Research Article

A Novel Teaching-Learning-Based Optimization with Laplace Distribution and Experience Exchange

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Teaching-Learning-Based Optimization (TLBO) algorithm is an evolutionary powerful algorithm that has better global searching capability. However, in the later period of evolution of the TLBO algorithm, the diversity of learners will be degraded with the increasing iteration of evolution and the smaller scope of solutions, which lead to a trap in local optima and premature convergence. This paper presents an improved version of the TLBO algorithm based on Laplace distribution and Experience exchange strategy (LETLBO). It uses Laplace distribution to expand exploration space. A new experience exchange strategy is applied to make good use of experience information to identify more promising solutions to make the algorithm converge faster. The experimental performances verify that the LETLBO algorithm enhances the solution accuracy and quality compared to original TLBO and various versions of TLBO and is very competitive with respect to other very popular and powerful evolutionary algorithms. Finally, the LETLBO algorithm is also applied to parameter estimation of chaotic systems, and the promising results show the applicability of the LETLBO algorithm for problem-solving.

1. Introduction

Evolutionary algorithms (EAs) are population-based optimization search techniques based on swarm intelligence (SI) and Darwin's theory of evolution which have been widely used in all kinds of complex real-valued optimization problems [1]. There are kinds of typical evolutionary optimization algorithms, such as PSO [2], DE [3], GSO [4], ABC [5], FSA [6], WCA [7], CS [8], DSA [9], ICA [10], BSA [11], ISA [12], and HTS [13].

Recently, Rao et al. [14] proposed the Teaching-Learning-Based Optimization (TLBO) algorithm. In the TLBO algorithm, the optimal individual is marked as "teacher" and the other individuals are "students", which simulates two behaviors of "Teacher Stage" and "Learner Stage" in a class and has advantages both simple computation and few controlling parameters except for the common controlling parameters to make it easy to implement and fast convergence speed [15]. TLBO has been paid much more attention and had a successful application to many real-world optimization problems [16–21].

However, relevant research shows that the higher the dimension of the problem to be optimized is, the easier the algorithm is to slow convergence. According to the above reason, a lot of TLBO variants are proposed in recent years. Venkata Rao et al. [22] incorporate an elitist strategy into the TLBO algorithm to identify its effect on the exploration and exploitation capacity. Feng Zou [23] maintains the diversity of the population in the teacher phase by using a ring neighborhood search. Farahani [14] presented the mutation operations of the differential and interactive to improve the exploration capability and maintain diversity. Debao [24] combines to renew individuals according to a random probability, and the remaining individuals have their positions renewed by learning knowledge from the best individual, the worst individual, and another random individual of the current generation. Feng Zou et al. [25] uses differential evolution (DE) operators to increase the diversity and repulsion learning method to make learners search knowledge from different directions. Sai H C [26] uses a confined TLBO (CTLBO) to eliminate the teaching factor and introduces eight new mutation strategies to the

teacher phase and four new mutation strategies to the student phase to enhance the algorithm's exploitation and exploration capabilities. Jiang et al. [27] designed the neighborhood topology and the fitness-distance-ratio mechanism to maintain the exploration ability of the population.

Although the aforementioned TLBO variants have shown a better performance than the original TLBO, there still exists a smaller scope of solution in the later stages of evolution. In addition, the blindness of the random self-learning method from another random learner in the Learner Stage of the original TLBO weakens the exploitation ability of the individuals. To address these issues, this paper proposes a novel version of TLBO that is augmented with Laplace distribution and Experience exchange strategy (LETLBO). The major contributions are as follows in this paper:

(i) Laplace distribution is introduced instead of standard Laplace distribution, which improves the mutation ability and broadens the scope of solutions in the later stage of evolution

(ii) Experience exchange strategy is designed in Learner Stage, which decreases the blindness of random self-learning method and improves the exploitation ability of the individuals

The paper firstly introduces the background and application of the original TLBO algorithm, then analyses the strengths and weaknesses of the original TLBO and the variant TLBO algorithms, and finally proposes a novel version of TLBO. In Section 2, the original TLBO algorithm is described. Section 3 presents TLBO with Laplace distribution and Experience exchange strategy (LETLBO). In Section 4, the results of LETLBO and related optimization algorithms are analysed via a comparative study. Section 5 applies the LETLBO algorithm to parameter estimation of chaotic systems. This paper concludes with Section 6.

2. Teaching-Learning-Based Optimization

The section aims to give a brief description of the TLBO algorithm proposed by Rao. The TLBO algorithm is a successful human-inspired method to mimic the teaching-learning ability between teacher and learners. The working of this algorithm is carried out during two key parts: Teacher Stage and Learner Stage. The Teacher Stage is referred to as learning from the teacher while the Learner Stage is considered as a mutual learning process between learners.

2.1. Teacher Stage. In Teacher Stage, the purpose is to increase their average grades depending on the teacher of the class to enhance the mean grade of the whole class. The fresh learner operator is recognized as follows:

$$X_{i,\text{new}} = X_i + r(X_{\text{teacher}} - T_F X_{\text{mean}}), \tag{1}$$

where X_i ($i = 1, 2, \ldots, N$, N is the number of learners) is a vector of learner; $X_{i,new}$ is a newly generated individual according to X_i; $X_{teacher}$ is the best individual of the current

population, X_{mean} is the mean of the learners; r indicates an arbitrary number which is distributed randomly in [0,1]; the parameter TF is a teaching factor deciding the value of the X_{mean} to be changed. The value of T_F is either 1 or 2, indicating the learner learns something or nothing from the teacher, respectively. It is obtained as rule in

$$T_F = \text{round}[1 + \text{rand}(0, 1)]. \tag{2}$$

2.2. Learner Stage. In Learner Stage, the learners obtain their knowledge by interacting with each other. A fresh learner $X_{i,new}$ is evaluated through a random learner X_j using the following expression:

$$X_{i,\text{new}} = \begin{cases} X_i + r(X_i - X_j) \text{ if } f(X_i) < f(X_j), \\ X_i + r(X_j - X_i) \text{ if } f(X_j) < f(X_i), \end{cases} \tag{3}$$

where i and j are mutually exclusive integers selected from 1 to N, N is the population size, r still indicates an arbitrary number which is distributed randomly in [0,1], and f is the fitness function, which is focused on minimum targets. A learner X_i learns something new if the other learner X_j has more knowledge than him or her.

3. A Novel Teaching-Learning-Based Optimization

There are some methods to improve the overall performance of TLBO by modifying the updating process of learners, but so far, there has been no method to introduce Laplace distribution and Experience exchange strategy of other learners. In this section, a novel version of TLBO is introduced which adopts Laplace distribution and Experience exchange strategy (LETLBO) is introduced. Figure 1 shows the flowchart of the LETLBO algorithm.

3.1. Motivation. The motivation of this method is to use the communication experience of other learners to reduce the blindness of random self-learning methods and improve learners' grades or scores, thus improving the overall performance of TLBO which both incorporates his own experience and contributes to his experience. In the real classroom teaching and learning process, learners can learn from the learning experience through mutual communication and discussion to improve their grades or scores between them. In this paper, the exchange experience of other learners is introduced into TLBO to make good use of experience information and identify more promising solutions to make the algorithm converge faster.

3.2. Laplace Distribution. Laplace distribution [28] is a continuous probability distribution that probability density function in dimension is as follows:
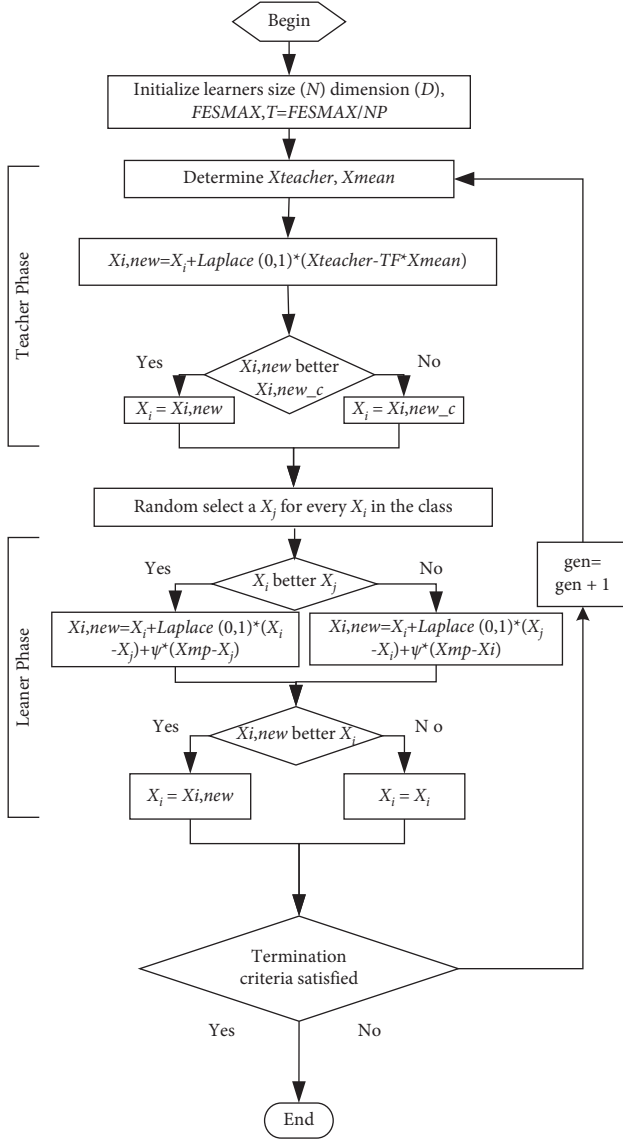
FIGURE 1: Flowchart of LETLBO algorithm.

$$\text{Laplace } (X|\mu, b) = \frac{1}{2b} \exp\left(-\frac{|X - \mu|}{b} \right)$$

$$= \frac{1}{2b} \begin{cases} \exp\left(-\frac{\mu - X}{b} \right), & if\ X < \mu, \\ \\ \exp\left(-\frac{X - \mu}{b} \right), & if\ X \geq \mu, \end{cases} \quad (4)$$

where μ is the position parameter; b is the scale parameter. It is the standard Laplace distribution when the parameter μ equals zero while the parameter b equals 1. Figure 2 shows the probability density curves of standard Gauss distribution, standard Uniform distribution, and standard Laplace distribution, respectively. As can be seen from Figure 2, the peak of Laplace distribution at the origin is the highest of three different distributions while the velocity of the long flat shape near to zero is the slowest. Therefore, if the variation
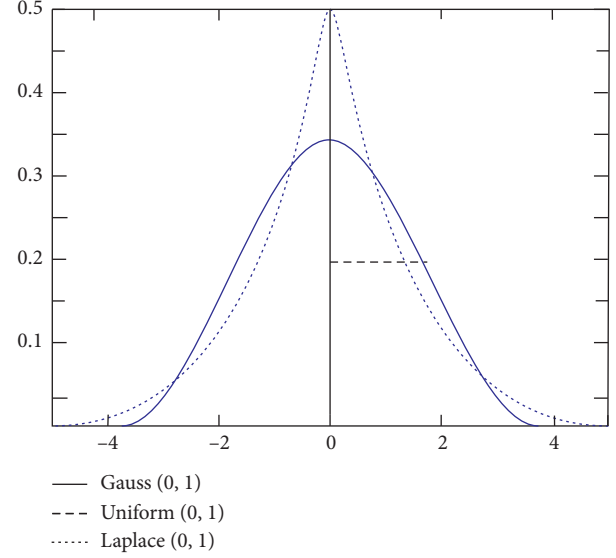

FIGURE 2: Probability density curve of standard Gauss, Uniform, and Laplace distribution.

ability of Laplacian distribution is used in the teacher stage and the learner stage, its interference ability or self-regulation ability is the strongest in the three different distributions; then the basic TLBO algorithm is more likely to jump out of local optimum. The updating equation of the Learner Phase is as follows:

$$X_{i,\text{new}} = X_i + \text{Laplace}\left(X_{\text{teacher}} - T_F X_{\text{mean}} \right). \quad (5)$$

3.3. Experience Exchange Strategy. It can be seen from (3) that each learner only learns randomly from other learners in the Learner Phase. This will lead to a certain degree of blindness in the learning direction, thus degrading the algorithm's accuracy. It is well known that exchange experience is an important way to learn. Experience exchange ensures that each learner not only incorporates his own experience but also contributes his experience to the entire group to maximize the utility of that experience. The updating equation of the Learner Phase is as follows:

$$X_{i,ne} = \begin{cases} X_i + \text{Laplace}\left(X_i - X_j \right) + \psi\left(X_{mp} - X_j \right) \text{if } f(X_i) < f(X_j), \\ X_i + \text{Laplace}\left(X_j - X_i \right) + \psi\left(X_{mp} - X_i \right) \text{if } f(X_i) < f(X_j), \end{cases} \quad (6)$$

where ψ is the exchange learning factor and denotes a vector whose elements are distributed randomly in the range [0,1.5] [29], X_{mp} is the mean of the current experience of all of the learners, and $X_{\text{pb}i}$ is also the experience of the ith learner. The X_{mp} updating equation of the Learner Phase is as follows:

$$X_{mp} = \frac{1}{N} \sum_{i=1}^{N} X_{pbi}. \quad (7)$$

3.4. Flowchart of Distribution of LETLBO Algorithm. As explained above, the flowchart of the novel version of TLBO with the Laplace distribution and Experience exchange strategy (LETLBO) is shown in Figure 1.

3.5. Analysis of Computational Complexity. Computational complexity [30] is usually considered as a method of measuring the input data set size. There are four kinds of operations including population initialization, Teacher Stage, Learner Stage, and Experience Exchange strategy for the LETLBO algorithm. Here, the total computational complexity time of the LETLBO algorithm in the iteration is as follows:

$$[O(ND) + O(ND) + O(ND) + \psi O(ND)]T_{\max}$$
$$= [(3 + \psi)O(ND)]T_{\max} \qquad (8)$$
$$= O(n^2)T_{\max},$$

where n denotes the larger value of N and D, and $T_{\max} = \lfloor FESMAX - N/2N \rfloor$.

According to the above analysis, the total time complexity consumed of the LETLBO algorithm in one cycle is no more than $O(n^2)T_{\max}$ while the total time complexity consumed by the original TLBO algorithm in one cycle is also $O(n^2)T_{\max}$. It shows that the LETLBO algorithm hardly increases the time complexity of the original TLBO algorithm.

4. Experimental Results and Discussion

The section first investigated control parameter population size about the proposed algorithm. Then, the performance of the LETLBO algorithm is evaluated by comparing with that of other variant TLBO algorithms and nine original intelligence optimization algorithms. The experimental results verify that the LETLBO algorithm is very competitive in terms of the solution accuracy and quality.

4.1. Experimental Designing. In order to study the performance of the proposed LETLBO algorithm, six benchmark functions [31] are numerically simulated in this experiment and listed in Table 1. Windows XP operating system with MATLAB 7.14.0 (R2012a) performed all test experiments in this paper, and the experiments were repeated 30 times independently with a Celeron 2.26 GHz CPU and 2 GB memory.

Parameters settings of all comparative experiments are listed. The dimension (D) of benchmark functions is set to 30. The others parameters of the compared algorithms are set the same value as the recommended value original paper. In addition, the stopping criterion is set to 300,000 Function Evaluations (FEs) [32].

To verify whether the overall optimization performance of various optimization algorithms is significantly different, the statistical method Wilcoxon Rank-Sum Test [33] with a significance level $\alpha = 0.05$ is conducted in this paper. The Wilcoxon Rank-Sum Test assesses whether the mean value (F_{mean}) of two solutions from any two algorithms are statistically different from each other. These marks "-", "+", and "≈" denote that the performance of the proposed algorithm is significantly worse than, significantly better than, and similar to that of LETLBO, respectively. The others parameters of the compared algorithms are set the same value as the recommended value original paper.

4.2. Population Size N Influence on LETLBO Performance. The population is now set from 10 to 100 in increments of 10 with the other parameters the same as previously, and the influence on LETLBO algorithm performance is investigated. All experiments are conducted on test functions F_1-F_6. Table 2 shows the results for different population sizes. From the statistical results in Table 2, it can be seen that the mean value (F_{mean}) of the LETLBO ($N = 30$) algorithm is better than that of other cases on functions F_1, F_2, F_3, F_4, and F_6 (5 out of the 6 functions). For other functions such as F_5, the mean value (F_{mean}) of the LETLBO ($N = 30$) algorithm is similar to other cases. In summary, LETLBO ($N = 30$) algorithm wins first place when compared with other cases according to the above analyses. As for the reason, the statistical results experiment shows that the smaller population size may lead to premature convergence and the larger population size will greatly decrease the probability of finding the correct search direction. Therefore, the population size N of the LETLBO algorithm is recommended to be set as the value 30.

4.3. Comparison LETLBO with the Original TLBO, LTLBO, and ETLBO Algorithm. The LETLBO is compared to the original version of TLBO and Teaching-Learning-based Optimization based on Laplace distribution (LTLBO)which is considered as only added Laplace distribution as well as Teaching-Learning-based Optimization based on Experience exchange strategy (ETLBO) which is considered as only added Experience exchange strategy for implementing conveniently. The parameter for the LTLBO and ETLBO are the same as LETLBO. Each algorithm runs independently 30 times, and the statistical results of F_{mean} and *SD* are provided in Table 3, the last three rows of which show the experimental results, and the best results are marked in bold. The evolution plots of TLBO, LTLBO, ETLBO, and LETLBO are illustrated in Figure 3. In addition, the semi-logarithmic convergence plots are used to analyze the relationship of the mean errors of the functions.

In this section, the LETLBO is compared to the original version of TLBO and LTLBO as well as ETLBO. From Table 3, it can be seen that LETLBO performs much better in most cases than TLBO, LTLBO, and ETLBO. In specific, LETLBO outperforms TLBO, LTLBO, and ETLBO five, four, and five test functions out of six test functions, respectively. For functions F₅, LETLBO performs the same as the TLBO, LTLBO, and ETLBO in terms of the statistical mean value (F_{mean}). Furthermore, Figure 3 reveals the convergence behaviors for TLBO, LTLBO, ETLBO, and LETLBO algorithms. As illustrated in Figure 3, according to these evolution plots, it can be seen that the LETLBO algorithm shows the fastest convergence rates for functions F_1, F_2, F_3, and F_5. Therefore, referring to the statistical results of F_{mean} and *SD*, the overall performance of LETLBO is significantly better than the original version of TLBO and LTLBO as well as ETLBO algorithms.

Considering the above-mentioned situations, the main reason is that Laplace distribution can expand the searching space and Experience exchange strategy to avoid detours to

TABLE 1: Six benchmark functions.

Name	Definition	Range	Optimum
F_1	Shifted Rosenbrock's function	[-100,100]	0
F_2	Shifted rotated Rastrigin's function	[-32,32]	0
F_3	Shifted rotated weierstrass function	[-600,600]	0
F_4	Schwefel's problem 2.13	[-2.048,2.048]	0
F_5	Expanded extended Griewank's plus Rosenbrock's function	[-32,32]	0
F_6	E expanded rotated extended Scaffe's F6	[-100,100]	0

TABLE 2: Results of LETLBO based on different populations over 30 independent runs on 6 test functions on 30 dimensions with 300,000 FEs.

Function	Result	$N = 10$	$N = 20$	$N = 30$	$N = 40$	$N = 50$	$N = 60$	$N = 70$	$N = 80$	$N = 90$	$N = 100$
F_1	F_{mean}	1.11 E+01	1.12 E+01	1.10 E+01	1.13 E+01	1.14 E+01	1.14 E+01	1.20 E+01	1.21 E+01	1.25 E+01	1.26 E+01
	SD	3.00E-10	3.14E-10	2.11E-10	2.09E-10	3.12E-09	2.77E-10	3.11E-08	3.55E-06	3.99E-06	3.21E-05
F_2	F_{mean}	6.05 E+01	6.11 E+01	6.03 E+01	6.06 E+01	6.06 E+01	6.07 E+01	6.20 E+01	6.18 E+01	6.23 E+01	6.25 E+01
	SD	3.44E-09	4.18E-07	3.24E-08	4.22E-08	4.55E-08	2.09E-08	4.54E-07	4.44E-08	4.34E-08	2.97E-08
F_3	F_{mean}	2.88 E+01	3.01 E+01	2.57 E+01	2.60 E+01	2.62 E+01	2.63 E+01	2.70 E+01	2.68 E+01	2.71 E+01	2.72 E+01
	SD	4.15E-01	3.14E-01	3.39E-01	4.43E-01	2.54E-01	4.66E-01	4.11E-01	4.55E-01	4.66E-01	4.30E-01
F_4	F_{mean}	4.90 E+03	4.90 E+03	4.89 E+03	4.90 E+03	4.90 E+03	4.91 E+03	4.91 E+03	4.93 E+03	4.95 E+03	4.95 E+03
	SD	1.87 E+03	2.43 E+03	1.10 E+03	1.12 E+03	2.13 E+03	2.43 E+03	2.33 E+03	2.14 E+02	3.15 E+03	3.43 E+03
F_5	F_{mean}	3.47 E+00	3.47 E+00	3.47 E+00	3.47 E+00	3.47 E+00	3.47 E+00	3.47 E+00	3.47 E+00	3.47 E+00	3.47 E+00
	SD	2.00E-01	2.11E-01	2.93E-01	3.03E-01	3.11E-01	3.33E-01	1.43E-01	3.97E-01	3.33E-01	3.43E-01
F_6	F_{mean}	1.24 E+01	1.22 E+01	1.21 E+01	1.24 E+01	1.23 E+01	1.23 E+01	1.24 E+01	1.24 E+01	1.24 E+01	1.24 E+01
	SD	1.65E-01	3.20E-01	1.30E-01	1.99E-01	2.10E-01	2.43E-01	2.30E-01	2.32E-01	4.09E-01	3.07E-01

TABLE 3: Results of four algorithms for 30 independent runs on 6 test functions of 30 dimensions.

| Function	Result	TLBO	LTLBO	ETLBO	LETLBO
F_1	F_{mean} SD	7.44 E+01–6.15e-09	1.11 E+01 ≈ 3.16E-09	1.25 E+01–6.79e-10	1.10 E+012.11E-10
F_2	F_{mean} SD	8.46 E+01–1.38e-09	8.35 E+01–1.25e-09	8.32 E+01–4.18e-10	6.03 E+013.24e-08
F_3	F_{mean} SD	3.09 E+01–1.29e-01	2.82 E+01–3.42e-01	3.02 E+01–7.87e-01	2.57 E+013.39E-01
F_4	F_{mean} SD	1.49 E+04–1.13 E+03	9.32 E+03–1.13 E+04	7.24 E+03–1.34 E+03	4.89 E+031.10 E+03
F_5	F_{mean} SD	3.47 E+00 ≈ 9.37E-01	3.47 E+00 ≈ 3.61E-01	3.47 E+00 ≈ 2.97E-01	3.47 E+002.93E-01
F_6	F_{mean} SD	1.27 E+01–1.89e-01	1.23 E+01–1.79e-01	1.26 E+01–1.18e-01	1.21 E+011.30E-01
	−	5	4	5	
	+	0	0	0	
	≈	1	2	0	

achieve a more accurate solution, helping to identify a more promising solution. Thus, exploration and exploitation are balanced better in the LETLBO. Therefore, it can be concluded that LETLBO performs most effectively for accuracy among original version TLBO of and LTLBO as well as ETLBO algorithms.

4.4. Comparison with Other Improved TLBO Variants.

In this section, we compare the LETLBO with four other improved TLBO variants including ETLBO [22], NSTLBO [23], TLBMO [34], and TLBODE [14]. The parameters for four other improved TLBO variants are taken from their reference listed above. Each algorithm runs independently 30 times, and the statistical results of F_{mean} and SD are provided in Table 4, the last three rows of which show the experimental results. The best results are shown in bold.

From the statistical mean value (F_{mean}) given in Table 4, the overall performance of LETLBO is significantly better than that of the other four algorithms. More specifically, the LETLBO outperforms ETLBO, NSTLBO TLBMO, and TLBODE on four, five, five, and four out of six test functions, respectively. As can be seen from the statistical mean value (F_{mean}) in Table 4, LETLBO is better than the other four algorithms for functions F_1, F_2, F_3, and F_4. LETLBO performs the same as the other four algorithms in terms of the statistical mean value (F_{mean}) for functions F_5. LETLBO performs the same as the TLBODE in terms of the statistical mean value (F_{mean}) for functions F_6. Therefore, referring to the statistical results of F_{mean} and SD, the overall performance of the LETLB algorithm is significantly better than other improved TLBO variants: ETLBO, NSTLBO, TLBMO, and TLBODE algorithms.

4.5. Comparison of LETLBO Algorithm with Nine Original Intelligence Optimization Algorithms.

In this section, LETLBO algorithm is compared with nine original intelligence optimization algorithms such as PSO [2], DE [3], GSO [4], ABC [5], WCA [7], CS [8], DSA [9], BSA [11], and ISA [12]. From the statistical mean value (F_{mean}) of Table 5, it can be seen that the LETLBO algorithm performs better

(F1)

(F2)

(F3)
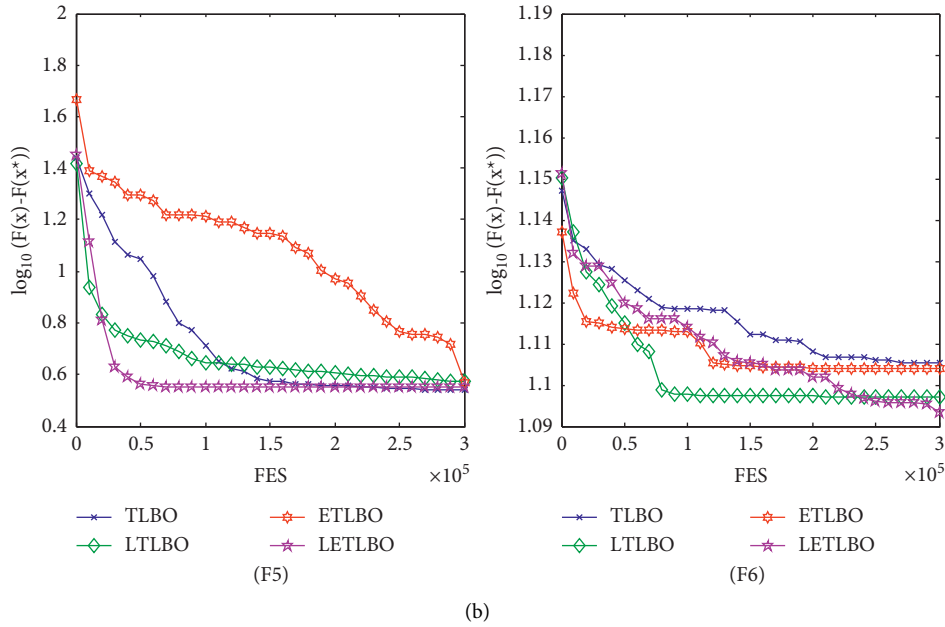
(F4)
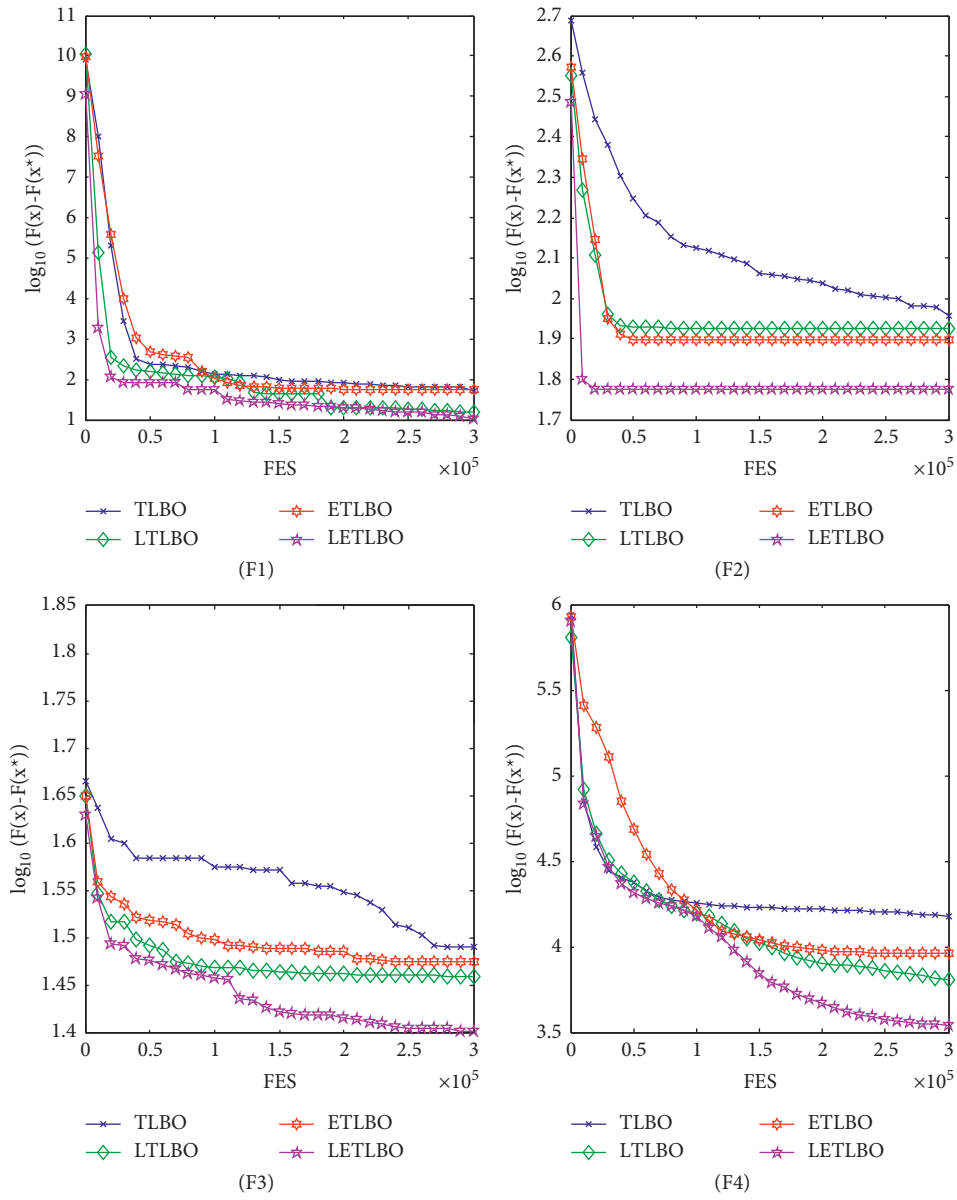
(a)

Figure 3: Continued.

(b)

FIGURE 3: Evolution of mean function error values derived from four algorithms.

TABLE 4: Results of five algorithms for 30 independent runs on 6 test functions of 30 dimensions.

Function	Result		ETLBO	NSTLBO	TLBMO	TLBODE	LETLBO
F_1	F_{mean}	SD	1.13 E+01−2.36e-06	1.23 E+01−1.07e-06	1.63 E+01−2.61e-10	1.18 E+01−3.18e-08	1.10 E+012.11E-10
F_2	F_{mean}	SD	6.02 E+01 + 1.24E-07	8.38 E+01−2.14e-08<	9.12 E+01−2.46e-09	6.26 E+01−1.98e-08	6.03 E+013.24E-08
F_3	F_{mean}	SD	2.61 E+01−1.43e-01	2.76 E+01−4.46e-01	2.88 E+01−6.38e-01	2.65 E+01−5.62e-01	2.57 E+013.39E-01
F_4	F_{mean}	SD	4.92 E+031.89 E+03	4.93 E+032.35 E+03	5.21 E+031.54 E+03	4.98 E+031.08 E+03	4.89 E+031.10 E+03
F_5	F_{mean}	SD	3.47 E+00 ≈ 4.65E-01	3.47 E+00 ≈ 6.38E-01	3.47 E+00 ≈ 7.48E-01	3.47 E+00 ≈ 1.67E-01	3.47 E+002.93E-01
F_6	F_{mean}	SD	1.20 E+01−2.24e-01	1.23 E+01−4.19e-01	1.27 E+01−5.56e-01	1.21 E+01 ≈ 4.13E-01	1.21 E+011.30E-01
	−		4	5	5	4	
	+		1	0	0	0	
	≈		1	1	1	2	

than the other nine original intelligence optimization algorithms based on the Wilcoxon Rank Sum Test results. More specifically, the LETLBO algorithm outperforms PSO, DE, ABC, CS, GSO, WCA, DSA, BSA, and ISA algorithms on two, four, five, six, five, six, five, three, and six, out of six test functions, respectively. For functions F_2, F_3, and F_4, the LETLBO algorithm especially outperforms above other nine original intelligence optimization algorithms. Table 5 reveals the standard deviation (SD) of ten algorithms in which the LETLBO algorithm is superior to all the other methods including nine original intelligence optimization algorithms on functions F_1, F_2, F_3, and F_4. This indicates that the robustness of the LETLBO algorithm outperforms nine original intelligence optimization algorithms on functions F_1, F_2, F_3, and F_4. Therefore, our approach is effective for solving optimization problems compared with PSO, DE, ABC, CS, GSO, WCA, DSA, BSA, and ISA algorithms.

5. Application LETLBO to Parameter Estimation of Chaotic System

In this section, we use the above LETLBO algorithm to solve the well-known Lorenz chaotic system and have an estimation of the unknown parameters of the chaotic system. Suppose that the chaotic system [35–37] is n-dimensional and described as follows:

$$F_{17}(x) = x_1^2 + 2x_2^2 - 0.3 \cos(3\pi x_1) + (4\pi x_2) + 0.3, \quad (9)$$

where $F_{18}(x) = -\sum_{i=1}^{5}[(x - a_i)(x - a_i)^T + c_i]^{-1}$ and $F_{19}(x) = -\sum_{i=1}^{7}[(x - a_i)(x - a_i)^T + c_i]^{-1}$, respectively, represent the state vector and initial state, and $F_{20}(x) = -\sum_{i=1}^{10}[(x - a_i)(x - a_i)^T + c_i]^{-1}$ denotes the set of the real structure parameters of the chaotic system.

While estimating the parameters of the chaotic system, it is presumed to be known the structure. Therefore, the estimated system can be denoted as follows:

Table 5: Results of 10 algorithms over 30 independent times on 6 test functions of 30 dimensions with 300,000 FES.

Function	Result	PSO	DE	ABC	CS	GSO	WCA	DSA	BSA	ISA	LETLBO
F_1	F_{mean}	1.05 E+02 +	1.21 E+02 −	2.01 E+02 −	3.10 E+02 −	3.10 E+02 −	2.00 E+02 −	7.33 E+02 −	6.30 E+02 −	7.01 E+02 −	1.10 E+01
	SD	1.01 E+01	2.18 E+01	1.45 E+02	4.72 E+01	5.23 E+01	1.01 E+02	1.07 E+02	2.32 E+02	2.61 E+02	2.11E-10
F_2	F_{mean}	1.05 E+02 −	1.20 E+02 −	3.25 E+02 −	1.21 E+02 −	3.11 E+02 −	1.32 E+02 −	7.52 E+02 −	4.43 E+01 ≈	1.21 E+02 −	6.03 E+01
	SD	3.32 E+01	6.22 E+01	3.57 E+01	1.07 E+01	2.13 E+01	3.43 E+01	3.75 E+01	2.43 E+01	2.97 E+01	3.24E-08
F_3	F_{mean}	2.11 E+01 +	1.85 E+01 +	2.89 E+01 −	2.63 E+01 −	3.20 E+01 −	3.00 E+01 −	2.60 E+01 −	2.57 E+01 ≈	3.01 E+01 −	2.57 E+01
	SD	3.64E-00	4.21E-00	2.32e-00	1.63e-00	3.02e-00	4.63e-00	1.11 E+00	2.52E-00	2.11e-00	3.39E-01
F_4	F_{mean}	7.88 E+04 −	5.04 E+03 −	8.89 E+03 −	3.04 E+04 −	4.54 E+04 −	2.23 E+04 −	2.32 E+04 −	1.33 E+04 −	1.54 E+06 −	4.89 E+03
	SD	7.65 E+04	2.32 E+03	3.23 E+03	1.01 E+04	3.43 E+04	3.53 E+04	7.44 E+03	4.32 E+03	8.21 E+04	1.10 E+03
F_5	F_{mean}	3.26E-00 +	2.72E-00 +	1.12E-00 +	6.65E-00 −	2.00E-00 +	9.78E-00 −	1.76E-00 +	1.69E-00 +	4.36E-00 −	3.47 e-00
	SD	2.40e-01	3.24e-01	9.54e-01	7.61e-01	4.43e-01	2.12e-00	8.88e-01	1.32e-00	1.97e-00	2.93e-01
F_6	F_{mean}	1.21 E+01 ≈	1.32 E+01 −	1.31 E+01 −	1.28 E+01	1.30 E+01	1.30 E+01 −	1.30 E+01 −	1.26 E+01 −	1.27 E+01 −	1.21 E+01
	SD	6.13E-01	1.25e-01	5.92e-01	6.03E-01	5.66E-01	7.42e-01	6.06e-01	5.90e-01	5.10e-01	1.30E-010
	−	2	4	5	6	5	6	5	3	6	
	+	3	2	1	0	1	0	1	2	0	
	≈	1	0	0	0	0	0	0	1	0	

"−", "+", and "≈" denote that the performance of the corresponding algorithm is significantly worse than, significantly better than, and similar to that of LETLBO, respectively.

$$\dot{X} = G\,(X, X_0, q_0)$$

X_0

$$\dot{Y} = G\,(Y, X_0, q_0)$$

Adjusting θ

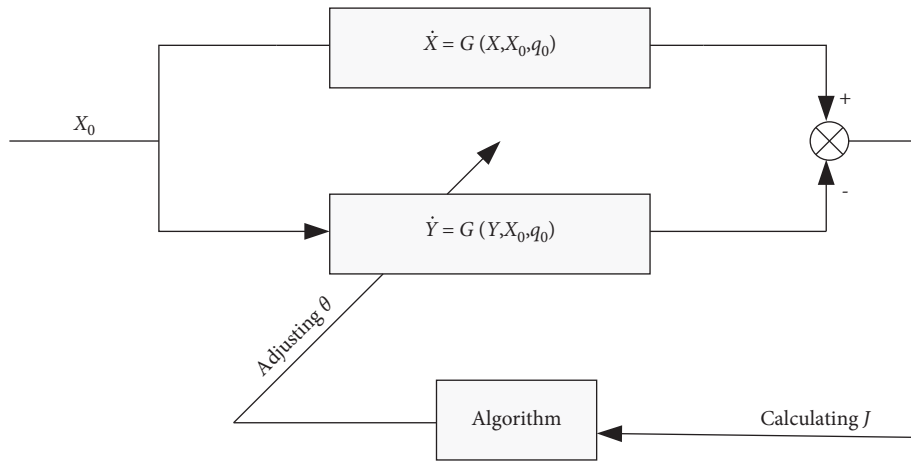Algorithm

Calculating J

+

−

FIGURE 4: The principle of parameters for a chaotic system via an optimization algorithm.
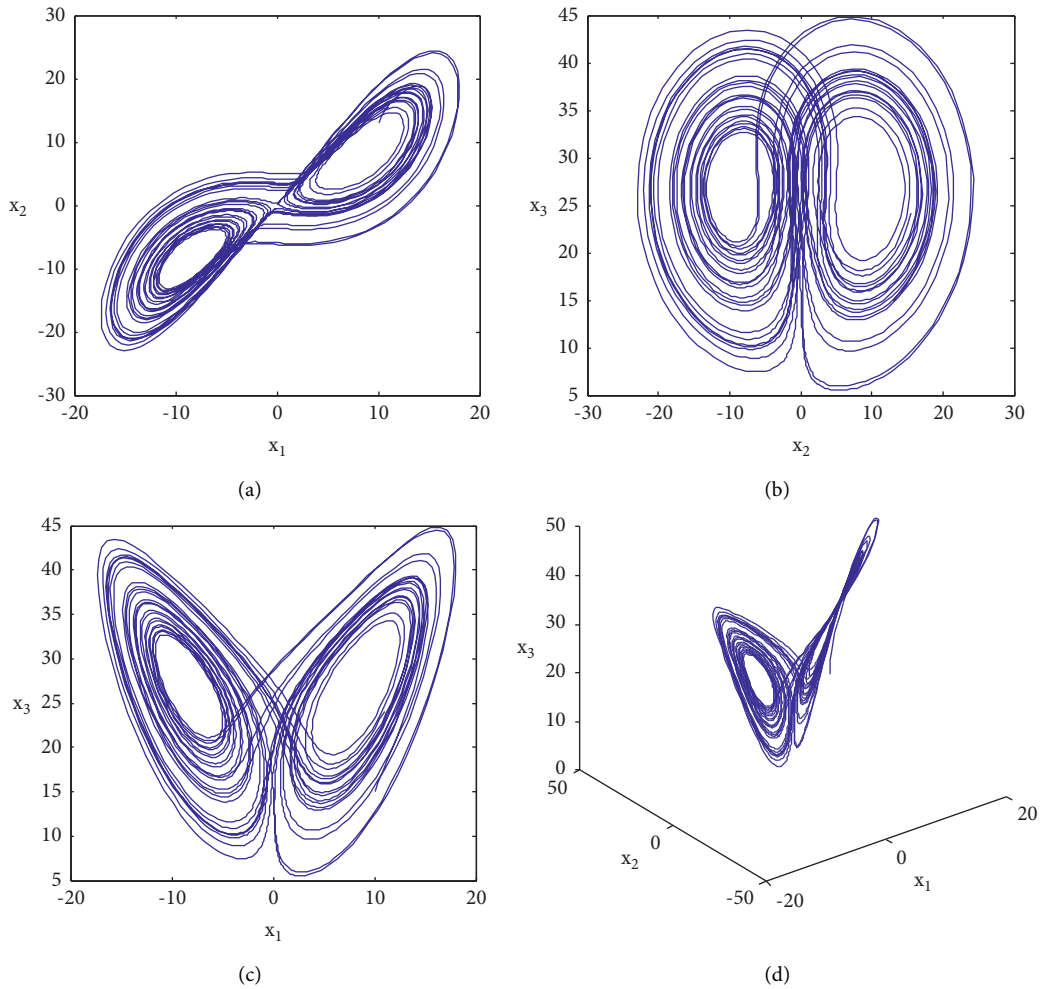
FIGURE 5: Running trajectories of the Lorenz chaotic system in each plane are as follows: (a) Running trajectories on the plane (x, y). (b) Running trajectories on the plane (x, z). (c) Running trajectories on the plane (y, z). (d) Running trajectories on the plane (x, y, z).

TABLE 6: The experimental results of above five different algorithms for the Lorenz chaotic system.

Level	Parameter	NSTLBO	ETLBO	TLBODE	TLBMO	LETLBO
Best	a					
Best	b	3.39E-27-1.49E-27	6.86E-09-8.83E-09	1.90E-27-2.69E-27	2.77E-21-2.66E-21	3.53E-281.43E-28
Best	c					
Best	J	1.56E-09-4.20E-09	5.46E-00-5.41E-00	5.08E-11-1.61E-10	6.81E-00-2.05E-00	2.42E-143.49E-14
Average	a					
Average	b	6.81 E+05−4.08 E+04	3.54 E+06−2.15 E+06	1.87 E+05−3.24 E+05	1.81 E+06−3.71 E+05	4.07E-226.05E-22
Average	c					
Average	J	6.81 E+05−4.08 E+04	3.54 E+06−2.15 E+06	1.87 E+05−3.24 E+05	1.81 E+06−3.71 E+05	4.07E-226.05E-22
Worst	a					
Worst	b	7.35 E+01 + 9.78 E+01	3.63 E+02−2.79 E+02	5.88 E+01 + 1.86 E+02	7.01 E+03−2.64 E+03	3.38 E+023.72 E+02
Worst	c					
Worst	J	3.16 E+03−6.77 E+02	4.79 E+03−1.19 E+03	3.53 E+02 ≈ 1.11 E+03	3.96 E+03−6.07 E+02	3.45 E+023.06 E+02

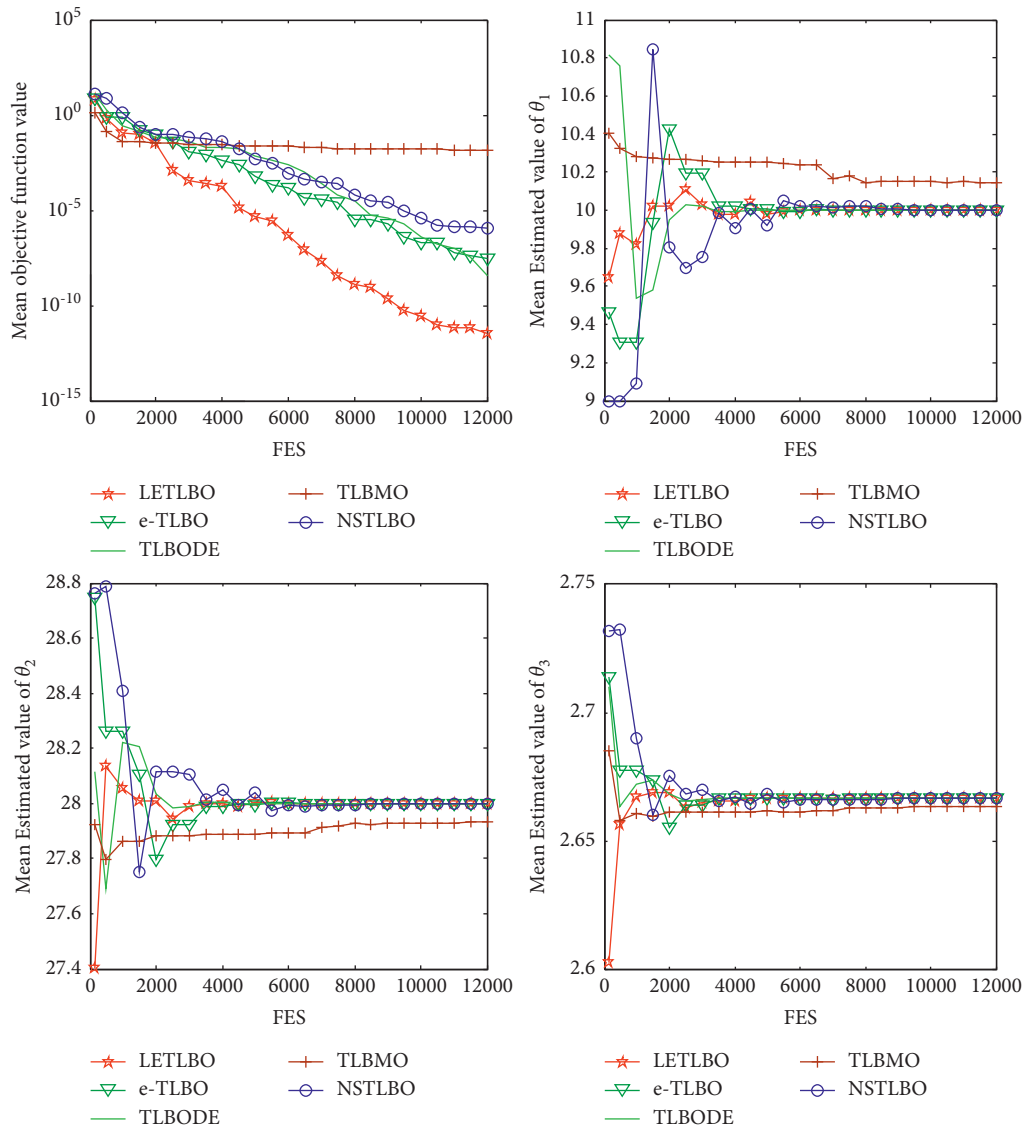FIGURE 6: Evolution running trajectories on each parameter of the Lorenz chaotic system.

$$f_1(x) = x^2,$$
$$f_2(x) = (x-2)^2, \tag{10}$$

where $x_i \in [-10^3, 10^3]$ and Y_0, respectively, represent the state vector and initial state and $f_1(x) = x_1$, $f_2(x) = (1 + 10x_2)[1 - (x_1/(1 + 10x_2))^2 - (x_1/(1 + 10x_2))\sin(8\pi x_1)]$ denotes the set of the estimated parameters of the chaotic system. Assume that the observation data and the state of the estimated systems are represented as $x_i \in [0, 1]$, $x_1 \in [0, 1], x_i = 0 (i \neq 1)$ at time k, respectively. The total number of the observation data is denoted by Alphabet M. It is obvious that the value of J would be denoted as follows [35] if all the estimated parameters are equal to their real values:

$$f_1(x) = 1 - \exp\left(-\sum_{i=1}^{3} \frac{x_i - 1}{\sqrt{3}}\right)^2,$$

$$f_2(x) = 1 - \exp\left(-\sum_{i=1}^{3} \frac{x_i + 1}{\sqrt{3}}\right)^2. \tag{11}$$

It is obvious that this problem is a multidimensional optimization problem. The unknown system parameter is the decision parameter $x_i \in [-4, 4]$ and minimizing $x_1 = x_2 = x_3$, $\in [-(1/\sqrt{3}), (1/\sqrt{3})]$ is the optimization goal. Figure 4 represents the principle of parameters for a chaotic system via an optimization algorithm. Traditional optimization methods usually have a lot of calculational cost and cannot obtain the global optima or satisfactory solution. In this section, we use the above LETLBO algorithm to solve the well-known Lorenz chaotic system to estimate the unknown parameters in the chaotic system. The Lorenz system is as follows with equations:

$$f_1(x) = x_1,$$

$$f_2(x) = g(x)\left(1 - \sqrt{\frac{f_1(x)}{g(x)}}\right),$$

$$g(x) = 1 + 9\sum_{i=2}^{n} \frac{x_i}{(n-1)}, \tag{12}$$

where the parameters $x_i \in [0, 1]$, $x_1 \in [0, 1], x_i = 0 (i \neq 1)$, $f_1(x) = x_1$, $f_2(x) = g(x)(1 - \sqrt{f_1(x)/g(x)} - (f_1(x)/g(x))\sin(10\pi x_1))$, $g(x) = 1 + 9\sum_{i=2}^{n} x_i/(n-1)$ of the system decide this system behavior $x_i \in [0, 1]$, $x_1 \in [0, 1], x_i = 0 (i \neq 1)$, $f_1(x) = 1 - \exp(-4x_1)\sin^6(6\pi x_1)$, $f_2(x) = g(x)(1 - (f_1(x)/g(x))^2)$, $g(x) = 1 + 9[\sum_{i=2}^{n} x_i/(n-1)]^{0.25}$ are the real values of the original system parameters, respectively. Running trajectories of the Lorenz chaotic system in each plane is as shown in Figure 5.

In this section, LETLBOETLBO, TLBODE, TLBMO, and NSTLBO algorithms are used to estimate parameters. The parameter searching ranges are set as follows: new $X_i = X_i + \text{Mutation\_vector}_i$, $\min_{X \in S} f(X)$, $X \in R^n$. In the experiment, the population N of the LETLBO algorithm is set to 50, and each comparison algorithm runs independently 30 times with 12000 function evaluations. The setting of other parameters is the same as that of the original algorithms. Table 6 shows a lot of lists about the best result, the worst result, the average result, and the estimation results of each algorithm. Figure 6 illustrates the evolution running trajectories of the Lorenz chaotic system on each parameter.

As can be seen from Table 6, parameter estimation values of the Lorenz chaotic system obtained by the LETLBO algorithm are very close to the true values, and the parameter estimation accuracy is high. As for the best result, the worst result, and the average result, the LETLBO algorithm is better than other algorithms. From Figure 6, it is shown that the LETLBO algorithm performs better than four version TLBO algorithms in terms of searching quality and convergence rate. This shows effectiveness and robustness of the LETLBO algorithm for the parameter estimation of the Lorenz chaotic system.

6. Summary and Conclusions

In this paper, a new version of the TLBO algorithm, namely the LETLBO algorithm, is proposed to solve unconstrained optimization problems. Laplace distribution and Experience exchange strategy are incorporated in the proposed algorithms. The influence of control parameters on the performance of these proposed algorithms is also analysed in detail. By comparing with that of other variant TLBO algorithms and nine original intelligence optimization algorithms, the performance of the LETLBO algorithm is evaluated. The experimental results verify that the LETLBO algorithm is superior to other variant TLBO algorithms in terms of the quality solution in most cases. In addition, the proposed algorithms have clear significant advantages compared with nine original intelligence optimization algorithms. Besides that, we also applied it to parameter estimation of chaotic systems and it can be regarded as a new choice that has high practical utility for solving parameter estimation of the Lorenz chaotic system.

Our future work will focus on the real-world application of our proposed LETLBO algorithm in areas of power system, material structure design, unmanned aerial vehicle route (UAV), and robotic path planning. These are all of great value for future research.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

Acknowledgments

References

[1] C. Backtracking, "Search Optimization Algorithm for numerical optimization problems," *Applied Mathematics and Computation*, vol. 219, no. 15, pp. 8121–8144, 2013.

[2] J. Kennedy and R. Eberhart, "Particle swam optimization," in *Proceedings of the IEEE International Conference on Neural Networks Path Australia IEEE Service Center Piscataway*, vol. 107, no. 12, pp. 1–4, Perth, WA, Australia, November 1995.

[3] R. Storn and K. Price, "Differential evolution-A simple and efficient heuristic for global optimization over continuous spaces," *Glob Optim*, vol. 11, no. 4, pp. 341–359, 1997.

[4] S. He, "Group search optimizer: an optimization algorithm inspired by animal searching behavior," *IEEE Transactions on Evolutionary Computation*, vol. 13, no. 5, pp. 973–990, 2009.

[5] D. Karaboga, "A powerful and efficient algorithm for numerical function optimization: artificial bee colony (ABC) algorithm," *Journal of Global Optimization*, vol. 39, no. 3, pp. 459–471, 2007.

[6] H. C. Tsai, "Modification of the fish swarm algorithm with particle swarm optimization formulation and communication behavior," *Elsevier Science Publishers B. V.*vol. 11, no. 8, pp. 5367–5374, 2011.

[7] H. Eskandar, A. Sadollah, A. Bahreininejad, and M. Hamdi, "Water cycle algorithm-A novel metaheuristic optimization method for solving constrained engineering optimization problems," *Computers & Structures*, vol. 110-111, no. 1, pp. 151–166, 2012.

[8] X. -S. Yang and D. Suash, "Cuckoo Search via Lévy Flights," in *Proceedings of the 2009 World Congress on Nature & Biologically Inspired Computing (NaBIC)*, pp. 210–214, Coimbatore, India, November 2009.

[9] P. Civicioglu, "Transforming geocentric cartesian coordinates to geodetic coordinates by using differential search algorithm," *Pergamon Press, Inc.*vol. 46, no. 3, pp. 229–247, 2012.

[10] H. Moradi, "An imperialist competitive algorithm for a mixed-model assembly line sequencing problem," *Journal of Manufacturing Systems*, vol. 32, no. 1, pp. 46–54, 2013.

[11] C. Backtracking, "Search Optimization Algorithm for numerical optimization problems," *Applied Mathematics and Computation*, vol. 219, no. 15, pp. 8121–8144, 2013.

[12] Gandomi, "Interior search algorithm(ISA):A novel approach for global optimization," *ISA Transactions*, vol. 53, no. 4, pp. 1168–1183, 2014.

[13] V. K. Patel, "Heat transfer search (HTS): a novel optimization algorithm," *Information Sciences*, vol. 324, pp. 217–246, 2015.

[14] H. F. Farahani, "An improved teaching-learning-based optimization with differential evolution algorithm for optimal power flow considering HVDC system," *Journal of Renewable and Sustainable Energy*, vol. 9, no. 3, pp. 697–711, 2017.

[15] Waghmare, "Comments on "A note on teaching–learning-based optimization algorithm," *Journal of Information Science*, vol. 229, pp. 159–169, 2013.

[16] K. L. Wang, "Teaching-learning-based optimization algorithm for dealing with real-parameter optimization problems," *Applied Mechanics and Materials*, vol. 380-384, pp. 1342–1345, 2013.

[17] V. Kumar, "Optimization of cropping patterns using elitist-jaya and elitist-TLBO algorithms," *Water Resources Management: An International Journal, Published for the European Water Resources Association (EWRA)*, vol. 33, no. 5, pp. 1817–1833, 2019.

[18] R. V. Rao, *Teaching Learning Based Optimization Algorithm: And its Engineering Applications*Springer Publishing Company, Manhattan, NY, USA, 2015.

[19] R. V. Rao, "Multi-objective optimization of heat exchangers using a modified teaching-learning-based optimization algorithm," *Applied Mathematical Modelling*, vol. 37, no. 3, pp. 1147–1162, 2013.

[20] T. Niknam, "Multiobjective teaching–learning-based optimization for dynamic economic emission Dispatch," *IEEE Systems Journal*, vol. 6, no. 2, 2012.

[21] D. Tansel and S. Ender, "Memetic Teaching-Learning-Based Optimization Algorithms for Large Graph Coloring Problems," *Engineering Applications of Artificial Intelligence*, vol. 102, 2021.

[22] R. V. Rao, "An elitist teaching-learning-based optimization algorithm for solving complex constrained optimization problems," *International Journal of Industrial Engineering Computations*, vol. 3, no. 4, pp. 535–560, 2012.

[23] L. Wang, "An improved teaching–learning-based optimization with neighborhood search for applications of ANN," *Neurocomputing*, vol. 143, no. nov.2, pp. 231–247, 2014.

[24] C. Debao, Z. Feng, L. Renquan, and W. Peng, "Learning Backtracking Search Optimisation Algorithm and its application," *Information Sciences*, vol. 376, 2017.

[25] C. Debao, Z. Feng, L. Renquan, and W. Peng, "Learning backtracking search optimisation algorithm and its application," *Information Sciences*, vol. 376, pp. 71–94, 2017.

[26] F. Zou, C. Debao, L. Renquan, W. Lehui, and L. Suwen, "Teaching–learning-based optimization with differential and repulsion learning for global optimization and nonlinear modeling," *Soft Computing*, vol. 22, no. 1, pp. 1–29, 2017.

[27] H. C. Tsai, "Confined teaching-learning-based optimization with variable search strategies for continuous optimization," *Information Sciences*, vol. 500, pp. 34–47, 2019.

[28] Z. Jiang, F. Zou, D. Chen, and J. Kang, "An improved teaching–learning-based optimization for multilevel thresholding image segmentation," *Arabian Journal for Science and Engineering*, vol. 2021, 2021.

[29] F. Kemp, *The Laplace Distribution and Generalizations: A Revisit with Applications to Communications, Economics, Engineering, and Finance*, Journal of the Royal Statistical Society: Series D (The Statistician), vol. 52, no. 2003, Hoboken, NJ, USA, 2001.

[30] G. Zhu, "Gbest-guided artificial bee colony algorithm for numerical function optimization," *Applied Mathematics and Computation*, vol. 217, no. 7, pp. 3166–3173, 2010.

[31] D. M. Curry, "Computational complexity measures for many-objective optimization problems," *Procedia Computer Science*, vol. 36, no. 4, pp. 185–191, 2014.

[32] P. N. Suganthan, "Problem Definitions and Evaluation Criteria for the CEC 2005 Special Session on Real-Parameter Optimization," *Natural Computing*, pp. 341–357, 2005.

[33] J. Liang, "JComprehensive learning particle swarm optimizer for global optimization of multimodal functions," *IEEE Transactions on Evolutionary Computation*, vol. 10, pp. 281–295, 2006.

[34] W. Yong, "Differential evolution with composite trial vector generation strategies and control parameters," *IEEE Transactions on Evolutionary Computation*, vol. 15, no. 1, pp. 55–66, 2011.

[35] Q. Zhang, "A hybrid bird mating optimizer algorithm with teaching-learning-based optimization for global numerical optimization," *Statistics*, vol. 3, no. 1, pp. 54–65, 2015.

[36] L. Wang, "An effective hybrid quantum-inspired evolutionary algorithm for parameter estimation of chaotic systems," *Expert Systems with Applications*, vol. 37, no. 2, pp. 1279–1285, 2010.

[37] W. Wang, "Complex patterns in a predator-prey model with self and cross-diffusion," *Communications in Nonlinear Science and Numerical Simulation*, vol. 16, no. 4, pp. 2006–2015, 2011.