Hindawi
Mathematical Problems in Engineering
Volume 2022, Article ID 4182740, 17 pages
https://doi.org/10.1155/2022/4182740

Research Article

Innovative Bipolar Fuzzy Sine Trigonometric Aggregation Operators and SIR Method for Medical Tourism Supply Chain

Muhammad Riaz,[1] **Dragan Pamucar**,[2] **Anam Habib**,[1] **and Nimra Jamil**[1]

[1]*Department of Mathematics, University of the Punjab, Lahore, Pakistan*
[2]*Department of Logistics, Military Academy, University of Defence in Belgarde, Belgarde 11000, Serbia*

Correspondence should be addressed to Dragan Pamucar; dragan.pamucar@va.mod.gov.rs

Received 13 February 2022; Revised 4 April 2022; Accepted 15 April 2022; Published 20 June 2022

Academic Editor: Ardashir Mohammadzadeh

Bipolar fuzzy sets (BFSs) are effective tool for dealing with bipolarity and fuzziness. The sine trigonometric functions having two significant features, namely, periodicity and symmetry about the origin, are helping in decision analysis and information analysis. Taking the advantage of sine trigonometric functions and significance of BFSs, innovative sine trigonometric operational laws (STOLs) are proposed. New aggregation operators (AOs) are developed based on proposed operational laws to aggregate bipolar fuzzy information. Certain characteristics of these operators are also discussed, such as boundedness, monotonicity, and idempotency. Moreover, a modified superiority and inferiority ranking (SIR) method is proposed to cope with multicriteria group decision-making (MCGDM) with bipolar fuzzy (BF) information. To exhibit the relevance and feasibility of this methodology, a robust application of best medical tourism supply chain is presented. Finally, a comprehensive comparative and sensitivity analysis is evaluated to validate the efficiency of suggested methodology.

1. Introduction

Multicriteria group decision-making (MCGDM) is a process to seek an optimal alternative and ranking of feasible alternatives by a group of decision-experts under several stages and several criteria. However, this process is desperate with uncertainty due to data imprecision and vague perception. As a result, crisp theory is insufficient for dealing with MCGDM problems. To deal with these matters, Zadeh [1] initiated the conception of fuzzy set (FS) and membership function. Later on, different researchers presented different extensions of FSs including, intuitionistic fuzzy sets (IFSs) [2], Pythagorean fuzzy sets (PyFSs) [3, 4], q-rung orthopair fuzzy sets (q-ROFSs) [5], hesitant fuzzy sets (HFSs) [6], neutrosophic sets (NSs) [7], single-valued NSs [8], picture fuzzy sets (PFSs) [9], and spherical fuzzy sets (SFSs) [10–12].

The fuzzy models are extremely useful in dealing with uncertain MCGDM problems, and they have been widely used by decision makers. Nevertheless, they all have one flaw in common: they can only deal with one property and its not-property at a time. They are unable to cope with any property's counter property. It is quite common in decision analysis to have to consider both the positive and negative aspects of a specific object. Some well-known contradictory features in decision analysis include effects and side effects, profit and loss, health and sickness, and so on. Zhang [13, 14] propounded the abstraction of bipolar fuzzy sets (BFSs) which deal with both a property and its counter property. Lee [15] studied operations on bipolar-valued fuzzy sets. Tehrim and Riaz [16] introduced connection numbers of SPA theory for the decision support system by using the IVBF linguistic VIKOR method. Jana and Pal [17] proposed the BF-EDAS method for MCGDM problems. Liu et al. [18] suggested an integrated bipolar fuzzy SWARA-MABAC technique and utilized it for the safety risk and occupational health diagnosis. Jana et al. [19] introduced BF-Dombi AOs and Wei et al. [20] developed bipolar fuzzy Hamacher AOs.

Han et al. [21] proposed the TOPSIS method for YinYang bipolar fuzzy cognitive TOPSIS. Wei et al. [22] established MADM with IVBF information. Hamid et al.

[23] initiated weighted aggregation operators for q-rung orthopair m-polar fuzzy set. Akram et al. [24] proposed the notion of complex fermatean fuzzy N-soft sets. AOs are crucial in information aggregation and are subject to a variety of operational laws. Based on algebraic operational laws, Xu [25] and Xu and Yager [26] propounded weighted averaging and geometric AOs for IFSs. Garg [27] introduced interactive operators for IFSs. Huang [28] proposed intuitionistic fuzzy Hamacher aggregation operators. Gou and Xu [29] suggested exponential operational laws (EOLs) for IFSs.

Li and Wei [30] proposed logarithmic operational laws (LOLs) for IFSs. Peng et al. [31] proposed EOLs for q-ROFSs. Similarly, the LOLs for PFSs [32] are also defined. Aside from the exponential and logarithmic functions, sine trigonometric function is another suitable choice for information fusion. The two main characteristics are periodicity and symmetry about the origin which aid in meeting the decision makers' expectations during object evaluation. Abdullah et al. [33] developed STOLs for PFSs. Kabani [34] studied Pakistan as a medical tourism destination. Muzaffar and Hussain [35] investigated medical tourism to discuss the challenge: are we ready to take the challenge. Zhang and Xu [36] proposed TOPSIS for PFSs and PFNs with MCDM.

Mahmood et al. [37] proposed an innovative MCDM method with spherical fuzzy soft rough (SFSR) average aggregation operators. Ihsan et al. [38] presented the MADM support model based on bijective hypersoft expert set. Karaaslan and Karamaz [39] introduced an innovative decision-making approach with HFPHFS. Alcantud [40] introduced the novel concepts of soft topologies and fuzzy soft topologies and investigated their relationships. Liu et al. [41] introduced the idea of mining temporal association rules based on temporal soft sets. Riaz et al. [42] introduced a novel TOPSIS approach based on cosine similarity measures and CBF-information. Zararsiz and Riaz [43] introduced the notion of bipolar fuzzy metric spaces with application. Riaz et al. [44] proposed distance and similarity measures for bipolar fuzzy soft sets with application to pharmaceutical logistics and supply chain management.

In 2021, Gergin et al. [45] modified the TOPSIS method to deal the supplier selection for automotive industry. Karamasa et al. [46] introduced the weighting factors which affect the logistics out-sourcing decision-making problem. Ali et al. [47] introduced Einstein geometric AO to deal complex IVPFS, and its novel principles and its operational laws are defined. Muhammad et al. [48] and Biswas et al. [49] propounded multicriteria decision-making techniques to deal real world problems. Milovanovic et al. [50] developed uncertainty modeling using intuitionistic fuzzy numbers.

In 2021, Garg [51] introduced some robust STOLs, its operational laws for PFSs, and AOs and algorithms to interpret MCDM. In 2021, Mahmood et al. [52] interpreted BCFHWA, BCFHOWA, BCFHHA, BCFHWG, BCFHOWG, and BCFHHG operators. Palanikumar et al. [53] proposed some new methods to solve MCDM based on PNSNIVS. A notion of PNSNIVWA, PNSNIVWG, GPNSNIVWA, and GPNSNIVWG is also discussed in the article. In 2021, Jana et al. [54] applied IFDHWA and IFDHWG AO to evaluate enterprise financial performance. In 2021, Jana et al. [55] extended Dombi operations towards single-valued trapezoidal neutrosophic numbers (SVTrNNs). They also presented Dombi operation on SVTrNNs, and they proposed some new averaging and geometric averaging operators named as SVTrN Domi weighted averaging (SVTrNDWA) operator, SVTrN Dombi ordered weighted averaging (SVTrNDOWA) operator, SVTrN Dombi hybrid weighted averaging (SVTrNDHWA) operator, SVTrN Dombi weighted geometric (SVTrNDWGA) operator, SVTrN Dombi ordered weighted geometric (SVTrNDOWGA) operator, and SVTrN Dombi hybrid weighted geometric (SVTrNDHWGA) operator. In 2022, Ajay et al. [56] extended the STOLs for NSs and CNSs and defined the operational laws and their functionality. They also defined distance measures and ST-AOs. In 2022, Qiyas et al. [57] defined some reliable STOLs for SFNs and defined ST-OAs to deal real world problems.

The superiority and inferiority ranking (SIR) technique is a generalization of the eminent PROMETHEE method. This technique employs superiority and inferiority information to represent decision makers' behavior toward each criterion and to determine the degrees of domination and subordination of each alternative, from which superiority and inferiority flows are derived. It was introduced by Xu [58]. Chai and Liu [59] proposed the IF-SIR method to deal with MCGDM problems. Peng and Yang [60] extended the SIR technique to pythagorean fuzzy data. Zhu et al. [61] proposed the SIR approach for q-ROFSs.

Keeping in mind the importance of sine trigonometric function and SIR method, the aims and perks of this manuscript are as follows:

(1) To address bipolarity and uncertainty, innovative sine trigonometric operational laws (STOLs) are proposed for bipolar fuzzy sets (BFSs).

(2) Averaging AOs are developed named as sine trigonometric bipolar fuzzy weighted averaging (ST-BFWA) operator, sine trigonometric bipolar fuzzy ordered weighted averaging (ST-BFOWA) operator, and sine trigonometric bipolar fuzzy hybrid weighted averaging (ST-BFHWA) operator.

(3) Geometric AOs are proposed including sine trigonometric bipolar fuzzy weighted geometric (ST-BFWG) operator, sine trigonometric bipolar fuzzy ordered weighted geometric (ST-BFOWG) operator, and sine trigonometric bipolar fuzzy hybrid weighted geometric (ST-BFHWG) operator.

(4) Certain aspects of proposed operators are also discussed, such as idempotency, boundedness, and monotonicity.

(5) A modified SIR method by using features of proposed operators is proposed to cope with MCGDM problems.

(6) A robust application of best medical tourism supply chain is presented by using a modified SIR technique involving sine trigonometric AOs.

The layout of the remaining manuscript is as follows. In Section 2, some fundamental concepts about BFSs are reviewed. In Section 3, we define STOLs for BFSs and discuss their properties. In Sections 4 and 5, we introduce novel AOs based on BF-STOLs and explore their characteristics. Section 6 provides an extended version of the SIR technique for dealing with MCGDM problems using bipolar fuzzy data. A numerical illustration and a comparative analysis are also proffered to validate the efficaciousness of the propounded technique. Finally, in Section 7, there are some closing remarks.

2. Preliminaries

This section includes some rudimentary abstractions related to BFSs. Throughout this manuscript, we consider $\mathbb{Y}$ as universe of discourse.

Definition 1 (see [13]). A BFS $\mathfrak{B}$ on $\mathbb{Y}$ can be described as

$$\mathfrak{B} = \{\langle y, \aleph^+_{\mathfrak{B}}(y), \aleph^-_{\mathfrak{B}}(y)\rangle : y \in \mathbb{Y}\}, \tag{1}$$

where $\aleph^+_{\mathfrak{B}}(y) \in [0,1]$ denotes positive membership degree and $\aleph^-_{\mathfrak{B}}(y) \in [-1,0]$ denotes negative membership degree of an element $y \in \mathbb{Y}$. A bipolar fuzzy number (BFN) can be expressed as $\mathfrak{B} = \langle \aleph^+_{\mathfrak{B}}, \aleph^-_{\mathfrak{B}}\rangle$.

In 2015, Gul proposed operational laws of BFNs in his M.Phil Thesis.

Definition 2 [62]. Let $\mathfrak{B}_1 = \langle \aleph^+_{\mathfrak{B}_1}, \aleph^-_{\mathfrak{B}_1}\rangle$ and $\mathfrak{B}_2 = \langle \aleph^+_{\mathfrak{B}_2}, \aleph^-_{\mathfrak{B}_2}\rangle$ be two BFNs and $\sigma \in (0,\infty)$, then operational laws between them can be defined as

(i) $\mathfrak{B}_1 \oplus \mathfrak{B}_2 = \langle \aleph^+_{\mathfrak{B}_1} + \aleph^+_{\mathfrak{B}_2} - \aleph^+_{\mathfrak{B}_1}\aleph^+_{\mathfrak{B}_2}, -\aleph^-_{\mathfrak{B}_1}\aleph^-_{\mathfrak{B}_2}\rangle$

(ii) $\mathfrak{B}_1 \otimes \mathfrak{B}_2 = \langle \aleph^+_{\mathfrak{B}_1}\aleph^+_{\mathfrak{B}_2}, -((-\aleph^-_{\mathfrak{B}_1}) + (-\aleph^-_{\mathfrak{B}_2}) - \aleph^-_{\mathfrak{B}_1}\aleph^-_{\mathfrak{B}_2})\rangle$

(iii) $\sigma\mathfrak{B}_1 = \langle 1 - (1 - \aleph^+_{\mathfrak{B}_1})^\sigma, -(-\aleph^-_{\mathfrak{B}_1})^\sigma\rangle$

(iv) $\mathfrak{B}_1^\sigma = \langle (\aleph^+_{\mathfrak{B}_1})^\sigma, -(1 - (1 - (-\aleph^-_{\mathfrak{B}_1}))^\sigma)\rangle$

(v) $\mathfrak{B}_1^c = \langle 1 - \aleph^+_{\mathfrak{B}_1}, -1 - \aleph^-_{\mathfrak{B}_1}\rangle$

(vi) $\mathfrak{B}_1 \prec \mathfrak{B}_2$ if $\aleph^+_{\mathfrak{B}_1} \leq \aleph^+_{\mathfrak{B}_2}$ and $\aleph^-_{\mathfrak{B}_1} \geq \aleph^-_{\mathfrak{B}_2}$

(vii) $\mathfrak{B}_1 = \mathfrak{B}_2$ if $\mathfrak{B}_1 \prec \mathfrak{B}_2$ and $\mathfrak{B}_2 \prec \mathfrak{B}_1$

Definition 3 (see [20]). For a BFN $\mathfrak{B} = \langle \aleph^+_{\mathfrak{B}}, \aleph^-_{\mathfrak{B}}\rangle$, score and accuracy functions can be expressed as

$$\mathrm{Scr}(\mathfrak{B}) = \frac{1 + \aleph^+_{\mathfrak{B}} + \aleph^-_{\mathfrak{B}}}{2}, \tag{2}$$

$$\mathrm{Acr}(\mathfrak{B}) = \frac{\aleph^+_{\mathfrak{B}} - \aleph^-_{\mathfrak{B}}}{2}. \tag{3}$$

The values of score and accuracy functions are used to compare two BFNs. For two BFNs $\mathfrak{B}_1$ and $\mathfrak{B}_2$,

(i) If $\mathrm{Scr}(\mathfrak{B}_1) < \mathrm{Scr}(\mathfrak{B}_2)$, then $\mathfrak{B}_1 < \mathfrak{B}_2$

(ii) If $\mathrm{Scr}(\mathfrak{B}_1) > \mathrm{Scr}(\mathfrak{B}_2)$, then $\mathfrak{B}_1 > \mathfrak{B}_2$

(iii) If $\mathrm{Scr}(\mathfrak{B}_1) = \mathrm{Scr}(\mathfrak{B}_2)$, then $\mathfrak{B}_1 < \mathfrak{B}_2$ if $\mathrm{Acr}(\mathfrak{B}_1) < \mathrm{Acr}(\mathfrak{B}_2)$

(iv) If $\mathrm{Scr}(\mathfrak{B}_1) = \mathrm{Scr}(\mathfrak{B}_2)$, then $\mathfrak{B}_1 > \mathfrak{B}_2$ if $\mathrm{Acr}(\mathfrak{B}_1) > \mathrm{Acr}(\mathfrak{B}_2)$

(v) If $\mathrm{Scr}(\mathfrak{B}_1) = \mathrm{Scr}(\mathfrak{B}_2)$, then $\mathfrak{B}_1 = \mathfrak{B}_2$ if $\mathrm{Acr}(\mathfrak{B}_1) = \mathrm{Acr}(\mathfrak{B}_2)$

Definition 4 (see [21]). If $\mathfrak{B}_1$ and $\mathfrak{B}_2$ are two BFSs on $\mathbb{Y} = \{y_1, y_2, \ldots, y_n\}$, then the normalized Hamming distance between them is calculated as

$$d(\mathfrak{B}_1, \mathfrak{B}_2) = \frac{1}{2n}\sum_{i=1}^{n}\left(\left|\aleph^+_{\mathfrak{B}_1}(y_i) - \aleph^+_{\mathfrak{B}_2}(y_i)\right| + \left|\aleph^-_{\mathfrak{B}_1}(y_i) - \aleph^-_{\mathfrak{B}_2}(y_i)\right|\right). \tag{4}$$

3. Sine Trigonometric Operational Laws for BFSs

In this section, we suggest sine trigonometric operational laws (STOLs) for BFNs and investigate some useful results.

Definition 5. Let $\mathfrak{B} = \{\langle y, \aleph^+_{\mathfrak{B}}(y), \aleph^-_{\mathfrak{B}}(y)\rangle : y \in \mathbb{Y}\}$ be a BFS on $\mathbb{Y}$. A sine trigonometric operator on $\mathfrak{B}$ can be defined as

$$\sin\mathfrak{B} = \left\{\left\langle y,\ \sin\left(\frac{\pi}{2}\aleph^+_{\mathfrak{B}}(y)\right),\ \sin\left(\frac{\pi}{2}\left(1 + \aleph^-_{\mathfrak{B}}(y)\right)\right) - 1\right\rangle : y \in \mathbb{Y}\right\}. \tag{5}$$

Clearly, $\sin\mathfrak{B}$ is again a BFS on $\mathbb{Y}$ because $\sin((\pi/2)\aleph^+_{\mathfrak{B}}(y)) \in [0,1]$ and $\sin((\pi/2)(1 + \aleph^-_{\mathfrak{B}}(y))) - 1 \in [-1,0]$ serve as positive and negative membership degrees, respectively, for every element $y \in \mathbb{Y}$. The set $\sin\mathfrak{B}$ is called sine trigonometric-BFS (ST-BFS).

Definition 6. Let $\mathfrak{B} = \langle \aleph^+_{\mathfrak{B}}, \aleph^-_{\mathfrak{B}}\rangle$ be a BFN, then

$$\sin\mathfrak{B} = \left\langle \sin\left(\frac{\pi}{2}\aleph^+_{\mathfrak{B}}\right),\ \sin\left(\frac{\pi}{2}\left(1 + \aleph^-_{\mathfrak{B}}\right)\right) - 1\right\rangle, \tag{6}$$

is called ST-BFN.

Definition 7 For two BFNs $\mathfrak{B}_1 = \langle \aleph^+_{\mathfrak{B}_1}, \aleph^-_{\mathfrak{B}_1}\rangle$ and $\mathfrak{B}_2 = \langle \aleph^+_{\mathfrak{B}_2}, \aleph^-_{\mathfrak{B}_2}\rangle$, we propose STOLs as follows:

(i) $\sin\mathfrak{B}_1 \oplus \sin\mathfrak{B}_2 = 1 - (1 - \sin((\pi/2)\aleph^+_{\mathfrak{B}_1}))(1 - \sin((\pi/2)\aleph^+_{\mathfrak{B}_1})), -(\sin((\pi/2)(1 + \aleph^-_{\mathfrak{B}_1})) - 1)\rangle \langle(\sin((\pi/2)(1 + \aleph^-_{\mathfrak{B}_2})) - 1)$

(ii) $\sin\mathfrak{B}_1 \otimes \sin\mathfrak{B}_2 = \langle \sin((\pi/2)\aleph^+_{\mathfrak{B}_1})\sin((\pi/2)\aleph^+_{\mathfrak{B}_1}), -(1 - \sin((\pi/2)(1 + \aleph^-_{\mathfrak{B}_1}))\sin((\pi/2)(1 + \aleph^-_{\mathfrak{B}_2})))\rangle$

(iii) $\sigma\sin\mathfrak{B}_1 = \langle 1 - (1 - \sin((\pi/2)\aleph^+_{\mathfrak{B}_1}))^\sigma, -(-(\sin((\pi/2)(1 + \aleph^-_{\mathfrak{B}_1}))) - 1)^\sigma\rangle;\ \sigma > 0$

(iv) $(\sin\mathfrak{B}_1)^\sigma = \langle(\sin((\pi/2)\aleph^+_{\mathfrak{B}_1}))^\sigma, -(1 - (\sin((\pi/2)(1 + \aleph^-_{\mathfrak{B}_1})))^\sigma)\rangle;\ \sigma > 0$

Theorem 1. *Let $\mathfrak{B}_1 = \langle \aleph^+_{\mathfrak{B}_1}, \aleph^-_{\mathfrak{B}_1}\rangle$ and $\mathfrak{B}_2 = \langle \aleph^+_{\mathfrak{B}_2}, \aleph^-_{\mathfrak{B}_2}\rangle$ be two BFNs and $\sigma > 0$, $\sigma_1 > 0$, $\sigma_2 > 0$ be three real numbers, then*

(i) $\sigma(\sin\mathfrak{B}_1 \oplus \sin\mathfrak{B}_2) = \sigma\sin\mathfrak{B}_1 \oplus \sigma\sin\mathfrak{B}_2$

(ii) $(\sin\mathfrak{B}_1 \otimes \sin\mathfrak{B}_2)^\sigma = (\sin\mathfrak{B}_1)^\sigma \otimes (\sin\mathfrak{B}_2)^\sigma$

(iii) $\sigma_1\sin\mathfrak{B}_1 \oplus \sigma_2\sin\mathfrak{B}_1 = (\sigma_1 + \sigma_2)\sin\mathfrak{B}_1$

(iv) $(\sin \mathfrak{B}_1)^{\sigma_1} \otimes (\sin \mathfrak{B}_1)^{\sigma_2} = (\sin \mathfrak{B}_1)^{\sigma_1+\sigma_2}$

Proof. We substantiate (i) and (iv), and others can be substantiated similarly.

(i) For $\sigma > 0$,

$$\begin{aligned}\sigma(\sin \mathfrak{B}_1 \oplus \sin \mathfrak{B}_2) &= \left\langle 1-\left(1-\sin\left(\frac{\pi}{2}\aleph^+_{\mathfrak{B}_1}\right)\right)^{\sigma}\left(1-\sin\left(\frac{\pi}{2}\aleph^+_{\mathfrak{B}_2}\right)\right)^{\sigma},\right.\\ &\left.-\left(-\left(\sin\left(\frac{\pi}{2}\left(1+\aleph^-_{\mathfrak{B}_1}\right)\right)-1\right)\right)^{\sigma}\left(-\left(\sin\left(\frac{\pi}{2}\left(1+\aleph^-_{\mathfrak{B}_2}\right)\right)-1\right)\right)^{\sigma}\right\rangle\\ &= \left\langle 1-\left(1-\sin\left(\frac{\pi}{2}\aleph^+_{\mathfrak{B}_1}\right)\right)^{\sigma}, -\left(-\left(\sin\left(\frac{\pi}{2}\left(1+\aleph^-_{\mathfrak{B}_1}\right)\right)-1\right)\right)^{\sigma}\right\rangle\\ &\oplus\left\langle 1-\left(1-\sin\left(\frac{\pi}{2}\aleph^+_{\mathfrak{B}_2}\right)\right)^{\sigma}, -\left(-\left(\sin\left(\frac{\pi}{2}\left(1+\aleph^-_{\mathfrak{B}_2}\right)\right)-1\right)\right)^{\sigma}\right\rangle = \sigma \sin \mathfrak{B}_1 \oplus \sigma \sin \mathfrak{B}_2.\end{aligned} \tag{7}$$

(iv) For $\sigma_1, \sigma_2 > 0$,

$$\begin{aligned}(\sin \mathfrak{B}_1)^{\sigma_1} \otimes (\sin \mathfrak{B}_1)^{\sigma_2} &= \left\langle\left(\sin\left(\frac{\pi}{2}\aleph^+_{\mathfrak{B}_1}\right)\right)^{\sigma_1}, -\left(1-\left(\sin\left(\frac{\pi}{2}\left(1+\aleph^-_{\mathfrak{B}_1}\right)\right)\right)^{\sigma_1}\right)\right\rangle\\ &\otimes\left\langle\left(\sin\left(\frac{\pi}{2}\aleph^+_{\mathfrak{B}_1}\right)\right)^{\sigma_2}, -\left(1-\left(\sin\left(\frac{\pi}{2}\left(1+\aleph^-_{\mathfrak{B}_1}\right)\right)\right)^{\sigma_2}\right)\right\rangle\\ &= \left(\sin\left(\frac{\pi}{2}\aleph^+_{\mathfrak{B}_1}\right)\right)^{\sigma_1}\left(\sin\left(\frac{\pi}{2}\aleph^+_{\mathfrak{B}_1}\right)\right)^{\sigma_2}, \rangle\\ &\langle -\left(1-\left(\sin\left(\frac{\pi}{2}\left(1+\aleph^-_{\mathfrak{B}_1}\right)\right)\right)^{\sigma_1}\left(\sin\left(\frac{\pi}{2}\left(1+\aleph^-_{\mathfrak{B}_1}\right)\right)\right)^{\sigma_2}\right)\rangle\\ &= \left\langle\left(\sin\left(\frac{\pi}{2}\aleph^+_{\mathfrak{B}_1}\right)\right)^{\sigma_1+\sigma_2}, -\left(1-\left(\sin\left(\frac{\pi}{2}\left(1+\aleph^-_{\mathfrak{B}_1}\right)\right)\right)^{\sigma_1+\sigma_2}\right)\right\rangle\\ &= (\sin \mathfrak{B}_1)^{\sigma_1+\sigma_2}.\end{aligned} \tag{8}$$

□

Definition 8. Let $\mathfrak{B} = \langle \aleph^+_{\mathfrak{B}}, \aleph^-_{\mathfrak{B}} \rangle$ be a BFN and $\sin \mathfrak{B}$ be the corresponding ST-BFN, then

$$(\sin \mathfrak{B})^c = \left\langle 1-\sin\left(\frac{\pi}{2}\aleph^+_{\mathfrak{B}}\right), -\sin\left(\frac{\pi}{2}\left(1+\aleph^-_{\mathfrak{B}}\right)\right)\right\rangle, \tag{9}$$

is called complement of $\sin \mathfrak{B}$.

Theorem 2. *Let* $\mathfrak{B}_1 = \langle \aleph^+_{\mathfrak{B}_1}, \aleph^-_{\mathfrak{B}_1} \rangle$ *and* $\mathfrak{B}_2 = \langle \aleph^+_{\mathfrak{B}_2}, \aleph^-_{\mathfrak{B}_2} \rangle$ *be two BFNs and* $\sigma > 0$*, then*

(i) $\sigma(\sin \mathfrak{B}_1)^c = ((\sin \mathfrak{B}_1)^{\sigma})^c$

(ii) $((\sin \mathfrak{B}_1)^c)^{\sigma} = (\sigma \sin \mathfrak{B}_1)^c$

(iii) $(\sin \mathfrak{B}_1 \oplus \sin \mathfrak{B}_2)^c = (\sin \mathfrak{B}_1)^c \otimes (\sin \mathfrak{B}_2)^c$

(iv) $(\sin \mathfrak{B}_1 \otimes \sin \mathfrak{B}_2)^c = (\sin \mathfrak{B}_1)^c \oplus (\sin \mathfrak{B}_2)^c$

Proof. We substantiate (i) and (iv), and others can be substantiated similarly.

(i)

$$\begin{aligned}(\sin \mathfrak{B}_1)^c &= \left\langle 1-\sin\left(\frac{\pi}{2}\aleph^+_{\mathfrak{B}_1}\right), -\sin\left(\frac{\pi}{2}\left(1+\aleph^-_{\mathfrak{B}_1}\right)\right)\right\rangle,\\ \sigma(\sin \mathfrak{B}_1)^c &= \left\langle 1-\left(\sin\left(\frac{\pi}{2}\aleph^+_{\mathfrak{B}_1}\right)\right)^{\sigma}, -\left(-\left(-\sin\left(\frac{\pi}{2}\left(1+\aleph^-_{\mathfrak{B}_1}\right)\right)\right)\right)^{\sigma}\right\rangle.\end{aligned} \tag{10}$$

Now, (iv)

$$(\sin \mathfrak{B}_1)^{\sigma} = \left\langle \left(\sin\left(\frac{\pi}{2}\aleph^{+}_{\mathfrak{B}_1}\right)\right)^{\sigma}, -\left(1-\left(\sin\left(\frac{\pi}{2}\left(1+\aleph^{-}_{\mathfrak{B}_1}\right)\right)\right)^{\sigma}\right)\right\rangle,$$

$$\begin{aligned}((\sin \mathfrak{B}_1)^{\sigma})^{c} &= \left\langle 1-\left(\sin\left(\frac{\pi}{2}\aleph^{+}_{\mathfrak{B}_1}\right)\right)^{\sigma}, -\left(\sin\left(\frac{\pi}{2}\left(1+\aleph^{-}_{\mathfrak{B}_1}\right)\right)\right)^{\sigma}\right\rangle \\ &= \sigma(\sin \mathfrak{B}_1)^{c}.\end{aligned} \tag{11}$$

$$\begin{aligned}(\sin \mathfrak{B}_1 \otimes \sin \mathfrak{B}_2)^{c} &= \left\langle 1-\sin\left(\frac{\pi}{2}\aleph^{+}_{\mathfrak{B}_1}\right)\sin\left(\frac{\pi}{2}\aleph^{+}_{\mathfrak{B}_2}\right), -\sin\left(\frac{\pi}{2}\left(1+\aleph^{-}_{\mathfrak{B}_1}\right)\right)\sin\left(\frac{\pi}{2}\left(1+\aleph^{-}_{\mathfrak{B}_2}\right)\right)\right\rangle, \\ (\sin \mathfrak{B}_1)^{c} \oplus (\sin \mathfrak{B}_2)^{c} &= \left\langle 1-\sin\left(\frac{\pi}{2}\aleph^{+}_{\mathfrak{B}_1}\right), -\sin\left(\frac{\pi}{2}\left(1+\aleph^{-}_{\mathfrak{B}_1}\right)\right)\right\rangle \oplus \left\langle 1-\sin\left(\frac{\pi}{2}\aleph^{+}_{\mathfrak{B}_2}\right), -\sin\left(\frac{\pi}{2}\left(1+\aleph^{-}_{\mathfrak{B}_2}\right)\right)\right\rangle \\ &= \left\langle 1-\sin\left(\frac{\pi}{2}\aleph^{+}_{\mathfrak{B}_1}\right)\sin\left(\frac{\pi}{2}\aleph^{+}_{\mathfrak{B}_2}\right), -\left(-\sin\left(\frac{\pi}{2}\left(1+\aleph^{-}_{\mathfrak{B}_1}\right)\right)\right)\left(-\sin\left(\frac{\pi}{2}\left(1+\aleph^{-}_{\mathfrak{B}_2}\right)\right)\right)\right\rangle \\ &= (\sin \mathfrak{B}_1 \otimes \sin \mathfrak{B}_2)^{c}.\end{aligned} \tag{12}$$

□

Theorem 3. . *Let* $\mathfrak{B}_1$ *and* $\mathfrak{B}_2$ *be two BFNs with* $\mathfrak{B}_1 \prec \mathfrak{B}_2$, *i.e.,* $\aleph^{+}_{\mathfrak{B}_1} \leq \aleph^{+}_{\mathfrak{B}_2}$ *and* $\aleph^{-}_{\mathfrak{B}_1} \geq \aleph^{-}_{\mathfrak{B}_2}$, *then* $\sin \mathfrak{B}_1 \prec \sin \mathfrak{B}_2$.

Proof. Since sine is an increasing function on the interval $[0, (\pi/2)]$ so for $\aleph^{+}_{\mathfrak{B}_1} \leq \aleph^{+}_{\mathfrak{B}_2}$, we have $\sin((\pi/2)\aleph^{+}_{\mathfrak{B}_1}) \leq \sin((\pi/2)\aleph^{+}_{\mathfrak{B}_2})$. Likewise, for $\aleph^{-}_{\mathfrak{B}_1} \geq \aleph^{-}_{\mathfrak{B}_2}$, we obtain $1+\aleph^{-}_{\mathfrak{B}_1} \geq 1+\aleph^{-}_{\mathfrak{B}_2}$. This implicates that $\sin((\pi/2)(1+\aleph^{-}_{\mathfrak{B}_1})) \geq \sin((\pi/2)(1+\aleph^{-}_{\mathfrak{B}_2}))$ which further implicates that $\sin((\pi/2)(1+\aleph^{-}_{\mathfrak{B}_1}))-1 \geq \sin((\pi/2)(1+\aleph^{-}_{\mathfrak{B}_2}))-1$. Hence, by Definition 2 (part (vi)), we have $\sin \mathfrak{B}_1 = \langle \sin((\pi/2)\aleph^{+}_{\mathfrak{B}_1}), \sin((\pi/2)(1+\aleph^{-}_{\mathfrak{B}_1}))-1\rangle \prec \langle \sin((\pi/2)\aleph^{+}_{\mathfrak{B}_2}), \sin((\pi/2)(1+\aleph^{-}_{\mathfrak{B}_2}))-1\rangle = \sin \mathfrak{B}_2$. □

4. Bipolar Fuzzy Sine Trigonometric Averaging Aggregation Operators

In this section, some new averaging AOs have been proposed on the basis of STOLs of BFNs. These aggregation operators include (i) ST-BFWA operator, (ii) ST-BFOWA operator, and (iii) ST-BFHWA operator.

4.1. ST-BFWA Operator

Definition 9. . Let $\mathfrak{B}_i$, $i = 1, 2, \ldots, n$, be a compendium of BFNs and $\varphi = (\varphi_1, \varphi_2, \ldots, \varphi_n)$ be the weights of $\mathfrak{B}_i$, $i = 1, 2, \ldots, n$, with $\varphi_i > 0$ and $\sum_{i=1}^{n} \varphi_i = 1$. Then, ST-BFWA operator is described as

$$\mathrm{ST-BFWA}(\mathfrak{B}_1, \mathfrak{B}_2, \ldots, \mathfrak{B}_n) = \varphi_1 \sin \mathfrak{B}_1 \oplus \varphi_2 \sin \mathfrak{B}_2 \oplus \cdots \oplus \varphi_n \sin \mathfrak{B}_n. \tag{13}$$

Theorem 4. . *Let* $\mathfrak{B}_i = \langle \aleph^{+}_{\mathfrak{B}_i}, \aleph^{-}_{\mathfrak{B}_i}\rangle$ *be n BFNs, then their cumulative value acquired by using (13) is again a BFN and is given by*

$$\mathrm{ST-BFWA}(\mathfrak{B}_1, \mathfrak{B}_2, \ldots, \mathfrak{B}_n) = \left\langle 1-\prod_{i=1}^{n}\left(1-\sin\left(\frac{\pi}{2}\aleph^{+}_{\mathfrak{B}_i}\right)\right)^{\varphi_i}, -\prod_{i=1}^{n}\left(-\left(\sin\left(\frac{\pi}{2}\left(1+\aleph^{-}_{\mathfrak{B}_i}\right)\right)-1\right)\right)^{\varphi_i}\right\rangle. \tag{14}$$

Proof. To prove the theorem, we employ mathematical induction on n. For $n = 2$, we have

$$\begin{aligned}\mathrm{ST}-\mathrm{BFWA}(\mathfrak{B}_1,\mathfrak{B}_2) &= \varphi_1 \sin \mathfrak{B}_1 \oplus \varphi_2 \sin \mathfrak{B}_2\\ &= \left\langle 1-\left(1-\sin\left(\frac{\pi}{2}\aleph^{+}_{\mathfrak{B}_1}\right)\right)^{\varphi_1}, -\left(-\left(\sin\left(\frac{\pi}{2}\left(1+\aleph^{-}_{\mathfrak{B}_1}\right)\right)-1\right)\right)^{\varphi_1}\right\rangle\\ &\oplus\left\langle 1-\left(1-\sin\left(\frac{\pi}{2}\aleph^{+}_{\mathfrak{B}_2}\right)\right)^{\varphi_2}, -\left(-\left(\sin\left(\frac{\pi}{2}\left(1+\aleph^{-}_{\mathfrak{B}_2}\right)\right)-1\right)\right)^{\varphi_2}\right\rangle\\ &= \left\langle 1-\prod_{i=1}^{2}\left(1-\sin\left(\frac{\pi}{2}\aleph^{+}_{\mathfrak{B}_i}\right)\right)^{\varphi_i}, -\prod_{i=1}^{2}\left(-\left(\sin\left(\frac{\pi}{2}\left(1+\aleph^{-}_{\mathfrak{B}_i}\right)\right)-1\right)\right)^{\varphi_i}\right\rangle.\end{aligned} \tag{15}$$

This shows that our assertion is correct for $n = 2$. Assume that the result holds true for $n = k$, i.e.,

$$\begin{aligned}\mathrm{ST}-\mathrm{BFWA}(\mathfrak{B}_1,\mathfrak{B}_2,\ldots,\mathfrak{B}_k) &= \varphi_1 \sin \mathfrak{B}_1 \oplus \varphi_2 \sin \mathfrak{B}_2 \oplus \cdots \oplus \varphi_k \sin \mathfrak{B}_k\\ &= \left\langle 1-\prod_{i=1}^{k}\left(1-\sin\left(\frac{\pi}{2}\aleph^{+}_{\mathfrak{B}_i}\right)\right)^{\varphi_i}, -\prod_{i=1}^{k}\left(-\left(\sin\left(\frac{\pi}{2}\left(1+\aleph^{-}_{\mathfrak{B}_i}\right)\right)-1\right)\right)^{\varphi_i}\right\rangle.\end{aligned} \tag{16}$$

Now, for $n = k + 1$, we have

$$\begin{aligned}ST-BFWA(\mathfrak{B}_1,\mathfrak{B}_2,\ldots,\mathfrak{B}_{k+1}) &= \varphi_1 \sin \mathfrak{B}_1 \oplus \varphi_2 \sin \mathfrak{B}_2 \oplus \cdots \oplus \varphi_k \sin \mathfrak{B}_k \oplus \varphi_{k+1} \sin \mathfrak{B}_{k+1}\\ &= \left\langle 1-\prod_{i=1}^{k}\left(1-\sin\left(\frac{\pi}{2}\aleph^{+}_{\mathfrak{B}_1}\right)\right)^{\varphi_i}, -\prod_{i=1}^{k}\left(-\left(\sin\left(\frac{\pi}{2}\left(1+\aleph^{-}_{\mathfrak{B}_1}\right)\right)-1\right)\right)^{\varphi_i}\right\rangle\\ &\oplus\left\langle 1-\left(1-\sin\left(\frac{\pi}{2}\aleph^{+}_{\mathfrak{B}_{k+1}}\right)\right)^{\varphi_{k+1}}, -\left(-\left(\sin\left(\frac{\pi}{2}\left(1+\aleph^{-}_{\mathfrak{B}_{k+1}}\right)\right)-1\right)\right)^{\varphi_{k+1}}\right\rangle\\ &= \left\langle 1-\prod_{i=1}^{k+1}\left(1-\sin\left(\frac{\pi}{2}\aleph^{+}_{\mathfrak{B}_i}\right)\right)^{\varphi_i}, -\prod_{i=1}^{k+1}\left(-\left(\sin\left(\frac{\pi}{2}\left(1+\aleph^{-}_{\mathfrak{B}_i}\right)\right)-1\right)\right)^{\varphi_i}\right\rangle.\end{aligned} \tag{17}$$

Hence, the result holds $\forall n$. □

Example 1. . Let $\mathfrak{B}_1 = (0.41, -0.39)$, $\mathfrak{B}_2 = (0.66, -0.21)$, $\mathfrak{B}_3 = (0.59, -0.46)$, and $\mathfrak{B}_4 = (0.72, -0.56)$ be four BFNs and $\varphi = (0.23, 0.31, 0.27, 0.19)$ be the corresponding weight vector, then

$$\begin{aligned}
\prod_{i=1}^{4}\left(1-\sin\left(\frac{\pi}{2}\aleph_{\mathfrak{B}_i}^{+}\right)\right)^{\varphi_i} &= \left(1-\sin\left(\frac{\pi}{2}\aleph_{\mathfrak{B}_1}^{+}\right)\right)^{\varphi_1}\times\left(1-\sin\left(\frac{\pi}{2}\aleph_{\mathfrak{B}_2}^{+}\right)\right)^{\varphi_2}\times\left(1-\sin\left(\frac{\pi}{2}\aleph_{\mathfrak{B}_3}^{+}\right)\right)^{\varphi_3}\times\left(1-\sin\left(\frac{\pi}{2}\aleph_{\mathfrak{B}_4}^{+}\right)\right)^{\varphi_4}\\
&= \left(1-\sin\left(\frac{\pi}{2}(0.41)\right)\right)^{0.23}\times\left(1-\sin\left(\frac{\pi}{2}(0.66)\right)\right)^{0.31}\times\left(1-\sin\left(\frac{\pi}{2}(0.59)\right)\right)^{0.27}\times\left(1-\sin\left(\frac{\pi}{2}(0.72)\right)\right)^{0.19}\\
&= 0.1821,\\
\prod_{i=1}^{4}\left(-\left(\sin\left(\frac{\pi}{2}\left(1+\aleph_{\mathfrak{B}_i}^{-}\right)\right)-1\right)\right)^{\varphi_i} &= \left(-\left(\sin\left(\frac{\pi}{2}\left(1+\aleph_{\mathfrak{B}_1}^{-}\right)\right)-1\right)\right)^{\varphi_1}\times\left(-\left(\sin\left(\frac{\pi}{2}\left(1+\aleph_{\mathfrak{B}_2}^{-}\right)\right)-1\right)\right)^{\varphi_2}\\
&\quad\times\left(-\left(\sin\left(\frac{\pi}{2}\left(1+\aleph_{\mathfrak{B}_3}^{-}\right)\right)-1\right)\right)^{\varphi_3}\times\left(-\left(\sin\left(\frac{\pi}{2}\left(1+\aleph_{\mathfrak{B}_4}^{-}\right)\right)-1\right)\right)^{\varphi_4}\\
&= \left(-\left(\sin\left(\frac{\pi}{2}(1-0.39)\right)-1\right)\right)^{0.23}\times\left(-\left(\sin\left(\frac{\pi}{2}(1-0.21)\right)-1\right)\right)^{0.31}\times\left(-\left(\sin\left(\frac{\pi}{2}(1-0.46)\right)-1\right)\right)^{0.27}\\
&\quad\times\left(-\left(\sin\left(\frac{\pi}{2}(1-0.56)\right)-1\right)\right)^{0.19}\\
&= 0.1550.
\end{aligned}\tag{18}$$

Now,

$$\begin{aligned}
\text{ST}-\text{BFWA}\left(\mathfrak{B}_1,\mathfrak{B}_2,\mathfrak{B}_3,\mathfrak{B}_4\right) &= \left\langle 1-\prod_{i=1}^{4}\left(1-\sin\left(\frac{\pi}{2}\aleph_{\mathfrak{B}_i}^{+}\right)\right)^{\varphi_i},\right.\\
&\qquad\left. -\prod_{i=1}^{4}\left(-\left(\sin\left(\frac{\pi}{2}\left(1+\aleph_{\mathfrak{B}_i}^{-}\right)\right)-1\right)\right)^{\varphi_i}\right\rangle\\
&= \langle 1-0.1821, -0.1550\rangle\\
&= \langle 0.8179, -0.1550\rangle.
\end{aligned}\tag{19}$$

Theorem 5. . *Let $\mathfrak{B}_i = \langle \aleph_{\mathfrak{B}_i}^{+}, \aleph_{\mathfrak{B}_i}^{-}\rangle$, $i = 1, 2, \ldots, n$, be a compendium of BFNs and $\varphi = (\varphi_1, \varphi_2, \ldots, \varphi_n)$ be the weight vector with $\varphi_i > 0$ and $\sum_{i=1}^{n}\varphi_i = 1$, then ST-BFWA operator holds the properties listed as follows:*

(i) *Idempotency. If all BFNs are equal, i.e., $\mathfrak{B}_i = \mathfrak{B} = \langle \aleph_{\mathfrak{B}}^{+}, \aleph_{\mathfrak{B}}^{-}\rangle$, then*

$$\text{ST}-\text{BFWA}\left(\mathfrak{B}_1,\mathfrak{B}_2,\ldots,\mathfrak{B}_n\right) = \sin\mathfrak{B}.\tag{20}$$

(ii) *Monotonicity. Let $\mathfrak{B}_i^{*} = \langle \aleph_{\mathfrak{B}_i^{*}}^{+}, \aleph_{\mathfrak{B}_i^{*}}^{-}\rangle$, $i = 1, 2, \ldots, n$, be another collection of BFNs such that $\mathfrak{B}_i \prec \mathfrak{B}_i^{*}$, $\forall i = 1, 2, \ldots, n$, then*

$$\begin{aligned}
&\text{ST}-\text{BFWA}\left(\mathfrak{B}_1,\mathfrak{B}_2,\ldots,\mathfrak{B}_n\right)\\
&\quad\preceq \text{ST}-\text{BFWA}\left(\mathfrak{B}_1^{*},\mathfrak{B}_2^{*},\ldots,\mathfrak{B}_n^{*}\right).
\end{aligned}\tag{21}$$

(iii) *Boundedness. Let $\mathfrak{B}_{-} = \langle \min_i(\aleph_{\mathfrak{B}_i}^{+}), \max_i(\aleph_{\mathfrak{B}_i}^{-})\rangle$ and $\overline{\mathfrak{B}} = \langle \max_i(\aleph_{\mathfrak{B}_i}^{+}), \min_i(\aleph_{\mathfrak{B}_i}^{-})\rangle$, then*

$$\sin\mathfrak{B}_{-} \prec \text{ST}-\text{BFWA}\left(\mathfrak{B}_1,\mathfrak{B}_2,\ldots,\mathfrak{B}_n\right) \prec \sin\overline{\mathfrak{B}}.\tag{22}$$

Proof

(i) Let $\mathfrak{B}_i = \mathfrak{B}\, \forall i = 1, 2, \ldots, n$. Then, by using (13), we have

$$\begin{aligned} ST-BFWA(\mathfrak{B}_1, \mathfrak{B}_2, \ldots, \mathfrak{B}_n) &= \left\langle 1-\prod_{i=1}^{n}\left(1-\sin\left(\frac{\pi}{2}\aleph_{\mathfrak{B}_i}^{+}\right)\right)^{\varphi_i} - \prod_{i=1}^{n}\left(-\left(\sin\left(\frac{\pi}{2}\left(1+\aleph_{\mathfrak{B}_i}^{-}\right)\right)-1\right)\right)^{\varphi_i}\right\rangle \\ &= \left\langle 1-\left(1-\sin\left(\frac{\pi}{2}\aleph_{\mathfrak{B}}^{+}\right)\right)^{\sum_{i=1}^{n}\varphi_i} - \left(-\left(\sin\left(\frac{\pi}{2}\left(1+\aleph_{\mathfrak{B}}^{-}\right)\right)-1\right)\right)^{\sum_{i=1}^{n}\varphi_i}\right\rangle \\ &= \left\langle 1-\left(1-\sin\left(\frac{\pi}{2}\aleph_{\mathfrak{B}}^{+}\right)\right) - \left(-\left(\sin\left(\frac{\pi}{2}\left(1+\aleph_{\mathfrak{B}}^{-}\right)\right)-1\right)\right)\right\rangle \\ &= \left\langle \sin\left(\frac{\pi}{2}\aleph_{\mathfrak{B}}^{+}\right), \ \sin\left(\frac{\pi}{2}\left(1+\aleph_{\mathfrak{B}}^{-}\right)\right)-1\right\rangle \\ &= \sin \mathfrak{B}. \end{aligned} \tag{23}$$

(ii) Since $\mathfrak{B}_i \prec \mathfrak{B}_i^{*}$, this implies that $\aleph_{\mathfrak{B}_i}^{+} \leq \aleph_{\mathfrak{B}_i^{*}}^{+}$ and $\aleph_{\mathfrak{B}_i}^{-} \geq \aleph_{\mathfrak{B}_i^{*}}^{-}$, $\forall i = 1, 2, \ldots, n$. Suppose that $\mathrm{ST}-\mathrm{BFWA}(\mathfrak{B}_1, \mathfrak{B}_2, \ldots, \mathfrak{B}_n) = \langle \widetilde{\aleph}^{+}, \widetilde{\aleph}^{-}\rangle$ and $\mathrm{ST}-\mathrm{BFWA}(\mathfrak{B}_1^{*}, \mathfrak{B}_2^{*}, \ldots, \mathfrak{B}_n^{*}) = \langle \widetilde{\aleph}^{*^{+}}, \widetilde{\aleph}^{*^{-}}\rangle$. Due to the monotonicity of sine function, we get

$$\begin{aligned} &\sin\left(\frac{\pi}{2}\aleph_{\mathfrak{B}_i}^{+}\right) \leq \sin\left(\frac{\pi}{2}\aleph_{\mathfrak{B}_i^{*}}^{+}\right), \\ &\Longrightarrow 1-\sin\left(\frac{\pi}{2}\aleph_{\mathfrak{B}_i}^{+}\right) \geq 1-\sin\left(\frac{\pi}{2}\aleph_{\mathfrak{B}_i^{*}}^{+}\right), \\ &\Longrightarrow \left(1-\sin\left(\frac{\pi}{2}\aleph_{\mathfrak{B}_i}^{+}\right)\right)^{\varphi_i} \geq \left(1-\sin\left(\frac{\pi}{2}\aleph_{\mathfrak{B}_i^{*}}^{+}\right)\right)^{\varphi_i}, \\ &\Longrightarrow \prod_{i=1}^{n}\left(1-\sin\left(\frac{\pi}{2}\aleph_{\mathfrak{B}_i}^{+}\right)\right)^{\varphi_i} \geq \prod_{i=1}^{n}\left(1-\sin\left(\frac{\pi}{2}\aleph_{\mathfrak{B}_i^{*}}^{+}\right)\right)^{\varphi_i}, \\ &\Longrightarrow \widetilde{\aleph}^{+} = 1-\prod_{i=1}^{n}\left(1-\sin\left(\frac{\pi}{2}\aleph_{\mathfrak{B}_i}^{+}\right)\right)^{\varphi_i} \leq 1-\prod_{i=1}^{n}\left(1-\sin\left(\frac{\pi}{2}\aleph_{\mathfrak{B}_i^{*}}^{+}\right)\right)^{\varphi_i} = \widetilde{\aleph}^{*^{+}}. \end{aligned} \tag{24}$$

Similarly,

$$\begin{aligned} &\sin\left(\frac{\pi}{2}\left(1+\aleph_{\mathfrak{B}_i}^{-}\right)\right) \geq \sin\left(\frac{\pi}{2}\left(1+\aleph_{\mathfrak{B}_i^{*}}^{-}\right)\right), \\ &\Longrightarrow \sin\left(\frac{\pi}{2}\left(1+\aleph_{\mathfrak{B}_i}^{-}\right)\right)-1 \geq \sin\left(\frac{\pi}{2}\left(1+\aleph_{\mathfrak{B}_i^{*}}^{-}\right)\right)-1, \\ &\Longrightarrow \left(-\left(\sin\left(\frac{\pi}{2}\left(1+\aleph_{\mathfrak{B}_i}^{-}\right)\right)-1\right)\right)^{\varphi_i} \leq \left(-\left(\sin\left(\frac{\pi}{2}\left(1+\aleph_{\mathfrak{B}_i^{*}}^{-}\right)\right)-1\right)\right)^{\varphi_i}, \\ &\Longrightarrow \widetilde{\aleph}^{-} = -\prod_{i=1}^{n}\left(-\left(\sin\left(\frac{\pi}{2}\left(1+\aleph_{\mathfrak{B}_i}^{-}\right)\right)-1\right)\right)^{\varphi_i} \geq -\prod_{i=1}^{n}\left(-\left(\sin\left(\frac{\pi}{2}\left(1+\aleph_{\mathfrak{B}_i^{*}}^{-}\right)\right)-1\right)\right)^{\varphi_i} = \widetilde{\aleph}^{*^{-}}. \end{aligned} \tag{25}$$

Since $\widetilde{\aleph}^{+} \leq \widetilde{\aleph}^{*^{+}}$ and $\widetilde{\aleph}^{-} \geq \widetilde{\aleph}^{*^{-}}$, we have

$$\text{ST}-\text{BFWA}\left(\mathfrak{B}_1,\mathfrak{B}_2,\ldots,\mathfrak{B}_n\right) \leq \text{ST}-\text{BFWA}\left(\mathfrak{B}_1^*,\mathfrak{B}_2^*,\ldots,\mathfrak{B}_n^*\right). \tag{26}$$

(iii) It is similar to the preceding proof, so we exclude it. □

4.2. ST-BFOWA Operator

Definition 10. Let $\mathfrak{B}_i$, $i = 1,2,\ldots,n$, be a compendium of BFNs, then ST-BFOWA operator is explicated as

$$\begin{aligned}&\text{ST}-\text{BFOWA}\left(\mathfrak{B}_1,\mathfrak{B}_2,\ldots,\mathfrak{B}_n\right)\\&= \varphi_1 \sin \mathfrak{B}_{\eta(1)} \oplus \varphi_2 \sin \mathfrak{B}_{\eta(2)} \oplus \cdots \oplus \varphi_n \sin \mathfrak{B}_{\eta(n)},\end{aligned} \tag{27}$$

where $(\eta(1),\eta(2),\ldots,\eta(n))$ is an arrangement of $(1,2,\ldots,n)$ with the constraint that $\mathfrak{B}_{\eta(i-1)} \geq \mathfrak{B}_{\eta(i)}\ \forall i = 2,3,\ldots,n$. It is noteworthy that the weights φ_i with $\varphi_i > 0$ and $\sum_{i=1}^{n}\varphi_i = 1$ are associated with the ordered positions of BFNs $\mathfrak{B}_i$.

Theorem 6. *The cumulative value of n BFNs $\mathfrak{B}_i = \langle \aleph^{+}_{\mathfrak{B}_i}, \aleph^{-}_{\mathfrak{B}_i}\rangle$ acquired by utilizing ST-BFOWA operator is still a BFN and is given by*

$$\begin{aligned}&\text{ST}-\text{BFOWA}\left(\mathfrak{B}_1,\mathfrak{B}_2,\ldots,\mathfrak{B}_n\right)\\&= \left\langle 1-\prod_{i=1}^{n}\left(1-\sin\left(\frac{\pi}{2}\aleph^{+}_{\mathfrak{B}_{\eta(i)}}\right)\right)^{\varphi_i},\right.\\&\left.-\prod_{i=1}^{n}\left(-\left(\sin\left(\frac{\pi}{2}\left(1+\aleph^{-}_{\mathfrak{B}_{\eta(i)}}\right)\right)-1\right)\right)^{\varphi_i}\right\rangle.\end{aligned} \tag{28}$$

Proof. Straightforward. □

Theorem 7. *Let $\mathfrak{B}_i = \langle \aleph^{+}_{\mathfrak{B}_i}, \aleph^{-}_{\mathfrak{B}_i}\rangle$, $i = 1,2,\ldots,n$, be a compendium of BFNs and $\varphi = (\varphi_1,\varphi_2,\ldots,\varphi_n)$ be the weight vector with $\varphi_i > 0$ and $\sum_{i=1}^{n}\varphi_i = 1$, then ST-BFOWA operator satisfies the following properties:*

(i) Idempotency. If $\mathfrak{B}_i = \mathfrak{B} = \langle \aleph^{+}_{\mathfrak{B}}, \aleph^{-}_{\mathfrak{B}}\rangle$, $\forall i = 1,2,\ldots,n$, then

$$\text{ST}-\text{BFOWA}\left(\mathfrak{B}_1,\mathfrak{B}_2,\ldots,\mathfrak{B}_n\right) = \sin \mathfrak{B}. \tag{29}$$

(ii) Monotonicity. Let $\mathfrak{B}_i^* = \langle \aleph^{+}_{\mathfrak{B}_i^*}, \aleph^{-}_{\mathfrak{B}_i^*}\rangle$, $i = 1,2,\ldots,n$, be another collection of BFNs such that $\mathfrak{B}_i \prec \mathfrak{B}_i^*$, $\forall i = 1,2,\ldots,n$, then

$$\text{ST}-\text{BFOWA}\left(\mathfrak{B}_1,\mathfrak{B}_2,\ldots,\mathfrak{B}_n\right) \leq \text{ST}-\text{BFOWA}\left(\mathfrak{B}_1^*,\mathfrak{B}_2^*,\ldots,\mathfrak{B}_n^*\right). \tag{30}$$

(iii) Boundedness. If $\underline{\mathfrak{B}} = \langle \min_i(\aleph^{+}_{\mathfrak{B}_i}), \max_i(\aleph^{-}_{\mathfrak{B}_i})\rangle$ and $\overline{\mathfrak{B}} = \langle \max_i(\aleph^{+}_{\mathfrak{B}_i}), \min_i(\aleph^{-}_{\mathfrak{B}_i})\rangle$, then

$$\sin \underline{\mathfrak{B}} \preceq \text{ST}-\text{BFOWA}\left(\mathfrak{B}_1,\mathfrak{B}_2,\ldots,\mathfrak{B}_n\right) \preceq \sin \overline{\mathfrak{B}}. \tag{31}$$

Proof. It is obvious. □

4.3. ST-BFHWA Operator

Definition 11. Let $\mathfrak{B}_i$, $i = 1,2,\ldots,n$, be a compendium of BFNs and $\varphi = (\varphi_1,\varphi_2,\ldots,\varphi_n)$ be the weight vector of $\mathfrak{B}_i$ with $\varphi_i > 0$ and $\sum_{i=1}^{n}\varphi_i = 1$. A ST-BFHWA operator with associated weight vector $\gamma = (\gamma_1,\gamma_2,\ldots,\gamma_n)$ with $\gamma_i > 0$ and $\sum_{i=1}^{n}\gamma_i = 1$ can be described as

$$\begin{aligned}&\text{ST}-\text{BFHWA}\left(\mathfrak{B}_1,\mathfrak{B}_2,\ldots,\mathfrak{B}_n\right)\\&= \gamma_1 \sin \dot{\mathfrak{B}}_{\eta(1)} \oplus \gamma_2 \sin \dot{\mathfrak{B}}_{\eta(2)} \oplus \cdots \oplus \gamma_n \sin \dot{\mathfrak{B}}_{\eta(n)},\end{aligned} \tag{32}$$

where $\dot{\mathfrak{B}}_i = n\varphi_i\mathfrak{B}_i$ and $(\eta(1),\eta(2),\ldots,\eta(n))$ is an arrangement of $(1,2,\ldots,n)$ with the stipulation that $\dot{\mathfrak{B}}_{\eta(i-1)} \geq \dot{\mathfrak{B}}_{\eta(i)}\ \forall i = 2,3,\ldots,n$.

Theorem 8. *The cumulative value of n BFNs $\mathfrak{B}_i = \langle \aleph^{+}_{\mathfrak{B}_i}, \aleph^{-}_{\mathfrak{B}_i}\rangle$ acquired by utilizing ST-BFHWA operator is still a BFN and is given by*

$$\begin{aligned}&\text{ST}-\text{BFHWA}\left(\mathfrak{B}_1,\mathfrak{B}_2,\ldots,\mathfrak{B}_n\right)\\&= \left\langle 1-\prod_{i=1}^{n}\left(1-\sin\left(\frac{\pi}{2}\aleph^{+}_{\dot{\mathfrak{B}}_{\eta(i)}}\right)\right)^{\gamma_i},\right.\\&\left.-\prod_{i=1}^{n}\left(-\left(\sin\left(\frac{\pi}{2}\left(1+\aleph^{-}_{\dot{\mathfrak{B}}_{\eta(i)}}\right)\right)-1\right)\right)^{\gamma_i}\right\rangle.\end{aligned} \tag{33}$$

Proof. Straightforward. □

5. Bipolar Fuzzy Sine Trigonometric Geometric Aggregation Operators

In this section, we propose geometric aggregation operators including (i) ST-BFWG operator, (ii) ST-BFOWG operator, and (iii) ST-BFHWG operator.

5.1. ST-BFWG Operator

Definition 12. For n BFNs $\mathfrak{B}_i$, a ST-BFWG operator is explicated as

$$\begin{aligned}\text{ST}-\text{BFWG}\left(\mathfrak{B}_1,\mathfrak{B}_2,\ldots,\mathfrak{B}_n\right) &= \left(\sin \mathfrak{B}_1\right)^{\varphi_1}\\&\otimes \left(\sin \mathfrak{B}_2\right)^{\varphi_2} \otimes \cdots \otimes \left(\sin \mathfrak{B}_n\right)^{\varphi_n}.\end{aligned} \tag{34}$$

Theorem 9. *Let $\mathfrak{B}_i = \langle \aleph^{+}_{\mathfrak{B}_i}, \aleph^{-}_{\mathfrak{B}_i}\rangle$ be n BFNs, then their cumulative value obtained by utilizing ST-BFWG operator is expressed as*

$$\text{ST} - \text{BFWG}(\mathfrak{B}_1, \mathfrak{B}_2, \ldots, \mathfrak{B}_n) = \left\langle \prod_{i=1}^{n} \left(\sin\left(\frac{\pi}{2} \aleph_{\mathfrak{B}_i}^{+} \right) \right)^{\varphi_i}, -\left(1 - \prod_{i=1}^{n} \left(\sin\left(\frac{\pi}{2} \left(1 + \aleph_{\mathfrak{B}_i}^{-} \right) \right) \right)^{\varphi_i} \right) \right\rangle. \tag{35}$$

Proof. It is obvious. □

Example 2. Let $\mathfrak{B}_1 = (0.41, -0.39)$, $\mathfrak{B}_2 = (0.66, -0.21)$, $\mathfrak{B}_3 = (0.59, -0.46)$, and $\mathfrak{B}_4 = (0.72, -0.56)$ be four BFNs and $\varphi = (0.23, 0.31, 0.27, 0.19)$ be the corresponding weight vector, then

$$\begin{aligned}
\prod_{i=1}^{4} \left(\sin\left(\frac{\pi}{2} \aleph_{\mathfrak{B}_i}^{+} \right) \right)^{\varphi_i} &= \left(\sin\left(\frac{\pi}{2} \aleph_{\mathfrak{B}_1}^{+} \right) \right)^{\varphi_1} \times \left(\sin\left(\frac{\pi}{2} \aleph_{\mathfrak{B}_2}^{+} \right) \right)^{\varphi_2} \times \left(\sin\left(\frac{\pi}{2} \aleph_{\mathfrak{B}_3}^{+} \right) \right)^{\varphi_3} \times \left(\sin\left(\frac{\pi}{2} \aleph_{\mathfrak{B}_4}^{+} \right) \right)^{\varphi_4} \\
&= \left(\sin\left(\frac{\pi}{2} (0.41) \right) \right)^{0.23} \times \left(\sin\left(\frac{\pi}{2} (0.66) \right) \right)^{0.31} \times \left(\sin\left(\frac{\pi}{2} (0.59) \right) \right)^{0.27} \times \left(\sin\left(\frac{\pi}{2} (0.72) \right) \right)^{0.19} \\
&= 0.7841 \\
\prod_{i=1}^{4} \left(\sin\left(\frac{\pi}{2} \left(1 + \aleph_{\mathfrak{B}_i}^{-} \right) \right) \right)^{\varphi_i} &= \left(\sin\left(\frac{\pi}{2} \left(1 + \aleph_{\mathfrak{B}_1}^{-} \right) \right) \right)^{\varphi_1} \times \left(\sin\left(\frac{\pi}{2} \left(1 + \aleph_{\mathfrak{B}_2}^{-} \right) \right) \right)^{\varphi_2} \times \left(\sin\left(\frac{\pi}{2} \left(1 + \aleph_{\mathfrak{B}_3}^{-} \right) \right) \right)^{\varphi_3} \times \left(\sin\left(\frac{\pi}{2} \left(1 + \aleph_{\mathfrak{B}_4}^{-} \right) \right) \right)^{\varphi_4} \\
&= \left(\sin\left(\frac{\pi}{2} (1 - 0.39) \right) \right)^{0.23} \times \left(\left(\sin\left(\frac{\pi}{2} (1 - 0.21) \right) \right)^{0.31} \\
&\quad \times \left(\sin\left(\frac{\pi}{2} (1 - 0.46) \right) \right)^{0.27} \times \left(\left(\sin\left(\frac{\pi}{2} (1 - 0.56) \right) \right)^{0.19} = 0.7973.
\end{aligned} \tag{36}$$

Now,

$$\begin{aligned}
\text{ST} - \text{BFWG}(\mathfrak{B}_1, \mathfrak{B}_2, \mathfrak{B}_3, \mathfrak{B}_4) &= \left\langle \prod_{i=1}^{4} \left(\sin\left(\frac{\pi}{2} \aleph_{\mathfrak{B}_i}^{+} \right) \right)^{\varphi_i}, -\left(1 - \prod_{i=1}^{4} \left(\sin\left(\frac{\pi}{2} \left(1 + \aleph_{\mathfrak{B}_i}^{-} \right) \right) \right)^{\varphi_i} \right) \right\rangle \\
&= \langle 0.7841, -(1 - 0.7973) \rangle \\
&= \left\langle 0.7841, -0.2027 \right\rangle.
\end{aligned} \tag{37}$$

The properties mentioned in Theorem 5, namely, idempotency, monotonicity, and boundedness, also apply to the ST-BFWG operator.

5.2. ST-BFOWG Operator

Definition 13. A ST-BFOWG operator is defined as

$$\text{ST} - \text{BFOWG}(\mathfrak{B}_1, \mathfrak{B}_2, \ldots, \mathfrak{B}_n) = \left(\sin \mathfrak{B}_{\eta(1)} \right)^{\varphi_1} \otimes \left(\sin \mathfrak{B}_{\eta(2)} \right)^{\varphi_2} \otimes \cdots \otimes \left(\sin \mathfrak{B}_{\eta(n)} \right)^{\varphi_n}, \tag{38}$$

where $(\eta(1), \eta(2), \ldots, \eta(n))$ is an arrangement of $(1, 2, \ldots, n)$ such that $\mathfrak{B}_{\eta(i-1)} \geq \mathfrak{B}_{\eta(i)}\ \forall i = 2, 3, \ldots, n$.

Theorem 10. *The cumulative value of n BFNs $\mathfrak{B}_i = \langle \aleph_{\mathfrak{B}_i}^{+}, \aleph_{\mathfrak{B}_i}^{-} \rangle$ acquired by utilizing ST-BFOWG operator is expressed as*

$$\text{ST} - \text{BFOWG}(\mathfrak{B}_1, \mathfrak{B}_2, \ldots, \mathfrak{B}_n) = \left\langle \prod_{i=1}^{n} \left(\sin\left(\frac{\pi}{2} \aleph_{\mathfrak{B}_{\eta(i)}}^{+} \right) \right)^{\varphi_i}, -\left(1 - \prod_{i=1}^{n} \left(\sin\left(\frac{\pi}{2} \left(1 + \aleph_{\mathfrak{B}_{\eta(i)}}^{-} \right) \right) \right)^{\varphi_i} \right) \right\rangle. \tag{39}$$

Proof. It is obvious.

Idempotency, monotonicity, and boundedness are all satisfied by the ST-BFOWG operator. □

5.3. ST-BFHWG Operator

Definition 14. A ST-BFHWG operator with associated weight vector $\gamma = (\gamma_1, \gamma_2, \ldots, \gamma_n)$ with $\gamma_i > 0$ and $\sum_{i=1}^{n} \gamma_i = 1$ can be described as

$$\begin{aligned}\text{ST} - \text{BFHWG}(\mathfrak{B}_1, \mathfrak{B}_2, \ldots, \mathfrak{B}_n) \\ = \left(\sin \dot{\mathfrak{B}}_{\eta(1)}\right)^{\gamma_1} \otimes \left(\sin \dot{\mathfrak{B}}_{\eta(2)}\right)^{\gamma_2} \otimes \cdots \otimes \left(\sin \dot{\mathfrak{B}}_{\eta(n)}\right)^{\gamma_n}.\end{aligned} \tag{40}$$

where $\dot{\mathfrak{B}}_i = (\mathfrak{B}_i)^{n\varphi_i}$ and $(\eta(1), \eta(2), \ldots, \eta(n))$ is an arrangement of $(1, 2, \ldots, n)$ such that $\dot{\mathfrak{B}}_{\eta(i-1)} \geq \dot{\mathfrak{B}}_{\eta(i)}\ \forall i = 2, 3, \ldots, n$.

Theorem 11. *The cumulative value of n BFNs* $\mathfrak{B}_i = \langle \aleph^+_{\mathfrak{B}_i}, \aleph^-_{\mathfrak{B}_i} \rangle$ *obtained by utilizing ST-BFHWG operator is expressed as*

$$\begin{aligned}\text{ST} - \text{BFHWG}(\mathfrak{B}_1, \mathfrak{B}_2, \ldots, \mathfrak{B}_n) \\ = \left\langle \prod_{i=1}^{n} \left(\sin\left(\frac{\pi}{2}\aleph^+_{\dot{\mathfrak{B}}_{\eta(i)}}\right)\right)^{\gamma_i}, \right. \\ \left. -\left(1 - \prod_{i=1}^{n} \left(\sin\left(\frac{\pi}{2}\left(1 + \aleph^-_{\dot{\mathfrak{B}}_{\eta(i)}}\right)\right)\right)^{\gamma_i}\right) \right\rangle.\end{aligned} \tag{41}$$

Proof. Straightforward. □

6. Bipolar Fuzzy SIR Method

An MCGDM problem is made up of a finite number of alternatives, a set of criteria, and a set of decision makers. To solve an MCGDM problem, the most apposite alternative must be chosen among those available. Let $\mathbb{A} = \{\hat{a}_1, \hat{a}_2, \ldots, \hat{a}_m\}$ be a set of alternatives and $\mathbb{C} = \{\hat{c}_1, \hat{c}_2, \ldots, \hat{c}_n\}$ be a set of criteria. Suppose that the set of decision makers is $\mathbb{E} = \{\hat{e}_1, \hat{e}_2, \ldots, \hat{e}_l\}$ and their weight vector is $\vartheta = \{\vartheta_1, \vartheta_2, \ldots, \vartheta_l\}$ where all the weights are BFNs. Construct the individual decision matrices $\mathbb{H}_k = (h^k_{ij})_{m\times n}$ in which h^k_{ij} denotes the evaluation information of the alternative $\hat{a}_i$ w.r.t the criterion $\hat{c}_j$ provided by the decision maker $\hat{e}_k$ in the form of BFNs. Assume that $\varphi = (\varphi^k_j)_{l\times n}$ is the criterion weight matrix in which φ^k_j is the weight of criterion $\hat{c}_j$ assigned by the decision maker $\hat{e}_k$ in the form of BFNs. In this section, we set up the BF-SIR technique to address this MCGDM problem. The steps in this technique are outlined as follows:

Step 1. Compute the relative propinquity coefficient of each ϑ_k, $k = 1, 2, \ldots, l$, by the equation

$$\delta_k = \frac{d(\vartheta_k, \underline{\vartheta})}{d(\vartheta_k, \underline{\vartheta}) + d(\vartheta_k, \overline{\vartheta})}. \tag{42}$$

where $\underline{\vartheta} = \langle \min_k (\aleph^+_{\vartheta_k}), \max_k (\aleph^-_{\vartheta_k}) \rangle$ and $\overline{\vartheta} = \langle \max_k (\aleph^+_{\vartheta_k}), \min_k (\aleph^-_{\vartheta_k}) \rangle$. It is evident that if $\vartheta_k \longrightarrow \underline{\vartheta}$, then $\delta_k \longrightarrow 0$, and if $\vartheta_k \longrightarrow \overline{\vartheta}$, then $\delta_k \longrightarrow 1$.

Step 2. Normalize δ_k, $k = 1, 2, \ldots, l$, by the equation

$$\zeta_k = \frac{\delta_k}{\sum_{k=1}^{l} \delta_k}. \tag{43}$$

In this way, we get a normalized vector $\zeta = \{\zeta_1, \zeta_2, \ldots, \zeta_l\}$ of relative propinquity coefficients.

Step 3. Acquire the accumulated bipolar fuzzy decision matrix and the criterion weight vector by utilizing ST-BFWA operator as follows:

$$\begin{aligned}\tilde{h}_{ij} &= \text{ST} - \text{BFWA}_{\zeta_k}\left(h^1_{ij}, h^2_{ij}, \ldots, h^l_{ij}\right) \\ &= \left\langle 1 - \prod_{k=1}^{l} \left(1 - \sin\left(\frac{\pi}{2}\aleph^+_{h^k_{ij}}\right)\right)^{\zeta_k}, -\prod_{k=1}^{l} \left(-\left(\sin\left(\frac{\pi}{2}\left(1 + \aleph^-_{h^k_{ij}}\right)\right) - 1\right)\right)^{\zeta_k} \right\rangle.\end{aligned} \tag{44}$$

$$\begin{aligned}\tilde{\varphi}_j &= \text{ST} - \text{BFWA}_{\zeta_k}\left(\varphi^1_j, \varphi^2_j, \ldots, \varphi^l_j\right) \\ &= \left\langle 1 - \prod_{k=1}^{l} \left(1 - \sin\left(\frac{\pi}{2}\aleph^+_{\varphi^k_j}\right)\right)^{\zeta_k}, -\prod_{k=1}^{l} \left(-\left(\sin\left(\frac{\pi}{2}\left(1 + \aleph^-_{\varphi^k_j}\right)\right) - 1\right)\right)^{\zeta_k} \right\rangle.\end{aligned} \tag{45}$$

Step 4. Obtain the relative efficiency function f_{ij} as follows:

$$f_{ij} = \frac{d\left(\tilde{h}_{ij}, \underline{\tilde{h}}\right)}{d\left(\tilde{h}_{ij}, \underline{\tilde{h}}\right) + d\left(\tilde{h}_{ij}, \overline{\tilde{h}}\right)}, \tag{46}$$

where $\underline{\tilde{h}} = \langle \min_i (\aleph^+_{\tilde{h}_{ij}}), \max_i (\aleph^-_{\tilde{h}_{ij}}) \rangle$ and $\overline{\tilde{h}} = \langle \max_i (\aleph^+_{\tilde{h}_{ij}}), \min_i (\aleph^-_{\tilde{h}_{ij}}) \rangle$. It follows that if $\tilde{h}_{ij} \longrightarrow \underline{\tilde{h}}$, then $f_{ij} \longrightarrow 0$, and if $\tilde{h}_{ij} \longrightarrow \overline{\tilde{h}}$, then $f_{ij} \longrightarrow 1$.

Step 5. Compute the preference intensity $PI_j(\widehat{a}_i, \widehat{a}_t)$ $(i, t = 1, 2, \ldots, m, i \neq t)$ which provides the degree of preference of alternative $\widehat{a}_i$ over alternative $\widehat{a}_t$ w.r.t the criterion $\widehat{c}_j$ and it can be defined as follows:

$$\mathrm{PI}_j(\widehat{a}_i, \widehat{a}_t) = \lambda_j\left(f_{ij} - f_{tj}\right), \tag{47}$$

where λ_j is a threshold function given by

$$\lambda_j(x) = \begin{cases} 0.01, & x > 0, \\ 0, & x \leq 0. \end{cases} \tag{48}$$

Step 6. Construct the superiority matrix $S = (S_{ij})_{m\times n}$ and inferiority matrix $I = (I_{ij})_{m\times n}$ by utilizing the following equations:

$$\begin{aligned}(S - \text{index})\, S_{ij} &= \sum_{t=1}^{m} \mathrm{PI}_j(\widehat{a}_i, \widehat{a}_t) \\ &= \sum_{t=1}^{m} \lambda_j\left(f_{ij} - f_{tj}\right),\end{aligned} \tag{49}$$

$$\begin{aligned}(I - \text{index})\, I_{ij} &= \sum_{t=1}^{m} \mathrm{PI}_j(\widehat{a}_t, \widehat{a}_i) \\ &= \sum_{t=1}^{m} \lambda_j\left(f_{tj} - f_{ij}\right).\end{aligned} \tag{50}$$

Step 7. Calculate the superiority flow (S-flow) and inferiority flow (I-flow) as follows:

$$\begin{aligned}\lambda^{>}(\widehat{a}_i) &= \mathrm{ST} - \mathrm{BFWA}_{S_{ij}}(\widetilde{\varphi}_1, \widetilde{\varphi}_2, \ldots, \widetilde{\varphi}_n) \\ &= \langle 1 - \prod_{j=1}^{n}\left(1 - \sin\left(\frac{\pi}{2}\aleph^{+}_{\varphi_j}\right)\right)^{S_{ij}}, \\ &\quad - \prod_{j=1}^{n}\left(-\left(\sin\left(\frac{\pi}{2}\left(1 + \aleph^{-}_{\varphi_j}\right)\right) - 1\right)\right)^{S_{ij}}\rangle.\end{aligned} \tag{51}$$

$$\begin{aligned}\lambda^{<}(\widehat{a}_i) &= \mathrm{ST} - \mathrm{BFWA}_{I_{ij}}(\widetilde{\varphi}_1, \widetilde{\varphi}_2, \ldots, \widetilde{\varphi}_n) \\ &= \left\langle 1 - \prod_{j=1}^{n}\left(1 - \sin\left(\frac{\pi}{2}\aleph^{+}_{\varphi_j}\right)\right)^{I_{ij}}, \right. \\ &\quad - \prod_{j=1}^{n}\left(-\left(\sin\left(\frac{\pi}{2}\left(1 + \aleph^{-}_{\varphi_j}\right)\right) - 1\right)\right)^{I_{ij}}\rangle.\end{aligned} \tag{52}$$

Step 8. Compute the score functions of $\lambda^{>}(\widehat{a}_i)$ and $\lambda^{<}(\widehat{a}_i)$, $i = 1, 2, \ldots, m$, by using (2).

Step 9. Apply the superiority ranking laws (SR-laws) and inferiority ranking laws (IR-laws) as follows:

SR-Law 1. If $\lambda^{>}(\widehat{a}_i) \succ \lambda^{>}(\widehat{a}_t)$ and $\lambda^{<}(\widehat{a}_i) \prec \lambda^{<}(\widehat{a}_t)$, then $\widehat{a}_i \succ \widehat{a}_t$

SR-Law 2. If $\lambda^{>}(\widehat{a}_i) \succ \lambda^{>}(\widehat{a}_t)$ and $\lambda^{<}(\widehat{a}_i) = \lambda^{<}(\widehat{a}_t)$, then $\widehat{a}_i \succ \widehat{a}_t$

SR-Law 3. If $\lambda^{>}(\widehat{a}_i) = \lambda^{>}(\widehat{a}_t)$ and $\lambda^{<}(\widehat{a}_i) \prec \lambda^{<}(\widehat{a}_t)$, then $\widehat{a}_i \succ \widehat{a}_t$

IR-Law 1. If $\lambda^{>}(\widehat{a}_i) \prec \lambda^{>}(\widehat{a}_t)$ and $\lambda^{<}(\widehat{a}_i) \succ \lambda^{<}(\widehat{a}_t)$, then $\widehat{a}_i \prec \widehat{a}_t$

IR-Law 2. If $\lambda^{>}(\widehat{a}_i) \prec \lambda^{>}(\widehat{a}_t)$ and $\lambda^{<}(\widehat{a}_i) = \lambda^{<}(\widehat{a}_t)$, then $\widehat{a}_i \prec \widehat{a}_t$

IR-Law 3. If $\lambda^{>}(\widehat{a}_i) = \lambda^{>}(\widehat{a}_t)$ and $\lambda^{<}(\widehat{a}_i) \succ \lambda^{<}(\widehat{a}_t)$, then $\widehat{a}_i \prec \widehat{a}_t$

Step 10. Integrate the SR-laws with the IR-laws to determine the optimal alternative.

6.1. Case Study. The process of seeking medical treatment supply chain from a foreign country is known as medical tourism. In the past, patients from underdeveloped parts of the world used to travel to Europe and America for diagnostics and treatment. However, in recent years, this trend has flipped as medical tourism, in which individuals from developed countries travel to developing countries for medical treatment. There are a variety of reasons why people from developed countries prefer less developed countries. The low cost of treatment is the main factor. Healthcare prices are dependent on a country's per capita gross domestic product (GDP), which serves as a procurator for income levels. The low cost of offshore medical care is indebted to low medicolegal and administrative costs. Second, people seek medical guidance from outside the country for the procedures for which health insurance is not provided, such as cosmetic surgery, fertility therapy, dental reconstruction, gender reassignment surgeries, and so on. Patients in countries where access to healthcare is regulated by the government, such as Canada and the United Kingdom, desire to avoid the delays that come with extensive waiting lists. Another factor could be the lack of availability of a certain operation in their home country, such as stem cell therapy, which may be inaccessible or limited in developed countries but widely available in emerging markets. Some patients believe that their privacy will be better protected in a remote location. Another motive for offshore treatment is the recreational aspect. As a result of these factors, medical tourism is expanding globally. Medical tourism was worth 54.4 billion US dollars in 2020, and by 2027, it was expected to be worth more than 200 billion US dollars (https://www.statista.com/statistics/1084720/medical-tourism-market-size-worldwide/). Figure 1 depicts the gradual expansion of the medical tourism industry from 2016 to 2020, with projections for 2027.

The medical tourism market in Asia-Pacific has a lot of room for expansion. Due to economic development, this region is expected to see rapid market expansion. Singapore, Japan, India, Thailand, and the Philippines are among the most popular medical tourism destinations. Singapore and India are well-known for their cardiac and orthopaedic surgery. Thailand is well-known for its dental procedures and gender reassignment surgeries. Japan has one of the best oncology treatment facilities in the world. The Philippines is famous for its cosmetic surgery, dentistry, and fertility treatment. The Medical Tourism Index (MTI) evaluates a country's suitability as a medical tourism destination by taking into account its overall environment, healthcare costs, tourist attractions, and the standard of medical amenities and services. The higher the MTI, the better the destination.

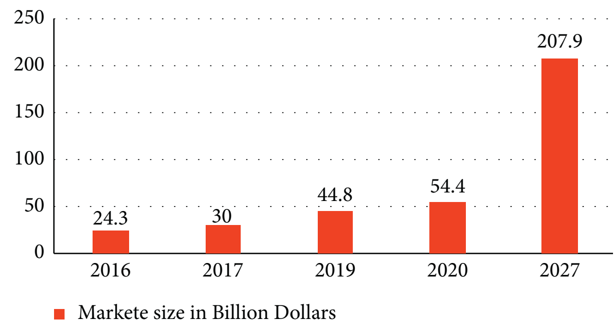

FIGURE 1: Medical tourism market size.

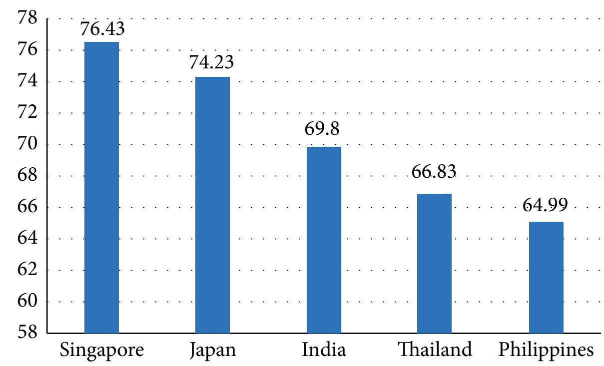

FIGURE 2: Medical tourism index (2020-2021).

Figure 2 depicts the medical tourism index for the aforementioned Asian countries (https://www.medicaltourism.com/mti/2020-2021/region/asia).

Medical tourism is seen as an unexplored sector in Pakistan that might be transformed into a lucrative potential if the government addresses some critical issues such as security, brain drain, and service quality. According to Pakistani medical professionals, Pakistan has "huge potential" to become a competitive medical tourism hub in Asia. In what follows, we will use the BF-SIR method to determine the best medical tourism destination in Pakistan.

6.2. Numerical Illustration. Suppose that ministry of health of a Pakistan needs to assess some true potential of medical tourism supply chain. For this purpose, the ministry hires three decision makers $\hat{e}_1, \hat{e}_2$, and $\hat{e}_3$ and assigns them weights which are given in Table 1. Let $\mathbb{A} = \{\hat{a}_1, \hat{a}_2, \hat{a}_3, \hat{a}_4\}$ be the set of alternatives where $\hat{a}_1 =$ Islamabad, $\hat{a}_2 =$ Karachi, $\hat{a}_3 =$ Lahore, and $\hat{a}_4 =$ Peshawar. Table 2 lists the criteria for determining the best alternative. The weights of criteria $\hat{c}_j$ given by the decision makers $\hat{e}_k$ are given in Table 3. The decision makers evaluate each alternative $\hat{a}_i$ w.r.t each criterion $\hat{c}_j$ and give their assessment via BFNs. Three decision matrices are given in Tables 4–6.

Step 1. The relative propinquity coefficients δ_k $(k = 1, 2, 3)$ are computed using (42) as follows:

$$\delta = \{0.12, 0.72, 0.52\}. \tag{53}$$

Step 2. The normalized vector is obtained using (43)as follows:

$$\zeta = \{0.0882, 0.5294, 0.3824\}. \tag{54}$$

Step 3. The accumulated bipolar fuzzy decision matrix is acquired using (44), which is given in Table 7. Equation (45) is used to determine accumulated weights of criteria, which are as follows:

$$\begin{aligned}\widetilde{\varphi}_1 &= \langle 0.9357, -0.1744\rangle,\\ \widetilde{\varphi}_2 &= \langle 0.9423, -0.0799\rangle,\\ \widetilde{\varphi}_3 &= \langle 0.9183, -0.0632\rangle.\end{aligned} \tag{55}$$

TABLE 1: Bipolar fuzzy weights of decision makers.

Decision makers	Weights
$\hat{e}_1$	$\vartheta_1 = \langle 0.79, -0.28\rangle$
$\hat{e}_2$	$\vartheta_2 = \langle 0.85, -0.37\rangle$
$\hat{e}_3$	$\vartheta_3 = \langle 0.92, -0.25\rangle$

Step 4. The relative efficiency function is calculated using (46)as follows:

$$f_{ij} = \begin{pmatrix} 0.3427 & 0.2234 & 0.4693\\ 0.4320 & 0.4241 & 0.5963\\ 1 & 0.8834 & 0.4820\\ 0.7463 & 0.2807 & 0.0246 \end{pmatrix}. \tag{56}$$

Step 5, 6. The superiority and inferiority matrices are constructed using (49) and (50) as follows:

$$\begin{aligned} S &= \begin{pmatrix} 0 & 0 & 0.01\\ 0.01 & 0.02 & 0.03\\ 0.03 & 0.03 & 0.02\\ 0.02 & 0.01 & 0 \end{pmatrix},\\ I &= \begin{pmatrix} 0.03 & 0.03 & 0.02\\ 0.02 & 0.01 & 0\\ 0 & 0 & 0.01\\ 0.01 & 0.02 & 0.03 \end{pmatrix}. \end{aligned} \tag{57}$$

Step 7, 8. The S-flow and I-flow are computed using (51) and (52), which are given in Table 8.

Step 9. Applying SR-laws to Table 8 yields the following ranking order:

$$\hat{a}_3 \succ \hat{a}_2 \succ \hat{a}_4 \succ \hat{a}_1. \tag{58}$$

Applying IR-laws to Table 8 yields the following ranking order:

$$\hat{a}_3 \succ \hat{a}_2 \succ \hat{a}_4 \succ \hat{a}_1. \tag{59}$$

Step 10. According to both SR and IR-laws, $\hat{a}_3$ is the best alternative.

Table 2: Criteria for the selection of the best medical tourism destination.

Criteria	Description
(i) Service quality ($\widehat{c}_1$)	This includes modern equipment, qualified staff, and variety of medical treatments.
(ii) Security ($\widehat{c}_2$)	This includes life and fiscal security of the tourists.
(iii) Infrastructure facilities ($\widehat{c}_3$)	This includes transportation and maintenance of hospitals and equipment.

Table 3: Bipolar fuzzy weights of criteria.

	$\widehat{c}_1$	$\widehat{c}_2$	$\widehat{c}_3$
$\widehat{e}_1$	$\langle 0.73, -0.26\rangle$	$\langle 0.65, -0.36\rangle$	$\langle 0.81, -0.29\rangle$
$\widehat{e}_2$	$\langle 0.82, -0.38\rangle$	$\langle 0.76, -0.19\rangle$	$\langle 0.78, -0.27\rangle$
$\widehat{e}_3$	$\langle 0.69, -0.42\rangle$	$\langle 0.83, -0.36\rangle$	$\langle 0.65, -0.17\rangle$

Table 4: BF decision matrix $\mathbb{H}_1$.

	$\widehat{c}_1$	$\widehat{c}_2$	$\widehat{c}_3$
$\widehat{a}_1$	$\langle 0.82, -0.21\rangle$	$\langle 0.92, -0.23\rangle$	$\langle 0.78, -0.26\rangle$
$\widehat{a}_2$	$\langle 0.76, -0.19\rangle$	$\langle 0.52, -0.41\rangle$	$\langle 0.66, -0.24\rangle$
$\widehat{a}_3$	$\langle 0.86, -0.17\rangle$	$\langle 0.87, -0.18\rangle$	$\langle 0.79, -0.34\rangle$
$\widehat{a}_4$	$\langle 0.67, -0.31\rangle$	$\langle 0.42, -0.38\rangle$	$\langle 0.76, -0.12\rangle$

Table 5: BF decision matrix $\mathbb{H}_2$.

	$\widehat{c}_1$	$\widehat{c}_2$	$\widehat{c}_3$
$\widehat{a}_1$	$\langle 0.69, -0.33\rangle$	$\langle 0.82, -0.19\rangle$	$\langle 0.89, -0.17\rangle$
$\widehat{a}_2$	$\langle 0.82, -0.21\rangle$	$\langle 0.66, -0.29\rangle$	$\langle 0.77, -0.32\rangle$
$\widehat{a}_3$	$\langle 0.91, -0.36\rangle$	$\langle 0.79, -0.26\rangle$	$\langle 0.87, -0.29\rangle$
$\widehat{a}_4$	$\langle 0.79, -0.29\rangle$	$\langle 0.56, -0.21\rangle$	$\langle 0.82, -0.26\rangle$

Table 6: BF decision matrix $\mathbb{H}_3$.

	$\widehat{c}_1$	$\widehat{c}_2$	$\widehat{c}_3$
$\widehat{a}_1$	$\langle 0.76, -0.22\rangle$	$\langle 0.88, -0.13\rangle$	$\langle 0.96, -0.41\rangle$
$\widehat{a}_2$	$\langle 0.89, -0.16\rangle$	$\langle 0.62, -0.24\rangle$	$\langle 0.69, -0.56\rangle$
$\widehat{a}_3$	$\langle 0.73, -0.29\rangle$	$\langle 0.92, -0.26\rangle$	$\langle 0.71, -0.31\rangle$
$\widehat{a}_4$	$\langle 0.81, -0.32\rangle$	$\langle 0.63, -0.46\rangle$	$\langle 0.56, -0.21\rangle$

Table 7: Accumulated BF decision matrix.

	$\widehat{c}_1$	$\widehat{c}_2$	$\widehat{c}_3$
$\widehat{a}_1$	$\langle 0.9128, -0.0895\rangle$	$\langle 0.9747, -0.0342\rangle$	$\langle 0.9922, -0.0740\rangle$
$\widehat{a}_2$	$\langle 0.9713, -0.0431\rangle$	$\langle 0.8396, -0.0938\rangle$	$\langle 0.9135, -0.1775\rangle$
$\widehat{a}_3$	$\langle 0.9751, -0.1162\rangle$	$\langle 0.9762, -0.0771\rangle$	$\langle 0.9585, -0.1102\rangle$
$\widehat{a}_4$	$\langle 0.9459, -0.1111\rangle$	$\langle 0.7886, -0.1074\rangle$	$\langle 0.9183, -0.0611\rangle$

Table 8: The BF-SIR flows.

Alternatives	$\lambda^{>}(\widehat{a}_i)$	$Scr(\lambda^{>}(\widehat{a}_i))$	$\lambda^{<}(\widehat{a}_i)$	$Scr(\lambda^{<}(\widehat{a}_i))$
$\widehat{a}_1$	$\langle 0.0469, -0.9482\rangle$	0.0494	$\langle 0.3425, -0.7045\rangle$	0.319
$\widehat{a}_2$	$\langle 0.2641, -0.7488\rangle$	0.2576	$\langle 0.1483, -0.8920\rangle$	0.1282
$\widehat{a}_3$	$\langle 0.3425, -0.7045\rangle$	0.319	$\langle 0.0469, -0.9482\rangle$	0.0494
$\widehat{a}_4$	$\langle 0.1483, -0.8920\rangle$	0.1282	$\langle 0.2641, -0.7488\rangle$	0.2576

Table 9: Comparative analysis of the suggested and existing methodologies.

Methods	Ranking of alternatives	Optimal alternative
Algorithm (Jana and Pal [17])	$\widehat{a}_3 \succ \widehat{a}_2 \succ \widehat{a}_1 \succ \widehat{a}_4$	$\widehat{a}_3$
Algorithm (Wei et al. [20])	$\widehat{a}_3 \succ \widehat{a}_2 \succ \widehat{a}_4 \succ \widehat{a}_1$	$\widehat{a}_3$
Algorithm (Hamid et al. [23])	$\widehat{a}_3 \succ \widehat{a}_4 \succ \widehat{a}_2 \succ \widehat{a}_1$	$\widehat{a}_3$
Algorithm (Akram et al. [24])	$\widehat{a}_3 \succ \widehat{a}_2 \succ \widehat{a}_4 \succ \widehat{a}_1$	$\widehat{a}_3$
Algorithm (Peng and Yang [60])	$\widehat{a}_3 \succ \widehat{a}_2 \succ \widehat{a}_4 \succ \widehat{a}_1$	$\widehat{a}_3$
Algorithm (Zhang and Xu [36])	$\widehat{a}_3 \succ \widehat{a}_1 \succ \widehat{a}_2 \succ \widehat{a}_4$	$\widehat{a}_3$
Algorithm (Proposed)	$\widehat{a}_3 \succ \widehat{a}_2 \succ \widehat{a}_4 \succ \widehat{a}_1$	$\widehat{a}_3$

6.3. Comparative and Sensitivity Analysis. In this section, we compare our suggested BF-SIR technique to some existing approaches in order to evaluate its validity. Table 9 summarizes the comparative study of various techniques. It can be seen from Table 9 that our suggested approach is compatible with the existing techniques.

7. Conclusion

In daily life, we encounter many situations where we must deal with uncertainty as well as bipolarity when making a decision. Taking this into consideration, the bipolar fuzzy set (BFS) is a sophisticated model that can handle bipolarity and fuzziness at the same time. The main contributions of this manuscript are listed as follows:

(1) Since the sine trigonometric function is periodic and symmetric about the origin, it can accommodate the decision expert's choices during object appraisal. Therefore, we proposed sine trigonometric operational laws (STOLs) for BFSs. We explored some of their properties as well.

(2) Based on BF-STOLs, we suggested the following averaging AOs: bipolar fuzzy sine trigonometric weighted averaging (BF-STWA) operator; bipolar fuzzy sine trigonometric ordered weighted averaging (BF-STOWA) operator; and bipolar fuzzy sine trigonometric hybrid weighted averaging (BF-STHWA) operator.

(3) Based on BF-STOLs, we suggested the following geometric AOs: bipolar fuzzy sine trigonometric weighted geometric (BF-STWG) operator; bipolar fuzzy sine trigonometric ordered weighted geometric (BF-STOWG) operator; and bipolar fuzzy sine trigonometric hybrid weighted geometric (BF-STHWG) operator.

(4) We investigated some important characteristics of these operators, such as idempotency, monotonicity, and boundedness.

(5) We established an extended superiority and inferiority ranking (SIR) method to handle MCGDM problems in a bipolar fuzzy environment. We applied this technique to the selection of the best medical tourism supply chain.

(6) We compared our suggested model with some existing ones to exhibit its validity and efficiency.

In the future, we will develop bipolar fuzzy sine trigonometric power aggregation operators, bipolar fuzzy sine trigonometric Hamy mean operators, bipolar fuzzy sine trigonometric Bonferroni mean operators, bipolar fuzzy sine trigonometric prioritized operators, and bipolar fuzzy sine trigonometric Dombi operators.

Data Availability

No data were used in this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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