Research Article

Effects of Homogeneous-Heterogeneous Reactions on Maxwell Ferrofluid in the Presence of Magnetic Dipole along a Stretching Surface: A Numerical Approach

W. Tahir,1 Nesreen Althobaiti,2 N. Kousar,1 Sharifah E. Alhazmi,3 S. Bilal,1 and A. Riaz

1Department of Mathematics, AIR University, Sector E-9 Islamabad, Pakistan
2Department of Mathematics and Statistics, College of Science, Taif University, Taif 21944, Saudi Arabia
3Mathematics Department, Al-Qunfudah University College, Umm Al-Qura University, Mecca, Saudi Arabia
4Department of Mathematics, Division of Science and Technology, University of Education, Lahore 54770, Pakistan

Correspondence should be addressed to A. Riaz; arshad-riaz@ue.edu.pk

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The document is presented to investigate momentum and thermal attributes of Maxwell liquid flow over a stretchable surface by inducting ferrite particles along with taking account of homogeneous-heterogeneous reactions. Two types of ferrite particles, namely, nickel zinc ferrite (NiZnFe2O4) and magnetite ferrite (Fe3O4), are considered and non-Newtonian fluid represented by Maxwell model is decided as base fluid blood is used. To elaborate effective use of ferrite particles, magnetization is provided by placing a single dipole. Heat transfer aspects are estimated through Cattaneo-Christov model, which includes thermal relaxation phenomena. The governing equations are changed into ODEs setup by obliging suitable variables. Subsequently, a solution is attained numerically by implementing shooting and RK methods jointly. The impact of involved pertinent parameters on associated momentum and thermal profiles is analyzed in graphical and tabular manner. It is measured that large intensity of homogeneous reaction weakens the concentration field, while ferrohydrodynamic interaction declines the flow velocity.

1. Introduction

In heat and mass transfer analysis, researchers have shown prodigious attention due to their superb industries and engineering utilizations common in freezing nuclear reactor, exchanger of heat, refrigerator, etc. The most frequently used Fourier’s [1] law illustrates heat and mass transfer mechanism in a given medium. Fourier’s law yields temperature field of parabolic type, which implies that heat transport has infinite speed and initial disturbance, notifying all over the medium. To solve this difficulty in the heat transfer paradox, Fourier’s law needs modifications. Fourier’s law is remodeled involving thermal inertial aspects by Cattaneo [2] in 1948. After that Christov [3] gave new formulation possessing Oldroyd’s derivative of upper-convected to maintain the invariance, which is renowned as Cattaneo-Christov model. Straughan [4] manifested singularity of the solutions heeded for most popular Cattaneo-Christov equations. Diffusive aspects in the motion of nanoliquids were pondered by Hayat et al. [5] with handling Cattaneo-Christov model. Sandeep et al. [6] commence latest model of Kinetic viscosity in the stagnant flow with the help of interior heat source. Flow of Ag-ethylene glycol mixed convective nanofluid was studied by Muhammad et al. [7] with centered heated cavity. He showed comparison between two popular models and found higher rank of heat transferring in case of classical heat flux model instead of Cattaneo-Christov formulation. Khan and Alzahrani [8] measured the transportation of heat in Jeffry fluid by employing double diffusive model. By taking account of generalized Cattaneo-Christov scheme, Ijaz and Ayub [9] studied the novel features of nonlinearly convective movement of Maxwell nanofluid convinced by inclined cylinder. The variable thermophysical attributes of squeezing Newtonian liquid
were probed by Farooq et al. [10] by encompassing Cattaneo-Christov double diffusive law. Heat flux with modified heat flux model for flow past over a cone and a wedge with nonuniformly provided heat source/sink was examined by Anantha Kumar et al. [11], in which they anticipated that excessive heat transfer occurred in case of cone as compared to wedge.

The studies relevant to non-Newtonian fluids have acquired honorable consideration in recent time due to their tremendous utilization in industrial products. Single constitutive equation failed to describe such fluids; hence, to illustrate non-Newtonian fluids, diverse models have been recommended. Amidst them, differential type and rate type gained popular attention. Maxwell fluid lies in the division of rate type viscoelastic model, which includes the special factors of fluid relaxation time. The scrutiny of transportation of heat over prolonged surface has received fabulous importance owning to its abounding application in chemical and constructing process adding polymer processing, metal casting, ejection of copper lead, paper production and many more. Sakiadis flux generated due to the motion of Maxwell liquid over moving flat plate was planned by Sadeghy et al. [12]. The usage of boundary layer equation is obtained from 2-dimensional flow in the case of Maxwell fluid utilized by Harris [13]. With the implementation of finite difference method, Kumari and Nath [14] depicted Maxwell fluid in mixed convection stagnation point flow to enumerate numerical solution of boundary value problem. Noor [15] deliberated the thermophoresis effects in Maxwell hydrodynamic passed over a vertical covering. The impact of stretching sheet in stagnation flow of Maxwell liquid is detailed by Hayat et al. [16], obtaining series solution based on homotopy. Effect of exterior temperature in the existence of heat source/sink Mukhopadhay [17] summarized the 2-dimensional MHD non-Newtonian Maxwell fluid with unsteady case progress over an elongated sheet. In 2D steady flow of an upper-convected incompressible Maxwell liquid was analyzed under the consequences of MHD and thermal radiation by Subhas Abel et al. [18]. Mustafa [19] compared two types as numerical and homotopy solutions in the way of rotating flow in Maxwell fluid. Sequel of thermal radiation in the nomination of stagnation point flow of Maxwell liquid slopping in extendable sheet was checked by Mushtaq et al. [20]. Kara et al. [21] studied the dependence of pressure on relaxation time and viscosity in the flow of Maxwell liquid. Saleem and Sulochana [22] studied theoretical analysis of upper-convected Maxwell liquid flow by incorporating modified Fourier heat model. Magnetically affected flow of nonviscous liquid over a stretched surface with diffusion was examined by Kumar et al. [23].

Due to this fact, finding of heat-mass transfer the effects of reacting chemical plays a significance role due to its distinct engineering appeal like procedure of food, hydrometallurgical industry, ceramics formulations, yield medication utilizing freezing, etc. Chemical reaction divided either homogeneous or heterogeneous built upon where they exist in quantity of the fluid, or they exist on the surfaces of catalytic. When transport of reactions and catalyst is in the same/distinct phase, reactions are termed as homogeneous/heterogeneous. Basically, occurrence of homogeneous reaction is normally in the complete given phase, whereas the confined region, or within the boundary of phase, the heterogeneous reaction takes place. In viscous fluid flow, Merkin [24] explored the homogeneous and heterogeneous reactions. He utilized cubic autocatalysis for the case of homogeneous reaction and also heterogeneous reactions in catalyst surface. He also concludes that homogeneous and heterogeneous rate parameters could build multiple solutions. Equal diffusivities in case of homogeneous and heterogeneous reactions are detailed by Chaudhary and Merkin [25]. They also attained the valid solution for larger and smaller values of δ. They concluded that, for smaller δ, the homogeneous reactions become more extensive mechanism.

In a stagnation point, the flow with regard to an elongated surface is examined by Bachok et al. [26]. In his paper, they discussed that, with fluid having smaller kinetic viscosity, a boundary layer is obtained in such case when extending velocity is less as compared to speed of free-stream and also inverted-boundary layer attained when elongated velocity shoots up free stream velocity. Flow of nanofluid bounded by porous stretchable flow with homogeneous and heterogeneous reactions takes place as announced by Kameswaran et al. [27]. They discussed the steady state of this system when auto catalyst and same diffusion coefficients of reactants are used. Homogeneous-heterogeneous reactions are taken by Hayat et al. [28] in the exploration of unfreezing heat in stretched flow of carbon nanotubes. They revealed that the temperature dispersal diminishes immediately as radiation parameter augmented. Imtiaz et al. [29] revealed the outturn of heterogeneous and homogeneous reactions in the study of MHD flow occurring in curved elongated surface. They observed that enhancement in curvature parameter fluid velocity also is enhanced. Also, enhancements occur in surface heat transmission for sizeable Prandtl number. Hayat et al. [30] scrutinized Cattaneo-Christov heat flux effects in the liquid flow by taking into account the Jeffery fluid model with the utilization of heterogeneous and homogeneous reactions. They concluded that for immense variation in strength of homogeneous reaction parameter concentration decreases, it is spiral in case of heterogeneous reaction parameter. Some recent developments on analysis of heat and flow attributes of liquids in multiple physical domain along with consideration of heterogeneous-homogeneous reactions are enclosed in refs [31–34].

In 1965, Stephen [35] is the first one who invented ferrofluid. These fluids lie in the category of magnetic fluid carrying very low viscosity, which is manufactured by mixing deeply potent colloidal suspension of refine magnetic particles in to the nonregulating carrier fluid. Ferrofluid became strongly magnetized due to the attendance of magnetic field. An astonishing and appealing attribute of ferromagnetism upon temperature is its credence on magnetization and this thermomagnetic coupling assemble ferromagnetic fluid functioning in distinct practical applications in [36–39]. Odenbach [40] performed the reliability of normal liquids with suspension of ferrite particles in special and medical lines. Shliomis [41] marked the influence of magnetization on ferrofluid viscosity and regulated it by
the cooperation of reversible thermodynamic law. By supplying radiative heat energy, Rani Titus [42] performed thermal analysis on ferromagnetic fluid, which is moving over extendable sheet. Applicability of variable surface temperature over nonlinear stretched sheet with the attendance of non-uniform magnetic field in ferrofluid is characterized by Bognar [43]. He expressed that, with increment in ferromagnetic parameter or power law exponent, the skin friction multiplier decreases, while esteem of heat transfer is shot out. In the flow of ferrofluid, the variable strength of magnetic dipole prompt due to magnetic domain was examined by Anderson and Valnes [44]. They concluded that, as compared to hydrodynamics case, primary effects of magnetic field diminished the fluid motion. Majeed et al. [45] enhanced the heat transfer effects in ferromagnetic fluid stream over extended sheet. In his paper, they discussed the different aspects of boundary conditions such as prescribed heat flux and prescribed surface temperature. Incorporation of three distinct ferrite particles in carrier fluid was discussed by Nadeem et al. [46] who also discussed in detail the heat transfer phenomena with the utilization of convective heat transfer coefficient. They conclude with the Nusselt number intimate swelling behavior with respect to broad solid volume fraction. The results of convective flow of ferrofluid slipped in curved prolonged belt in the occupancy of magnetic dipole were exposed by Imtiaz et al. [47]. They described that, due to enhancement in radiation parameter and Biot number, temperature also is enhanced. Microscopic rotational attributes of magnetic nanoparticles in side ferrofluid are examined by Hussanan et al. [48]. They also mentioned that the profile near the flow domain accelerates and then decreases in the presence of mass transfer parameter.

The motivation of this work is to analyze the impact of Maxwell ferrofluid by inducing ferrite particles (Nickel Zinc Ferrite and Magnetite ferrite) due to its vast applications in medical and industrial sciences. The phenomenon underlines the utilization of the study being related to the curing of cancer disease. This effort is not done so far and will definitely provide direction to researchers. Here, blood is considered as represented by Maxwell model, and flow is considered over a linearly stretched surface by incorporating modified version of Fourier heat flux model along with assumption of heterogenous-homogenous reactions. A numerical investigation for the problem taken has been developed by using bvp4c tool. The results are validated through tabular justification made with the existing literature.

2. Mathematical Formulation

Let us assume electrically nonconducting two-dimensional incompressible Maxwell ferromagnetic fluid (base fluid Blood with suspended two ferrite particles Nickel Zinc Ferrite and Magnetite Ferrite) over a linearly elongated surface (Figure 1) with stretching velocity \( U_{w} (x) \). Dipole is placed at \( y \)-axis at a distance \( c \) from the sheet, \( T_{w} \) is the wall temperature, Curie temperature is \( T_{c} \) and temperature away from the sheet is \( T_{w, \infty} \). Also, we have taken that \( T_{w} > T_{c} \), fluid having no capability of being magnetization. The thermophysical properties of the fluid and particles are mentioned in Table 1.

The Ferro particles and base fluid are assumed to be in thermal equilibrium. The model for momentum and energy equation is utilized by Muhammad and Nadeem [49] with the addition of Maxwell fluid and Cattaneo-Christov heat flux model. Also Chaudhary and Merkin [25] proposed the following form for heterogeneous and homogeneous reactions:

\[
A + 2B \rightarrow 3B, \quad \text{rate} = k_{ab}b^{2},
\]

\[
A \rightarrow B, \quad \text{rate} = k_{a}a.
\]

Using boundary layer approximation, the constitutive equations are as follows:

\[
\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0,
\]

\[
\left( \frac{\partial u}{\partial x} + \nu \frac{\partial u}{\partial y} \right) + \lambda \left( u \frac{\partial^{2} u}{\partial x^{2}} + v \frac{\partial^{2} u}{\partial y^{2}} + 2\nu \frac{\partial^{2} u}{\partial x \partial y} \right) = \frac{\mu_{t}}{\rho_{nf}} \frac{\partial H}{\partial x} + \frac{\mu_{nf}}{\rho_{nf}} \frac{\partial^{2} u}{\partial y^{2}},
\]

\[
\left( \frac{\partial T}{\partial x} + \nu \frac{\partial T}{\partial y} \right) + \lambda \left( u \frac{\partial^{2} T}{\partial x^{2}} + v \frac{\partial^{2} T}{\partial y^{2}} + 2\nu \frac{\partial^{2} T}{\partial x \partial y} \right)
+ \frac{\mu_{t} T}{(\rho C)_{nf}} \frac{\partial^{2} T}{\partial y^{2}} = \frac{k_{nf}}{(\rho C)_{nf}} \frac{\partial^{2} T}{\partial y^{2}},
\]

\[
\frac{\partial a}{\partial x} + \nu \frac{\partial a}{\partial y} = D_{a} \frac{\partial^{2} a}{\partial y^{2}} - k_{ab}b^{2}.
\]

\[
\frac{\partial b}{\partial x} + \nu \frac{\partial b}{\partial y} = D_{b} \frac{\partial^{2} b}{\partial y^{2}} + k_{ab}b^{2},
\]
where $M$ is the magnetization, and $H$ is the magnetic field, $\mu_0$ is characterized as magnetic permeability, $\mu_{nf}$ is defined as dynamic viscosity, $\lambda_a$ represents relaxation parameter, $\lambda_c$ expresses thermal relaxation time of heat flux, $k_1$ and $k_2$ are the diffusion rates, and the respective diffusion coefficients are $D_A$ and $D_B$.

The boundary constraints are as follows:
\begin{align*}
  u(0) &= U_w = Sx, \\
  v(0) &= 0, \\
  T(0) &= T_w, \\
  D_A \frac{\partial a}{\partial y}(0) &= k_a a(0), \\
  D_B \frac{\partial a}{\partial y}(0) &= -k_a a(0), \\
  |u|_{y=-\infty} &\longrightarrow 0, \\
  |T|_{y=-\infty} &\longrightarrow T_{\infty} = T_c, \\
  |a|_{y=-\infty} &\longrightarrow a_0, \\
  |b|_{y=-\infty} &\longrightarrow 0.
\end{align*} 

(7)

Anderson and Valnes [44] showed that there is linear connection between temperature and magnetization:
\[
  M = K(T_c - T). \tag{8}
\]

For finding the similarity solutions of equations (3)–(6) along with boundary conditions given in equation (7), the following transformations are used:
\begin{align*}
  \psi(\eta, \xi) &= \frac{\mu_f}{\rho_f} \eta f(\xi), \\
  \theta(\eta, \xi) &= \frac{T_c - T}{T_c - T_w} = \theta_1(\xi) + \eta^2 \theta_2(\xi), \\
  a &= a_0 g(\xi), \\
  b &= a_0 h(\xi), \\
  \xi &= y \left( \frac{\rho_f S}{\mu_f} \right)^{1/2}, \\
  \eta &= x \left( \frac{\rho_f S}{\mu_f} \right)^{1/2}. \tag{9}
\end{align*}

Express the velocity components, which govern the flow problem as
\begin{align*}
  u &= Sx f'(\xi), \\
  v &= \left(S v_f \right)^{1/2} f(\xi). \tag{11}
\end{align*}

By substitution of equations (8)–(11) into equations (3)–(6), we obtain the following system of nonlinear ODEs as

\begin{table}[h]
\centering
\caption{Thermophysical properties of blood, nickel zinc ferrite and magnetite ferrite.}
\begin{tabular}{|l|c|c|c|}
\hline
Thermo-physical properties & $\rho$(kg/m$^3$) & $C_p$(J/kgK) & $k$(W/mK) \\
\hline
Blood & 1060.0 & 3770 & 0.52 \\
Nickel zinc ferrite & 4800 & 710 & 6.3 \\
Magnetite ferrite & 5180 & 670 & 9.7 \\
\hline
\end{tabular}
\end{table}
\[
\left( \frac{1}{(1 - \phi)^2} \right) \left( -\alpha_a f^2 \right) f''' - f'^2 + 2\alpha_a f f'' - \frac{2\beta \theta_1}{(\xi + \gamma)^4} = 0, \\
(12)
\]

\[
\left( \frac{k_{nf}/k_f}{(1 - \phi + \phi (\rho C_p)/(\rho C_p))} - \alpha_c f^2 \right) \frac{\theta_1'}{\Phi^2} + \Phi (f \theta_1' - \alpha_c f f' \theta_1') + \frac{2\lambda \beta f (\xi - \theta_1)}{(\xi + \gamma)^2} = 0, \\
(13)
\]

\[
\left( \frac{k_{nf}/k_f}{(1 - \phi + \phi (\rho C_p)/(\rho C_p))} - \alpha_c f^2 \right) \frac{\theta_2'}{\Phi^2} - \Phi \left( \frac{2 f' \theta_2 - f \theta_2'}{\Phi^2} - \frac{4 \alpha_c f^2 \theta_2}{(\xi + \gamma)^2} \right) - \frac{\lambda \beta (\theta_1 - c) \frac{2 f \theta_2}{(\xi + \gamma)^2}}{\Phi^2} = 0, \\
(14)
\]

\[
\frac{1}{\phi_c} g' + f g' - k_1 g h^2 = 0, \\
(15)
\]

\[
\frac{\delta h''}{\phi_c} + f h' + k_1 g h^2 = 0. \\
(16)
\]

The associated boundary conditions (7) take the form

\[
f (\xi) = 0, \\
f' (\xi) = 1, \\
\theta_1 (\xi) = 1, \\
\theta_2 (\xi) = 0, \\
g' (\xi) = k_2 g (\xi), \\
\delta h' (\xi) = k_2 g (\xi)at \xi = 0, \\
f' (\xi) \rightarrow 0, \\
\theta_1 (\xi) \rightarrow 0, \\
\theta_2 (\xi) \rightarrow 0, \\
g (\xi) = 1, \\
h (\xi) = 0, when \xi \rightarrow \infty, \\
(17)
\]

where \( \epsilon \) is the Curie temperature, \( \beta \) represents ferrohydrodynamic interaction parameter, \( Pr \) is the Prandtl number, \( \alpha_a \) is the Deborah number, \( \alpha_c \) is denoted as nondimensional thermal relaxation parameter, \( \lambda \) indicates viscous dissipation parameter, \( S_c \) shows Schmidt number, and \( k_1 \) and \( k_2 \) are the strength of homogeneous and heterogeneous reaction parameters.
In most applications, it is expected that diffusion rate of chemical species $A$ and $B$ be of comparable size, which makes us make further assumptions that diffusion coefficients $D_A$ and $D_B$ are equal and about to take $\delta = 1$. In this case, we can write
\[
g(\xi) + h(\xi) = 1. \tag{19}
\]

The equations (15) and (16) reduce to the following form:
\[
\frac{1}{\delta_c} g'' + f g' - k_1 g (1 - g)^2 = 0, \tag{20}
\]
subject to the following B.C.s:
\[
g'(\xi) = k_2 g(\xi), \quad \xi \longrightarrow 0,
\]
\[
g(\xi) = 1, \quad \xi \longrightarrow \infty. \tag{21}
\]

The physical quantities of interests are
\[
C_f = \frac{-2\tau_w}{\rho_{nf} U_w}, \quad \text{Nu}_x = \frac{xq_w}{k_f(T_c - T_w)}, \tag{22}
\]
with surface shear stress and surface heat flux being
\[
\tau_w = \mu_{nf} \left( \frac{\partial u}{\partial y} \right) |_{y=0},
\]
\[
q_w = -k_{nf} \left( \frac{\partial T}{\partial y} \right) |_{y=0}. \tag{23}
\]

The dimensionless expressions of equation (21) are attained

\[
y_1' = y_2, \tag{26}
\]
\[
y_2' = y_3, \tag{27}
\]
\[
y_3' = \left[ \frac{1}{1/(1 - \phi)^{2.5} * (1 - \phi + \phi \gamma_p/r_f)} - \alpha_\gamma y_{2/2} \right] \left[ y_2^2 - y_2 y_3 + y_3^2 - 2\alpha_\gamma y_1 y_2 y_3 + \frac{2\gamma y_4}{(\xi + \gamma)^2 * (1 - \phi + \phi r_f/r_f)} \right] \tag{28}
\]
\[
y_4' = y_5, \tag{29}
\]
\[
y_5' = \left( \frac{1 - \phi + \phi \gamma_p}{k_{nf}/k_f} \right) \left( 1 - \frac{1}{\Pr \alpha_p y_2' \gamma} \right) \Pr \alpha_p (y_1 y_2 y_3 - y_1 y_5) - \frac{2\gamma y_4 (y_4 - \epsilon)}{(\xi + \gamma)^2}, \tag{30}
\]
\[
y_6' = y_5, \tag{31}
\]

\[\frac{1}{2} \Re_{\infty}^{1/2} C_f = \frac{1}{1 - \phi} \frac{f''(0)}{2}, \tag{24}\]

\[\Re_{\infty}^{1/2} \text{Nu}_x = \frac{k_{nf}}{k_f} \left( \theta_1'(0) + \eta^2 \theta_2(\xi) \right), \]
where $\Re = (Sx^2/\nu_{nf})$.

3. Solution Procedure

This segment is presented to disclose attributes of Maxwell liquid by inducing two different types of the ferrite particles investigated. PDEs obtained in the process of mathematical formulation are turned out into a set of nonlinear ODEs with the usage of similarity transforms, and then these equations are handled numerically by using Runge-Kutta and shooting method. In shooting method, the formulated boundary-value problems are figured out in the form of initial value problems by choosing appropriate finite values of $\xi$ (say $\xi_\infty$). Initially, the equations (12)–(14) and (20) are converted in first-order system by selecting the set of new variables. For solving the current system with the support of shooting method, one must need the guess of missing values, which will hit the boundary conditions at every end. The acquired solution is valid or not relying upon velocity, temperature, and concentration profiles. Profiles are needed to approach boundary conditions at $\xi = \xi_\infty$ asymptotically.

To transform boundary-value problems defined in equations (12)–(14), and (20) into initial value problems, suppose that
\[
(y_1, y_2, y_3, y_4, y_5, y_6, y_7, y_8, y_9 = f, f', f'', \theta_1, \theta_1', \theta_2, \theta_2', g, g'). \tag{25}
\]

Subjecting these values, our system will become
f(x) = \left\{ \begin{array}{ll}
1 - \phi + \phi p_{Cp} / p_{Cp} & \text{(i)} \\
1 - p_{Cp} / p_{Cp} & \text{(ii)} \\
1 & \text{(iii)}
\end{array} \right.

\text{with prescribed conditions}

y_1(0) = 0,
y_3(0) = \omega_1 \text{ (unknown initial condition)},
y_4(0) = 1,
y_5(0) = \omega_2 \text{ (unknown initial condition)},
y_6(0) = 0,
y_7(0) = \omega_3 \text{ (unknown initial condition)},
y_9(0) = k_2 y_8(0),
y_{10}(0) = \omega_4 \text{ (unknown initial condition)}.

As we are employing RK method in association with shooting scheme, so we required fulfillment of boundary conditions at infinity. The important factor here is to select \( \xi_\infty \) values. The \( \xi_\infty \) estimation is selected to opposite estimation of the limit at \( \xi_\infty \) for specified set of variables. The procedure is repeated until we obtain convergent solution with in an acceptable limit, i.e., \( 10^{-5} \).

4. Graphical Analysis

In this section, we have explained the impact of flow variables on associated profiles.

4.1. Impact Physical Parameters on Velocity Profile. Figures 2–5 represent change in velocity profile against ferrohydrodynamic interaction (\( \beta \)), Deborah number (\( \alpha_\eta \)), thermal relaxation parameter (\( \alpha_c \)) and Prandtl number (\( \text{Pr} \)). Variation in velocity field against ferrohydrodynamic parameter (\( \beta \)) is probed in Figure 2. By increasing (\( \beta \)) velocity profile delineates is observed. The reason behind this fact is that by incrementing (\( \beta \)) interaction between magnetic field and particles also increases and more solid particles attracted towards magnetic field also due to presence of Lorentz forces velocity that is reduced. Also, momentum boundary layer decreases in case of Magnetite ferrite (Fe\(_2\)O\(_4\)) because magnetite ferrite more magnetized as compared to Nickel Zinc Ferrite so more attracted towards the magnetic field and velocity slow down quickly as compared to Nickel Zinc ferrite. Figure 3 interprets the velocity profile against Deborah number (\( \alpha_\eta \)). It is monitored that with elongation in (\( \alpha_\eta \)) fluid velocity decelerates. As Deborah number characterizes the fluidity of materials, so when we enhance (\( \alpha_\eta \)) the fluid behaves like solid, and more viscosity is inherited, which reduces the velocity distribution. Figure 4 discusses the behavior of velocity distribution against thermal relaxation parameter (\( \alpha_c \)). It is observed that by increasing (\( \alpha_c \)) velocity increases because by increasing thermal relaxation parameter (\( \alpha_c \)) momentum of fluid increases and an outcome average kinetic energy increases, which uplifts velocity. It is also focused that uplifting profile of velocity seemed in case of Magnetite ferrite (Fe\(_2\)O\(_4\)). Sketch for momentum distribution against Prandtl number (\( \text{Pr} \)) is scrutinized in Figure 5. Positive trend in velocity found against (\( \text{Pr} \)). Since (\( \text{Pr} \)) shows the ratio of momentum to thermal diffusivities, so with increment in (\( \text{Pr} \)) the momentum diffusivities are enhanced, and fluid velocity profile exceeded, which is higher in case of Magnetite ferrite (Fe\(_2\)O\(_4\)) as compared to Nickel Zinc ferrite (NiZnFe\(_2\)O\(_4\)).

4.2. Impact Physical Parameters on Temperature Profile. Temperature profile against ferrohydrodynamic interaction (\( \beta \)), Deborah number (\( \alpha_\eta \)), thermal relaxation parameter (\( \alpha_c \)) and Prandtl number (\( \text{Pr} \)) is observed in Figures 6–9. Figure 6 describes the influence of ferrohydrodynamic interaction (\( \beta \)) on temperature field. This parameter shows interlink between the motion of fluid and action of active magnetic field. It is observed that increment in (\( \beta \)) elevates the frictional heating in the fluid layer, which is directly accountable for the augmentation in temperature profile. The augmented profile of temperature was observed in case of Magnetite ferrite (Fe\(_2\)O\(_4\)). Sketch for momentum distribution against Prandtl number (\( \text{Pr} \)) is accomplished in Figure 7. The effect of \( \alpha_\eta \) on temperature profile is observed that higher Deborah number expresses higher relaxation time, which protests fluid motion and is responsible of generating heat and increasing thermal boundary layer thickness. Figure 8 displays the inverse connection between thermal relaxation parameter (\( \alpha_c \)) and temperature field. As soon as we increase \( \alpha_c \) the decorum of temperature field demolished, because materials particles require exciting time to confer energy to their neighboring particles. Here, it is noticed that lower temperature is built in case Magnetite ferrite (Fe\(_2\)O\(_4\)). Sketch of temperature field with escalating values of Prandtl number can be seen in Figure 9. Escalating values of Prandtl number decline temperature profile, which concludes that, physically, Prandtl number contains the expression of...
momentum diffusivity to thermal diffusivity, due to which it is obvious that as Prandtl number varies, thermal diffusivity is going to be decreased, which means diffused heat slow.

4.3. Impact Physical Parameters on Concentration Profile.
The behavior of Schmidt number ($S_c$) on concentration field is depicted in Figure 10. Schmidt number shows identical behavior against concentration field. As in case of Schmidt number, there is a ratio between momentum and mass diffusivity, so for advancement ($S_c$) corresponds to
reduction in diffusivity and as consequence the concentration field is enhanced. Figure 11 discloses the relation between strength of homogeneous reaction parameter \( k_1 \) and concentration field, which is quite opposite as compared to Schmidt number. From the figure, it is obvious that inclination in \( k_1 \) results in declination in concentration field. This kind of behavior is due to the fact that during homogeneous reaction the reactants are consumed. Graph of concentration field against different values of strength of heterogeneous reaction parameter \( k_2 \) is shown in Figure 12. It is concluded that magnifying values of strength of heterogeneous parameter reduce concentration field. Also, it is perceived that in both graphs 4.10 and 4.11 higher concentration field is achieved by Nickel Zinc ferrite \((\text{NiZnFe}_2\text{O}_4)\).

4.4. Impact Physical Parameters on Skin Friction Parameter and Nusselt Number. Graphical results between ferrohydrodynamic interaction \( \beta \), Prandtl number \( \text{Pr} \) and skin friction coefficient are taken out in Figure 13. It is delineated that progressive attribute occurred in case of ferrohydrodynamic interaction \( \beta \) whereas reverse response is observed in case of Prandtl number \( \text{Pr} \) against skin friction. This situation arises because due to enhancing Prandtl number, the number rate of transferring momentum also increases due to increasing kinetic energy, which in term reduces skin friction at wall. Figure 14 shows the direct relation between ferrohydrodynamic interaction \( \beta \), Prandtl number \( \text{Pr} \) and heat transfer coefficient. It is inferred that, by broadening Prandtl number \( \text{Pr} \), Nusselt number also blooms. Physically, we can explain this phenomenon as by increasing Prandtl number momentum diffusivity also magnifies, which give rise to kinetic energy and convective heat transfer rate which is responsible of augmenting Nusselt number. Expanding profile seemed to occur in case of Magnetite ferrite \((\text{Fe}_3\text{O}_4)\) because thermal...
conductivity of Magnetite ferrite is higher as compared to Nickel Zinc ferrite. Variation in mass flux coefficient ($g'(0)$) against strength of homogeneous parameter and Schmidt number ($S_c$) is evaluated in Figure 15. It is depicted that by increasing strength of homogeneous parameter ($k_1$) change in mass distribution with in flow domain is enhanced, whereas contrary behavior is depicted in case of up surging magnitude of Schmidt number ($S_c$). Incrementing amplitude of Sherwood number ($g'(0)$) at wall against strength of homogeneous parameter ($k_1$) justified that by augmenting ($k_1$) homogeneous mixture is generated and accumulation of particles takes place which as an outcome raises the concentration magnitude. In addition Schmidt number ($S_c$) is the ratio of momentum to mass diffusivity so, by increasing Schmidt number ($S_c$) momentum of fluid molecules raises and concentration field reduces. Hence, it is divulged that magnitude of ($g'(0)$) is more in case of Nickel Zinc ferrite. Elevation in strength of heterogeneous reaction and Schmidt number ($S_c$) against mass flux coefficient is interrogated in Figure 16. The figure shows that strength of heterogeneous reaction parameter ($k_2$) upgrades the mass flux coefficient, while a contrasting behavior is noticed in case of Schmidt number ($S_c$). This mechanism is explained as in case of incrementing strength of heterogeneous reaction ($k_2$), the heterogeneous reaction give rise to and accretion of particles held, which in turn increases mass flux coefficient. Also ($S_c$) contain inverse relation with mass diffusivity so when we augmented ($S_c$) mass flux reduces. The skin friction coefficient, Nusselt number and mass flux coefficient are displayed in tabulated form with increasing values of ferrohydrodynamic interaction ($\beta$), Prandtl number (Pr), strength of homogeneous and heterogeneous parameters ($k_1$ and $k_2$) and Schmidt number ($S_c$). In Table 2 it is observed that by increasing ferrohydrodynamic interaction parameter, skin friction increases whereas contrary effect is observed in case of increasing values of Prandtl number. Enhancement in skin friction is observed against ferrohydrodynamic interaction ($\beta$) due to the generation of excessive resistive forces between molecules of fluid. In addition it is worthy to mention that skin friction is more increases in case of Nickel Zinc ferrite as compared to Magnetite ferrite.

In Table 3 magnifying values of Prandtl number (Pr) cause enhancement in ($-\theta'(0)$) as perceived. Table 4 represents the response of strength of homogeneous and heterogeneous parameters ($k_1$ and $k_2$) against ($g'(0)$). Table 5 shows that our present work shows excellent agreement with Rashidi and Noor results that increasing Pr number also increases Nusselt number. Table 6 shows the grid independence test for different values of tolerance by taking $\beta = 0.1$, $\lambda = 0.01$, $\gamma = 1.0$, Pr = 21 It suggests that, by reducing tolerance, Nusselt number and Skin friction show convergent behavior.

![Figure 14: Nusselt number with fluctuation in $\beta$ and Pr.](image1)

![Figure 15: Mass flux coefficient with fluctuation in $k_1$ and $S_c$.](image2)

![Figure 16: Mass flux coefficient with fluctuation in $k_2$ and $S_c$.](image3)

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>Pr</th>
<th>$f''(0)$</th>
</tr>
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<td>21</td>
<td>0.0296</td>
</tr>
<tr>
<td>0.3</td>
<td>22</td>
<td>0.0625</td>
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<tr>
<td>0.5</td>
<td>23</td>
<td>1.1261</td>
</tr>
<tr>
<td>0.1</td>
<td>21</td>
<td>0.0336</td>
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<tr>
<td>0.5</td>
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<td>1.1335</td>
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</table>

Table 2: Values of $f''(0)$ by altering values of $\beta$ and Pr.

<table>
<thead>
<tr>
<th>$(NiZnFe_2O_4)$</th>
<th>Pr</th>
<th>$f''(0)$</th>
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<tr>
<td>0.1</td>
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</tr>
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<td>0.3</td>
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<tr>
<td>0.1</td>
<td>21</td>
<td>0.0336</td>
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<tr>
<td>0.5</td>
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<td>1.1335</td>
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</table>

In Table 3 magnifying values of Prandtl number (Pr) cause enhancement in ($-\theta'(0)$) as perceived. Table 4 represents the response of strength of homogeneous and heterogeneous parameters ($k_1$ and $k_2$) against ($g'(0)$). Table 5 shows that our present work shows excellent agreement with Rashidi and Noor results that increasing Pr number also increases Nusselt number. Table 6 shows the grid independence test for different values of tolerance by taking $\beta = 0.1$, $\lambda = 0.01$, $\gamma = 1.0$, Pr = 21 It suggests that, by reducing tolerance, Nusselt number and Skin friction show convergent behavior.
### Table 3: Observations of \((-\theta' (0))\) by altering values of \(\beta\) and \(Pr\).

<table>
<thead>
<tr>
<th>((\text{NiZnFe}_2\text{O}_4))</th>
<th>(\beta)</th>
<th>(Pr)</th>
<th>((-\theta' (0)))</th>
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<tr>
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<table>
<thead>
<tr>
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<th>(\beta)</th>
<th>(Pr)</th>
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### Table 4: Obtained values of \(g' (0)\) by altering values of \(k_1\), \(k_2\) and \(S_c\).

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<th>(k_2)</th>
<th>(S_c)</th>
<th>(g' (0))</th>
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</table>

<table>
<thead>
<tr>
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<th>(k_1)</th>
<th>(k_2)</th>
<th>(S_c)</th>
<th>(g' (0))</th>
</tr>
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</table>

### Table 5: Agreement of results for heat flux coefficient by considering \(\beta = \lambda = \epsilon = \alpha_c, \alpha_a, S_c, k_1\) and \(k_2 = 0\).

<table>
<thead>
<tr>
<th>(Pr)</th>
<th>Rashidi [50]</th>
<th>OHAM results (Re^{1/2}X^{-1}Nu) Muhammad and Nadeem [49]</th>
<th>BVph2-midpoint (Re^{1/2}X^{-1}Nu) Muhammad and Nadeem [49]</th>
<th>Present work</th>
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5. Conclusion

Current communication explicates phenomenon of Maxwell ferro liquid flow over a linear stretching sheet with heterogeneous-homogeneous reactions. The impact of important parameters related to flow problem like the strength of chemical reaction parameters \( k_1 \) and \( k_2 \), ferro hydrodynamic interaction \( \beta \), Schmidt number \( S_c \), Deborah number \( \alpha_d \), and thermal relaxation time \( \alpha_t \) is graphically analyzed for momentum, thermal and concentration fields as well as for skin friction and heat flux coefficients. Significant observations are deliberated as follows:

(i) Ferrohydrodynamic interaction parameter \( \beta \) cut down the velocity profile while accelerating temperature field and concentration field.

(ii) Variation in Deborah number \( \alpha_d \) declined velocity profile whereas boosting temperature field.

(iii) Intensifying thermal relaxation parameter \( \alpha_t \) results in declination in temperature profile.

(iv) By augmenting, Prandtl number shows flourishing behavior towards velocity and shrinkage in temperature profile.

(v) Growing values of Schmidt number \( S_c \) generate reduction in concentration in field.

(vi) Strength of homogeneous reaction parameter \( k_1 \) weakens concentration field [51].

Data Availability

Data can be obtained upon request to the authors.

Conflicts of Interest

The authors declare that there are no conflicts of interest.

Authors’ Contributions

W. Tahir has done the drafting, Nesreen Althobaiti has presented the validity of the results, N. Kousar has produced the solutions and graphs, Sharifah E. Alhazmi has helped in conceptualization and language improvement, S. Bilal has done the methodology, and A. Riaz has supervised and improved the results.

References


