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Retraction

Retracted: Method for Fuzzy Number Intuitionistic Fuzzy Multiple Attribute Decision Making and Its Application to Blended Classroom Teaching Reform Effect Evaluation

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This article has been retracted by Hindawi following an investigation undertaken by the publisher [1]. This investigation has uncovered evidence of one or more of the following indicators of systematic manipulation of the publication process:

(1) Discrepancies in scope
(2) Discrepancies in the description of the research reported
(3) Discrepancies between the availability of data and the research described
(4) Inappropriate citations
(5) Incoherent, meaningless and/or irrelevant content included in the article
(6) Peer-review manipulation

The presence of these indicators undermines our confidence in the integrity of the article's content and we cannot, therefore, vouch for its reliability. Please note that this notice is intended solely to alert readers that the content of this article is unreliable. We have not investigated whether authors were aware of or involved in the systematic manipulation of the publication process.

Wiley and Hindawi regrets that the usual quality checks did not identify these issues before publication and have since put additional measures in place to safeguard research integrity.

We wish to credit our own Research Integrity and Research Publishing teams and anonymous and named external researchers and research integrity experts for contributing to this investigation.

The corresponding author, as the representative of all authors, has been given the opportunity to register their agreement or disagreement to this retraction. We have kept a record of any response received.

References

[1] X. Sun, "Method for Fuzzy Number Intuitionistic Fuzzy Multiple Attribute Decision Making and Its Application to Blended Classroom Teaching Reform Effect Evaluation," *Mathematical Problems in Engineering*, vol. 2022, Article ID 3907871, 14 pages, 2022.

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Research Article

Method for Fuzzy Number Intuitionistic Fuzzy Multiple Attribute Decision Making and Its Application to Blended Classroom Teaching Reform Effect Evaluation

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With the continuous development of mobile Internet and educational technology, the deep integration of education and Internet has a great impact on the education concept and teaching mode. The mixed teaching based on Rain Classroom is the specific application of new educational technology in the teaching of Internet plus education. Most of the students in ethnic universities come from the central and western ethnic regions. In addition, the evaluation of blended classroom teaching reform effect is looked as the classical multiple attribute decision making (MADM). We extend the dual MSM (DMSM) operator with fuzzy number intuitionistic fuzzy numbers (FNIFNs) to build the fuzzy number intuitionistic fuzzy dual MSM (FNIFDMSM) operator and fuzzy number intuitionistic fuzzy weighted DMSM (FNIFWDMSM) equation in this study. A few MADM operators are built with FNIFWDMSM operator. Finally, taking blended classroom teaching reform effect evaluation as an useful example, this paper illustrates the depicted approach.

1. Introduction

In 1965, Zadeh [1] proposed the fuzzy sets (FSs) to fuse information in the fuzzy information domain [2–6]. To extend the FSs, the intuitionistic fuzzy sets (IFSs) [7] were also proposed. Subsequently, FS and its corresponding extension are investigated in more and more decision-making analysis domains [8–15]. At the same time, more and more methods are built to solve the different MADM issues [16–22] and MAGDM issues [23–28]. Su et al. [29] proposed the interactive dynamic IF-MAGDM. Arya and Yadav [30] proposed the intuitionistic fuzzy super-efficiency measure. Tian et al. [31] defined the partial derivative and complete differential of binary intuitionistic fuzzy functions. Garg [32] proposed the cosine similarity measure under given IFSs. Tan [33] proposed the Choquet-TOPSIS tools for IF-MADM. Zhao et al. [34] defined the Interactive intuitionistic fuzzy methods for multilevel programming problems. Li [35] proposed the GOWA operators to MADM under IFSs. Buyukozkan et al. [36] selected the transportation decision schemes with the intuitionistic fuzzy Choquet method. Joshi et al. [37] proposed the given dissimilarity measure along with IFSs. De and Sana [38] solved the random demand models along with Bonferroni operator under IFSs. Li et al. [39] devised the given VIKOR-based dynamic IF-MADM. Niroomand [40] solved the multiobjective linear programming with IFSs. Yu et al. [41] defined the derivatives and differentials for multiplicative IFSs. Yu [42] proposed the prioritized geometric means under IFSs. Wu and Zhang [43] solved the IF-MADM with weighted entropy. Verma and Sharma [44] built the inaccuracy measure for IF-MADM. Furthermore, Liu and Yuan [45] proposed the fuzzy number IFSs (FNIFSs) along with the IFSs. The membership and non-membership of IFSs are [0, 1], and the sum of membership and non-membership of IFSs is less than one. Different from the IFSs, the membership and non-membership of FNIFSs are triangular fuzzy sets (TFSs). Thus, The FNIFSs could well depict the uncertainties and fuzziness during the real-life decision issues. Fan [46] built the FNIFHPWG function to make evaluation about knowledge innovation ability. Wang and Wang [47] defined the FNIFHCG operator under FNIFSs. Chen and Wang [48] defined the IFNIFHOWA operator for project performance evaluation. Wang and Yu [49] defined the FNIFHCA function to evaluate the rural landscape design projects. Lu [50] built the IFNIFHCG operator for international competitiveness assessment. Wang [51] defined some useful operational laws along with FNIFSs based on the arithmetic operators. Zhao et al. [52] defined the FNIFHPWA function for appraising the software performance. Li et al. [53] built the information entropy and similarity measure with FNIFSs. Wang [54] proposed the geometric means under FNIFSs. Li [55] expanded the generalized Maclaurin symmetric mean (GMSM) equation under FNIFNs to establish the fuzzy number intuitionistic fuzzy GMSM (FNIFGMSM) equation and fuzzy number intuitionistic fuzzy weighted GMSM (FNIFWGMSM) equation.

Nevertheless, all the given functions and useful tools proposed by these scholars do not take into account the relationship between given parameters [56–60]. To conquer these given shortcomings, the crucial purpose of such given article is to combine the FNIFSs with DMSM means [61] to propose several novel fused tools under FNIFSs. The motivations of the paper can be summarized as follows: (1) the DMSM formula is utilized to build several DMSM fused formulas with FNIFNs: FNIFDMSM operator and FNIFWDMSM operator; (2) the FNIFDMSM operator and FNIFWDMSM operator method is proposed to solve the MADM problems with FNIFNs; (3) a case study for blended classroom teaching reform effect evaluation is supplied to show the developed approach; and (4) some comparative studies are provided with the existing methods. The rest of the paper is organized as follows. Several basic concepts of FNIFSs and DMSM means are given in Section 2. The DMSM means with FNIFSs are given in Section 3. An instance about blended classroom teaching reform effect evaluation is given in Section 4. The conclusions are drawn in Section 5.

2. Preliminaries

In this section, we introduce the definition of FNIFSs [32] and the DMSM operator.

2.1. FNIFSs. Liu and Yuan [45] built the FNIFS, and the membership and non-membership are depicted under TFNs.

Definition 1 (see [45]). Suppose $E = \{e_1, e_2, \ldots, e_n\}$ is a given fixed set and B is a given FNIFS on E, and its mathematical expression is

$$B = \in \{\langle e, T_B(e), F_B(e)\rangle e \in E\}. \tag{1}$$

$T_B(e)$, $F_B(e)$ are two given TFNs between 0 and 1, and $T_B(e) = (X(e), Y(e), Z(e)), e \longrightarrow [0,1]$, $F_B(e) = (A(e), S(e), D(e)), e \longrightarrow [0,1]$, $0 \leq Z(e) + D(e) \leq 1$, $\forall e \in E$. Let $T_B(e) = (X(e), Y(e), Z(e))$, $F_B(e) = (A(e), S(e), D(e))$, so $Q(e) = \langle (X(e), Y(e), Z(e)), (A(e), S(e), D(e))\rangle$, and $Q(e)$ is viewed as a given FNIFN.

Definition 2 (see [51, 54]). $Q(e_i) = \langle (X(e_i), Y(e_i), Z(e_i)), (A(e_i), S(e_i), D(e_i))\rangle$ and $Q(e_j) = \langle (X(e_j), Y(e_j), Z(e_j)), (A(e_j), S(e_j), D(e_j))\rangle$ are two given FNIFNs; consequently,

(1) $Q(e_i) \oplus Q(e_j) = \left\{\begin{pmatrix} X(e_i) + X(e_j) - X(e_i)X(e_j) \\ Y(e_i) + Y(e_j) - Y(e_i)Y(e_j) \\ Z(e_i) + Z(e_j) - Z(e_i)Z(e_j) \end{pmatrix}, \begin{pmatrix} A(e_i)A(e_j) \\ S(e_i)S(e_j) \\ D(e_i)D(e_j) \end{pmatrix}\right\}$.

(2) $Q(e_i) \otimes Q(e_j) = \left\{\left(\begin{pmatrix} X(e_i)X(e_j) \\ Y(e_i)Y(e_j) \\ Z(e_i)Z(e_j) \end{pmatrix}\right), \left(\begin{pmatrix} A(e_i) + A(e_j) - A(e_i)A(e_j) \\ S(e_i) + S(e_j) - S(e_i)S(e_j) \\ D(e_i) + D(e_j) - D(e_i)D(e_j) \end{pmatrix}\right)\right\}$.

(3) $\lambda Q(e_i) = \left\{\begin{pmatrix} 1 - (1 - X(e_i))^\lambda \\ 1 - (1 - Y(e_i))^\lambda \\ 1 - (1 - Z(e_i))^\lambda \end{pmatrix}, \begin{pmatrix} (A(e_i))^\lambda \\ (S(e_i))^\lambda \\ (D(e_i))^\lambda \end{pmatrix}\right\}, \lambda \geq 0.$

(4) $(Q(e_i))^\lambda = \left\{\begin{pmatrix} (X(e_i))^\lambda \\ (Y(e_i))^\lambda \\ (Z(e_i))^\lambda \end{pmatrix}, \begin{pmatrix} 1 - (1 - A(e_i))^\lambda \\ 1 - (1 - S(e_i))^\lambda \\ 1 - (1 - D(e_i))^\lambda \end{pmatrix}\right\}, \lambda \geq 0.$

Definition 3 (see [51, 54]). Let $Q(e) = \langle (X(e), Y(e), Z(e)), (A(e), S(e), D(e))\rangle$ be a given FNIFN, and the score function of $Q(e)$ is depicted as

$$SF(Q(e)) = \frac{X(e) + 2Y(e) + Z(e)}{4} - \frac{A(e) + 2S(e) + D(e)}{4}, \quad SF(Q(e)) \in [-1, 1]. \tag{2}$$

Definition 4 (see [51, 54]). Let $Q(e) = \langle (X(e), Y(e), Z(e)), (A(e), S(e), D(e))\rangle$ be the given FNIFN, and an accuracy function of $Q(e)$ is defined as

$$AH(Q(e)) = \frac{X(e) + 2Y(e) + Z(e)}{4} + \frac{A(e) + 2S(e) + D(e)}{4}, \quad AH(Q(e)) \in [0, 1]. \tag{3}$$

Based on $SF(Q(e))$ and $AH(Q(e))$, let us look at the size comparison of the two FNIFNs.

Definition 5 (see [51, 54]). Let $Q(e_1)$ and $Q(e_2)$ be two FNIFNs; then, if $SF(Q(e_1)) < SF(Q(e_2))$, then $Q(e_1) < Q(e_2)$; if $SF(Q(e_1)) = SF(Q(e_2))$, then

(1) If $AH(Q(e_1)) = AH(Q(e_2))$, then $Q(e_1) = Q(e_2)$.

(2) If $AH(Q(e_1)) < AH(Q(e_2))$, then $Q(e_1) < Q(e_2)$.

2.2. DMSM Operators. Qin and Liu [61] proposed the dual MSM (DMSM) operator considering both the MSM and the dual operation.

Definition 6. (see [61]). Let $a_i\,(i = 1, 2, \ldots, n)$ be a couple of non-negative real numbers, and $\lambda_1, \lambda_2, \ldots, \lambda_k \geq 0$. If

$$DMSM^{(k)}(x_1, x_2, \ldots, x_n) = \frac{1}{k}\left(\prod_{1 \leq i_1 < \ldots < i_k \leq i_n}\left(\sum_{j=1}^{k} x_{i_j}\right)^{(1/C_n^k)}\right), \tag{4}$$

then we name $DM\,SM^{(k)}$ as dual MSM (DMSM) operator, where $(i_1, i_2, \ldots, i_k)$ changes all the k-tuple information combinations of $(1, 2, \ldots, n)$ and C_n^k is the mathematical binomial coefficient value.

Definition 7. Let $Q(e_m) = \langle (X(e_m), Y(e_m), Z(e_m)), (A(e_m), S(e_m), D(e_m))\rangle, m = 1, 2, \ldots k$, be a couple of given FNIFNs. The FNIFGMSM operator could be defined as

3. FNIFDMSM and FNIFWDMSM Operators

3.1. The FNIFDMSM Operator. The DMSM is built to coalesce all given FNIFNs, and the fuzzy number intuitionistic fuzzy DMSM (FNIFDMSM) operators are built.

$$FNIFDMSM^{(k)}(Q(e_1), Q(e_2), \ldots, Q(e_n)) = \frac{1}{k}\left(\underset{1 \leq i_1 < \ldots < i_k \leq i_n}{\otimes}\left(\bigoplus_{j=1}^{n} Q\left(e_{i_j}\right)\right)^{(1/C_n^k)}\right). \tag{5}$$

Theorem 1. *Let $Q(e_m) = \langle (X(e_m), Y(e_m), Z(e_m)), (A(e_m), S(e_m), D(e_m))\rangle, m = 1, 2, \ldots k$, be a set of FNIFNs. The obtained value from FNIFDMSM is still an FNIFN.*

$$\begin{aligned}
&FNIFDMSM^{(k)}(Q(e_1), Q(e_2), \ldots, Q(e_n))\\
&= \frac{1}{k}\left(\underset{1 \leq i_1 < \cdots < i_k \leq i_n}{\otimes}\left(\bigoplus_{j=1}^{n} Q\left(e_{i_j}\right)\right)^{(1/C_n^k)}\right),\\
&= \left\{\begin{pmatrix} 1-\left(1-\prod_{1 \leq i_1 < \cdots < i_k \leq i_n}\left(1-\prod_{j=1}^{k}\left(1-X\left(e_{i_j}\right)\right)\right)^{(1/C_n^k)}\right)^{(1/k)},\\ 1-\left(1-\prod_{1 \leq i_1 < \cdots < i_k \leq i_n}\left(1-\prod_{j=1}^{k}\left(1-Y\left(e_{i_j}\right)\right)\right)^{(1/C_n^k)}\right)^{(1/k)},\\ 1-\left(1-\prod_{1 \leq i_1 < \ldots < i_k \leq i_n}\left(1-\prod_{j=1}^{k}\left(1-Z\left(e_{i_j}\right)\right)\right)^{(1/C_n^k)}\right)^{(1/k)} \end{pmatrix}, \begin{pmatrix} \left(1-\prod_{1 \leq i_1 < \cdots < i_k \leq i_n}\left(1-\prod_{j=1}^{k}\left(A\left(e_{i_j}\right)\right)\right)^{(1/C_n^k)}\right)^{(1/k)},\\ \left(1-\prod_{1 \leq i_1 < \cdots < i_k \leq i_n}\left(1-\prod_{j=1}^{k}\left(S\left(e_{i_j}\right)\right)\right)^{(1/C_n^k)}\right)^{(1/k)},\\ \left(1-\prod_{1 \leq i_1 < \cdots < i_k \leq i_n}\left(1-\prod_{j=1}^{k}\left(D\left(e_{i_j}\right)\right)\right)^{(1/C_n^k)}\right)^{(1/k)} \end{pmatrix}\right\}.
\end{aligned} \tag{6}$$

Proof. From Definition 2, we can derive

$$\bigoplus_{j=1}^{n} Q\left(e_{i_j}\right) = \left\{\begin{matrix} \left(1-\prod_{j=1}^{k}\left(1-X\left(e_{i_j}\right)\right), 1-\prod_{j=1}^{k}\left(1-Y\left(e_{i_j}\right)\right), 1-\prod_{j=1}^{k}\left(1-Z\left(e_{i_j}\right)\right)\right),\\ \left(\prod_{j=1}^{k}\left(A\left(e_{i_j}\right)\right), \prod_{j=1}^{k}\left(S\left(e_{i_j}\right)\right), \prod_{j=1}^{k}\left(D\left(e_{i_j}\right)\right)\right) \end{matrix}\right\}. \tag{7}$$

Thus,

$$\left(\bigoplus_{j=1}^{n} Q\left(e_{i_j}\right)\right)^{\left(1/C_n^k\right)} = \left\{ \begin{array}{l} \left(\left(1-\prod_{j=1}^{k}\left(1-X\left(e_{i_j}\right)\right)\right)^{\left(1/C_n^k\right)}, \left(1-\prod_{j=1}^{k}\left(1-Y\left(e_{i_j}\right)\right)\right)^{\left(1/C_n^k\right)}, \left(1-\prod_{j=1}^{k}\left(1-Z\left(e_{i_j}\right)\right)\right)^{\left(1/C_n^k\right)}\right) \\ \left(1-\left(1-\prod_{j=1}^{k}\left(A\left(e_{i_j}\right)\right)\right)^{\left(1/C_n^k\right)}, 1-\left(1-\prod_{j=1}^{k}\left(S\left(e_{i_j}\right)\right)\right)^{\left(1/C_n^k\right)}, 1-\left(1-\prod_{j=1}^{k}\left(D\left(e_{i_j}\right)\right)\right)^{\left(1/C_n^k\right)}\right) \end{array} \right\}. \tag{8}$$

Thereafter,

$$\bigotimes_{1\le i_1<\cdots<i_k\le i_n}\left(\bigoplus_{j=1}^{n} Q\left(e_{i_j}\right)\right)^{\left(1/C_n^k\right)} = \left\{ \left(\begin{array}{l} \prod_{1\le i_1<\cdots<i_k\le i_n}\left(1-\prod_{j=1}^{k}\left(1-X\left(e_{i_j}\right)\right)\right)^{\left(1/C_n^k\right)} \\ \prod_{1\le i_1<\cdots<i_k\le i_n}\left(1-\prod_{j=1}^{k}\left(1-Y\left(e_{i_j}\right)\right)\right)^{\left(1/C_n^k\right)} \\ \prod_{1\le i_1<\cdots<i_k\le i_n}\left(1-\prod_{j=1}^{k}\left(1-Z\left(e_{i_j}\right)\right)\right)^{\left(1/C_n^k\right)} \end{array} \right), \left(\begin{array}{l} 1-\prod_{1\le i_1<\cdots<i_k\le i_n}\left(1-\prod_{j=1}^{k}\left(A\left(e_{i_j}\right)\right)\right)^{\left(1/C_n^k\right)} \\ 1-\prod_{1\le i_1<\cdots<i_k\le i_n}\left(1-\prod_{j=1}^{k}\left(S\left(e_{i_j}\right)\right)\right)^{\left(1/C_n^k\right)} \\ 1-\prod_{1\le i_1<\cdots<i_k\le i_n}\left(1-\prod_{j=1}^{k}\left(D\left(e_{i_j}\right)\right)\right)^{\left(1/C_n^k\right)} \end{array} \right) \right\}. \tag{9}$$

Furthermore,

$$FNIFDMSM^{(k)}\left(Q\left(e_1\right), Q\left(e_2\right), \ldots, Q\left(e_n\right)\right) = \frac{1}{k}\left(\bigotimes_{1\le i_1<\cdots<i_k\le i_n}\left(\bigoplus_{j=1}^{n} Q\left(e_{i_j}\right)\right)^{\left(1/C_n^k\right)}\right)$$

$$= \left\{ \left(\begin{array}{l} 1-\left(1-\prod_{1\le i_1<\cdots<i_k\le i_n}\left(1-\prod_{j=1}^{k}\left(1-X\left(e_{i_j}\right)\right)\right)^{\left(1/C_n^k\right)}\right)^{(1/k)}, \\ 1-\left(1-\prod_{1\le i_1<\cdots<i_k\le i_n}\left(1-\prod_{j=1}^{k}\left(1-Y\left(e_{i_j}\right)\right)\right)^{\left(1/C_n^k\right)}\right)^{(1/k)}, \\ 1-\left(1-\prod_{1\le i_1<\cdots<i_k\le i_n}\left(1-\prod_{j=1}^{k}\left(1-Z\left(e_{i_j}\right)\right)\right)^{\left(1/C_n^k\right)}\right)^{(1/k)} \end{array} \right), \left(\begin{array}{l} \left(1-\prod_{1\le i_1<\cdots<i_k\le i_n}\left(1-\prod_{j=1}^{k}\left(A\left(e_{i_j}\right)\right)\right)^{\left(1/C_n^k\right)}\right)^{(1/k)}, \\ \left(1-\prod_{1\le i_1<\cdots<i_k\le i_n}\left(1-\prod_{j=1}^{k}\left(S\left(e_{i_j}\right)\right)\right)^{\left(1/C_n^k\right)}\right)^{(1/k)}, \\ \left(1-\prod_{1\le i_1<\cdots<i_k\le i_n}\left(1-\prod_{j=1}^{k}\left(D\left(e_{i_j}\right)\right)\right)^{\left(1/C_n^k\right)}\right)^{(1/k)} \end{array} \right) \right\} \tag{10}$$

Hence, (6) is kept.

Then, we can prove that (11) is an FNIFN. We shall check the following two conditions:

(i) $(X(e), Y(e), Z(e)) \subseteq [0,1]$, $(A(e), S(e), D(e)) \subseteq [0,1]$.

(ii) $0 \le Z(e) + D(e) \le 1$. □

Proof

(i) Since $0 \leq X(e_{i_j}) \leq 1$, we get

$$0 \leq \prod_{j=1}^{k}\left(1 - X\left(e_{i_j}\right)\right) \leq 1 \text{ and } 0 \leq 1 - \prod_{j=1}^{k}\left(1 - X\left(e_{i_j}\right)\right) \leq 1. \tag{11}$$

Then,

$$0 \leq 1 - \prod_{1 \leq i_1 < \cdots < i_k \leq i_n}\left(1 - \prod_{j=1}^{k}\left(1 - X\left(e_{i_j}\right)\right)\right)^{\left(1/C_n^k\right)} \leq 1. \tag{12}$$

Thus,

$$0 \leq 1 - \left(1 - \prod_{1 \leq i_1 < \cdots < i_k \leq i_n}\left(1 - \prod_{j=1}^{k}\left(1 - X\left(e_{i_j}\right)\right)\right)^{\left(1/C_n^k\right)}\right)^{(1/k)} \leq 1. \tag{13}$$

That means $X(e) = 1 - (1 - \prod_{1 \leq i_1 < \cdots < i_k \leq i_n}(1 - \prod_{j=1}^{k}(1 - X(e_{i_j})))^{(1/C_n^k)})^{(1/k)} \subseteq [0,1]$. Similarly, we can get $(Y(e), Z(e)) \subseteq [0,1]$, and $(A(e), S(e), D(e)) \subseteq [0,1]$, so (1) is kept.

(2) For $Z(e_{i_j}) + D(e_{i_j}) \leq 1$, then we can have $D(e_{i_j}) \leq 1 - Z(e_{i_j})$, and thus

$$\begin{aligned}
0 &\leq Z(e) + D(e) \\
&= 1 - \left(1 - \prod_{\substack{1 \leq i_1 < \ldots \\ < i_k \leq i_n}}\left(1 - \prod_{j=1}^{k}\left(1 - Z\left(e_{i_j}\right)\right)\right)^{\left(1/C_n^k\right)}\right)^{(1/k)} + \left(1 - \prod_{\substack{1 \leq i_1 < \ldots \\ < i_k \leq i_n}}\left(1 - \prod_{j=1}^{k}\left(D\left(e_{i_j}\right)\right)\right)^{\left(1/C_n^k\right)}\right)^{(1/k)} \\
&\leq 1 - \left(1 - \prod_{\substack{1 \leq i_1 < \ldots \\ < i_k \leq i_n}}\left(1 - \prod_{j=1}^{k}\left(1 - Z\left(e_{i_j}\right)\right)\right)^{\left(1/C_n^k\right)}\right)^{(1/k)} + \left(1 - \prod_{\substack{1 \leq i_1 < \ldots \\ < i_k \leq i_n}}\left(1 - \prod_{j=1}^{k}\left(1 - Z\left(e_{i_j}\right)\right)\right)^{\left(1/C_n^k\right)}\right)^{(1/k)} \\
&= 1.
\end{aligned} \tag{14}$$

□

Example 1. Let $Q(e_1) = \langle(0.1, 0.2, 0.3), (0.2, 0.5, 0.6)\rangle$, $Q(e_2) = \langle(0.2, 0.3, 0.3), (0.2, 0.5, 0.5)\rangle$, and $Q(e_3) = \langle(0.4, 0.5, 0.5), (0.3, 0.4, 0.4)\rangle$ be three FNIFNs, and suppose $k = 2$; then, according to (10), we derive

$$\begin{aligned}
&FNIFDMSM^{(k)}(Q(e_1), Q(e_2), \ldots, Q(e_n)) = \frac{1}{k}\left(\underset{1 \leq i_1 < \cdots < i_k \leq i_n}{\otimes}\left(\bigoplus_{j=1}^{n} Q\left(e_{i_j}\right)\right)^{\left(1/C_n^k\right)}\right) \\
&= \left\{\begin{array}{c}
\left(\begin{array}{l}
1 - \left(1 - ((1 - (1 - 0.1) \times (1 - 0.2)) \times (1 - (1 - 0.1) \times (1 - 0.4)) \times (1 - (1 - 0.2) \times (1 - 0.4)))^{\left(1/C_3^2\right)}\right)^{(1/2)} \\
1 - \left(1 - ((1 - (1 - 0.2) \times (1 - 0.3)) \times (1 - (1 - 0.2) \times (1 - 0.5)) \times (1 - (1 - 0.3) \times (1 - 0.5)))^{\left(1/C_3^2\right)}\right)^{(1/2)} \\
1 - \left(1 - ((1 - (1 - 0.3) \times (1 - 0.3)) \times (1 - (1 - 0.3) \times (1 - 0.5)) \times (1 - (1 - 0.3) \times (1 - 0.5)))^{\left(1/C_3^2\right)}\right)^{(1/2)}
\end{array}\right), \\
\left(\begin{array}{l}
\left(1 - ((1 - 0.2 \times 0.2) \times (1 - 0.2 \times 0.3) \times (1 - 0.2 \times 0.3))^{\left(1/C_3^2\right)}\right)^{(1/2)} \\
\left(1 - ((1 - 0.5 \times 0.5) \times (1 - 0.5 \times 0.4) \times (1 - 0.5 \times 0.4))^{\left(1/C_3^2\right)}\right)^{(1/2)} \\
\left(1 - ((1 - 0.6 \times 0.5) \times (1 - 0.6 \times 0.4) \times (1 - 0.5 \times 0.4))^{\left(1/C_3^2\right)}\right)^{(1/2)}
\end{array}\right)
\end{array}\right\} \\
&= \langle(0.2294, 0.3335, 0.3672), (0.2311, 0.4659, 0.4978)\rangle.
\end{aligned} \tag{15}$$

Then, we discuss some better properties for FNIFDMSM.

Property 1 (idempotency). If $Q(e_i) = \langle (X(e_i), Y(e_i), Z(e_i)), (A(e_i), S(e_i), D(e_i))\rangle$ $(i = 1, 2, \ldots, n)$ are same, then

$$FNIFDMSM^{(k)}(Q(e_1), Q(e_2), \ldots, Q(e_n)) = Q(e). \quad (16)$$

Property 2 (monotonicity). Let $Q(e_i) = \langle (X(e_i), Y(e_i), Z(e_i)), (A(e_i), S(e_i), D(e_i))\rangle$ $(i = 1, 2, \ldots, n)$ and $Q(e_j) = \langle (X(e_j), Y(e_j), Z(e_j)), (A(e_j), S(e_j), D(e_j))\rangle$ $(j = 1, 2, \ldots, n)$ be two sets of FNIFNs. If $X(e_i) \le X(e_j), Y(e_i) \le Y(e_j), Z(e_i) \le Z(e_j), A(e_i) \ge A(e_j), S(e_i) \ge S(e_j), D(e_i) \ge D(e_j)$ hold for all i, j, then

$$\begin{aligned} &FNIFDMSM^{(k)}\left(Q\left(e_{i_1}\right), Q\left(e_{i_2}\right), \ldots, Q\left(e_{i_n}\right)\right) \\ &\quad \le FNIFDMSM^{(k)}\left(Q\left(e_{j_1}\right), Q\left(e_{j_2}\right), \ldots, Q\left(e_{j_n}\right)\right). \end{aligned} \quad (17)$$

Property 3 (boundedness). Let $Q(e_i) = \langle (X(e_i), Y(e_i), Z(e_i)), (A(e_i), S(e_i), D(e_i))\rangle$ $(i = 1, 2, \ldots, n)$ be a group of FNIFNs. If $Q(e)^+ = \left\{\begin{matrix}(\max(X(e_i)), \max(Y(e_i)), \max(Z(e_i))), \\ (\min(A(e_i)), \min(S(e_i)), \min(D(e_i)))\end{matrix}\right\}$ $(i = 1, 2, \ldots, n)$ and $Q(e)^- = \left\{\begin{matrix}(\min(X(e_i)), \min(Y(e_i)), \min(Z(e_i))), \\ (\max(A(e_i)),\end{matrix}\right.$ $\max(S(e_i)), \max(D(e_i)))\}$ $(i = 1, 2, \ldots, n)$, then

$$Q(e)^- \le FNIFDMSM^{(k)}(Q(e_1), Q(e_2), \ldots, Q(e_n)) \le Q(e)^+. \quad (18)$$

Property 4 (commutativity). Let $Q(e_i)$ $(i = 1, 2, \ldots, n)$ be a set of given FNIFNs, and $Q(e_i')$ $(i = 1, 2, \ldots, n)$ is any permutation of $Q(e_i)$ $(i = 1, 2, \ldots, n)$; then,

$$\begin{aligned} &FNIFDMSM^{(k)}(Q(e_1), Q(e_2), \ldots, Q(e_n)) \\ &\quad = FNIFDMSM^{(k)}(Q(e_1'), Q(e_2'), \ldots, Q(e_n')). \end{aligned} \quad (19)$$

3.2. The FNIFWDMSM Operator. In real-life MADM, it is very crucial to take weights into account. We shall solve the fuzzy number intuitionistic fuzzy weighted DMSM (FNIFWDMSM) mean.

Definition 8. Let $Q(e_m) = \langle (X(e_m), Y(e_m), Z(e_m)), (A(e_m), S(e_m), D(e_m))\rangle$ $(m = 1, 2, \ldots, k)$ be a couple of built FNIFNs with given weight values $\xi_m = (\xi_1, \xi_2, \ldots, \xi_k)^T$, and $\xi_m \in [0, 1]$, $\sum_{m=1}^{k} \xi_m = 1$. If

$$\begin{aligned} &FNIFWDMSM_{nw}^{(k)}(Q(e_1), Q(e_2), \ldots, Q(e_n)) \\ &\quad = \frac{1}{k}\left(\underset{1 \le i_1 < \cdots < i_k \le i_n}{\otimes}\left(\bigoplus_{j=1}^{k}\left(n\xi_{i_j} \otimes Q\left(e_{i_j}\right)\right)\right)^{(1/C_n^k)}\right), \end{aligned} \quad (20)$$

then we call $FNIFWDMSM_{k\xi}^{(k,\lambda_1,\lambda_2,\ldots,\lambda_k)}$ the fuzzy number intuitionistic fuzzy weighted DMSM (FNIFWDMSM) formula.

Theorem 2. *Let $Q(e_m) = \langle (X(e_m), Y(e_m), Z(e_m)), (A(e_m), S(e_m), D(e_m))\rangle$ $(i = 1, 2, \ldots, n)$ be a couple of built FNIFNs. The obtained value from FNIFWDMSM formula is still a FNIFN.*

$$\begin{aligned} &\mathrm{FNIFWDMSM}_{n\xi}^{(k)}(Q(e_1), Q(e_2), \ldots, Q(e_n)) = \frac{1}{k}\left(\underset{1 \le i_1 < \cdots < i_k \le i_n}{\otimes}\left(\bigoplus_{j=1}^{k}\left(n\xi_{i_j} \otimes Q\left(e_{i_j}\right)\right)\right)^{(1/C_n^k)}\right) \\ &= \left\{\begin{pmatrix} 1 - \left(1 - \prod_{1 \le i_1 < \cdots < i_k \le i_n}\left(1 - \prod_{j=1}^{k}\left(1 - X\left(e_{i_j}\right)\right)^{n\xi_{i_j}}\right)^{(1/C_n^k)}\right)^{(1/k)} \\ 1 - \left(1 - \prod_{1 \le i_1 < \cdots < i_k \le i_n}\left(1 - \prod_{j=1}^{k}\left(1 - Y\left(e_{i_j}\right)\right)^{n\xi_{i_j}}\right)^{(1/C_n^k)}\right)^{(1/k)} \\ 1 - \left(1 - \prod_{1 \le i_1 < \cdots < i_k \le i_n}\left(1 - \prod_{j=1}^{k}\left(1 - Z\left(e_{i_j}\right)\right)^{n\xi_{i_j}}\right)^{(1/C_n^k)}\right)^{(1/k)} \end{pmatrix}, \begin{pmatrix} \left(1 - \prod_{1 \le i_1 < \cdots < i_k \le i_n}\left(1 - \prod_{j=1}^{k}\left(A\left(e_{i_j}\right)\right)^{n\xi_{i_j}}\right)^{(1/C_n^k)}\right)^{(1/k)} \\ \left(1 - \prod_{1 \le i_1 < \cdots < i_k \le i_n}\left(1 - \prod_{j=1}^{k}\left(S\left(e_{i_j}\right)\right)^{n\xi_{i_j}}\right)^{\frac{1}{C_n^k}}\right)^{(1/k)} \\ \left(1 - \prod_{1 \le i_1 < \cdots < i_k \le i_n}\left(1 - \prod_{j=1}^{k}\left(D\left(e_{i_j}\right)\right)^{n\xi_{i_j}}\right)^{(1/C_n^k)}\right)^{(1/k)} \end{pmatrix}\right\}. \end{aligned} \quad (21)$$

Proof. With Definition 2, we can derive

$$n\xi_{i_j} \otimes Q\left(e_{i_j}\right) = \left\{ \begin{array}{c} \left(1-\left(1-X\left(e_{i_j}\right)\right)^{n\xi_{i_j}}, 1-\left(1-Y\left(e_{i_j}\right)\right)^{n\xi_{i_j}}, 1-\left(1-Z\left(e_{i_j}\right)\right)^{n\xi_{i_j}}\right), \\ \left(\left(A\left(e_{i_j}\right)\right)^{n\xi_{i_j}}, \left(S\left(e_{i_j}\right)\right)^{n\xi_{i_j}}, \left(D\left(e_{i_j}\right)\right)^{n\xi_{i_j}}\right) \end{array} \right\}. \tag{22}$$

Thus,

$$\begin{aligned} &\bigoplus_{j=1}^{k}\left(n\xi_{i_j} \otimes Q\left(e_{i_j}\right)\right) \\ &= \left\{ \begin{array}{c} \left(1-\prod_{j=1}^{k}\left(1-X\left(x_{i_j}\right)\right)^{n\xi_{i_j}}, 1-\prod_{j=1}^{k}\left(1-Y\left(x_{i_j}\right)\right)^{n\xi_{i_j}}, 1-\prod_{j=1}^{k}\left(1-Z\left(x_{i_j}\right)\right)^{n\xi_{i_j}}\right), \\ \left(\prod_{j=1}^{k}\left(A\left(e_{i_j}\right)\right)^{n\xi_{i_j}}, \prod_{j=1}^{k}\left(S\left(e_{i_j}\right)\right)^{n\xi_{i_j}}, \prod_{j=1}^{k}\left(D\left(e_{i_j}\right)\right)^{n\xi_{i_j}}\right) \end{array} \right\}. \end{aligned} \tag{23}$$

Thereafter,

$$\begin{aligned} &\left(\bigoplus_{j=1}^{k}\left(n\xi_{i_j} \otimes Q\left(e_{i_j}\right)\right)\right)^{\left(1/C_n^k\right)} \\ &= \left\{ \begin{array}{c} \left(\left(1-\prod_{j=1}^{k}\left(1-X\left(e_{i_j}\right)\right)^{n\xi_{i_j}}\right)^{\left(1/C_n^k\right)}, \left(1-\prod_{j=1}^{k}\left(1-Y\left(e_{i_j}\right)\right)^{n\xi_{i_j}}\right)^{\left(1/C_n^k\right)}, \left(1-\prod_{j=1}^{k}\left(1-Z\left(e_{i_j}\right)\right)^{n\xi_{i_j}}\right)^{\left(1/C_n^k\right)}\right) \\ \left(1-\left(1-\prod_{j=1}^{k}\left(A\left(e_{i_j}\right)\right)^{n\xi_{i_j}}\right)^{\left(1/C_n^k\right)}, 1-\left(1-\prod_{j=1}^{k}\left(S\left(e_{i_j}\right)\right)^{n\xi_{i_j}}\right)^{\left(1/C_n^k\right)}, 1-\left(1-\prod_{j=1}^{k}\left(D\left(e_{i_j}\right)\right)^{n\xi_{i_j}}\right)^{\left(1/C_n^k\right)}\right) \end{array} \right\}. \end{aligned} \tag{24}$$

Furthermore,

$$\begin{aligned} &\bigotimes_{1 \le i_1 < \ldots < i_k \le i_n}\left(\bigoplus_{j=1}^{k}\left(n\xi_{i_j} \otimes Q\left(e_{i_j}\right)\right)\right)^{\left(1/C_n^k\right)} \\ &= \left\{ \left(\begin{array}{c} \prod_{1 \le i_1 < \ldots < i_k \le i_n}\left(1-\prod_{j=1}^{k}\left(1-X\left(e_{i_j}\right)\right)^{n\xi_{i_j}}\right)^{\left(1/C_n^k\right)} \\ \prod_{1 \le i_1 < \ldots < i_k \le i_n}\left(1-\prod_{j=1}^{k}\left(1-Y\left(e_{i_j}\right)\right)^{n\xi_{i_j}}\right)^{\left(1/C_n^k\right)} \\ \prod_{1 \le i_1 < \ldots < i_k \le i_n}\left(1-\prod_{j=1}^{k}\left(1-Z\left(e_{i_j}\right)\right)^{n\xi_{i_j}}\right)^{\left(1/C_n^k\right)} \end{array} \right), \left(\begin{array}{c} 1-\prod_{1 \le i_1 < \ldots < i_k \le i_n}\left(1-\prod_{j=1}^{k}\left(A\left(e_{i_j}\right)\right)^{n\xi_{i_j}}\right)^{\left(1/C_n^k\right)} \\ 1-\prod_{1 \le i_1 < \ldots < i_k \le i_n}\left(1-\prod_{j=1}^{k}\left(S\left(e_{i_j}\right)\right)^{n\xi_{i_j}}\right)^{\left(1/C_n^k\right)} \\ 1-\prod_{1 \le i_1 < \ldots < i_k \le i_n}\left(1-\prod_{j=1}^{k}\left(D\left(e_{i_j}\right)\right)^{n\xi_{i_j}}\right)^{\left(1/C_n^k\right)} \end{array} \right) \right\}. \end{aligned} \tag{25}$$

Therefore,

$$FNIFWDMSM_{n\xi}^{(k)}\left(Q(e_1),Q(e_2),\ldots,Q(e_n)\right)=\frac{1}{k}\left(\underset{1\le i_1<\ldots<i_k\le i_n}{\otimes}\left(\bigoplus_{j=1}^{k}\left(n\xi_{i_j}\otimes Q\left(e_{i_j}\right)\right)\right)^{\left(1/C_n^k\right)}\right)$$

$$=\left\{\begin{pmatrix}1-\left(1-\prod_{\substack{1\le i_1<\ldots\\<i_k\le i_n}}\left(1-\prod_{j=1}^{k}\left(1-X\left(e_{i_j}\right)\right)^{n\xi_{i_j}}\right)^{\left(1/C_n^k\right)}\right)^{(1/k)}\\ 1-\left(1-\prod_{\substack{1\le i_1<\ldots\\<i_k\le i_n}}\left(1-\prod_{j=1}^{k}\left(1-Y\left(e_{i_j}\right)\right)^{n\xi_{i_j}}\right)^{\left(1/C_n^k\right)}\right)^{(1/k)}\\ 1-\left(1-\prod_{\substack{1\le i_1<\ldots\\<i_k\le i_n}}\left(1-\prod_{j=1}^{k}\left(1-Z\left(e_{i_j}\right)\right)^{n\xi_{i_j}}\right)^{\left(1/C_n^k\right)}\right)^{(1/k)}\end{pmatrix},\begin{pmatrix}\left(1-\prod_{\substack{1\le i_1<\ldots\\<i_k\le i_n}}\left(1-\prod_{j=1}^{k}\left(A\left(e_{i_j}\right)\right)^{n\xi_{i_j}}\right)^{\left(1/C_n^k\right)}\right)^{(1/k)}\\ \left(1-\prod_{\substack{1\le i_1<\ldots\\<i_k\le i_n}}\left(1-\prod_{j=1}^{k}\left(S\left(e_{i_j}\right)\right)^{n\xi_{i_j}}\right)^{\left(1/C_n^k\right)}\right)^{(1/k)}\\ \left(1-\prod_{\substack{1\le i_1<\ldots\\<i_k\le i_n}}\left(1-\prod_{j=1}^{k}\left(D\left(e_{i_j}\right)\right)^{n\xi_{i_j}}\right)^{\left(1/C_n^k\right)}\right)^{(1/k)}\end{pmatrix}\right\}. \tag{26}$$

Hence, (23) is kept.

Then, we shall prove that (23) is an FNIFN. We shall check the following two conditions:

(i) $(X(e),Y(e),Z(e))\subseteq[0,1]$, $(A(e),S(e),D(e))\subseteq[0,1]$.

(ii) $0\le Z(e)+D(e)\le 1$. □

Proof. (1) Since $0\le X(e_{i_j})\le 1$, we get

$$0\le\prod_{j=1}^{k}\left(1-X\left(e_{i_j}\right)\right)^{n\xi_{i_j}}\le 1\text{ and }0\le 1-\prod_{j=1}^{k}\left(1-X\left(e_{i_j}\right)\right)^{n\xi_{i_j}}\le 1. \tag{27}$$

Then,

$$0\le 1-\prod_{1\le i_1<\cdots<i_k\le i_n}\left(1-\prod_{j=1}^{k}\left(1-X\left(e_{i_j}\right)\right)^{n\xi_{i_j}}\right)^{\left(1/C_n^k\right)}\le 1. \tag{28}$$

Thus,

$$0\le 1-\left(1-\prod_{1\le i_1<\cdots<i_k\le i_n}\left(1-\prod_{j=1}^{k}\left(1-X\left(e_{i_j}\right)\right)^{n\xi_{i_j}}\right)^{\left(1/C_n^k\right)}\right)^{(1/k)}\le 1. \tag{29}$$

That means $X(e) = 1 - (1 - \prod_{1 \le i_1 < \cdots < i_k \le i_n} (1 - \prod_{j=1}^{k} (1 - X(e_{i_j}))^{n\xi_{i_j}})^{(1/C_n^k)})^{(1/k)} \subseteq [0,1]$. Similarly, we can get $(Y(e), Z(e)) \subseteq [0,1]$, and $(A(e), S(e), D(e)) \subseteq [0,1]$, so (1) is maintained.

(2) For $Z(e_{i_j}) + D(e_{i_j}) \le 1$, then we can derive $D(e_{i_j}) \le 1 - Z(e_{i_j})$, and thus

$$
\begin{aligned}
0 &\le Z(e) + D(e) \\
&= 1 - \left(1 - \prod_{1 \le i_1 < \cdots < i_k \le i_n} \left(1 - \prod_{j=1}^{k} \left(1 - Z\left(e_{i_j}\right)\right)^{n\xi_{i_j}}\right)^{(1/C_n^k)}\right)^{(1/k)} + \left(1 - \prod_{1 \le i_1 < \cdots < i_k \le i_n} \left(1 - \prod_{j=1}^{k} \left(D\left(e_{i_j}\right)\right)^{n\xi_{i_j}}\right)^{(1/C_n^k)}\right)^{(1/k)} \\
&\le 1 - \left(1 - \prod_{1 \le i_1 < \cdots < i_k \le i_n} \left(1 - \prod_{j=1}^{k} \left(1 - Z\left(e_{i_j}\right)\right)^{n\xi_{i_j}}\right)^{(1/C_n^k)}\right)^{(1/k)} + \left(1 - \prod_{1 \le i_1 < \cdots < i_k \le i_n} \left(1 - \prod_{j=1}^{k} \left(1 - Z\left(e_{i_j}\right)\right)^{n\xi_{i_j}}\right)^{(1/C_n^k)}\right)^{(1/k)} \\
&= 1.
\end{aligned}
\tag{30}
$$

□

Example 2. Let $Q(e_1) = \langle(0.1, 0.2, 0.3), (0.2, 0.5, 0.6)\rangle$, $Q(e_2) = \langle(0.2, 0.3, 0.3), (0.2, 0.5, 0.5)\rangle$, and $Q(e_3) = \langle(0.4, 0.5, 0.5), (0.3, 0.4, 0.4)\rangle$ be three FNIFNs, and suppose $k = 2\xi = (0.2, 0.3, 0.5)$; then according to (21), we have

$$
\begin{aligned}
&FNIFW\ DM\ SM_{n\xi}^{(k)}(Q(e_1), Q(e_2), \ldots, Q(e_n)) \\
&= \frac{1}{k}\left(\underset{1 \le i_1 < \cdots < i_k \le i_n}{\otimes}\left(\bigoplus_{j=1}^{k}\left(n\xi_{i_j} \otimes Q\left(e_{i_j}\right)\right)\right)^{(1/C_n^k)}\right) \\
&= \left\{
\begin{array}{l}
\left(
\begin{array}{l}
1 - \left(1 - \left(\left(1 - (1 - 0.1)^{0.6} \times (1 - 0.2)^{0.9}\right) \times \left(1 - (1 - 0.1)^{0.6} \times (1 - 0.4)^{1.5}\right) \times \left(1 - (1 - 0.2)^{0.9} \times (1 - 0.4)^{1.5}\right)\right)^{(1/C_3^2)}\right)^{(1/2)}, \\
1 - \left(1 - \left(\left(1 - (1 - 0.2)^{0.6} \times (1 - 0.3)^{0.9}\right) \times \left(1 - (1 - 0.2)^{0.6} \times (1 - 0.5)^{1.5}\right) \times \left(1 - (1 - 0.3)^{0.9} \times (1 - 0.5)^{1.5}\right)\right)^{(1/C_3^2)}\right)^{(1/2)} \\
1 - \left(1 - \left(\left(1 - (1 - 0.3)^{0.6} \times (1 - 0.3)^{0.9}\right) \times \left(1 - (1 - 0.3)^{0.6} \times (1 - 0.5)^{1.5}\right) \times \left(1 - (1 - 0.3)^{0.9} \times (1 - 0.5)^{1.5}\right)\right)^{(1/C_3^2)}\right)^{(1/2)}
\end{array}
\right) \\
\left(
\begin{array}{l}
\left(1 - \left(\left(1 - 0.2^{0.6} \times 0.2^{0.9}\right) \times \left(1 - 0.2^{0.6} \times 0.3^{1.5}\right) \times \left(1 - 0.2^{0.9} \times 0.3^{1.5}\right)\right)^{(1/C_3^2)}\right)^{(1/2)} \\
\left(1 - \left(\left(1 - 0.5^{0.6} \times 0.5^{0.9}\right) \times \left(1 - 0.5^{0.6} \times 0.4^{1.5}\right) \times \left(1 - 0.5^{0.9} \times 0.4^{1.5}\right)\right)^{(1/C_3^2)}\right)^{(1/2)} \\
\left(1 - \left(\left(1 - 0.6^{0.6} \times 0.5^{0.9}\right) \times \left(1 - 0.6^{0.6} \times 0.4^{1.5}\right) \times \left(1 - 0.5^{0.9} \times 0.4^{1.5}\right)\right)^{(1/C_3^2)}\right)^{(1/2)}
\end{array}
\right)
\end{array}
\right\} \\
&= \langle(0.2469, 0.3463, 0.3706), (0.2526, 0.4744, 0.4976)\rangle.
\end{aligned}
\tag{31}
$$

Then, we shall discuss some better properties for FNIFWDMSM.

Property 5 (idempotency). If $Q(e_i) = \langle (X(e_i), Y(e_i), Z(e_i)), (A(e_i), S(e_i), D(e_i)) \rangle$ $(i = 1, 2, \ldots, n)$ are same, then

$$FNIFWDMSM_{n\xi}^{(k)}(Q(e_1), Q(e_2), \ldots, Q(e_n)) = Q(e). \tag{32}$$

Property 6 (monotonicity). Let $Q(e_i) = \langle (X(e_i), Y(e_i), Z(e_i)), (A(e_i), S(e_i), D(e_i)) \rangle$ $(i = 1, 2, \ldots, n)$ and $Q(e_j) = \langle (X(e_j), Y(e_j), Z(e_j)), (A(e_j), S(e_j), D(e_j)) \rangle$ $(j = 1, 2, \ldots, n)$ be two sets of FNIFNs. If $X(e_i) \leq X(e_j), Y(e_i) \leq Y(e_j), Z(e_i) \leq Z(e_j), A(e_i) \geq A(e_j), S(e_i) \geq S(e_j), D(e_i) \geq D(e_j)$ hold for all i, j, then

$$\begin{aligned} &FNIFWDMSM_{n\xi}^{(k)}\left(Q\left(e_{i_1}\right), Q\left(e_{i_2}\right), \ldots, Q\left(e_{i_n}\right)\right) \\ &\quad \leq FNIFWDMSM_{n\xi}^{(k)}\left(Q\left(e_{j_1}\right), Q\left(e_{j_2}\right), \ldots, Q\left(e_{j_n}\right)\right). \end{aligned} \tag{33}$$

Property 7 (boundedness). Let $Q(e_i) = \langle (X(e_i), Y(e_i), Z(e_i)), (A(e_i), S(e_i), D(e_i)) \rangle$ $(i = 1, 2, \ldots, n)$ be a couple of FNIFNs. If $Q(e)^+ = \left\{ \begin{array}{c} (\max(X(e_i)), \max(Y(e_i)), \max(Z(e_i))) \\ (\min(A(e_i)), \min(S(e_i)), \min(D(e_i))) \end{array} \right\}$ $(i = 1, 2, \ldots, n)$ and $Q(e)^- = \left\{ \begin{array}{c} (\min(X(e_i)), \min(Y(e_i)), \min(Z(e_i))) \\ (\max(A(e_i)), \end{array} \right.$ $\max(S(e_i)), \max(D(e_i))) \}$ $(i = 1, 2, \ldots, n)$, then

$$\begin{aligned} Q(e)^- &\leq FNIFWDMSM_{n\xi}^{(k)}(Q(e_1), Q(e_2), \ldots, Q(e_n)) \\ &\leq Q(e)^+. \end{aligned} \tag{34}$$

Property 8 (commutativity). Let $Q(e_i)$ $(i = 1, 2, \ldots, n)$ be a couple of FNIFNs, and $Q(e_i')$ $(i = 1, 2, \ldots, n)$ is any permutation of $Q(e_i)$ $(i = 1, 2, \ldots, n)$; then,

$$\begin{aligned} &FNIFWDMSM_{n\xi}^{(k)}(Q(e_1), Q(e_2), \ldots, Q(e_n)) \\ &\quad = FNIFWDMSM_{n\xi}^{(k)}(Q(e_1'), Q(e_2'), \ldots, Q(e_n')). \end{aligned} \tag{35}$$

4. Data Instance and Comparative Analysis

4.1. Data Instance. With the rapid development of information technology in the world and the new form of knowledge economy under "Internet +," more and more traditional industries are carrying science and technology for further development and communication. Of course, education is no exception. However, at the present stage, all kinds of schools only achieve "Internet +" rather than "Internet +" in the real sense. Most of the information-based teaching reform is just to add some network means to traditional classroom for auxiliary teaching, without truly integrating high-quality teaching content. Since the emergence of MOOC, a large-scale online network platform, in 2008, it has had a significant impact on the education of all countries in the world. Although this platform has not been introduced to China for a long time, it has been rapidly applied in the field of higher education. At the same time, flipped classroom, as a new classroom mode, overturns the traditional language teaching method, changes the role of teachers and students, and sets up the student-centered teaching concept. By selecting high-quality resources from all over the world and using flipped classroom teaching, the education in colleges and universities has taken on a new look. Vocational education, as an important branch of the education system, is rarely applied in this aspect. Especially in the accounting course, which plays an important role in secondary vocational education, students with low level of teaching informatization, low comprehensive quality of classes and teachers, and poor learning motivation lack learning motivation in the current teaching process, and the overall teaching quality has problems. Therefore, with the help of the MOOC+ rolling classroom teaching mode, the current predicament of accounting courses can be effectively alleviated, and accounting education can be effectively promoted. A point in case about the blended classroom teaching reform effect evaluation with FNIFNs would be used to illustrate the built methods. We shall give 5 possible schools H_i $(i = 1, 2, 3, 4, 5)$ to select. The experts shall select four given attributes to assess the management quality level of teacher education of these given schools: (1) J_1 represents the teaching environment and studying environment; (2) J_2 depicts curriculum design; (3) J_3 depicts the teaching practice; (4) J_4 depicts the student satisfaction degree. Several schools are depicted with FNIFNs by the given DMs with 4 given criteria (whose weighting vector $\xi = (0.25, 0.20, 0.15, 0.40)$), and the FNIFN decision values are depicted in Table 1.

Then, the FNIFWDMSM operator is used to deal with blended classroom teaching reform effect evaluation with FNIFNs.

Step 1. From Table 1, we can fuse all FNIFNs r_{ij} by FNIFWDMSM mean to calculate the FNIFNs H_i $(i = 1, 2, 3, 4, 5)$ of the given schools H_i; the obtained values are shown in Table 2 $(n = 2)$.

Step 2. The SF of schools is calculated in Table 3.

Step 3. From Table 3, the order of the schools is given in Table 4. Note that ">" depicts "preferred to." The best college school is H_1.

4.2. Argument Analysis. Table 5 describes the corresponding calculation results of different n values in FNIFWGMSM formula. From Table 5, it could be seen that the results are stable along with different parameter alteration. The best optimal alternative is still H_1.

4.3. Comparative Analysis. In this section, we will compare the technology depicted with other technologies, and the conclusions are shown in Table 6.

From above analysis, comparing the result of the proposed FNIFWDMSM operator with FNIFWA and FNIFWG formulas, these schemes rank a little differently and the

Table 1: The FNIFN DM.

	J_1	J_2
H1	⟨(0.3, 0.4, 0.4), (0.4, 0.5, 0.5)⟩	⟨(0.2, 0.2, 0.4), (0.1, 0.2, 0.2)⟩
H2	⟨(0.4, 0.5, 0.5), (0.3, 0.4, 0.4)⟩	⟨(0.2, 0.3, 0.3), (0.4, 0.4, 0.6)⟩
H3	⟨(0.4, 0.4, 0.6), (0.2, 0.2, 0.3)⟩	⟨(0.5, 0.6, 0.6), (0.2, 0.3, 0.3)⟩
H4	⟨(0.5, 0.6, 0.6), (0.1, 0.2, 0.2)⟩	⟨(0.6, 0.6, 0.7), (0.1, 0.1, 0.2)⟩
H5	⟨(0.4, 0.6, 0.6), (0.1, 0.2, 0.3)⟩	⟨(0.1, 0.4, 0.5), (0.2, 0.3, 0.4)⟩
	J_3	J_4
H1	⟨(0.1, 0.1, 0.4), (0.3, 0.3, 0.4)⟩	⟨(0.4, 0.5, 0.5), (0.3, 0.3, 0.4)⟩
H2	⟨(0.1, 0.2, 0.3), (0.2, 0.5, 0.6)⟩	⟨(0.2, 0.3, 0.3), (0.2, 0.5, 0.5)⟩
H3	⟨(0.5, 0.7, 0.7), (0.1, 0.2, 0.2)⟩	⟨(0.3, 0.4, 0.5), (0.1, 0.2, 0.4)⟩
H4	⟨(0.4, 0.5, 0.6), (0.2, 0.3, 0.3)⟩	⟨(0.6, 0.6, 0.7), (0.1, 0.1, 0.2)⟩
H5	⟨(0.2, 0.2, 0.4), (0.3, 0.3, 0.4)⟩	⟨(0.1, 0.3, 0.4), (0.2, 0.3, 0.5)⟩

Table 2: The fused values by FNIFWMSM.

Fused operator	Schools	Results
	H_1	<(0.5167, 0.5616, 0.6303), (0.1797, 0.1963, 0.2468)>
	H_2	<(0.1791, 0.3712, 0.4633), (0.2166, 0.2952, 0.4191)>
FNIFWDMSM	H_3	<(0.2552, 0.3039, 0.4154), (0.2877, 0.3451, 0.3907)>
	H_4	<(0.2229, 0.3226, 0.3411), (0.3008, 0.4649, 0.5321)>
	H_5	<(0.4002, 0.4986, 0.5769), (0.1804, 0.2509, 0.3218)>

Table 3: The SF of the schools.

	FNIFWDMSM
H_1	0.3628
H_2	0.0397
H_3	−0.0226
H_4	−0.1383
H_5	0.2426

Table 4: Order of the schools.

	Order
FNIFWDMSM	$H_1 > H_5 > H_2 > H_3 > H_4$

Table 5: Ranking results under different parameters of FNIFWMSM operator.

	SF (H_1)	SF (H_2)	SF (H_3)	SF (H_4)	SF (H_5)	Ordering
$n = 1$	0.2985	-0.0189	−0.1036	−0.1946	0.2246	$H_1 > H_5 > H_2 > H_3 > H_4$
$n = 2$	0.3628	0.0397	−0.0226	−0.1383	0.2426	$H_1 > H_5 > H_2 > H_3 > H_4$
$n = 3$	0.3999	0.0668	0.0228	−0.1134	0.2563	$H_1 > H_5 > H_2 > H_3 > H_4$
$n = 4$	0.4416	0.0866	0.0572	-0.0943	0.2745	$H_1 > H_5 > H_2 > H_3 > H_4$

Table 6: Comparative analysis.

	Order
FNIFWA operator [51]	$H_1 > H_5 > H_2 > H_3 > H_4$
FNIFWG operator [54]	$H_1 > H_5 > H_2 > H_3 > H_4$
FNIFWHM operator [62]	$H_1 > H_5 > H_2 > H_3 > H_4$
FNIFHPWG operator [46]	$H_1 > H_5 > H_3 > H_2 > H_4$
IFNIFHCA operator [63]	$H_1 > H_5 > H_2 > H_3 > H_4$
FNIFWDMSM	$H_1 > H_5 > H_2 > H_3 > H_4$

optimal alternative is not different. The FNIFWHM, FNIFHPWG, and IFNIFHCA operators only consider relationship between two given arguments. Nevertheless, the FNIFWA and FNIFWG formulas do not consider the relationship between given arguments, which cannot correctly estimate the effect of different given values of n arguments on the final ranking results. The FNIFWDMSM formula could perfectly consider the relationship between different values of n being fused.

5. Conclusion

With the progress of science and technology, the information age has come, and information technology plays an important role in many fields. The social environment of informationization promotes the development of educational information, and new educational ideas and methods are emerging, which makes the development of education informatization more indepth. The introduction of information-based means into teaching based on the blended teaching mode of flipped classroom helps to realize the concept of student-centered education, which can enrich the content of classroom teaching and effectively improve students' enthusiasm for learning. In order to solve the difficult problems faced in the current mathematics teaching in rural junior high schools, the study draws on the experience of carrying out blended teaching mode based on flipped classroom at home and abroad. After the teaching experiment, through the analysis of experimental results, it is concluded that the implementation of the blended teaching mode based on flipped classroom in rural junior high schools is conducive to improving the effect of mathematics teaching and improving students' ability of self-learning. In this paper, we deal with the MADM issues under FNIFNs and utilize the DMSM means to devise several GMSM fused means with FNIFNs: FNIFDMSM tool and FNIFWDMSM tool. The characteristic of these two built operators is also analyzed. The FNIFWDMSM tool is utilized to cope with MADM issues under FNIFNs. Finally, a point in case for blended classroom teaching reform effect evaluation is used to show the built method. In our future work, the application and expansion of the FNIFN information fused operators will be debated in another MADM research direction [64–71] and also taking into account the decision makers' risk attitude [72–74] for MAGDM with FNIFNs.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The author declares that there are no conflicts of interest.

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