfully identifies fabric in these images, demonstrating robustness and validity of RHWT.

1. Introduction

High explosives are one type of energetic materials and they are widely used in rocket motors and munitions [1]. High explosives are mixtures of a large volume of hard explosive particles (90–95% by weight) and a small percentage of soft binder [2]. Young’s modulus of soft binders is typically three to four orders of magnitude lower than that of explosive grains [3, 4]. The key macro properties of high explosives including sensitivity to shock, the possibility of initiation, and the subsequent chemical reaction are known to be controlled by processes occurring at their microstructure level [5]. Under shock/impact loadings, the ignition of chemical reactions in explosive particles is controlled by localized regions of elevated temperature, that is, hot spots [6]. These hot spots are formed by various dissipative processes occurring at microstructures, including collapsing pores [7], plastic deformation [8], and frictional sliding [9], which convert mechanical work into thermal energy [10]. A hot spot produces a small burning region that propagates in high explosives which is responsible for the buildup to detonation [9]. High explosives have complex, stochastic microstructures, which significantly affect formations of hot spots [11, 12]. The microstructure of high explosives includes high explosive particle size and shape, as well as the orientations of high explosive particles.

The explosive particle size and shape are the important parameters controlling the formation, ignition, and growth of hot spots in the shocked high explosives and then the macroscopic behavior of the high explosives in the shock-to-
detonation transition process. Therefore, the explosive particle size and shape effects on the shock-to-detonation transition process have received increasing attention [13]. Many experimental studies have shown that shock sensitivity of heterogeneous high explosives increases when particle size is decreased [14]. Eyring et al. [15] proposed the particle burn concept to explain such observations. The particle burn concept implies that the characteristic reaction rate in high explosives is proportional to the product of characteristic burning rate and specific surface area of high explosive particles. A large amount of data is available on the critical diameter which is proportional to the duration of the reaction zone and is also a characteristic of the chemical peak [16]. Ershov et al. [17, 18] investigated influences of particle sizes for cyclotrimethylene-trinitramine (RDX), cyclotetramethylene-tetranitramine (HMX), and pentaerythritol tetranitrate (PETN). They found that smaller particles significantly reduced reaction zone and the maximum mass velocity. In explosive formulations, the shape of the particles also plays a key role in realizing better packability and processability. Kim [19] reported that spherical particles of explosives improved the mix-fluidity of formulations and had a great impact on the performance and insensitivity towards a sudden shock compared to nonspherical crystals. Sarangapani [20] found that spherical particles of 3-nitro-1,2,4-triazol-5-one (NTO) reduced the viscosity by about 75% for the same explosive content to binder ratio in comparison with nonspherical NTO particles. Porosity data also supported the same conclusion of improvement in explosive loading by 10% by the usage of spherical NTO particles instead of nonspherical NTO particles for the given explosive to binder ratio.

Orientations of high explosive grains are termed as fabric anisotropy [21]. Analytical, experimental, and numerical results indicate anisotropic nature of high explosives being critical in formation of hot spots and shock-to-detonation transition process [22, 23]. As such, the anisotropic constitutive models of high explosives have been developed by Olokan et al. [5] to predict performance of high explosives which fitted well into a statistically determined range of values obtained from both numerical and experimental methods presented in literature. Conroy et al. [24] modelled the behavior of HMX grains using first principles density functional theory (DFT). They found that the anisotropic material model predicted experimental results that adequately matched existing data. Zamiri and De [25] developed a constitutive model for the anisotropic plastic response of HMX, which successfully predicts the experimentally observed highly anisotropic and orientation dependent yield surfaces of HMX having different particle orientations. The model also correctly predicts the anisotropic plastic yielding of b-HMX under uniaxial compression at different temperatures along different particle directions.

The explosive particle size and shape can be quantified by image processing techniques [16, 20]. However, there is a lack of an easy, effective, and accurate method to quantify fabric anisotropy despite an abundance of evidence regarding the importance of it on mechanical and chemical behavior of high explosives. The fabric anisotropy should be quantified from two aspects, the primary fabric direction and the degree of anisotropy. These two parameters can alternatively be computed based on the orientations of particle long axes, from the orientations of interparticle contacts, or the orientations of voids (Oda and Nakayama [26]). The fabric computed from the orientations of particle long axes is the focus of this study.

A large body of literature has demonstrated the effects of fabric anisotropy on macro mechanical behavior of particulate materials, for example, Hansen and Gibson [27], Mitchell [28], Duncan and Seed [29], Tatsuoka [30], Oda et al. [31], Azami et al. [32], Arthur and Menzies [33], Lade [34], Yang et al. [35], Hosseininia [36], Fonseca [37], Zhao and Guo [38, 39], Alam et al. [40], and Imseeh et al. [41]. As such, many research works have worked on methods for quantifying fabric anisotropy. Oda [42] impregnated resin into sand specimens and then cut them into thin sections of thickness between 0.04 and 0.07 mm. He manually measured the long axis orientations of 200 particles to determine fabric orientation. Oda’s [42] method was later augmented by digital image processing techniques. Yang et al. [35] cut specimens at different locations (but not thin sections). They polished the surfaces before capturing images. Fonseca et al. [37, 43] captured the section images of specimens using micro X-ray computed tomography. Instead of manually counting particle orientations, Yang et al. [35] and Fonseca et al. [37, 43] used image thresholding techniques to identify individual particles and compute the orientation distributions of particle long axes. This method required that sand particles must have a different color from the surrounding background. It is also challenging to computationally determine the boundaries of contacting particles so that their orientations could be assessed individually. In addition, these techniques were developed for particulate materials, which cannot be readily used for analyzing high explosive images.

This research aims to develop a wavelet transform technique for analyzing microstructure of high explosives. The wavelet transform was firstly introduced to analyze particulate material images by Shih and Hryciw [44]. They found that particle sizes in a particulate image can be computed by wavelet transform. Hryciw et al. [45] formed the theoretical framework for optical granulometry of sands by wavelet transform method. The wavelet analysis is explained by using simple mathematics with comparisons drawn to the use of wavelet decomposition in image compression and image enhancement. Later, Zheng and Hryciw [46, 47] demonstrated that the wavelet transform technique is capable of analyzing microstructure of particulate materials. Alhasan et al. [48], Li et al. [49], and Yang et al. [50] demonstrated that the wavelet transform technique can be used to analyze microstructure of pavement to determine pavement surface roughness and skid resistance.

Based on the above works, the wavelet transform technique is introduced to analyze microscopic images of high explosives and determine their fabric anisotropy by this study. This technique analyzes texture patterns of high explosive images to determine statistical distributions of particle long axis orientations. The proposed technique is
2. Introduction to Wavelet Transform

In computer vision, wavelet transform is often used for quantifying grayscale changes in an image. The most commonly used wavelet analysis is called Haar wavelet transform (HWT), proposed by Haar in the 1910s [51]. The detailed theoretical derivation of the HWT is found in many textbooks on wavelet mathematics. Therefore, this paper will not repeat it but will explain the concept of the HWT.

A simple $8 \times 8$ pixel grayscale image $A_0$ is shown in Figure 1. Both 8-bit and 16-bit grayscale images can be used in the analysis. The grayscale values are superimposed on the image. Larger values correspond to brighter areas in the image. In $A_0$, the grayscale values change only in horizontal direction while remaining constant in the vertical direction. The HWT firstly divides the image into $2 \times 2$ blocks. There are a total of 16 such $2 \times 2$ blocks in the $8 \times 8$ image in Figure 1. For each $2 \times 2$ block $(i, j)$, assume that the four pixel values are

$$ A_0(i, j) = \begin{bmatrix} a & b \\ c & d \end{bmatrix}. $$

(1)

The HWT computes four values $A_1(i, j), H_1(i, j), V_1(i, j),$ and $D_1(i, j)$. $A_1(1, 1)$ is twice the average of the four numbers:

$$ A_1(i, j) = \frac{a + b + c + d}{2}. $$

(2)

$H_1(i, j)$ is the average difference between two columns:

$$ H_1(i, j) = \frac{(a + c) - (b + d)}{2}. $$

(3)

$V_1(i, j)$ is the average difference between two rows:

$$ V_1(i, j) = \frac{(a + b) - (c + d)}{2}. $$

(4)

$D_1(i, j)$ is the average difference between two diagonals:

$$ D_1(i, j) = \frac{(a + d) - (b + c)}{2}. $$

(5)

In Figure 1, the computed results for the first subarea $A_0(1, 1)$ are $A_1(1, 1) = 350, H_1(1, 1) = 50, V_1(1, 1) = 0,$ and $D_1(1, 1) = 0$. These computations are repeated for all 16 subareas which yield the $4 \times 4$ matrices $A_1, H_1, V_1,$ and $D_1$ as shown in Figure 1(a). Only $A_1, H_1,$ and $V_1$ are useful for fabric characterization. Matrix $A_1$ (which can be thought of as a $4 \times 4$ image) is essentially a downscaling of $A_0$ by a factor of 2. Matrix $H_1$ quantifies the grayscale change in the horizontal direction, while $V_1$ quantifies the grayscale change in the vertical direction. Since there is no grayscale change in the vertical direction, $V_1$ is a zero matrix.

In computer vision, the sum of the squares of the values in a matrix is called its energy ($E$). The computed energies for subareas $A_1, H_1, V_1,$ and $D_1$ are $E_{A1}, E_{H1}, E_{V1},$ and $E_{D1}$ as shown in Figure 1. $E_{H1}$ and $E_{V1}$ quantify the grayscale changes of adjacent pixels in $A_0$ in horizontal and vertical directions, respectively. $E_{D1}$ is not particularly useful for our needs but it is necessary to "preserve the energy" in a HWT. The energy of the original image $A_0$ is preserved in the four submatrices $A_1, H_1, V_1,$ and $D_1$ of the decomposition:

$$ E_{A0} = E_{A1} + E_{H1} + E_{V1} + E_{D1}. $$

(6)

The HWT can now be applied to $A_1$ in a second decomposition as shown in Figure 1(b). It computes four other submatrices $A_2, H_2, V_2,$ and $D_2$ whose corresponding energy values are also shown. $E_{H2}$ and $E_{V2}$ now quantify the grayscale changes of adjacent $2 \times 2$ pixel regions in $A_0$ in the horizontal and vertical directions. Finally, $A_2$ can be further decomposed into $A_3, H_3, V_3,$ and $D_3$ as shown in Figure 1(c). $E_{H3}$ and $E_{V3}$ quantify the grayscale changes of adjacent $4 \times 4$ pixel regions in $A_0$ in the horizontal and vertical directions. Again, the energy is preserved at each decomposition.

The original size of $A_0$ is $8 \times 8$ pixels. Therefore, three HWT decompositions could be performed on the image. It is easy to see that a $2^n \times 2^n$ pixel image can be decomposed $L$ times. At the $i$-th decomposition level, the computed energies $E_{H1}$ and $E_{V1}$ quantify the magnitudes of grayscale changes in adjacent $2^{2^{i-1}} \times 2^{2^{i-1}}$ pixel regions in the horizontal and vertical directions in image $A_0$.

3. The Relationship between Fabric and HWT

A rice image is used to illustrate HWT results of real particulate material. The rice is used due to the large aspect ratio, facilitating visual fabric evaluations. For example, observing Figure 2(a), we can perceive that the dominant particle long axis direction of rice particles is horizontal. A box having a size of $1024 \times 1024$ pixels is used to cut the rice image in Figure 2(a). Then, the HWT is performed on the cut image. A total of ten decomposition levels are performed as shown in Figure 2(b). At each decomposition level, the energy values $E_{H1}$ and $E_{V1}$ are computed and plotted in Figure 2(c). Two important properties of HWT are revealed by Figure 2 and analyzed as follows.

First, both $E_{H1}$ and $E_{V1}$ values display bell shape curves along with the increasing decomposition levels and both $E_{H1}$ and $E_{V1}$ values reach the peak values at the decomposition level of 5 as shown in Figure 2(c). In the particulate image, image grayscale changes at particle edges and particle interiors due to particle textures. It is evident that the grayscale changes at particle edges are stronger than the grayscale changes at particle interiors.

As the decomposition level $i$ increases, the weak features in the image (i.e., the grayscale changes at particle interiors) are filtered out gradually, while the grayscale changes at...
edges become dominant as shown from $A_0$ to $A_4$ in Figure 2(b). As a result, both $E_{Hi}$ and $E_{Vi}$ increase and reach their peaks at $A_4$, implying that the majority of the internal texture is eliminated and the grayscale changes only occur at edges.

At higher postpeak decomposition levels, the grayscale changes at edges will also be filtered out. Thus, the entire image becomes progressively blurred as shown in $A_5$ to $A_7$. $E_{Hi}$ and $E_{Vi}$ reduce in these higher levels.

The maximum $E_{Hi}$ and $E_{Vi}$ values essentially quantify the grayscale change in the horizontal and vertical directions only due to the occurrences of particle edges. Obviously, only the edges should define the material fabric. Therefore, only the maximum $E_{Hi}$ and $E_{Vi}$ values are used to define the horizontal and vertical energies $E_H$ and $E_V$ for this image:

$$E_H = \max_{i=1...L} (E_{Hi}).$$  \hspace{1cm} (7)

$$E_V = \max_{i=1...L} (E_{Vi}).$$  \hspace{1cm} (8)

For the example image in Figure 2, $E_H$ and $E_V$ are computed as $E_{H10}$ and $E_{V10}$ as shown in Figure 2(c).

Second, $E_V$ is larger than $E_H$ as shown in Figure 2(c). As discussed above, the $E_H$ and $E_V$ values quantify the magnitudes of grayscale changes due to occurrence of particle edges in horizontal and vertical directions. In Figure 2(d), we can observe more frequent occurrence of long edges in the horizontal direction, yielding more grayscale changes in the vertical direction. Therefore, larger vertical energy $E_V$ and smaller horizontal energy $E_H$ values are expected.

Generally, the grayscale changes are less frequent in parallel to the fabric direction than the grayscale changes perpendicular to the fabric direction, resulting in smaller energy in fabric direction and larger energy perpendicular to fabric direction. As such, if we can identify a direction simultaneously satisfying the conditions that the energy is the minimum in this direction while the energy is the maximum perpendicular to this direction, this direction must be the fabric direction. Therefore, an energy ratio ($ER$) is proposed by this study:

$$ER = \frac{E_V}{E_H}.$$  \hspace{1cm} (9)
Figure 2: The results of Haar wavelet transform for analyzing particulate material image (the images are original images before RHWT). (a) The original image and cutting box. (b) Haar wavelet transformation of region in cutting box. (c) The energy distributions at different decomposition levels. (d) The energies of the image in horizontal and horizontal directions.
The value of $ER$ ranges from 0 to 1. $\theta$ is the orientation of cutting box. For example, in Figure 2(d), $\theta$ of cutting box is zero. $ER$ is computed as

$$ER(\theta) = \frac{2}{\pi} \arctan \left( \frac{E(\theta + 90^\circ)}{E(\theta)} \right).$$

(9)

$$ER(\theta) = \frac{2}{\pi} \arctan \left( \frac{E(\theta + 90^\circ)}{E(\theta)} \right) = \frac{2}{\pi} \arctan \left( \frac{E_V}{E_H} \right),$$

(10)

$$= 0.81.$$

$E_v$ and $E_H$ can be determined by Figure 2(c) following (7) and (8). For actual particulate materials, the fabric directions are in a range of 0° to 180°. If we could compute the $ER$ values of an image over the range 0° $\leq \theta \leq$ 180°, $\theta$ corresponding to the maximum $ER$ is the fabric orientation. However, the HWT computes energy only in the horizontal and vertical directions in an image and cannot identify fabric in other directions. Therefore, a rotational Haar wavelet transform (RHWT) is proposed to determine energy values in all directions.

4. Rotational Haar Wavelet Transform

The rotational Haar wavelet transform (RHWT) is proposed by this study to address the limitation of HWT that can only compute energies in vertical and horizontal directions. The rice image in Figure 2 continues to be used to illustrate RHWT algorithm as shown in Figure 3. The cutting box is rotated counterclockwise by $\theta$ degrees as shown in Figure 3(a) and the cut image is shown in Figure 3(b). The cut image is then rotated clockwise by $\theta$ degrees as shown in Figure 3(c). Then, the ten-level HWT is performed on the image in Figure 3(c) to compute $E_H$ and $E_V$ values based on (7) and (8). The $E_H$ and $E_V$ values are plotted back to the original image as shown in Figure 3(d), which determine the energy values at two directions of $\theta$ and $\theta + 90^\circ$ simultaneously: $E(\theta) = E_H$ and $E(\theta + 90^\circ) = E_V$.

The energy ratio $ER$ at two directions of $\theta$ and $\theta + 90^\circ$ can be computed simultaneously following (9) as shown in Figure 3(e):

$$ER(\theta) = \frac{2}{\pi} \arctan \left( \frac{E(\theta + 90^\circ)}{E(\theta)} \right).$$

(11)

and

$$ER(\theta + 90^\circ) = \frac{2}{\pi} \arctan \left( \frac{E(\theta)}{E(\theta + 90^\circ)} \right),$$

(12)

If we increase the rotation angle $\theta$ of cutting box from 0° to 89° by increments of 1°, a total of 90 cut images will be generated. For each rotation, the above computations are repeated to compute $E_H$ and $E_V$. Taking computed $E_H$ and $E_V$ into (11), the $ER$ for 0° $\leq \theta \leq$ 89° can be computed as shown in Figure 3(e). Taking computed $E_H$ and $E_V$ into (12), the $ER$ for 90° $\leq \theta \leq$ 179° can be computed. Due to rotational symmetry, $ER(\theta + 180^\circ) = ER(\theta)$. Therefore, we can plot $ER$ in polar coordinates over the full range 0° $\leq \theta \leq$ 360° in Figure 3(f). An important property of (11) and (12) is that the sum of the computed $ER$ values in any two orthogonal directions equals 1:

$$ER(\theta) + ER(\theta + 90^\circ) = 1.$$

(13)

Therefore, $ER_{\text{max}} + ER_{\text{min}} = 1$ will also always be true.

The ER curve in Figure 3(f) is equivalent to a rose diagram traditionally developed after statistically counting the number of particles orientations in each direction. The $ER$ values increase as $\theta$ approaches the fabric direction and the peak $ER$ ($ER_{\text{max}}$) identifies the fabric direction. Meanwhile, orthogonal to the fabric direction, $ER$ will be at a minimum ($ER_{\text{min}}$). As shown in Figure 3(f), the fabric direction is close to horizontal direction, which agrees well with the visual observation of Figure 3(a).

However, it is difficult to determine the fabric direction precisely in Figure 3(f) due to the zigzagged $ER$ plot, which is caused by the randomness of particle orientations, particle internal texture, and image noise. To address this issue, Fourier series is introduced to smooth the $ER$ plot such as the result in Figure 3(f). A second-order Fourier series is used in this study:

$$ER(\theta) = a_0 + a_1 \cos(\theta) + b_1 \sin(\theta) + a_2 \cos(2\theta) + b_2 \sin(2\theta).$$

(14)

Since $ER(\theta + 180^\circ) = ER(\theta)$, $a_1$ and $b_1$ must be zero. Equation (14) must also satisfy (13). Therefore, $a_0$ must be 0.5. With these two constraints, (14) simplifies to

$$ER(\theta) = 0.5 \cos(2\theta) + b_2 \sin(2\theta).$$

(15)

Then, $ER_{\text{max}}$ and $ER_{\text{min}}$ are computed as

$$\begin{bmatrix} ER_{\text{max}} \\ ER_{\text{min}} \end{bmatrix} = 0.5 \pm \sqrt{a_2^2 + b_2^2}.$$
The coefficients $a_2$ and $b_2$ can be determined by fitting (15) to the $ER$ plot in Figure 3(f). For example, in Figure 3, $a_2$ and $b_2$ are computed as 0.350 and $-0.077$, respectively. Then, $ER_{\text{max}}$ and $ER_{\text{min}}$ are computed as 0.86 and 0.14 following (15). The smoothed Fourier $ER$ plot is superimposed to the original $ER$ plot in Figure 3(g). The Fourier $ER$ curve removes the noise of original $ER$ curve, eliminating the uncertainty and difficulty in the determination of fabric direction. Based on Figure 3(g), the direction of $ER_{\text{max}}$ is computed as 174°, which precisely determines the fabric direction in Figure 3(a).

**Figure 3**: The concept of rotational Haar wavelet transform (RHWT) (the images are original images before RHWT).
5. Key Parameters of RHWT

The key parameters controlling the accuracy of RHWT include rotational increment of cutting box $\Delta \theta$ and the analyzed image size $L$ as shown in Figure 4(a). In previous example in Figure 3, the image size was $1450 \times 1450$ pixels of which its central $1024 \times 1024$ area was analyzed. Generally, if the size of the analyzed central region is $2^L \times 2^L$ on which an L-level HWT is performed, the corresponding image size must be at least $2^{L+0.5} \times 2^{L+0.5}$ as shown in Figure 4(a). Therefore, $L$ is used to quantify the image size.

In previous example illustrated in Figure 3, $L$ was set as 10 and $\Delta \theta$ was set as $1^\circ$ to compute ER values. To investigate the influence of $\Delta \theta$ on the fabric characterization results, we keep $L=10$ but use small $\Delta \theta$ of $0.1^\circ$ and large $\Delta \theta$ of $10^\circ$ to repeat the RHWT on the rice image. The computed ER curves are shown in Figures 4(b) and 4(c). Comparing the ER curves in Figure 3(g), Figure 4(b), and Figure 4(c), it is evident that $\Delta \theta$ could affect computational results. However, once the rotational increment is smaller than $1^\circ$, the fabric characterization results are not affected but computational time could increase significantly because more cut images should be analyzed. Therefore, $\Delta \theta = 1^\circ$ is suggested.

The image size $L$ is affected by the particle size. The large particles naturally require large image size $L$ to contain particles for performing a reliable RHWT analysis. Therefore, this paper proposed a parameter PPW (pixels per width of particle) to quantify the relationship between particle size and image size. The minimum PPW for ensuring accurate RHWT should be established. The long-grain rice can develop a strong fabric; Figure 4 is used to investigate the effects of PPW. If we could determine the minimum PPW that would correctly characterize such a strong fabric anisotropy, we would be able to use this minimum PPS as the criterion for other natural materials, such as high explosives, whose particles are less elongated. The long-grain rice image was upscaled and downscaled to generate a series of images having different PPW values. The images were $46 \times 46$, $91 \times 91$, $181 \times 181$, and $2897 \times 2897$ pixels. The analyzed central areas were correspondingly $2^3 \times 2^3$, $2^6 \times 2^6$, $2^2 \times 2^2$, and $2^{11} \times 2^{11}$ pixels. The corresponding PPW values are 2, 4, 7, and 110 pixels. The computed original and Fourier ER plots for different PPSs are shown in Figure 4. The results are stable and consistent once PPS exceeds 7 pixels. Therefore, the required minimum PPS to achieve reliable results is 7 pixels. The next issue is how to determine PPW value for a given image automatically. The image size is a known value for a given image. Therefore, we only need to determine particle size. Hryciw et al. [45] proposed an excellent image processing technique based on wavelet analysis technique to determine particle sizes automatically, which can be used to evaluate PPW values of the given image.

6. Fabric Tensor by RHWT

In constitutive modeling, the fabric of particulate material is usually quantified by a fabric tensor [52, 53]:

$$\varphi_{ij} = \int_{\Omega} P(\Omega)n_i n_j d\Omega, \quad (i, j = 1, 2, 3).$$

$n_i$ $(i = 1, 2, 3)$ are the components of a unit vector $n$ describing the orientations of particle long axes; $P(\Omega)$ is a probability density function describing the distribution of $n$; $O$ is the solid angle corresponding to the entire surface of a unit sphere; and $P(\Omega)d\Omega$ is the fraction of $n$ oriented within a small solid angle $d\Omega$. The computed fabric tensor by Eq. (21) is a symmetric, third-rank matrix, which can be represented by three principal values $\varphi_1$, $\varphi_2$, and $\varphi_3$ at corresponding principal directions $\theta_1$, $\theta_2$, and $\theta_3$. For a two-dimensional image, (17) can be simplified to

$$\varphi_{ij} = \int_{0}^{2\pi} P(\theta)n_i n_j d\theta, \quad (i, j = 1, 2),$$

where $(n_1, n_2) = (\cos \theta, \sin \theta)$ and the bar indicates that it is for a 2D image. $P(\theta)$ can be any function that satisfies

$$\int_{0}^{2\pi} P(\theta)d\theta = 1.$$

With a minor mathematical adjustment to satisfy (19), our Fourier idealization of the ER distribution given by (15) becomes the function $P(\theta)$:

$$P(\theta) = \frac{2}{\pi} \text{ER}(\theta) - \frac{1}{2\pi} = \frac{1}{2\pi} + \frac{2a_2}{\pi} \cos(2\theta) + \frac{2b_2}{\pi} \sin(2\theta).$$

Integrating (20) and (22), the two-dimensional fabric tensor is computed to be

$$\varphi = \begin{bmatrix} 0.5 + a_2 & b_2 \\ b_2 & 0.5 - a_2 \end{bmatrix}.$$

$\varphi$ is a symmetric, second-rank tensor and its two principal values $\varphi_1$ and $\varphi_3$ are

$$\begin{bmatrix} \varphi_1 \\ \varphi_3 \end{bmatrix} = \frac{(\varphi_{11} + \varphi_{33}) \pm \sqrt{(\varphi_{11} - \varphi_{33})^2 + 4\varphi_{13}^2}}{2}$$

$$= 0.5 \pm \sqrt{a_2^2 + b_2^2}.$$

Comparing (16) and (22), the two principal values $\varphi_1$ and $\varphi_3$ are exactly $ER_{\max}$ and $ER_{\min}$.

Knowing that the sum of $ER_{\max}$ and $ER_{\min}$ is unity, this paper introduces a single factor $\Psi$ for replacing $ER_{\max}$ and $ER_{\min}$. $\Psi$ is set as $ER_{\max}$ because of its importance. Therefore, $ER_{\min}$ equals $1-\Psi$. The physical meaning of $\Psi$ is the degree of fabric anisotropy as shown in Figure 5. $\Psi$ is in a range of 0.5 to 1. When $\Psi$ is 0.5, both $ER_{\max}$ and $ER_{\min}$ are 0.5, indicating that the particle long axes are completely in random directions and the ER curve is a circle as shown in Figure 5(a). The material develops an isotropic fabric in this case. When $\Psi$ approaches 1, $ER_{\max}$ approaches 1 and $ER_{\min}$ approaches 0, indicating that particles display stronger preferred orientations as shown in Figures 5(b)–5(d) and the degrees of fabric anisotropy become stronger. In the extreme condition, $\Psi$ equals 1, indicating that all the
If \( \text{ER}_{\text{max}} \) and \( \text{ER}_{\text{min}} \) are replaced by \( \Psi \) and \( 1 - \Psi \), respectively, we have \( \varphi_1 \propto \Psi \) and \( \varphi_3 \propto 1 - \Psi \). Therefore, the two-dimensional fabric tensor of (21) can be rewritten as

\[
\varphi = \begin{bmatrix} \Psi & 0 & 0 \\ 0 & 0 & \Psi \\ 0 & 0 & 1 - \Psi \end{bmatrix} \quad \text{(23)}
\]

Most particulate material exhibits cross-anisotropic fabric. Therefore, three principal values \( \varphi_1, \varphi_2, \) and \( \varphi_3 \) in three-dimensional specimen can be determined by \( \Psi \) and \( 1 - \Psi \) values computed from two-dimensional images following the procedure proposed by Oda and Nakayama [26]. Assume that principal values \( \varphi_1 \) and \( \varphi_3 \) in two-dimensional images are proportional to the principal values \( \varphi_1 \) and \( \varphi_3 \) in three dimensions for cross-anisotropic soil [26]:

\[
\frac{\varphi_1}{\varphi_3} = \frac{\Psi}{1 - \Psi} \quad \text{(24)}
\]
Increasing fabric anisotropy ($0.5 < \varphi < 1.0$)

Since $\varphi_1 = \varphi_2$ and $\varphi_1 + \varphi_2 + \varphi_3 = 1$, the three principal values of the fabric tensor $\varphi_1$, $\varphi_2$, and $\varphi_3$ can be obtained. The three-dimensional fabric tensor of (17) can be rewritten as

$$
\varphi = \begin{bmatrix}
\varphi_1 & 0 & 0 \\
0 & \varphi_2 & 0 \\
0 & 0 & \varphi_3
\end{bmatrix}
$$

(25)

$$
\Psi = \frac{1}{1 + \varphi_3} = \frac{1}{1 + \Psi}
$$

(25)

$\Psi$ can be determined using the RHWT technique detailed in this paper. (24) can be readily used in development of constitutive models and statistically correlated to various anisotropic properties of high explosive materials. For isotropic fabric, $\Psi = 0.5$ and, therefore, $\varphi_1 = \varphi_2 = \varphi_3 = 1/3$. For complete anisotropy, $\Psi = 1.0$ and then $\varphi_1 = \varphi_2 = 0.5$ and $\varphi_3 = 0$. As $\Psi$ increases from 0.5 to 1, $\varphi_1$ and $\varphi_2$ increase from $1/3$ to 0.5, while $\varphi_3$ decreases from $1/3$ to 0.

7. Fabric Characterizations of High Explosives

The overall technical flow of the proposed RHWT is shown in Figure 6. The input image could be an RGB image or a grayscale image (8-bit or 16-bit). Then, the key parameter PPW is computed by using the image processing algorithm developed by Hryciw et al. [45]. If the PPW is smaller than 7 pixels, the RHWT program generates an error notifying users that the results would be inaccurate due to low image resolution. If the PPW is larger than 7 pixels, the RHWT continues to crop the image using a square with a size of $2^L \times 2^L$. Then, the cropping square is rotated by an incremental angle of $\Delta \theta = 1^\circ$. For each rotation, the cropped central area is analyzed by the Haar wavelet transform to determine the energy ratio ($ER$). All the $ER$ values form a zigzagged curve, which is smoothed by Fourier analysis. The smoothed $ER$ curve is used to compute the fabric tensor.

This study collected three types of explosive particles, including HMX (octahydro-1,3,5,7-tetranitro-1,3,5,7-tetrazocine), RDX (1,3,5-trinitro-1,3,5-triazacyclohexane), and AP (Ammonium Perchlorate). These explosive particles were randomly distributed and photographed by scanning electrical microscopy (SEM, JEOL JSM-5800) as shown in Figures 7(a), 7(c), and 7(e), respectively.

The image sizes were set as $728 \times 728$ pixels and the central $512 \times 512$ areas were analyzed by a nine-level RHWT. The fabric characterization results are shown in Figures 7(b), 7(d), and 7(f). For simplicity of illustration, only the fabric direction and degree of fabric anisotropy $\Psi$ are shown in Figure 7. The two-dimensional and three-dimensional fabric tensors defined in (23) and (25) can be readily computed knowing $\Psi$. For the three images in Figure 7, the $\Psi$ values are close to 0.5, indicating that nearly isotropic fabric is developed in the image. This is expected because particles are randomly distributed without preferred orientations.

Three high explosive formulations were investigated in this study: PBX-109, GHL-Q, and GO-1. PBX-109 contained, by weight, 65% RDX, 21% Aluminum, and 14% HTPB binder plasticized with DOA (dioctyl adipate). GHL contained, by weight, 64% RDX, 20% Aluminum, and 16% HTPB binder. GO-1 contained, by weight, 88% HMX and 12% polymeric binder. Three high explosive specimens with three formations were prepared in a mold with a diameter of 10 mm. Then, we cut the specimen up to a depth of 1 mm to
create a groove and broke it into two pieces. The fractured surfaces were not polished to keep intact particles and binder. The SEM was used to photograph the fracture surfaces as shown in Figures 8(a), 8(c), and 8(e). The image sizes were 728 × 728 pixels, in which the central 512 × 512 areas were analyzed by the RHWT. The fabric characterization results are shown in Figures 8(b), 8(d), and 8(f), indicating that the fracture surfaces of high explosives develop a level of fabric anisotropy.

There are many microstructural images of high explosives in the published literature. Some of these images from prestigious laboratories were collected and used to validate the RHWT algorithm and fabric tensor computational methods proposed by this study. Skidmore et al. [54]...
captured a series of microstructural images of high explosives PBX 9501 under different compression pressures. PBX 9501 contained, by weight, 95% HMX, 2.5% Estane 5703 (a polyurethane binder), and 2.5% bis-dinitropropyl acetal/formal (a nitroplasticizer).

As described by Skidmore et al. [54], the first PBX 9501 specimen in Figure 9(a) was compressed in a fixed volume mold (approximately 41 mm in diameter, 55 mm high) with a pressure of 138 MPa under the temperature of 90°C until the porosity achieved 16%. The second PBX 9501 specimen in Figure 9(b) was pressed using the same mold under the same temperature as the first specimen but using a higher pressure of 262 MPa. Therefore, a smaller porosity of 2% was achieved. We could observe that fractures develop in explosive particles in Figure 9(b) due to the high compression pressure.

Microstructural examination was performed on pieces of each specimen cut near the center of the compact in order to avoid skin effects. Polarized light microscopy (PLM) was conducted by mounting material in a low-viscosity epoxy, polishing the cured mount, examination in reflected light, and photography by digital CCD camera as described by Skidmore et al. [54].

The captured images are shown in Figures 9(a) and 9(c). The image sizes were 728 × 728 pixels, in which the central 512 × 512 areas were analyzed by a nine-level RHWT. The fabric characterization results are shown in Figures 9(b) and 9(d). The first specimen developed a fabric direction of 167° and a degree of fabric anisotropy $\Psi$ of 0.53, while the second specimen developed a fabric direction of 173° and a degree of fabric anisotropy $\Psi$ of 0.60. Under the higher pressure, more explosive particles are mobilized so that their long axes rotate to the horizontal direction. Therefore, the fabric direction changes from 167° to 173°. Meanwhile, the preferred particle orientations become stronger, yielding a stronger fabric anisotropy. Therefore, $\Psi$ increased from 0.53 to 0.60.

Manner et al. [55] formulated a series of HMX-based PBXs using low-density binders allowing for micro X-ray computed tomography (microCT) characterization. Two formulations were prepared and tested in their work: CF75 L and CF75-AMO1. CF75 L was comprised of 88% HMX, 5.4% HTPB, 5.4% DOA, 0.7% lecithin, and trace dibutyltin dilaurate as a catalyst. A low level of 0.5% diphenylmethane diisocyanate (isonate) was used to cure the HTPB and form a polyurethane binder system surrounding the HMX crystals. CF75-AMO1 was comprised of 88% HMX bound with mixtures of 3.0% glycidyl azide polymer (GAP) cured with 9.0% acrylic monomers/oligomers (AMO) capable of cycloaddition with the azide functionality on the GAP.

Cylindrical specimens with a diameter of 7 mm and a height of 8 ~ 11 mm were prepared. These cylindrical specimens were used to conduct strain-controlled uniaxial compression tests at strain rate of 0.45% per second. Specimens were sequentially 3D imaged using CT with an in situ load stage during compression. Each sample was compressed to identical strain of 10% and 20%. They were...
held for 10–15 min after compression to allow any residual plastic flow to occur and then imaged at this strain [21, 22]. Each full 3D measurement required approximately 2 h. The microCT images of CF75 L are shown in Figures 10(a) to 10(c), while the microCT images of CF75-AMO1 are shown in Figures 11(a) to 11(c).

For each microCT image, the center part having a size of 728×728 pixels was cropped, in which 512×512 pixels were analyzed by a nine-level RHWT. The fabric characterization results are shown in Figures 10(d) to 10(f) and 11(d) to 11(f). Both CF75 L and CF75-AMO1 specimens developed a horizontal fabric. Under compression, more explosive particles are mobilized so that their long axes rotate to the horizontal direction. Therefore, the preferred particle orientations become stronger, yielding a stronger fabric anisotropy. Therefore, Ψ increases along with compressive strain.

In the above examples, the specimens of high explosives were prepared by different methods including directly fracturing (Figure 8) and cutting and polishing (Figure 9). Various imaging technologies are used for capturing microstructure of specimens, including SEM (Figures 7 and 8), PLM (Figure 9), and microCT (Figures 10 and 11). Despite these variables, the proposed RHWT successfully identifies fabric in these images, demonstrating robustness and capability of RHWT.
Unlike the rice image having elongated particles allowing for visual analysis of fabric, the fabric in the high explosive images (Figures 8 to 11) is not perceivable due to less elongated explosive particles, presence of plastic binders between particles, and small grayscale contrast between particle and binders. However, the RHWT successfully detects and identifies these microstructures.

The microstructures of high explosives under different compression levels (Figures 9–11) are key indicators for preparation of high explosives, which affects formation, ignition, and growth of hot spots in the shocked high explosives and then the macroscopic behavior of the high explosives in the shock-to-detonation transition process. To the knowledge scope of authors, this paper, at the first time, captures the microstructure evolutions under compression, providing valuable results for numerical simulation or constitutive model developments for high explosives.

8. Conclusions

This paper develops an RHWT for analyzing fabric anisotropy of high explosives. The basic concept of RHWT is to evaluate grayscale changes in the image. The preferred direction of particle long axes possesses minimum grayscale changes, which defines the fabric direction in the image. An energy ratio \((ER)\) is defined to quantify grayscale changes, which is plotted in the traditional rose diagram to perform fabric characterizations. For a purely isotropic material, the \(ER\) plot is a circle having a value of 0.5 in all directions. However, for anisotropic material, the \(ER\) plot is elongated in the fabric direction. The Fourier series method is used to filter out the noises of the original \(ER\) curve, facilitating accurate computations of \(ER_{\text{max}}, ER_{\text{min}}\, \text{and fabric direction.}

The key parameters controlling the accuracy of RHWT include the rotational angle increment of cutting box and image size. Thorough investigations based on rice image illustrate that the appropriate rotational angle increment is 1° and the minimum required image size is \(2^7 \times 2^7\) pixels.

Due to the importance of \(ER_{\text{max}}\) value, this paper defines \(ER_{\text{max}}\) as the degree of fabric anisotropy \(\Psi\). \(\Psi\) has a theoretical range of 0.5 to 1.0. As \(\Psi\) increases, the fabric anisotropy becomes stronger in the image, indicating a stronger preferred particle orientation exhibited in the image. This paper also demonstrates that \(\Psi\) can be used to compute the two-dimensional and three-dimensional fabric tensor, commonly used in numerical simulations and constitutive models.

The RHWT based microstructure characterization methodology is utilized to analyze rice images, yielding accurate results which agree with visual observations well. Then, three explosive particles without binder and three typical high explosives are photographed by scanning electrical microscopy and analyzed by RHWT. In addition to the image captured by this study, various microscopic high explosive images in the published literature from three laboratories are also extracted. Those images captured by polarized light microscopy and micro X-ray computed tomography are also analyzed by RHWT. All the results demonstrate that the RHWT is capable of analyzing fabric anisotropy of high explosives and capturing the microstructure evolutions of high explosive during compressions.
Data Availability
The article contains all the data.

Conflicts of Interest
The authors declare that they have no conflicts of interest.

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