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*Research Article*

# Using ARIMA-GARCH Model to Analyze Fluctuation Law of International Oil Price

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It is meaningful and of certain theoretical value for the development of economy through analyzing fluctuation rules of international oil prices and forecasting the future trend of international oil prices. By composing the autoregressive integrated moving average (ARIMA) model and the combination model of autoregressive integrated moving average model-generalized autoregressive conditional heteroskedasticity (ARIMA-GARCH) for analyzing and forecasting international oil prices, study shows that the combination model of ARIMA (1,1,0)-GARCH (1,1) is more suitable for short-term forecasting of international oil prices with higher accuracy that the MAPE of forecasting has reduced from 1.549% to 0.045% and the RMSE of forecasting has reduced from 1.032 to 0.071.

## 1. Introduction

Oil, gold in black, "the blood of industry," is such a kind of important industrial source and power source and indispensable strategic resource for nations to survive and develop. It plays an immeasurable role in safeguarding national economic and social development and defense security. To some extent, the effects of oil on economy are achieved by the price fluctuation in oil and the report of "Strategic Energy Policy Challenges in the 21st Century" of USA has pointed out that "almost every recession in economy since the late 1940s happened with a spike in oil prices as a prelude." It is just like what economists pointed out that, for oil consumers or importers, the deduction in oil prices is the "driver" of their economies, and for oil producers or exporters, it is regarded as the "victim" of their economies and it is opposite to the rise in oil prices. This shows that the fluctuation in oil prices will definitely bring effects on global economic development. Thus, it is meaningful and of certain theoretical value for the development of economy through analyzing fluctuation rules of international oil prices and forecasting the future trend of international oil prices.

The main contributions of this study are as follows:

(1) ARIMA model and ARIMA-GARCH combined model have been constructed
(2) The future trend of international oil price is predicted, which has certain theoretical value and significance for economic development
(3) Comparing with other traditional models, we obtain that the proposed model has higher prediction accuracy

The rest of the study is organized as follows. In Section 2, we devote to the study of related work, and references related to ARIMA and GARCH models have been analyzed. In Section 3, we focus on the introduction of ARIMA and GARCH models. In Section 4, we devote to positive analysis of international oil price forecasting. In Section 5, we summarize and forecast the work in this study.

## 2. Review of Literature

In recent years, many scholars have made outstanding achievements in applications of ARIMA and GARCH models. De Oliveira and FL Cyrino Oliveira [1] forecasted long term electricity consumptions by using the ARIMA model in 2017. Krba et al. [2] made an analysis and forecasting of COVID-19 between European countries by using the ARIMA model in 2020. Lanling Liu et al. [3] used the ARIMA model to forecast fiscal revenue in 2020 with high prediction accuracy. Panigrahi et al. [4] made a forecasting of wind speed over the sea surface and time series of sunspots by using the ARIMA model in 2021; other scholars also made a short-term forecasting of COVID-19 in India by using the ARIMA model. Arora et al. [5–8] have also made splendid achievements in applications of the GARCH model [9–11]. Kim and Algieri [12, 13] analyzed and forecasted the network traffic and extreme price changes by applying the INGARCH model in 2020. Renjie Zhu made a positive analysis of fluctuation ratio of stock market based on the GARCH model in 2020 [14]. Other scholars conducted research on the improved GARCH model in time series [15–19]. Zolfaghari Mehdi applied the combination of AWT, LSTM, and ARIMA-GARCH models to forecast stock index in 2021. For forecasting stock fluctuation ratio of USA stock market, Dow Jones industrial index and Nasdaq composite index, two major index of stock fluctuation ratio, robustness analysis has a higher accuracy [20]. Lin and Huang [21] obtained corresponding fluctuation characteristics and achieved forecasting of transport flow by combining ARIMA and GARCH models in 2021. The experiment indicates that the ARIMA-GARCH model has a good performance to meet requirements of practical applications. Ding and Duan [22] forecasted short-term passengers' flow volume of three subway stations in Beijing by applying the ARIMA-GARCH model in 2018, and the combination model significantly improved the reliability of predicted point's value and the coverage probability of prediction interval by decreasing length of the average prediction interval.

In the aspect of international oil price forecasting, many scholars have also made outstanding progress. Ali Safari et al. [23]combined the exponential smoothing model, the autoregressive integrated moving average model, and nonlinear autoregressive neural network in a structure of the state space model in 2018 to increase accuracy of forecasting. In [24], Aimei Hu built ARIMA(5,1,3) and GARCH(1,1) models in accordance with monthly data of WTI crude oil and made a forecasting of oil prices in 2012 which showed that the forecasting results' accuracy of GARCH is higher than that of the ARIMA model, and the mean relative error decreased from 8.2157% to 5.4791%, and the root mean square error decreased from 9.449168 to 7.25275. In [25], Jue Wang raised a semiheterogeneous approach to combine forecasting of crude oil prices in 2018 by decomposing the original price series using four decomposition methods plus four different forecasting technologies such as AR and ARIMA models to predict components of each disposition methods and finally rebuilding price forecasting based on the predicted components. The result showed that comprehensive forecasting errors decreased obviously. In [26], scholars apply different models to predict on international oil prices aiming to figure out the fluctuation rules of oil prices in order to take appropriate measures when strike occurs to reduce negative effects on economic development. For forecasting modeling issues of international oil prices which have complex fluctuation characteristics, the combination model theory of ARIMA-GARCH has great potential for improving forecasting performance and stationarity of international oil prices.

## 3. Brief Introduction of ARIMA and GARCH Models

*3.1. General Form of the ARMA Model.* The structure of the ARMA model is as follows:

$$X_t = \sum_{j=1}^{p} \phi_j X_{t-j} + \sum_{j=0}^{q} \theta_j \varepsilon_{t-j}, \quad t \in Z, \qquad \begin{cases} \theta_0 = 1, \\ \phi_p \theta_q \neq 0, \end{cases} \tag{1}$$

where $\{\varepsilon_t\}$ represents a flat noise in zero-mean , real polynomial.

$\phi(z) = 1 - \phi_1 z - \cdots - \phi_p z^p$ and $\theta(z) = \theta_0 + \theta_1 z + \cdots + \theta_q z^q$ meet the requirements of stationarity and reversibility, respectively.

*3.2. General Form of the ARIMA Model.* In the ARIMA($p$, $d$, $q$), AR represents autoregressive, $p$ represents the number of autoregressive terms, MA represents average move, $q$ represents the average number of terms of moving, and $d$ represents the difference number. If

$$Y_t = (1 - B)^d X_t, \tag{2}$$

is a sequence of ARMA($p$, $q$), it indicates that $\{X_t\}$ is a sequence of ARMA($p$, $q$)and the model is shown as follows:

$$\phi(B)(1 - B)^d X_t = \theta(B)\varepsilon_t, \quad t \in Z, \tag{3}$$

where $B$ represents the operator, $(1 - B)$ represents finite difference operator, $\{\varepsilon_t\}$ represents a flanoise in zero-mean, and real polynomial $\phi(z) = 1 - \phi_1 z - \cdots - \phi_p z^p$ and $\theta(z) = \theta_0 + \theta_1 z + \cdots + \theta_q z^q$ meet the requirements of stationarity and reversibility, respectively.

The modeling steps of ARIMA(p, $d$, q) model are as follows:

① The stationarity test is carried out on the original time series. If the series does not meet the stationarity condition, the difference transformation is needed to make the series meet the stationarity condition, so as to obtain the value of $d$ in the model.

② The values of $p$ and $q$ in the model are determined by using ACF and PACF.

③ The unknown parameters of the model were estimated and the significance of the parameters and the applicability of the diagnostic model were tested.

④ Predict the future value of time series.

*3.3. ARCH Model.*

$$\begin{cases} x_t = f(t, x_{t-1}, x_{t-2}, \ldots) + \varepsilon_t, \\ \varepsilon_t = \sqrt{h_t}\, e_t, \\ h_t = w + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2, \\ e_t \sim II\ D(0, 1), \end{cases} \tag{4}$$

where $\alpha_i$ is nonnegative and $f(t, x_{t-1}, x_{t-2}, \ldots)$ is the deterministic information fitting model of $\{x_t\}$.

*3.4. GARCH Model.*

$$\begin{cases} x_t = f(t, x_{t-1}, x_{t-2}, \ldots) + \varepsilon_t, \\ \varepsilon_t = \sqrt{h_t}\, e_t, \\ h_t = w + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \gamma_j h_{t-j}, \\ e_t \sim II\ D(0, 1), \end{cases} \tag{5}$$

where $\alpha_i$ and $\gamma_j$ are nonnegative and $f(t, x_{t-1}, x_{t-2}, \ldots)$ is the deterministic information fitting model of $\{x_t\}$. It is an extension of the ARCH model and claims that $h_t$ has AR $\sum_{j=1}^{p} \gamma_j h_{t-j}$ and ARCH term is $\sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2$. In general, the GARCH model is easier to identify and estimate, and the GARCH model can capture the flat period and fluctuation period of time series.

## 4. Positive Analysis of International Oil Price Forecasting

This study collects closing price data of WTI crude oil in total of 125 days from July 1, 2021, to December 22, 2021, as samples for analyzing and forecasting and sets the last 10 days of closing price data as a forecasting sample, and data originate from IN-EN.COM.

*4.1. Test of Stationarity.* Firstly, observe the sequence diagram of samples; partial fluctuation is obvious in the figure, and it shows a trend of decline, rise, and decline as a whole which does not represent seasonality and singular point and can be preliminarily judged that the time series is nonstationary series, as seen in Figure 1. As the time series is not stationary, so there are differences on it, the sequence diagram fluctuates up and down in 0 after being differencing at a time and can be preliminarily judged that the time series is stationary after being differencing at a time, as seen in Figure 2.

To assure the correctness of judgment, we follow and use unit root test of ADF to further conduct experiment. The original hypothesis that ADF tests is that there is at least one unit root; the alternate hypothesis is there is no unit root. If the statistic tested by ADF is above the marginal value, the original hypothesis is accepted which means that there is a unit root, and the series is nonstationary. Otherwise, there is no unit root, and the series is stationary.

To use unit root of ADF for testing, firstly, we need to ensure the lag intervals for endogenous of rational regressive definition. Through BIC criteria, to ensure the lag intervals for endogenous of rational regressive definition, we need to choose constants and temporal trend and observe the sequence diagram, and we can choose figures that do not contain constant terms and temporal trend to conduct ADF test, and tests results can be seen in Table 1.

The result shows that statistics of ADF for WTI is −1.7004 which is above 10% of the marginal value and accepts the original hypothesis which means that WTI series is nonstationary. Statistics of ADF for dWTI is −7.6897 which is above 1% of the marginal value and refuses the original hypothesis which means that dWTI series is stationary. Thus, WTI series is stationary after being differencing at a time.

*4.2. Build the ARIMA Model.* The time series of WTI has transferred as a stationary series after being differencing at a time, so we need to ensure the value of $p$ and $q$. Thus, we observe first the difference figure of ACF and PACF, as shown in Figure 3, and we can judge that the value of $p$ is 1 and the value of $q$ is 0. So, we can build the ARIMA(1, 1, 0) model as below:

$$(1 - B)\mathrm{WTI}_t = -0.0474\mathrm{WTI}_{t-1} + \varepsilon_t, \varepsilon_t \sim N(0, 3.072). \tag{6}$$

*4.3. Test of the ARCH Effect.* For the test result of the ARCH effect which shows that, under the two situations of 4 orders-lag and 8 orders-lag, both WTI series refuse the original hypothesis with 1% significant level and consider the ARCH effect, as seen in Table 2; we can further build the ARIMA-GARCH model.

*4.4. ARIMA-GARCH Model Estimation.* The build of the ARIMA-GARCH model firstly needs to create the GARCH model of WTI. The average equation that we choose is ARIMA(1,1,0) and the chosen fluctuation ratio equation is GARCH(1, 1); estimated parameters are listed in Table 3, and the models are as below:

$$\begin{cases} (1 - B)\mathrm{WTI}_t = 74.562559 + 0.95346\mathrm{WTI}_{t-1} + \varepsilon_t, \\ \varepsilon_t = \sqrt{h_t}\, e_t, \\ h_t = 0.095549 + 0.0000\varepsilon_{t-1}^2 + 0.969848 h_{t-1}, \\ e_t \sim \mathrm{IID}(0, 1). \end{cases} \tag{7}$$

The parameter of $\varepsilon_{t-1}^2$ is small, which is close to 0.

*4.5. Comparison of Predictive Validity between ARIMA and ARIMA-GARCH.* The forecasting figures of ARIMA(1, 1, 0) and ARIMA-GARCH are shown as Figures 4 and 5. It is not clearly distinguished from the forecasting figures whether ARIMA or ARIMA-GARCH is better for prediction; thus, the study divides the sample data into two parts: one part for the training set from July 1, 2021, to December 8, 2021,

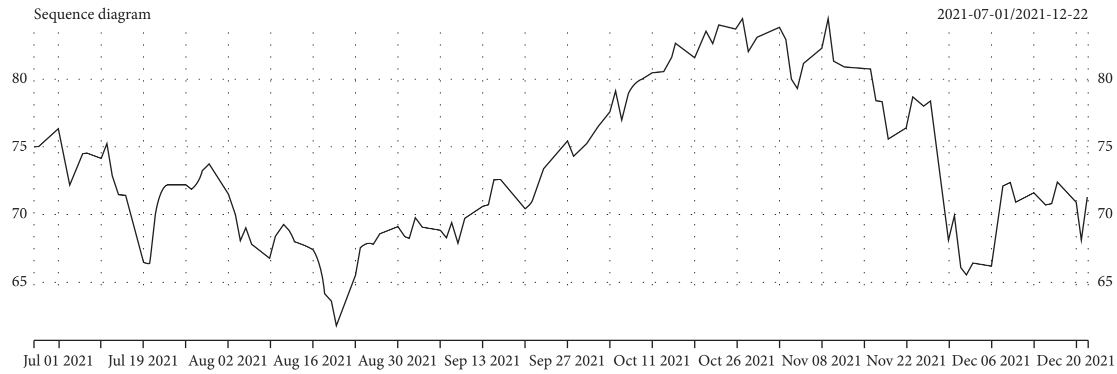


Figure 1: WTI sequence diagram.

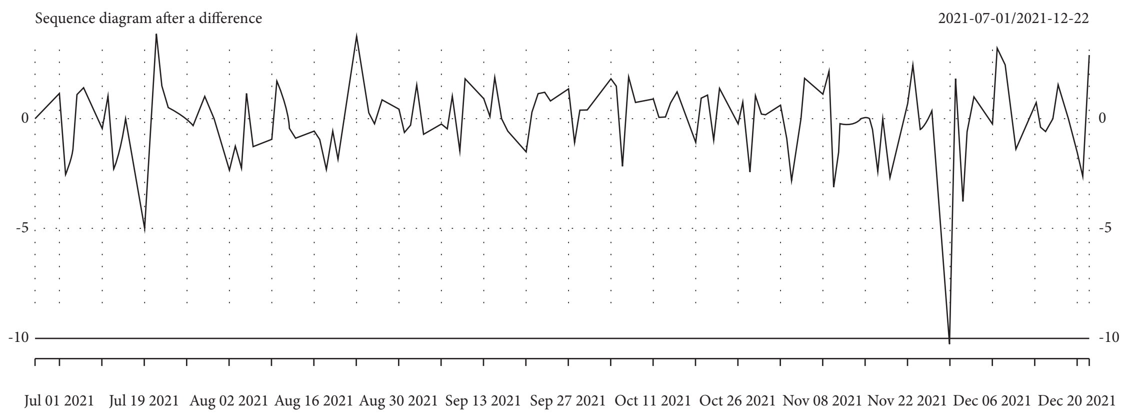


Figure 2: WTI-order difference sequence diagram.

Table 1: Test results of stationarity of each variable.

| Variable | D | $(C, T, K)^*$ | ADF test value | Critical value of ADF at significance level | | | Stationarity |
|---|---|---|---|---|---|---|---|
| | | | | 1% | 5% | 10% | |
| WTI | 0 | (C, 0, 1) | −1.7004 | −3.46 | −2.88 | −2.57 | Nonstationarity |
| dWTI | 1 | (0, 0, 1) | −7.6897 | −2.58 | −1.95 | −1.62 | Stationarity |

Note: the unit root test assumes that a sequence has a unit root. $(C, T, K)$ denotes the constant term, the time trend, and the best delay term of the unit root test equation, respectively. $C$ (or $t$) means excluding constant or trend terms. The delay term is added to make the residual term white noise. The selection criteria of lag period refer to SC criterion.

and another part for the testing set from 9 December 2021 to 22 December 2021. The training set is used to forecast the future data of WTI by applying in the ARIMA(1, 1, 0) model and the ARIMA(1, 1, 0)-GARCH(1, 1) model. To compare forecasting results with the real value, with forecasting results being represented in Figure 4, the results show that the forecasting MAPE and RMSE of the ARIMA-GARCH model are 0.045% and 0.071, and those of the ARIMA model are 1.540% and 1.032. So, the forecasting effect of the ARIMA-GARCH model is better. The MAPE and RMSE of the forecasts are shown in Table 4. It can be seen that the ARIMA-GARCH model solves the heteroscedasticity of the ARIMA model residual and improves the prediction accuracy. In addition, the ARIMA-GARCH model has solved the prediction modeling issue that the time series can be affected by complex factors, and it is represented as abnormal leptokurtosis and fat-tail distribution.

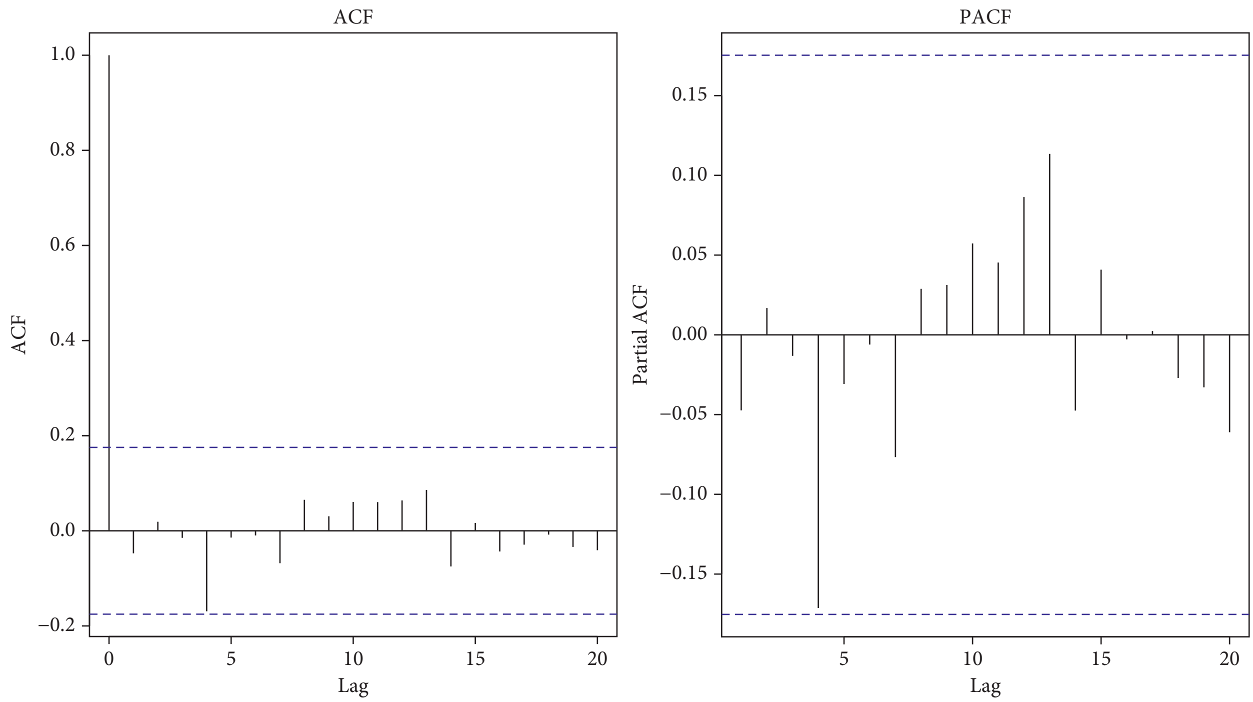


Figure 3: ACF and PACF.

Table 2: The test result of the ARCH effect.

| | ARCH(4) | | ARCH(8) | |
|---|---|---|---|---|
| | Chi-squared | $p$ value | Chi-squared | $p$ value |
| **WTI** | 110.17 | 2.20E-16 | 107.08 | 2.20E-16 |

Table 3: Coefficient significance test table.

| | Estimate | Std. error | $t$ value | $\Pr(>\lvert t\rvert)$ |
|---|---|---|---|---|
| **mu** | 74.562559 | 1.536693 | 48.521 | 0.000000 |
| **ar1** | 0.953460 | 0.028053 | 33.988 | 0.000000 |
| **omega** | 0.095549 | 0.049584 | 1.927 | 0.053975 |
| **alpha1** | 0.000000 | 0.008002 | 0.000 | 1.000000 |
| **beta1** | 0.969848 | 0.029566 | 32.803 | 0.000000 |

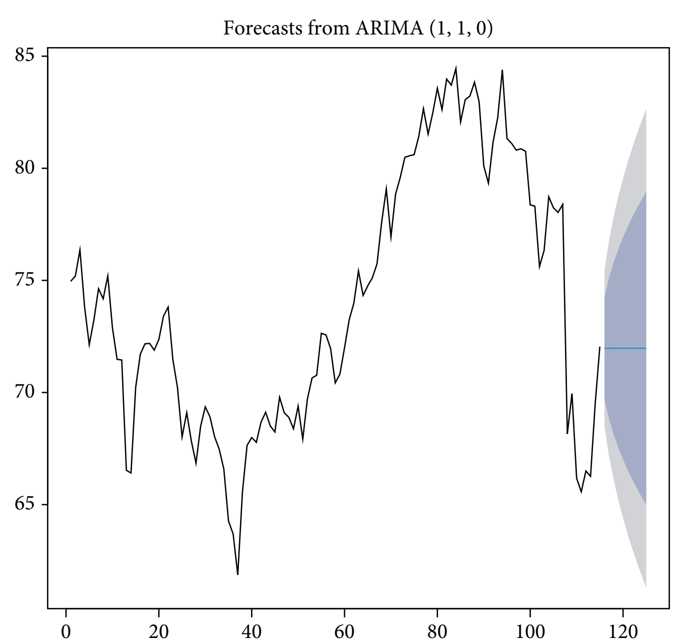


Figure 4: Forecasts from ARIMA(1, 1, 0).

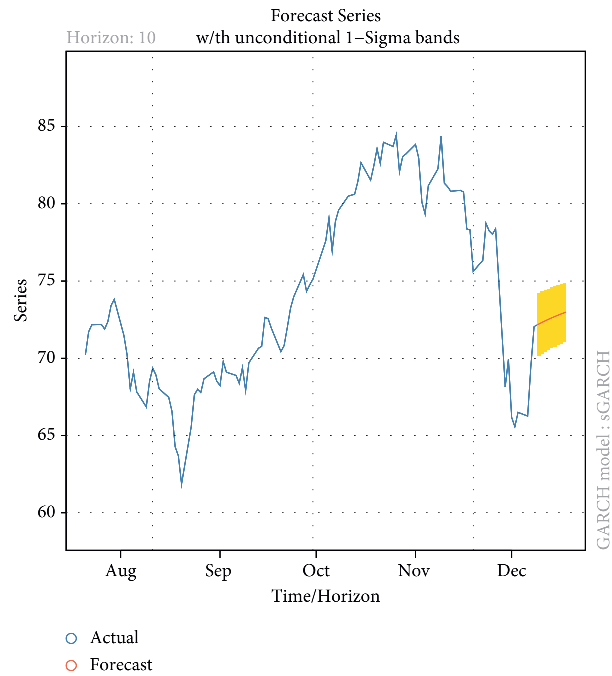


Figure 5: Forecasts from ARIMA(1, 1, 0)-GARCH(1, 1).

Table 4: MAPE and RSME.

| Date | Actual value | ARIMA predicted value | ARIMA relative error (%) | ARIMA-GARCH predicted value | ARIMA-GARCH relative error (%) |
|---|---|---|---|---|---|
| 2021/12/9 | 72.36 | 71.9682748 | 0.541 | 72.26946 | 0.125 |
| 2021/12/10 | 70.94 | 71.97088379 | 1.453 | 70.91556 | 0.034 |
| 2021/12/13 | 71.67 | 71.9708005 | 0.420 | 71.64392 | 0.036 |
| 2021/12/14 | 71.29 | 71.97080316 | 0.955 | 71.26396 | 0.037 |
| 2021/12/15 | 70.73 | 71.97080307 | 1.754 | 70.70396 | 0.037 |
| 2021/12/16 | 70.87 | 71.97080308 | 1.553 | 70.84396 | 0.037 |
| 2021/12/17 | 72.38 | 71.97080308 | 0.565 | 72.35396 | 0.036 |
| 2021/12/20 | 70.86 | 71.97080308 | 1.568 | 70.83396 | 0.037 |
| 2021/12/21 | 68.23 | 71.97080308 | 5.483 | 68.20396 | 0.038 |
| 2021/12/22 | 71.12 | 71.97080308 | 1.196 | 71.09396 | 0.037 |
| MAPE | | 1.549% | | 0.045% | |
| RMSE | | 1.032 | | 0.071 | |

## 5. Conclusion

To make up for arch that exists in the ARIMA model, known as ARCH effect, the study has applied the ARIMA-GARCH model to analyze and forecast the forward price of WTI crude oil based on MAPE and RMSE as evaluation; the predicted result shows that the combination model of ARIMA(1, 1, 0)-GARCH(1, 1) has increased forecast accuracy. The MAPE of forecasting has reduced from 1.549% to 0.045% and the RMSE of forecasting has reduced from 1.032 to 0.071. In addition, the ARIMA-GARCH model has solved the prediction modeling issue that the forward price of WTI crude oil can be affected by complex factors, and it is represented abnormal leptokurtosis and fat-tail distribution. In the future, the following two aspects are planned: one is to distinguish the prediction effect of the ARIMA model and the ARIMA-GARCH model by integrating various evaluation indexes. The second is to popularize the ARIMA-GARCH model, for example, to analyze and forecast international gold price, stock price index, Sino-US exchange rate, and short-term passenger flow of subway station.

## Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

## Conflicts of Interest

The author declares no conflicts of interest or personal relationships that could have appeared to influence the work reported in this paper.

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