A Similarity-Based Hesitant Fuzzy Group Decision Making Approach and Its Application in Hydraulic Engineering Project Management

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Hesitancy and uncertainty features of experts are common in the decision-making process, especially for the project management events. To solve this problem, a novel similarity-based decision-making approach is put forward, as well as an application to the hydraulic engineering project management. Several experts, who are invited in the decision-making process, are suggested to adopt hesitant fuzzy preference relations (HFPRs) to show their evaluations. To measure the similarity degree of experts, a novel integrated similarity index \( SI \) is given combining the alternative ranking-based similarity index \( SI_{AR} \) and distance-based similarity index \( SI_{D} \) between HFPRs. The \( SI_{AR} \) can be derived from the comparison of the alternative rankings, while the \( SI_{D} \) depends on evaluations’ distance degree. After that, on the basic of opinion transition probabilities, experts’ weights are allocated, which is necessary for the aggregation process. Then, the collective preferences can be aggregated from the individuals’ evaluations. Afterwards, the above methods along with a score function are adopted to obtain the optimal solution for an actual hydraulic engineering project management event. Finally, for verifying the feasible and effective features of the presented methods, some significative discussions and comparative analyses are provided.

1. Introduction

Group decision-making (GDM) is concerned with deriving an optimal decision from a group, where a number of experts or decision makers are involved in and evaluations regarding several alternatives are provided in the decision-making process [1]. In general, experts or participators always take pairwise comparison to reveal their evaluation information in GDM [2]. As the most common preference structures, preference relations can be used to express experts’ opinion accurately and model decision process [3–5]. Due to the various decision-making circumstances, a range of targeted preference relations and approaches were researched in [6–9].

In some practical GDM situations, due to the hesitant and uncertain features in human judgments, experts are likely to be hesitant. As a result, two or more values will be appeared in their numerical comparisons [10]. Hesitant fuzzy preference relations (HFPRs), as one kind of effective preference relations, always draw decision-making organizations’ attention. Therefore, HFPRs and its extension are widely researched in GDM problems [8, 11–16]. For the values in hesitant fuzzy elements (HFEs) of an HFPR, which represent experts’ multiple possible results in the alternative evaluating process.

As we all know, human’s living environment, production practice, as well as the process of social development are suffered from the flood disaster, which is deemed to be a common natural disaster around the world [17–20]. Unfortunately, with a large population, China has afflicted with numerous serious flood disasters in recent years [21]. For instance, alarming rainstorm-induced flood disaster
occurred in East China in 2010 [22], residential area waterlogging near the Huangpu river in 2013 [23], the urban flood disaster in the Zhengzhou city [24, 25], almost 627 thousand of people suffered from floods in Wuhan in 2016 [26], more than RMB 50 billion direct economic loss results from flood disaster in the Hebei province in 2016 [27], and so on. As occurred in the last year, the flood swept most cities in the Henan province, a number of people died in this misfortune. It is worth noting that the flood disaster has caused a great economic loss. Although much attention has been paid to flood disaster and hydraulic engineering construction, the flood disaster still caused a great deal of economic losses, casualties, and collapsed houses. The detailed flood disaster occurred per year from 2016 to 2020 as shown in Table 1.

It means that effectively hydraulic engineering project management is still an attractive topic and it should not be ignored in China. Concerned with the hydraulic engineering project management, the most crucial problem is to adopt applicable plans or alternatives to reduce the loss and probability of risk. It means that these management behaviors could be regarded as a practical GDM problem in some circumstances. Thus, applying a GDM-based perspective, and take the hesitant and uncertain features of experts’ judgments into consideration, to research the hydraulic engineering project management is a topic worth researching.

For this article, a similarity-based hesitant fuzzy GDM approach is developed, as well as the application is performed to hydraulic engineering project management events. The experts’ hesitancy and uncertainty are considered in the decision-making process, and HFPRs are utilized to express their evaluations. An integrated similarity index (SI) is presented to measure the similarity degree between experts, which is researched in the following hydraulic engineering project management. Subsequently, the SI-based experts’ weights determining method is developed for the GDM approach. Furthermore, based on the weighted averaging (WA) operator, decision-making organizers could form participants’ evaluations into a collective one. Besides, a score function of HFPRs is designed, which can be utilized to achieve the optimal solution for hydraulic engineering project management events. The main contributions and attractive novelties are summarized as follows:

1. Considering the hesitancy and uncertainty features, experts invited are allowed to use hesitant fuzzy preference relations (HFPRs) to express their evaluations. Besides, score function and distance measure method are proposed for an application of hydraulic engineering project.

2. An integrated SI is developed to define the similarity degree between involved experts. The SI is given combining the SI_{AR} and SI_{D} of experts. SI_{AR} can be derived from the comparison of alternative rankings, while SI_{D} is depending on the evaluations’ distance degree.

3. The SI-based experts’ weights determining method is developed for the GDM approach. For determining experts’ weights, which is necessary for the aggregation process in GDM, an opinion transition probabilities-based method is utilized. According to this called stationary vector method, experts’ weights are determined reasonably without any subjective factor.

A hydraulic engineering project management event is provided for verifying the effectiveness of the method. Besides, some extended comparative discussions and analyses are presented for assessing hesitancy and uncertainty of experts, which always have a significant impact on results in realistic decision-making events.

The remainder of this article is designed as follows. For the following second section, a review referred to fuzzy preference relations (FPRs), HFS, and HFPRs, as well as a critical score function is provided. In Section 3, an integrated SI and SI-based experts’ weights determining method are presented for the GDM approach. The presented GDM method is applied to resolve a hydraulic engineering project management in Section 4. Furthermore, some analyses and comparisons are provided in Section 5. In the end, concluding remarks, which involve some future research that are put forward.

2. Preliminaries

For this part, some basic knowledge of FPRs, HFS, and HFPRs are introduced in Section 2.1. Then, the score functions of HFE and HFPRs are presented in second section. For simplicity, some symbol descriptions which used in the whole paper are shown in Table 2.

2.1. Concepts of FPRs, HFS, and HFPRs. For the sake of simplicity, let sets $M = \{1, 2, \ldots, m\}$ and $N = \{1, 2, \ldots, n\}$. Besides, $E = \{e_1, e_2, \ldots, e_m\}$, $X = \{x_1, x_2, \ldots, x_n\}$ ($n \geq 2$) be the sets of experts and alternatives, respectively.

Definition 1 (see [6]). An FPR $P$ on a finite set of alternatives $X = \{x_1, x_2, \ldots, x_n\}$ is a fuzzy relation on the product set $X \times X$ with membership function $\mu_p: X \times X \rightarrow [0, 1]$, $\mu_p(x_i, x_j) = p_{ij}$, satisfying

$$0 \leq p_{ij} \leq 1, \quad p_{ii} = 0.5, \quad p_{ij} + p_{ji} = 1, \quad i, j \in N. \quad (1)$$

HFS was originally introduced as a generation of intuitionistic fuzzy sets (IFS), where a set of possible values are involved [15].

Definition 2 (see [15]). For a fixed preference set $X$, HFS is defined on $X$ in terms of a function that when applied to $X$ outputs a subset of $[0, 1]$. The mathematical format of HFS is developed in [28]:

Definition 3 (see [28]). Let $X = \{x_1, x_2, \ldots, x_n\}$ be a fixed set of alternatives. Then, HFS $E$ on $X$ is characterized by a membership function $\mu_E(x)$ that when applied to $X$ returns a subset of $[0, 1]$, which can be described by a mathematical expression:
For a fixed set of alternatives, denoted as \(X = \{x_1, x_2, \ldots, x_n\}\), HFPR \(H\) on the finite set \(X\) is expressed by a matrix \(H = (h_{ij})_{n \times n} \subseteq X \times X\), where \(h_{ij} = \{h_{ij}^{(l)} | l = 1, 2, \ldots, n\}\) is called HFE. \#\(h_{ij}\) denotes the number of elements in \(h_{ij}\). Besides, the \(l\)th element \(h_{ij}^{(l)}\) in \(h_{ij}\) (\(i, j \in \mathbb{N}\)) should also verify the conditions as

\[
h_{ij}^{(l)} + h_{ji}^{(l)} = 1, h_{ii} = 0.5, \#h_{ij} = \#h_{ji}.
\]  

(3) If \(h_{ij}^{(l)} = 0.5\), it indicates that expert \(e_k\) considers there is indifference between alternatives \(x_i\) and \(x_j\), denoted as \(x_i \sim x_j\).

(4) If \(h_{ij}^{(l)} = 1\) (or \(h_{ji}^{(l)} = 0\)), it indicates that expert \(e_k\) considers alternative \(x_i\) is definitely preferred to alternative \(x_j\), denoted as \(x_i \succ x_j\).

Similar to the Definition 3 in [5], we measure the distance between HFPRs as follows:

\[
d(H_k, H_r) = \frac{1}{\#h_{ij}} \sum_{l=1}^{\#h_{ij}} \left( \frac{2}{n(n-1)} \sum_{l'=1}^{n-1} \sum_{j'=1}^{n} \left( h_{ij}^{(l)} - h_{ij'}^{(l')} \right)^2 \right). 
\]  

(4) Remark 1. For HFPRs in GDM, there are two research points that deserve attention. Firstly, consistency is an important issue for HFPR, it can ensure the reliability of experts’ assessment information. An effective additive consistency analysis and an improvement method are presented for HFPRs [30]. According to adjustment rules, the consistency level of evaluations is improved.

Secondly, the number of values in different \(h_{ij}\) is diverse. There are several normalization methods proposed in [14, 31], which guarantees the same number of values in
different HFEs. In our method, experts are suggested to provide reliable HFPRs for the decision process, which is applied to hydraulic engineering project management problems.

2.2. The Score Function of HFE and HFPRs. For further researching, several score functions are designed in [28, 32]. The score function is presented as follows.

**Definition 6 (see [28]).** For HFE \( h \), the score function of \( h \) is

\[
s(h) = \frac{1}{\#h} \sum_{\gamma \in h} \gamma.
\]

Referring to the number of values in \( h \), denoted as \( \#h \), \( s(h) \) is so called the score function of HFE \( h \).

Apply such a score function in HFPR \( H = (h_{ij})_{n \times n} \), where \( h_{ij} \) implicates the preference degree between alternatives \( x_i \) and \( x_j \). For the alternatives set \( X = \{x_1, x_2, \ldots, x_n\} \), the score value of alternative \( s(x_i) \), \( i \in N \), can be computed as follows:

\[
s(x_i) = \sum_{j=1}^{n} \left( \frac{1}{\#h_{ij}} \sum_{\gamma \in h_{ij}} \gamma \right).
\]

For two alternatives \( x_1 \) and \( x_2 \), if \( s(x_1) > s(x_2) \), then \( x_1 \succ x_2 \), which means that \( x_1 \) is superior to \( x_2 \); \( s(x_1) = s(x_2) \), then \( x_1 \sim x_2 \), which means that \( x_1 \) is identical to \( x_2 \).

It is obvious that, the score values \( s(x_i) \) \((i = 1, 2, \ldots, n)\) could be adopted for the alternative ranking procedure.

3. A GDM Approach Considering Hesitancy and Uncertainty

A GDM approach considering hesitancy and uncertainty is presented in this part. Firstly, an integrated SI is developed to measure the similarity degree between experts in Section 3.1. Next, the SI-based method is provided for determining experts’ weights, which is a necessary step for the aggregation process in GDM problems. Finally, in the last section, the complete decision-making process is shown.

3.1. An Integrated SI for GDM. For this part, an integrated SI is developed to measure inter-experts’ similarity degree in GDM. The index consists of two parts, the alternative ranking-based similarity index \( SI_{AR} \) and the distance-based similarity index \( SI_D \) of experts.

3.1.1. SI\(_{AR}\) between Experts. Referring to the score function method, alternatives’ score values can be acquired, also the alternative rankings of experts, i.e., \( R(e_k) \) \((k = 1, 2, \ldots, m)\). Example 1 is presented as follows.

**Example 1.** For a fixed set of alternatives, expressed as \( X = \{x_1, x_2, x_3, x_4\} \), experts \( e_1 \) and \( e_2 \) are allowed to utilize HFPRs to present their evaluations, which are indicated as \( H_1 = (h_{ij,1})_{4 \times 4} \) and \( H_2 = (h_{ij,2})_{4 \times 4} \):

\[
H_1 = \begin{bmatrix}
0.5 & 0.4, 0.3 & 0.2, 0.3 & 0.6, 0.5 \\
0.6, 0.7 & 0.5 & 0.4, 0.4 & 0.6, 0.7 \\
0.8, 0.7 & 0.6, 0.6 & 0.5 & 0.6, 0.5 \\
0.4, 0.5 & 0.4, 0.3 & 0.4, 0.5 & 0.5
\end{bmatrix},
\]

\[
H_2 = \begin{bmatrix}
0.5 & 0.4, 0.5 & 0.4, 0.3 & 0.7, 0.6 \\
0.6, 0.5 & 0.5 & 0.6, 0.6 & 0.5, 0.6 \\
0.6, 0.7 & 0.4, 0.4 & 0.5 & 0.6, 0.8 \\
0.3, 0.4 & 0.5, 0.4 & 0.4, 0.2 & 0.5
\end{bmatrix}.
\]

By equation (6), we get the score values of each alternative for \( e_1 \) and \( e_2 \), denoted as \( s_1(x_i) \) and \( s_2(x_i) \), \( i = 1, 2, 3, 4 \).

\[
s_1(x_1) = 1.65, s_1(x_2) = 2.2, s_1(x_3) = 2.4, s_1(x_4) = 1.75,
\]

\[
s_2(x_1) = 1.95, s_2(x_2) = 2.2, s_2(x_3) = 2.25, s_2(x_4) = 1.6.
\]

Thus, we can get the alternative ranking of \( e_1 \) and \( e_2 \) as

\[
R(e_1): x_1 \succ x_2 \succ x_3 \succ x_4; R(e_2): x_1 \succ x_2 \succ x_4 \succ x_3.
\]

**Remark 2.** Under normal circumstances, experts’ alternative rankings present a strict priority sequence. However, some equal score values of different alternatives will exist in a few cases. For example, \( s_1(x_1) = s_2(x_1) \) and then \( x_1 \sim x_2 \); it can be considered that \( x_1 \succ x_2 \) or \( x_1 < x_2 \). To avoid confusion, all possible ranking information is taken into consideration.

Furthermore, inter-experts’ similarity degree can be obtained from comparison of their alternative ranking information. Thus, the inter-experts’ similarity degree could be measured, denoted as \( SI_{AR} \):

**Definition 7.** Let \( n \in N \) be the number of alternatives, \( SI_{AR} \) between \( e_k \) and \( e_r \) is defined as

\[
SI_{AR}(e_k, e_r) = \frac{\#R(e_k), R(e_r)}{n}.
\]

For rankings \( R(e_k) \) and \( R(e_r) \), the quantity of them with same position is denoted as \( \#R(e_k), R(e_r) \). Referring to two experts \( e_1 \) and \( e_2 \) in Example 1, \( R(e_1): x_1 \succ x_2 \succ x_3 \succ x_4 \) and \( R(e_2): x_3 \succ x_2 \succ x_4 \succ x_1 \), then \( SI_{AR}(e_1, e_2) = 2/4 = 0.5 \). It means a similarity level with half alternative ranking information consistent between experts \( e_1 \) and \( e_2 \).

It is obvious that \( SI_{AR}(e_k, e_r) \in [0, 1] \). Meanwhile, a higher similarity level exists between \( e_k \) and \( e_r \), along with a higher \( SI_{AR}(e_k, e_r) \) value.

3.1.2. SI\(_D\) between Experts. In this section, another SI\(_D\) in GDM is developed to investigate the inter-experts’ similarity degree. According to the distance degree between \( H_k \) and \( H_r \) provided in Definition 5, SI\(_D\) between \( e_k \) and \( e_r \) can be defined as
\[ SI_D(e_k, e_r) = 1 - d(H_k, H_r) \]
\[ = 1 - \frac{1}{\# h_{ik}} \sum_{j=1}^{n} \frac{2}{n(n-1)} \sum_{l=1}^{n} (h_{lj}^{(l)} - h_{lj}^{(0)})^2. \] 

(11)

It is obvious that index \( SI_D(e_k, e_r) \), on the other hand, describes the similarity level of the evaluations that belong to \( e_k \) and \( e_r \), respectively. Based on equation (11), the similarity degree of \( e_k \) and \( e_r \) could be obtained. For example, for the experts \( e_1 \) and \( e_2 \) in Example 1, we can obtain that \( SI_D(e_1, e_2) = 1 - d(H_1, H_2) = 1 - 0.1535 = 0.8465 \). It also means a higher similarity level between \( e_1 \) and \( e_2 \) based on their evaluations.

In a same way, we can get that \( SI_D(e_k, e_r) \) is not always consistent. Example 2 is provided as follows.

**Example 2.** Except for two experts \( e_1 \) and \( e_2 \), another expert \( e_3 \) is also involved in the GDM problem. The evaluation provided by \( e_3 \) is denoted as \( H_3 = (h_{ij}, k) \):

\[
H_3 = \begin{bmatrix}
0.5 & 0.4, 0.6 & 0.4, 0.5 & 0.3, 0.4 \\
0.6, 0.4 & 0.5 & 0.2, 0.3 & 0.6, 0.9 \\
0.6, 0.5 & 0.8, 0.7 & 0.5 & 0.3, 0.7 \\
0.7, 0.6 & 0.4, 0.1 & 0.7, 0.3 & 0.5 
\end{bmatrix}
\] 

(12)

Similar to Example 1, alternatives’ score values can be obtained, denoted as \( s_j(x_i) \) \((i = 1, 2, 3, 4)\), as well as the alternative ranking for \( e_3 \):

\[
s_j(x_1) = 1.8, s_j(x_2) = 2.0, \quad s_j(x_3) = 2.3, s_j(x_4) = 1.9,
\]
\[
R(e_3): x_3 > x_2 > x_4 > x_1.
\]

Thus, for experts \( e_1 \), \( e_2 \), and \( e_3 \), we have that

\[
SI_{AR}(e_1, e_2) = \frac{2}{4} = 0.5; SI_{AR}(e_1, e_3) = \frac{4}{4} = 1,
\]
\[
SI_D(e_1, e_2) = 1 - 0.1535 = 0.8465; SI_D(e_1, e_3) = 1 - 0.2020 = 0.7980.
\]

(14)

It is obvious that \( SI_{AR}(e_1, e_3) > SI_{AR}(e_1, e_2) \) and it means that \( e_1 \) possess a higher similarity degree with \( e_3 \) than \( e_2 \) in GDM. However, there also exists a comparison that \( SI_D(e_1, e_3) < SI_D(e_1, e_2) \), which indicates that a higher similarity degree between \( e_1 \) and \( e_3 \) compared with \( e_1 \) and \( e_2 \). It means that there are some differences between two indices in measuring the similarity degree of experts.

Based on the above analysis, we developed an integrated \( SI \) to measure the similarity degree between experts in GDM. It can be calculated as follows:

\[
SI(e_k, e_r) = \alpha SI_{AR}(e_k, e_r) + \beta SI_D(e_k, e_r).
\]

(15)

It can be seen that the integrated \( SI \) is a weighted combination of \( SI_{AR} \) and \( SI_D \), which can be utilized to investigate the inter-experts’ similarity level in GDM effectively. The parameters \( \alpha \) and \( \beta \) are introduced to weigh the two parts in SI, \( \alpha, \beta \in [0, 1] \) and \( \alpha + \beta = 1 \).

For equation (15), two special cases are considered as follows:

(1) If \( \alpha = 1, \beta = 0 \), it indicates that the similarity degree of experts does not consider the \( SI_D \) value. In some complicated or emergency circumstances, decision-making organizers can directly adopt index \( SI_{AR} \) to describe experts’ similarity relation.

(2) If \( \alpha = 0, \beta = 1 \), it represents that \( SI_{AR} \) is not taken into consideration. In this situation, organizers can more clearly delineate the distance between preference relationships of experts, especially when their \( SI_{AR} \) values are equal.

It means that parameters \( \alpha \) and \( \beta \) can be determined flexible depending on different cases. For general research GDM problems, \( SI_{AR} \) and \( SI_D \) are considered with equal status, when they are investigated in the inter-experts’ similarity level determining process. Thus, we have \( \alpha = \beta = 0.5 \). For example, we can calculate the \( SI \) value of \( e_1 \) and \( e_3 \) in Example 2:

\[
SI(e_1, e_3) = 0.5 \times 1 + 0.5 \times 0.7980 = 0.8990.
\]

(16)

Meanwhile, for equation (15), index \( SI \) possesses the following properties:

(1) \( 0 \leq SI(e_k, e_r) \leq 1 \). Specially, \( SI(e_k, e_k) = 1 \).

(2) \( SI(e_k, e_r) = SI(e_r, e_k) \).

In general, \( SI_D \) and other related indices are usually adopted to investigate the inter-experts’ similarity degree. Based on some distance equations, the distance degree between experts is described, as well as the similarity degree. However, when much experts and alternatives are included in the decision-making process, the above method always needs a great deal of calculations. Compared with \( SI_D \), index \( SI_{AR} \) is developed, which depends on alternative rankings of experts. In this method, decision-making organizers or leaders can easily identify the similarity degree of experts. But in some cases, as shown in Remark 2, there will exist equal score values of different alternatives. Besides, Example 2 means that there are some differences between two indices in measuring the similarity degree of experts. In conclusion, we consider the pros and cons of both approaches, we present the above integrated index \( SI \) to measure the similarity degree, along with two parameters to weigh indices \( SI_{AR} \) and \( SI_D \) in \( SI \).
3.2. An SI-Based Method Determining Experts’ Weights.

In this section, according to experts’ opinion transition probabilities information, an experts’ weights determining method is developed. As shown in equation (15), an integrated index \(SI(e_k, e_r)\) \((k, r = 1, 2, \ldots, m)\) is built to depict the inter-experts’ similarity level. It is worth noting that, the experts’ similarity degree implies opinion transition possibilities among them. This opinion transition process in GDM, from the perspective of information spread, can be regarded as a finite state space Markov chain to a certain extent. Referring to integrated index \(SI(e_k, e_r)\), inter-experts’ opinion transition probabilities could be calculated as follows:

\[
p_{kr} = \frac{SI(e_k, e_r)}{\sum_{r=1}^{m} SI(e_k, e_r)}, r = 1, 2, \ldots, m. \tag{17}
\]

Following the inter-experts’ opinion transition probabilities determining process, a Markov matrix \(P = (p_{kr})_{m \times m}\) can be obtained, which implies some opinion transfer information:

\[
\sum_{r=1}^{m} p_{kr} = 1, p_{kr} > 0. \tag{18}
\]

For this significant matrix \(P\), consists of \(k\) probability vectors, which is called a transition probability matrix. Paying attention to this Markov matrix \(P\), its limiting distribution is denoted as an unique fixed vector or point, which could be regarded as experts’ weights allocation results [33]. Furthermore, stationary vector of matrix \(P\), denoted as \(v = (v_1, v_2, \ldots, v_m)\), satisfies the condition that \(vP = v\). \tag{19}

For equation (19), a transposition operation is applied, then we get

\[
P^T v = 1v^T. \tag{20}
\]

Concerned about transposed matrix \(P^T\), corresponding to eigenvalue \(\lambda = 1\), the vector \(v\) is regarded as the fixed vector. Assume that \(w_h = v_k\), then the normalized experts’ weight vector \(w = (w_1, w_2, \ldots, w_m)^T\) can be achieved.

Our presented weights allocating method based on the inter-experts’ similarity degree. For GDM problems, during our decision-making weights allocating process, some unnecessary subjective prejudice could be avoided effectively. On the basis of SI-based way above, weights of experts \(e_1\), \(e_2\), and \(e_3\) in Example 2 can be obtained. Three required steps are shown as follows:

Step 1. Calculate inter-experts’ similarity degree values, denoted as \(SI(e_k, e_r)\) \((k, r = 1, 2, 3)\).

\[
SI(e_1, e_2) = 0.5 \times 0.5 + 0.5 \times 0.8465 = 0.6733, \\
SI(e_1, e_3) = 0.5 \times 1 + 0.5 \times 0.7980 = 0.8990, \\
SI(e_2, e_3) = 0.5 \times 0.5 + 0.5 \times 0.7597 = 0.6299. \tag{21}
\]

Step 2. Calculate the opinion transition probability between experts, and then we can eventually obtain the matrix of opinion transfer \(P = (p_{kr})_{3 \times 3} (k, r = 1, 2, 3)\).

\[
P = (p_{kr})_{3 \times 3} = \begin{bmatrix}
0.3888 & 0.2617 & 0.3495 \\
0.2923 & 0.4342 & 0.2735 \\
0.3555 & 0.2491 & 0.3954
\end{bmatrix}. \tag{22}
\]

Step 3. According to equations (19) and (20), experts’ standardized weight vector \(w = (w_1, w_2, w_3)^T\) can be obtained as

\[
w = (0.3474, 0.3111, 0.3415)^T. \tag{23}
\]

Finally, we can get the decision weights of experts in Example 2.

3.3. Detailed Decision Processes for Hesitant Fuzzy GDM. Referring to the SI-based way above, experts’ decision weights in GDM problems can be achieved. Then, according to the method presented in [34], the normalized collective evaluation \(H_c = (h_{ij,k})_{m \times n}\) can be calculated as follows:

\[
H_c = \frac{m}{\sum_{k=1}^{n} w_k h_{ij,k}} = \left(\frac{m}{\sum_{k=1}^{n} w_k h_{ij,k}}\right)_{m \times n}, \tag{24}
\]

where \(h_{ij,k}^{(l)} = \sum_{k=1}^{n} w_k h_{ij,k}^{(l)}, l = 1, 2, \ldots, n\).

As a result, we can get the collective evaluation of the group. Then, the score function method in Section 2.2 is used to obtain the alternative ranking, as well as the best alternative for the group.

In conclusion, the detailed decision process for hesitant fuzzy GDM is developed in the following algorithm.

4. GDM Problem: A Hydraulic Engineering Project Management

A hydraulic engineering project management event is provided in this part, according to the proposed GDM method considering hesitance and uncertainty. Firstly, the description of hydraulic engineering project management is given in Section 4.1. Then, an application of the presented hesitant fuzzy GDM model to hydraulic engineering project management is shown in Section 4.2.

4.1. The Description of Hydraulic Engineering Project Management. Because of some geographical and climatic change reasons, city Y has suffered much flood disasters during the past few years, especially in the rainy season. For improving the level of emergency supervision and reducing direct economic loss of hydraulic engineering facilities as soon as possible, city Y determined to formulate a hydraulic engineering project management plan.

According to local condition and special circumstances, city Y has researched abundant actions and decisions adopted in the other places. And then, the local government plans to redesign and improve the whole city’s hydraulic engineering facilities. After a deep investigation and much discussion, five possible management solutions are selected as alternatives, denoted as \(x_1, x_2, x_3, x_4, x_5\). Subsequently, numerous experts are involved, which are from the Institute of Water Resources Security Strategic, Flood Disaster...
Emergency Management Department, Risk Assessment Department, and the Research Institute of Universities, respectively. With the goal of improving the quality of decision, some important criteria are determined as follows:

(i) Affordable financial and material budget
(ii) The supervision of the potential disaster area
(iii) The impact to the local ecological environment
(iv) Time required for renovating the hydraulic engineering facilities
(v) Other necessary safeguards

Referring to above criteria, invited experts are suggested to use HFPRs to express their professional evaluations. Before the GDM process, the evaluation information should be processed according to Remark 1 in Section 2.1. At last, four experts’ evaluations are adopted to proceed the process. For these four experts $e_1, e_2, e_3, e_4$, their reliable evaluations can be denoted as $H_k = (h_{ij,k})_{n\times m}$, $h_{ij,k} = \{h_{ij,k}^{(p)} | l = 1, 2\}$, $k = 1, 2, \ldots, 4$.

4.2. An Application of Hesitant Fuzzy GDM. For an application of a hydraulic engineering project management event, the proposed hesitant fuzzy GDM approach, i.e., Algorithm 1, is adopted for achieving an optimal solution (alternative). Four steps are provided below as follows:

**Step 1.** Primary HFPRs utilized by involved experts to express their reliable evaluations as

$$H_1 = \begin{bmatrix}
{[0.5]} & {[0.7,0.3]} & {[0.9,0.6]} & {[0.7,0.6]} & {[0.5,0.2]} \\
{[0.3,0.7]} & {[0.5]} & {[0.6,0.8]} & {[0.6,0.7]} & {[0.1,0.2]} \\
{[0.1,0.4]} & {[0.4,0.2]} & {[0.5]} & {[0.8,0.9]} & {[0.4,0.1]} \\
{[0.3,0.4]} & {[0.4,0.3]} & {[0.2,0.1]} & {[0.5]} & {[0.4,0.2]} \\
{[0.5,0.8]} & {[0.9,0.8]} & {[0.6,0.9]} & {[0.6,0.8]} & {[0.5]} \n\end{bmatrix},$$

$$H_2 = \begin{bmatrix}
{[0.5]} & {[0.9,0.4]} & {[0.8,0.7]} & {[0.6,0.3]} & {[0.4,0.2]} \\
{[0.1,0.6]} & {[0.5]} & {[0.6,0.7]} & {[0.4,0.7]} & {[0.2,0.3]} \\
{[0.2,0.3]} & {[0.4,0.3]} & {[0.5]} & {[0.9,0.4]} & {[0.4,0.2]} \\
{[0.4,0.7]} & {[0.6,0.3]} & {[0.1,0.6]} & {[0.5]} & {[0.4,0.3]} \\
{[0.6,0.8]} & {[0.8,0.7]} & {[0.6,0.8]} & {[0.6,0.7]} & {[0.5]} \n\end{bmatrix},$$

$$H_3 = \begin{bmatrix}
{[0.5]} & {[0.9,0.6]} & {[0.8,0.7]} & {[0.8,0.9]} & {[0.4,0.1]} \\
{[0.1,0.4]} & {[0.5]} & {[0.7,0.9]} & {[0.5,0.3]} & {[0.3,0.2]} \\
{[0.2,0.3]} & {[0.3,0.1]} & {[0.5]} & {[0.4,0.2]} & {[0.4,0.2]} \\
{[0.2,0.1]} & {[0.5,0.7]} & {[0.6,0.8]} & {[0.5]} & {[0.2,0.1]} \\
{[0.6,0.9]} & {[0.7,0.8]} & {[0.6,0.8]} & {[0.8,0.9]} & {[0.5]} \n\end{bmatrix},$$

$$H_4 = \begin{bmatrix}
{[0.5]} & {[0.9,0.7]} & {[0.9,0.7]} & {[0.7,0.6]} & {[0.4,0.3]} \\
{[0.1,0.3]} & {[0.5]} & {[0.7,0.8]} & {[0.8,0.7]} & {[0.3,0.2]} \\
{[0.1,0.3]} & {[0.3,0.2]} & {[0.5]} & {[0.9,0.8]} & {[0.4,0.2]} \\
{[0.3,0.4]} & {[0.2,0.3]} & {[0.1,0.2]} & {[0.5]} & {[0.4,0.1]} \\
{[0.6,0.7]} & {[0.7,0.8]} & {[0.6,0.8]} & {[0.6,0.9]} & {[0.5]} \n\end{bmatrix} \tag{25}$$

**Step 2.** By equations (10) and (11), we can calculate $SI_{AR}(e_1, e_2)$ and $SI_{D}(e_1, e_4)$ of the experts $e_1$ and $e_4$, respectively. Then, the integrated $SL(e_k, e_l)$ can be calculated by equation (15). For example:

$$SI_{AR}(e_1, e_2) = 0.6, SI_{D}(e_1, e_4) = 0.843, SI_{D}(e_1, e_2) = 0.843. \tag{26}$$

For further analysis, the calculations are presented in matrix form as follows:

$$SI_{AR} = \begin{bmatrix} 1 & 0.6 & 0.6 & 1 \\ 0.6 & 1 & 0.6 & 1 \\ 1 & 0.6 & 0.6 & 1 \n\end{bmatrix},$$

$$SI_{D} = \begin{bmatrix} 0.8430 & 0.7608 & 0.8663 \\
0.7608 & 0.7728 & 0.8280 \\
0.8663 & 0.8280 & 0.7706 \n\end{bmatrix}, \tag{27}$$

$$SI_{AR} = \begin{bmatrix} 1 & 0.7215 & 0.6804 & 0.9331 \\
0.6804 & 0.8864 & 1 & 0.6853 \\
0.9331 & 0.7140 & 0.6853 & 1 \n\end{bmatrix}.$$  

**Step 3.** Use the $SI$-based approach to obtain experts’ weights. Based on values in (SI)$_{4\times 4}$, we can obtain the matrix of opinion transfer by equation (17), denoted as $P = (p_{kr})_{n\times m}$:

$$P = \begin{bmatrix}
0.2998 & 0.2163 & 0.2040 & 0.2798 \\
0.2172 & 0.3010 & 0.2668 & 0.2149 \\
0.2092 & 0.2726 & 0.3075 & 0.2107 \\
0.2800 & 0.2143 & 0.2056 & 0.3001 \n\end{bmatrix} \tag{28}$$

Then, by equations (19) and (20), we can get the weights of experts as

$$w_1 = 0.2519, w_2 = 0.2509, w_3 = 0.2456, w_4 = 0.2516. \tag{29}$$

**Step 4.** Calculate the collective evaluation $H_c = (h_{ij,c})_{n\times m}$ by equation (24). For simplicity, the final calculated values retain two decimal places.

$$H_c = \begin{bmatrix}
{[0.5]} & {[0.85,0.50]} & {[0.85,0.67]} & {[0.70,0.60]} & {[0.43,0.20]} \\
{[0.15,0.50]} & {[0.5]} & {[0.65,0.80]} & {[0.58,0.60]} & {[0.22,0.23]} \\
{[0.15,0.33]} & {[0.35,0.20]} & {[0.5]} & {[0.75,0.58]} & {[0.40,0.17]} \\
{[0.30,0.40]} & {[0.42,0.40]} & {[0.25,0.42]} & {[0.5]} & {[0.35,0.18]} \\
{[0.57,0.80]} & {[0.78,0.77]} & {[0.60,0.83]} & {[0.65,0.82]} & {[0.5]} \n\end{bmatrix} \tag{30}$$
Finally, by equation (6), alternatives scores of $H_c$ can be obtained as
\[
s_c(x_1) = 2.899, s_c(x_2) = 2.3636, s_c(x_3) = 1.965, s_c(x_4) = 1.8607, s_c(x_5) = 3.4118.
\] (31)

Thus, the alternative ranking of the group is $R(H_c): x_5 > x_1 > x_2 > x_3 > x_4$, and $x_5$ is regarded as the optimal alternative for the group.

5. Analyses and Comparisons
Some analyses and comparative discussions are provided in this part. In Section 5.1, a sensitivity analysis of the SI weight is provided. And then, the considerable impact of experts’ hesitancy and uncertainty features on decision-making results is investigated in Section 5.2.

5.1. Sensitivity Analysis of SI Weight. For parameters $\alpha$ and $\beta$ in equation (15), some related sensitivity analyses are provided for investigating their impact on experts’ weights. We have elaborated on the computational results for readability; please see Table 3 for details.

As a result, it can be easily found that caused by different values of parameters $\alpha$ and $\beta$, experts’ weights have an inevitable change. It is obvious that the weights of experts $e_1$, $e_2$, and $e_4$ are gradually get bigger with the change of parameters, while the $e_3$’ weight got smaller. Moreover, Figure 1 shows several results of experts’ weights with three different combinations of values $\alpha$ and $\beta$. We pay attention to the blue line in Figure 1, where $\alpha = \beta = 0.5$ is used to obtain the experts’ weights.

Due to diverse professional knowledge background and decision-making experience, experts’ weights always need be allocated reasonable according to practical GDM problems. However, in [29, 35–38], experts’ weights are determined in advance, while the weights in [29, 38] are even set as exactly equal. There is no doubt that the effectiveness of the consensus reaching process will be greatly suffered from these arbitrary acts. To solve this problem, our proposed opinion transition probabilities-based method is used for determining experts’ weights during the consensus reaching process. This method only relies on the similarity information between experts without any subjective bias. Besides, compared with the approach in [5], the similarity index in our model is improved effectively. Furthermore, expert’s decision weights can change dynamically according to several parameters, making our method can deal with different decision-making situations.

5.2. The Impact of Experts’ Hesitancy and Uncertainty on Alternative Ranking in GDM. The influence of experts’ hesitancy and uncertainty features on decision-making results is analyzed in this section. As previously mentioned, FPRs can be considered as one special presentation of HFPs with no hesitancy or uncertainty in experts’ evaluations. All possible values in HFE are same, that is, $h^{(1)}_{ij,k} = h^{(2)}_{ij,k} = \cdots = h^{(l)}_{ij,k}, \forall i, j = 1, \ldots, n$. For comparison, four experts here are suggested to use FPRs to express their evaluations. For example, four FPRs are denoted as $P_k = p_{ij,k} = h_{ij,k}^{(k)}$ ($k = 1, 2, 3, 4$):
\[
\begin{align*}
P_1 &= \begin{bmatrix}
0.5 & 0.7 & 0.9 & 0.5 & 0.4 \\
0.3 & 0.5 & 0.6 & 0.1 & 0.2 \\
0.3 & 0.4 & 0.2 & 0.5 & 0.4 \\
0.5 & 0.9 & 0.6 & 0.6 & 0.5
\end{bmatrix}, \\
P_2 &= \begin{bmatrix}
0.5 & 0.9 & 0.8 & 0.4 \ \\
0.1 & 0.5 & 0.6 & 0.4 & 0.2 \\
0.4 & 0.6 & 0.1 & 0.5 & 0.4 \\
0.6 & 0.8 & 0.6 & 0.6 & 0.5
\end{bmatrix}, \\
P_3 &= \begin{bmatrix}
0.5 & 0.9 & 0.8 & 0.4 \ \\
0.1 & 0.5 & 0.7 & 0.5 & 0.3 \\
0.2 & 0.3 & 0.5 & 0.4 & 0.4 \\
0.6 & 0.7 & 0.6 & 0.8 & 0.5
\end{bmatrix}, \\
P_4 &= \begin{bmatrix}
0.5 & 0.9 & 0.9 & 0.7 & 0.4 \\
0.1 & 0.5 & 0.7 & 0.8 & 0.3 \\
0.3 & 0.2 & 0.1 & 0.5 & 0.4 \\
0.6 & 0.7 & 0.6 & 0.6 & 0.5
\end{bmatrix}.
\end{align*}
\]

We can calculate $SI_{AB}(e_1, e_3)$ and $SI_D(e_2, e_1)$ of experts $e_1$ and $e_3$, respectively. Then, the integrated SI ($e_k, e_r$) can be also obtained. The calculations are presented in matrix form as follows:
\[
(SI_{AB})_{4 \times 4} = \begin{bmatrix}
1 & 0.6 & 0.4 & 0.6 \\
0.6 & 1 & 0.6 & 0.4 \\
0.4 & 0.6 & 1 & 0.6 \\
0.6 & 0.4 & 0.6 & 1
\end{bmatrix}, \\
(SI_D)_{4 \times 4} = \begin{bmatrix}
0.8860 & 0.8183 & 0.8775 \ \\
0.8860 & 1 & 0.8103 & 0.8586 \\
0.8183 & 0.8103 & 1 & 0.8000 \\
0.8775 & 0.8586 & 0.8000 & 1
\end{bmatrix}, \\
(SI)_{4 \times 4} = \begin{bmatrix}
0.7430 & 0.6092 & 0.7388 \\
0.7430 & 1 & 0.7051 & 0.6293 \\
0.6092 & 0.7051 & 1 & 0.7000 \\
0.7388 & 0.6293 & 0.7000 & 1
\end{bmatrix}.
\]

Then, we can get experts’ weights based on SI-based method as
\[
\omega_1 = 0.2523, \omega_2 = 0.2512, \omega_3 = 0.2461, \omega_4 = 0.2504.
\]

Finally, group evaluation can be obtained as
\[
P_\varepsilon = \begin{bmatrix}
0.5 & 0.85 & 0.85 & 0.70 & 0.43 \\
0.15 & 0.5 & 0.65 & 0.58 & 0.22 \\
0.15 & 0.35 & 0.5 & 0.75 & 0.40 \\
0.30 & 0.42 & 0.25 & 0.5 & 0.35 \\
0.57 & 0.78 & 0.60 & 0.65 & 0.5
\end{bmatrix}.
\]
Step 1. For a set of alternatives, denoted as $X = \{x_1, x_2, \ldots, x_n\}$ ($n \geq 2$). Subsequently, several experts with professional knowledge, denoted as $E = \{e_1, e_2, \ldots, e_m\}$, are invited in GDM. According to practical application, invited experts are suggested to adopt HFPRs to provide their evaluations. These professional evaluations can be expressed as $H_k = (h_{ij,k})_{n \times n}$ ($k = 1, 2, \ldots, m$).

Step 2. Calculating the inter-experts’ similarity degree of experts $e_k$ and $e_r$, denoted as $SI_{AR}(e_k, e_r)$, $SI_{DR}(e_k, e_r)$, based on equations (10) and (11). Then, adopt equation (15) to obtain the integrated $SI(e_k, e_r)$.

Step 3. Adopting the SI-based method provided in Section 3.2 to obtain the decision weights of experts in GDM.

Step 4. Based on equation (24), the whole group preference evaluation can be got, denoted as $H_c = (h_{ij,c})_{n \times n}$. Then, the score function method in Section 2.2 is used to obtain the alternative rankings, as well as the best alternative for the group.

Step 5. Close.

**Algorithm 1: Hesitant fuzzy decision-making approach.**

**Figure 1:** The experts’ weights with different values of $\alpha$ and $\beta$.

**Table 3:** Experts’ weights with different values of $\alpha$ and $\beta$.

<table>
<thead>
<tr>
<th>$\alpha$ and $\beta$</th>
<th>$w_1$</th>
<th>$w_2$</th>
<th>$w_3$</th>
<th>$w_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha = 0$, $\beta = 1$</td>
<td>0.2500</td>
<td>0.2500</td>
<td>0.2500</td>
<td>0.2500</td>
</tr>
<tr>
<td>$\alpha = 0.1$, $\beta = 0.9$</td>
<td>0.2504</td>
<td>0.2502</td>
<td>0.2491</td>
<td>0.2503</td>
</tr>
<tr>
<td>$\alpha = 0.2$, $\beta = 0.8$</td>
<td>0.2508</td>
<td>0.2504</td>
<td>0.2482</td>
<td>0.2507</td>
</tr>
<tr>
<td>$\alpha = 0.3$, $\beta = 0.7$</td>
<td>0.2511</td>
<td>0.2505</td>
<td>0.2473</td>
<td>0.2510</td>
</tr>
<tr>
<td>$\alpha = 0.4$, $\beta = 0.6$</td>
<td>0.2515</td>
<td>0.2507</td>
<td>0.2465</td>
<td>0.2513</td>
</tr>
<tr>
<td>$\alpha = 0.5$, $\beta = 0.5$</td>
<td>0.2519</td>
<td>0.2509</td>
<td>0.2456</td>
<td>0.2517</td>
</tr>
<tr>
<td>$\alpha = 0.6$, $\beta = 0.4$</td>
<td>0.2522</td>
<td>0.2510</td>
<td>0.2448</td>
<td>0.2520</td>
</tr>
<tr>
<td>$\alpha = 0.7$, $\beta = 0.3$</td>
<td>0.2526</td>
<td>0.2512</td>
<td>0.2439</td>
<td>0.2523</td>
</tr>
<tr>
<td>$\alpha = 0.8$, $\beta = 0.2$</td>
<td>0.2529</td>
<td>0.2514</td>
<td>0.2431</td>
<td>0.2526</td>
</tr>
<tr>
<td>$\alpha = 0.9$, $\beta = 0.1$</td>
<td>0.2533</td>
<td>0.2515</td>
<td>0.2423</td>
<td>0.2529</td>
</tr>
<tr>
<td>$\alpha = 1$, $\beta = 0$</td>
<td>0.2536</td>
<td>0.2517</td>
<td>0.2415</td>
<td>0.2532</td>
</tr>
</tbody>
</table>

**Table 4:** Detailed comparison between HFPRs and FPRs.

<table>
<thead>
<tr>
<th>Preference relation</th>
<th>Weight vector of experts</th>
<th>Alternative ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_k = (h_{ij,k})_{5 \times 5}$</td>
<td>$w = (0.2519, 0.2509, 0.2456, 0.2516)$</td>
<td>$x_3 &gt; x_1 &gt; x_2 &gt; x_4 &gt; x_2$</td>
</tr>
<tr>
<td>$P_k = (p_{ij,k})<em>{5 \times 5} = (h</em>{ij,k}^{(1)})_{5 \times 5}$</td>
<td>$w = (0.2523, 0.2512, 0.2461, 0.2504)$</td>
<td>$x_4 &gt; x_3 &gt; x_2 &gt; x_1 &gt; x_5$</td>
</tr>
<tr>
<td>$P_k = (p_{ij,k})<em>{5 \times 5} = (h</em>{ij,k}^{(2)})_{5 \times 5}$</td>
<td>$w = (0.2519, 0.2417, 0.2438, 0.2626)$</td>
<td>$x_2 &gt; x_3 &gt; x_1 &gt; x_4 &gt; x_3$</td>
</tr>
</tbody>
</table>
By equation (6), we can obtain the alternative scores of $P_c$ as follows:

$$s_c(x_1) = 3.3245, s_c(x_2) = 2.0998, s_c(x_3) = 2.1518,$$

$$s_c(x_4) = 1.8243, s_c(x_5) = 3.0996.$$  \hspace{1cm} (36)

Thus, the alternative ranking for the group is $R(P_c)$:

$$x_1 > x_2 > x_3 > x_4 > x_5.$$  

In a similar way, the results of FPRs $P_k = p_{ij,k} = h^{(k)}_{ij}$ ($k = 1, 2, 3, 4$) can also be obtained. For analyzing the influence of hesitancy and uncertainty in experts’ evaluations, some comparisons between HFPRs and FPRs are shown in Table 4.

It is obvious that the alternative rankings calculated from the FPRs result in some differences. Compared with the approaches developed in [5, 39, 40], FPRs are considered in their models, while the hesitant and uncertain behaviors of experts are ignored. In most practical problems, experts’ hesitancy and uncertainty features in their evaluations are widespread and have some profound impacts on decision-making results. In our model, these behaviors are involved in the whole GDM process. Besides, score function and similarity index of HFPRs are developed to apply to a realistic hydraulic engineering project management event. Compared with FPRs, the HFPRs-based GDM method can make decision results complete and reasonable.

### 6. Conclusion

GDM plays a significant role in project management events. In our model, a similarity-based hesitant fuzzy GDM approach is provided, as well as its application in hydraulic engineering project management.

Firstly, experts’ hesitancy and uncertainty features are considered, as a result, experts are suggested to adopt HFPRs to express their evaluations information. Besides, score function of HFPRs and distance measure method are proposed to apply to a hydraulic engineering project management event.

Secondly, an integrated SI is developed to measure the inter-experts’ similarity degree. The index is given combining the alternative ranking similarity degree and distance degree between HFPRs.

Thirdly, the SI-based experts’ weights determining method is developed for the GDM approach. An inter-experts’ opinion transition probabilities-based method is developed, as a result, the fixed vector which represents experts’ weights can be obtained. The experts’ weights are determined reasonably without any subjective factor.

Except for the hesitancy and uncertainty of experts, much psychological behaviors are involved in GDM problems, such as noncooperative behaviors [38, 41–43] and over-confidence behaviors [4, 44]. It is worth researching these behaviors in decision-making science. Along with the study of these decision behaviors, some related adjustments need to be made in consensus models [45, 46]. Due to the large number of participants invited, large-scale group decision-making (LSGDM) has been getting more and more attention [47–50]. Besides, social network analysis (SNA) has emerged as an effective method in GDM [51–54]. Based on SNA related methods, experts and their behavior characteristics can be clearly researched on corresponding social networks. Therefore, for LSGDM problems, whether the SNA-based methods can be integrated into the consensus reaching models is noteworthy in the future GDM researching contents.

### Data Availability

The data used to support the findings of this study are available within the article.

### Conflicts of Interest

The authors declare no conflicts of interest.

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