Study on Energy Dissipation and Fuel Consumption in Lattice Hydrodynamic Model under Traffic Control

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A lattice hydrodynamic traffic model considering the average optimal flow of multiple grids downstream as a feedback control is proposed. The energy dissipation and fuel consumption are investigated under the feedback control based on the lattice hydrodynamic traffic model. Through linear stability analysis, the stability condition of the model is obtained. The mKdV equation and its kink-antikink density wave solution are derived by using the reduced perturbation method of nonlinear analysis. The variation trends of density wave, energy dissipation, and fuel consumption under traffic control are studied by numerical simulations. The research shows that exerting the feedback control can effectively suppress traffic congestion and improve the stability of traffic system. Meanwhile, it can also reduce the energy dissipation and fuel consumption of traffic system.

1. Introduction

With the rapid development of social economy and the accelerating process of urbanization, the relative lag of traffic construction has caused serious traffic congestion, resulting in increased travel time, safety hazards, energy, and fuel consumption etc. In 2020, the number of motor vehicles in China reached 372 million and gasoline consumption reached 147 million tons [1]. These problems have aroused great concern in the academic circles and induced intensive investigation of traffic flow theory [2]. Therefore, how to employ scientific scheme to alleviate traffic congestion and reduce vehicle fuel consumption is worthy of careful study and discussion by scholars [3]. In the whole traffic research field, many recognized and effective traffic models have been proposed, such as cellular automata models [4–6], car-following models [7–11], continuum models [12–15], and lattice hydrodynamic models [16–18]. They give us a good look at different perspectives on different types of traffic congestion, such as stop-and-go traffic and synchronous traffic. Among these traffic models, the lattice hydrodynamics model has the properties of the macroscopic and microscopic traffic models, so the lattices hydrodynamics model has been widely studied.

In the 1950s, Lightill, Whitham, and Richard independently proposed the hydrodynamic model by treating traffic flow as compressible fluid, which was called LWR theory [12, 13]. By integrating the idea of optimal velocity model and hydrodynamic model, Nagatani et al. [16] first proposed the lattice hydrodynamic model of traffic flow in 1998. Konishi et al. [19] used the delayed feedback control (DFC) method to suppress the traffic jams in the car-following model, which is a convenient tool for controlling chaotic system. Ge et al. [20] found that feedback control can also alleviate congestion even in the macroscopic lattice hydrodynamic model. Redhu et al. [21] studied the delay feedback control method in lattice hydrodynamic model. Later, scholars proposed many extended lattice hydrodynamic models by considering control methods under different traffic factors such as density change rate difference [22], honk environment [23], interruption probability [24], the next-nearest-neighbor interactions [25]. Recently, Zhai et al. [26] studied delay feedback control for lattice hydrodynamic model under cyber-attacks and connected vehicle environment. Based on the lattice hydrodynamic model, Zhao et al. [27] studied the influence of delayed-time for two-lane freeway. Kuang et al. [28] proposed a novel lattice hydrodynamic model with consideration of multi-
anticipative average flux effect under ITS environment. With the development of connected automated vehicle (CAV) technologies, traffic information control and utilizations have become more widely investigated like cyber-physical system (CPS) with optimal transmission reliability enhancement mechanism (OTREM) [29], seamless connectivity-based message propagation mechanism (SC-MPM) in V2X communications, [30] and so on.

Moreover, energy consumption and fuel consumption in traffic flow have also attracted great attention. Ahn [31, 32] proposed a VT-Micro model to predict vehicle emissions and fuel consumption by considering the effects of instantaneous velocity and acceleration. Treiber et al. [3] proposed the instantaneous fuel consumption model and found that traffic congestion would lead to increased fuel consumption of vehicles. Shi et al. [33] studied the relation of the stability and energy consumption by analysis and comparison of energy consumption for the several typical car-following models. The research studies show that when the traffic flow is stable, the lower the energy consumption. Zhang et al. investigated the relations of energy dissipation to the speed limit, stochastic noise, boundary conditions, and “stop-and-go” traffic in NaSch model [34, 35]. The energy dissipation of the mixed traffic flow was studied by using cellular automata models [36, 37]. Furthermore, Tang et al. used the car-following model to study the impacts of fuel consumption and emissions on the trip cost without late arrival at the equilibrium state and the effect if signal light on fuel consumption and emissions [38, 39]. Madani and Moussa simulated fuel consumption and engine pollutant of traffic flow by using the cellular automaton model [40]. More recently, Peng et al. investigated energy consumption in a lattice hydrodynamic model that considered the effect of cooperative information transfer delay in a V2X environment [41]. Pollutant emission and fuel consumption of mixed traffic flow are also studied based on cellular automata model [42, 43].

In this paper, to study the influence of the average optimal flow of multiple grids downstream on traffic flow, a new lattice hydrodynamic model with feedback control is proposed in Section 2. Linear stability analysis and nonlinear analysis is carried out in Sections 3 and 4, respectively. Section 5 conducts numerical simulations, especially the investigation of the energy dissipation and fuel consumption under traffic control. Finally, some conclusions are yielded in Section 6.

2. The Model

The original lattice hydrodynamics model was proposed by Nagatani [16], which is composed of the following flow equation and conservation equation as follows:

\[ \partial_t \rho_j + \rho_0 (\rho_j v_j - \rho_{j-1} v_{j-1}) = 0, \]

\[ \partial_t (\rho_j v_j) = a \rho_0 V(\rho_{j+1}) - a \rho_j v_j, \]

where the constant parameter \( a = 1/3 \) is sensitivity of a driver; \( \rho_j \) and \( v_j \), respectively, represent the local density and velocity at position \( j \) at time \( t \); \( \rho_0 \) denotes the total average density; the optimal velocity function \( V(\rho) \) is usually defined as [44] follows:

\[ V(\rho) = \frac{v_{\text{max}}}{2} \left[ \tanh \left( \frac{2 - \rho - \rho_0}{\rho_c} \right) + \tanh \left( \frac{1}{\rho_c} \right) \right], \]

where \( \rho_c \) is the critical density, which is an inflection point of the optimal velocity function and \( v_{\text{max}} \) is the maximum velocity of traffic flow.

Under the actual traffic conditions, the downstream flow has an important impact on the stability of the current traffic flow. Different traffic conditions downstream lead to different impacts on the current traffic. This effect is reflected by considering the average optimal flow at multiple locations downstream. Thus, an extended lattice hydrodynamic model is proposed by introducing the difference between the average optimal flow of multiple grids downstream and the current flow as a feedback control. The governing equation is expressed as follows:

\[ \partial_t \rho_j + \rho_0 (\rho_j v_j - \rho_{j-1} v_{j-1}) = 0, \]

\[ \partial_t (\rho_j v_j) = a \rho_0 V(\rho_{j+1}) - a \rho_j v_j + a \lambda \left[ \sum_{n=1}^{s} \rho_0 V(\rho_{j+n}) - \rho_j v_j \right], \]

where \( \lambda \) represents the feedback gain which is a dimensionless quantity, and \( s \) denotes the number of selected grids downstream. In the ideal case, the range of variable \( s \) is \( 1 \sim n-1 \). However, due to the visual field limitation of the driver, the range of variable \( s \) is generally \( 1 \sim 3 \). When the feedback gain \( \lambda = 0 \), the model transforms into Nagatani’ model. The following governing equation is obtained by eliminating the instantaneous velocity in combination with equations (4) and (5).

\[ \partial_t^2 \rho_j(t) + a \partial_t \rho_j(t) + a \rho_0^2 V(\rho_{j+1}(t) - \rho_j(t)) \]

\[ + a \lambda \frac{1}{3} \left( \sum_{n=0}^{s} V(\rho_{j+n}(t)) - \sum_{n=0}^{s} V(\rho_{j+n}(t)) \right) \]

\[ + a \lambda \partial_t \rho_j(t) = 0. \]

3. Linear Stability Analysis

The stability of traffic flow is mainly based on the investigation of the evolution process of small disturbance in the system near steady state. It is assumed that all vehicles move at an optimal speed under uniform initial conditions. Then, the steady-state solution of the traffic system is

\[ \rho_j(t) = \rho_0, \]

\[ v_j(t) = V(\rho_0). \]

A small disturbance \( y_j(t) \) is added to (6) of the steady-state flow solution:

\[ \rho_j(t) = \rho_0 + y_j(t). \]
Let \( y_j(t) = \exp(\imath k t + \varepsilon t) \), by linearizing (5), we get the following equation:

\[
z^2 + az + a \rho_0^2 V' (\rho_0) (\varepsilon^k - 1) + a \lambda \frac{1}{\rho_0^2} \rho_0 V'' (\rho_0) \left[ \sum_{n=1}^{\infty} e^{\imath n k} - \sum_{n=1}^{\infty} e^{\imath n k} \right] + a \lambda \varepsilon = 0,
\]

where \( V' (\rho_0) = \frac{\partial V (\rho)}{\partial \rho} |_{\rho = \rho_0} \). By setting \( z = z_1 (ik) + z_2 (ik)^2 + \ldots \), and substituting it into (8), the first-order and second-order terms of \((ik)\) are derived as follows:

\[
z_1 = -\rho_0^2 V' (\rho_0),
\]

\[
z_2 = -\frac{z_1^2}{a (1 + \lambda)} - \frac{\rho_0^2 V'' (\rho_0)}{2 (1 + \lambda)} - \frac{\lambda \rho_0^2 V''' (\rho_0)}{2 (1 + \lambda)}.
\]

When \( z_2 < 0 \), uniform steady-state flow develops into the unstable state. Conversely, when \( z_2 > 0 \), the traffic flow remains stable. Therefore, the critical stable condition of the new model is deduced as

\[
a > -\frac{2 \rho_0^2 V' (\rho_0)}{1 + \lambda s}.
\]

Consequently, the critical stable condition is derived by \( z_2 = 0 \):

\[
ed^2 \left[ a (b + \rho_0^2 V') + a \lambda \left( b + \rho_0^2 V' \right) \right] \partial_{\lambda} R + \varepsilon^3 \left[ b^2 + \frac{1}{2} a \rho_0^2 V' (1 + \lambda s) \right] \partial_{\lambda}^2 R,
\]

\[
+ \varepsilon^5 \left[ 2b \partial_{\lambda} R + \frac{1}{24} a \rho_0^2 V' (1 + \lambda s) \partial_{\lambda}^2 R + \frac{1}{12} a \rho_0^2 V'' (1 + \lambda s) \partial_{\lambda}^3 R \right],
\]

where \( V' = \frac{\partial V (\rho)}{\partial \rho} |_{\rho = \rho_0} \), \( V'' = \frac{\partial^2 V (\rho)}{\partial \rho^2} |_{\rho = \rho_0} \). There is a relationship \( a = a_c (1 - \varepsilon^2) \) near the critical point \((\rho_c, a_c)\). Let \( b = -\rho_0^2 V' \) and eliminate the second-order and third-order terms of \( \varepsilon \) in (14), then it can be simplified to

\[
a = -\frac{2 \rho_0^2 V' (\rho_0)}{1 + \lambda s}.
\]

Obviously, when \( \lambda = 0 \), the stability condition of the new model is consistent with that of Nagatani’s model [16]. By analyzing (10), with the increase of feedback gain \( \lambda \) and variable \( s \), the driver’s sensitivity requirement is easier to meet, which demonstrate that considering feedback control and downstream average optimal flow can effectively improve the stability of traffic system.

### 4. Nonlinear Stability Analysis

The spatial slow variable \( X \) and slow time variable \( T \) near the critical point in the neutral stability curve are defined as

\[
X = \varepsilon (j + bt), \quad T = \varepsilon^2 t,
\]

where \( \varepsilon \) is the slow scales for \( 0 < \varepsilon \ll 1 \); \( b \) is an undetermined coefficient. The density at position \( j \) is

\[
\rho_j (t) = \rho_c + \varepsilon X (j, t).
\]

Substituting (12) and (13) into (5) and making the Taylor expansions to the fifth-order of \( \varepsilon \), the resulting equation is

\[
ed^4 \left( \partial_{\lambda} R - g_4 \partial_{\lambda}^2 R + g_5 \partial_{\lambda}^3 R \right)
\]

\[
+ \varepsilon^5 \left( g_3 \partial_{\lambda}^2 X + g_4 \partial_{\lambda} X + g_5 \partial_{\lambda}^3 X \right) = 0,
\]

where

\[
g_1 = \frac{(1 + \lambda s)}{6 (1 + \lambda)} a \rho_0^2 V',
\]

\[
g_2 = \frac{1}{8} \rho_0^2 V'',
\]

\[
g_3 = \frac{1}{a_c (1 + \lambda)} b^2,
\]

\[
g_4 = \frac{(1 + \lambda s)}{24 (1 + \lambda)} \rho_0^2 V' - \frac{(1 + \lambda s)}{6 a_c (1 + \lambda)} \rho_0^2 V' \times 2b,
\]

\[
g_5 = \frac{(1 + \lambda s)}{12 (1 + \lambda)} \rho_0^2 V' - \frac{1}{6 a_c (1 + \lambda)} \rho_0^2 V'' \times 2b.
\]
To obtain the standard mKdV equation, we adopt the following transformation:

$$T' = g_1T, \quad R = \sqrt{g_1/g_2}. \quad (17)$$

Thus, the transformation form of (15) can be obtained as

$$\frac{\partial}{\partial x} R' - \frac{\partial^3}{\partial x^3} R' + \frac{\partial^2}{\partial x^2} R' + \frac{\partial}{\partial x} R^3 + \varepsilon M[R'] = 0. \quad (18)$$

where

$$M[R'] = \sqrt{4g_1} g_3^3 R' + (g_1 g_2/g_3) \frac{\partial R'^3}{\partial x} + g_2 \frac{\partial^3}{\partial x^3} R'.$$

Ignoring the term O(\varepsilon) of (18), the solution of kink-antikink wave is given by

$$R_0'(X, T') = \sqrt{c} \tanh \sqrt{c/2}(X - cT'). \quad (19)$$

where $R_0'(X, T')$ should satisfy the solvable condition:

$$(R_0', M[R']) = \int_{-\infty}^{\infty} dX R_0'(X, T')M([X, T'])). \quad (20)$$

By integrating, the value of propagation velocity $c$ is

$$c = 5 \frac{g_4 g_7}{(2g_7 g_4 - 3g_3 g_5)}. \quad (21)$$

Hence, the kink-antikink solution of the mKdV (18) is obtained as follows:

$$\rho_j = \rho_c + \varepsilon \sqrt{\frac{g_1 c}{g_2}} \tanh \left( \frac{c}{2} (X - c g_1 T) \right). \quad (22)$$

The amplitude $A$ of kink-anti-kink wave is

$$A = \sqrt{\frac{g_1 c}{g_2} \left( \frac{\alpha_c}{\alpha_c} - 1 \right)}. \quad (23)$$

According to the derived solution, $\rho_j = \rho_c - A$ represents the traffic phases in free flow, $\rho_j = \rho_c + A$ represents the congested phases in congestion flow.

Figure 1 shows the phase diagrams on $(\rho, \alpha)$-plane of the traffic model. The solid line in the figure is the neutral stability line, and the dashed line is the coexisting curve. As can be seen from Figures 1(a) and 1(b), the feedback gain $\lambda$ increases or the variable $s$ increases, the neutral stability line and coexisting curve decrease, that is, the stability region increases. This proves that the stability of the traffic system has been effectively improved.

5. Energy Dissipation and Fuel Consumption

In traffic transportation, the driving of vehicles is often accompanied by the change of energy dissipation and fuel consumption. Therefore, how to reduce unnecessary energy dissipation and fuel consumption in the traffic system is particularly important.

5.1. Energy Dissipation. In the lattice hydrodynamics model, the energy change of each grid on a single lane is expressed as

$$\Delta E_j(t) = \frac{1}{2} \rho_j(t + \Delta t) v_j^2(t + \Delta t) \Delta x - \frac{1}{2} \rho_j(t) v_j^2(t) \Delta x, \quad (24)$$

where each term on the right-hand side of (24) is represented as kinetic energy of the $j$th grid at time $t + \Delta t$ and at time $t$, respectively. $\Delta t$ is the time interval between time $t$ and time $t + \Delta t$ and $\Delta x$ is the size or length of the $j$th grid on a one-dimensional lane.

When the kinetic energy of the $j$th grid at time $t + \Delta t$ is less than the kinetic energy at time $t$, $|\Delta E_j(t)|$ represents the energy dissipation per grid mass of the $j$th grid at time $t$ as

$$|\Delta E_j(t)| = \frac{1}{2} \rho_j(t + \Delta t) v_j^2(t + \Delta t) \Delta x - \frac{1}{2} \rho_j(t) v_j^2(t) \Delta x. \quad (25)$$

According to formula (25), the total energy dissipation ($\Delta E(t)$) on the whole road can be written as

$$\Delta E(t) = \sum_{j=1}^{N} |\Delta E_j(t)|. \quad (26)$$

5.2. Fuel Consumption. In 2002, Ahn et al. [31, 32] proposed an improved VT-Micro model to study the fuel consumption of each vehicle as

$$\ln(MOE_v) = \sum_{\alpha=0}^{3} \sum_{\beta=0}^{3} \left( L_{\alpha \beta}^{e} \times v_n^{\alpha} \times \left( \frac{dv_n}{dt} \right)^\beta \right) \frac{dv_n}{dt} \geq 0, \quad (27)$$

where $v_n$ and $dv_n/dt$ are the instantaneous speed (km/h) and instantaneous acceleration (km/h/s) of the nth vehicle, respectively. $L_{\alpha \beta}^{e}$ and $M_{\alpha \beta}^{e}$ are the regression coefficients (see Table 1) when the instantaneous acceleration is greater than 0 and less than 0, respectively, and $MOE_v$ is the instantaneous fuel consumption rate (mL/s). Yang et al. made a systematic summary of the research on fuel consumption and emissions of vehicles under the following state [45].

Equation (27) is the fuel consumption of each vehicle and has microcosmic properties. The fuel consumption of the $n$th vehicle can be transformed into the traffic fuel consumption of the $j$th grid by means of the following conversion relationship between micro variables and macro variables [46]. This transformation to estimate fuel consumption and emissions in VT-macro model was carried out in [47, 48].

$$v_j(t) \rightarrow v_n(t), \quad \frac{dv_n}{dt} = \frac{\partial v_j}{\partial t} v_j + \frac{\partial v_j}{\partial \rho_j} \rho_j = v_{j+1} - v_{j-1} \frac{\Delta x}{v_j} + \frac{\partial v}{\partial c} \frac{\partial v_j}{\partial t}. \quad (28)$$

By combining (3) and (4) and using the forward difference scheme, the flow rate of the $j$th grid of the grid is obtained.

$$\frac{\partial \rho_j v_j}{\partial t} = \frac{\partial \rho_j}{\partial t} v_j + \frac{\partial v_j}{\partial \rho_j} = -\rho_0 (\rho_j v_j - \rho_{j-1} v_{j-1}) v_j + \frac{\partial v}{\partial c} \frac{\partial v_j}{\partial t}. \quad (29)$$
Using equations (5), (29), and (30), the acceleration of the \( n \)th vehicle can be converted into the acceleration of the \( j \)th grid, which is expressed as follows:

\[
\frac{dv_j}{dt} = v_j \frac{v_{j+1} - v_j}{\Delta x} + \frac{1}{\rho_j} \left[ a_0 \rho_j V(\rho_{j+1}) - a_0 \rho_j v_j + a_\lambda \left( \frac{1}{s} \sum_{n=1}^{s} \rho_0 V(\rho_{j+n}) - \rho_j v_j \right) \right] + \frac{1}{\rho_j^s} (\rho_j v_j - \rho_{j-1} v_{j-1}) v_j.
\]

(30)

Then, substituting (30) into (27), the VT-Micro model becomes the traffic fuel consumption of the \( j \)th grid.

\[
\ln(MOE_j) = \begin{cases} 
\sum_{a=0}^{3} \sum_{\beta=0}^{3} I_{a,\beta}^c \times v_j^a \times \left( \frac{dv_j}{dt} \right)^\beta \cdot \frac{dv_j}{dt} \geq 0, \\
\sum_{a=0}^{3} \sum_{\beta=0}^{3} M_{a,\beta}^c \times v_j^a \times \left( \frac{dv_j}{dt} \right)^\beta \cdot \frac{dv_j}{dt} < 0.
\end{cases}
\]

(31)

Table 1: Parameters of formula (28) [32].

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>Constant</th>
<th>Speed</th>
<th>Speed(^2)</th>
<th>Speed(^3)</th>
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<td></td>
<td></td>
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<td>Acceleration(^3)</td>
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<td>0.000468</td>
<td>-1.79E-05</td>
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<td></td>
<td></td>
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<td>Acceleration(^3)</td>
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<td>2.42E-06</td>
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</table>

(a) \( s = 1 \), (b) \( \lambda = 0.1 \).

Figure 1: Phase diagrams in parameter space \((\rho, a)\).
Figure 2: Short-term density change of different feedback gain $\lambda$ and variable $s$. (a) $\lambda = 0.1, s = 1$, (b) $\lambda = 0.3, s = 1$, (c) $\lambda = 0.2, s = 2$, and (d) $\lambda = 0.2, s = 3$. 

Figure 3: Continued.
6. Numerical Simulation

In this section, we will conduct numerical simulation to explore whether traffic control can be achieved and traffic congestion can be suppressed through numerical simulation, and then study the energy dissipation and fuel consumption of traffic flow with periodic boundary under traffic control conditions.
6.1. System Evolution under Traffic Control. The numerical simulation starts with the following initial conditions: a lane is divided into grids of \( N = 100 \), the initial state of traffic flow is uniform, and the initial density is \( \rho_0 = \rho_c = 0.25 \). Sensitivity coefficient and maximum velocity are selected as \( a = 1.2 \) and \( V_{\text{max}} = 2 \), respectively. The small disturbance imposed on the traffic flow is

\[
\rho_j(w) = \begin{cases} 
\rho_0, & j \neq 50, 51, \\
\rho_0 + 0.1, & j = 50, \\
\rho_0 - 0.1, & j = 51.
\end{cases}
\]

where \( w = 1, 2, \ldots, 5 \) denotes the initial five-time steps.

Figures 2 and 3 show the short-term and long-term behavior of density under control, respectively. Compared with Figures 2(a) and 2(b) and Figures 3(a) and 3(b), when the number of downstream selection grids \( s \) is fixed and the feedback gain \( \lambda \) increases, the amplitude fluctuation of density wave decreases. As shown in Figures 2(c) and 2(d) or Figures 3(c) and 3(d), when the feedback gain \( \lambda \) is fixed and the number \( s \) of selected grids is increased, the fluctuation of density wave also decreases. The results show that increasing the feedback gain \( \lambda \) or variable \( s \) is conducive to suppressing traffic jamming.

Figure 4 displays the kink-anti-kink density waves under the action of small perturbations in the system. As can be seen from Figure 4, it is obvious that the amplitude of density wave decreases with the increase of \( \lambda \) or \( s \), which indicates that feedback control considering the average optimal flow of multiple grids downstream can effectively inhibit traffic congestion.

In Figure 5, the scatter plots of density difference \( \rho_{25}(t) - \rho_{25}(t-1) \) against \( \rho_{25}(t) \) in phase space for different feedback gain \( \lambda \) and multiple grids \( s \) display the hysteresis loop. The state of traffic system in unstable region can jump from one periodic orbit to another and exhibits chaotic behavior. Compared with Figures 5(a) and 5(b) and Figure 5(c) and 5(d), the hysteresis loop is gradually shrunk with the increase of the feedback gain \( \lambda \) or the number of grids \( s \). This indicates that the disappearance of the traffic jams corresponding to
the spatiotemporal density wave in Figure 4 reflects that traffic congestion has been alleviated.

6.2. Energy Dissipation and Fuel Consumption. The energy dissipation formula and fuel consumption model have dimensional characteristic. Then, in order to compute energy dissipation and fuel consumption, the dimensional initial density function proposed by Herrmann and Kerner [49] is used as follows:

$$
\rho(x, 0) = \rho_0 + \Delta \rho_0 \left\{ \cosh^{-2} \left[ \frac{160}{L} \left( x - \frac{5L}{16} \right) \right] - \frac{1}{4} \cosh^{-2} \left[ \frac{40}{L} \left( x - \frac{11L}{32} \right) \right] \right\},
$$

(33)

Figure 6: Spatiotemporal pattern of energy dissipation from 2000 s to 8000 s. (a) $\lambda = 0.1, s = 1, \rho_0 = 0.01 \text{veh/m}$, (b) $\lambda = 0.1, s = 1, \rho_0 = 0.05 \text{veh/m}$, (c) $\lambda = 0.3, s = 1, \rho_0 = 0.05 \text{veh/m}$, and (d) $\lambda = 0.3, s = 3, \rho_0 = 0.05 \text{veh/m}$.

Figure 7: The total energy dissipation and fuel consumption of the whole road at $t = 5000$ s.
where $L (= N \times \Delta x)$ stands for the total length of the road and $\Delta \rho_0$ is the density fluctuation. We assume that the periodic boundary condition is adopted:

$$\rho_0(0,t) = \rho_0(L,t), \quad v_0(0,t) = v_0(L,t). \quad (34)$$

The equilibrium velocity function $V_e(\rho)$ [50] reads

$$V_e(\rho) = v_f \left[ \left( 1 + \exp\frac{\rho - \rho_m - 0.25}{0.06} \right)^{-1} - 3.72 \times 10^{-6} \right], \quad (35)$$

where $v_f$ and $\rho_m$ are denoted as the free-flow velocity $v_f = 30 m/s$ and maximum density $\rho_m = 0.2 veh/m$, respectively. The other related parameters are listed as follows:

$$a = 0.2 s^{-1}, \quad \Delta x = 100m, \quad L = 1.0 \times 10^4 m, \quad \Delta \rho_0 = 0.01 veh/m, \quad \Delta t = 1s. \quad (36)$$

Figure 6 shows the spatiotemporal pattern of kinetic energy dissipation from 2000 s to 8000 s. By comparing with Figures 6(a) and 6(b), it can be concluded that when under the traffic control condition and the number of grids $s$ are constant, with the increase of initial density $\rho_0$ of the traffic system, which results in the decrease of kinetic energy dissipation. In numerical simulation, when the initial density is larger, the number of vehicles on the road is larger under periodic boundary conditions, resulting in a decrease in energy consumption in Figure 7(a). As can be seen from Figures 6(b)–6(d), when $\lambda$ or $s$ is fixed, increasing the value of $s$ or $\lambda$, the smaller the kinetic energy dissipation, the more stable the traffic flow is. This indicates that the feedback control can suppress traffic congestion and decrease the kinetic energy dissipation of traffic flow.

Figure 8 shows the spatiotemporal pattern of fuel consumption during the evolution of traffic flow. By comparing Figures 8(a) and 8(b), the number of grids $s$ remains unchanged, the initial density $\rho_0$ increases, and the fuel consumption of the traffic system decreases under periodic boundary condition and considering the same feedback control coefficients. The curve trend in Figure 7(b) can also reflect that fuel consumption will decrease in a high-density traffic situation, which is consistent with the research result by Ren et al. [51]. When the density is constant, the fuel consumption decreases with the increase of feedback gain $\lambda$ and the variable $s$, and the fluctuation of fuel consumption is more stable.

For different control parameters $\lambda$ and $s$, Figures 7(a) and 7(b), respectively, represent the total energy dissipation and total fuel consumption of the whole road at $t = 5000 s$ under different initial densities. It can be concluded from the two figures that when the initial density $\rho_0$ increases, the total energy dissipation and total fuel consumption decrease. When the initial density remains constant and the feedback gain $\lambda$ and variable $s$ are increased, the stability of the transportation system is improved, thus reducing the total energy dissipation and total fuel consumption.

7. Conclusion

In this paper, an extended one-dimensional lattice hydrodynamic model is proposed by considering the average optimal flow of multiple grids downstream as a feedback
control. The stability condition is obtained by linear stability analysis. Meanwhile, the kink-antikink density wave solution of the mKdV equation is derived through nonlinear analysis. Furthermore, energy dissipation of the $j^{th}$ grid at time $t$ is presented and fuel consumption of each vehicle in VT-Micro model is converted to the fuel consumption of the $j^{th}$ grid at time $t$. Then, the results of theoretical analysis are further verified by numerical simulation and the following conclusions are drawn:

(i) Exerting the feedback control can improve the stability of traffic flow and effectively suppress traffic congestion. In addition, when the feedback gain $\lambda$ is increased, the stability of traffic flow is better.

(ii) When feedback gain $\lambda$ is fixed, with the increase of the number $s$ of multiple grids downstream, the stability of traffic flow is also enhanced. It indicates that considering the average optimized flow of multiple grids downstream can realize the control strategy of suppressing traffic jams.

(iii) When the feedback gain $\lambda$ and the number $s$ of multiple grids downstream are taken into account, the stability of traffic system can be greatly improved. Thus, energy dissipation and fuel consumption can also be greatly reduced under traffic control.

Although the extended one-dimensional lattice hydrodynamic model introduced the average optimal flow of multiple grids downstream as a feedback control to realize the control strategy of suppressing traffic jams, the traffic conditions far downstream cannot be observed due to the visual field limitation of drivers. Therefore, the model of this paper can actually implement the monitoring of multiple vehicles on the road ahead through the intelligent transportation system and so as to achieve traffic forecast and management.

**Data Availability**

All data generated or analyzed during this study are included in this article.

**Conflicts of Interest**

The authors declare that they have no conflicts of interest.

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