

Research Article

Dynamic Equilibrium Strategy of Power Battery Closed-Loop Supply Chain Based on Stochastic Differential Game

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The industrialization of new energy vehicles has accelerated, and the recycling industry driven by a large number of retired power batteries has exploded rapidly. How to control and deal with decommissioned batteries and balance economic and environmental benefits has become an urgent problem to be solved. Considering the influence of uncertain interference factors, the dynamic equilibrium strategy of power battery closed-loop supply chain members composed of leading manufacturer, recycler, and cascade utilization enterprise was studied. (1) This paper describes the stochastic evolution process of cascade utilization effort level by using Ito process and constructs a stochastic differential game model of closed-loop supply chain dynamic system combined with cost-sharing coordination mechanism. (2) Based on the stochastic differential game theory, the optimal profit function of the participants is given, and the dynamic equilibrium strategy of each participant is obtained. (3) In order to grasp the statistical characteristics of cascade utilization effort level, the random evolution characteristics of cascade utilization effort level are revealed. (4) Combined with numerical example, the influence of relevant parameters on the dynamic system of power battery closed-loop supply chain is analyzed. The results found that with the increase of the proportion of manufacturer sharing the innovation cost of cascade utilization enterprise, the cascade utilization effort level will improve, the profits of cascade utilization enterprise and recycler will increase significantly, the profits of manufacturer will decrease slightly, and the overall profits of the supply chain will increase. The cost-sharing coordination mechanism is conducive to ensuring the steady development of the industry and effectively improving the utilization efficiency of industrial resources.

1. Introduction

At the United Nations General Assembly in 2020, China made a commitment to achieve carbon peak by 2030 and carbon neutralization by 2060. The discussion on the dual-carbon goal is heating up. Under this background, the new energy industry is developing rapidly [1], among which the most representative is the rapid development of new energy vehicles. With the rapid growth of production and sales of new energy vehicles, the output of power batteries has surged, and the earlier batch of power batteries put into use have ushered in the retirement period. According to relevant calculations, the recycling volume of power batteries will reach 120 GWh in 2025 [2]. The decommissioned power

batteries are recycled and reused in whole package or disassembled and screened and then reused in other scenarios with low requirements on battery performance, including static scenarios of chemical energy storage such as power generation side, distribution side and power consumption side, and dynamic scenarios such as low-speed scooters and logistics vehicles. This kind of cascade utilization can effectively improve the utilization rate of resources, avoid large-scale idling and abandonment of power batteries before they have exerted their due effect, reduce the harm of harmful metal leakage in batteries to the environment and human body, and effectively protect the ecological environment. At present, due to the various specifications of decommissioned power batteries, the testing standards need

to be clarified, the industry value evaluation has not reached a consensus and for other reasons, the cascade utilization industry is still in the initial stage of extremely unstable development. How to effectively coordinate relevant subjects to balance economic and environmental benefits, and jointly undertake the important responsibility of promoting the improvement of the comprehensive utilization level of industrial resources has become one of the hot problems of common concern of all sectors of society.

At the macro level, in August 2021, the Ministry of Industry and Information Technology issued the “Administrative Measures for the Cascade Utilization of New Energy Vehicle Power Batteries,” which put forward relevant requirements for the recycling and processing of cascade utilization enterprises and the design and production of cascade products. “Vehicle Power Battery Recycling and Utilization Cascade Utilization Part 3: Cascade Utilization Requirements,” which was implemented in March 2022, regulates the remaining energy requirements, cycle life requirements, safety requirements, and other performance requirements of cascade utilization of retired batteries. It aims to ensure the highest cycle value and safety of cascade utilization products, and lay the foundation for the standardized and low-carbon cascade utilization of waste power batteries in China. In general, the cascade utilization of power batteries is gradually changing from “national recommended standards” to “national mandatory standards,” and cascade utilization has become the general trend. At the micro level, there are many demonstration projects of cascade utilization at home and abroad. More than 100 enterprises at home and abroad have carried out application research and business model exploration related to cascade utilization. Domestically, there is a dual-state trend of technological upgrading from the single application of battery disassembly to the application of the whole module and scale expansion from the kilowatt level to the megawatt level. China Southern Power Grid build energy storage power stations, and used local and Beijing-Tianjin-Hebei electric vehicle retired power batteries in a centralized manner; State Grid Corporation of China (SGCC) build a kilowatt-level cascade energy storage project in Daxing, Beijing to cut peak and fill valleys; Jiangsu Nantong “industrial and commercial energy storage system based on cascade utilization of retired power batteries” and related enterprises led by iron tower company and Bi-Ya-Di also actively responded to promote the whole module and large-scale cascade utilization of power batteries. The foreign cascade utilization market mainly focuses on the user side and its economic benefits. For example, 4R Energy Company, General Electric Company, ABB (Asea Brown Boveri) Group, EnerDel Company and Bosch Group use retired power batteries for home energy storage, commercial energy storage or integrated wind and solar energy storage systems. In response to the management problems caused by the classification and parameter differences of batteries in the cascade utilization, new energy vehicle companies such as Fengfan, BAIC New Energy, Chehejia, and Weilai automobile have successively incorporated the “battery swap mode” into their new development strategies to explore

opening up the entire value chain of cascade utilization, which has laid the foundation for the development of cascade utilization. With the promotion of the “battery swap mode” and the development of a circular economy, the cascade utilization will show a blowout trend in the future.

This paper takes the power battery closed-loop supply chain members composed of leading manufacturer, recycler, and cascade utilization enterprise as the study objects, aiming at studying the feedback dynamic balance strategy of power battery CLSC members by introducing the cascade utilization market and combining the cost-sharing coordination mechanism, in the case of considering the influence of uncertain factors. The contribution of this paper is to analyze the dynamic equilibrium strategy of members of power battery CLSC system with cascade utilization market by using stochastic differential game theory, in view of uncertain competitive environment and combined with the cost-sharing coordination mechanism, this paper provides a theoretical reference for the behavior decision-making of relevant enterprises, and is also a beneficial supplement and expansion of existing research in relevant fields, so as to help the efficient utilization of power batteries. In addition, at the practice implication level, combined with the actual situation, this paper provides ideas for improving and innovating the operation mechanism of internal coordination and interaction of power battery CLSC. The research results have important guiding significance for improving the utilization efficiency of industrial resources and reducing environmental pollution.

2. Literature Review

The research in related fields of power battery cascade utilization is also carried out with the emergence of practical problems. From the perspective of industry operation, Li et al. sorted out the relevant policies and standards for the cascade utilization of domestic retired power batteries, investigated and analyzed the effects and roles of relevant typical demonstration enterprises and pilot enterprises [2]. Yano pointed out that the recovery of retired power batteries does have great value through the actual investigation of the electric vehicle industry [3]. Heymans et al. adopted the life cycle evaluation system, and further considered the impact of power battery recycling on the manufacturing cost, and the results showed that the cost can be greatly saved [4]. From the perspective of power battery raw material supply, Ahmad et al. analyzed the global output and prices of metals such as nickel, cobalt, and lithium, and further elaborated on the hidden benefits of power battery recycling [5]. Harper et al. combed the current lithium battery recycling methods and policy measures, classified them and made predictions on future recycling trends [6]. Huang et al. first introduced the recycling products of power batteries and the development of recycling technology [7]. Zeng et al. analyzed the opportunities and challenges faced by the power battery recycling process [8]. Dong et al. sorted out the recycling modes of new energy vehicle power batteries at home and abroad, and compared and analyzed the development advantages of independent recycling of production enterprises,

industry alliances, and third-party recycling modes [9]. Mu et al. built a model based on the theory of system dynamics to analyze the recovery and utilization of power batteries of new energy vehicles [10]. Wang and Wu proposed an improvement strategy to optimize the recovery method and improve the recovery rate of scarce metals according to China's actual situation [11]. Under the background of circular economy, Hao et al. put forward countermeasures such as combination of forward and reverse, joint consultation and joint construction for recycling reverse logistics [12].

From the perspective of micro-operation, the research on the CLSC (closed-loop supply chain) of power batteries should include multi-cycle and multi-level recycling [13, 14], dual-channel recycling [15, 16], and supply chain coordination. Although literature [13, 14] considered multiple uncertain production processes, it did not consider the impact of the internal coordination mechanism of the supply chain. Literature [15, 16] studied the contract coordination under different recovery modes, but they were all based on static vision and did not study the dynamic game process of the supply chain in depth. Natkunarajah et al. first predicted the sales volume of electric vehicles, and analyzed the recovery rate of the power batteries in different scenarios based on the life cycle of the power batteries, laying the foundation for further discussion on the power batteries recovery mode. Although this study explored different recycling modes, it did not include the cascade utilization market in the research process, which has certain limitations [17]. Yang et al. combined the two recycling methods of "cascade utilization" and "regenerative dismantling" into the reverse supply chain of power batteries, built a reverse supply chain model, and studied the recycling mode of used power batteries for new energy vehicles. However, this study did not consider the impact of uncertainty factors on stakeholders' decision-making [18]. Aiming at maximizing the profit of the supply chain, Lin et al. analyzed the influence of different battery quality, processing cost and other factors on the profitability of the supply chain, but the study did not consider the internal coordination of the supply chain [19]. Heydari et al. constructed a secondary reverse supply chain of a single manufacturer and retailer, increased consumers' willingness to recycle through quantity discounts and payment fees, and designed cost contracts to achieve CLSC coordination [20]. Zhang and Chen took the third-party recyclers whose power battery recycling is the core business as the main body of recycling, and combine the characteristics of the remanufacturing, cascade utilization, and dismantling and recycling of waste power batteries to build a CLSC with multiple recycling channels, and studied the strategy and coordination of decision-makers [21]. Both of them deeply studied the internal coordination problem of supply chain, but they both carried out research from static vision, without considering the dynamic decision-making process of supply chain under the influence of random factor disturbance. Han et al. studied the impact of the "trade-in" strategy in the Cournot duopoly competitive secondary CLSC on the competitiveness of enterprises and product market share, but did not consider the coordinated decision-making of

internal agents in the supply chain [22]. Hoyer et al. established a linear analysis model for the CLSC of lithium power batteries to optimally select multiple given production and recycling schemes, but did not consider the important role of the cascade utilization market in the entire recycling process [23]. For remanufacturing cost and recovery rate, Wang and Deng used a dynamic game to compare the optimal decision-making of recycling and remanufacturing supply chain members in three situations, but ignored the influence of random factors [24]. Guo et al. introduced the valuable metal recycling station to build a supply chain model and analyzed the effectiveness of the revenue sharing contract, but further research from a dynamic perspective is needed [25]. Based on the third-party remanufacturing model considering outsourcing and authorization, Zhang et al. constructed a competitive CLSC of duopoly manufacturers and remanufacturers, and studied the strategic trends of each subject [26]. Guan and Hou considered the internal and external coordination of the supply chain under a certain environment, and analyzed the equilibrium decision-making problem of enterprises in the power battery CLSC which introduced the cascade utilization market [27]. However, the impact of uncertain factors on the system needs to be further explored.

To sum up, the cascade utilization of power batteries is still in the early stage of development. At present, the research on the power battery CLSC mainly focuses on the interpretation of the cascade utilization policy, the battery recycling mode, the determination of recycling channels, and the discussion of the CLSC coordination mechanism, both are static and deterministic methodological research, all of them belong to the research on the equilibrium strategy of the decision-making subject in a certain environment, without taking into account the influence of various random disturbances on decision-making subjects in the decision-making process. In essence, the operation process of CLSC system will be affected by a large number of uncontrollable factors [28], such as the personality and emotion of decision-makers, the ability to obtain information, the political and cultural environment where the decision-making subject is located, industry background and humanistic factors, and the evolution process of the system state is difficult to predict. Therefore, in view of the uncertainty of internal and external interference factors in the system, studying the internal coordination mechanism of power battery CLSC and the dynamic equilibrium strategy of decision-makers have become an important field of current research, which also has important theoretical and practical guiding significance for improving the efficiency of comprehensive utilization of resources.

Considering the influence of uncertain factors, this paper introduces cascade utilization market and combines the cost-sharing coordination mechanism to study the feedback dynamic equilibrium strategy of the power battery CLSC members. The structure of the study is arranged as follows: first, the Ito process is used to describe the stochastic evolution process of the cascade utilization effort level, and the objective function of each participant's pursuit of profit maximization is constructed, and the stochastic differential

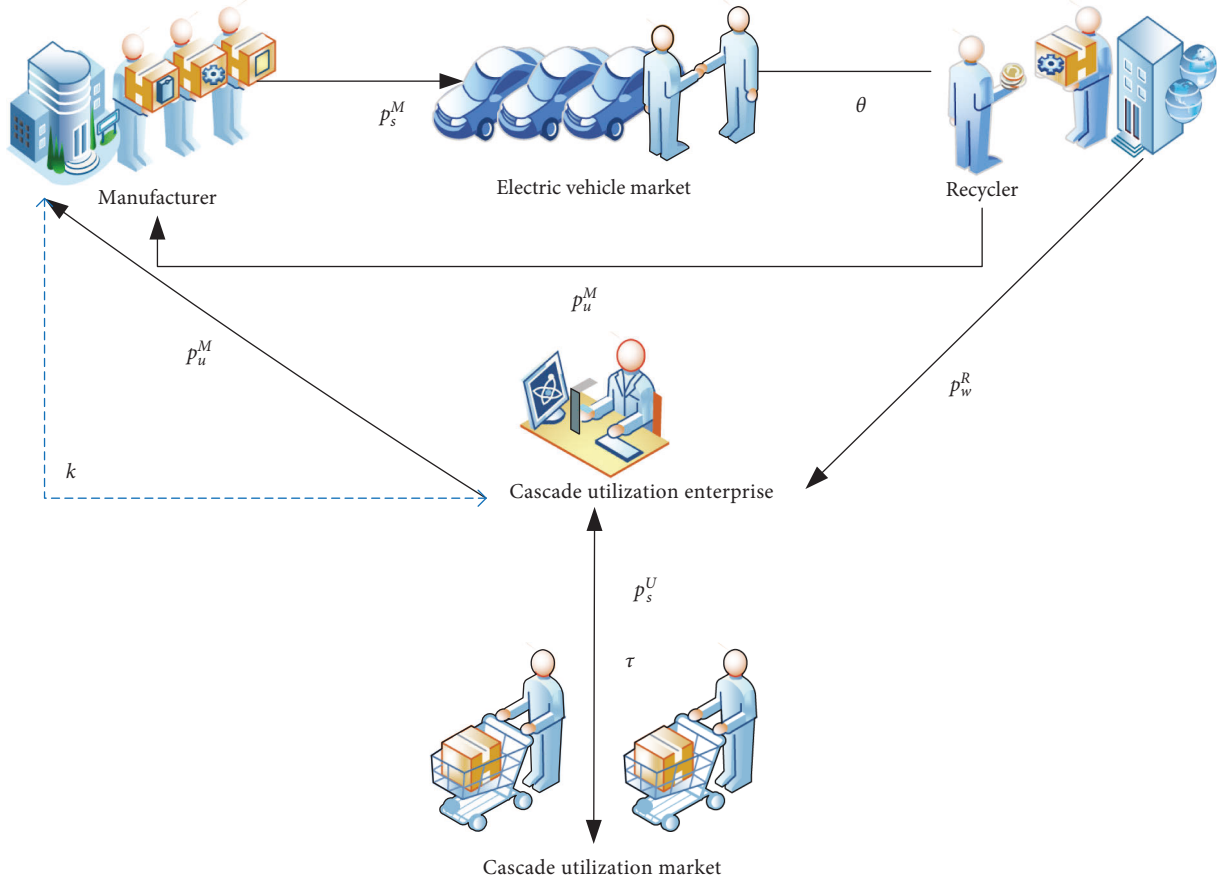


FIGURE 1: Power battery CLSC model considering cascade utilization.

game model of the CLSC dynamic system is constructed by combining the cost-sharing coordination mechanism; secondly, the stochastic differential game theory and continuous-time dynamic programming theory are used to solve the dynamic equilibrium control strategy of each participant; next, in order to reveal the dynamic evolution characteristics of the random cascade utilization effort level, the stability of the expectation and variance of the random cascade utilization effort level is analyzed; finally, combined with a numerical example, the sensitivity of the relevant parameters is analyzed and verified, and the influence of the relevant parameters on the dynamic system of the power battery CLSC is determined.

3. Stochastic Differential Game Model

3.1. Problem Description. In this paper, a CLSC dynamic system of power battery is constructed, which is composed of manufacturer, recycler, and cascade utilization enterprise. Among them, the manufacturer is in a leading position in the system, responsible for the production and sales (at price p_s^M) of the power battery for electric vehicles, and at the same time, in order to promote the healthy development of the industry and promote the recycling of resources, the manufacturer bears the innovation cost of the cascade utilization of retired batteries (at cost-sharing ratio k). The recycler recycles, sorts, and processes the retired power

batteries in the market (at recycling rate θ), and makes profits by selling the waste batteries with high-energy density to the cascade utilization enterprise (at wholesale price p_w^R), and hands over low-energy density waste batteries to manufacturers for remanufacturing (at uniform recycle price p_u^M); Facing the cascade utilization market, cascade utilization enterprise processes and reorganizes the purchased high-energy density batteries into cascade utilization products to sell (at price p_s^U), and is responsible for recycling all the discarded batteries after cascade utilization and handing them over to the manufacturer for remanufacturing (at uniform recycle price p_u^M). Figure 1 describes the basic model of power battery CLSC.

3.2. Variable Description and Model Assumption. In order to simplify the analysis and build the model, the symbols and meanings of variables used in this paper are shown in Table 1.

Based on the product supply chain and resource flow trajectory of the real power battery industry, combined with the basic model and reference to existing studies, the model assumptions are made as follows:

Assumption 1. The demand function of the electric vehicle market for new power batteries [29] is $L(p_s^M(t)) = a - bp_s^M(t)$, and $c_r^M < c_m^M < p_s^M(t)$.

TABLE 1: Description of variables.

Variable	Description	Variable	Description
$p_s^M(t)$	Unit sales price of new power battery	$\tau(t)$	Cascade utilization effort level
c_m^M	Unit production cost of new power battery	a	Electric vehicle market's market size
c_r^M	Unit reproduction cost of waste power battery	b	Electric vehicle market's price sensitivity coefficient
$\theta(t)$	Recovery rate of waste battery	f	Cascade utilization market's market size
i	Unit price of retired battery recovered by recycler	g	Cascade utilization market's price sensitivity coefficient
$p_w^R(t)$	Unit wholesale price at which recycler sells high-energy density battery	$A(t)$	Cascade utilization effort utility
$p_u^M(t)$	Uniform price for manufacturer to recycle waste battery	δ	Cascade utilization effort utility cost coefficient
$p_s^U(t)$	Unit sales price of cascade utilization battery sold by cascade utilization enterprise	α	Influence coefficient of cascade utilization effort utility on effort level
c_r^U	Unit recovery cost of cascade utilization enterprise	β	Attenuation coefficient of cascade utilization effort level
k	Cascade utilization effort utility cost-sharing coefficient	λ	Discount rate
$dz(t)$	Standard wiener process	$\sigma(\tau(t))$	Volatility of cascade utilization effort level
$\Delta = c_m^M - c_r^M$	Unit cost-saving of remanufactured products produced by manufacturer	B	Recovery difficulty coefficient

Assumption 2. The number of retired power batteries recovered by recyclers is $\theta(t)L(p_s^M(t)) = \theta(t)(a - bp_s^M(t))$. The recovery fixed cost [30] is $1/2B\theta(t)^2$.

Assumption 3. The demand function for high-energy-density waste batteries in the cascade utilization market [31] is: $L(p_s^U(t), \tau(t)) = (f - gp_s^U(t))\tau(t)$, and $p_w^R(t) < p_s^U(t) < p_s^M(t)$. cascade utilization effort utility cost [32] is $1/2\delta A(t)^2$. The cascade utilization market is different from the ordinary reuse market, and its demand is independent of the demand for new power batteries in the electric vehicle market. The cascade utilization enterprise is responsible for recycling all used batteries after cascade utilization (at unit recovery cost c_r^U).

Assumption 4. As an emerging market, the scale of the cascade utilization market is still small, and the number of high-energy-density waste batteries recovered and sorted by recycler can fully meet the demand of the cascade utilization market.

Assumption 5. Manufacturer with remanufacturing production lines recycle and remanufacture all used batteries at a uniform price $p_u^M(t)$, and $0 < i < p_u^M(t) < p_w^R(t)$, $p_u^M(t) < \Delta$. The power batteries produced with recycled materials and new materials have the same quality and characteristics, that is, there is no difference in sales price and consumer preference.

Assumption 6. In the model, the forward electric vehicle market sales and the reverse decommissioned battery recycling, cascade utilization, and remanufacturing are all completed in one cycle. That is, the model only considers the cycle process of the power battery resource in a single cycle.

Assumption 7. In a single cycle, each game participant discounts its future earnings at a fixed discount rate $\lambda > 0$.

3.3. Dynamic Model of Cascade Utilization Effort Level. $\tau(t)$ is the effort made by cascade utilization enterprises to improve the level of cascade utilization, such as developing high-tech and using new media for publicity. In fact, the level of cascade utilization effort is affected by many factors, such as effort effectiveness, consumers' awareness of environmental protection, and other uncontrollable factors. Similar to the advertising capital model [33], it is assumed that the utility of cascade utilization effort affects the evolutionary drift rate of cascade utilization effort level, that is, $d\tau(t) = \alpha A(t)dt$. In addition, it is assumed that the decline of consumers' awareness of environmental protection and the aging of relevant facilities affect the cascade utilization effort level, which decreases exponentially at a rate $\tau(t)/\tau(t) = -\beta$, that is, $d\tau(t) = -\beta n(t)dt$. In addition, similar to Prasad and Sethi [34], it is assumed that the cascade utilization process is influenced by the standard Wiener process, and the volatility of the cascade utilization process is directly proportional to the square root of the cascade utilization effort level, that is, $d\tau(t) = \sigma\sqrt{\tau(t)}dz$. Based on the above factors, the evolution process of cascade utilization effort level is described as the Ito process given by

$$d\tau(t) = [\alpha A(t) - \beta\tau(t)]dt + \sigma\sqrt{\tau(t)}dz(t). \quad (1)$$

3.4. Fundamental Relation Expression and Stochastic Differential Game Model. Based on the cost-sharing mechanism, the goal of supply chain members is all to maximize profits in the planned period. The manufacturer's income comes from the sales of new power batteries, and its cost comes from the cost of recycling power batteries and sharing part of the utility cost of cascade utilization enterprise. The revenue of the recycler comes from selling recycled power batteries, and its cost is the fixed cost of recycling. The revenue of the cascade utilization enterprise comes from selling cascade utilization products to the cascade utilization market, and its

cost is the effort utility cost of cascade utilization. For the sake of brevity of the model, the time variable t will be omitted in the following statement.

$$\max \pi^M = E \left\{ \int_0^\infty e^{-\lambda t} \left[(p_s^M - c_m^M)(a - bp_s^M) + (\Delta - p_u^M)(\theta(a - bp_s^M)) - k \frac{1}{2} \delta A^2 \right] dt \right\}. \quad (2)$$

To sum up, the profit target function of the manufacturer is given by

The profit target function of the recycler is given by

$$\max \pi^R = E \left\{ \int_0^\infty e^{-\lambda t} \left[(p_w^R(t) - i)(f - gp_s^U)\tau + (p_u^M - i)(\theta(a - bp_s^M) - (f - gp_s^U)\tau) - \frac{1}{2} B\theta^2 \right] dt \right\}. \quad (3)$$

The profit target function of the cascade utilization enterprise is given by

$$\max \pi^U = E \left\{ \int_0^\infty e^{-\lambda t} \left[(p_s^U - c_r^U + p_u^M - p_w^R)(f - gp_s^U)\tau - (1 - k) \frac{1}{2} \delta A^2 \right] dt \right\}. \quad (4)$$

Under the power structure in which the manufacturer is the system leader and the recycler and cascade utilization enterprise are the system followers, the

Stackelberg stochastic differential game model of power battery CLSC system equilibrium strategy is summarized as given by

$$\begin{cases} \max_{[p_s^M, p_u^M]} \pi^M [p_s^M, p_u^M, \theta, p_w^R, p_s^U, A], \\ \text{s.} \begin{cases} \max_{[p_w^R, \theta]} \pi^R [p_s^M, p_u^M, \theta, p_w^R, p_s^U, A], \\ \max_{[p_s^U, A]} \pi^U [p_s^M, p_u^M, \theta, p_w^R, p_s^U, A], \end{cases} \\ d\tau(t) = [\alpha A(t) - \beta \tau(t)]dt + \sigma \sqrt{\tau(t)} dz. \end{cases} \quad (5)$$

4. Dynamic Equilibrium Strategy

In order to obtain the dynamic equilibrium strategy of power battery CLSC system, first, the reverse induction method was used to obtain the sales price and its own effort utility of cascade utilization enterprise, and then the recovery rate of recycler and the wholesale price of high-energy density waste batteries sold to cascade utilization enterprise are obtained based on the response strategy of cascade utilization enterprise; Finally, based on the response strategies of recycler and cascade utilization enterprise, the manufacturer's sales price of new power

batteries and the unified price of recycled waste batteries are obtained.

Proposition 1. *The response strategy of cascade utilization enterprise, sales price and their own effort utility, is given by*

$$p_s^U(t) = \frac{3f + gc_r^U}{4g}, \quad (6)$$

$$A = \frac{\alpha V_\tau^U}{\delta(1 - k)}.$$

Proof. The HJB (Hamilton-Jacobian-Bellman) partial differential equation that the cascade utilization enterprise equilibrium strategy should satisfy can be obtained by

$$\lambda V^U(\tau) - \frac{1}{2}\sigma^2\tau V_{\tau\tau}^U(\tau) = \max\left\{(p_s^U - c_r^U + p_u^M - p_w^R)(f - gp_s^U)\tau - (1-k)\frac{1}{2}\delta A^2 + V_\tau^U(\tau)(\alpha A - \beta\tau)\right\}, \quad (7)$$

where $V^U(\tau)$ is the optimal value function of the cascade utilization enterprise, and $V_\tau^U(\tau), V_{\tau\tau}^U(\tau)$ is the first and second order partial derivatives of the cascade utilization effort level. In order to ensure the existence of the optimal solution of the model, it is assumed that $2B - b(\Delta - i)^2 > 0$.

Solving the optimization of the right-hand side of equation (7), we obtain

$$\begin{cases} \frac{\partial \lambda V^U(\tau)}{\partial p_s^U(t)} = f\tau - p_u^M g\tau + p_w^R g\tau - 2p_s^U g\tau + gc_r^U \tau, \\ \frac{\partial \lambda V^U(\tau)}{\partial A} = -A\delta(1-k) + \alpha V_\tau^U(\tau). \end{cases} \quad (8)$$

The response strategy of cascade utilization enterprise is obtained, as given by equation (9), by combining the formulas given in

$$\begin{cases} p_s^U(t) = \frac{3f + gc_r^U}{4g}, \\ A = \frac{\alpha V_\tau^U}{\delta(1-k)}. \end{cases} \quad (9) \quad \square$$

using the continuous dynamic programming theory, as given by

Proposition 2. *The response strategy of recycler, the recovery rate, and the wholesale price of high-energy density waste batteries sold to cascade utilization enterprise, expression is given by*

$$\begin{cases} \theta = \frac{(p_u^M(t) - i)(a - bp_s^M(t))}{A}, \\ p_w^R(t) = \frac{f - gc_r^U}{2g} + p_u^M(t). \end{cases} \quad (10)$$

Proof. The HJB partial differential equation that the recycler equilibrium strategy should satisfy can be obtained by using the continuous dynamic programming theory.

$$\lambda V^R(\tau) - \frac{1}{2}\sigma^2\tau V_{\tau\tau}^R(\tau) = \max\left\{(p_w^R - i)(f - gp_s^U)\tau + (p_u^M - i)(\theta(a - bp_s^M) - (f - gp_s^U)\tau) - \frac{1}{2}B\theta^2 + V_\tau^R(\tau)(\alpha A - \beta\tau)\right\}, \quad (11)$$

where $V^R(\tau)$ is the optimal value function of the recycler, and $V_\tau^R(\tau), V_{\tau\tau}^R(\tau)$ are the first and second order partial derivatives of the cascade utilization effort level.

To obtain the reaction strategy of the recycler, equation (9) is brought into equation (11), and the optimization of its right-hand side is solved, as given by

$$\begin{cases} \frac{\partial \lambda V^R(\tau)}{\partial p_w^R(t)} = (f - gp_s^U)\tau - \frac{1}{2}ng(p_w^R - p_u^M), \\ \frac{\partial \lambda V^R(\tau)}{\partial \theta} = (p_u^M - i)(a - bp_s^M) - B\theta. \end{cases} \quad (12)$$

The response strategy of recycler, as given by equation (13), is obtained by combining the formulas given in

$$\begin{cases} \theta = \frac{(p_u^M - i)(a - bp_s^M)}{B}, \\ p_w^R = \frac{f - gc_r^U}{2g} + p_u^M. \end{cases} \quad (13) \quad \square$$

Proposition 3. *The optimal strategy of manufacturer, sales price of new power batteries, and the unified price of recycled waste batteries, expression is given by*

$$\begin{cases} p_s^{M*} = \frac{2B(a + bc_m^M) - ab(\Delta - i)^2}{4Bb - b^2(\Delta - i)^2}, \\ p_u^{M*} = \frac{(\Delta + i)}{2}. \end{cases} \quad (14)$$

Proof. The HJB partial differential equation that the manufacturer equilibrium strategy should satisfy can be obtained

by using the continuous dynamic programming theory, as given by

$$\lambda V^M(\tau) - \frac{1}{2} \sigma^2 \tau V_{\tau\tau}^M(\tau) = \max \left\{ \begin{array}{l} (p_s^M - c_m^M)(a - bp_s^M) + (\Delta - p_u^M)(\theta(a - bp_s^M)) \\ -k \frac{1}{2} \delta A^2 + V_{\tau}^M(\tau)(\alpha A - \beta \tau) \end{array} \right\}, \quad (15)$$

where $V^M(\tau)$ is the optimal value function of the recycler, and $V_{\tau}^M(\tau)V_{\tau\tau}^M(\tau)$ are the first and second order partial derivatives of the cascade utilization effort level.

To obtain the optimal strategy of manufacturer, equations (9) and (13) is brought into equation (15), and the optimization of its right-hand side is solved, as given by

$$\left\{ \begin{array}{l} \frac{\partial \lambda V^M(\tau)}{\partial p_s^M} = (a - 2bp_s^M) + bc_m^M + (\Delta - p_u^M)\theta(-b) + (\Delta - p_u^M)(a - bp_s^M) \frac{(p_u^M - i)(-b)}{B}, \\ \frac{\partial \lambda V^M(\tau)}{\partial p_u^M} = -\theta(a - bp_s^M) + (\Delta - p_u^M)(a - bp_s^M) \frac{(a - bp_s^M)}{B}. \end{array} \right. \quad (16)$$

The optimal strategy of manufacturer, as given by equation (17), is obtained by combining the formulas given in

$$\left\{ \begin{array}{l} p_s^{M*} = \frac{2B(a + bc_m^M) - ab(\Delta - i)^2}{4Bb - b^2(\Delta - i)^2}, \\ p_u^{M*} = \frac{(\Delta + i)}{2}. \end{array} \right. \quad (17) \quad \square$$

Proposition 4. *The dynamic equilibrium strategy of power battery CLSC system is given by*

$$\left\{ \begin{array}{l} p_s^{M*} = \frac{2B(a + bc_m^M) - ab(\Delta - i)^2}{4Bb - b^2(\Delta - i)^2}, \\ p_u^{M*} = \frac{(\Delta + i)}{2}, \\ \theta^* = \frac{(a - bc_m^M)(\Delta - i)}{4B - b(\Delta - h)^2}, \\ p_w^{R*} = \frac{f + g(\Delta - c_r^U + i)}{2g}, \\ p_s^{U*} = \frac{3f + gc_r^U}{4g}, \\ A^* = \frac{\alpha V_{\tau}^U}{\delta(1 - k)}. \end{array} \right. \quad (18)$$

Proof. Taking (18) into the recycler's reaction strategy (13), the optimal strategy of recycler, θ^*, p_w^{R*} , can be obtained. For the sake of completeness, Theorem 1 also lists the optimal strategy of cascade utilization enterprise, p_s^{U*}, A^* .

To obtain the dynamic equilibrium strategy of the power battery CLSC system using Proposition 4, it is necessary to determine the specific form of the optimal value function of the manufacturer, cascade utilization enterprise, and recycler in Proposition 4. In fact, this will face the problem of partial differential equation solving. Therefore, the specific solution process of the optimal value function is given by Proposition 5. \square

Proposition 5. *The optimal value function of manufacturer, cascade utilization enterprise, and recycler is given by*

$$\left\{ \begin{array}{l} V^M(\tau)^* = x_1^* \tau^2 + x_2^* \tau + x_3^*, \\ V^R(\tau)^* = y_1^* \tau^2 + y_2^* \tau + y_3^*, \\ V^U(\tau)^* = j_1^* \tau^2 + j_2^* \tau + j_3^*, \end{array} \right. \quad (19)$$

where

$$\begin{cases} j_1^* = -\frac{\delta(1-k)(\lambda+2\beta)}{2\alpha^2}, \\ j_2^* = \frac{\alpha^2(f-gc_r^U) - 8g\delta\sigma^2(\lambda+2\beta)(1-k)}{16g\alpha^2(2\lambda+3\delta)}, \\ j_3^* = \frac{[\alpha^2(f-gc_1) - 8g\delta\sigma^2(\lambda+2\beta)(1-k)]^2}{512\delta\lambda g^2\alpha^2(2\lambda+3\beta)^2(1-k)}, \end{cases} \begin{cases} y_1^* = 0, \\ y_2^* = \frac{(f-gc_r^U)^2}{8g(2\lambda+3\beta)}, \\ y_3^* = \frac{(f-gc_r^U)^2[\alpha^2(f-gc_r^U) - 8g\delta\sigma^2(\lambda+2\beta)(1-k)]}{128\delta\lambda g^2(2\lambda+3\beta)^2(1-k)} + \frac{B(a-bc_m^M)^2(\Delta-i)^2}{2\lambda[4B-b(\Delta-i)^2]^2}, \end{cases} \begin{cases} x_1^* = -\frac{\delta k(\lambda+2\beta)}{6\alpha^2}, \\ x_2^* = \frac{2k(\lambda+2\beta)j_2^*}{3(2\lambda+3\beta)(1-\mu_m)} + \frac{\sigma^2 x_1^*}{(2\lambda+3\beta)}, \\ x_3^* = \frac{B(a-bc_m^M)^2}{\lambda[4Bb-b^2(\Delta-h)^2]} + \frac{\alpha^2 j_2^* x_2^*}{\delta\lambda(1-k)} - \frac{\alpha^2 k j_2^{*2}}{2\delta\lambda(1-k)^2}, \end{cases} \quad (20)$$

Proof. To determine the profit optimal value function of manufacturer, cascade utilization enterprise, and recycler, the system dynamic equilibrium strategy in Proposition 4 is

brought into equations (7), (11), and (15), and the partial differential equations that should be satisfied by the optimal value functions of manufacturer, cascade utilization enterprise, and recycler are obtained.

$$\lambda V^M(\tau) = \frac{B(a-bc_m^M)^2}{[4Bb-b^2(\Delta-i)^2]} - \frac{\alpha^2 k V_\tau^U(\tau)^2}{2\delta(1-k)^2} + \frac{\alpha^2 V_\tau^U(\tau)V_\tau^M(\tau)}{\delta(1-k)} - \delta V_\tau^M(\tau)\tau + \frac{1}{2}\sigma^2 V_{\tau\tau}^M(\tau)\tau, \quad (21)$$

$$\lambda V^R(\tau) = \frac{[f-gc_1]^2\tau}{8g} + \frac{B(a-bc_m^M)^2(\Delta-i)^2}{2[4B-b(\Delta-i)^2]^2} + \frac{\alpha^2 V_\tau^U(\tau)V_\tau^R(\tau)}{\delta(1-\mu_m)} - \beta V_\tau^R(\tau)\tau + \frac{1}{2}\sigma^2 V_{\tau\tau}^R(\tau)\tau, \quad (22)$$

$$\lambda V^U(\tau) = \frac{[f-gc_1]^2\tau}{16g} + \frac{\alpha^2 V_\tau^U(\tau)^2}{2\delta(1-k)} - \beta V_\tau^U(\tau)\tau + \frac{1}{2}\sigma^2 V_{\tau\tau}^U(\tau)\tau. \quad (23)$$

In order to obtain the solutions of the partial differential equations (15) to (17), try the following optimal value function form of manufacturer, cascade utilization enterprise, and recycler with undetermined coefficients.

$$\begin{cases} V^M(\tau) = x_1\tau^2 + x_2\tau + x_3, \\ V^R(\tau) = y_1\tau^2 + y_2\tau + y_3, \\ V^U(\tau) = j_1\tau^2 + j_2\tau + j_3. \end{cases} \quad (24)$$

The first and second derivatives are given by

$$\begin{cases} V_\tau^M(\tau) = 2x_1\tau + x_2, & V_{\tau\tau}^M(\tau) = 2x_1, \\ V_\tau^R(\tau) = 2y_1\tau + y_2, & V_{\tau\tau}^R(\tau) = 2y_1, \\ V_\tau^U(\tau) = 2j_1\tau + j_2, & V_{\tau\tau}^U(\tau) = 2j_1. \end{cases} \quad (25)$$

Substituting equations (24) and (25) into equations (21) to (23), and use the identity relationship to obtain the nonlinear equations that the undetermined coefficient should satisfy

$$\begin{cases} \lambda x_1 = \frac{4\alpha^2 j_1 x_1}{\delta(1-k)} - \frac{4\alpha^2 k j_1^2}{2\delta(1-k)^2} - 2\beta x_1, \\ \lambda x_2 = \frac{2\alpha^2 j_1 x_2}{\delta(1-k)} + \frac{2\alpha^2 j_2 x_1}{\delta(1-k)} - \frac{4\alpha^2 k j_1 j_2}{2\delta(1-k)^2} - \beta x_2 + \sigma^2 x_1, \\ \lambda x_3 = \frac{B(a-bc_m^M)^2}{4Bb-b^2(\Delta-h)^2} + \frac{\alpha^2 x_2 j_2}{\delta(1-k)} - \frac{\alpha^2 k j_2^2}{2\delta(1-k)^2}, \end{cases} \quad (26)$$

$$\begin{cases} \lambda y_1 = \frac{4\alpha^2 j_1 y_1}{\delta(1-k)} - 2\beta y_1, \\ \lambda y_2 = \frac{[f-gc_1]^2}{8g} + \frac{2\alpha^2 j_1 y_2}{\delta(1-k)} + \frac{2\alpha^2 j_2 y_1}{\delta(1-k)} - \beta y_2 + \sigma^2 y_1, \\ \lambda y_3 = \frac{B(a-bc_m^M)^2(\Delta-i)^2}{2[4Bb-b^2(\Delta-i)^2]^2} + \frac{\alpha^2 j_2 y_2}{\delta(1-k)}, \end{cases} \quad (27)$$

$$\begin{cases} \lambda j_1 = \frac{4\alpha^2 j_1^2}{2\delta(1-k)} - 2\beta j_1, \\ \lambda j_2 = \frac{[f - gc_1]^2}{16g} + \frac{4\alpha^2 j_1 j_2}{2\delta(1-k)} - \beta j_2 + \sigma^2 j_1, \\ \lambda j_3 = \frac{\alpha^2 j_2^2}{2\delta(1-k)}. \end{cases} \quad (28)$$

Next, the undetermined coefficients are obtained by solving equations (26) to (28) of the nonlinear equations. The process is: first calculate j_1^*, j_2^*, j_3^* , then calculate y_1^*, y_2^*, y_3^* , and finally determine x_1^*, x_2^*, x_3^* . Hence, Proposition 5 is proved. \square

5. Dynamic Evolution Characteristics of Cascade Utilization Effort Level

Since the effort level of cascade utilization is affected by the random disturbance of uncontrollable factors in reality, whether the stochastic evolution characteristics of cascade utilization effort level can be grasped and what kind of evolution characteristics it has. This section discussed the expectation and variance of cascade utilization effort level and the stability of the expectation and variance of cascade utilization effort level.

Proposition 6. *Expectation and variance of random cascade utilization effort level, and when $t \rightarrow \infty$, the expectation and variance of cascade utilization effort level are given by*

$$\begin{aligned} E(\tau) &= e^{Qt}(\tau_0 + NQ^{-1} - NQ^{-1}e^{-Qt}), \\ \lim_{t \rightarrow \infty} E(\tau) &= -NQ^{-1}, \\ D(\tau) &= e^{2Qt}(\tau_0 + NQ^{-1} - NQ^{-1}e^{-Qt})^2 - e^{2Qt} \left(\begin{array}{c} \tau_0^2 + (Q\tau_0 + N)(2N + \sigma^2)Q^{-2} - \\ N(2N + \sigma^2)(2Q^2)^{-1} \end{array} \right) \\ &\quad - e^{Qt}(Q\tau_0 + N)(2N + \sigma^2)Q^{-2} + N(2N + \sigma^2)(2Q^2)^{-1}, \\ \lim_{t \rightarrow \infty} D(\tau) &= N(2N + \sigma^2)(2Q^2)^{-1} - N^2Q^{-2}, \end{aligned} \quad (29)$$

where $Q = -(3\beta + \lambda)$, $N = \alpha^2 j_2^* [\delta(1-k)]^{-1}$.

Proof. Bringing the optimal effort utility of the cascade utilization enterprise in Proposition 4 and the optimal value function of the cascade utilization enterprise in Proposition 5 into equation (1), the dynamic model of cascade utilization effort level, and the Ito process of the change of cascade utilization effort level are obtained.

$$d\tau = \left[\left(\frac{2\alpha^2 j_1^*}{\delta(1-k)} - \beta \right) \tau + \frac{\alpha^2 j_2^*}{\delta(1-k)} \right] dt + \sigma \sqrt{\tau(t)} dz, \quad \tau(0) = \tau_0. \quad (30)$$

Assuming $Q = -(3\beta + \lambda)$, $N = \alpha^2 j_2^* [\delta(1-k)]^{-1}$, equation (30) can be written as

$$d\tau = [Q\tau + N]dt + \sigma \sqrt{\tau(t)} dz, \quad \tau(0) = \tau_0. \quad (31)$$

Solving the differential equation (31), we obtain

$$E(\tau) = e^{Qt}(\tau_0 + NQ^{-1} - NQ^{-1}e^{-Qt}). \quad (32)$$

Due to $Q < 0$, when $t \rightarrow \infty$,

$$\lim_{t \rightarrow \infty} E(\tau) = -NQ^{-1}. \quad (33)$$

The random change process of the square of cascade utilization effort level is obtained by using ITO lemma.

$$d\tau^2 = [2Q\tau^2 + (2N + \sigma^2)\tau]dt + 2\sigma\tau\sqrt{\tau(t)}dZ(t), \quad \tau^2(0) = \tau_0^2. \quad (34)$$

Integrating both sides of the equation (34) and using the boundary conditions, we obtain

$$\tau^2 = \tau_0^2 + \int_0^t (2Q\tau^2 + (2N + \sigma^2)\tau)dt + \int_0^t 2\sigma\tau\sqrt{\tau(t)}dZ(t). \quad (35)$$

Taking the expectation on both sides, and using the zero-expectation property of Wiener process, we obtain

$$E(\tau^2) = \tau_0^2 + \int_0^t (2QE(\tau^2) + (2N + \sigma^2)E(\tau))dt. \quad (36)$$

Substituting equation (25) into equation (29), we obtain

$$E(\tau^2) = e^{2Qt} \left(\tau_0^2 + \frac{(Q\tau_0 + N)(2N + \sigma^2)}{Q^2} - \frac{N(2N + \delta^2)}{2Q^2} \right) - e^{Qt} \frac{(Q\tau_0 + N)(2N + \sigma^2)}{Q^2} + \frac{N(2N + \sigma^2)}{2Q^2}. \tag{37}$$

Then, the variance of cascade utilization effort level is obtained by using the relationship $D(\tau) = E(\tau^2) - [E(\tau)]^2$. Due to $Q < 0$, when $t \rightarrow \infty$,

$$\lim_{t \rightarrow \infty} E(\tau^2) = \frac{N(2N + \sigma^2)}{(2Q^2)}. \tag{38}$$

From equations (33) and (38), we obtain $\lim_{t \rightarrow \infty} D(\tau) = \lim_{t \rightarrow \infty} E(\tau^2) - \lim_{t \rightarrow \infty} [E(\tau)]^2 = N(2N + \sigma^2)/(2Q^2) - N^2/Q^2$. In particular, in the absence of random interference ($\sigma = 0$), $\lim_{t \rightarrow \infty} E(\tau^2) = N^2/Q^2$, therefore $D(\tau) = E(\tau^2) - [E(\tau)]^2 = 0$. Hence, Proposition 6 is proved. \square

6. Sensitivity Analysis of Parameters

Analyze the impact of the price sensitivity of the cascade utilization market g , the recycling price of recycler i , and the cost-saving of manufacturer in producing remanufactured products Δ on the sales price p_s^U and effort utility A of cascade utilization enterprise, the wholesale price p_w^R of selling high-energy density batteries and the recovery rate θ of recycler, the sales price of new power batteries p_s^M , and the unified price of repurchasing used batteries p_u^M of manufacturer. Relevant conclusions are summarized in Table 2 and subsequent inferences.

Inference 1. As the price sensitivity degree of the cascade utilization market g increases, p_s^{M*} , A^* , and p_w^{R*} will decrease. θ^* , p_s^{U*} , and p_u^{M*} remain unchanged.

Proof

$$\frac{\partial A}{\partial g} = -\frac{\alpha(f - gc_r^U)^2}{16g^2(2\lambda + 3\beta)\delta(1 - k)} < 0; \frac{\partial p_s^U}{\partial g} = -\frac{3f}{4g^2} < 0; \frac{\partial p_w^R}{\partial g} = -\frac{f}{2g^2} < 0, \tag{39}$$

$$\frac{\partial \theta}{\partial g} = \frac{\partial p_s^M}{\partial g} = \frac{\partial p_u^M}{\partial g} = 0.$$

\square decrease, and the manufacturer's strategy p_s^{M*} and p_u^{M*} will increase. p_s^{U*} and A^* remain unchanged.

Inference 2. With the increase in the price of recycler recycling retired batteries i , p_w^{R*} will increase, θ^* will

Proof

$$\frac{\partial p_w^R}{\partial i} = \frac{1}{2} > 0; \frac{\partial \theta}{\partial i} = -\frac{(a - bc_m^M)[4A + b(\Delta - i)^2]}{[4A - b(\Delta - i)^2]^2} < 0,$$

$$2ab(\Delta - i)[4Bb - b^2(\Delta - i)^2] -$$

$$\frac{\partial p_u^M}{\partial i} = \frac{1}{2} > 0; \frac{\partial p_s^M}{\partial i} = \frac{2b^2(\Delta - i)[2B(a - bc_m^M) - ab(\Delta - i)^2]}{[4Bb - b^2(\Delta - i)^2]^2} > 0, \tag{40}$$

$$\frac{\partial p_s^U}{\partial i} = \frac{\partial A}{\partial i} = 0.$$

Inference 3. With the increase in cost-saving of remanufacturing by manufacturer Δ , p_w^{R*} and θ^* will increase, p_s^{M*}

will decrease, and p_u^{M*} will increase. p_s^{U*} and A^* remain unchanged. \square

TABLE 2: Sensitivity analysis.

	P_s^{U*}	A^*	P_w^R*	θ^*	P_s^{M*}	P_u^{M*}
g	↓	↓	↓	→	→	→
i	→	→	↑	↓	↑	↑
Δ	→	→	↑	↑	↓	↑

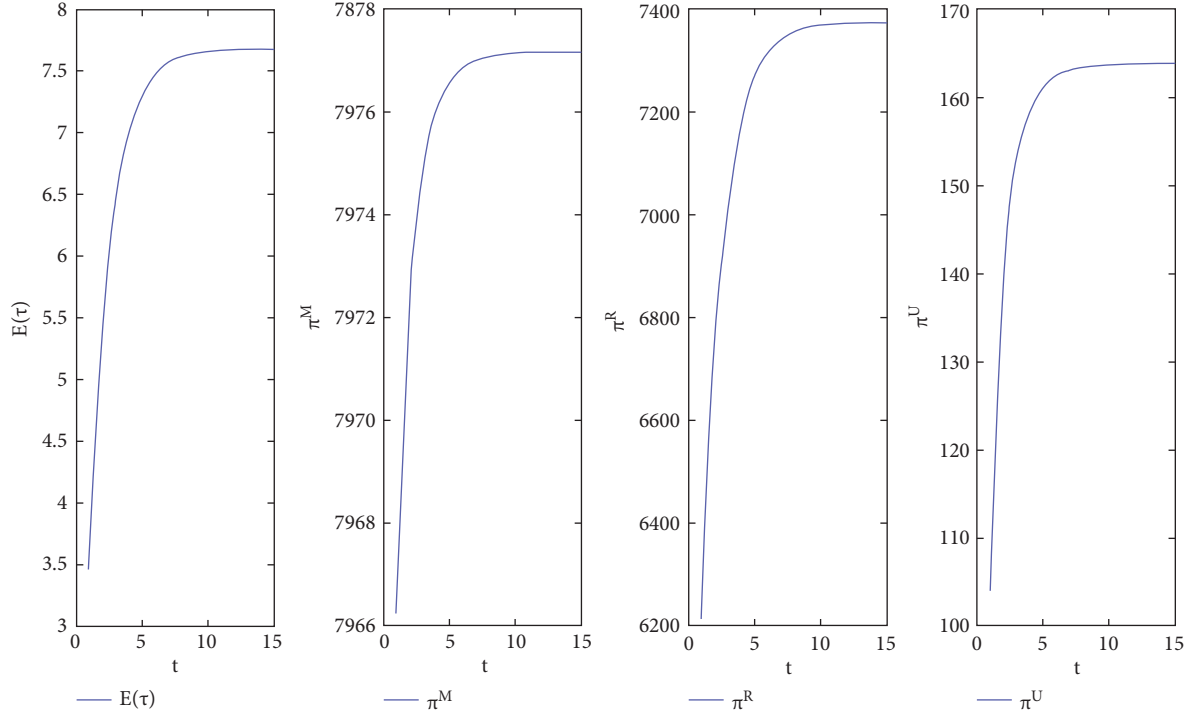


FIGURE 2: Evolution trajectory of system state variables.

Proof

$$\begin{aligned} \frac{\partial P_w^R}{\partial \Delta} &= \frac{1}{2} > 0; \quad \frac{\partial \theta}{\partial \Delta} = \frac{(a - bc_m^M)[4B + b(\Delta - i)^2]}{[4B - b(\Delta - i)^2]^2} > 0, \\ &\quad -2ab(\Delta - i)[4Bb - b^2(\Delta - i)^2] + \\ \frac{\partial P_u^M}{\partial \Delta} &= \frac{1}{2} > 0; \quad \frac{\partial P_s^M}{\partial \Delta} = \frac{2b^2(\Delta - i)[2B(a - bc_m^M) - ab(\Delta - i)^2]}{[4Bb - b^2(\Delta - i)^2]^2} < 0, \\ \frac{\partial A}{\partial \Delta} &= \frac{\partial P_s^U}{\partial \Delta} = 0. \end{aligned} \quad (41)$$

□

7. Numerical Example

In order to describe the model and reveal the law more intuitively, Section (7) analyzes the dynamic equilibrium strategy of the power battery CLSC system under the influence of random disturbance factors. The system parameters are selected as follows: $B = 2$, $\lambda = 0.3$, $a = 70$, $b = 0.5$, $\Delta = 4$, $\beta = 0.3$, $c_r^U = 1$, $\delta = 100$, $\alpha = 3$, $f = 35$, $k = 0.1$, $i = 2$, $g = 0.5$, and $\sigma = 0.01$.

7.1. Evolution Path Analysis. In Figure 2 shows the evolution trajectories of the expectation of cascade utilization effort

level, manufacturer, recycler, and cascade utilization enterprise profits under the cost-sharing scenario considering the impact of uncertain factors.

The expectation of cascade utilization effort and the profit trajectory of each participant show a nonlinear upward trend with a gradually decreasing growth rate over time, and will eventually reach a steady state. At the same time, the profit of the manufacturer has always been the highest, and the profit of the cascade utilization enterprise has been in a low state.

7.2. Sensitivity Analysis

7.2.1. Cost-Sharing Ratio. Keeping other parameters unchanged, Figure 3 shows the impact of the change in the cost-sharing ratio, k , on the expectation of cascade utilization effort level and the profit of each participant.

With the increase of k , the cascade utilization effort level, the profits of recycler, and cascade utilization enterprise all show a nonlinear upward trend, while the manufacturer's profit shows a nonlinear downward trend. As manufacturer shares the increase in the innovation cost of cascade utilization enterprise, the overall profits of manufacturer are decline. However, from the perspective of the overall supply chain, manufacturer still occupy a large market share, and

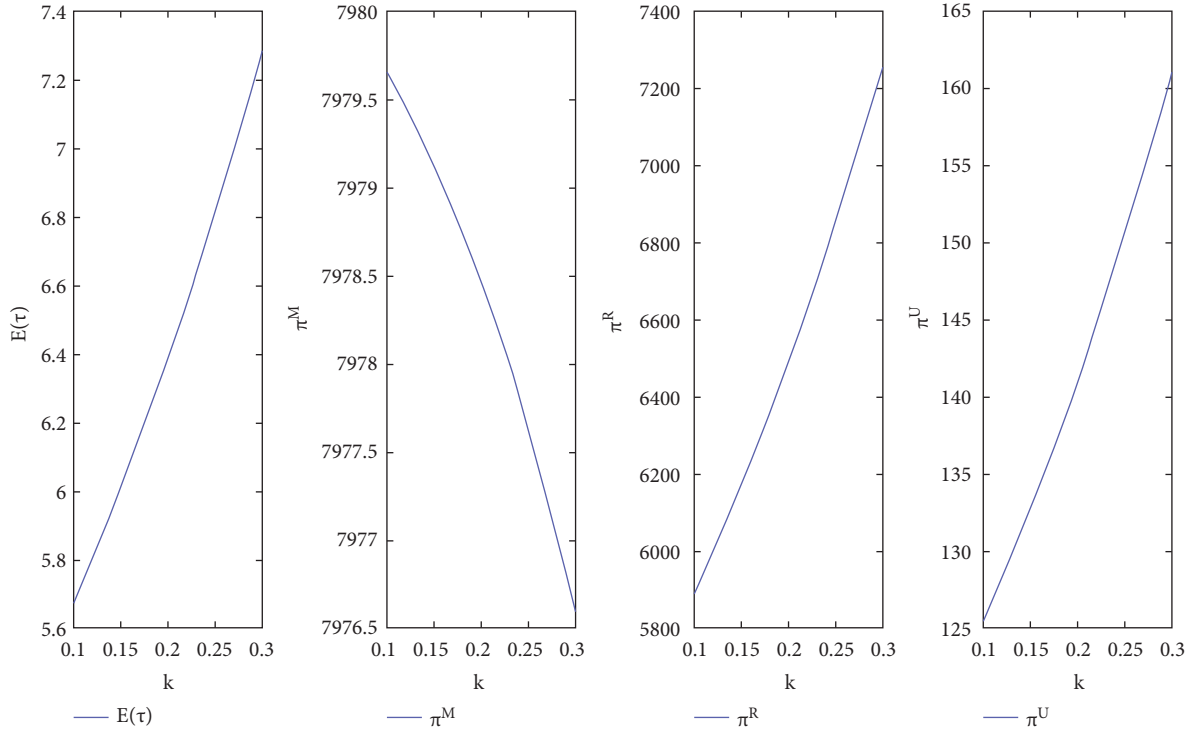


FIGURE 3: Sensitivity analysis of cost-sharing ratio.

the profit is always higher than that of recycler and cascade utilization enterprises. At the same time, the profits of recycler and cascade utilization enterprises are increasing, which will greatly improve their enthusiasm for recycling resources and effectively promote the steady improvement of cascade utilization levels.

7.2.2. Price Sensitivity Coefficient of Cascade Utilization Market. Keeping other parameters unchanged, Figure 4 shows the impact of the change in the price sensitivity coefficient of the cascade utilization market, g , on the dynamic system of the power battery CLSC.

With the increase of g , A^* , p_s^{U*} , p_w^{R*} , π^R , and π^U all show a nonlinear downward trend, and π^R is always greater than π^U , only π^M show a nonlinear upward trend. The increasing sensitivity of consumers to the price of cascade utilization products has a negative impact on both cascade profit and recycling market, and the impact on cascade utilization enterprises is more severe. The demand change of cascade utilization market does not affect the manufacturer's production and recycling decision, but due to the reduction of the cascade utilization effort utility, the cost of cascade utilization effort utility sharing by the manufacturer decreases accordingly. Therefore, when other parameters remain unchanged and g increase, only manufacturers can make more profits.

7.2.3. Recycled Battery Price of Recycler. Keeping other parameters unchanged, Figure 5 shows the impact of

changes in the price of recycled batteries by recycler, i , on the dynamic system of the power battery CLSC.

With the increase of i , p_u^{M*} , and p_w^{R*} show a linear upward trend, p_s^{M*} shows a nonlinear upward trend, θ , π^R , and π^M show a nonlinear downward trend, π^R has a larger decline, π^M is always greater than π^R , and π^U is the smallest and remains unchanged. The increase in i enables recycler to increase p_w^{R*} and reduce θ , but such decision-making changes have not stopped the decline of their own profits; Manufacturer thus raises p_u^{M*} , and then increase p_s^{M*} , so as to reduce consumer demand and eventually damage their own interests; The increase of i has little impact on cascade utilization enterprise, and the decision-making and profit of cascade utilization enterprise will not change.

7.2.4. Cost-Saving for Manufacturers in Remanufacturing. Keeping other parameters constant, Figure 6 shows the impact of changes in cost-saving in the manufacture of remanufactured products, Δ , on the dynamic system of the power battery CLSC.

With the increase of Δ , p_u^{M*} , and p_w^{R*} show a linear upward trend, p_s^{M*} shows a nonlinear downward trend, θ , π^R , and π^M show a nonlinear upward trend, π^M is always greater than π^R , π^U is the smallest and remains unchanged. The increase in Δ has greatly improved the enthusiasm and motivation of manufacturer to recycle resources, so that manufacturer can still be profitable whereas reducing p_s^{M*} to attract consumers, which reflecting the importance of manufacturer's relevant technology research and development and innovation; Recycler earns large profits by

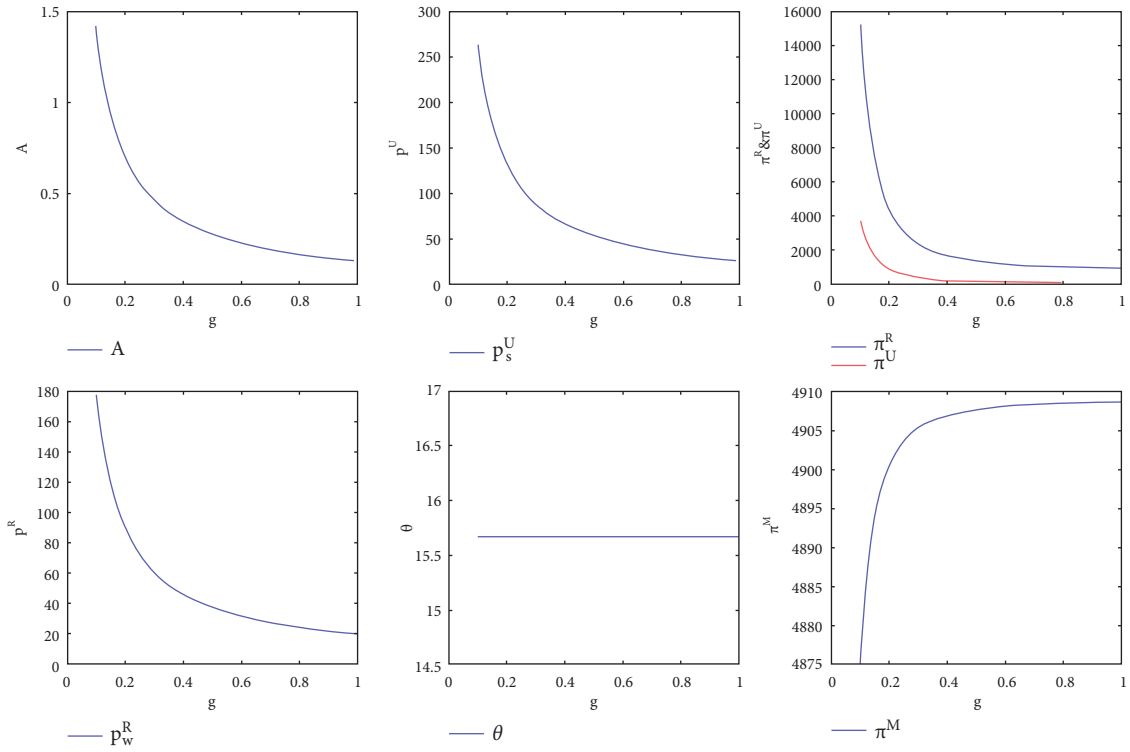


FIGURE 4: Sensitivity analysis of the price sensitivity coefficient of cascade utilization market.

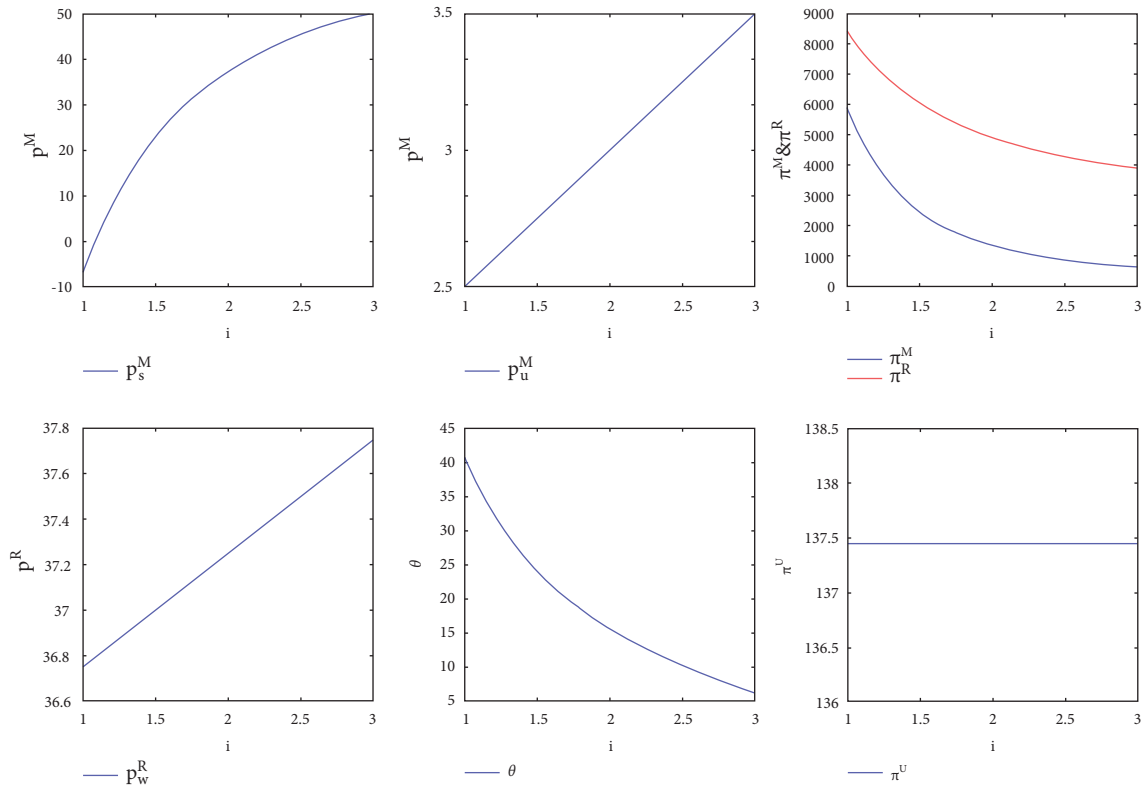


FIGURE 5: Sensitivity analysis of recycled battery price of recycler.

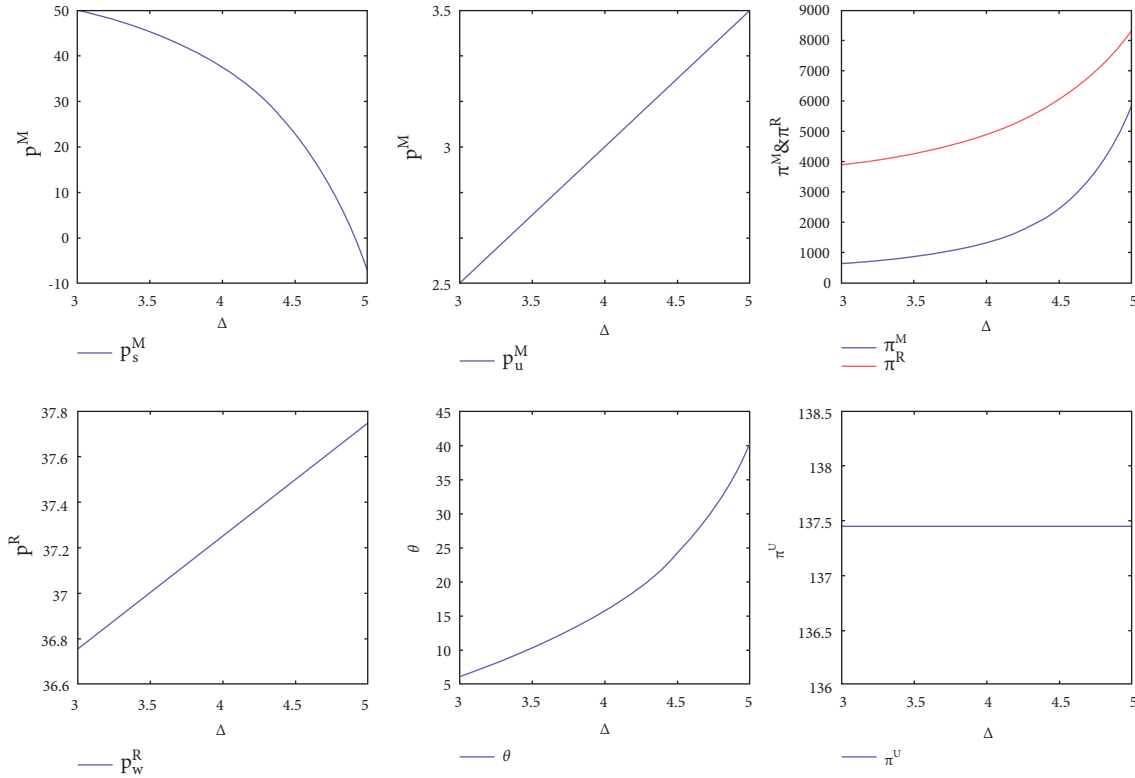


FIGURE 6: Sensitivity analysis of cost-saving for manufacturer in remanufacturing.

increasing p_w^{R*} and θ ; cascade utilization enterprise is not affected by Δ , the decisions and profits remain unchanged.

8. Conclusions

This paper studies the dynamic equilibrium strategy of power battery CLSC members under the interference of uncertain factors. The stochastic evolution process characteristics of cascade utilization effort level of power battery CLSC dynamic system are described by using Ito process. According to the profit composition of participants, the target function of each participant pursuing profit maximization is constructed. Aiming at the dynamic system of power battery CLSC with the manufacturer as the leader, the stochastic differential game model is established, and the dynamic equilibrium control strategy of each participant is solved by using stochastic differential game theory and continuous-time dynamic programming theory. In order to reveal the evolution characteristics of random cascade utilization effort level, the stability of expectation and variance of random cascade utilization effort level is analyzed. Combined with numerical examples, the evolution path analysis of state variables and parameter sensitivity analysis of dynamic systems are carried out to verify the influence of relevant parameters on dynamic systems.

The results found that with the increase of cost-sharing ratio, the effort level of cascade utilization enterprise is improved, the manufacturer’s profit is decreased slightly, but it does not affect its industry dominant position, the profits of recycler and cascade utilization enterprise are increased,

which effectively improves the enthusiasm of resource recycling in the supply chain. Manufacturer shares part of the innovation cost of cascade utilization enterprise that can effectively improve the profit space of relevant subjects, stimulate the enthusiasm of relevant subjects to recycle resources, reduce environmental pollution, and guarantee the effective improvement of cascade utilization level, so as to promote the prosperity and development of the industry. At the same time, the price sensitivity degree of cascade utilization market has a significant negative impact on the members of the supply chain. The administrative supervision department should actively improve the industry regulations and strictly supervise the whole process of power battery production, recycling, and reuse, and at the same time, it can also enhance consumers’ green awareness by carrying out relevant publicity work, supply chain members should also jointly build and improve the battery recycling platform and network system, formulate relevant industry standards and other measures to deepen customer trust, make joint efforts to effectively resist the interference of internal and external uncertain factors, reduce the potential risks of the industry, and ensure the long-term and steady development of the battery recycling industry. In addition, the increase in the cost-saving of manufacturer’s remanufactured products can effectively improve the enthusiasm of manufacturers for recycling resources, reflect the importance of technology research and development and innovation, drive the development of the industry with innovation, and promote the effective improvement of resource recycling efficiency.

All parties should strengthen information exchange and cooperation, jointly build and share, link interests, achieve mutual benefit and win-win results, give full play to the effectiveness of the internal coordination mechanism of the supply chain, promote the coordinated development of the supply chain, effectively resist the interference of internal and external uncertain factors, and jointly undertake the responsibility of improving the level of industrial resource recycling. The contribution of this paper is to study and analyze the dynamic equilibrium strategy of the members of the power battery CLSC from the dynamic perspective, in view of the influence of uncertain factors and the cost-sharing coordination mechanism. At the theoretical level, it provides theoretical reference for the behavior decision-making of related enterprises, enriches, and expands the theoretical research in related fields. At the practical level, it provides ideas for improving and innovating the operation mechanism of the power battery CLSC. The research results have important guiding significance for relevant enterprises to fully and efficiently use the cascade, improve the level of resource utilization, and reduce environmental pollution.

However, this paper only considers the business strategy selection of enterprises in a single channel, and does not consider the impact of online and offline dual-channel recycling and sales, competitive cooperation game and other situations. The next research direction is to establish a dynamic cooperative game model considering factors such as consumers and different recycling and sales channels, and to study the influence of multi-channel sales and recycling of cascade products on the cascade utilization of power batteries.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare no conflicts of interest.

Acknowledgments

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